ECE557 Lab4 Report

Introduction

In this lab session, we continue our exploration of the cart-pendulum system, an interesting challenge in robotics and control theory. Building upon our work from Lab 3, we shifted our focus to developing an output feedback controller. This advanced controller enables the cart to oscillate between two fixed points, effectively tracking a square wave signal, while maintaining the pendulum in a near-upright position. Previously, we developed the Matlab function 'cartpend.m' to simulate the nonlinear differential equations of the system and integrated this function with Simulink for dynamic modelling. Our efforts also included the design of state feedback controllers using eigenvalue assignment and linear-quadratic regulator (LQR) techniques, with the aim of stabilizing the system at a central equilibrium point with the pendulum upright.

This lab's objectives are multifaceted and expand on our previous work. Firstly, we will modify the state feedback controllers from Lab 3 to make the cart track a square wave trajectory while keeping the pendulum upright. Secondly, we plan to design an observer for the linearized model, which will serve as a foundation for developing an output feedback controller suitable for square wave tracking. This controller will be simulated in Simulink using the nonlinear model and implemented on an Arduino for experimental testing with the physical cart-pendulum system. Additionally, we will design and test an output feedback integral controller, assessing its performance compared to the previously developed controller. This lab represents a critical step in advancing our understanding and control of dynamic systems, combining theoretical design, simulation, and practical application.

