To calculate the neutron production we simply integrate all of the neutrons producing rates on each radii for a sphere. Calculating the neutron production rate on each radii is actually also quite simple. This is the fusion cross section times the energy spectrum of the particles on that radii. The difficult part is in calculation of the energy spectrum. This is done a the next page.

$$NPR = 4\pi \int_0^b \{S_{fi}(r) + S_{fn}(r)\}dr \tag{1}$$

$$S_{fi}(r) = n_{gas} \int_0^{E_{max}} \sigma_f(E)\{f_i^-(r,E) + f_i^+(r,E)\}dE \tag{2}$$

$$= n_{gas}\left\{ \int_0^{E_{max}} \sigma_f(E) f_i^-(r,E)dE + \int_0^{E_{max}} \sigma_f(E) f_i^+(r,E)dE \right\} \tag{3}$$

$$S_{fn}(r) = n_{gas} \int_0^{E_{max}} \sigma_f(E)\{f_n^-(r,E) + f_n^+(r,E)\}dE \tag{4}$$

$$= n_{gas}\left\{ \int_0^{E_{max}} \sigma_f(E) f_n^-(r,E)dE + \int_0^{E_{max}} \sigma_f(E) f_n^+(r,E)dE \right\} \tag{5}$$

$$f_n^-(r,E) = f_{n1}^-(r,E) + f_{n2}^-(r,E) \tag{6}$$

$$f_n^+(r,E) = f_{n1}^+(r,E) + f_{n2}^+(r,E) \tag{7}$$

so

$$S_{fn}(r) = n_{gas}\left\{ \int_0^{E_{max}} \sigma_f(E) f_{n1}^-(r,E)dE + \int_0^{E_{max}} \sigma_f(E) f_{n1}^+(r,E)dE + \int_0^{E_{max}} \sigma_f(E) f_{n2}^-(r,E)dE + \int_0^{E_{max}} \sigma_f(E) f_{n2}^+(r,E)dE \right\} \tag{8}$$

On the next page all $f_x^\pm$ functions are displayed. There is however a catch.

$$S_{fn} = n_{gas}\{S_{fn1}^- + S_{fn1}^+ + S_{fn2}^- + S_{fn1}^+\} \tag{9}$$

with:

$$S_{fn1}^- = \int_0^{E_{max}} \sigma_f(E) f_{n1}^-(r,E)dE = \begin{cases} S_{fn1,a<r}^- = \int_0^{E_{max}} \sigma_f(E) f_{n1,a<r}^-(r,E)dE & \text{for } a<r \\ S_{fn1,r<a}^- = \int_0^{E_{max}} \sigma_f(E) f_{n1,r<a}^-(r,E)dE & \text{for } r>a \end{cases} \tag{10}$$

$$S_{fn1}^+ = \int_0^{E_{max}} \sigma_f(E) f_{n1}^+(r,E)dE = \begin{cases} S_{fn1,a<r}^+ = \int_0^{E_{max}} \sigma_f(E) f_{n1,a<r}^+(r,E)dE & \text{for } a<r \\ S_{fn1,r<a}^+ = \int_0^{E_{max}} \sigma_f(E) f_{n1,r<a}^+(r,E)dE & \text{for } r>a \end{cases} \tag{11}$$

$$S_{fn2}^- = \int_0^{E_{max}} \sigma_f(E) f_{n2}^-(r,E)dE = \begin{cases} S_{fn2,a<r}^- = \int_0^{E_{max}} \sigma_f(E) f_{n2,a<r}^-(r,E)dE & \text{for } a<r \\ S_{fn2,r<a}^- = \int_0^{E_{max}} \sigma_f(E) f_{n2,r<a}^-(r,E)dE & \text{for } r>a \end{cases} \tag{12}$$

