A brief introduction to Geostatistics

A lecture by Márk Somogyvári

07.05.2019

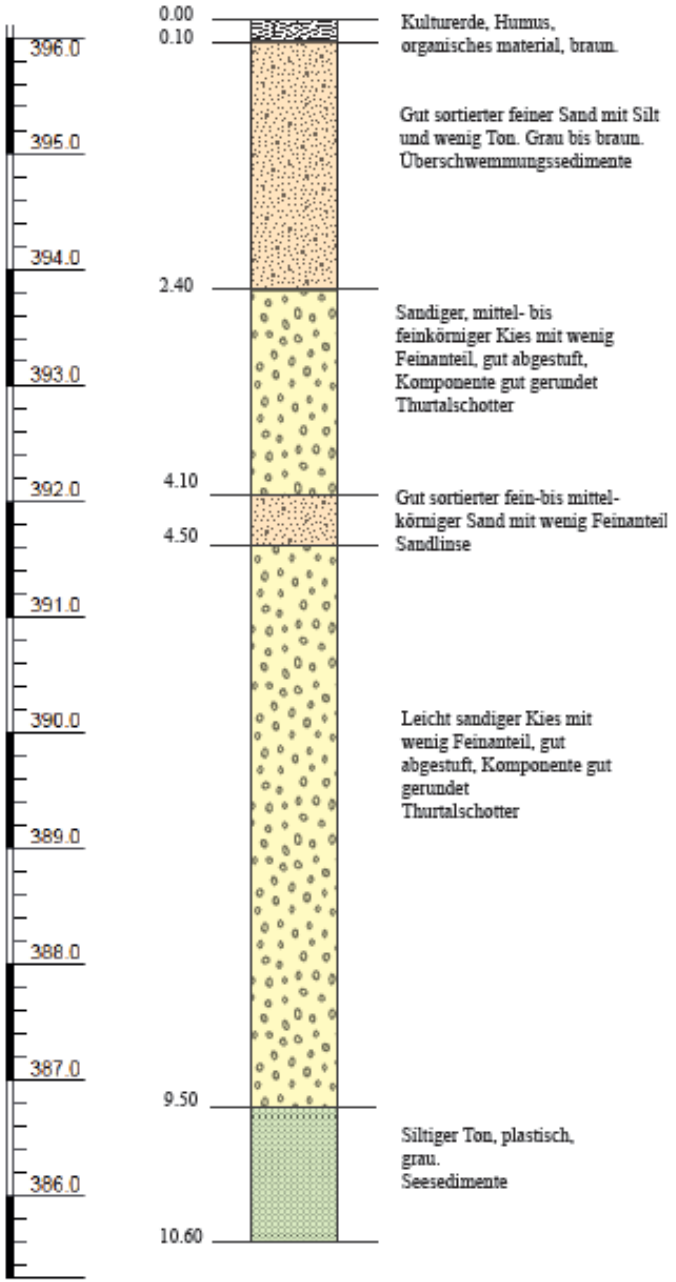
Slides and exercises

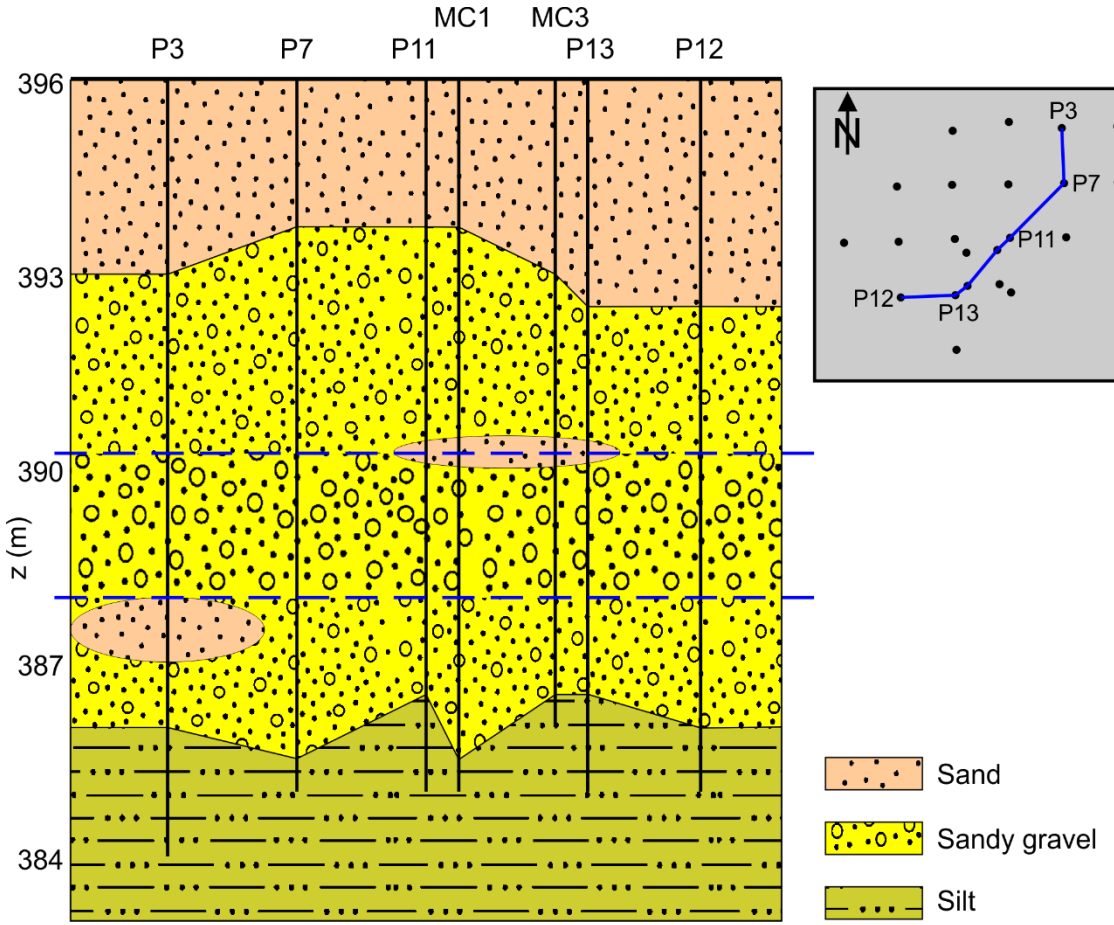
- https://github.com/marksomogyvari/geostatistics

What is geostatistics?

- Statistics of physical properties with spatio-temporal variations.

