THE CHINESE UNIVERSITY OF HONG KONG, SHENZHEN

DDA 3020

MACHINE LEARNING

Assignment 2 Report

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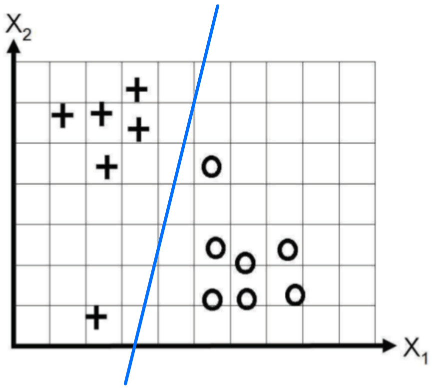
1 Written Problems

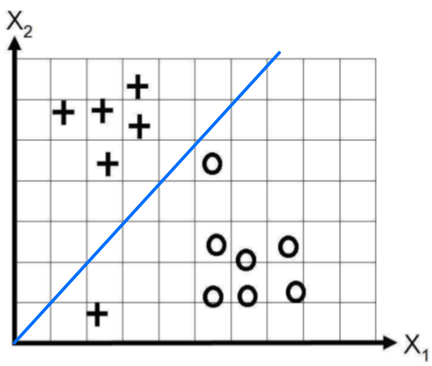
1、(1)



The answer is not unique.

There is no classification error made on the dataset.

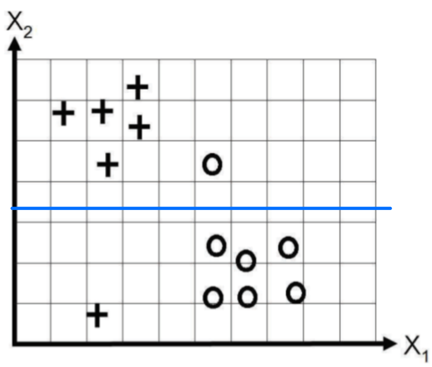
(2)



Since w_0 would be regularized to 0, so the boundary goes through the origin.

1 classification error has been made on the training set.

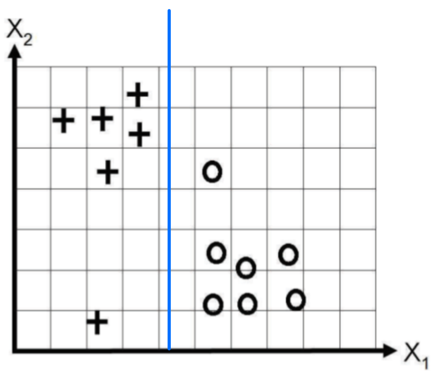
(3)



w_1 will be regularized to 0, the boundary will be horizontal.

2 classification error has been made on the training set.

(4)



w_2 will be regularized to 0, the boundary will be vertical.

0 classification error has been made on the training set.

2. (1) $\varphi(x_1)=[1,0,0]^T$ $\quad \varphi(x_2)=[1,2,2]^T$

$\varphi(x_2)-\varphi(x_1)=[0,2,2]^T$

Since w is orthogonal to the decision boundary, and $\varphi(x_1)$, $\varphi(x_2)$ have decided the decision boundary, so a possible vector parallel to w can be $[0,2,2]^T$

(2) $d_{12}=\|\varphi(x_2)-\varphi(x_1)\|_2 = \sqrt{0+4+4} = 2\sqrt{2}$

$\gamma=\frac{1}{2}d_{12}=\sqrt{2}$

Thus, the margin should be $\sqrt{2}$.

(3) $\frac{1}{\|w\|}=\sqrt{2} \Rightarrow \|w\| = \frac{\sqrt{2}}{2}$

From (1), we can set w to be $[0, 2a, 2a]^T$

$\|w\|_2=\sqrt{0+4a^2+4a^2} = 2\sqrt{2}\,|a| = \frac{\sqrt{2}}{2}$

$\Rightarrow |a|=\frac{1}{4} \Rightarrow a=\frac{1}{4}$ or $a=-\frac{1}{4}$

Since $y_i(w^T\varphi(x_i)+w_0)\geq 1$

$$\begin{cases} -1\cdot w_0\geq 1 \\ 1\cdot(4a+4a+w_0)\geq 1 \end{cases} \Rightarrow \begin{cases} -w_0\geq 1 \\ 8a+w_0\geq 1 \end{cases} \Rightarrow 8a\geq 2 \Rightarrow a\geq\frac{1}{4}$$