$$S_{fn2}^+ = \int_0^{E_{max}} \sigma_f(E) f_{n2}^+(r,E)dE = \begin{cases} S_{fn2,a<r}^+ = \int_0^{E_{max}} \sigma_f(E) f_{n2,a<r}^+(r,E)dE & \text{for } a<r \\ S_{fn2,r<a}^+ = \int_0^{E_{max}} \sigma_f(E) f_{n2,r<a}^+(r,E)dE & \text{for } r>a \end{cases} \tag{13}$$

$$\tag{14}$$

from equation 2 we have:

$$S_{fi}^- = \int_0^{E_{max}} \sigma_f(E) f_i^-(r,E)dE = \begin{cases} S_{fi,a<r}^- = \int_0^{E_{max}} \sigma_f(E) f_{i,a<r}^-(r,E)dE & \text{for } a<r \\ S_{fi,r<a}^- = \int_0^{E_{max}} \sigma_f(E) f_{i,r<a}^-(r,E)dE & \text{for } r>a \end{cases} \tag{15}$$

$$S_{fi}^+ = \int_0^{E_{max}} \sigma_f(E) f_i^+(r,E)dE = \begin{cases} S_{fi,a<r}^+ = \int_0^{E_{max}} \sigma_f(E) f_{i,a<r}^+(r,E)dE & \text{for } a<r \\ S_{fi,r<a}^+ = \int_0^{E_{max}} \sigma_f(E) f_{i,r<a}^+(r,E)dE & \text{for } r>a \end{cases} \tag{16}$$

$$\tag{17}$$

Inward traveling ions: $a < r$, (eq 28)

$$f^{-}_{i,a<r}(r,E) = \frac{1}{q} \quad \frac{r'^2}{r^2} \quad S(r') \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[\frac{1}{1-T_c^2 g_{cp}(r')}\right] \quad \left[g(r,r')\right] \quad + \quad \frac{b^2\Gamma_0}{r^2} \quad f(r) \quad \delta[E-E_0-q\phi(r)] \tag{18}$$

Inward traveling ions: $r < a$, (eq 30)

$$f^{-}_{i,r<a}(r,E) = \frac{1}{q} \quad \frac{r'^2}{r^2} \quad T_c \quad S(r') \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[\frac{1}{1-T_c^2 g_{cp}(r')}\right] \quad \left[g(a,r')\exp[n_g\sigma_{cs}[E(a,r')](r-a)]\right] \quad +T_c \quad \frac{b^2\Gamma_0}{r^2} \quad f(a)\exp[n_g\sigma_{cx}[E(a)](r-a)] \quad \delta[E-E_0-q\phi(r)] \tag{19}$$

Outward traveling ions: $r < a$, (eq 31)

$$f^{+}_{i,a<r}(r,E) = \frac{1}{q} \quad \frac{r'^2}{r^2} \quad T_c \quad S(r') \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[\frac{g_{cp}(r')}{1-T_c^2 g_{cp}(r')}\right] \quad \left[\frac{1}{g(a,r')\exp[n_g\sigma_{cs}[E(a,r')](r-a)]}\right] \quad +T_c \quad \frac{b^2\Gamma_0}{r^2} \quad \frac{f^2(0)}{f(a)\exp[n_g\sigma_{cx}[E(a)](r-a)]} \quad \delta[E-E_0-q\phi(r)] \tag{20}$$

Outward traveling ions: $a < r$, (eq 29)

$$f^{+}_{i,r<a}(r,E) = \frac{1}{q} \quad \frac{r'^2}{r^2} \quad T_c^2 \quad S(r') \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[\frac{g_{cp}(r')}{1-T_c^2 g_{cp}(r')}\right] \quad \left[\frac{1}{g(r,r')}\right] \quad +T_c^2 \quad \frac{b^2\Gamma_0}{r^2} \quad \frac{f^2(0)}{f(r)} \quad \delta[E-E_0-q\phi(r)] \tag{21}$$

Inward traveling neutrals from Class I: $a < r < r' < b$, (eq 40)

$$f^{-}_{n1,a<r}(r,E) = \frac{1}{q} \quad \frac{b^2}{r^2} \quad n_g\sigma_{cx}(E) \quad \Gamma_0 \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[f(r')\Theta(r'-r)\right] \tag{22}$$