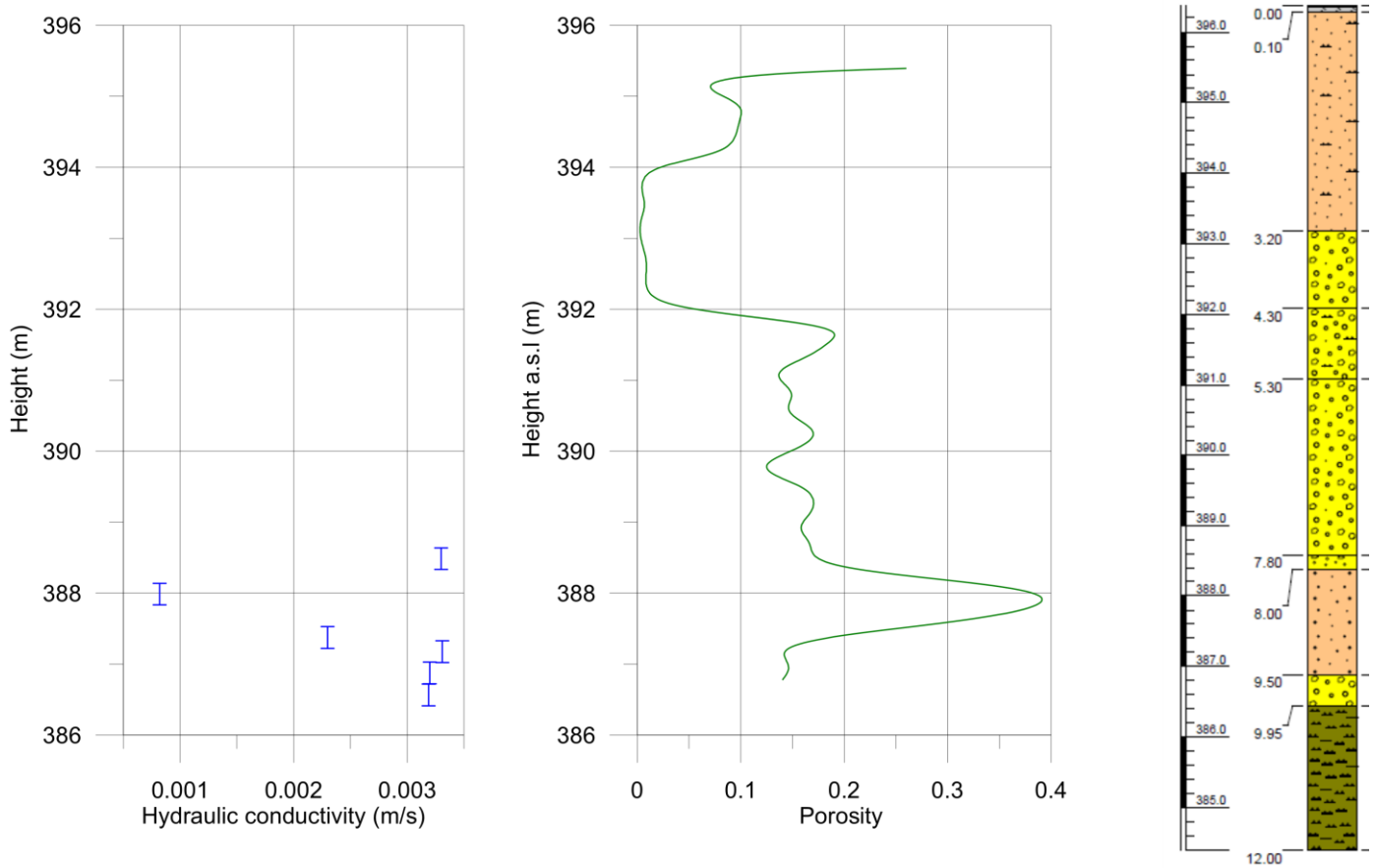
Widen field site, Switzerland





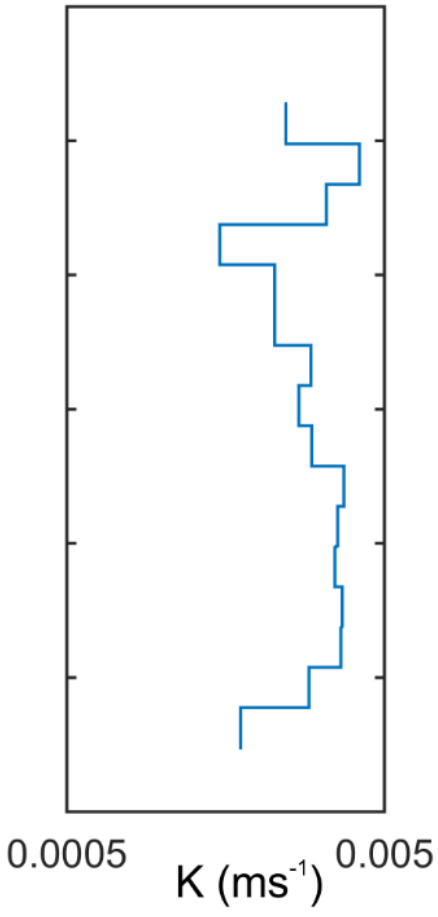
Problem in 1-D

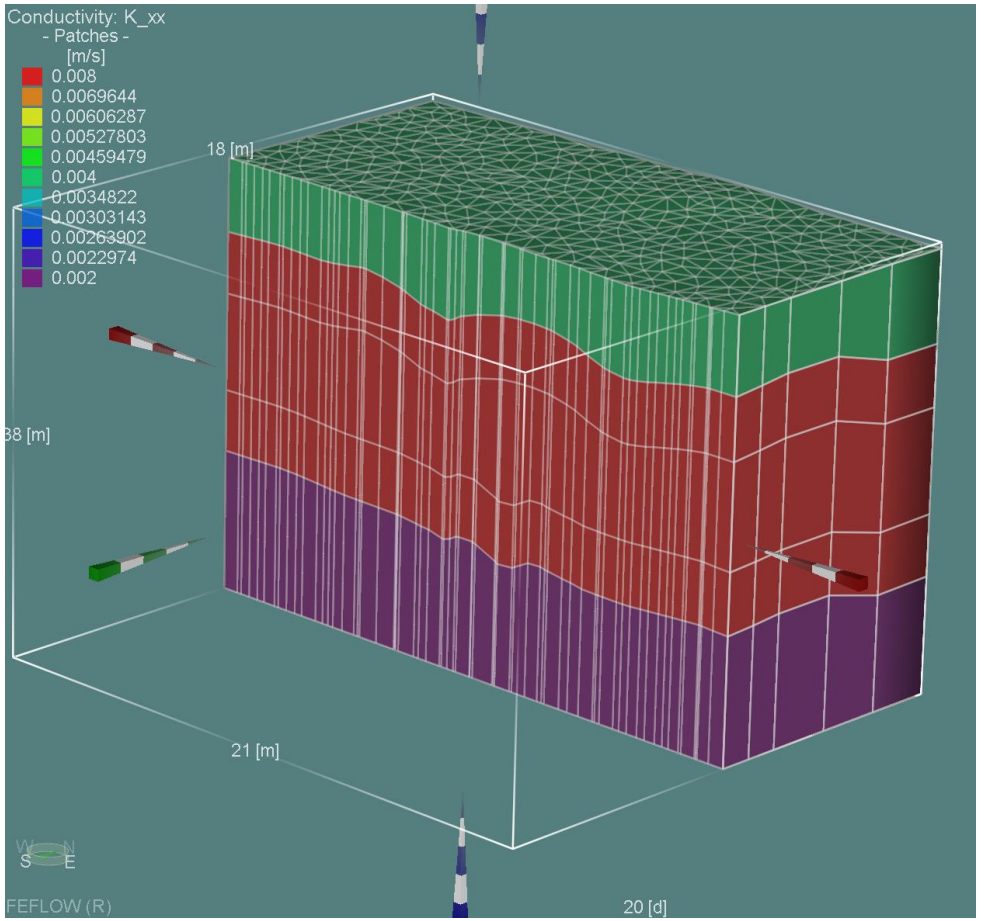
Widen field site, Switzerland



Problem in 3-D

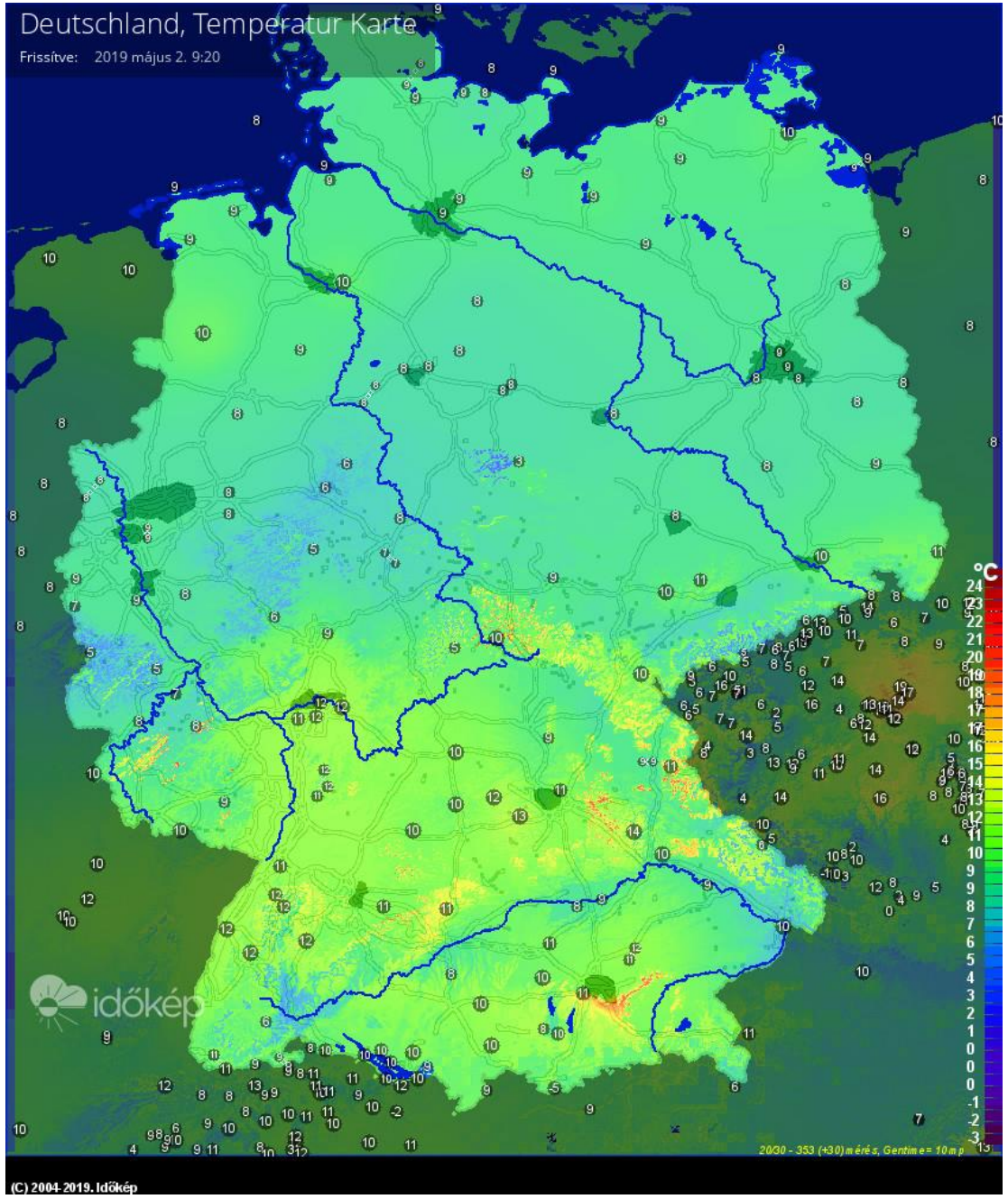
Widen field site, Switzerland



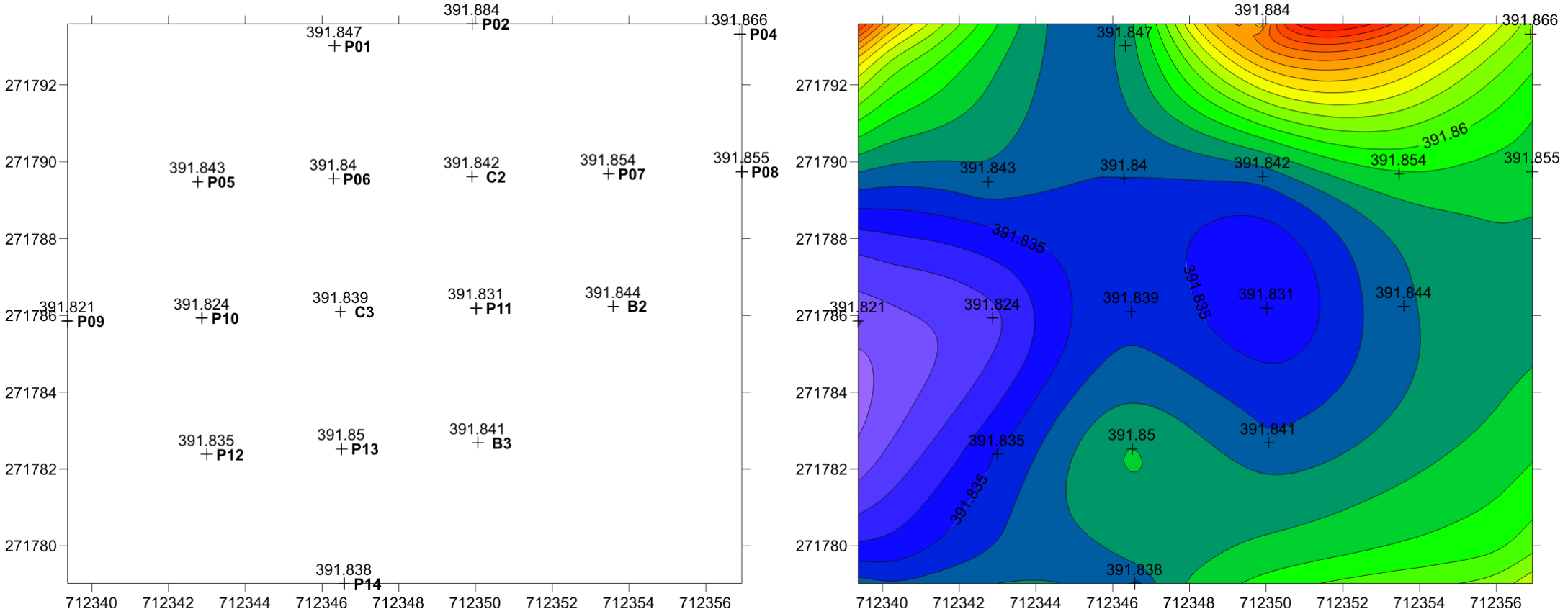


Another example

Hungarian website on German weather

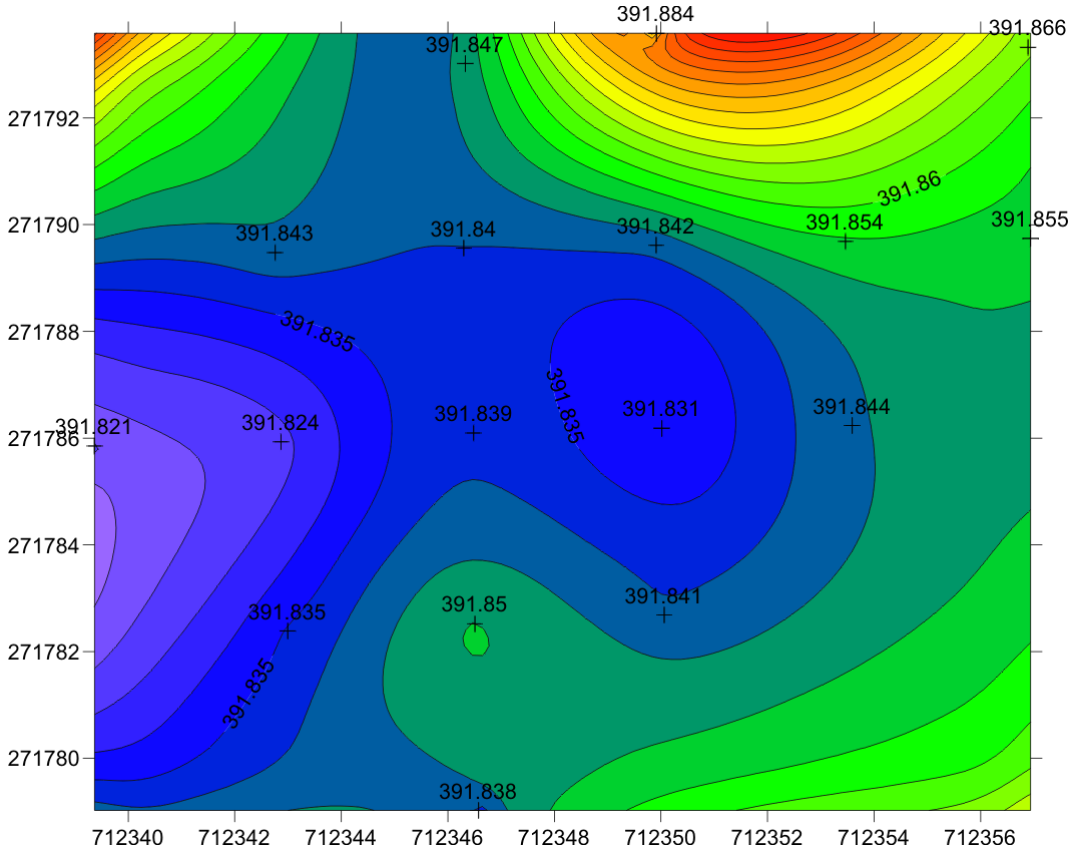


And another from hydrogeology



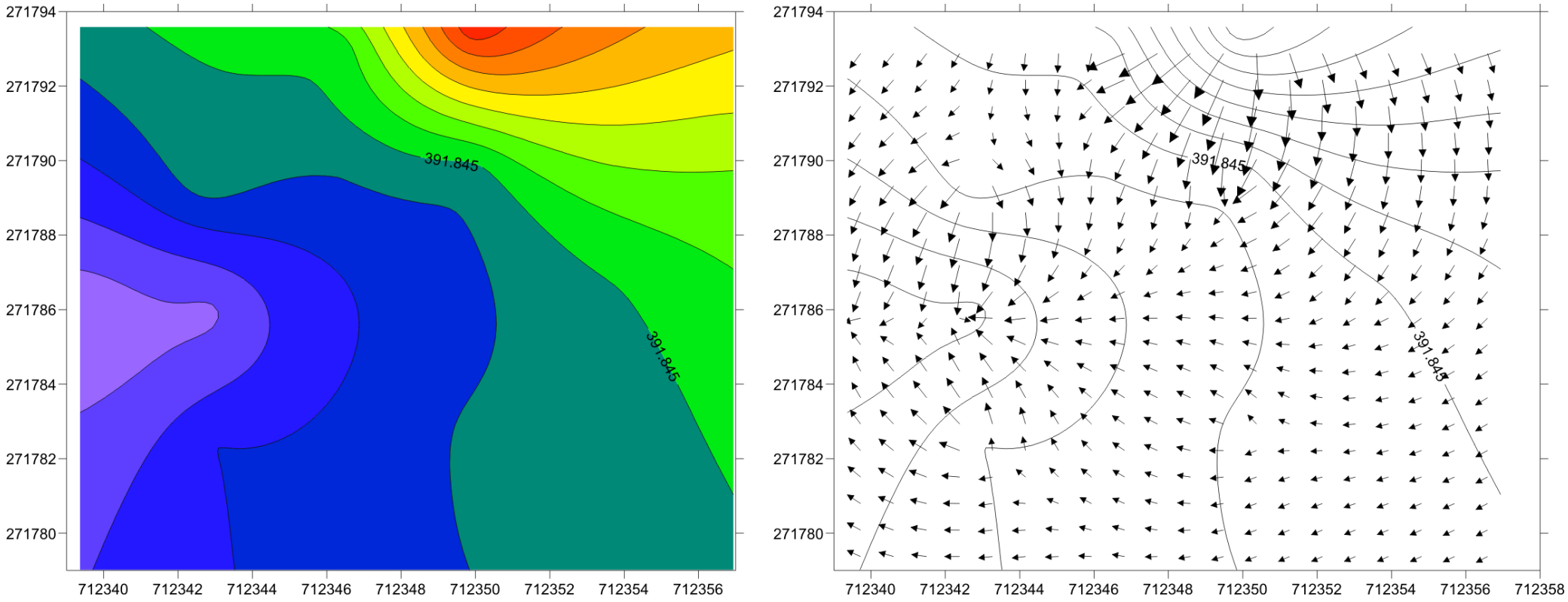
Hydraulic heads from a fluvial aquifer in Switzerland

