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Université de Lausanne – HEC
Empirical Methods in Finance

Project 2

Group 6

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April 21, 2023

1 Stationarity

Q1.1

Given that our regression is defined as $p_t = \mu + \phi p_{t-1} + \epsilon_t$, the null hypothesis ($H_0$) for the Dickey-Fuller (DF) test implies that we have a unit root $\phi_1 = 1$ conditional on the assumption that $\mu = 0$, meaning that our time series is non-stationary. The intuition behind H_0 is that if we have a unit root and $\mu = 0$, then our model simplifies to $p_t = p_{t-1} + \epsilon_t$ which is a random walk with no drift, hence a non-stationary time series.

The alternative hypothesis (H_a), since we did not take the first difference during the regression, is that the absolute value of our auto-regressive coefficient is less than 1, $|\phi_1| < 1$, and our $\mu \neq 0$, meaning that our time series is stationary, following an AR(1) process with a non-zero (hence, not a random walk).

Q1.2

The test statistic (τ_μ) for testing H_0 is estimated using the following formula:

$$\tau_\mu = \frac{\hat{\phi}_1 - 1}{\text{std}\left(\hat{\phi}_1\right)} = \frac{\hat{\phi}_1 - 1}{\left(\hat{\sigma}_\varepsilon^2 / \sum_{t=1}^{T} \left(p_{t-1} - \bar{p}\right)^2\right)^{1/2}} \tag{1}$$

where:

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{T-1} \sum_{t=1}^{T} \left(p_t - \hat{\mu} - \hat{\phi}_1 p_{t-1}\right)^2 \tag{2}$$

However, since our test statistic deviates significantly from the standard normal distribution, we need to compute the critical values using Monte Carlo simulations based on our sample size.

We can perform a one-sided test using the test statistics obtained from our price data and the critical values corresponding to the respective percentiles of the simulated Monte Carlo distribution. Our decision rule to reject H_0 is based on whether the test statistic is smaller than the critical value, and vice versa for H_a. Intuitively, we reject the null if the test statistic computed on our log-price data is outside the core of the numerically simulated distribution, where the boundaries of the core are the critical values (for different significance levels).

Critical Values

Q1.3

The random walk is a common model for financial time series. In a random walk, the value at each point in time depends only on the previous value plus a random shock. This is a useful model for asset prices because they typically show trends over time, but they are also subject to random fluctuations.

By generating random walks, we can empirically estimate the distribution of the test statistic under the null, namely that the time series is a (non-stationary) random walk. This provides us with critical values to test whether the actual test statistic computed on log-price data is significantly different from a test statistic drawn under the null (non-stationary time series).

Q1.4

By plotting the distribution of $t(\phi^{(i)} - 1)$ (Figure 1) we can observe that it is symmetrically distributed and centred around -2. The distribution represents the empirical distribution of the DF test statistic based on the Monte Carlo simulations under the null hypothesis of a unit root, hence non-stationarity, of the time series.

Q1.5

The critical values of the Dickey-Fuller test:

	Critical Value
0.01	-3.44
0.05	-2.89
0.10	-2.58

Table 1: Critical Values of the DF Test

In Figure 1 we have plotted the critical values of the DF test at the percentiles of 10% (green bar), 5% (orange bar), and 1% (red bar).

Testing Non-Stationarity

Q1.7

After computing the DF test statistics on our sample of commodities, we do not reject the null hypothesis (see Table 2) that the log-price of each commodity has a unit root, which means that the time series are non-stationary and follow a random walk process. This has important implications for forecasting and modelling price series, because it exhibits a trend that is unpredictable and may change randomly over time.

	Corn	Wheat	Soybean	Coffee	Cacao
DF_{TS}	-1.13	-1.86	-1.27	-1.40	-2.51
$CV_{1\%}$	-3.44	-3.44	-3.44	-3.44	-3.44
$CV_{5\%}$	-2.89	-2.89	-2.89	-2.89	-2.89
$CV_{10\%}$	-2.58	-2.58	-2.58	-2.58	-2.58
Reject H_0 1%	False	False	False	False	False
Reject H_0 5%	False	False	False	False	False
Reject H_0 10%	False	False	False	False	False
P-Value	0.71	0.36	0.65	0.59	0.12

Table 2: Testing Non-Stationarity for daily log-prices

Q1.8

As discussed above, log-price time series are non-stationary, and individually appear to be spurious. However, it is possible that there exist combinations of these that are stationary, which would imply cointegration, meaning that the pairs in question have a long-term equilibrium relationship.

2 Cointegration

Critical Values

Q2.1

After constructing an empirical distribution of $t(\phi^{(i)} - 1)$ under the alternative, we observe a similar distribution as in Q1.4. Since our test statistic is a ratio of two non-stationary variables, we observe in Figure 2 a distribution that is not centred at 0. The critical values of the Dickey-Fuller test under no cointegration are:

	Critical Value
0.01	-3.89
0.05	-3.32
0.10	-3.04

Table 3: Critical Values of the DF Test under no Cointegration

Testing for Cointegration

Q2.2

We found multiple pairs of assets which were cointegrated, notably Wheat-Corn and Soybean-Coffee. The full table of cointegration tests on asset pairs can be found in Table 11 in the Appendix.

	DF_{TS}	$CV_{1\%}$	$CV_{5\%}$	$CV_{10\%}$	P-Value	Reject H_0 1%	Reject H_0 5%	Reject H_0 10%
Corn-Wheat	-3.14	-3.89	-3.32	-3.04	0.08	False	False	True
Wheat-Corn	-3.54	-3.89	-3.32	-3.04	0.03	False	True	True
Soybean-Coffee	-3.14	-3.89	-3.32	-3.04	0.09	False	False	True
Coffee-Soybean	-3.11	-3.89	-3.32	-3.04	0.09	False	False	True

Table 4: Cointegrated Asset Pairs

Q2.3

The cointegration regression ($p_t^A = \alpha + \beta p_t^B + z_t$) represents the long-run relationship between our two non-stationary variables. The value of alpha is the intercept in the cointegration regression, representing the long-term difference between the two commodities. The beta is the regression coefficient, in our case since it is a regression in the form of ln-ln, it indicates the price elasticity of one commodity price with respect to another.

In general, a high value of beta indicates a stronger relationship between the two assets. Looking at the table, we can see that the pairs Corn-Wheat, Wheat-Corn, Corn-Soybean and Soybean-Corn all have large values of beta, whereas Corn-Cacao, Corn-Cacao, Cacao-Soybean, and Soybean-Cacao all have low values of beta, indicating a weaker relationship.

	Alpha	Beta
Corn-Wheat	0.60	0.86
Corn-Soybean	0.09	0.85
Corn-Coffee	4.21	0.36
Corn-Cacao	5.33	0.08
Wheat-Corn	1.53	0.79
Wheat-Soybean	1.56	0.68
Wheat-Coffee	4.56	0.35
Wheat-Cacao	4.43	0.23
Soybean-Corn	2.15	0.80
Soybean-Wheat	2.60	0.69
Soybean-Coffee	4.52	0.49
Soybean-Cacao	5.32	0.21
Coffee-Corn	1.36	0.59
Coffee-Wheat	1.05	0.61
Coffee-Soybean	-1.01	0.85
Coffee-Cacao	1.47	0.44
Cacao-Corn	7.47	0.06
Cacao-Wheat	6.65	0.19
Cacao-Soybean	6.70	0.17
Cacao-Coffee	6.85	0.20

Table 5: Cointegration regression parameters

Q2.4

The commodity pair with the strongest cointegration is Wheat-Corn. From Table 11 in Appendix, we can observe that Wheat-Corn is the only pair that rejects the H_0 at the 5% level. This strong cointegration can be intuitively explained by their use as substitute goods in various applications such as ethanol production and animal feed. For instance, livestock farmers may switch to wheat as substitute feed if corn prices rise, consequently increasing demand for wheat and converging their long-term price relationship.

Q2.5

See Figure 3 in the Appendix showing the Wheat-Corn log-price relationship.

3 Pair Trading

Trading Signal

Q3.1

In order to compute the spread z_t, we run a simple regression between the prices of Wheat (dependent variable), which is the most cointegrated pair in-sample.

The spread is defined as $z_t = P_t^{\text{Wheat}} - \left(\alpha + \beta P_t^{\text{Corn}}\right)$. The fact that the prices of Wheat and Corn are cointegrated means that there supposedly exists a long-term economic relationship between the values of these two assets. The long-run equilibrium price of Wheat with respect to the price of Corn is $\alpha + \beta P_t^{\text{Corn}}$. The spread z_t measures the deviation of the actual price of Wheat compared to the equilibrium price.

Since the spread z_t measures the deviation of the price compared to the equilibrium price, as long as the cointegration relation holds we expect z_t to be, on average, equal to 0. If the economic forces that drive the relations between two assets remain the same across time price should go back to the equilibrium price in the same way across time, which means that the autocorrelation between the z_t should be constant across time. Then, since the mean of z_t and the autocorrelation are time-invariant, z_t should be a weak stationary variable.