Block diagram

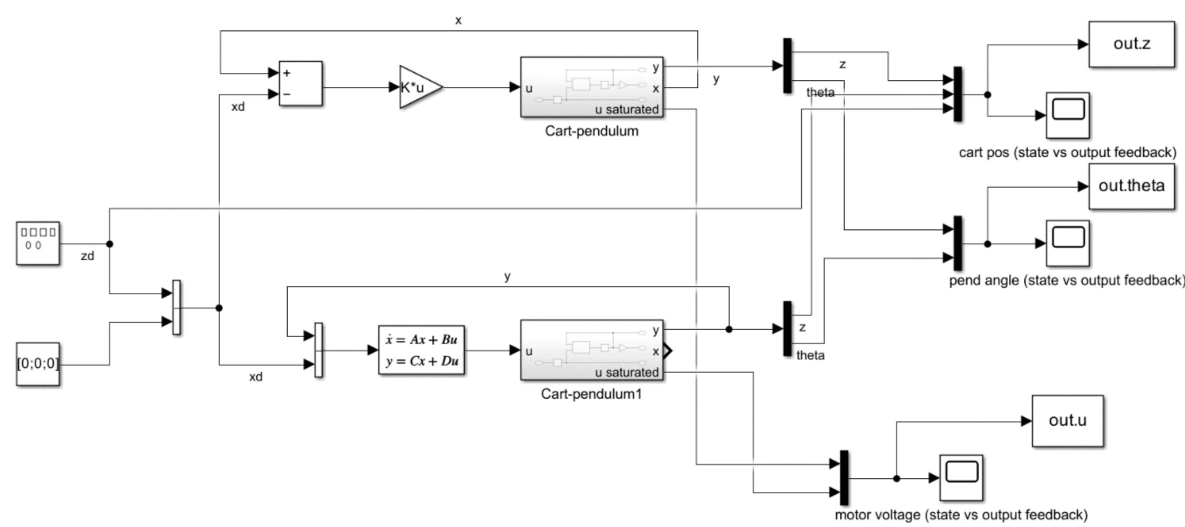

Figure 1: Block diagram for the cart pendulum system

Lab Results

Output 1

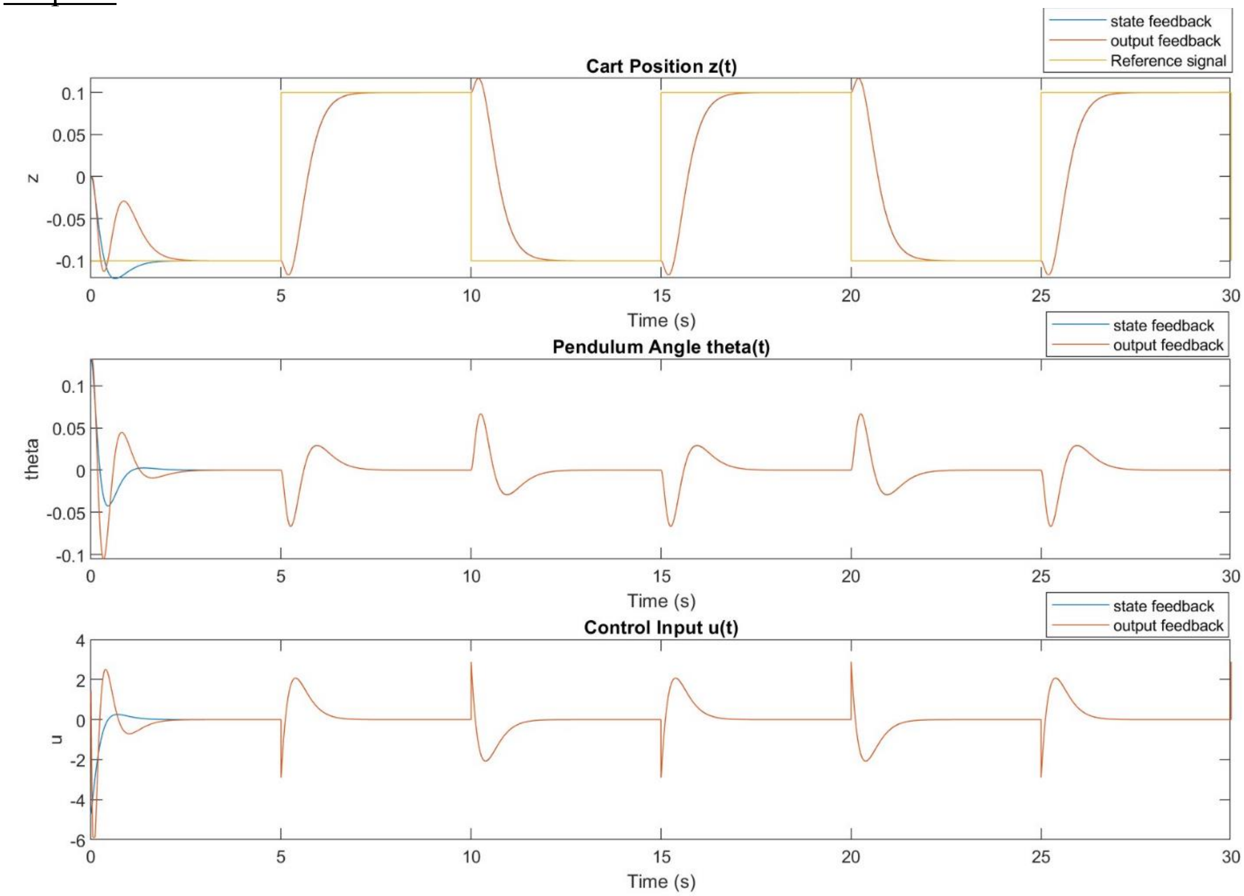

Figure 2: Pole assignment controller with eigenvalues of A + BK at -5 and A − LC at -10.

In the graph, both approaches exhibit rapid convergence. Notably, output feedback initially shows a slightly higher overshoot, resulting in an elevated controller input in the initial phase. However, this difference is only discernible in the first two seconds and rapidly diminishes after the initial loop.