Result is in conflict with reality



Where is the problem?

fixed

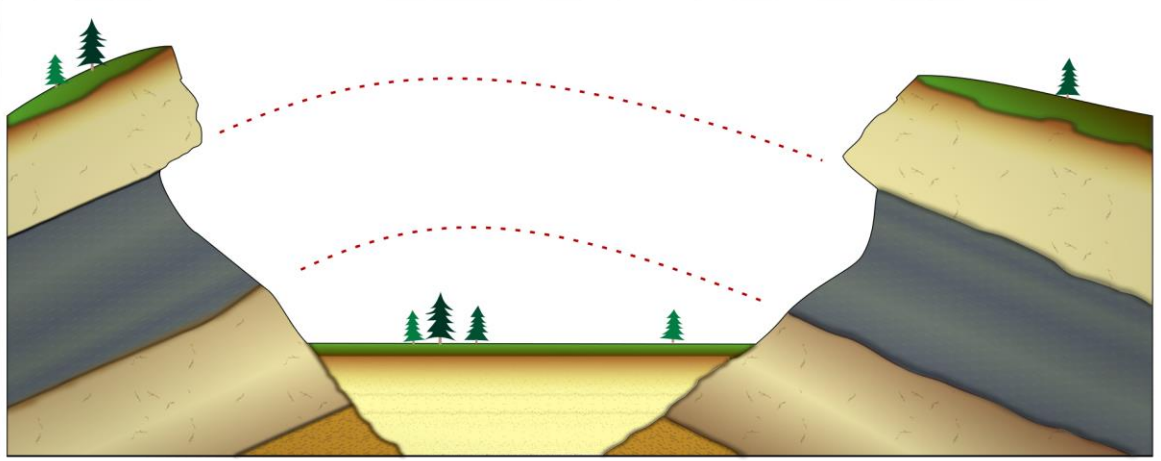


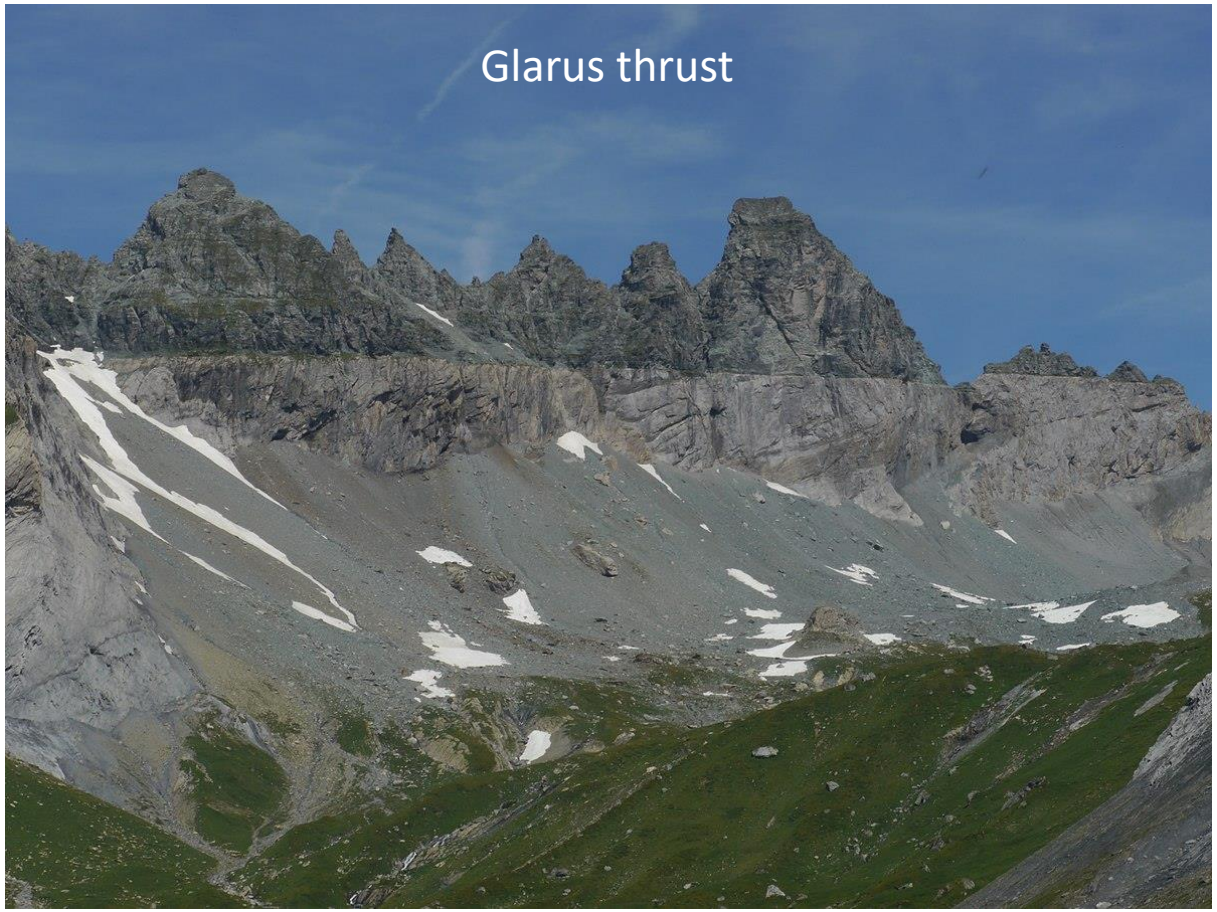
Hydraulic heads from a fluvial aquifer in Switzerland

General approaches

The geologist‘s perspective

- Continuity of layers



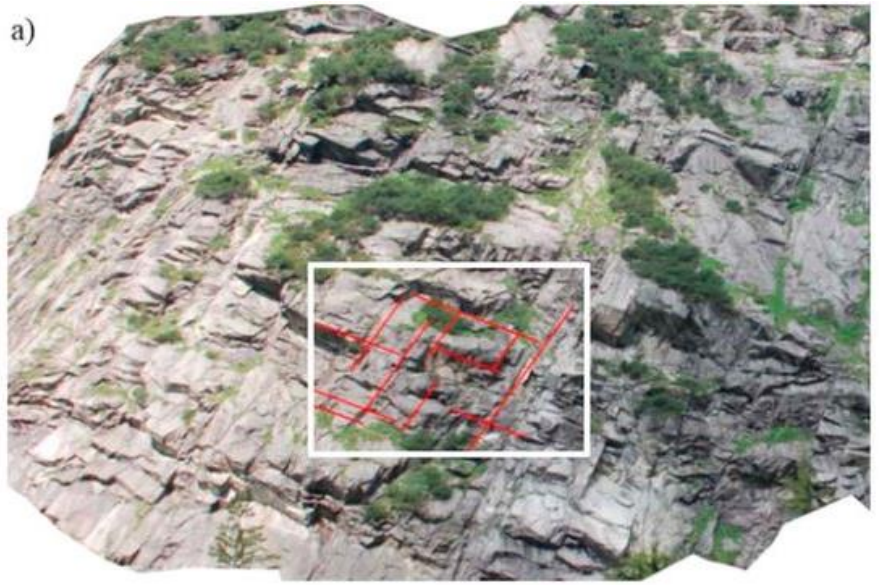


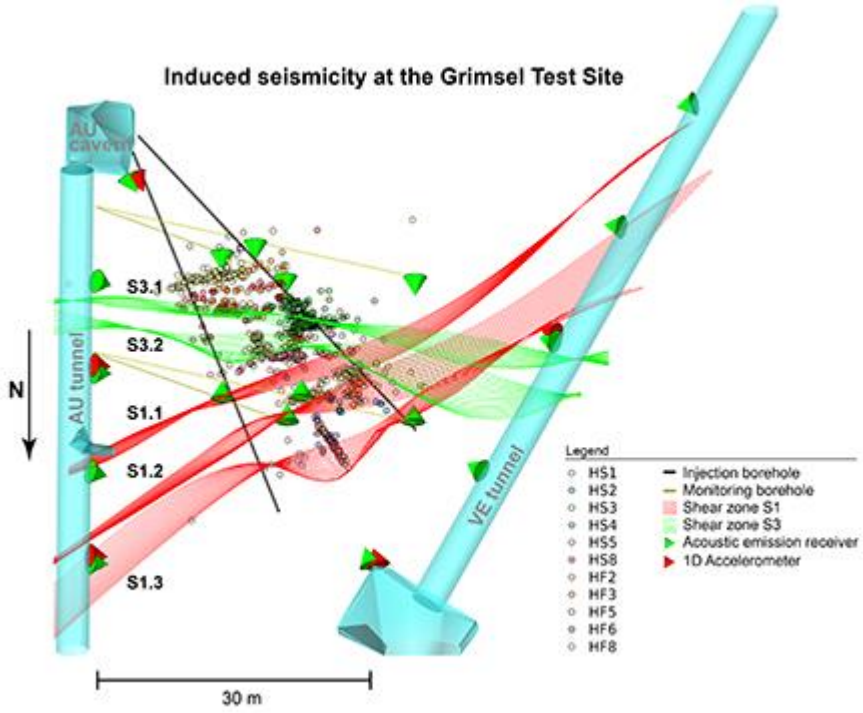
(Source: Wikipedia)

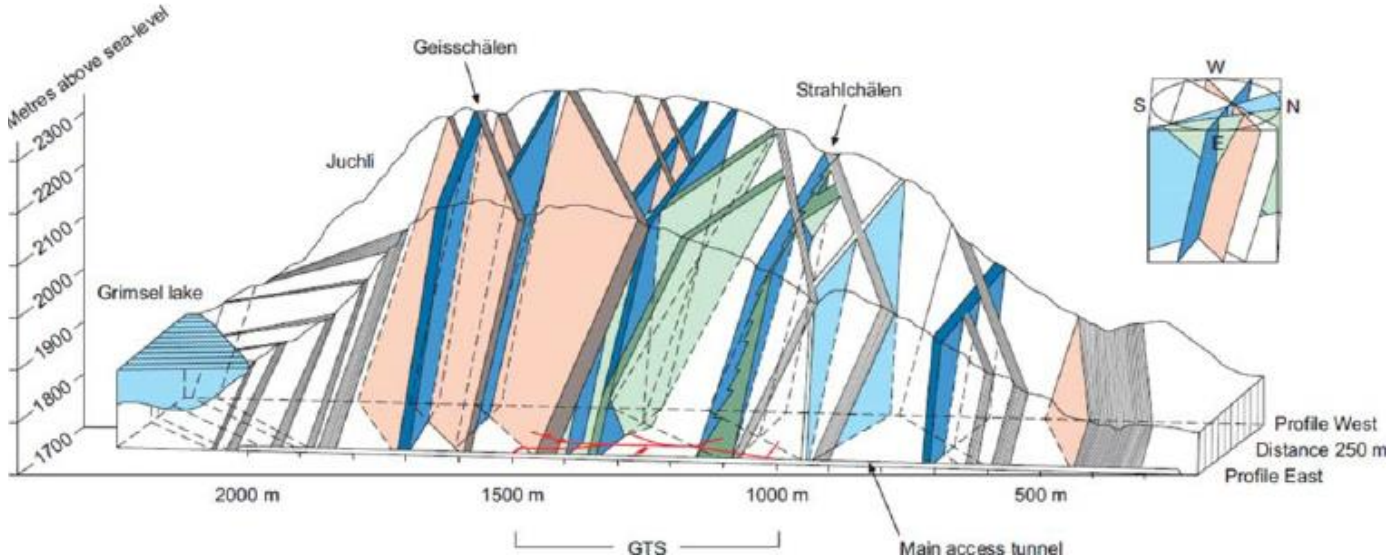
The geologist‘s perspective

- Continuity of structural elements

Tschingelmad outcrop
Afshari Moein et. al, (2018)





