$C[a,b]$
$f(x) \in$
$C[a,b]$
$f(x_i) =$
$y_i(i =$
$0,1,\ldots,n)$
$$\varphi(x) = \sum_{i=0}^{n} \alpha_i \varphi_i(x) f(x)$$

$\varphi(x_i) = y_i(i = 0,1,\ldots,n)$
(1)
$\varphi(x)$
n
$\varphi(x) = a_0 + a_1 x + \ldots + a_n x^n$
(2)
H_n
n
$\varphi(x) \in$
H_n
Theorem .1

??

??

??
$\varphi(x)$
??
$\{ \ a_0 + a_1 x_0 + \ldots + a_n x_0^n = y_0, a_0 + a_1 x_1 + \ldots + a_n x_1^n = y_1, \ \ldots\ldots\ldots a_0 + a_1 x_n + \ldots + a_n x_n^n = y_n.$
(3)
$_n(x_0, x_1, \ldots, x_n) =$
$\begin{vmatrix} 1 x_0 x_0^2 \ldots x_0^n \\ 1 x_1 x_1^2 \ldots x_1^n \\ \\ 1 x_n x_n^2 \ldots x_n^n \end{vmatrix} =$
$\prod_{i=1}^{n} \prod_{j=0}^{i-1} (x_i -$
$x_j) =$
$\prod_{0 \le j < i \le n} (x_i -$
$x_j)$
$x_i \neq$
$x_j(i \neq$
$j)$
Theorem .2 (Rolle Law)

R

$f(x)$

$[a,b]$

(a,b)

$f(a) =$
$f(b)$
$\xi \in$
(a,b)
$f'(\xi) =$
0
$n+$
$1n$
$_n(x_k) =$
$y_k(k =$
$1,2,\ldots,n)$
$l_j(x_k) =$
$\delta_{jk} =$
$\{ 1 \ , k =$
j
$0, k \neq$
j
$(j, k =$
$0,1,\ldots,n)$
$l(x) f(x) n L_n(x) f(x) n$

$$R_n(x) = f(x) - L_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\omega_{n+1}(x)\xi = \xi(x) \in (a,b)$$

(5)

$$|R_n(x)| \frac{M}{(n+1)!} |\omega_{n+1}(x)|$$

$f'(x) \in C[a,b] M = \max_{axb} \left|f^{(n+1)}(x)\right|$ ${}_n(x_i) = f(x_i) - L_n(x_i) = 0$ $R_n(x) = k(x)(x - x_0)(x - x_1)\dots(x - x_n) \overline{\overline{}} k(x)\omega_{n+1}(x)$

$x[a,b]$ $\varphi(t) = R_n(t) - k(x)\omega_{n+1}(t) \overline{\overline{}} f(t) - L_n(t) - k(x)(t - x_0)(t - x_1)\dots(t - x_n)$

$\varphi n + 2\{x_0, x_1, \dots, x_n, x\} \varphi^{(n+1)}(a,b)\xi, \xi(x) \in (a,b)$

$$\varphi^{(n+1)}(\xi) = f^{(n+1)}(\xi) - (n+1)!k(x) = 0$$

$$k(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}, \xi = \xi(x) \in (a,b)$$

1^{m} $m+$

$$P_{0,1,\ldots,m,l}(x) = P_{0,1,\ldots,m}(x) + \frac{P_{0,1,\ldots,m-1,l}(x) - P_{0,1,\ldots,m}(x)}{x_l - x_m}(x - x_m)$$

(6)

$$P_{0,1,\ldots,m+1}(x) = P_{0,1,\ldots,m}(x) + \frac{P_{1,2,\ldots,m+1}(x) - P_{0,1,\ldots,m}(x)}{x_{m+1} - x_0}(x - x_0)$$

(7)

$i = 0, 1, \ldots, m-1$

$$P_{0,1,\ldots,m,l}(x_i) = P_{0,1,\ldots,m}(x_i) = f(x_i)$$

m $i =$

$$P_{0,1,\ldots,m,l}(x_i) = P_{0,1,\ldots,m}(x_m) = f(x_m)$$

l $i =$

$$P_{0,1,\ldots,m,l}(x_i) = P_{0,1,\ldots,m}(x_l) + \frac{f(x_l) - P_{0,1,\ldots,m}(x_l)}{x_l - x_m}(x_l - x_m) = f(x_l)$$

$$f[x_i] = f(x_i)$$

$$f[x_i, x_{i+1}] = \frac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}$$

k

$$f[x_i, x_{i+1}, \ldots, x_{i+k}] = \frac{f[x_{i+1}, x_{i+2}, \ldots, x_{i+k}] - f[x_i, x_{i+1}, \ldots, x_{i+k-1}]}{x_{i+k} - x_i}$$

Theorem .3

$$f[x_0, x_1, \ldots, x_n] = \sum_{j=0}^{n} \frac{f(x_j)}{(x_j - x_0)(x_j - x_1)\ldots(x_j - x_{j-1})(x_j - x_{j+1})\ldots(x_j - x_n)}$$