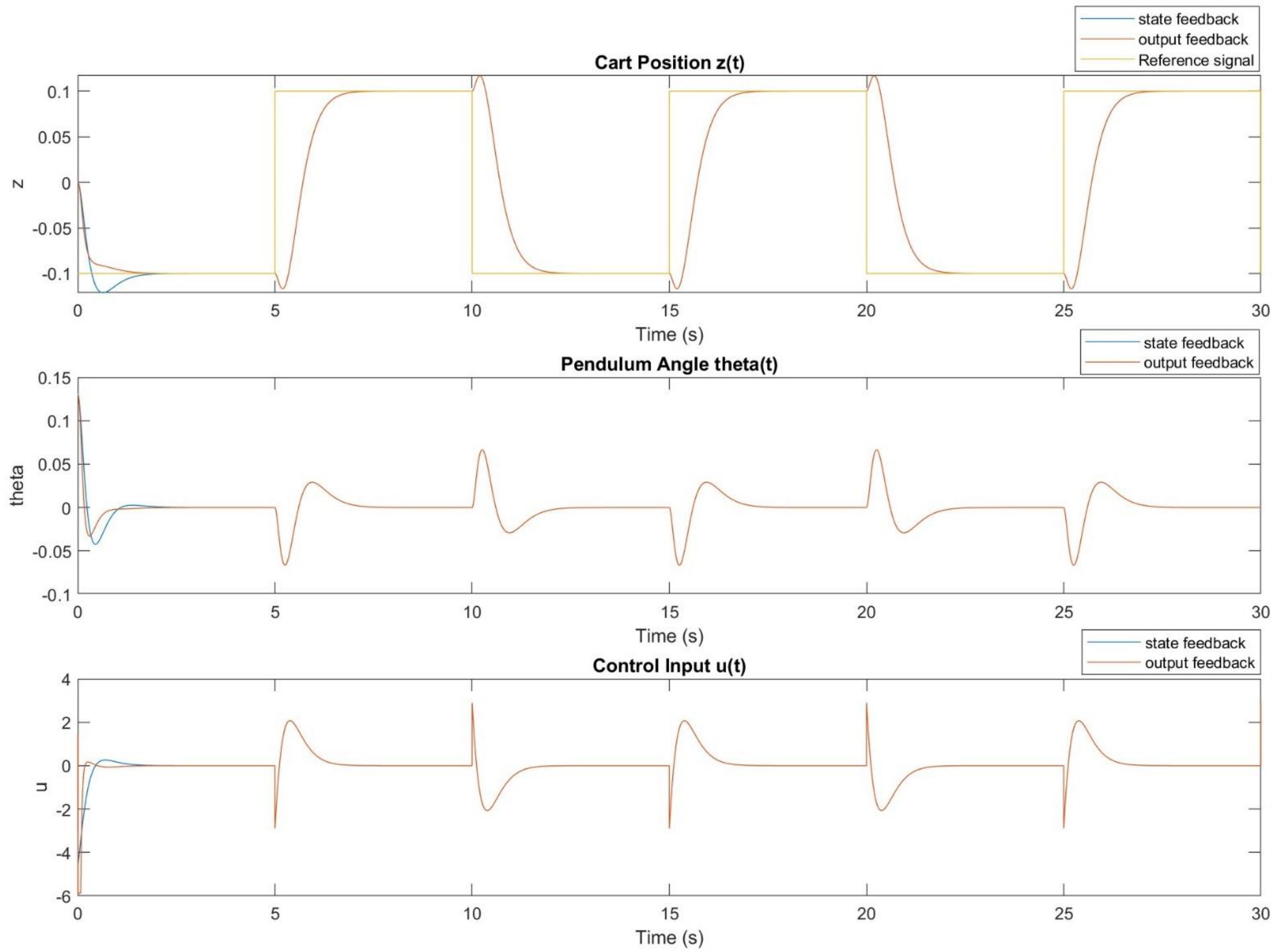

Figure 3: Pole assignment controller with eigenvalues of A + BK at -5 and A − LC at -40.

Upon changing the eigenvalues of A−LC to -40, we observed a significant reduction in the disparity between the two approaches. The resulting graphs became nearly identical, indicating a much smaller difference in their behavior. Furthermore, the adjustment of the eigenvalues to -40 resulted in an increase in the convergence speed for both approaches. T_s = 1.69 s and T_{sat} = 0s.