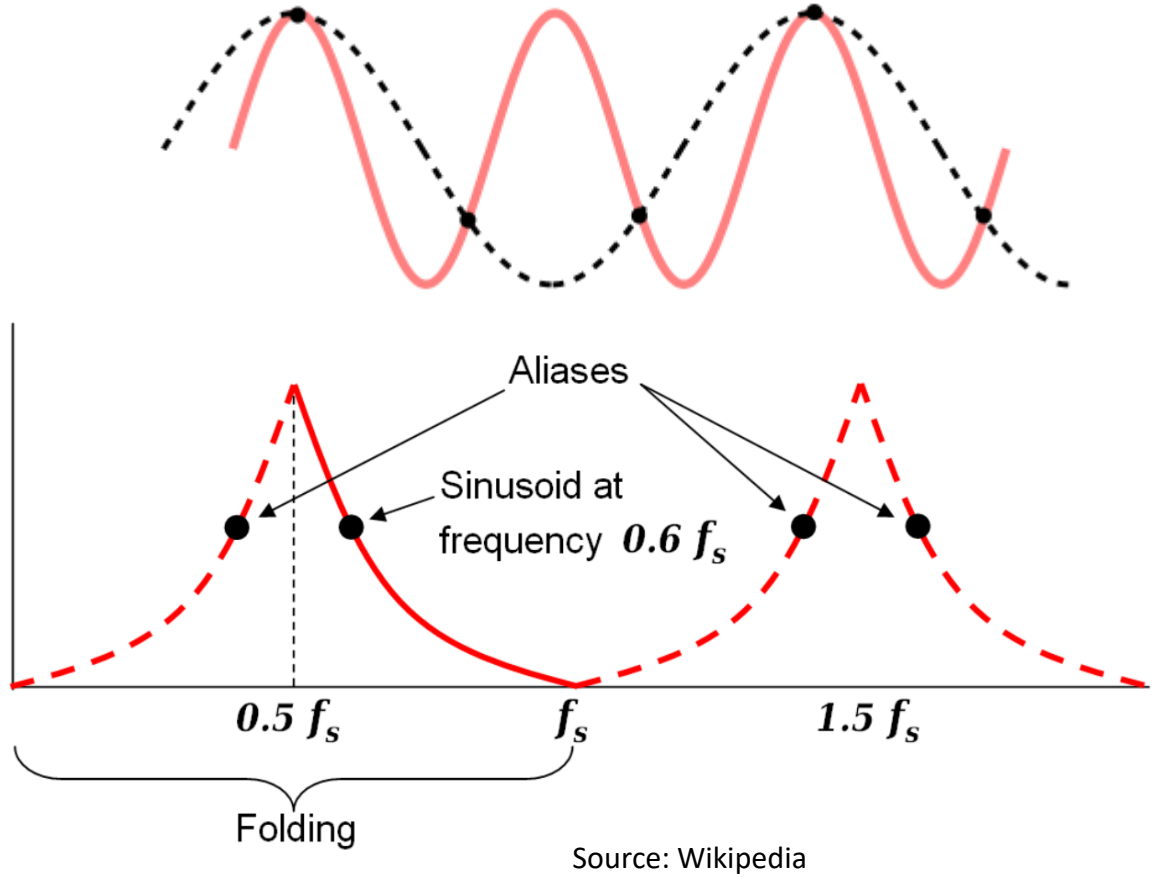


Sources: researchgate.org
tunneltalk.com

(Problem of scales)

Geophysics perspective

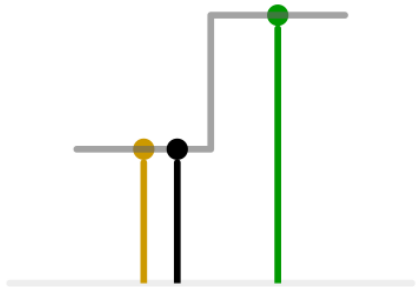
- Nyquist-Shannon sampling theorem
- Nyquist frequency: 2 x fmax



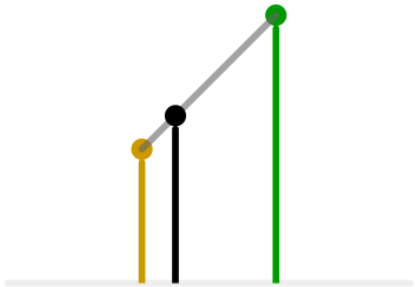
Source: Wikipedia

We cannot add more information by interpolation

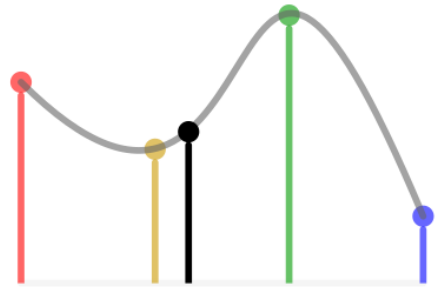
Interpolation



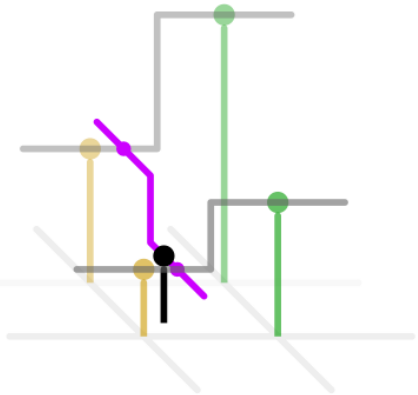
1D nearest-neighbour



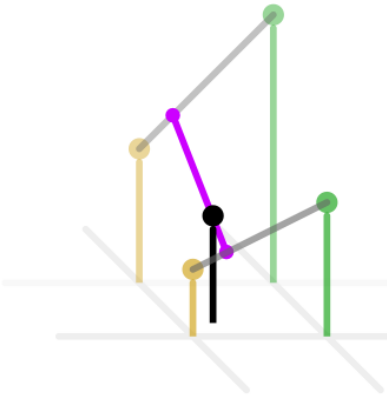
Linear



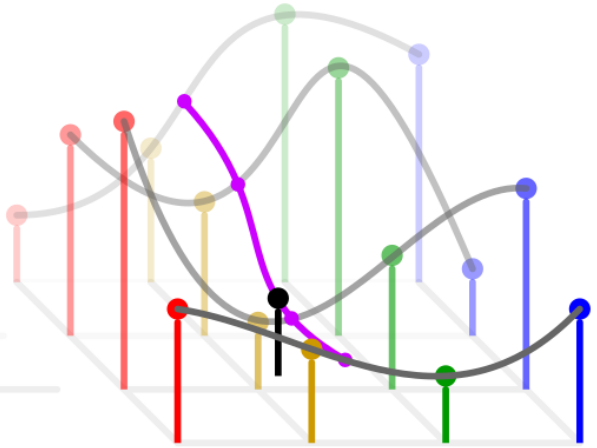
Cubic



2D nearest-neighbour



Bilinear



Bicubic

Objective:

Reconstruct spatially continuous attributes from discrete samples.

Interpolation syntax

$$Y(x_o)$$

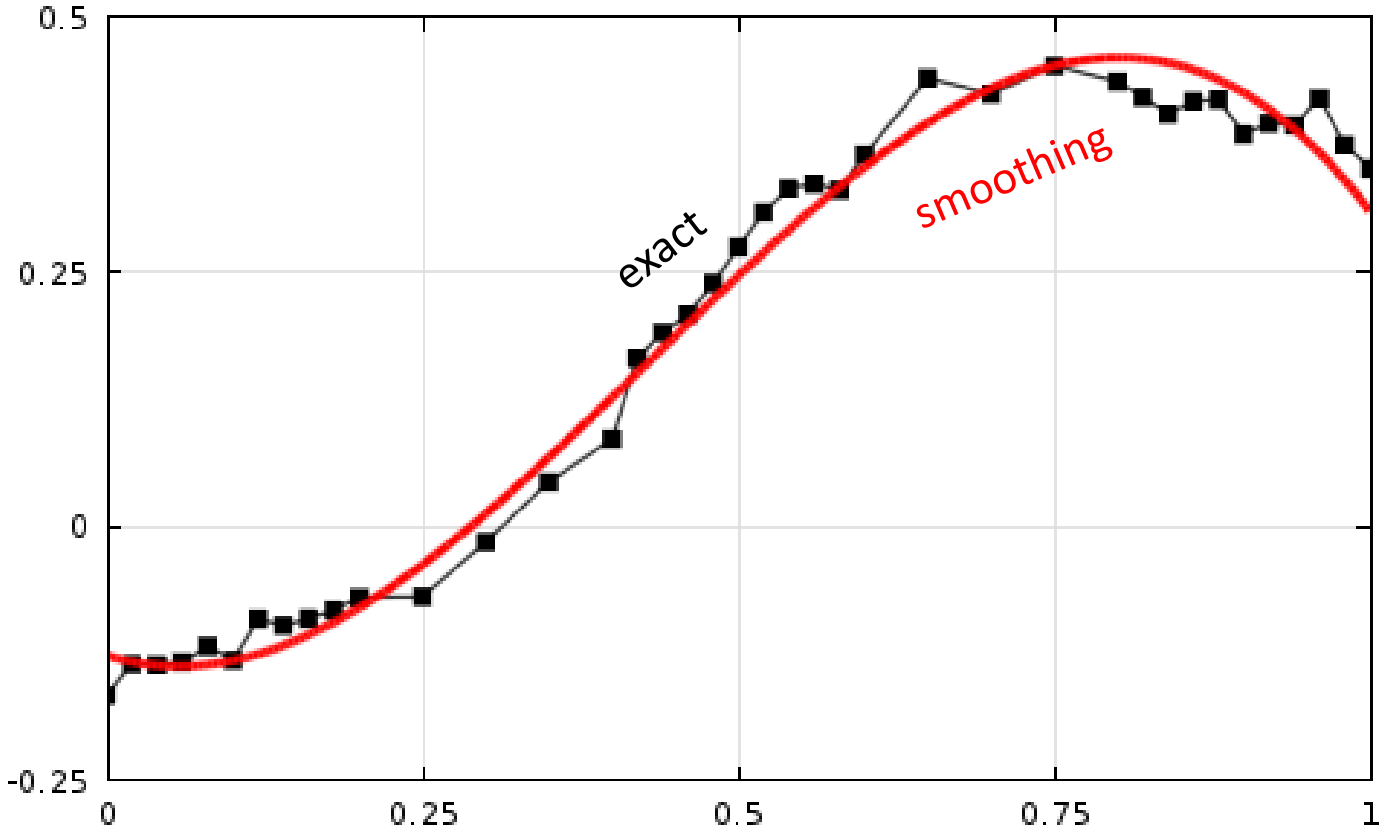
$$Y(x_i) = interp(Y(x_o), x_o)$$

- Sample points/observations
- Query points

Calculate complete field vs calculate query point

Interpolation properties

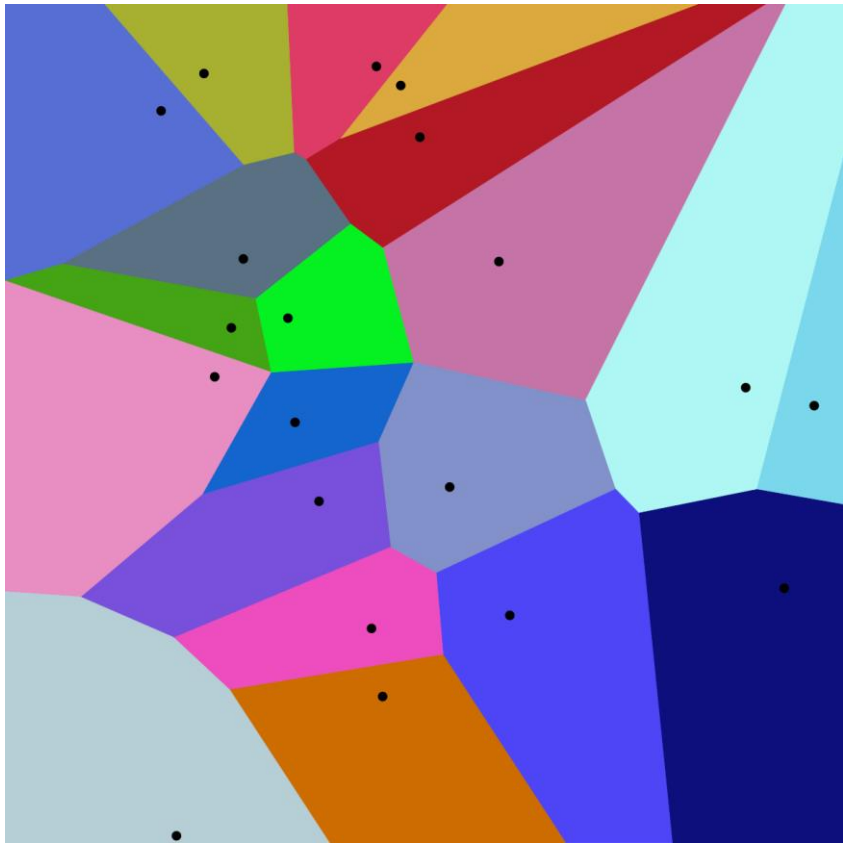
- 1D/2D
- Exact/smoothing
- Extrapolation



Nearest neighbor

$Y(x_i) = Y(x^*)$, where:
$|x_i - x^*| < |x_i - x_k| \; \forall k \in K$

Take the value of the closest sample



Source: Wikipedia (Balu Ertl)