$f[x_0, x_1, \ldots, x_n] =$
$f[x_1, x_0, x_2, \ldots, x_n]$
$f[x_0, x_1, \ldots, x_k] =$
$\frac{f[x_0, \ldots, x_{k-2}, x_k] - f[x_0, x_1, \ldots, x_{k-1}]}{x_k - x_{k-1}}$

2.0

f $[a, b]$ n $x_0, x_1, \ldots, x_n \in$
$[a, b]$ $\xi \in$
(a, b)

$$f[x_0, x_1, \ldots, x_n] = \frac{f^{(n)}(\xi)}{n!} \tag{8}$$

$$P_1(x) = f(x_0)+\frac{f(x_1) - f(x_0)}{x_1 - x_0}(x-x_0)$$

$n+1 (x_0, y_0)(x_1, y_1)\ldots(x_n, y_n)$

$$P_n(x) = a_0+a_1(x-x_0)+a_2(x-x_0)(x-x_1)+\ldots+a_n(x-x_0)\ldots(x-x_{n-1}) \tag{9}$$

Theorem .4

$P_n(x)$ n $N_n(x)$

$$N_n(x_i) = f(x_i) \quad (i = 0, 1, \ldots, n) \tag{10}$$

$$N_n(x) = f[x_0]+f[x_0, x_1](x-x_0)+\ldots+f[x_0, \ldots, x_n](x-x_0)\ldots(x-x_{n-1}) \tag{11}$$

$$R_n(x) = f(x)-N_n(x) = f[x, x_0, x_1, \ldots, x_n]\omega_{n+1}(x)$$

$n+1$ $f(x_i)=y_i, f'(x_i)=m_i, (i=0,1,\ldots,n)$ $2n+1$ $H_{2n+1}(x)$ $H_{2n+1}(x)$ $2n+1$ $H_{2n+1}(x_i)=y_i, H'_{2n+1}(x_i)=m_i, (i=0,1,\ldots,n)$

$$\{ \alpha_j(x)=[1-2(x-x_j)\sum_{k=0 k\neq j}^{n}\frac{1}{x_j-x_k}]l_j^2(x)\beta_j(x)=(x-x_j)l_j^2(x)(j=0,1,\ldots,n)$$

(12)

$$H_{2n+1}(x)=\sum_{j=0}^{n}[a_jf(x_j)+\beta_jf'(x_j)]$$

(13)

$$R_{2n+1}(x)=f(x)-H_{2n+1}(x)=\frac{f^{(2n+2)}(\xi)}{(2n+2)!}\omega_{n+1}^2(x)$$

(14)

$n=1$ x_i, x_i+1 $\alpha_i(x)=(1+2\frac{x-x_i}{x_{i+1}-x_i})(\frac{x-x_{i+1}}{x_i-x_{i+1}})^2$

$\alpha_{i+1}(x)=(1+2\frac{x-x_{i+1}}{x_i-x_{i+1}})(\frac{x-x_i}{x_{i+1}-x_i})^2$

$\beta_i(x)=(x-x_i)(\frac{x-x_{i+1}}{x_i-x_{i+1}})^2$

$\beta_{i+1}(x)=(x-x_{i+1})(\frac{x-x_i}{x_{i+1}-x_i})^2$

$H_3(x), \bar{H}_3(x)$ $g(x)=H_3(x)-\bar{H}_3(x)$ $g(x)$ 2 $g(x)\equiv 0$.

$f(x)$ $[a,b]$ $f(x)$ $a=x_0<x_1<\cdots\leq x_n=b$ $f_0, f_1, \ldots, f_n$ $h_k=x_{k+1}-x_k, h=\max_{0kn-1} h_k$ $I_h(x)$ $I_h(x)\in C^0[a,b]$ $I_h(x_k)=f_k (k=0,1,\ldots,n)$ $[x_k, x_{k+1}]$ $I_h(x)$ $I_h(x)$ $f(x)$

$$I_h(x)=\sum_{k=0}^{n}f_kl_{h,k}(x)$$

(15)

$[x_k, x_{k+1}]$

$$I_h(x)=f_k(\frac{x-x_{k+1}}{x_k-x_{k+1}})+f_{k+1}(\frac{x-x_k}{x_{k+1}-x_k})x\in[x_k,x_{k+1}]$$