To achieve a settling time of 1 second and a relatively short saturation time (T_{sat}), we fine-tune the parameters for both the pole assignment controller and the LQR controller. We found:

$$Q = \begin{bmatrix} 4000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad R = 0.5, \qquad P_K = [-7.5, -7.51, -7.52, -7.53]$$

The tuning process has yielded a settling time of 0.96 seconds for the LQR feedback controller and 0.92 seconds for the pole assignment feedback controller. Additionally, the saturation time in each control loop is less than 0.1 seconds, meeting our specified requirements.

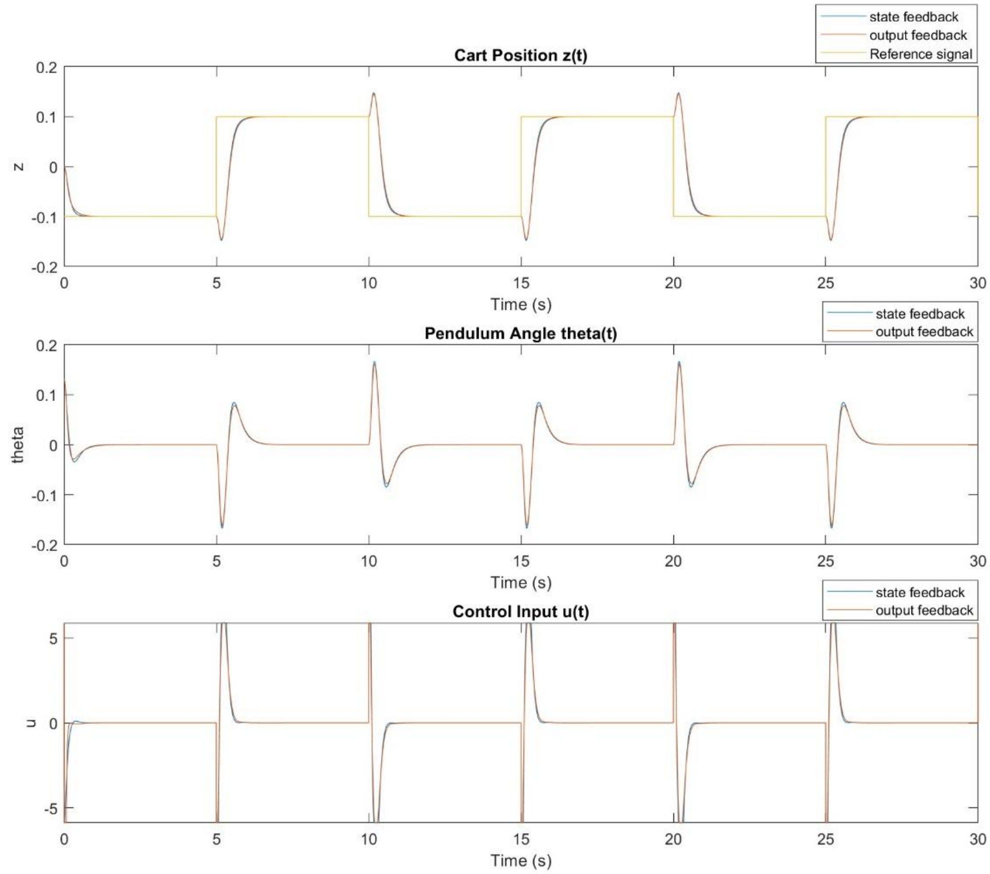

Figure 4: Pole assignment controller with eigenvalues of A + BK at -7.5 and A − LC at -40.