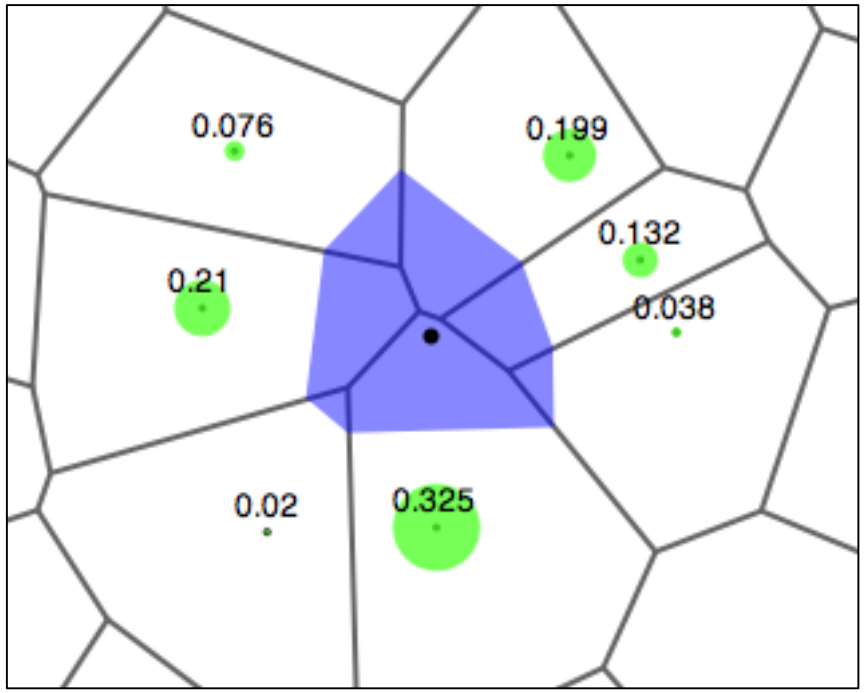
Voronoi cells

Natural neighbor

- Considers multiple points
- Each neighbor is considered

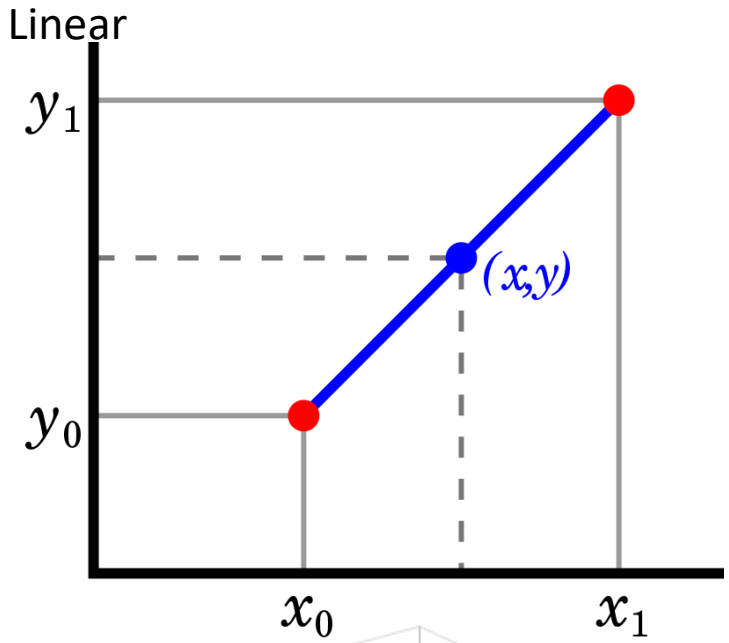
$$Y(x_i) = \sum_K Y(x_k) w_k$$

$$w_k = \frac{A_{ik}}{A_i}$$

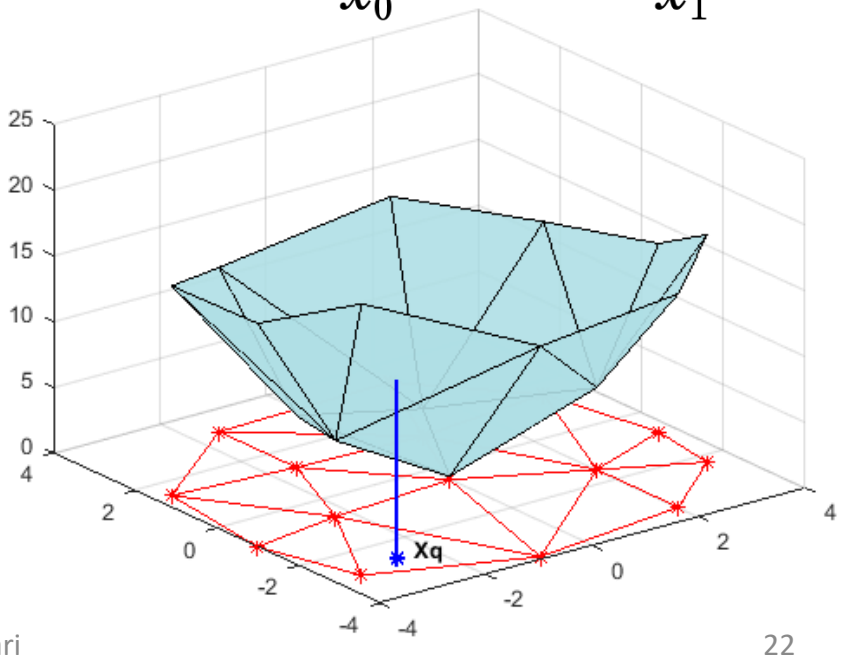


Source: Wikipedia

Linear



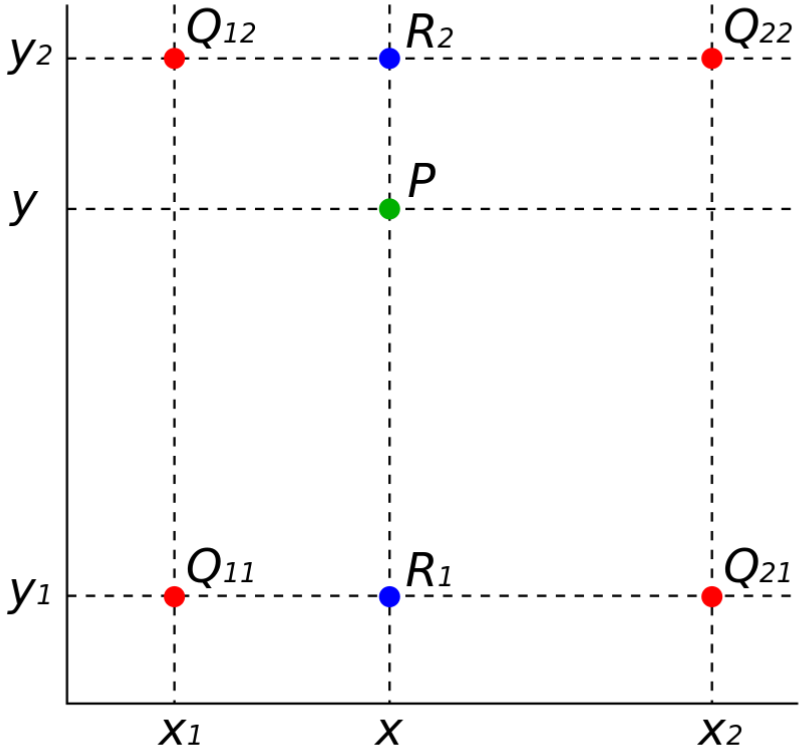
- Does not work in 2-D by definition
  - triangulation

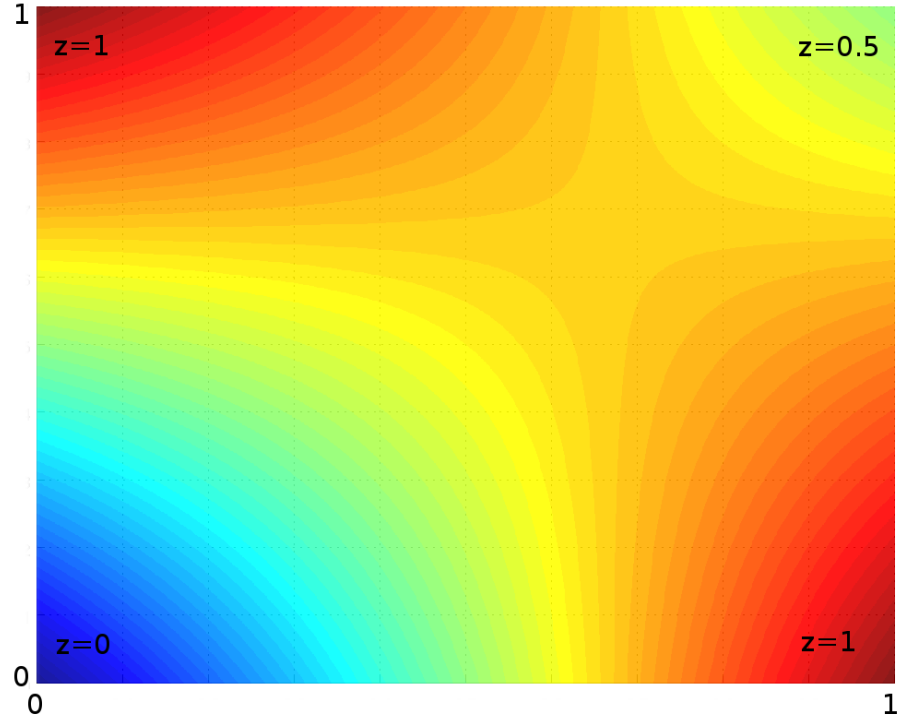


Source: Wikipedia

Bilinear

Needs data on a regular grid

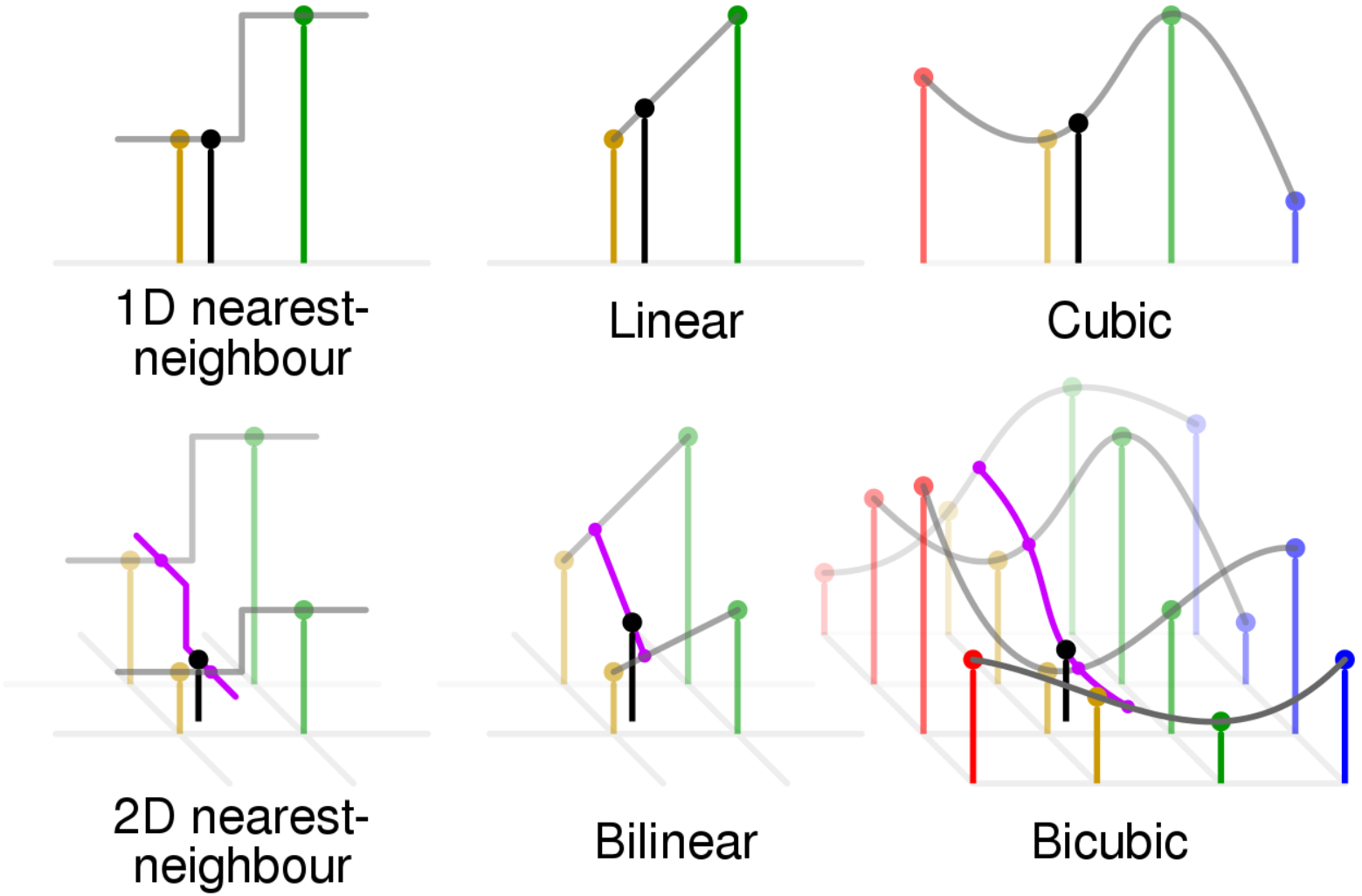




Source: Wikipedia

What is trilinear interpolation?