(16)

$$|R(x)|=|f(x)-I_h(x)|\frac{h^2}{}M$$

$f(x)[a,b]f(x)a = x_0 < x_1 < \cdots \leq x_n = b f_0, f_1, \ldots, f_n f'(x_k) = m_k (k = 0, 1, \ldots, n), h = \max\limits_{0kn-1}(x_{k+1} - x_k) I_h(x)$ $I_h(x) \in C^1[a,b]$ $I_h(x_k) = f_k, I_h'(x_k) = m_k (k = 0, 1, \ldots, n)$ $[x_k, x_{k+1}] I_h(x)$ $I_h(x) f(x)$ **Hermite**

$$I_h(x) = \sum_{k=0}^{n} [\alpha_k(x) f_k + \beta_k(x) m_k] \tag{18}$$

$[x_k, x_{k+1}]$

$$I_h(x) = \alpha_k(x) f_k + \alpha_{k+1}(x) f_{k+1} + \beta_k(x) m_k + \beta_{k+1}(x) m_{k+1} \tag{19}$$

$$|R(x)| = |f(x) - I_h(x)| \frac{h^4}{384} M_4 \tag{20}$$

$M_4 = \max\limits_{x \in [a,b]} \left| f^{(4)}(x) \right|$

Theorem .5

$\|D(f - I_h)\|_\infty \frac{\sqrt{3}}{216} h^3 \|D^4 f\|_\infty$

$\|D^2(f - I_h)\|_\infty \frac{\sqrt{1}}{12} h^2 \|D^4 f\|_\infty$

$\|D^3(f - I_h)\|_\infty \frac{\sqrt{1}}{12} h \|D^4 f\|_\infty$

$D^i (i = 1, 2, 3, 4)$ i $\|f\|_\infty = \max\limits_{x \in [a,b]} |f(x)|$.

$a = x_0 < x_1 < \cdots x_n \leqq b$ $[a, b]$ $s(x)$

$s(x) \in C^2[a, b]$

$[x_k, x_{k+1}]$ $(k = 0, 1, \ldots, n-1)$ $s(x)$

$s(x)$ $x_0, x_1, \ldots, x_n$

$s(x_k) = f(x_k) = y_k$ $(k = 0, 1, \ldots, n)$

$s(x)$

$s(x)$ $4n$ $n+1$ $(n-1)$ $4n-2$

,

$$s'(x_0) = f'_0 = f'(x_0), s'(x_n) = f'_n = f'(x_n) \tag{21}$$

$$s''(x_0) = f''_0 = f''(x_0), s''(x_n) = f''_n = f''(x_n) \tag{22}$$

$$s''(x_0) = s''(x_n) = 0 \tag{23}$$

$f(x)$ $x_n - x_0$

$$s^{(k)}(x_0+0) = s^{(k)}(x_n-0)(k = 0, 1, 2) \tag{24}$$

$h_k = x_{k+1} - x_k s(x)[x_k, x_{k+1}]$

$_{k+1})^2[h_k + 2(x - x_k)] \overline{h_k^3 y_k + \frac{(x-x_k)^2[h_k+2(x_{k+1}-x)]}{h_k^3} y_{k+1} + \frac{(x-x_{k+1})^2(x-x_k)}{h_k^2} m_k + \frac{(x-x_k)^2(x-x_{k+1})}{h_k^2} m_{k+1}}$

$_k = x_{k+1} - x_k$

$f[x_k, x_{k+1}] = \frac{y_{k+1}-y_k}{h_k}$

$\lambda_k = \frac{h_k}{h_{k-1}+h_k}$

$\mu_k = \frac{h_{k-1}}{h_{k-1}+h_k}$

$e_k = 3(\lambda_k f[x_{k-1}, x_k] + \mu_k f[x_k, x_{k+1}])$
$m_0, m_n m_1, m_2, \ldots, m_{n-1} 4n - 1$

$$2\mu_1 0 \ldots 000\lambda_2 2\mu_2 \ldots 0000\lambda_3 2 \ldots 000000 \ldots \lambda_{n-2} 2\mu_{n-2} 000 \ldots 0\lambda_{n-1} 2 m_1 m_2 m_3 m_{n-2} m_{n-1} = e_1 - \lambda_1 f_0' e_2 e_3 e_{n-2} e_{n-1} \quad (25)$$

$e_0 = 3f[x_0, x_1], e_n = 3f[x_{n-1}, x_n]$

$$210 \ldots 000\lambda_1 2\mu_1 \ldots 0000\lambda_2 2 \ldots 000000 \ldots \lambda_{n-1} 2\mu_{n-1} 000 \ldots 012 m_0 m_1 m_2 m_{n-1} m_n = e_0 e_1 e_2 e_{n-1} e_n \quad (26)$$

$$\{ 2\ m_0 + m_1 = 3f[x_0, x_1] - \frac{h_0}{2} f_0'' e_0 m_{n-1} + 2m_n = 3f[x_{n-1}, x_n] + \frac{h_{n-1}}{2} f_n'' e_n \quad (27)$$

$m_0 = m_n, \mu_n m_1 + \lambda_n m_{n-1} + 2m_n = e_n \mu_n = \frac{h_{n-1}}{h_0+h_{n-1}}, \lambda_n = \frac{h_0}{h_0+h_{n-1}}, e_n = 3(\mu_n f[x_0, x_1] + \lambda_n f[x_{n-1}, x_n])$

$$2\mu_1 0 \ldots 00\lambda_1 \lambda_2 2\mu_2 \ldots 000000 \ldots \lambda_{n-1} 2\mu_{n-1} \mu_n 00 \ldots 0\lambda_n 2 m_1 m_2 m_{n-1} m_n = e_1 e_2 e_{n-1} e_n \quad (28)$$