To implement the pair trading strategy, we have to monitor the value of z_t. If the absolute value of z_t becomes high this means that the price of Wheat has deviated significantly from its long-term equilibrium price relation with Corn, which means that we can take advantage of a statistical arbitrage by applying the following strategy:

- If $z_t > 0$: the price of Wheat is too high compared to the price of Corn.
  Strategy: short the Wheat future and long the Corn future.

- If $z_t < 0$: the price of Wheat is too low compared to the price of Corn.
  Strategy: long the Wheat future and short the Corn future.

Q3.2

See Appendix, Figure 4.

Q3.3

We can see that all the autocorrelations are significantly different from 0 up to 10 lags. In order to test if the spread is serially correlated, we run a Ljung-Box test: the null hypothesis that the return series is not serially correlated over the sample is rejected at 1% (Figure 5).

We notice that the autocorrelation is constantly decreasing with the number of lags. This entails that when the $\tilde{z}_t$ is high (in absolute value), it reverts back slowly to 0.

This behaviour is essential for the pair trading strategy, which is based on an assumption that the spread has a mean-reverting behaviour, which implies a decreasing autocorrelation in the spread when time elapses.

In-sample Pair Trading Strategy

Q3.4 (Table 12 - Strategy 1)

Please find the charts in Figure 6 (Appendix).

Q3.5 (Table 12 - Strategy 2)

Please find the charts in Figure 7 (Appendix).

Notice that, as we have not made any changes to our underlying strategy, the additional leverage only changes the magnitude of our strategy's payoff, namely we obtain a higher profit and a higher return on equity (ROE).

	Leverage = 2	Leverage = 20
Profit	1078.72	1759.65
ROE	107.87%	175.97%
Init wealth	1000.00	1000.00
Final wealth	2078.72	2759.65
Min wealth	976.82	966.89
Max wealth	2078.72	2759.65
Pos1 trades	4	4
Pos2 trades	2	2
Total trades	6	6

Table 6: Effect of leverage on strategy

Q3.6

The stop loss rule is based on $\tilde{z}^{stop}$ as we want to exit if we do not experience reversion, to limit losses if we find ourselves on the wrong side of the market.

In order to measure the probability of hitting the $\tilde{z}^{stop}$ the day after opening the position at $\tilde{z}^{in} = 1.5$, we assume that $\tilde{z}_{t+1}$ can be described with an AR(1) process:

$$\tilde{z}_{t+1} = \phi_0 + \phi_1 \tilde{z}_t + \epsilon_t$$

$$E[\tilde{z}_{t+1}] = \frac{\phi_0}{(1 - \phi_1)} \quad \text{and} \quad V[\tilde{z}_{t+1}] = \frac{\sigma_\epsilon^2}{1 - \phi^2}$$

Since we assume that $\epsilon_t \sim N(0, \sigma_\epsilon^2) \implies \tilde{z}_{t+1} \sim N(E[\tilde{z}_{t+1}], V[\tilde{z}_{t+1}])$. Then we can easily compute $P(\tilde{z}_{t+1} > \tilde{z}^{stop})$:

$\tilde{z}^{stop}$	$Pr(\tilde{z}_{t+1} > \tilde{z}^{stop})$
1.75	44.23%
2.75	41.03%

Table 7: Probability of breaching stop loss

Q3.7 (Table 12 - Strategy 3)

Imposing a stop loss lowers the overall performance of the strategy (Figure 8). Setting a stop loss reduces the risk of loss if the spread does not revert back to 0 in the case where the two assets are no longer cointegrated. However, like any risk reduction strategy, it comes with costs: we can lose potential gains if the spread goes above the stop loss and subsequently goes back to 0. Therefore there is a trade-off between security and capped upside, that a stop loss provides.

	No $\tilde{z}^{stop}$	$\tilde{z}^{stop} = 2.75$
Profit	1078.72	707.91
ROE	107.87%	70.79%
Init wealth	1000.00	1000.00
Final wealth	2078.72	1707.91
Min wealth	976.82	976.82
Max wealth	2078.72	1725.67
Pos1 trades	4	5
Pos2 trades	2	3
Total trades	6	8

Table 8: Strategy with no $\tilde{z}^{stop}$ vs. $\tilde{z}^{stop} = 2.75$

Out-of-sample Pair Trading Strategy

Q3.8

The rolling correlations of log returns are more robust, whereas correlations in log prices tend to fluctuate more vigorously. Indeed log returns are much more stationary than prices, implying that the correlation in returns of two assets will remain much more stable over time than the correlation in prices.

The decrease in price correlation can be viewed as a signal of a break down in the cointegration relation between the assets' prices (Figure 9).

Q3.9

During the out-of-sample analysis, we observed that our alpha value tended to fluctuate, reflecting a persistent difference between the prices of commodities. Specifically, when Corn prices remained consistently higher than Wheat prices, the value of alpha became negative, and when Wheat prices exceeded Corn prices, the alpha value became positive (see Figure 10).

Moreover, out-of-sample we found that the beta tended to fluctuate as well. This reflects the fact that the relations between the price of the two assets evolve. This fluctuation reflects in turn how the cointegrated relation changes over time (Figure 11). However, a decrease in beta should not be interpreted necessarily as a broke down of the cointegration relations, but only as a change in the cointegration relation: even with low beta, the two assets can have a high correlation. A change in the beta implies a need to recalibrate our strategy in order to measure the spread correctly.

We can see in Figure 12 that our out-of-sample spread is much noisier than our in-sample spread, we can also observe more extreme values of the signals, which increases the frequency at which we open positions. There are moments where we fluctuate between $\tilde{z}^{in}$ and $\tilde{z}^{stop}$ which leads us to open positions and then close them shortly after.

Q3.10

Our out-of-sample strategy performed worse, yielding an ROE of 33.4%. Since we have rolling windows of 500 days, our spread is noisier and more dispersed with respect to the in-sample spread. In Figure 13, we observe that many times our spread fluctuates around the $\tilde{z}_{stop}$, which creates a destructive dynamic whereby we open a position as we are above $\tilde{z}_{in}$ and close our position once the stop loss is hit at a loss, and as we drift back down we reopen a position and this dynamic repeats damaging the profitability.

However, this is not unexpected, as we are operating in a more realistic setting, namely we exploit only known information at each time, thus the uncertainty related to future price moves decreases the accuracy of our trading signals.

	Strategy 4
Profit	334.03
ROE	33.40%
Initial wealth	1000.00
Final wealth	1334.03
Min wealth	876.56
Max wealth	1426.83
Pos1 trades	29
Pos2 trades	4
Total trades	33

Table 9: Strategy with OS spread

Q3.11

As depicted in Figure 14, out-of-sample we observe consistently high p-values, indicating an absence of cointegration in the majority of the sub-samples. From the standpoint of an investor who would like to trade only when the cointegration relation is verified, this clearly reduces the number of viable trading periods.

Looking back at Figure 14, we observe that the inversions in the cointegration relation happen when the out-of-sample spread explodes, for instance in mid-2018. Intuitively, extreme price shocks cause the cointegration relationship to become highly insignificant, so out-of-sample it's not clear in these moments that asset prices will converge back to the long-run equilibrium.

Q3.12

As discussed in Q3.11, out-of-sample the cointegration relation is actually broken most of the time, which highlights the hazards for an investor trying to practically implement this strategy.

Since the cointegration between Wheat and Corn holds at a 5% significance level only for a very limited time frame, in our strategy we decided to use a 10% significance level to identify the intervals where to open positions, as to increase the number of viable time frames for trades. Still, in practice, we found only a very limited amount of such periods (namely, 12).

Moreover, this is exacerbated by the fact that, due to the rolling windows, large shocks propagate for many periods of time, therefore if our cointegration is broken it remains broken for a long period of time, effectively reducing further the number of viable entries. This further limited the number of trades we were able to place.

Finally, there were multiple times when we had brief moments of cointegration, which then broke shortly after, damaging the profitability of our strategy as can be seen in Figure 15.

	Strategy 5
Profit	-106.10
ROE	-10.61%
Initial wealth	1000.00
Final wealth	893.90
Min wealth	890.85
Max wealth	1067.88
Pos1 trades	12
Pos2 trades	0
Total trades	12

Table 10: Strategy with cointegration = 10%

Q3.13

Pair trading is a mean-reversing strategy, which is based on trading two assets (in our case, commodities) that move in a stationary fashion, and profit from short-term price discrepancies.

Taking our analysis cum grano salis, there are several market frictions we still have not accounted for in our model, notably margin calls, slippage and trading fees. It is easily conceivable that these factors could kill the little profitability achieved out-of-sample, either by reducing funds (slippage, trading fees) or by forcing us to close a position prematurely and realize a loss (margin calls). Also, we did not properly calibrate our strategy, in the sense that we did not decide on optimal parameters for entry or stop losses.

In conclusion, it was interesting to experiment with this statistical arbitrage strategy, however, we realize that there are still several factors to deal with before being able to set up shop.

