ECE557 Lab4 Report

Introduction:

The cart-pendulum system presents a unique challenge within the domain of robotics and control theory, offering a platform to apply and test control strategies that address both linear and nonlinear dynamics. Building on the foundational work completed in Lab 3, this session's goal is to enhance the cart-pendulum system's responsiveness through the development of an output feedback controller. This enhancement is designed to enable the cart to adhere to a square wave trajectory while maintaining the pendulum in a near-vertical alignment. The evolution of our control strategy is marked by the integration of a Matlab function, 'cartpend.m', that models the nonlinear behavior of the system, and the subsequent application of Simulink for dynamic system simulation. Our methodology includes not only the fine-tuning of state feedback controllers through eigenvalue assignment but also the implementation of the linear-quadratic regulator (LQR) for maintaining system stability.

In this lab, we take a comprehensive approach, aiming to refine and build upon our existing control systems. Our initial step involves modifying the state feedback controllers derived from Lab 3 to enable square wave tracking while ensuring the pendulum's vertical stability. Furthermore, we aim to design an observer for the linearized cart-pendulum model, which will be instrumental in developing a robust output feedback controller optimized for square wave tracking. The effectiveness of this controller will be evaluated through simulation in Simulink, using the nonlinear model, and through practical experimentation with the physical cart-pendulum system implemented on an Arduino platform. Additionally, we will explore the implementation of an output feedback integral controller, providing a comparative analysis of its performance against the non-integral controller. This lab serves as an important milestone in our educational journey, bridging theoretical concepts with practical experimentation to deepen our understanding of dynamic system control.

Block diagram:

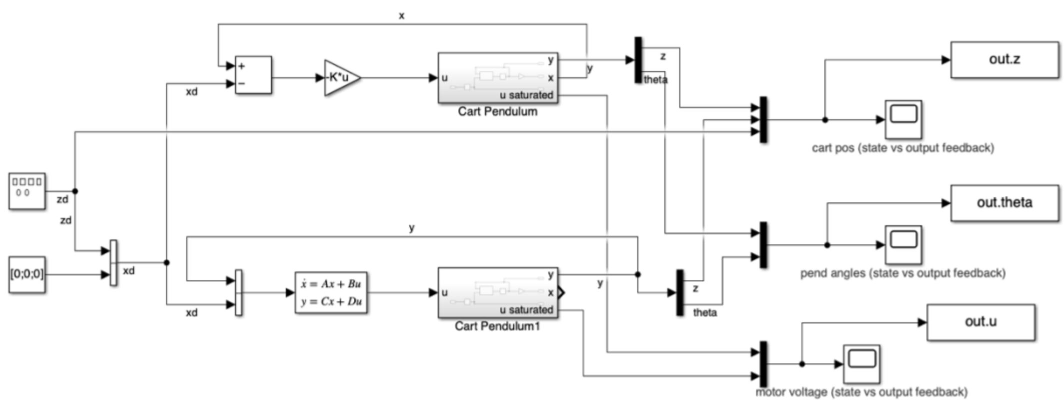

Figure 1: Block diagram for the cart pendulum system

Lab Result:

Output1:

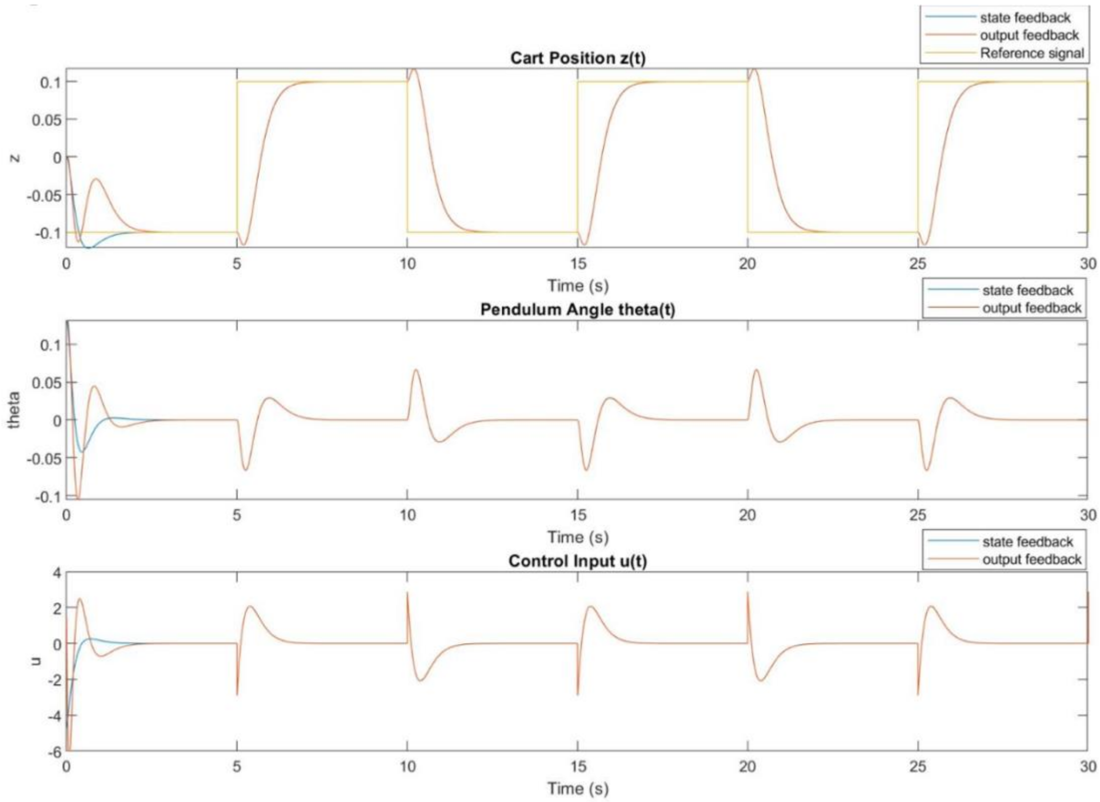

Figure 2: Response of state and output feedback controllers with eigenvalues of A + BK at -5 and A – LC at -10 for cart position, pendulum angle, and control input.

The graph reveals that both control strategies achieve rapid convergence to the desired state. Notably, the output feedback controller exhibits a marginally greater overshoot than the state feedback, which is particularly evident in the initial response phase. This results in a higher initial control effort. However, this overshoot and the resulting elevated control input from the output feedback controller are mostly noticeable within the first two seconds and tend to reduce significantly after the system completes the first cycle of the square wave.