Inward traveling neutrals from Class I: $r < a < r' < b$, (eq 47)

$$f^{-}_{n1,r<a}(r,E) = \frac{1}{q} \quad \frac{b^2}{r^2} \quad n_g\sigma_{cx}(E) \quad T_c \quad \Gamma_0 \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[f(r')\right] \quad +T_c \quad \frac{b^2\Gamma_0}{r^2} \quad f(a)\left[1-\exp[-2n_g\sigma_{cx}(E_{max})a]\right] \quad \delta(E-E_{max}) \tag{23}$$

Outward traveling neutrals from Class I: $r < a < r' < b$, (eq 51)

$$f^{+}_{n1,a<r}(r,E) = \frac{1}{q} \quad \frac{b^2}{r^2} \quad n_g\sigma_{cx}(E) \quad T_c \quad \Gamma_0 \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[f(r')\right] \quad +T_c \quad \frac{b^2\Gamma_0}{r^2} \quad f(a)\left[1-\exp[2n_g\sigma_{cx}(E_{max})(r-a)]\right] \quad \delta(E-E_{max}) \tag{24}$$

Outward traveling neutrals from Class I: $a < r < r' < b$, (eq 55)

$$f^{+}_{n1,r<a}(r,E) = \frac{1}{q} \quad \frac{b^2}{r^2} \quad n_g\sigma_{cx}(E) \quad T_c^2 \quad \Gamma_0 \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[f(r') + \frac{f^2(0)}{f(r')}\Theta(r'-r)\right] \quad +T_c^2 \quad \frac{b^2\Gamma_0}{r^2} \quad f(a)\left[1-\exp[-2n_g\sigma_{cx}(E_{max})a]\right] \quad \delta(E-E_{max}) \tag{25}$$

Inward traveling neutrals from Class II: $a < r < b$, (eq 58)

$$f^{-}_{n2,a<r}(r,E) = \frac{1}{q} \quad \frac{1}{r^2} \quad n_g\sigma_{cx}(E) \quad \int_r^b \quad S(r'') \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[\frac{g(r',r'')}{1-T_c^2 g_{cp}(r'')}\right] \quad \left[\Theta(r-r')\right] \quad r''^2 dr'' \tag{26}$$

Inward traveling neutrals from Class II: $r < a$, (eq 65)

$$f^{-}_{n2,r<a}(r,E) = \frac{1}{q} \quad \frac{1}{r^2} \quad n_g\sigma_{cx}(E) \quad T_c \quad \int_a^b \quad S(r'') \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[\frac{g(r',r'')}{1-T_c^2 g_{cp}(r'')}\right] \quad \left[\Theta(r'-a)\right] \quad r''^2 dr'' \quad +T_c \quad \frac{r''^2}{r^2}\left[\frac{g(a,r'')}{1-T_c^2 g_{cp}(r'')}\right] \quad \left[1-\exp[n_g\sigma_{cx}(E)(r-a)]\right] \quad \frac{S(r'')}{q\left|\frac{\partial\phi}{\partial r'}\right|} \tag{27}$$

Outward traveling neutrals from Class II: $r < a$, (eq 68)

$$f^{+}_{n2,a<r}(r,E) = \frac{1}{q} \quad \frac{1}{r^2} \quad n_g\sigma_{cx}(E) \quad T_c \quad \int_a^b \quad S(r'') \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[\frac{g(r',r'')}{1-T_c^2 g_{cp}(r'')}\right] \quad \left[\Theta(r'-a)\right] \quad r''^2 dr'' \quad +T_c \quad \frac{r''^2}{r^2}\left[\frac{g(a,r'')}{1-T_c^2 g_{cp}(r'')}\right] \quad \left[1-\exp[-n_g\sigma_{cx}(E)(r+a)]\right] \quad \frac{S(r'')}{q\left|\frac{\partial\phi}{\partial r'}\right|} \tag{28}$$