Appendix

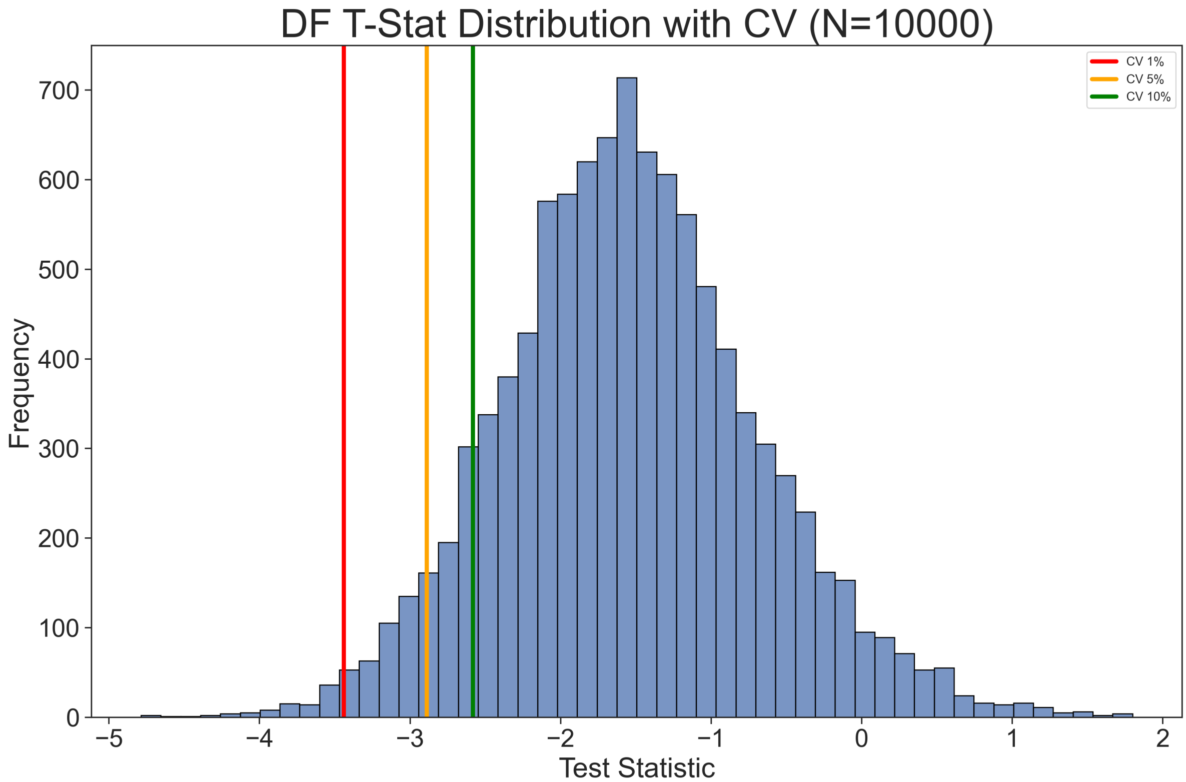

Figure 1: Distribution of Dickey-Fuller Test-Statistics

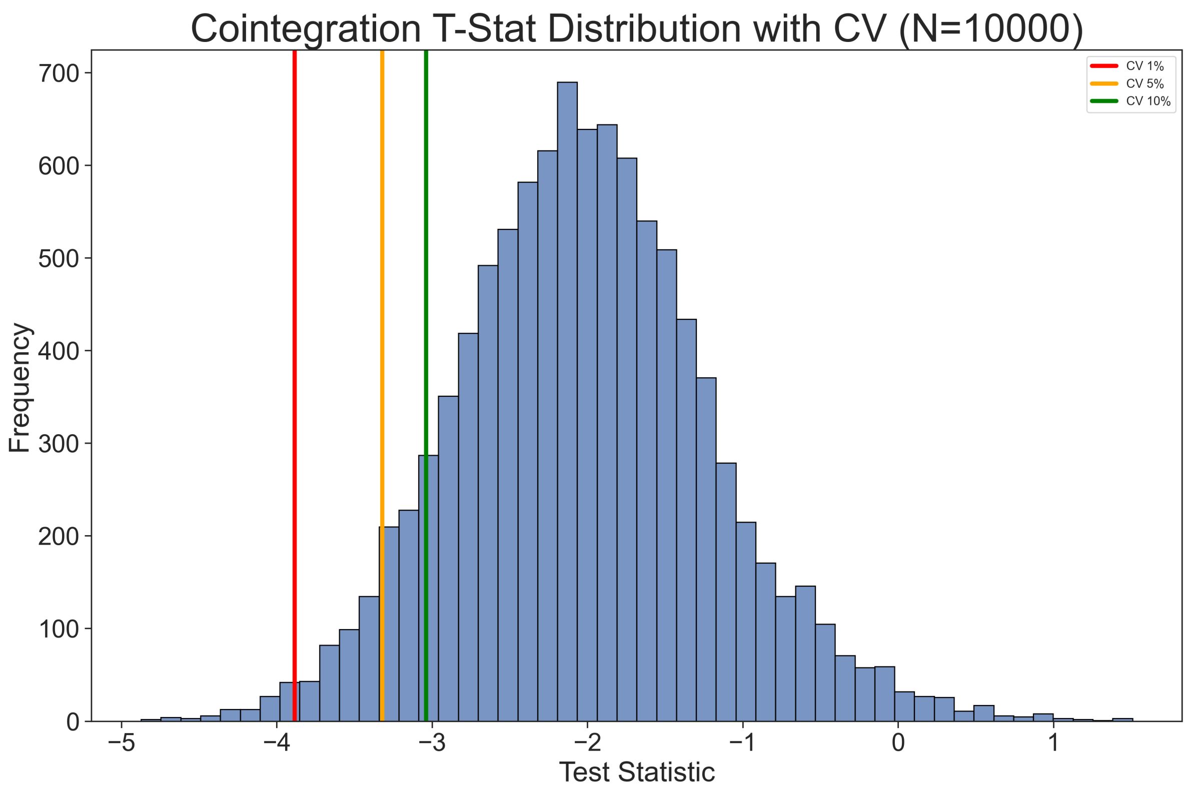

Figure 2: Distribution of Test-Statistics

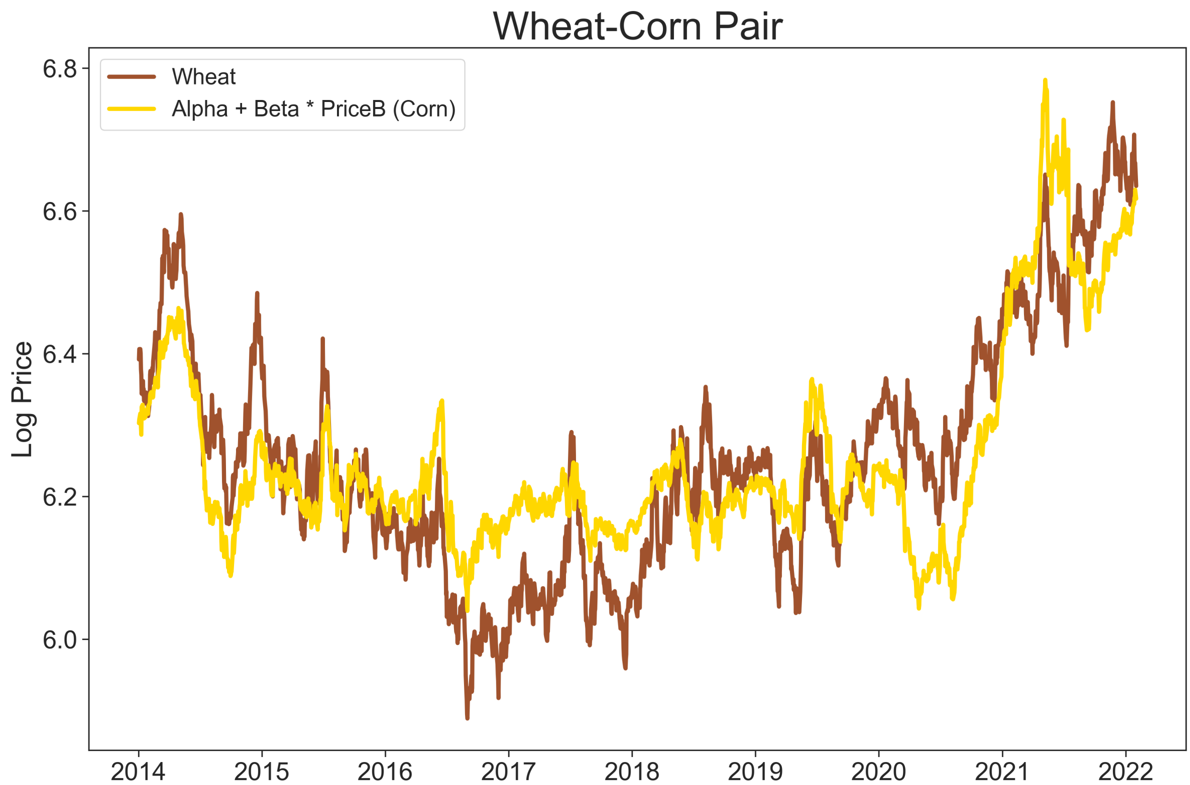