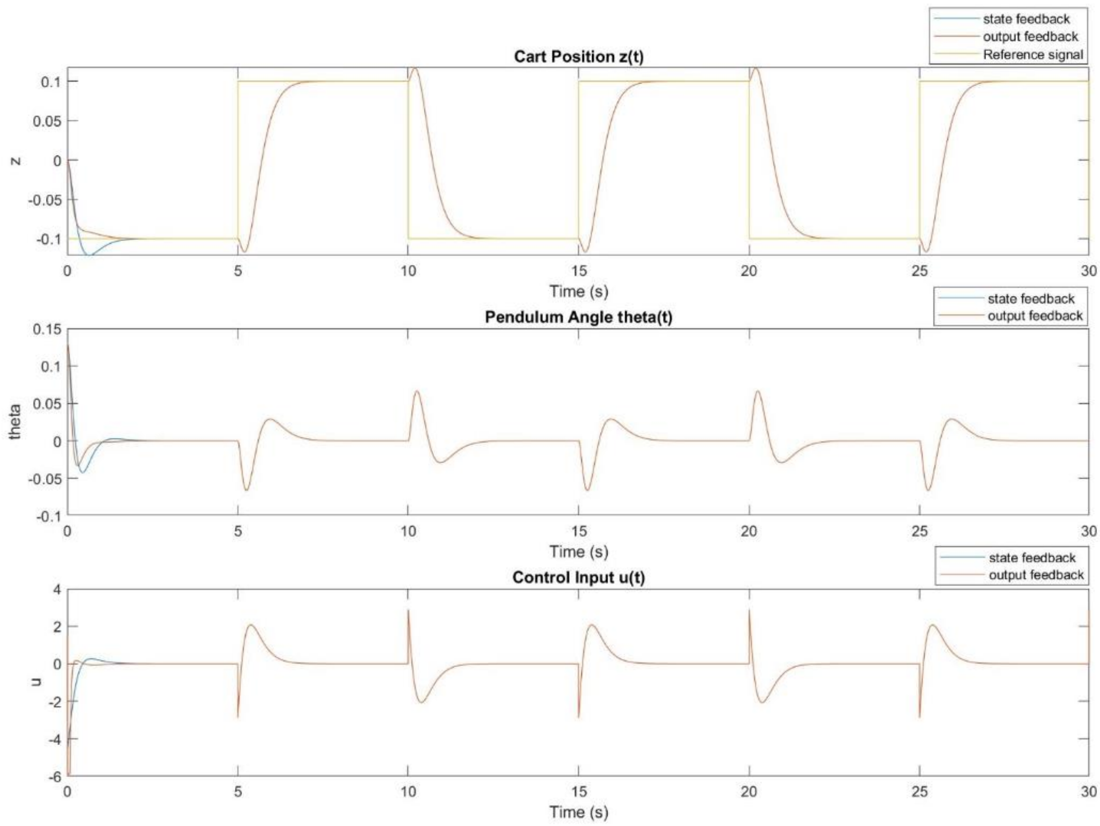

Figure 3: Response of state and output feedback controllers with eigenvalues of A + BK at -5 and A – LC at -40 for cart position, pendulum angle, and control input.

After adjusting the eigenvalues of A - LC to -40, there was a marked improvement in the performance of both the state feedback and output feedback controllers. The response graphs now show a closer resemblance, indicating that the behavior of the two controllers has become more similar. This adjustment to the eigenvalues has led to faster convergence for both control methods, with T_s now at 1.69 seconds and T_{sat} effectively at 0 seconds.

To meet the objective of a 1 second settling time and a short saturation time, the controllers were fine-tuned using a new set of parameters.

$$Q = \begin{bmatrix} 4000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ R = 0.5,\ P_k = [-7.5, -7.51, -7.52, -7.53]$$

The chosen matrices Q and R for the LQR controller, and the pole placement for P_k, resulted in an improved settling time of 0.96 seconds for the LQR feedback controller and 0.92 seconds for the pole assignment feedback controller. Moreover, the saturation time for each control loop was reduced to less than 0.1 seconds, satisfying the specified requirements.