Outward traveling neutrals from Class II: $a < r$ (eq 72)

$$f^{+}_{n2,r<a}(r,E) = \frac{1}{q} \quad \frac{1}{r^2} \quad n_g\sigma_{cx}(E) \quad T_c^2 \quad \int_a^b \quad S(r'') \quad \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \quad \left[\frac{1}{1-T_c^2 g_{cp}(r'')}\right] \quad \left[g(r',r'')\Theta(r'-a) + \frac{g_{cp}(r'')}{g(r',r'')}\Theta(r-r')\right] \quad r''^2 dr'' \quad +T_c^2 \quad \frac{r''^2}{r^2}\left[\frac{g(a,r'')}{1-T_c^2 g_{cp}(r'')}\right] \quad \left[1-\exp[-2n_g\sigma_{cx}(E)a]\right] \quad \frac{S(r'')}{q\left|\frac{\partial\phi}{\partial r'}\right|} \tag{29}$$

for all equations r' is related to E via $E(r') = q\phi(r')$.
In the last 4 equations r' is determined by r" and E. Equation 56 for the first inward going ions and equation 62 for the out going ions.

The potential inside the fusor is divined as:

$$\phi(r) = \begin{cases} V_0 & \text{for } r < a \\ V_0 \frac{a(b-r)}{r(b-a)} & \text{for } a < r \end{cases} \tag{30}$$

the equation to calculate r' (eq :

$$E(r, r') = q(\phi(r') - \phi(r)) \tag{31}$$

the equation to calculate r'' eq 62):

$$E = q(\phi(r'') - \phi(a)) \tag{32}$$

So when we want to know the radius where the particles are born we need the inverse of ϕ.

$$r = \phi^{-1}(E) = \begin{cases} V_0 & \text{for } r < a \\ \frac{ab}{\frac{E}{V_0}(a-b)-a} & \text{for } a < r \end{cases} \tag{33}$$

Inward traveling ions: $a < r$

$$S^-_{fi,a<r}(r) = \int_0^{E_{max}} \sigma_f(E) f^-_{i,a<r}(r,E)dE \tag{34}$$

$$= \int_0^{E_{max}} \sigma_f(E) \Bigg\{ \frac{1}{q} \frac{r'^2}{r^2} S(r') \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[\frac{1}{1 - T_c^2 g_{cp}(r')}\right] \Big[g_(r, r')\Big] + \sigma_f(E) \frac{b^2\Gamma_0}{r^2} f(r) \delta[E - E_0 - q\phi(r)] \Bigg\} dE \tag{35}$$

$$= \int_0^{E_{max}} \sigma_f(E) \frac{1}{q} \frac{r'^2}{r^2} S(r') \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[\frac{1}{1 - T_c^2 g_{cp}(r')}\right] \Big[g_(r, r')\Big] dE + \int_0^{E_{max}} \sigma_f(E) \frac{b^2\Gamma_0}{r^2} f(r) \delta[E - E_0 - q\phi(r)] \, dE \tag{36}$$

$$= \int_r^b \sigma_f(E) \frac{1}{q} \frac{r'^2}{r^2} S(r') \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[\frac{1}{1 - T_c^2 g_{cp}(r')}\right] \Big[g_(r, r')\Big] q\left|\frac{\partial\phi(r')}{\partial r'}\right| dr' + \frac{b^2\Gamma_0}{r^2} f(r) \sigma_f(q\phi(r)) \tag{37}$$

$$= \int_r^b \sigma_f(E) \frac{r'^2}{r^2} S(r') \left[\frac{1}{1 - T_c^2 g_{cp}(r')}\right] \Big[g_(r, r')\Big] dr' + \frac{b^2\Gamma_0}{r^2} f(r) \sigma_f(q\phi(r)) \tag{38}$$