Figure 3: Wheat-Corn log-price relationship

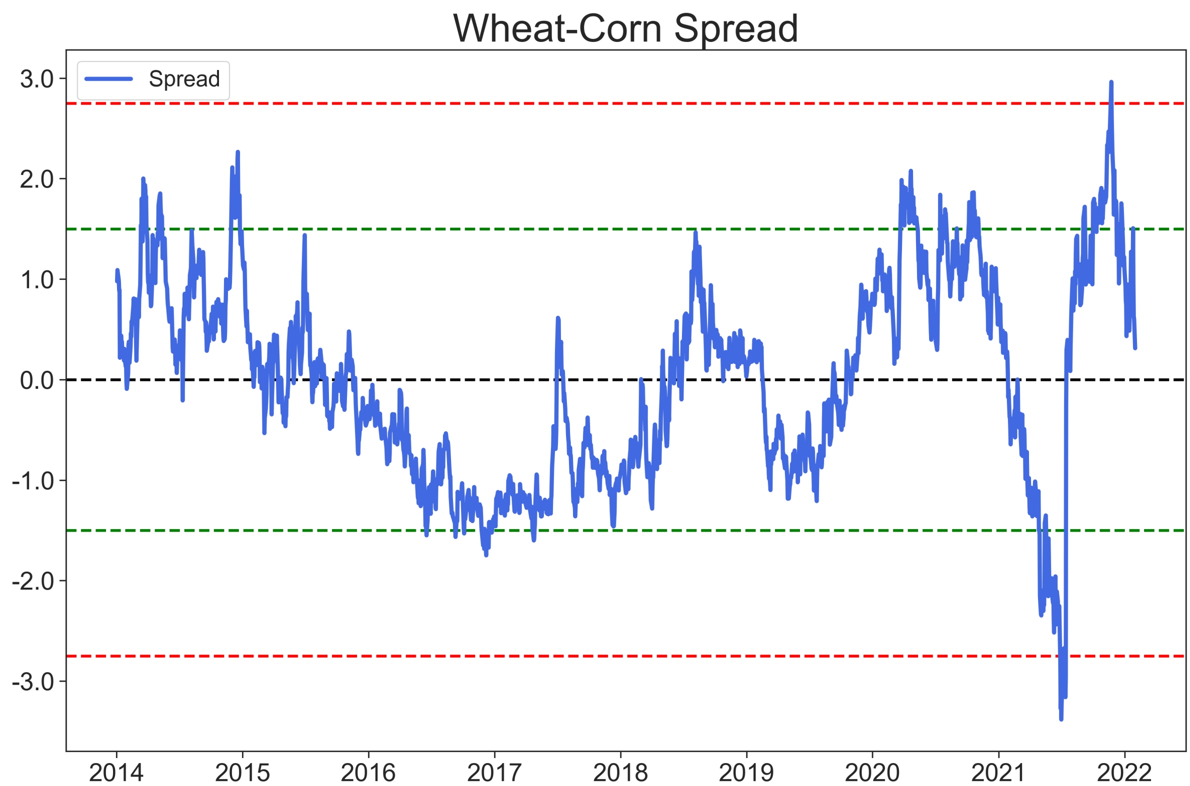

Figure 4: Wheat-Corn normalised spread $\tilde{z}_t$

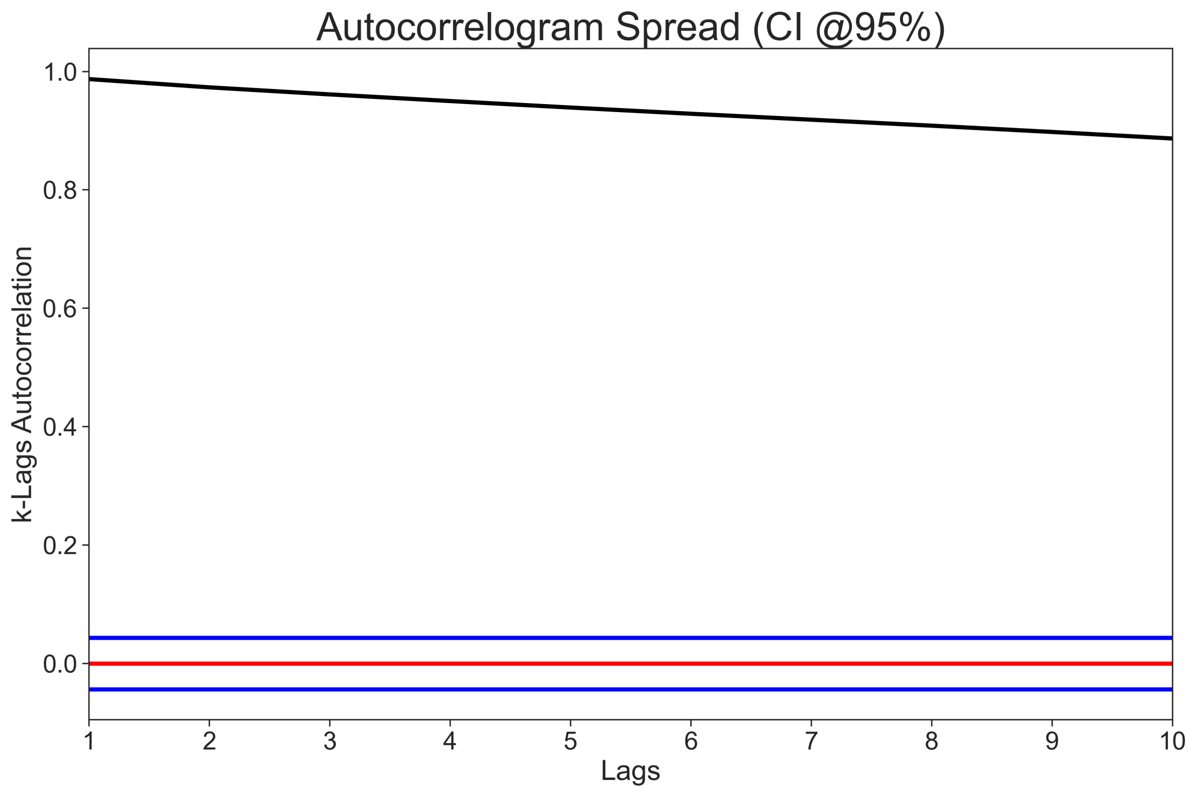

Figure 5: Auto-correlogram of $\tilde{z}_t$

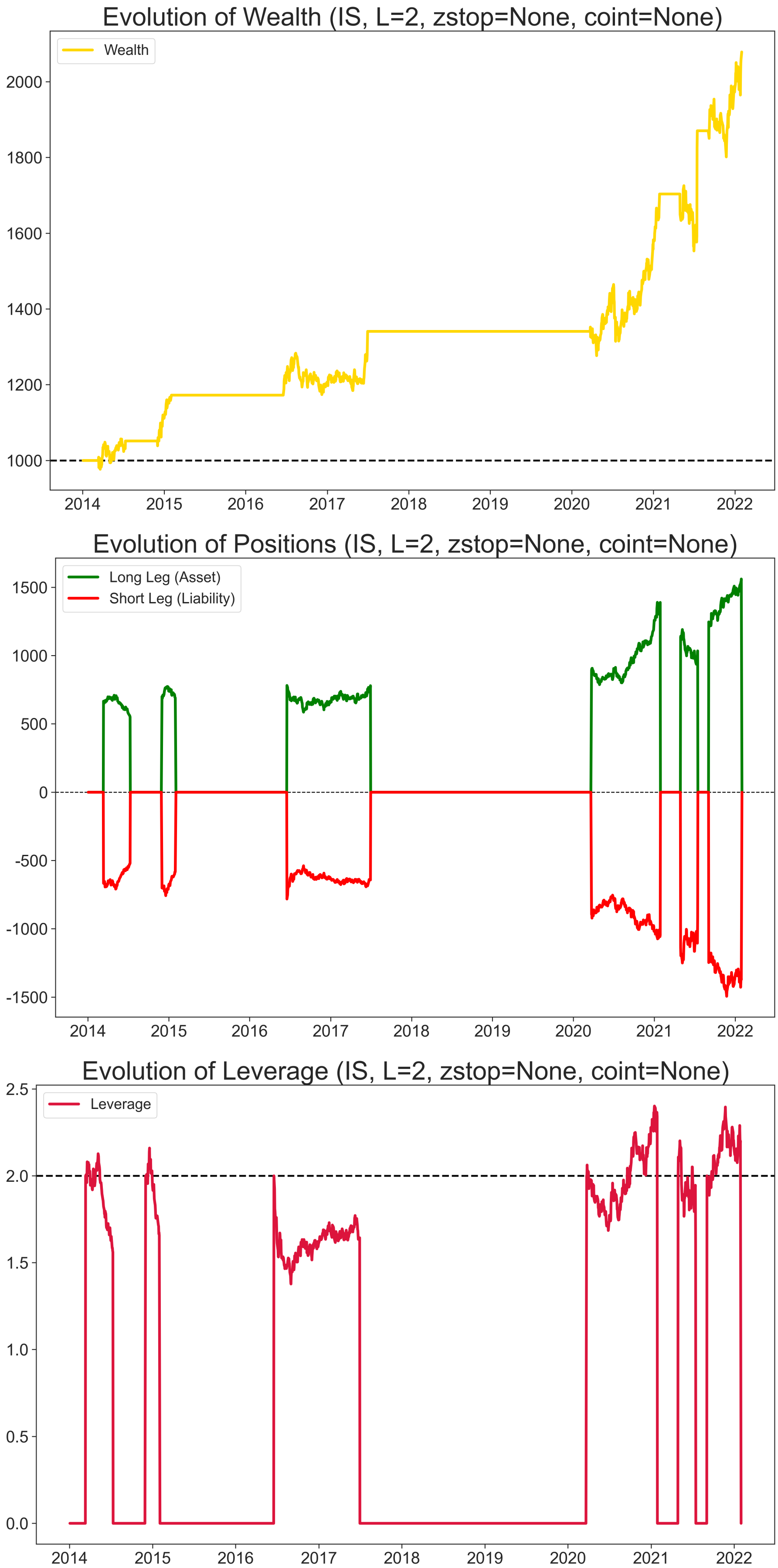