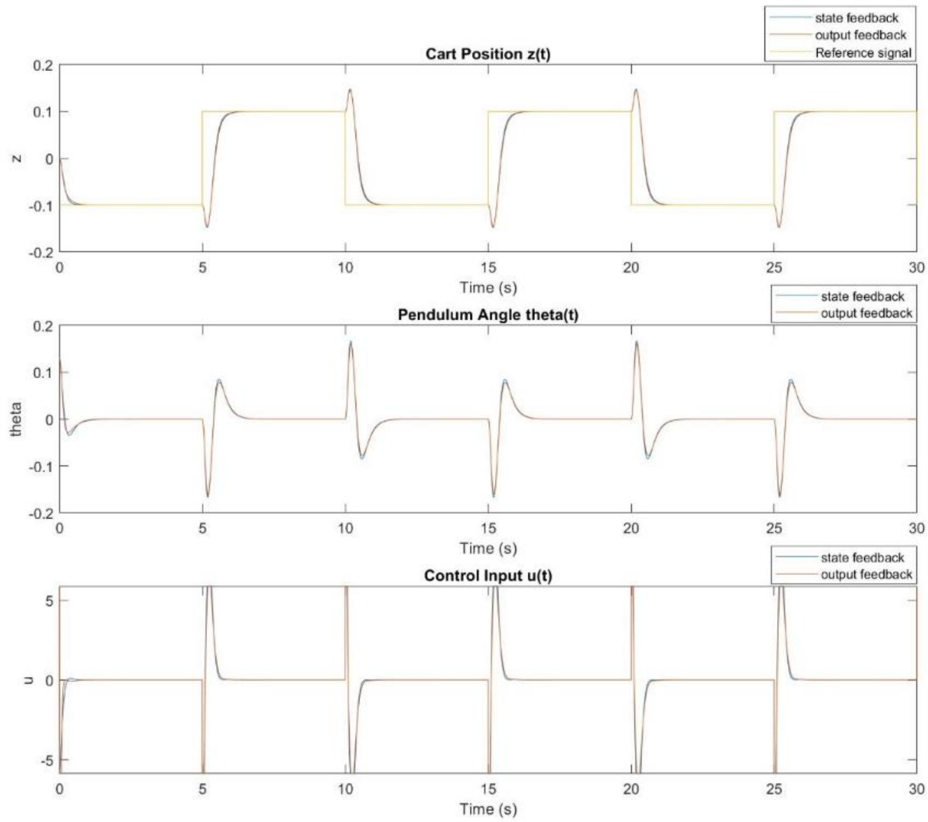

Figure 4: Response of state and output feedback controllers with eigenvalues of A + BK at -7.5 and A – LC at -40 for cart position, pendulum angle, and control input.

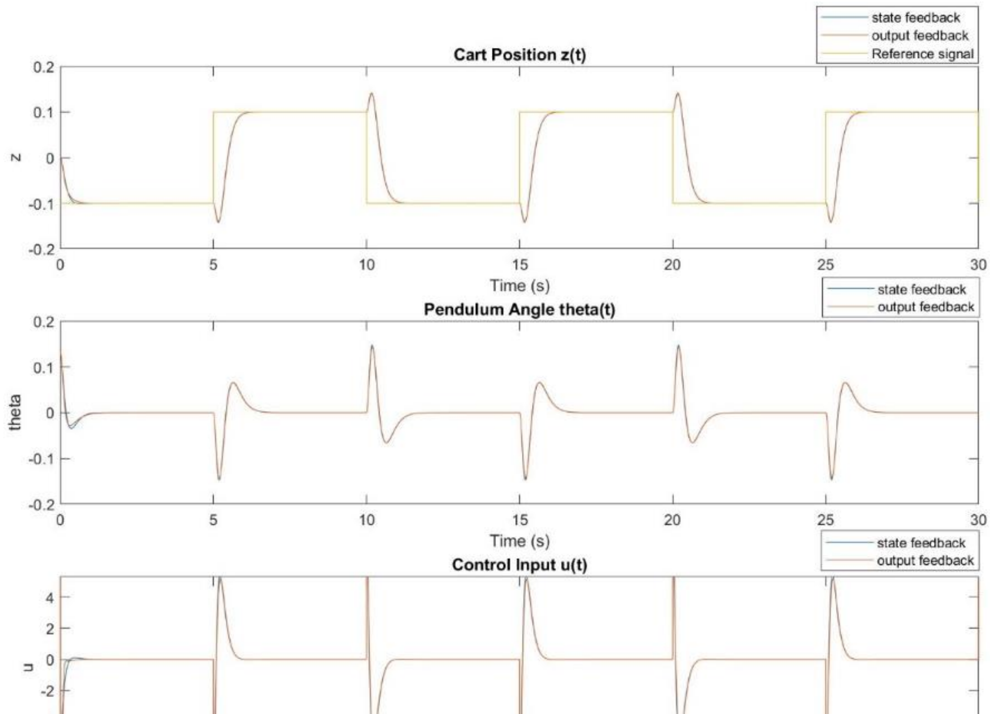

Figure 5: LQR controller with q1 = 4000, q2 = 10, R = 0.5

Output 2:

In the preliminary stages, informed by the pre-lab analysis, a higher weight on q1 was initially set to attain a fast-settling time. However, during the experimental runs, it became apparent that such a high value for q1 led to excessive responsiveness, causing pronounced oscillations due to the physical attributes of the pendulum. To mitigate these issues and achieve a balance between rapid settling and system stability, an iterative process of tuning was employed.