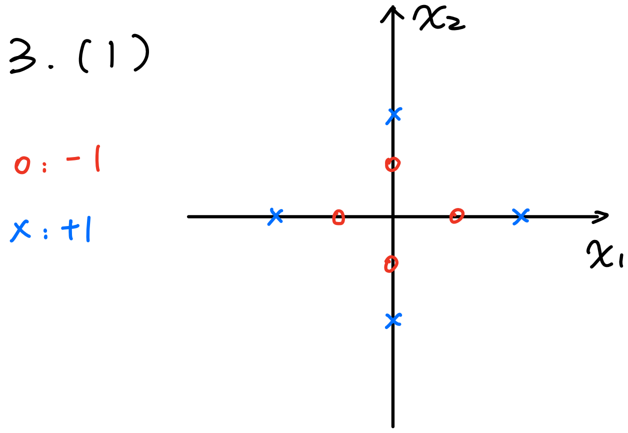
Thus, $a=\frac{1}{4}$, so $w=[0,\frac{1}{2},\frac{1}{2}]^T$

(4) $$\begin{cases} -1\cdot w_0\geq 1 \\ 1\cdot(2+w_0)\geq 1 \end{cases} \Rightarrow \begin{cases} w_0\leq -1 \\ w_0\geq -1 \end{cases} \Rightarrow w_0=-1$$

(5) $f(x)=w_0+w^T\varphi(x) = -1+\begin{bmatrix}0 & \frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}1\\ \sqrt{2}x \\ x^2\end{bmatrix}$

$=-1+\frac{\sqrt{2}}{2}x+\frac{1}{2}x^2$

3. (1)

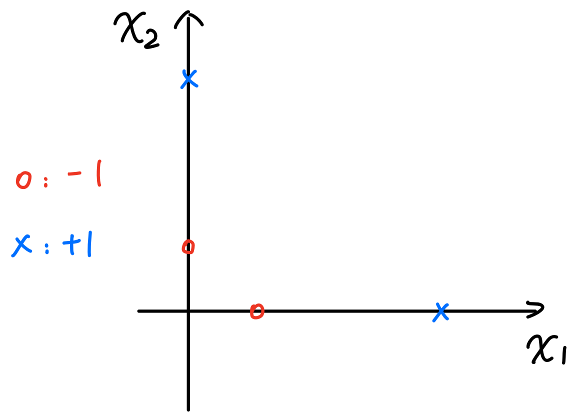


We cannot find a svm classifier without slack variable for this dataset, since the data is not linearly separable in the original dimension space.

(2)

Class −1 : $\begin{bmatrix} (1 & 0) \\ (0 & 1) \end{bmatrix}$ Class +1 : $\begin{bmatrix} (4 & 0) \\ (0 & 4) \end{bmatrix}$

Then we draw the plot below:



After the transformation by the kernel function, the data becomes linearly separable.

Then, we fit the SVM classifier, let $w = [w_1 \ w_2]^T$

$$\min_{w,b} \frac{1}{2}\|w\|^2$$

$$\begin{aligned} \text{s.t.} \quad & -1\cdot\left(w^T\cdot\begin{bmatrix}0\\1\end{bmatrix}+b\right)\geq 1 \qquad \alpha_1 \\ & -1\cdot\left(w^T\cdot\begin{bmatrix}1\\0\end{bmatrix}+b\right)\geq 1 \qquad \alpha_2 \\ & 1\cdot\left(w^T\cdot\begin{bmatrix}0\\4\end{bmatrix}+b\right)\geq 1 \qquad \alpha_3 \\ & 1\cdot\left(w^T\cdot\begin{bmatrix}4\\0\end{bmatrix}+b\right)\geq 1 \qquad \alpha_4 \end{aligned}$$

$$L = \frac{1}{2}(w_1^2+w_2^2) + \alpha_1(1+w_2+b) + \alpha_2(1+w_1+b) + \alpha_3(1-4w_2-b)$$
$$+\alpha_4(1-4w_1-b)$$

Stationarity:

$$\frac{\partial L}{\partial w_1} = w_1 + \alpha_2 - 4\alpha_4 = 0 \quad , \quad \frac{\partial L}{\partial w_2} = w_2 + \alpha_1 - 4\alpha_3 = 0$$

$$\frac{\partial L}{\partial b} = \alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 = 0$$

Feasibility: $\alpha_i \geq 0$, $1 - y_i(w^T x_i + b) \leq 0$, $i = 1, 2, 3, 4$.