Figure 6: Strategy 1: $\tilde{z}^{in} = 1.5$, $L = 2$

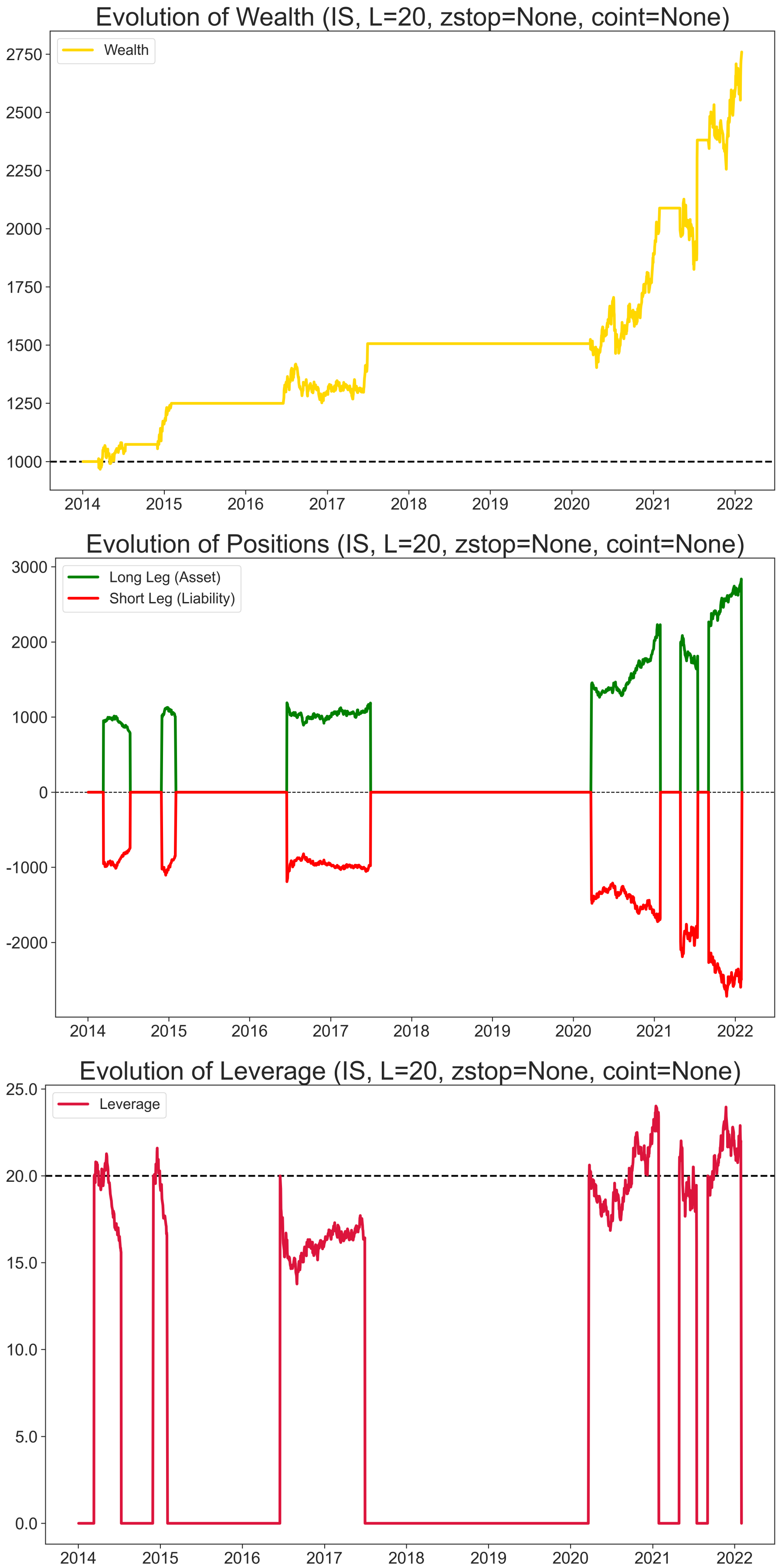

Figure 7: Strategy 2: $\tilde{z}^{in} = 1.5$, $L = 20$

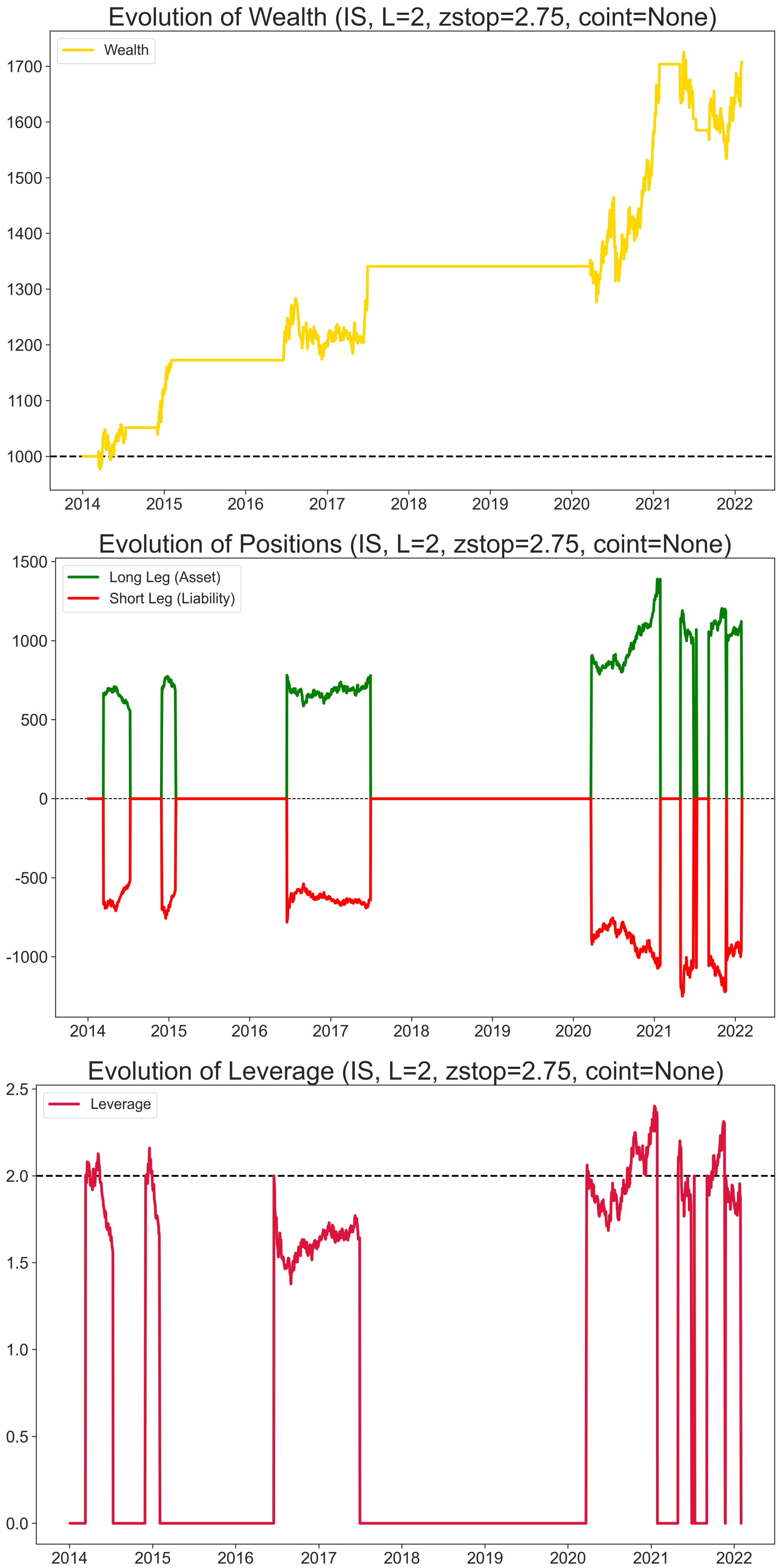

Figure 8: Strategy 3: $\tilde{z}^{in} = 