Inward traveling ions: $r < a$

$$S^-_{fi,r<a}(r) = \int_0^{E_{max}} \sigma_f(E) f^-_{i,r<a}(r,E)dE \tag{39}$$

$$= \int_0^{E_{max}} \sigma_f(E) \Bigg\{ \frac{1}{q} \frac{r'^2}{r^2} T_c S(r') \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[\frac{1}{1 - T_c^2 g_{cp}(r')}\right] \Big[g(a, r')\exp[n_g\sigma_{cs}[E(a,r')](r-a)]\Big] + T_c \frac{b^2\Gamma_0}{r^2} f(a)\exp[n_g\sigma_{cx}[E(a)](r-a)] \delta[E - E_0 - q\phi(r)] \Bigg\} dE \tag{40}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{r'^2}{r^2} T_c S(r') \left[\frac{1}{1 - T_c^2 g_{cp}(r')}\right] \Big[g(a, r')\exp[n_g\sigma_{cs}[E(a,r')](r-a)]\Big] dr' + T_c \frac{b^2\Gamma_0}{r^2} f(a)\exp[n_g\sigma_{cx}[E(a)](r-a)] \sigma_f(q\phi(r)) \tag{41}$$

Outward traveling ions: $r < a$

$$S^+_{fi,r<a}(r) = \int_0^{E_{max}} \sigma_f(E) f^+_{i,r<a}(r,E)dE \tag{42}$$

$$= \int_0^{E_{max}} \sigma_f(E) \Bigg\{ \frac{1}{q} \frac{r'^2}{r^2} T_c S(r') \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[\frac{g_{cp}(r')}{1 - T_c^2 g_{cp}(r')}\right] \left[\frac{1}{g(a, r')\exp[n_g\sigma_{cs}[E(a,r')](r-a)]}\right] + T_c \frac{b^2\Gamma_0}{r^2} \frac{f^2(0)}{f(a)\exp[n_g\sigma_{cx}[E(a)](r-a)]} \delta[E - E_0 - q\phi(r)] \Bigg\} dE \tag{43}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{r'^2}{r^2} T_c S(r') \left[\frac{g_{cp}(r')}{1 - T_c^2 g_{cp}(r')}\right] \left[\frac{1}{g(a, r')\exp[n_g\sigma_{cs}[E(a,r')](r-a)]}\right] dr' + T_c \frac{b^2\Gamma_0}{r^2} \frac{f^2(0)}{f(a)\exp[n_g\sigma_{cx}[E(a)](r-a)]} \sigma_f(q\phi(r)) \tag{44}$$

Outward traveling ions: $a < r$, (eq 29)

$$S^+_{fi,a<r}(r) = \int_0^{E_{max}} \sigma_f(E) f^+_{i,a<r}(r,E)dE \tag{45}$$

$$= \int_0^{E_{max}} \sigma_f(E) \Bigg\{ \frac{1}{q} \frac{r'^2}{r^2} T_c^2 S(r') \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[\frac{g_{cp}(r')}{1 - T_c^2 g_{cp}(r')}\right] \left[\frac{1}{g(r, r')}\right] + T_c^2 \frac{b^2\Gamma_0}{r^2} \frac{f^2(0)}{f(r)} \delta[E - E_0 - q\phi(r)] \Bigg\} dE \tag{46}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{r'^2}{r^2} T_c^2 S(r') \left[\frac{g_{cp}(r')}{1 - T_c^2 g_{cp}(r')}\right] \left[\frac{1}{g(r, r')}\right] dr' + T_c^2 \frac{b^2\Gamma_0}{r^2} \frac{f^2(0)}{f(r)} \sigma_f(q\phi(r)) \tag{47}$$

Inward traveling neutrals from Class I: $a < r < r' < b$

$$S^-_{fn1,a<r}(r) = \int_0^{E_{max}} \sigma_f(E) f^-_{n1,a<r}(r,E)dE \tag{48}$$

$$= \int_0^{E_{max}} \sigma_f(E) \frac{1}{q} \frac{b^2}{r^2} n_g\sigma_{cx}(E) \Gamma_0 \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \Big[f(r')\Theta(r' - r)\Big] dE \tag{49}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{b^2}{r^2} n_g\sigma_{cx}(q\phi(r')) \Gamma_0 \Big[f(r')\Theta(r' - r)\Big] dr' \tag{50}$$