Complementarity slackness:

$\alpha_1(1 + w_2 + b) = 0$, $\alpha_2(1 + w_1 + b) = 0$, $\alpha_3(1 - 4w_2 - b) = 0$, $\alpha_4(1 - 4w_1 - b) = 0$

After the calculation, $w = [\frac{2}{3} \ \frac{2}{3}]^T$, $b = -\frac{5}{3}$

The decision boundary should be $\frac{2}{3}x_1 + \frac{2}{3}x_2 - \frac{5}{3} = 0 \Rightarrow 2x_1 + 2x_2 - 5 = 0$

Let $x_1 = [1 \ 2]^T$, $\varphi(x_1) = [1 \ 4]^T$

$w^T \varphi(x_1) + b = \frac{2}{3} + \frac{8}{3} - \frac{5}{3} = \frac{5}{3} > 0$, so the label of $[1 \ 2]^T$ should be class $+1$.

4. The dual problem of the optimization problem in the question is.

$$\min_{w, b} \ \frac{1}{2}\|w\|^2$$

$$\text{s.t.} \ 1 - y_i(w^T x_i + b) \leq 0 \quad \forall i$$

$$L(w, b, \alpha) = \frac{1}{2}\|w\|^2 + \sum_i^m \alpha_i(1 - y_i(w^T x_i + b))$$

$$\frac{\partial L}{\partial w} = 0 \Rightarrow w = \sum_i^m \alpha_i y_i x_i$$

$$\frac{\partial L}{\partial b} = 0 \Rightarrow \sum_i^m \alpha_i y_i = 0$$

By strong duality theorem,

$$\sum_{n=1}^N \alpha_n - \frac{1}{2}\sum_{n=1}^N \sum_{m=1}^N \alpha_n \alpha_m y_n y_m x_n^T x_m = \frac{1}{2}\|w\|^2$$

Since $\sum_{n=1}^N \sum_{m=1}^N \alpha_n \alpha_m y_n y_m x_n^T x_m = (\sum_{i=1}^N \alpha_i y_i x_i)^T (\sum_{j=1}^N \alpha_j y_j x_j) = w^T w = \|w\|^2$,

so the above equation can be rewritten as:

$$\sum_{n=1}^{N} \alpha_n - \frac{1}{2}\|w\|^2 = \frac{1}{2}\|w\|^2$$

$$\|w\|^2 = \sum_{n=1}^{N} \alpha_n$$

Since γ is the margin, then $\gamma = \frac{1}{\|w\|}$

$$\Rightarrow \frac{1}{\gamma^2} = \|w\|^2 = \sum_{n=1}^{N} \alpha_n \qquad \text{Q.E.D.}$$

2 Programming

In this assignment, we construct several SVM models with different kernels and slack variables to classify the Iris dataset.
The basic form of SVM is given below:

$$min_{w,b} \frac{1}{2}\|w\|^2$$

$$s.t.\ 1 - y_i(w^T x_i + b) \le 0, \forall i$$

Where w is the coefficient of different features, b is the intercept of the hyperplane, $\boldsymbol{x}_i$ and y_i are the features and labels of the Iris data.

2.1 (SVM without slack variables)

1. The optimization problem

We first get the dual problem of the original problem stated above.
The dual Lagrange function is:

$$\mathcal{L}(w, b, \alpha) = \frac{1}{2}\|w\|^2 + \sum_i^m \alpha_i(1 - y_i(w^T x_i + b))$$

The primal and dual optimal solutions should satisfy KKT conditions:

Stationarity:

$$\frac{\partial \mathcal{L}}{\partial w} = 0 \Rightarrow w = \sum_i^m \alpha_i y_i x_i$$

$$\frac{\partial \mathcal{L}}{\partial b} = 0 \Rightarrow \sum_i^m \alpha_i y_i = 0$$