Reducing q1 to 175 and adjusting q2 to 30, while setting R to 1, produced a more stable system response. These modifications led to a more controlled reaction from the system, reducing oscillations while maintaining a swift settling time. The weight q2 was chosen to ensure the pendulum angle stability was emphasized in the system's performance.

The updated matrix is as follows:

$$Q = \begin{bmatrix} 175 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 30 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R = 1$$

With these parameters, our experimental results showed a marked improvement.

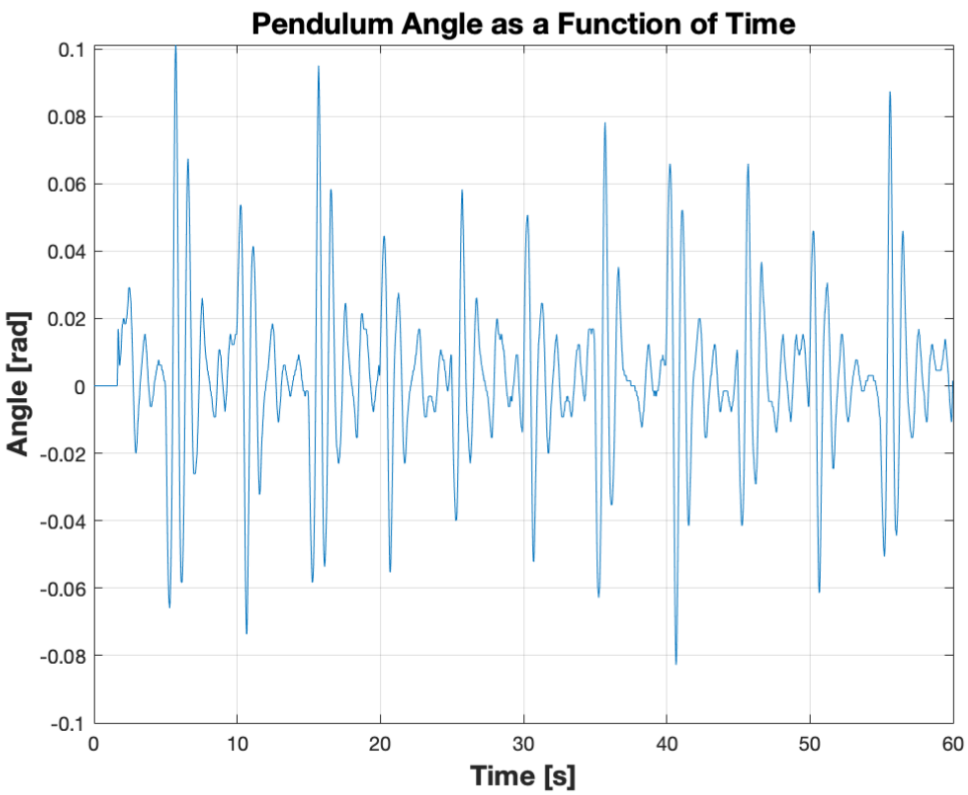

Figure 6: Pendulum Angle as a Function of Time with LQR controller q1 = 175, q2 = 30, R = 1