Inward traveling neutrals from Class I: $r < a < r' < b$

$$S^-_{fn1,r<a}(r) = \int_0^{E_{max}} \sigma_f(E) f^-_{n1,r<a}(r,E)dE \tag{51}$$

$$= \int_0^{E_{max}} \sigma_f(E) \Bigg\{ \frac{1}{q} \frac{b^2}{r^2} n_g\sigma_{cx}(E) T_c \Gamma_0 \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \Big[f(r')\Big] + T_c \frac{b^2\Gamma_0}{r^2} f(a)\Big[1 - \exp[-2n_g\sigma_{cx}(E_{max})a]\Big] \delta(E - E_{max}) \Bigg\} dE \tag{52}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{b^2}{r^2} n_g\sigma_{cx}(q\phi(r')) T_c \Gamma_0 \Big[f(r')\Big] dr' + T_c \frac{b^2\Gamma_0}{r^2} f(a)\Big[1 - \exp[-2n_g\sigma_{cx}(E_{max})a]\Big] \sigma_f(q\phi(r)) \tag{53}$$

Outward traveling neutrals from Class I: $r < a < r' < b$

$$S^-_{fn1,r<a}(r) = \int_0^{E_{max}} \sigma_f(E) f^+_{n1,a<r}(r,E)dE \tag{54}$$

$$= \int_0^{E_{max}} \sigma_f(E) \Bigg\{ \frac{1}{q} \frac{b^2}{r^2} n_g\sigma_{cx}(E) T_c \Gamma_0 \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \Big[f(r')\Big] + T_c \frac{b^2\Gamma_0}{r^2} f(a)\Big[1 - \exp[2n_g\sigma_{cx}(E_{max})(r-a)]\Big] \delta(E - E_{max}) \Bigg\} dE \tag{55}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{b^2}{r^2} n_g\sigma_{cx}(q\phi(r')) T_c \Gamma_0 \Big[f(r')\Big] + T_c \frac{b^2\Gamma_0}{r^2} f(a)\Big[1 - \exp[2n_g\sigma_{cx}(E_{max})(r-a)]\Big] \sigma_f(q\phi(r)) \tag{56}$$

Outward traveling neutrals from Class I: $a < r < r' < b$

$$S^-_{fn1,a<r}(r) = \int_0^{E_{max}} \sigma_f(E) f^+_{n1,r<a}(r,E)dE \tag{57}$$

$$= \int_0^{E_{max}} \sigma_f(E) \frac{1}{q} \frac{b^2}{r^2} n_g\sigma_{cx}(E) T_c^2 \Gamma_0 \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[f(r') + \frac{f^2(0)}{f(r')}\Theta(r' - r)\right] + T_c^2 \frac{b^2\Gamma_0}{r^2} f(a)\Big[1 - \exp[-2n_g\sigma_{cx}(E_{max})a]\Big] \delta(E - E_{max}) \tag{58}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{b^2}{r^2} n_g\sigma_{cx}(q\phi(r')) T_c^2 \Gamma_0 \left[f(r') + \frac{f^2(0)}{f(r')}\Theta(r' - r)\right] dE + T_c^2 \frac{b^2\Gamma_0}{r^2} f(a)\Big[1 - \exp[-2n_g\sigma_{cx}(E_{max})a]\Big] \sigma_f(q\phi(r)) \tag{59}$$

Inward traveling neutrals from Class II: $a < r < b$

$$S^{-}_{fn2,a<r}(r) = \int_0^{Emax} \sigma_f(E) f^{-}_{n2,a<r}(r,E)dE \tag{60}$$

$$= \int_0^{Emax} \sigma_f(E) \frac{1}{q} \frac{1}{r^2} n_g\sigma_{cx}(E) \int_r^b S(r'') \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[\frac{g(r',r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[\Theta(r-r')\Big] r''^2 dr'' dE \tag{61}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{1}{r^2} n_g\sigma_{cx}(q\phi(r')) \int_r^b S(r'') \left[\frac{g(r',r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[\Theta(r-r')\Big] r''^2 dr'' dr' \tag{62}$$