Feasibility:

$$\alpha_i \ge 0, 1 - y_i(w^T x_i + b) \le 0, \forall i$$

Complementary slackness:

$$\alpha_i(1 - y_i(w^T x_i + b)) = 0, \forall i$$

Then, the dual problem can be derived by substituting all the stationary conditions into the primal problem, finally we get:

$$\max_\alpha \sum_i^m \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i \alpha_j y_i y_j x_i^T x_j,$$

$$s.t. \sum_i^m \alpha_i y_i = 0, \alpha_i \ge 0, \forall i$$

Also, b is given by:

$$y_j(w^T x_i + b) = 1, \forall j \in S$$

$$\Rightarrow\ y_j\left(\sum_i^m \alpha_i y_i {x_i}^T x_j + b\right) = 1, \forall j \in S$$

Since $y_j{}^2 = 1$,

$$\sum_i^m \alpha_i y_i {x_i}^T x_j + b = y_j\ , \forall j \in S$$

$$\Rightarrow b = \frac{1}{|S|}\sum_{j\in S} y_j - \sum_i^m \alpha_j y_i {x_i}^T x_j$$

2. Data processing and results

I use OneVsRestClassifier and sklearn.svm.SVC to do the following problems. Since sklearn doesn't provide strict separation, we use C = 1e5, kernel = 'linear' to estimate the attributes and calculate errors. The result is shown below.

```
*SVM_linear - 记事本
文件(F) 编辑(E) 格式(O) 查看(V) 帮助(H)
training error: 0.041666666666666664
testing_error: 0.0
w_of_setosa: -0.04575352,0.52216766,-1.00294058,-0.46406882
b_of_setosa: 1.44746413
support_vector_indices_of_setosa: 78,13,31
w_of_versicolor: -0.75160959,-3.4187652,2.06714366,-4.63634689
b_of_versicolor: 11.31356887
support_vector_indices_of_versicolor: 1,2,3,14,15,20,28,31,32,81,82,83,84,86,88,89,
91,92,93,95,96,98,99,100,103,104,107,112,116,117,119,41,43,44,45,46,47,50,52,54,
55,56,57,58,59,62,64,65,66,68,69,71,73,74,75,76,77,78,79
w_of_virginica: -4.26389247,-6.19330415,8.64141632,12.56275266
b_of_virginica: -19.19066652
support_vector_indices_of_virginica: 50,52,57,63,97,99,103,108
```

When determining which class is linearly separable, we first calculate the train loss by 1-training score by the sklearn, then if the train loss is 0, it is linearly separable. The statistics are shown below:

```
Label 0 linearly separable: True
train_loss for label 0: 0.0
Label 1 linearly separable: False
train_loss for label 1: 0.21666666666666667
Label 2 linearly separable: False
train_loss for label 2: 0.016666666666666672
```

In conclusion, only label 0 (setosa) is linearly separable in the dataset.

2.2 (SVM with slack variables)

1. The optimization problem

For SVM with slack variables, we can simply modify it by adding a penalty term. The dual Lagrange function is:

$$\mathcal{L}(w,b,\alpha,\xi,\mu) = \frac{1}{2}\|w\|^2 + C\sum_i^m \xi_i + \sum_i^m [\alpha_i(1-\xi_i - y_i(w^T x_i + b)) - \mu_i\xi_i]$$

The primal and dual optimal solutions should satisfy KKT conditions:

Stationarity:

$$\frac{\partial \mathcal{L}}{\partial w} = 0 \Rightarrow w = \sum_{i}^{m} \alpha_i y_i x_i$$

$$\frac{\partial \mathcal{L}}{\partial b} = 0 \Rightarrow \sum_{i}^{m} \alpha_i y_i = 0$$

$$\frac{\partial \mathcal{L}}{\partial \xi_i} = 0 \Rightarrow \alpha_i = C - \mu_i, \forall i$$

Feasibility:

$$\alpha_i \geq 0, 1 - \xi_i - y_i(w^T x_i + b) \leq 0, \xi_i \geq 0, \mu_i \geq 0, \forall i$$

