$Ax =$
$b\{x^{(k)}\}x^*$
$^{(k+1)} =$
$Mx^{(k)}+$
f
$Ax =$
bA
$\{\ x_1 =$
$b_{12}x_2+$
$b_{13}x_3+$
$\cdots+$
$b_{1n}x_n+$
g_1
$x_2 =$
$b_{21}x_1+$
$b_{23}x_3+$
$\cdots+$
$b_{2n}x_n+$
g_2
$\cdots\cdots\cdots$
$x_n =$
$b_{n1}x_1+$
$b_{n2}x_2+$
$\cdots+$
$b_{nn-1}x_{n-1}+$
g_n
$\Rightarrow$
$x =$
$Bx+$
g
$^{-1}A =$
$0 b_{12} b_{13} \cdots b_{1n}$
$b_{21} 0 b_{23} \cdots b_{2n}$

$b_{n1} b_{n2} b_{n3} \cdots 0$
$, I =$
$D^{-1}b =$
g_1
g_2
$\cdots$
g_n
$, D =$
a_{11}
a_{22}

a_{nn}
$x^{(0)}$
$^{(k+1)} =$
$Bx^{(k)}+$
$g(k =$
$1, 2, \cdots)$
$x^{(1)}, x^{(2)}, \cdots$ **Jacobi** B
$x^* \lim_{k\to\infty} x^{(k)} =$
x^*
$S(B) <$
$1 \Leftrightarrow$
$\|B\| <$
$1 \Rightarrow$
$\|B\| B$
$\|x^{(k)}-$
$x^*\| \frac{\|B\|^k}{1-\|B\|} \|x^{(1)}-$
$x^{(0)}\|$
$\|x^{(k)}-$
$x^*\| \frac{\|B\|}{1-\|B\|} \|x^{(k)}-$
$x^{(k-1)}\|$
$\Rightarrow$
$S(B) <$
$1 I-$
$Bx =$
$Bx+$
gx^*
$\{\ x^* =$
Bx^*+
g
$x^{(k)} =$
$Bx^{(k-1)}+$
g
$\Rightarrow$
$x^{(k)}-$
$x^* =$
$B(x^{(k-1)}-$
$x^*)$
$^{(k)}-$
$x^* =$
$B^k(x^{(0)}-$
$x^*)$

$S(B) <$
$1 \lim_{k\to\infty} B^k =$
0

$\sum_{j=k}^{\infty}(x^{(j)}-x^{(j+1)})$

$\Rightarrow \|x^{(k)}-x^*\| \sum_{j=k}^{\infty}\|x^{(j)}-x^{(j+1)}\| = \sum_{j=k}^{\infty}\|x^{(j+1)}-x^{(j)}\|$

$(\sum_{j=k}^{\infty}\|B\|^{j})\|x^{(1)}-x^{(0)}\|$

$= \frac{\|B\|^k}{1-\|B\|}\|x^{(1)}-x^{(0)}\|$

$x^* = Bx^* + g, x^{(k)} = Bx^{(k-1)} + g$

$^{(k)} = Bx^{(k-1)} + (I-B)x^*$

$\Rightarrow (I-B)(x^{(k)}-x^*) = B(x^{(k-1)}-x^{(k)})$

$\Rightarrow x^{(k)}-x^* = (I-B)^{-1}B(x^{(k-1)}-x^{(k)})$

$\Rightarrow \|x^{(k)}-x^*\| \frac{\|B\|}{1-\|B\|}\|x^{(k)}-x^{(k-1)}\|$

$x^{(k+1)} x^{(k)}$

$\{ x_1^{(k+1)} = b_{12}x_2^{(k)} + b_{13}x_3^{(k)} + \cdots + b_{1n}x_n^{(k)} + g_1$
$x_2^{(k+1)} = b_{21}x_1^{(k)} + b_{23}x_3^{(k)} + \cdots + b_{2n}x_n^{(k)} + g_2$
$\cdots\cdots\cdots$
$x_n^{(k+1)} = b_{n1}x_1^{(k)} + b_{n2}x_2^{(k)} + \cdots + b_{nn-1}x_{n-1}^{(k)} + g_n$

$\Rightarrow x^{(k+1)} = Bx^{(k)} + g$

$x_1^{(k+1)}, x_2^{(k+1)}, \ldots, x_{i-1}^{(k+1)} x_1^{(k)}, x_2^{(k)}, \ldots, x_{i-1}^{(k)}$

$\{ x_1^{(k+1)} = b_{12}x_2^{(k)} + b_{13}x_3^{(k)} + \cdots +$

Theorem .1
$\forall\omega \in$
$\mathbf{C}$:

$S(L_\omega)|\omega-1|$

$\omega \in$
$\mathbf{R}$ SOR

$0 < \omega < 2$

Definations:
$Ann2nP$

$PAP^T = A_{11} A_{12} 0 A_{13}$

$A_{11} r A_{22} n-$
rA PA
A
$A0$
$BA0B$
$An3$
$A \neq$
0
$A =$
$(a_{ij})_{n\times n} \in$
$\mathbf{R}^{n\times n} \mathbf{C}^{n\times n}$

$$|a_{ii}| \sum_{j=1 j\neq i}^{n} |a_{ij}| \, (i = 1, 2, \ldots, n)$$

A
$i \, (i =$
$1, 2, \ldots, n) A$
$Ai \, (i =$
$1, 2, \ldots, n) A$
Theorem .2
A A .
Theorem .3
A A A .
Conclusions:
A $A2D-$
$A \Leftrightarrow$
A $\Rightarrow$
A $0 <$
$\omega <$
2
$\Rightarrow$
A $\Rightarrow$
A $\Rightarrow$
A $0 <$
$\omega 1$
$\Rightarrow$