Inward traveling neutrals from Class II: $r < a$

$$S^{-}_{fn2,r<a}(r) = \int_0^{Emax} \sigma_f(E) f^{-}_{n2,r<a}(r,E)dE \tag{63}$$

$$= \int_0^{Emax} \sigma_f(E) \left\{ \frac{1}{q} \frac{1}{r^2} n_g\sigma_{cx}(E)\, T_c \int_a^b S(r'') \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[\frac{g(r',r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[\Theta(r'-a)\Big] r''^2 dr'' + T_c \frac{r''^2}{r^2}\left[\frac{g(a,r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[1-\exp[n_g\sigma_{cx}(E)(r-a)]\Big] \frac{S(r'')}{q\left|\frac{\partial\phi}{\partial r'}\right|} \right\} dE \tag{64}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{1}{r^2} n_g\sigma_{cx}(q\phi(r'))\, T_c \int_a^b S(r'') \left[\frac{g(r',r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[\Theta(r'-a)\Big] r''^2 dr'' + T_c \int_0^{Emax} \sigma_f(E) \frac{r''^2}{r^2}\left[\frac{g(a,r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[1-\exp[n_g\sigma_{cx}(E)(r-a)]\Big] \left.\frac{S(r'')}{q\left|\frac{\partial\phi}{\partial r'}\right|} \right\} dE \tag{65}$$

Outward traveling neutrals from Class II: $r < a$

$$S^{-}_{fn2,r<a}(r) = \int_0^{Emax} \sigma_f(E) f^{+}_{n2,a<r}(r,E)dE \tag{66}$$

$$= \int_0^{Emax} \sigma_f(E) \left\{ \frac{1}{q} \frac{1}{r^2} n_g\sigma_{cx}(E)\, T_c \int_a^b S(r'') \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[\frac{g(r',r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[\Theta(r'-a)\Big] r''^2 dr'' + T_c \frac{r''^2}{r^2}\left[\frac{g(a,r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[1-\exp[-n_g\sigma_{cx}(E)(r+a)]\Big] \frac{S(r'')}{q\left|\frac{\partial\phi}{\partial r'}\right|} \right\} dE \tag{67}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{1}{r^2} n_g\sigma_{cx}(q\phi(r'))\, T_c \int_a^b S(r'') \left[\frac{g(r',r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[\Theta(r'-a)\Big] r''^2 dr'' + T_c \int_0^{Emax} \sigma_f(E) \frac{r''^2}{r^2}\left[\frac{g(a,r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[1-\exp[-n_g\sigma_{cx}(E)(r+a)]\Big] \frac{S(r'')}{q\left|\frac{\partial\phi}{\partial r'}\right|} dE \tag{68}$$

Outward traveling neutrals from Class II: $a < r$

$$S^{-}_{fn2,r<a}(r) = \int_0^{Emax} \sigma_f(E) f^{+}_{n2,r<a}(r,E)dE \tag{69}$$

$$= \int_0^{Emax} \sigma_f(E) \left\{ \frac{1}{q} \frac{1}{r^2} n_g\sigma_{cx}(E)\, T_c^2 \int_a^b S(r'') \frac{1}{\left|\frac{\partial\phi(r')}{\partial r'}\right|} \left[\frac{1}{1-T_c^2 g_{cp}(r'')}\right] \left[g(r',r'')\Theta(r'-a) + \frac{g_{cp}(r'')}{g(r',r'')}\Theta(r-r')\right] r''^2 dr'' + T_c^2 \frac{r''^2}{r^2}\left[\frac{g(a,r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[1-\exp[-2n_g\sigma_{cx}(E)a]\Big] \frac{S(r'')}{q\left|\frac{\partial\phi}{\partial r'}\right|} \right\} dE \tag{70}$$