Complementary slackness:

$$\alpha_i(1 - \xi_i - y_i(w^T x_i + b)) = 0, \mu_i \xi_i = 0, \forall i$$

Then, the dual problem can be derived by substituting all the stationary conditions into the primal problem, finally we get:

$$\max_\alpha \sum_{i}^{m} \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i \alpha_j y_i y_j x_i^T x_j,$$

$$s.t. \sum_{i}^{m} \alpha_i y_i = 0,\ 0 \leq \alpha_i \leq C, \forall i$$

Also, b is given by: $M = \{i|0 < \alpha_i < C\}$

$$y_j(w^T x_i + b) = 1, \forall j \in M$$

$$\Rightarrow y_j\left(\sum_{i}^{m} \alpha_i y_i {x_i}^T x_j + b\right) = 1, \forall j \in M$$

Since ${y_j}^2 = 1$,

$$\sum_{i}^{m} \alpha_i y_i {x_i}^T x_j + b = y_j, \forall j \in M$$

$$\Rightarrow b = \frac{1}{|M|}\sum_{j \in M} y_j - \sum_{i}^{m} \alpha_j y_i {x_i}^T x_j$$

For the slack variables' calculation, $\max(0, 1 - y_i(w^T x_i + b))$ can return the value of the slack variables, where y_i is the label {-1, +1}, $w^T x_i + b$ is the result calculated by the input vectors x_i and the estimated parameters w and b.

2. Data processing and results

We use a for loop to cyclically write into the output, when deriving the slack variables' value, we use the decision_function(X) provided by sklearn.svn.SVC to directly get the term $w^T x_i + b$ and then modify the y_train that it only contains labels of {-1, +1}, finally calculate the value of slack variables.

Below are two sample outputs C = 0.3 and C = 0.7:

```
----- SVM with slack variable C=0.3-----
training error: 0.05
testing_error: 0.13333333333333333
w_of_setosa: -0.08447998,0.44581505,-0.8401827,-0.39556637
b_of_setosa: 1.56536027
support_vector_indices_of_setosa: 73,78,13,14,31
slack_variable_of_setosa: 0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.02039173,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.00033244,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.00016572,0.0,0.0,0.0,0.0,0.0,0.2933789,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
w_of_versicolor: -0.27866492,-1.41,0.4881854,-0.75418015
b_of_versicolor: 4.26123751
support_vector_indices_of_versicolor: 2,3,15,19,20,24,27,28,31,32,35,37,81,82,83,84,86,88,89,
91,92,93,95,96,97,98,99,100,102,103,104,105,107,108,109,112,113,116,117,119,41,42,43,44,45,46,
47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79
slack_variable_of_versicolor: 0.0,0.0,0.30168746,0.2945643,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
0.26817355,0.0,0.0,0.0,0.0,0.06977302,0.18290653,0.0,0.0,0.0,0.0,0.18163951,0.0,0.0,0.0,0.18163951,0.28891688,
0.0,0.0,1.17263237,0.00691688,0.0,0.0,0.0,0.15085144,0.0,2.43e-06,0.0,0.0,0.0,0.0,1.69377703,0.31419053,1.2889993,
1.61125276,1.88465386,1.46372193,0.914639,0.50292088,0.74996111,1.90911983,1.41431106,0.75851321,
0.99716327,1.49245492,1.71578737,1.2942462,1.67699667,1.4341879,1.03868375,0.62991316,0.60331369,1.16311211,0.93469467,1.40798895,2.21460591,1.81
w_of_virginica: -0.12864428,-0.42231489,1.52634679,1.34681045
b_of_virginica: -7.68506986
support_vector_indices_of_virginica: 43,46,48,50,52,53,56,57,58,63,64,65,66,67,71,80,81,83,89,91,93,96,97,103,104,108,111,112,116,117,119
slack_variable_of_virginica: 0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0
```

```
----- SVM with slack variable C=0.7-----
training error: 0.05
testing_error: 0.1
w_of_setosa: -0.04307697,0.48820506,-0.93871798,-0.43435899
b_of_setosa: 1.42161509
support_vector_indices_of_setosa: 78,13,31
slack_variable_of_setosa: 0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,8.1e-07,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,7e-07,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.12848638,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
w_of_versicolor: -0.39293791,-1.9218171,0.84620772,-1.47414042
b_of_versicolor: 6.06715976
support_vector_indices_of_versicolor: 2,3,15,19,20,24,27,28,31,35,37,81,
82,83,84,86,88,89,91,92,93,95,96,97,98,99,100,102,103,104,105,107,108,112,
116,117,119,41,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79
slack_variable_of_versicolor: 0.0,0.0,0.45288328,0.39548991,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.3961232,0.0,0.0,0.0,0.0,0.12964115,0.28252907,0.0,0.0,0.0,0.0,0.3
w_of_virginica: -0.34650663,-0.40290808,1.90192497,1.77461362
b_of_virginica: -8.97370076
support_vector_indices_of_virginica: 43,46,48,50,52,56,57,58,63,64,66,80,89,91,93,96,97,103,104,108,112,116,119
slack_variable_of_virginica: 0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0
```

