

$$U_{i,rep}(\sigma_i^o) = \begin{cases} \frac{1}{2} n_i \left( \frac{1}{\|\sigma_i^o - \pi(\sigma_i^o)\|} - \frac{1}{b} \right)^2, & \|\sigma_i^o - \pi(\sigma_i^o)\| \leq b \\ 0, & \|\sigma_i^o - \pi(\sigma_i^o)\| > b \end{cases}$$

$$F_{i,rep}(\sigma_i^o) = - \nabla U_{i,rep}(\sigma_i^o) = \begin{cases} n_i \left( \frac{1}{\|\sigma_i^o - \pi(\sigma_i^o)\|} - \frac{1}{b} \right) \frac{\sigma_i^o - \pi(\sigma_i^o)}{\|\sigma_i^o - \pi(\sigma_i^o)\|^3} \\ 0 \end{cases}$$

$$\|\sigma_i^o - \pi(\sigma_i^o)\| = \max\{0, (\sigma_i^o - p^o)^T \vec{n}\}$$

$$\sigma_i^o - \pi(\sigma_i^o) = \|\sigma_i^o - \pi(\sigma_i^o)\| \vec{n}$$

$$F_{i,rep}(\sigma_i^o) = n_i \left( \frac{1}{\|\sigma_i^o - \pi(\sigma_i^o)\|} - \frac{1}{b} \right) \frac{1}{\|\sigma_i^o - \pi(\sigma_i^o)\|^2} \vec{n}, \text{ if } \|\sigma_i^o - \pi(\sigma_i^o)\| \leq b$$

and  $\|\sigma_i^o - \pi(\sigma_i^o)\| > b$

For workspace plane,  $\vec{n} = (0, 0, 1)^T$ ,  $p^o = (0, 0, 32)$

For a cylinder with radius  $R$



$$d_{\perp} = \max\{0, (\sigma_i^o - a^o)^T \vec{n}, -(\sigma_i^o - b^o)^T \vec{n}\}$$

$$d_{||} = \max\{0, \|\sigma_i^o - a^o\| - (\sigma_i^o - a^o)^T \vec{n} \cdot \vec{n} \| - R\}$$

$$\|\sigma_i^o - \pi(\sigma_i^o)\| = \sqrt{d_{\perp}^2 + d_{||}^2}, \quad \sigma_i^o - \pi(\sigma_i^o) = \max\{0, (\sigma_i^o - a^o)^T \vec{n}\} \vec{n} + \max\{0, -(\sigma_i^o - b^o)^T \vec{n}\} (-\vec{n}) + \frac{(\sigma_i^o - a^o) - (\sigma_i^o - a^o)^T \vec{n} \cdot \vec{n}}{\|\sigma_i^o - a^o\| - (\sigma_i^o - a^o)^T \vec{n} \cdot \vec{n}} d_{||}$$

$$a^0 = (a_x, a_y, h)^T, \quad b(a_x', a_y', 0)^T, \quad n = (0, 0, 1)^T$$

$$\Rightarrow d_{\perp} = \max\{0, (\sigma_i^0 - a^0)^T n\}$$

$$d_{\parallel} = \max\{0, |(\sigma_i^0 - a^0) - (\sigma_i^0 - a^0)^T n \cdot n| - R\}$$

$$\|\sigma_i^0 - \pi(\sigma_i^0)\| = \sqrt{d_{\perp}^2 + d_{\parallel}^2}$$

$$\sigma_i^0 - \bar{\kappa}(\sigma_i^0) = d_{\perp} \cdot n + d_{\parallel} \frac{(\sigma_i^0 - a^0) - (\sigma_i^0 - a^0)^T n \cdot n}{\|(\sigma_i^0 - a^0) - (\sigma_i^0 - a^0)^T n \cdot n\|}$$