1.5$, $L = 2$, $\tilde{z}_{stop} = 2.75$

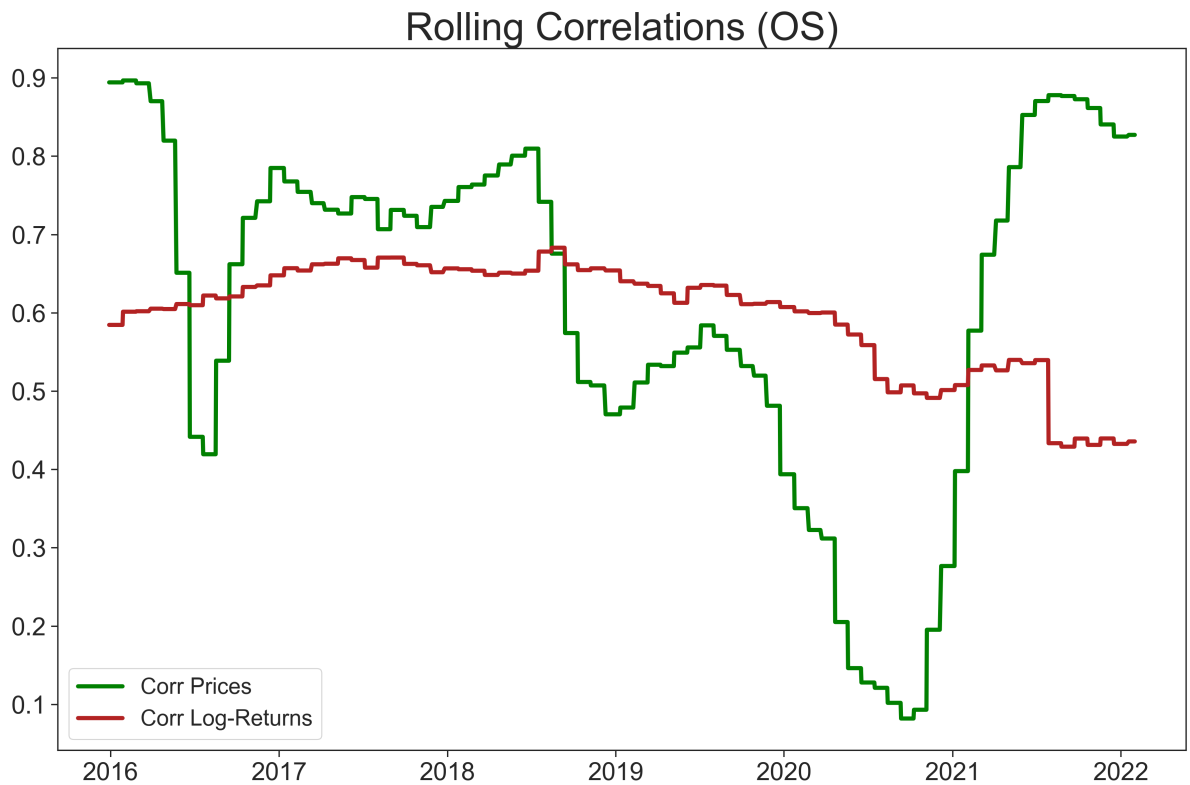

Figure 9: Rolling correlations (OS)

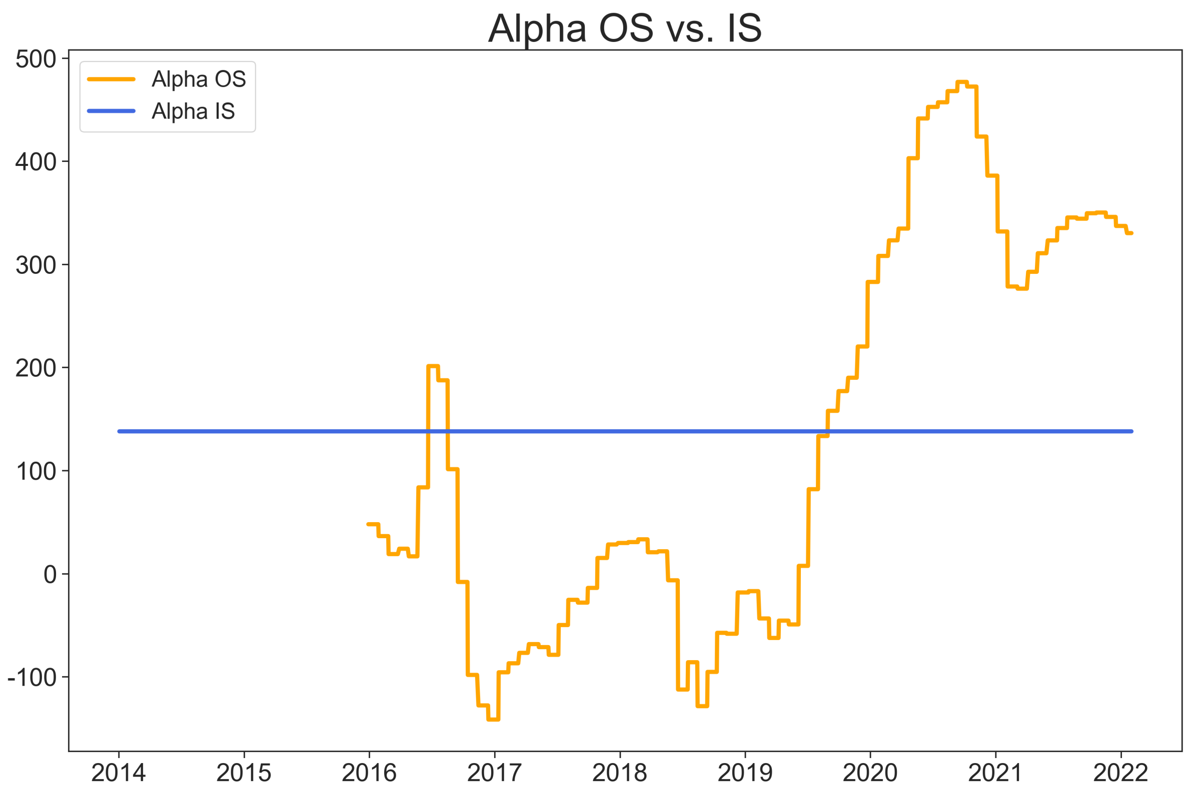

Figure 10: Alpha (OS)

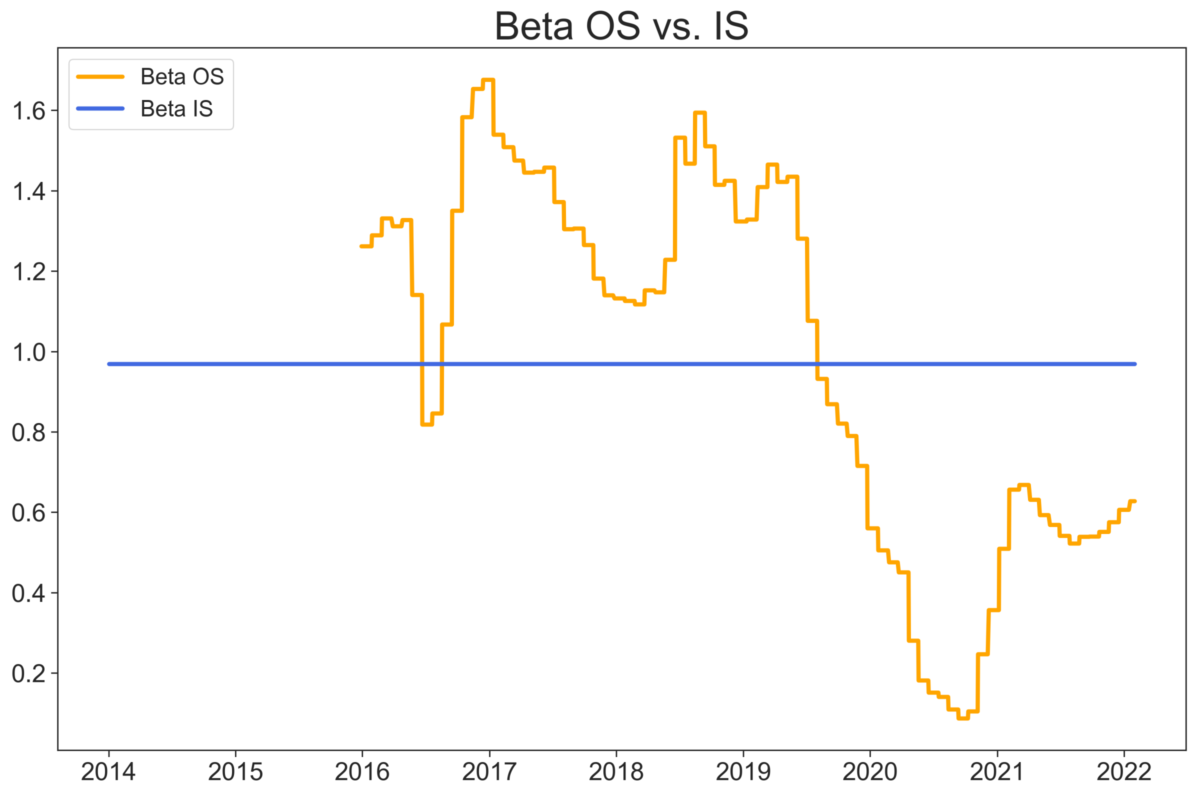

Figure 11: Beta (OS)

