4.1 Exponential Function (Part II)

In this lesson, we will focus on finding the equation of exponential function that describe real-world applications.

1. A population of 1000 is decreasing 35% each year. Find the function that represent the population after t year.

2. You invest \$3,000 in an investment account that pays 3% interest compounded quarterly.

a. Find the exponential function that represents the account value after t years.

b. How much will the account be worth in 10 years?

<u>Definitions:</u> Compounded Interest Formula

Compound Interest can be calculated using the formula

$$A(t) = a\left(1 + \frac{r}{k}\right)^{kt}$$

Where
$A(t)$ is the account value
t is measured in years
a is the starting amount on the account, often called the principle
r is the annual percentage rate (APR), also called the nominal rate
k is the number of compounding periods in one year.

3. Let's calculate the value of k which is the number of compounding periods in one year.

Frequency	Value of k
Annually	
Quarterly	
Monthly	
Daily	

4. You invest $3,000 in an investment account that pays 3% interest compounded in different compounding frequency.

a. Find the exponential function that represents the account value after t years for each frequency below. Calculate the account value after one year.

Frequency	Account value after t years	Account value after one year.
Annually		
Semiannually		
Quarterly		
Monthly		
Daily		

b. Which compounded frequency yields a higher account value after one year.

5. Let's us examine the value of $1 invested at 100% interest for 1 year.

Frequency	Account Value
Annually	$2
Quarterly	$2.441406
Monthly	$2.613035
Daily	$2.714567
Hourly	$2.718127
Once per minute	$2.718279
Once per second	$2.718282

As the frequency increases, the account values appear to approach ________ .

Definitions: Euler's Number, e

e is the letter used to represent the value that $\left(1+\frac{1}{k}\right)^k$ approaches as k gets big.

$$e \approx 2.718282$$

Continuous Growth Formula

Continuous growth can be calculated using the formula

$$f(x) = ae^{rx}$$

Where
a is the starting amount
r is the continuous growth rate

6. You invest $3,000 in an investment account that pays 3% interest compounded continuously. Find the function that represents the account value after t years.

7. Radon-222 decays at a continuous rate of 17.3% per day. How much will 100mg of Randon-222 decay to in 3 days.

8. If $1,000 were invested at 10%, the table below shows the value after 1 year at different compounding frequencies.

Frequency	Account value after 1 year
Annually	$1100
Semiannually	$1102.50
Quarterly	$1103.81
Monthly	$1104.71
Daily	$1105.16

What is the actual percentage increase for the daily compounding?

Definitions: Annual Percentage Yield

The annual percentage yield is the actual percentage a quantity increases in one year. It can be calculated as

$$APY = \left(1 + \frac{r}{k}\right)^k - 1$$

Where
r is the annual percentage rate (APR), also called the nominal rate
k is the number of compounding periods in one year.

9. Bank A offers an account paying 1.2% compounded quarterly. Bank B offers an account paying 1.1% compounded monthly. Which is offering a better rate?