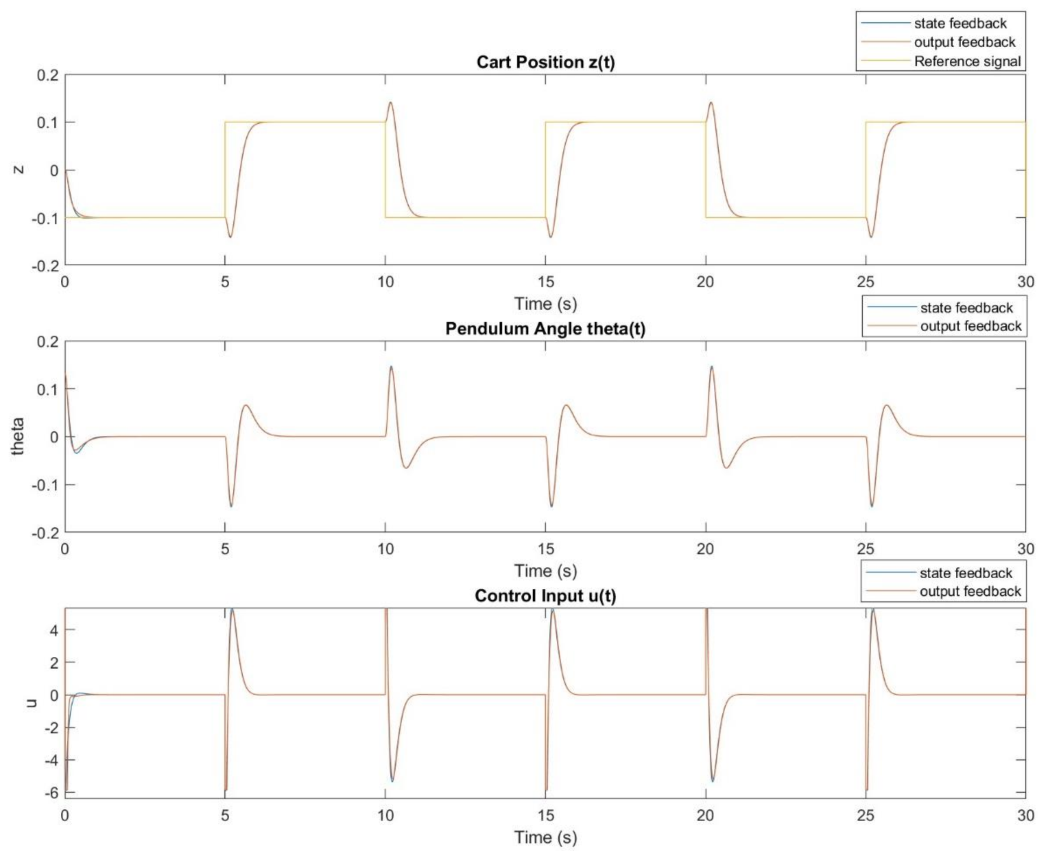

Figure 5: LQR controller with q1=4000, q2=10, R=0.5.

Output 2

Using insights gained from the prelab, our initial approach involved setting a large value for q1 to achieve a short settling time. However, we encountered issues with the system's intensive reaction, particularly due to the presence of the rod. Consequently, we adjusted our parameters to establish system stability. Through iterative tuning, we discovered that lowering q_1 and slightly increasing q_2 and R can enhance system's stability. Our resulting parameters are:

$$Q = \begin{bmatrix} 165 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 30 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad R = 1$$

With these parameters, our experiment results are illustrated below:

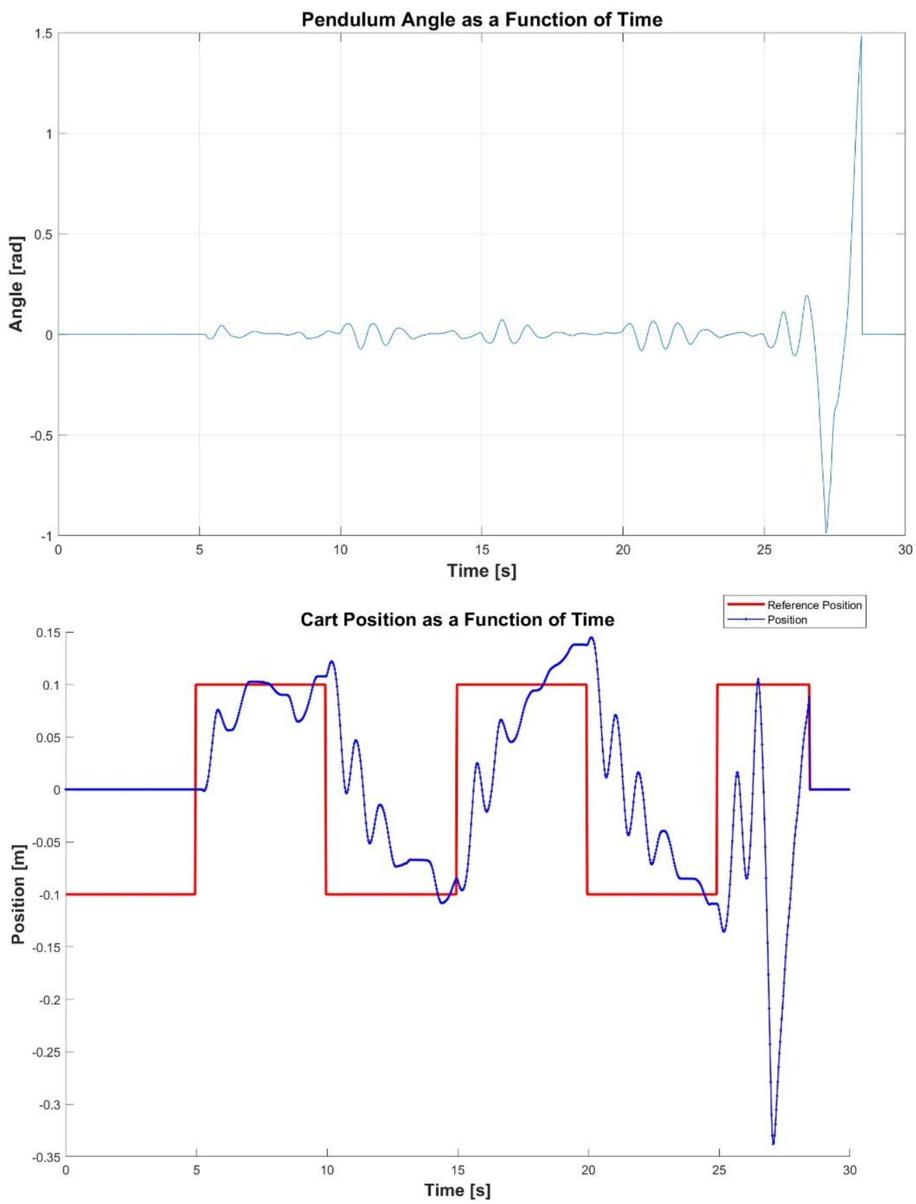

Figure 6: Experiment result for the LQR controller with q_1=165, q_2=20, R=1

The results align with our expectations, as the pendulum angle remains relatively small, and the cart movement exhibits a relatively square wave pattern. The settling time of around 2.5 seconds corresponds with our expectations, considering our deliberate reduction of the q_1 value to improve system stability. Despite the cart losing balance in the final moments, we anticipate that incorporating an integral control term will improve stability and address this issue.