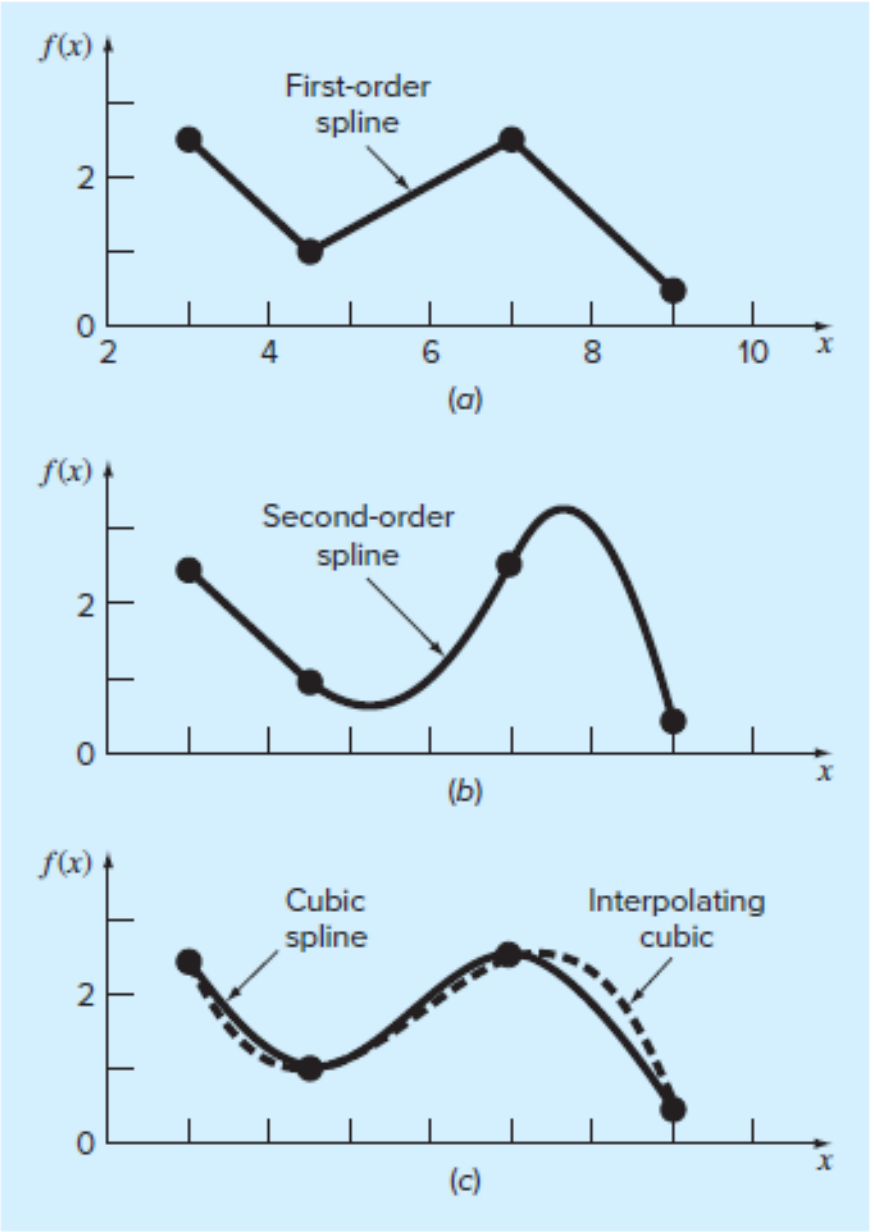
Interpolation on regular grids



Source: Wikipedia

Higher powers

- Cubic
- Bicubic
- Quadratic etc…

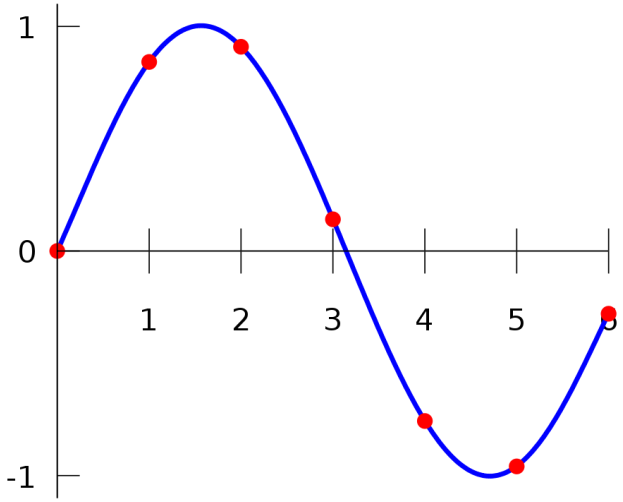


Source: Berlin Chen

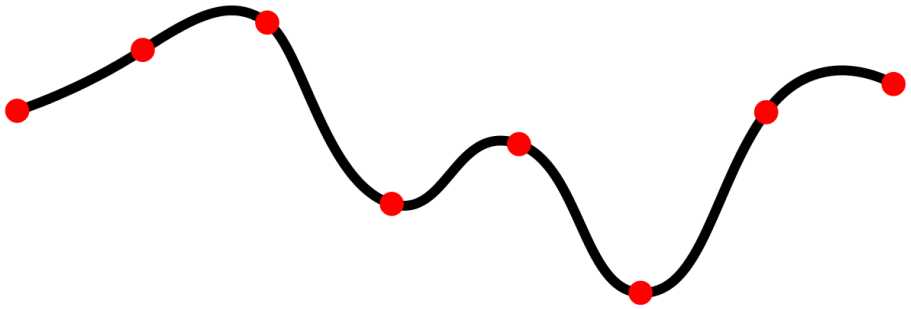
Using polynomials – spline interpolation

- Fit polinomials to all data – exact
  - Find lowest order polynomial that fits the data

$$a_0 + a_1 x + \cdots + a_n x^n$$



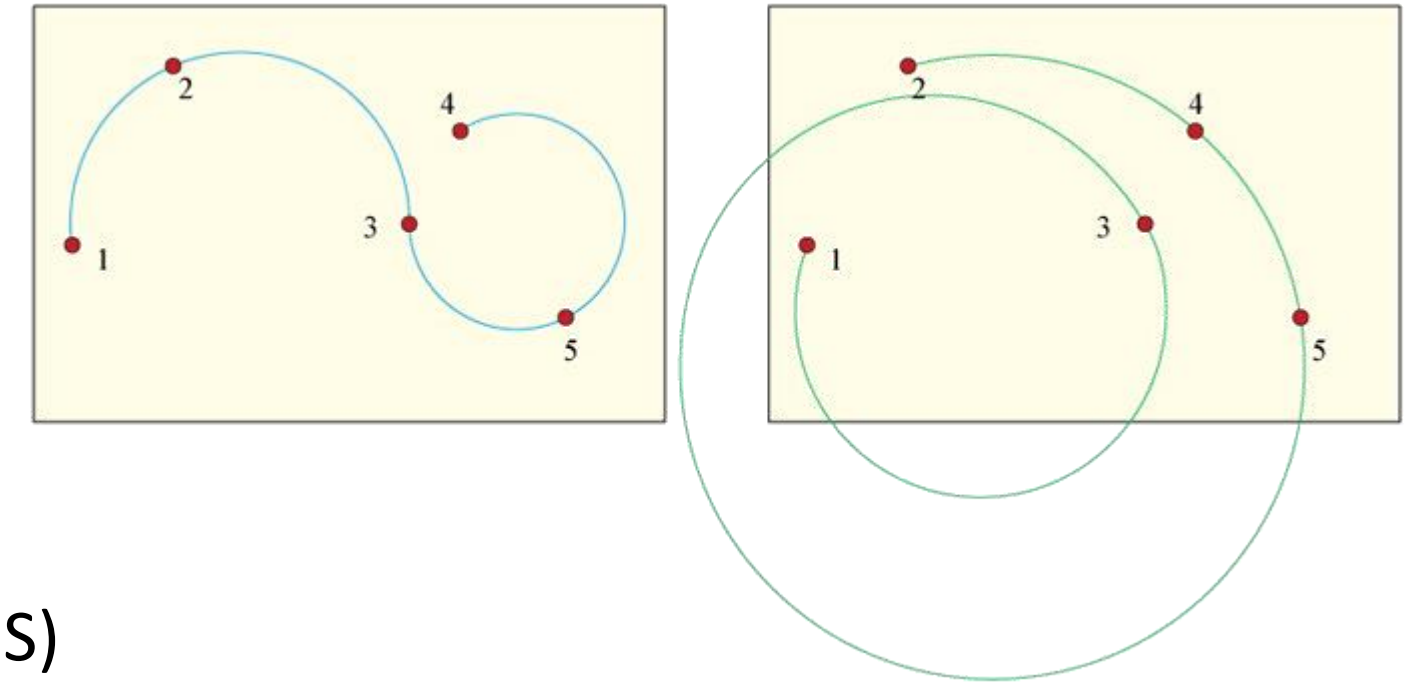
- Fit spline segments between
  - Continuus derivatives at points (dx and d2x)
  - Works better with a lot of points



Source: Wikipedia

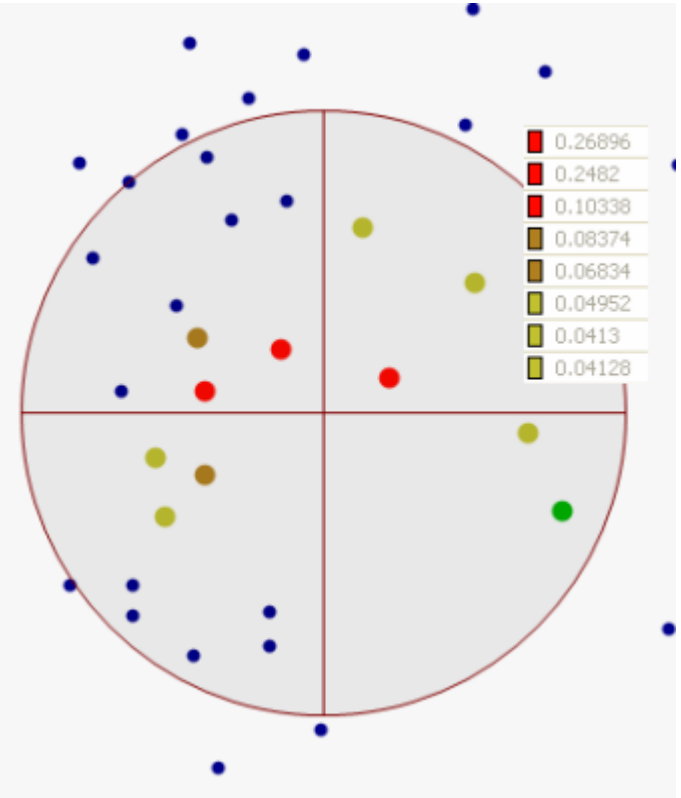
Derivative-based

- Minimum curvature



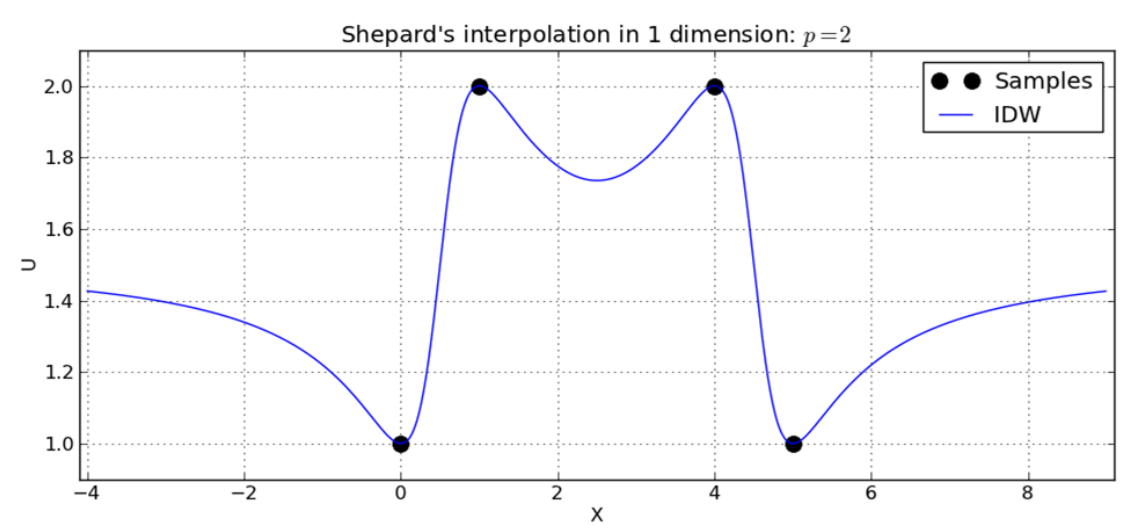
- Thin plate spline (TPS)