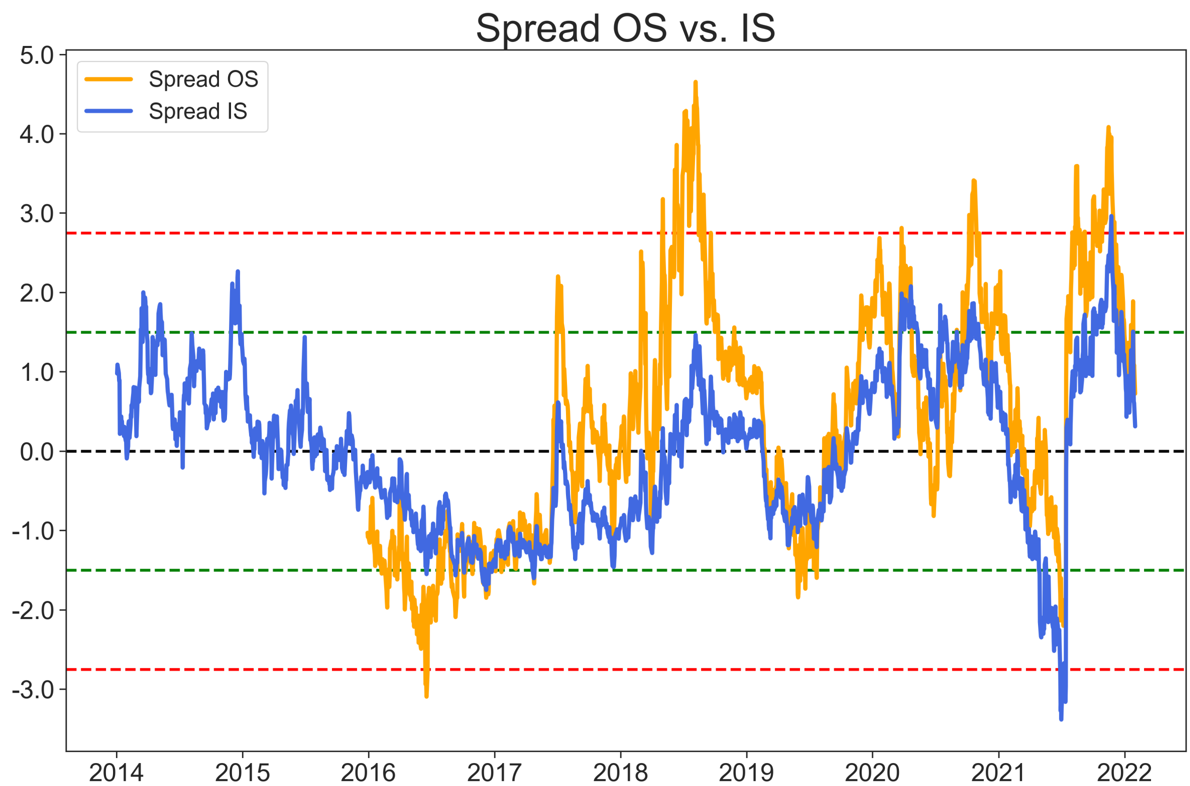

Figure 12: Spread OS vs IS

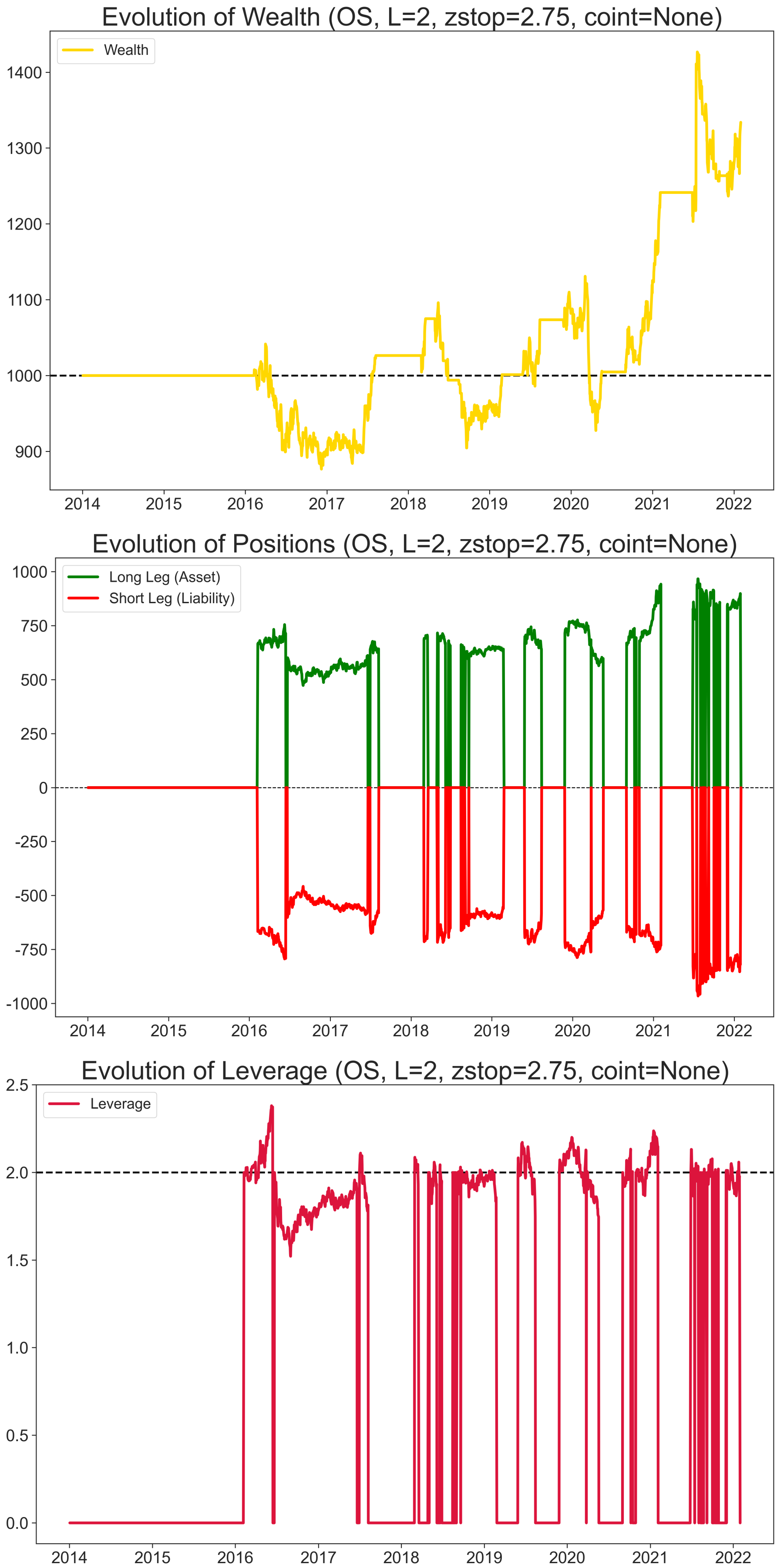

Figure 13: Strategy 4: $\tilde{z}^{in} = 1.5$, $L = 2$, $\tilde{z}_{stop} = 2.75$