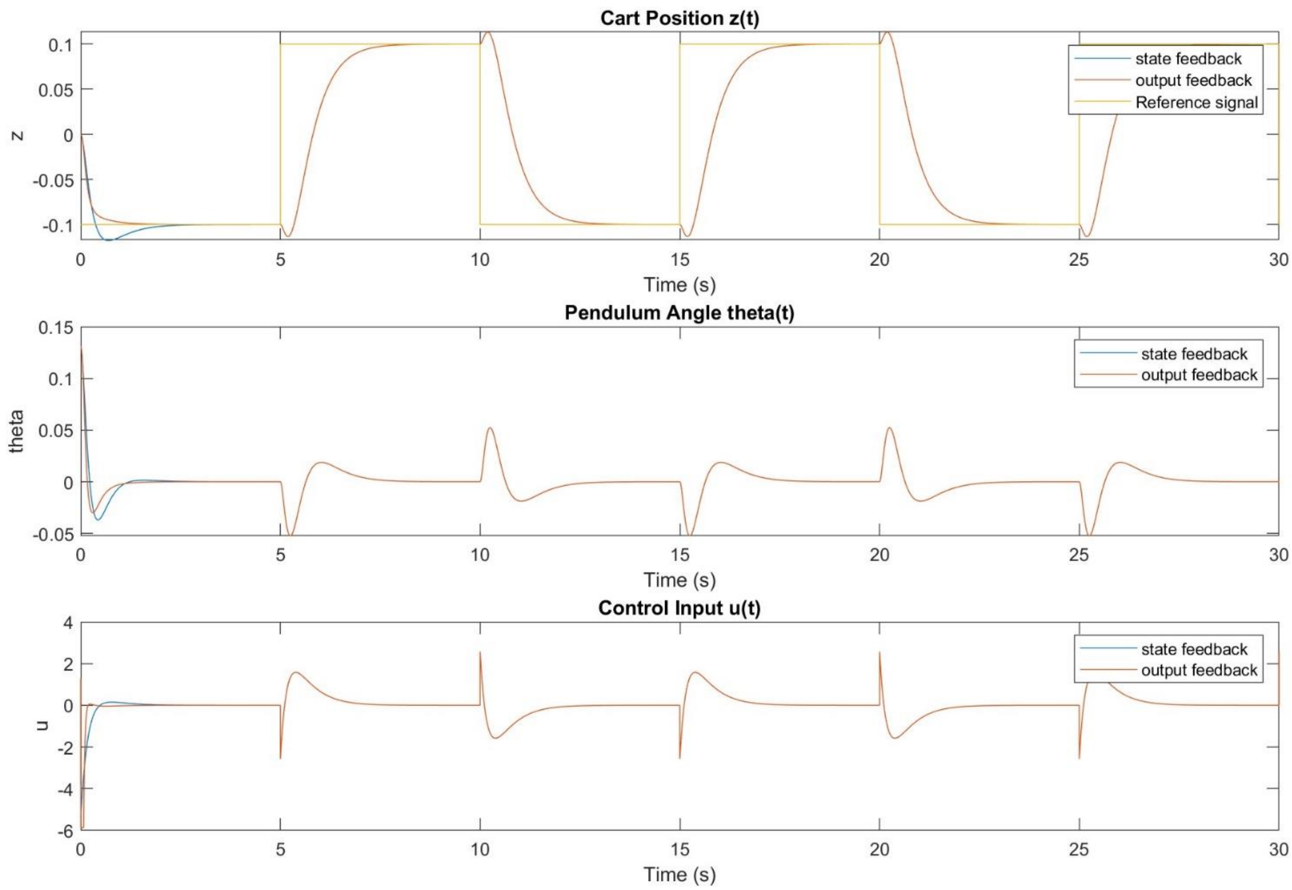

Figure 7: Simulation result for the LQR controller with q_1=165, q_2=20, R=1

Reviewing the simulation presented above, it generally aligns with our experimental results, particularly in anticipating a settling time of around 2.5 seconds. We can see slight differences between the experiment and simulation in the theta and cart position. This disparity may be attributed to sensor errors and input noise.

Output 3

Upon introducing the integral term, we noted its positive impact on system stability. However, similar to the parameter q_1, excessively increasing the integral term q_3 can lead to system overreaction and loss of balance. Therefore, we are in the process of identifying an optimal value that is relatively larger while maintaining system stability. After several rounds of adjustments, we identified our optimized parameters:

$$Q = \begin{bmatrix} 175 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 30 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}, \qquad R = 1$$