$$= \int_r^b \sigma_f(q\phi(r')) \frac{1}{r^2} n_g\sigma_{cx}(q\phi(r'))\, T_c^2 \int_a^b S(r'') \left[\frac{1}{1-T_c^2 g_{cp}(r'')}\right] \left[g(r',r'')\Theta(r'-a) + \frac{g_{cp}(r'')}{g(r',r'')}\Theta(r-r')\right] r''^2 dr'' dr' + T_c^2\, \sigma_f(E) \frac{r''^2}{r^2}\left[\frac{g(a,r'')}{1-T_c^2 g_{cp}(r'')}\right] \Big[1-\exp[-2n_g\sigma_{cx}(E)a]\Big] \frac{S(r'')}{q\left|\frac{\partial\phi}{\partial r'}\right|} dE \tag{71}$$

because we know:

$$dE = q\left|\frac{\partial\phi(r')}{\partial r'}\right|dr' \tag{72}$$

In the case of the ion spectrum inside the cathode, due to the constant potential in the cathode to following subsitude can be made:

$$f(r) = f(a)\exp[n_g \sigma_{cx}[E(a)](r-a)] \tag{73}$$
$$g(r,r') = g(a,r')\exp[n_g \sigma_{cs}[E(a,r')](r-a)] \tag{74}$$

$$f(r) = exp\bigg\{ - \int_r^b n_{gas}\sigma_{cx}[E(r')]dr' \bigg\} \tag{75}$$
$$g(r,r') = exp\bigg\{ - \int_r^{r'} n_{gas}\sigma_{cx}[E(r", r')]dr" \bigg\} \tag{76}$$

The source rate:

$$S(r) = A(r) + \int_r^b K(r,r')S('r)dr' \tag{77}$$

Where:

$$A(r) = \frac{b^2}{r^2} n_{gas}\Gamma_0 \sigma_{tot}[E(r)]\bigg[f(r) + T_c^2 \frac{f^2(0)}{f(r)} \bigg] \tag{78}$$

and

$$K(r,r') = n_{gas}\sigma_{tot}[E(r,r)]\frac{r'^2}{r^2}\bigg[g(r,r') + T_c^2 \frac{g_{cp}(r')}{g(r,r')} \bigg] \frac{1}{1 - T_c^2 g_{cp}(r')} \tag{79}$$

for $r < r'$ otherwise $K(r,r') = 0$.

The last part that remains is the ionflux at the anode. This can be calculated by the current:

$$I_{c1} = 4\pi q \quad (1-T_c) \quad \Gamma_0 \quad b^2 \quad \bigg[f(a) + T_c \frac{f^2(0)}{f(a)} \bigg] \quad (1+\gamma(qV_0)) \tag{80}$$
$$I_{c2} = 4\pi q \quad (1-T_c) \quad \int_a^b \quad \frac{S(r')}{1-T_c^2 g_{cp}(r')} \quad \bigg[g(a,r') + T_c \frac{g_{cp}(r')}{g(a,r')} \bigg] \quad (1+\gamma(r')) \quad r'^2 dr' \tag{81}$$
$$I_{c3} = 4\pi q \quad T_c \quad \Gamma_0 \quad n_{gas} \quad \frac{\sigma_{tot}[E_{max}]}{\sigma_{cx}[E_{max}]} \quad b^2 \quad \bigg[f(a) \bigg] \quad (1 - exp(-2n_{gas}\sigma_{cx}(E_{max})a)) \tag{82}$$
$$I_{c4} = 4\pi q \quad T_c \quad n_{gas} \quad \int_a^b \quad \frac{\sigma_{tot}[E(a,r')]}{\sigma_{cx}[E(a,r')]} \quad \frac{S(r')}{1-T_c^2 g_{cp}(r')} \quad \bigg[g(a,r') \bigg] \quad (1 - exp[-2n_{gas}\sigma_{cx}[E(a,r')a]) \quad r'^2 dr' \tag{83}$$

The Edge Ion flux is then:

$$\Gamma_0 = \frac{I_{tot} - I_{c2} - I_{c4}}{I_{c1} + I_{c3}} \tag{84}$$