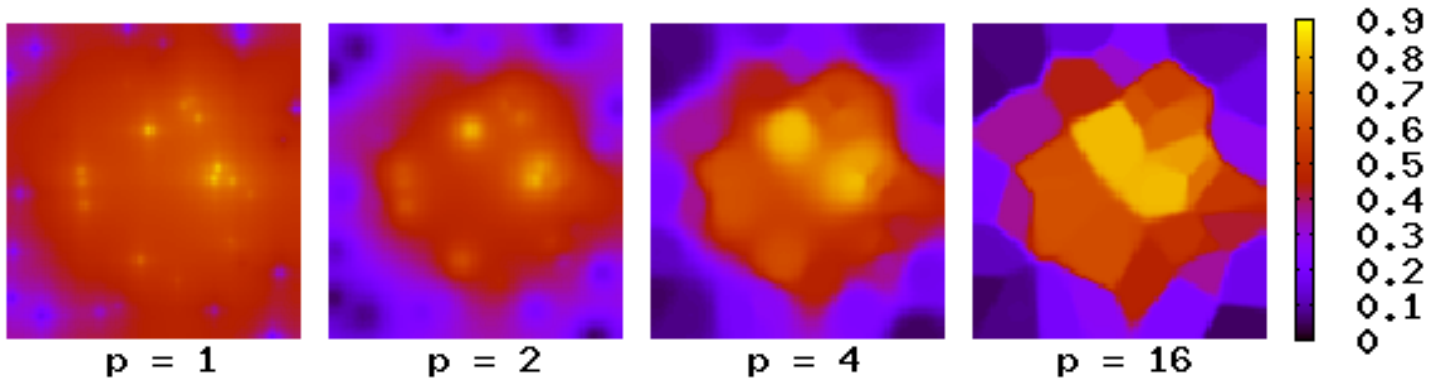
Inverse distance weighting (IDW)



Weights based on distance

Source: ArcGIS



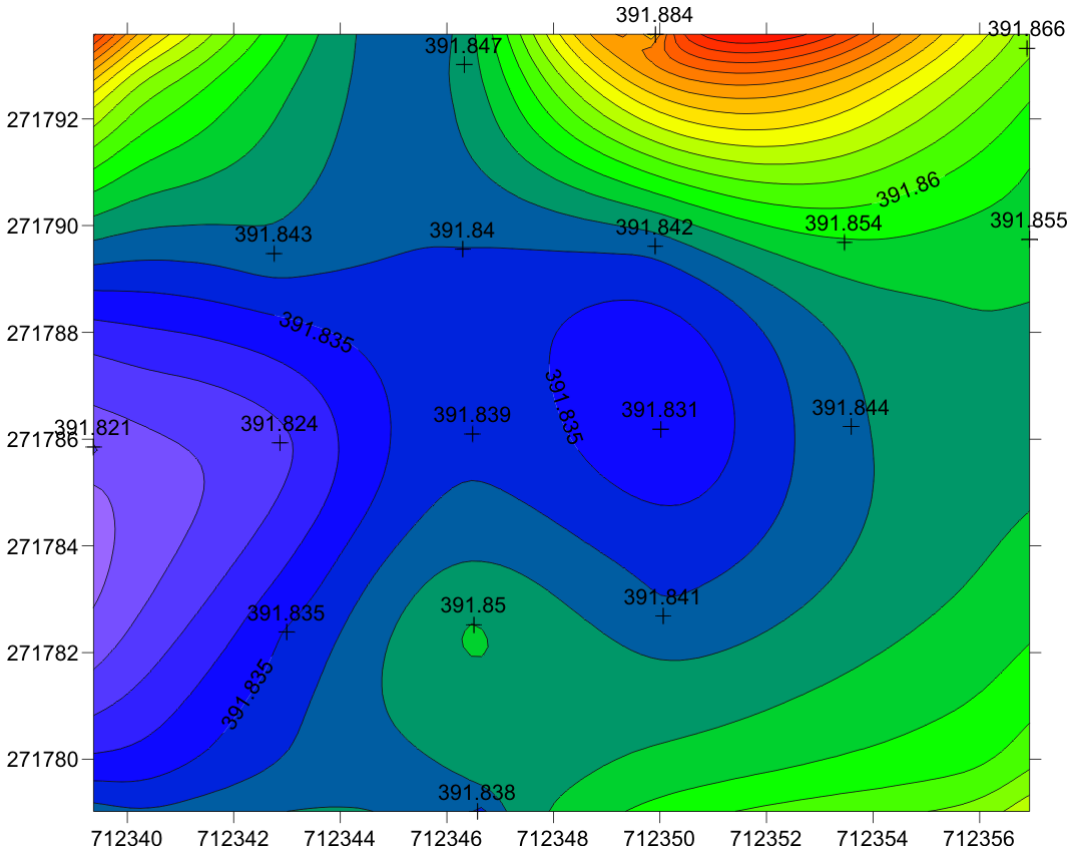


Z=exp(-x^2-y^2)

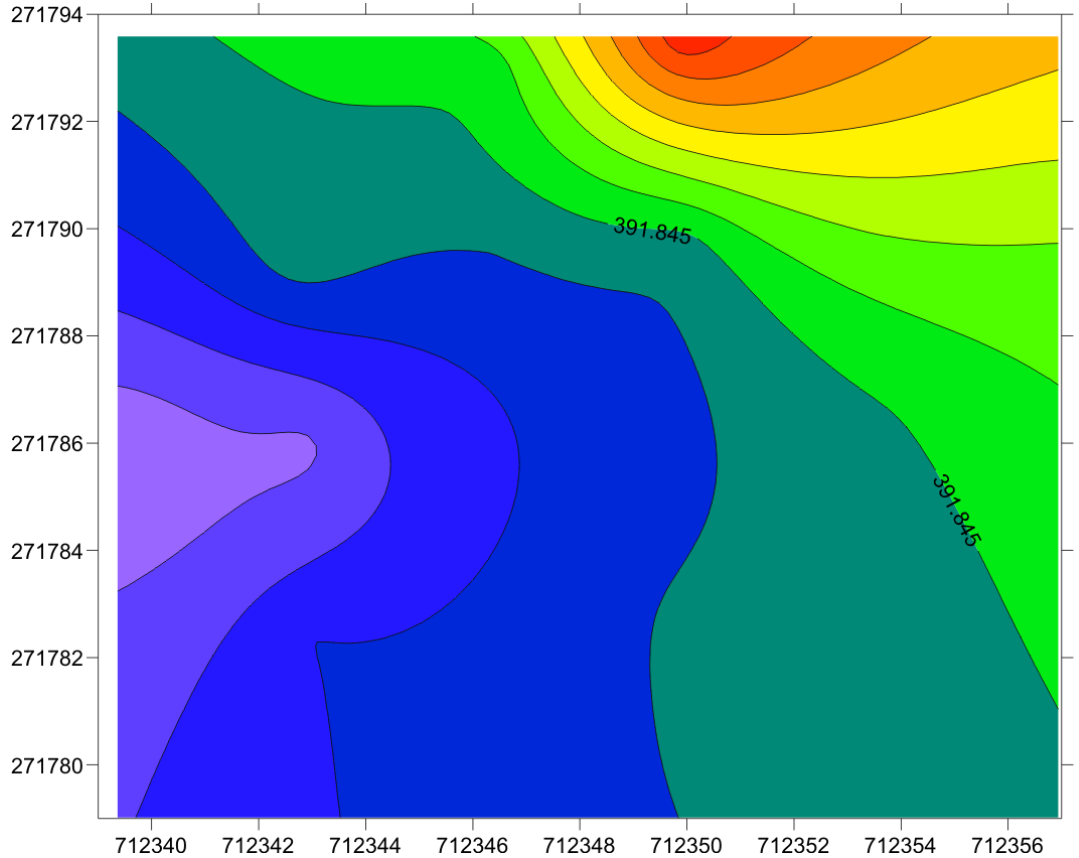
Source: Wikipedia

Special rules

- Modified Shepards method



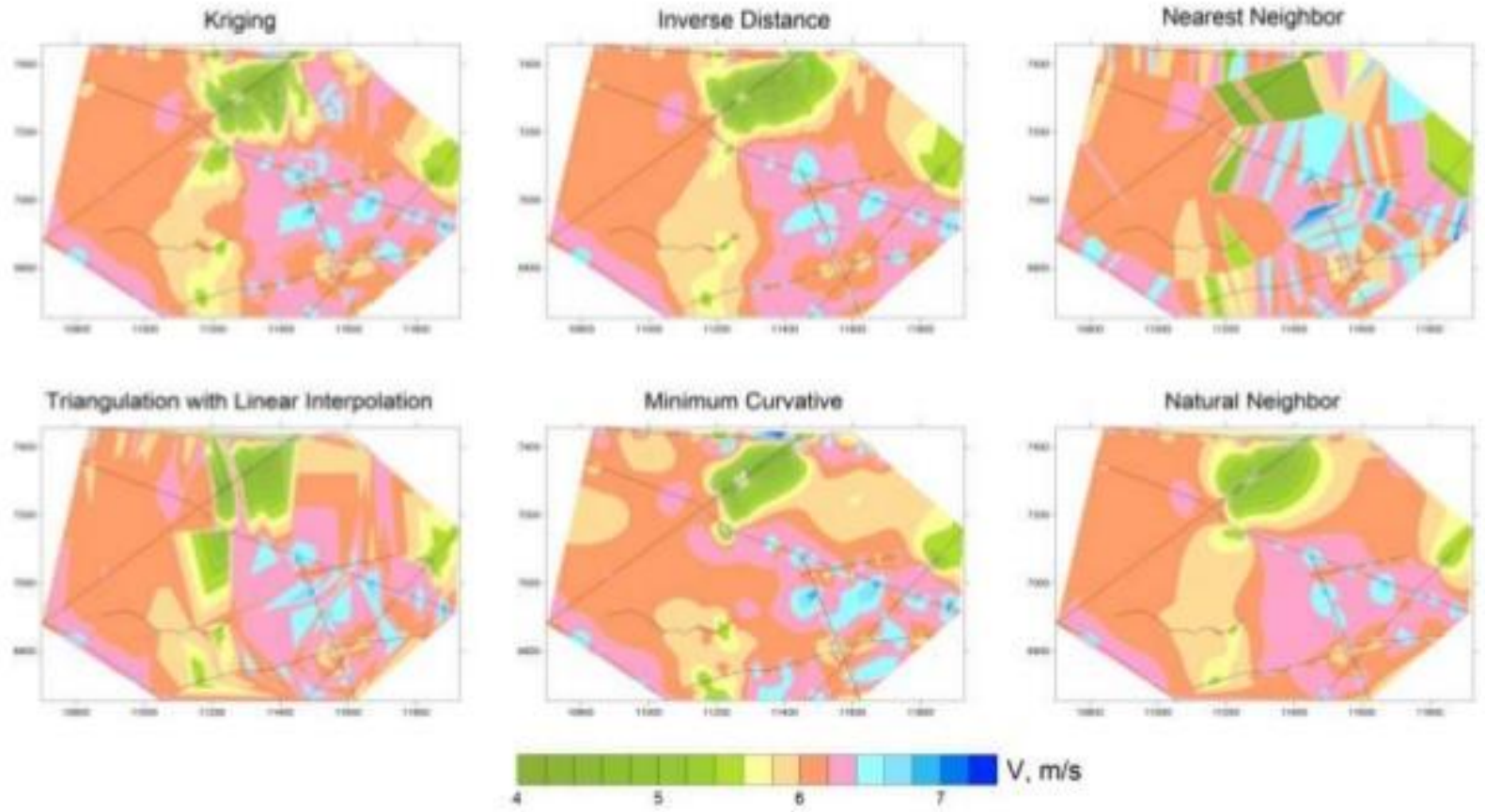
- No bulls-eye effect



TBH I am not sure if it is actually that...

Parallel Algorithm for Natural Neighbor Interpolation (slide 4 / 19)

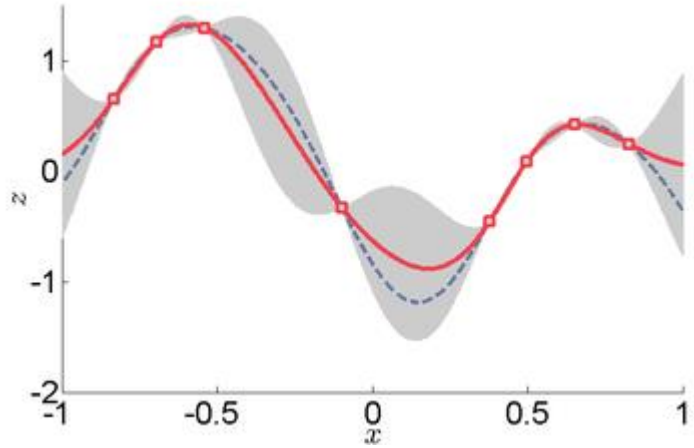
Interpolation methods

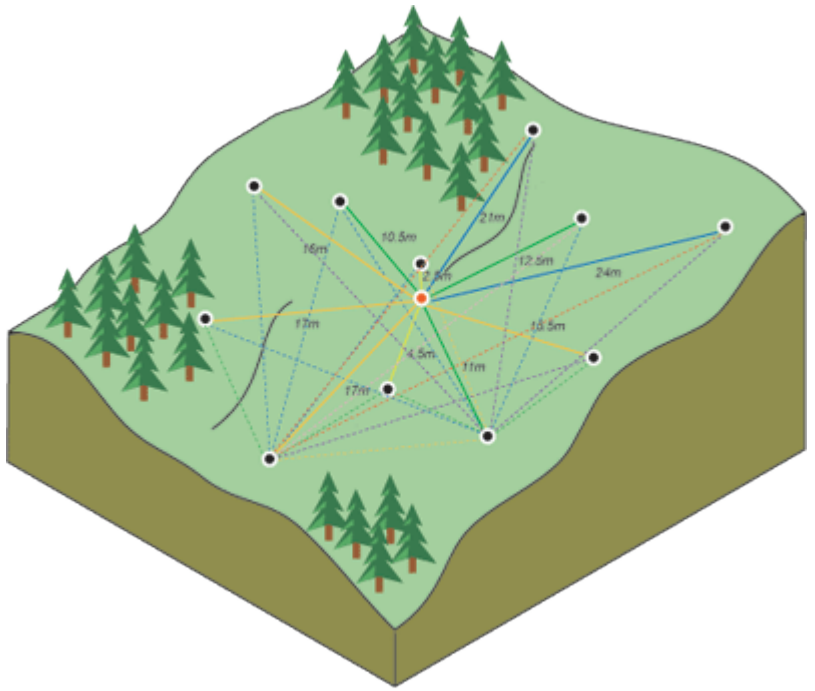


2nd Ural Workshop on Parallel, Distributed, and Cloud Computing for Young Scientists
Yekaterinburg, Russia, October 6, 2016

Alexander Tsidaev

Exercise part 1.