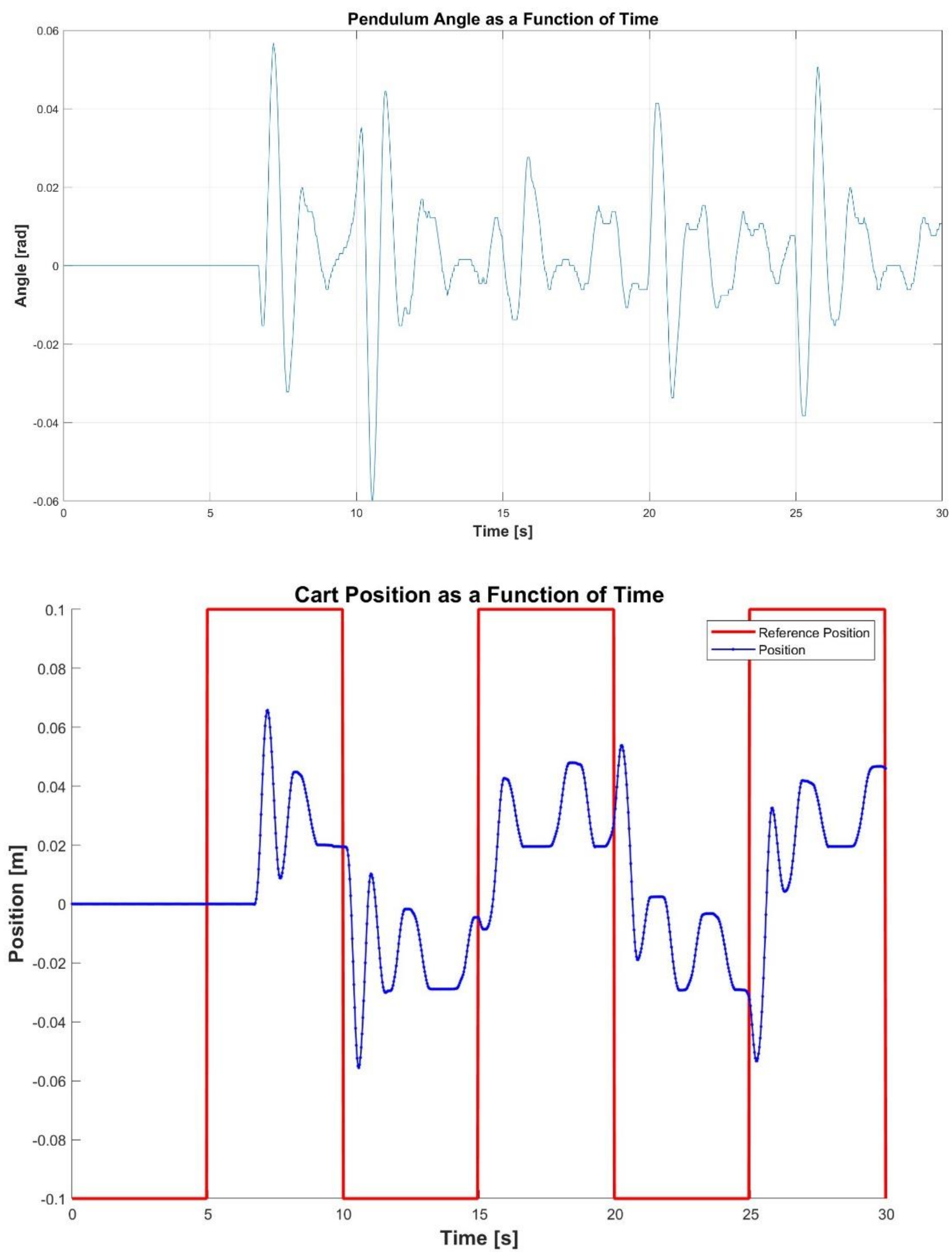

Figure 6: Experiment result for the LQR controller with integral term

We observed a substantial improvement in system stability upon incorporating the integral action. The pendulum angle is notably smaller compared to the previous experiment. However, as a trade-off, the cart's movement becomes less pronounced. While it still exhibits a square pattern, it fails to reach the settling point due to a lack of magnitude. The obtained results are satisfactory and align with our strategy, which prioritizes maintaining balance over the objective of reaching a specific position in the cart movement.