2.3 (SVM with kernels and slack variables)

1. The optimization problem

The derivation of the dual problem of the kernel is similar to 2.2:

$$\max_\alpha \sum_i^m \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i \alpha_j y_i y_j k(x_i, x_j)$$

$$s.t. \sum_i^m \alpha_i y_i = 0, \alpha_i \geq 0, \forall i$$

The related kernel functions are:

Polynomial kernel:

$$k(x, x_i) = (1 + \frac{x^T x_i}{\sigma^2})^p, p > 0$$

Radical Basis Function (RBF) kernel:

$$k(x, x_i) = \exp\left\{-\frac{\|x - x_i\|^2}{2\sigma^2}\right\}$$

Sigmoidal kernel:

$$k(x, x_i) = \frac{1}{1 + exp^{-\frac{x^T x_i + b}{\sigma^2}}}$$

2. Data processing and results

(a) In this scenario, we set C = 1, kernel = 'poly', degree = 2, gamma = 1, and get the following output:

```
training error: 0.025
testing_error: 0.0
b_of_setosa: 1.22094132
support_vector_indices_of_setosa: 78,13,31
b_of_versicolor: 4.33667006
support_vector_indices_of_versicolor: 14,31,89,93,96,97,99,103,108,48,50,52,57,58,63,64
b_of_virginica: -10.42876523
support_vector_indices_of_virginica: 50,52,57,63,96,97,99,103,108
```

(b) In this scenario, we set C = 1, kernel = 'poly', degree = 3, gamma = 1, and get the following output:

```
training error: 0.008333333333333333
testing_error: 0.0
b_of_setosa: 1.13434963
support_vector_indices_of_setosa: 78,13,31
b_of_versicolor: 1.54426028
support_vector_indices_of_versicolor: 31,89,97,99,101,103,108,119,50,52,57,63,70
b_of_virginica: -6.11788922
support_vector_indices_of_virginica: 50,52,57,63,89,103,108,119
```

(c) In this scenario, we set C = 1, kernel = 'rbf', gamma = 0.5, and get the following output:

```
training error: 0.03333333333333333
testing_error: 0.03333333333333333
b_of_setosa: -0.33592638
support_vector_indices_of_setosa: 42,45,78,84,87,88,89,101,104,106,4,5,12,14,31
b_of_versicolor: -0.41536232
support_vector_indices_of_versicolor: 4,5,13,14,31,80,88,89,91,93,96,97,99,101,103,108,116,119,40,43,46,48,50,52,56,57,58,63,64,65,66,78
b_of_virginica: -0.30449079
support_vector_indices_of_virginica: 3,4,5,12,14,31,40,43,46,48,50,52,56,57,58,63,64,65,66,80,87,88,89,91,93,96,97,99,101,103,104,108,111,116,119
```

(d) In this scenario, we set C = 1, kernel = 'sigmoid', gamma = ‘auto’ (gamma = 0.25), and get the following output:

training error: 0.825
testing_error: 0.7666666666666667
b_of_setosa: -1.0
support_vector_indices_of_setosa: 80,81,82,83,84,85,86,87,88,89,90,91,92,93,
94,95,96,97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,
116,117,118,119,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39
b_of_versicolor: -1.0
support_vector_indices_of_versicolor: 80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,
95,96,97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,
117,118,119,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79
b_of_virginica: -1.00000003
support_vector_indices_of_virginica: 0,1,2,3,7,9,10,11,12,13,14,15,16,17,18,19,20,21,
24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,60,61,73,78,79,80,81,82,83,84,85,
86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119