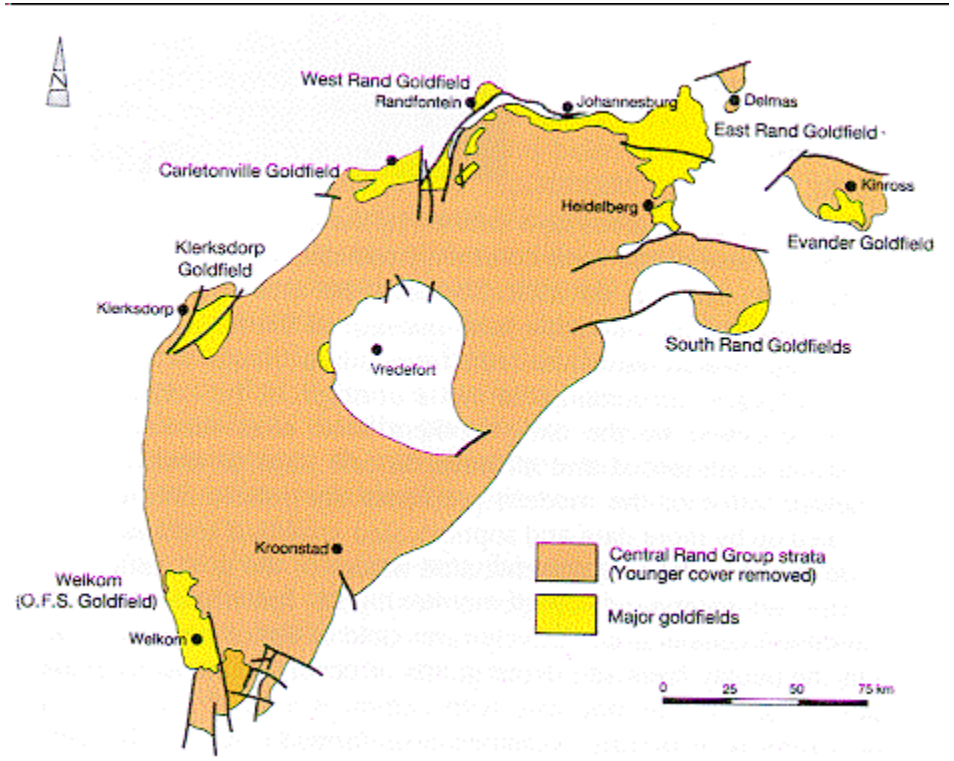


Kriging

Gold mines in South Africa

- Danie G. Krige (1951)
- Georges Matheron (1960)

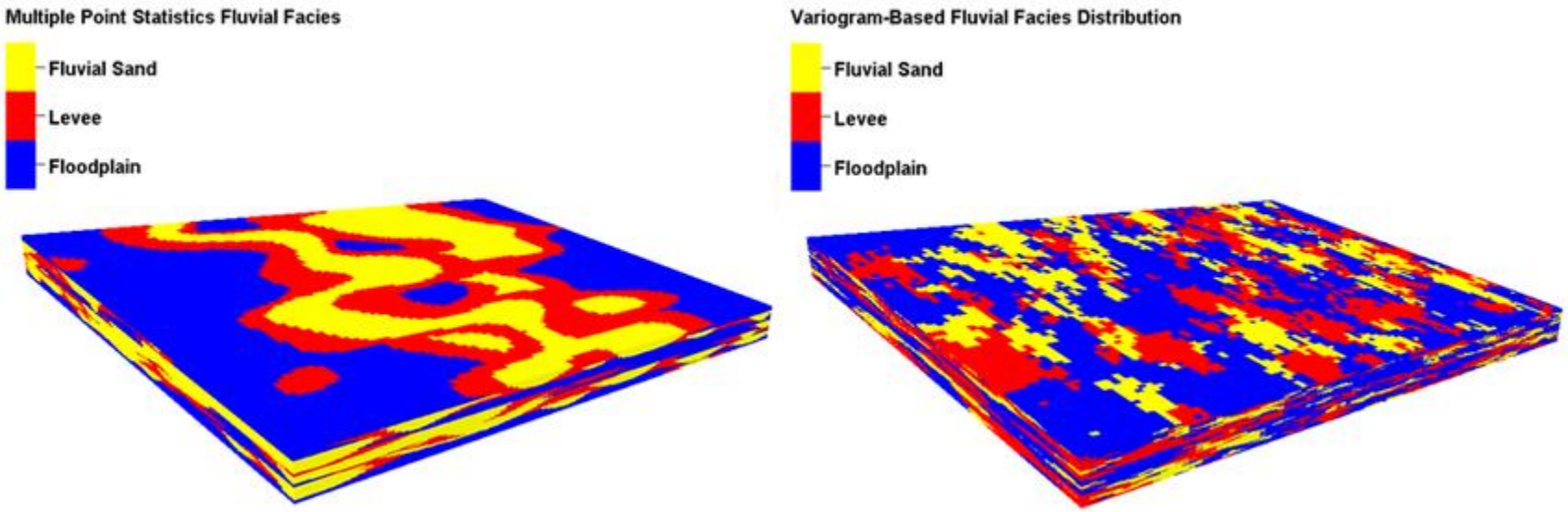
Statistical relation between measurement points



Source: http://wwwu.edu.uni-klu.ac.at/mmessner/sites/rsa/wits/wits.htm

Exercise part 2.

Multi-point statistics



Source: Researchgate – Matt Burton-Kelly

MPS with training images

- Patterns
- Formations
- Structures

- Training image
  - Outcrop
  - Borehole data
  - Geological model

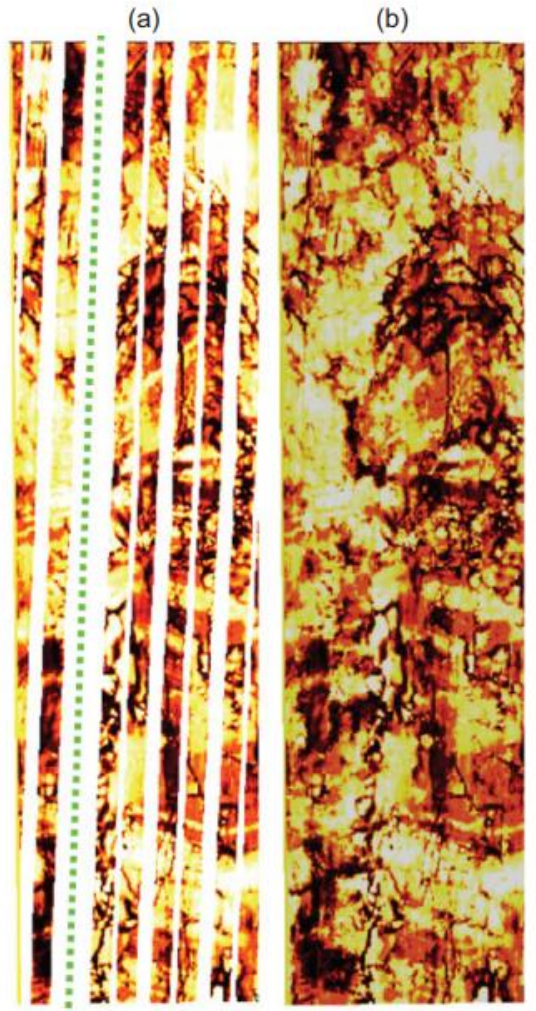


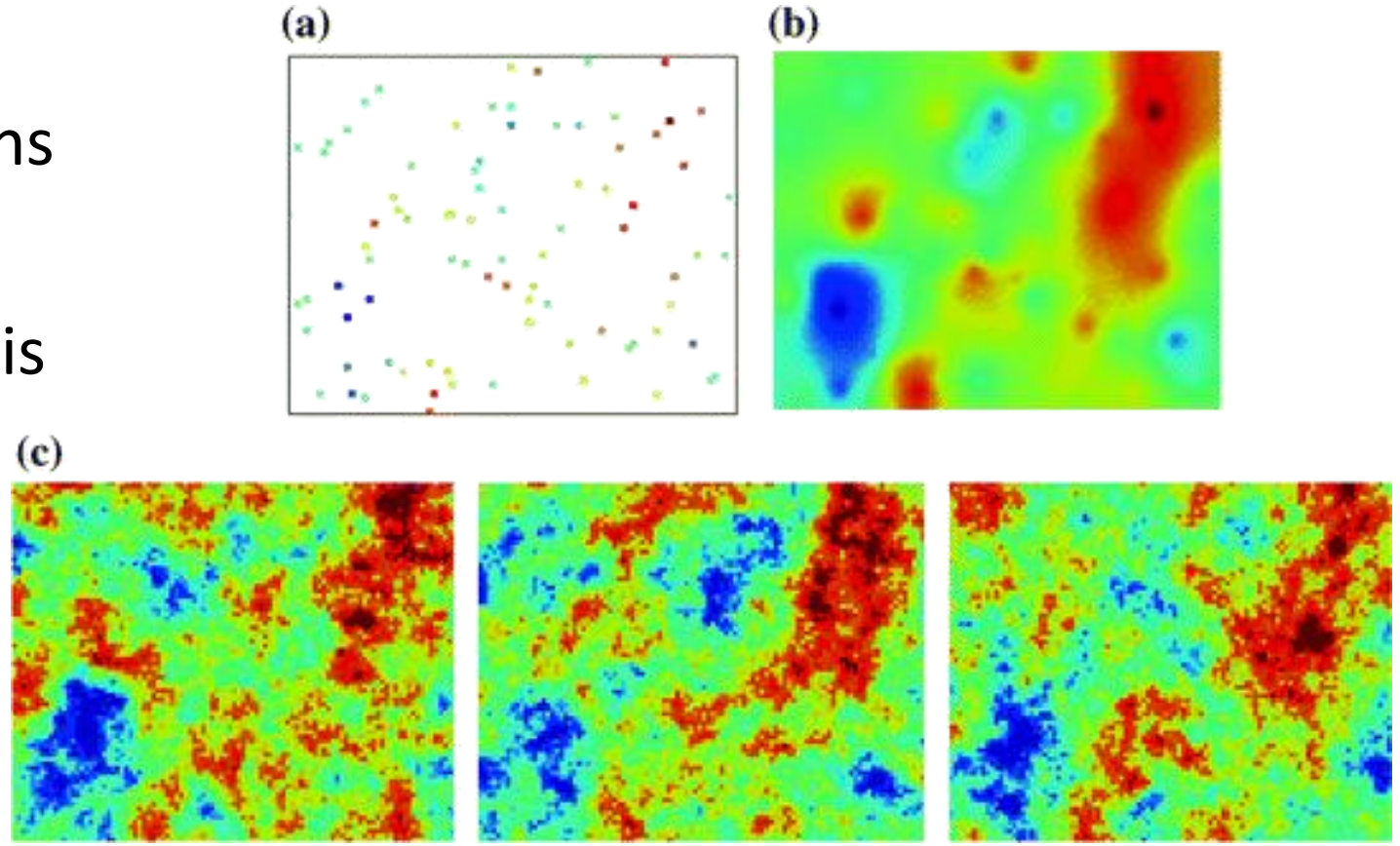
Fig. 10—Single-pass images with bad pad removed (a) and Filtersim full-bore images (b) in the same layered, fractured interval shown in Figs. 6 through 9. No vertical exaggeration. Bit size is 8.5 in. (21.5 cm). North is at the left and right edge, and south is in the center of each image. This is an 8-ft (2.4-m) vertical section.

Source: Hurley (2011)
Method to generate full-bore images using borehole images and multipoint statistics

Stochastic methods

- Create a lot of realizations

- Further statistical analysis
- UQ



Comparison between the results of Kriging (**b**) and stochastic simulation (**c**) using conditioning point data in (**a**). Figure from Tahmasebi (2018)

Other topics in geostatistics…

- Statistical analysis
- Monte Carlo simulations
- Gaussian simulation
- Bayesian inference
- Classification
- Machine learning

Software

- Surfer
- ArcGIS
- R
- GoCAD

Recommended `literature`

- ArcGIS/Surfer tutorials
- Youtube tutorials
- Handbook of Mathematical Geosciences (chapter on MPS)