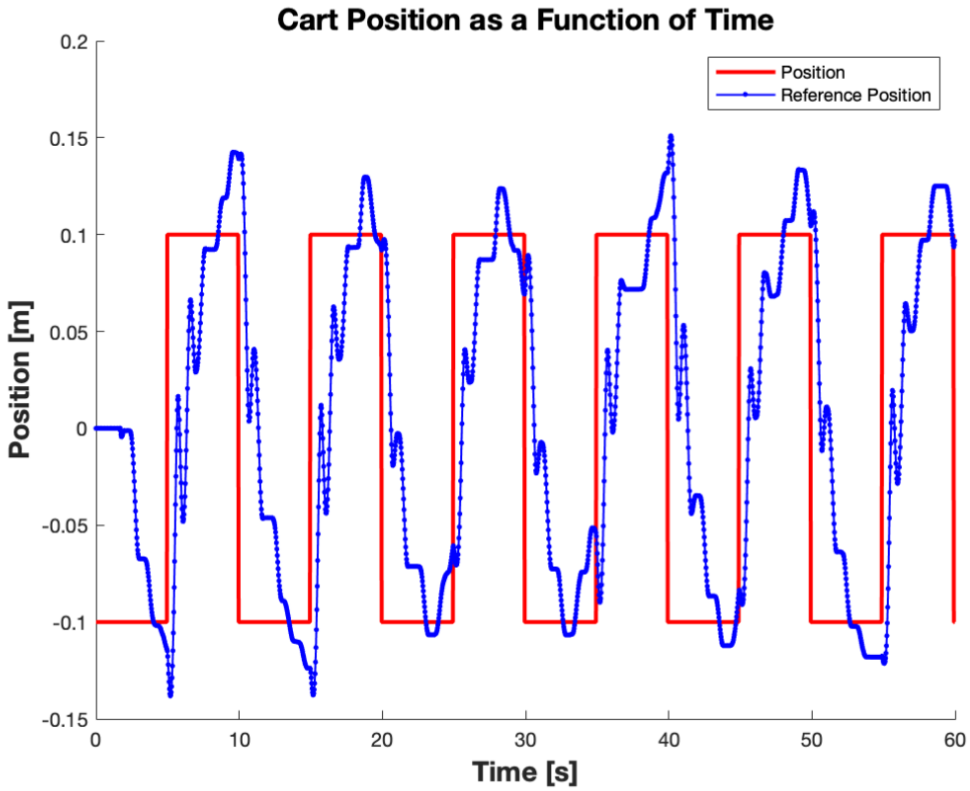

Figure 7: Cart Position as a Function of Time with LQR controller q1 = 175, q2 = 30, R = 1

The experimental results obtained with the LQR controller, configured with q1 = 175, q2 = 30, and R = 1, are depicted in the provided figures. The first figure illustrates the pendulum angle as a function of time, showing that the angle fluctuates within a relatively narrow band, indicating a stable pendulum with minimal deviations from the upright position. While there are sporadic spikes in the angle, these do not suggest a loss of control, and the pendulum remains stable throughout the duration of the experiment.

The second figure presents the cart position compared to the reference square wave signal. The cart follows the reference with a clear square wave pattern, albeit with some delay and minor overshoots at the turning points. The system appears to track the reference well, with the deviations between the actual and reference positions remaining consistent over time.

These results are satisfactory and largely meet our expectations for system performance. The pendulum maintains stability, and the cart tracks the reference signal with an acceptable degree of accuracy. The settling time of around 2.5 seconds corresponds with our expectations, considering our reduction of the q1 value to improve system stability. There is room for improvement in reducing the delay and overshoot in cart positioning, which could potentially be addressed by fine-tuning the controller parameters further or incorporating additional control strategies such as feedforward control or disturbance compensation.