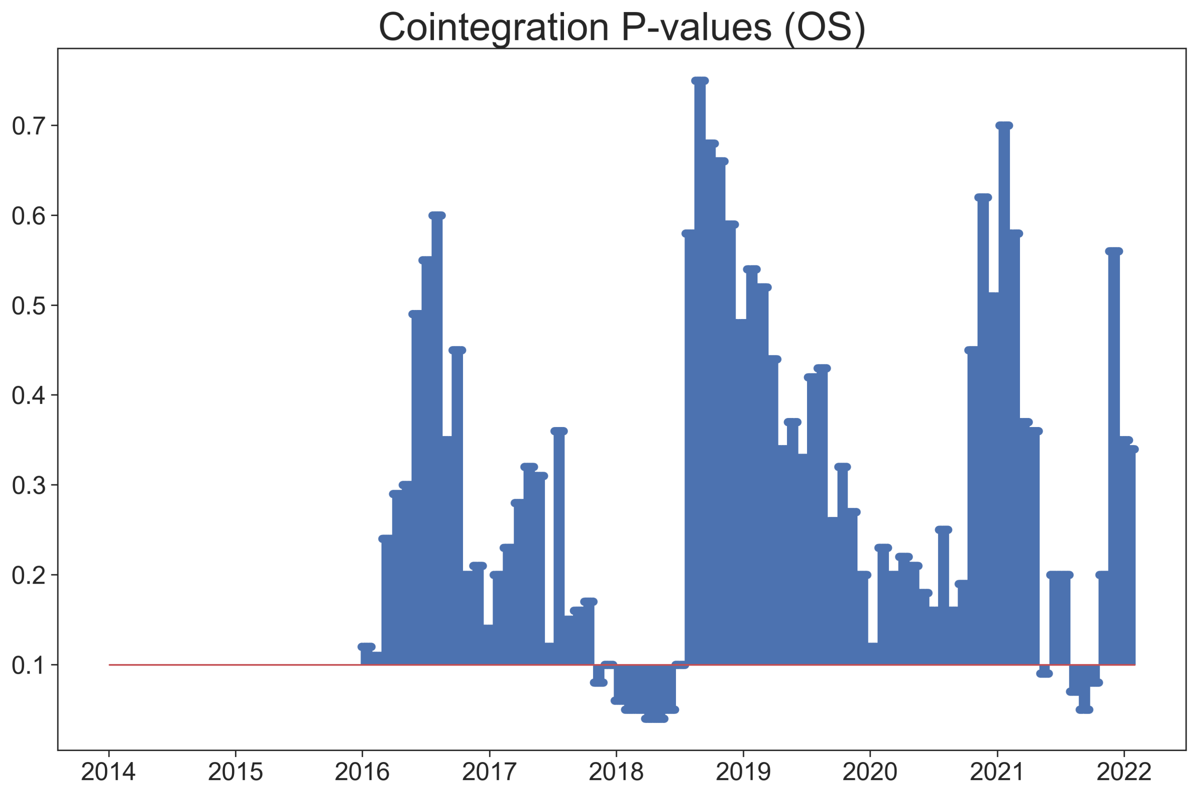

Figure 14: Rolling Cointegration (OS)

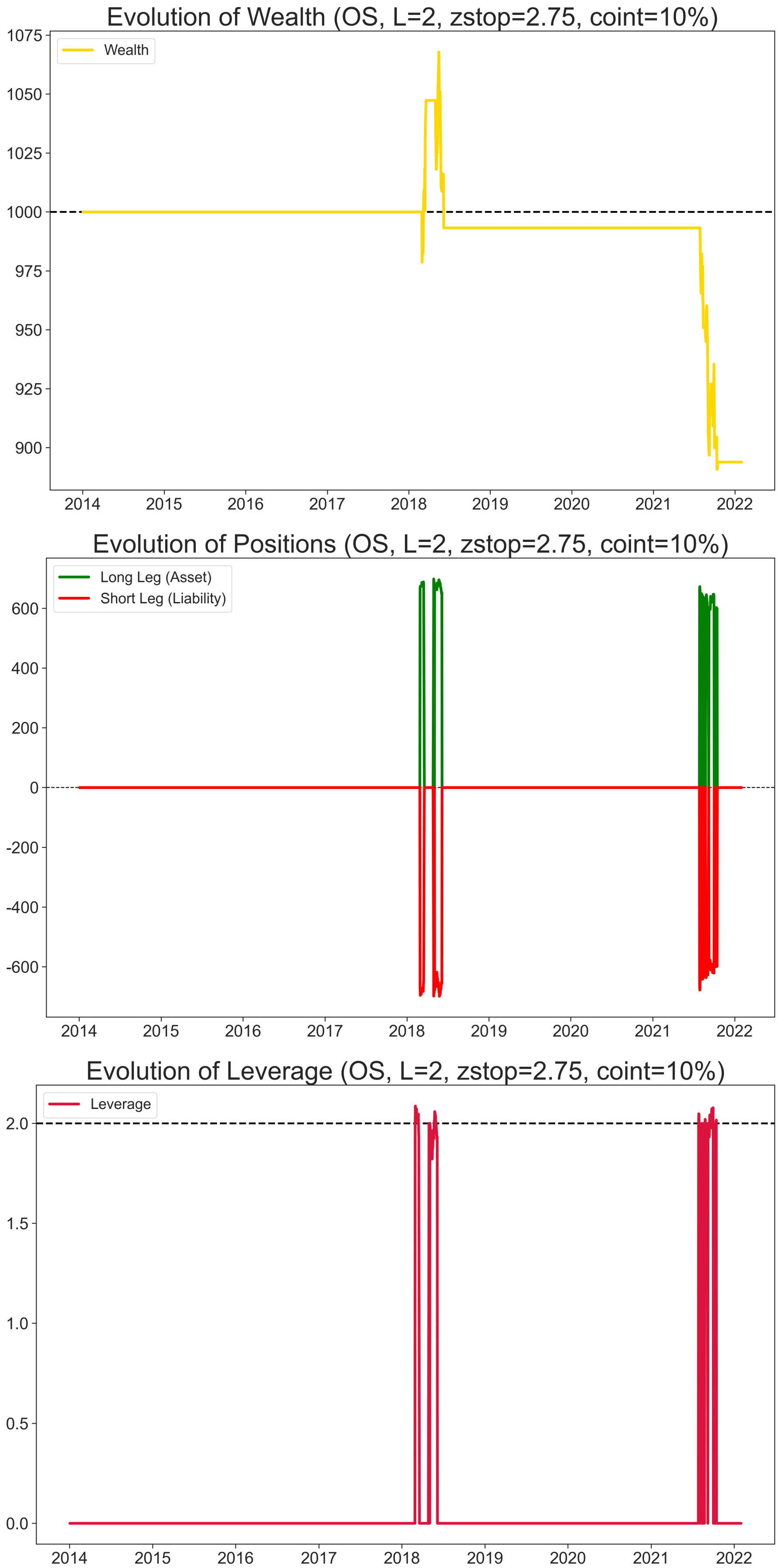

Figure 15: Strategy 5: $\tilde{z}^{in} = 1.5$, $L = 2$, $\tilde{z}_{stop} = 2.75$

	DF_TS	$CV_{1\%}$	$CV_{5\%}$	$CV_{10\%}$	P_Value	Reject H_0 1%	Reject H_0 5%	Reject H_0 10%
Corn-Wheat	-3.14	-3.89	-3.32	-3.04	0.08	False	False	True
Corn-Soybean	-2.97	-3.89	-3.32	-3.04	0.12	False	False	False
Corn-Coffee	-2.13	-3.89	-3.32	-3.04	0.46	False	False	False
Corn-Cacao	-1.14	-3.89	-3.32	-3.04	0.87	False	False	False
Wheat-Corn	-3.54	-3.89	-3.32	-3.04	0.03	False	True	True
Wheat-Soybean	-2.93	-3.89	-3.32	-3.04	0.13	False	False	False
Wheat-Coffee	-2.73	-3.89	-3.32	-3.04	0.19	False	False	False
Wheat-Cacao	-1.93	-3.89	-3.32	-3.04	0.56	False	False	False
Soybean-Corn	-3.12	-3.89	-3.32	-3.04	0.09	False	False	True
Soybean-Wheat	-2.60	-3.89	-3.32	-3.04	0.24	False	False	False
Soybean-Coffee	-3.14	-3.89	-3.32	-3.04	0.09	False	False	True
Soybean-Cacao	-1.34	-3.89	-3.32	-3.04	0.82	False	False	False
Coffee-Corn	-2.32	-3.89	-3.32	-3.04	0.36	False	False	False
Coffee-Wheat	-2.39	-3.89	-3.32	-3.04	0.33	False	False	False
Coffee-Soybean	-3.11	-3.89	-3.32	-3.04	0.09	False	False	True
Coffee-Cacao	-1.53	-3.89	-3.32	-3.04	0.75	False	False	False
Cacao-Corn	-2.51	-3.89	-3.32	-3.04	0.28	False	False	False
Cacao-Wheat	-2.56	-3.89	-3.32	-3.04	0.25	False	False	False
Cacao-Soybean	-2.55	-3.89	-3.32	-3.04	0.26	False	False	False
Cacao-Coffee	-2.58	-3.89	-3.32	-3.04	0.25	False	False	False

Table 11: Cointegration Results

	Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5
Profit	1078.72	1759.65	707.91	334.03	-106.10
ROE	107.87%	175.97%	70.79%	33.40%	-10.61%
Init wealth	1000.00	1000.00	1000.00	1000.00	1000.00
Final wealth	2078.72	2759.65	1707.91	1334.03	893.90
Min wealth	976.82	966.89	976.82	876.56	890.85
Max wealth	2078.72	2759.65	1725.67	1426.83	1067.88
Pos1 trades	4	4	5	29	12
Pos2 trades	2	2	3	4	0
Total trades	6	6	8	33	12

Table 12: Strategy Comparison

Strategy	Strategy Description
1	IS, L=2, $\tilde{z}_{in} = 1.5$
2	IS, L=20, $\tilde{z}_{in} = 1.5$
3	IS, L=2, $\tilde{z}_{in} = 1.5$, $\tilde{z}_{stop} = 2.75$
4	OS, L=2, $\tilde{z}_{in} = 1.5$, $\tilde{z}_{stop} = 2.75$
5	OS, L=2, $\tilde{z}_{in} = 1.5$, $\tilde{z}_{stop} = 2.75$, coint=10%

Table 13: Strategy Notes