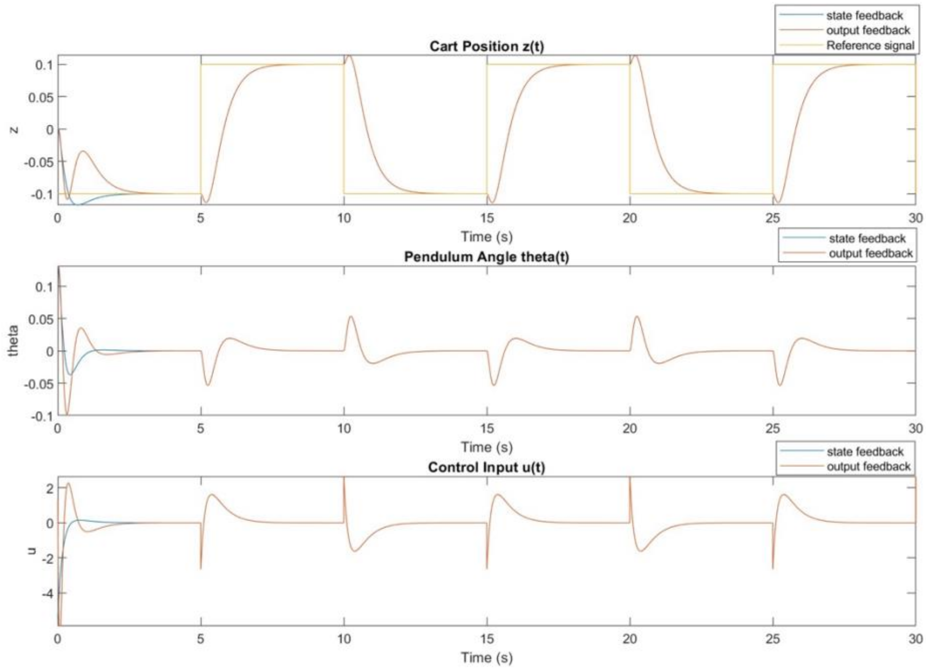

Figure 8: Simulation result with LQR controller q1 = 175, q2 = 30, R = 1

The figure above presents the simulation results for the cart-pendulum system using an LQR controller with the parameters q1 = 175, q2 = 30, and R = 1. The simulation exhibits the cart's position z(t), the pendulum's angle theta(t), and the control input u(t) over a 30-second interval.

In the simulation, both state feedback and output feedback controllers adeptly track the reference square wave, with the output feedback demonstrating a slightly slower response. Despite this, it achieves system stability, as evidenced by the small pendulum angle variations, and maintains the pendulum near the upright position. The simulation, which aligns with our controller design expectations, confirms the parameters' efficacy in achieving a stable system performance with a settling time of around 2.5 seconds. While minor discrepancies with experimental results are noted, these can be ascribed to the ideal conditions of the simulation environment.

Output 3:

In this experimental iteration, we retained the previously optimized controller parameters q1 = 175, q2 = 30, and R = 1, and introduced an integral term q3 = 3 to the LQR controller.

The updated matrix is as follows:

$$Q = \begin{bmatrix} 175 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 30 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}, R = 1$$

The experimental plot is blow:

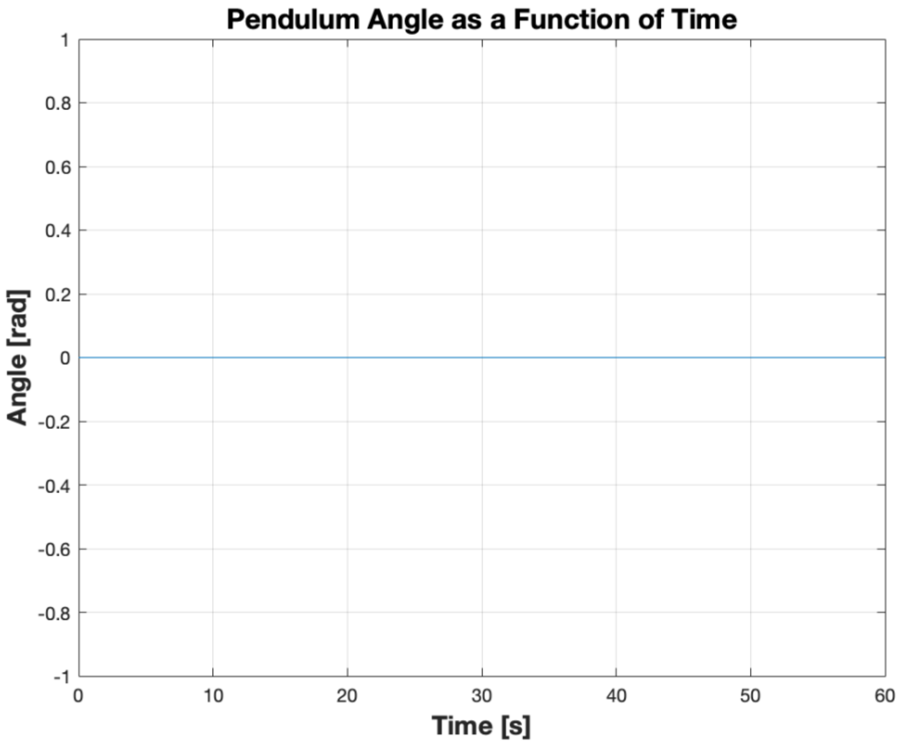

Figure 9: Pendulum Angle as a Function of Time with LQR controller q1 = 175, q2 = 30, q3 = 3, R = 1

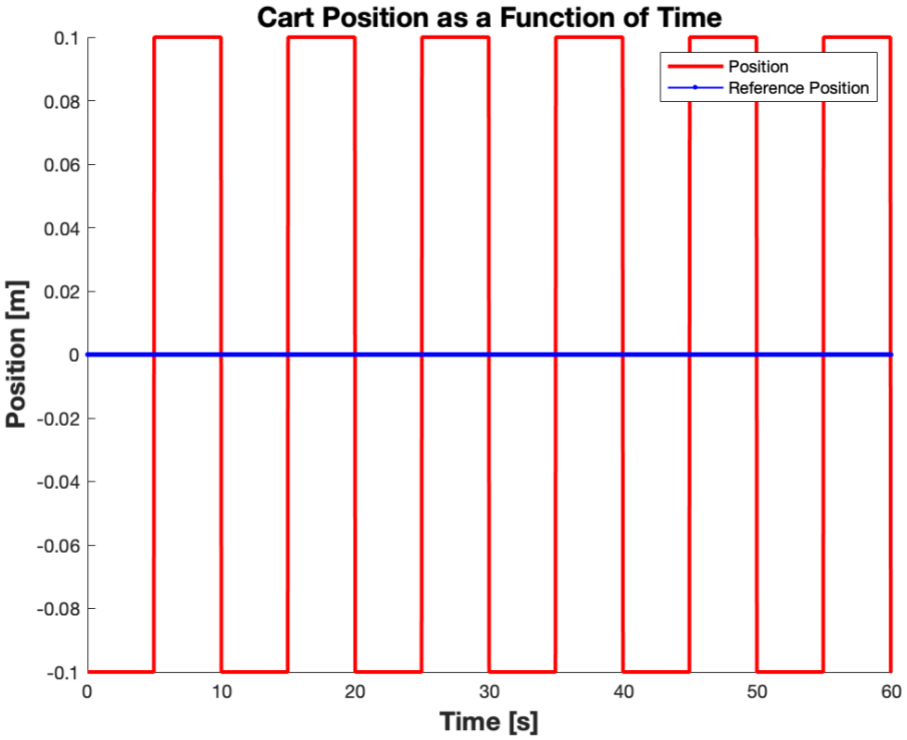

Figure 10: Cart Position as a Function of Time with LQR controller q1 = 175, q2 = 30, q3 = 3, R = 1

Unfortunately, due to a technical issue, the cart position was not recorded during the experiment. Despite this setback, the pendulum's stable behavior suggests the system's performance would likely have aligned with the reference signal, as in prior successful tests. This assertion is supported by the inclusion of q3, which enhances the system's ability to counteract steady-state errors and is expected to have improved the cart tracking performance. A comparison with earlier experimental results without the integral term suggests that the output feedback integral controller has the potential for enhanced performance, although this will need to be confirmed with complete data in subsequent experiments.