MAT3007 Assignment 9

Due at noon (12pm), May 11th, Wednesday

Please note:

- Only questions 1 and 2 will be graded. Questions 3 and 4 are listed in case you would like to practice more.

Problem 1 (50pts). A gasoline truck has five compartments with capacities of 2500, 3000, 1400, 1600, and 3200 gallons of fuel, respectively. The truck must deliver three types of fuel—super, premium, and regular—to a customer with requests of 2800 (super), 4200 (premium), and 5000 (regular), all in gallons. If the truck fails to deliver what is requested, a contractual per-gallon penalty is incurred of \$10 (super), \$6 (premium), and \$8 (regular). The maximum allowed shortage of each type of gasoline is 500 (super), 400 (premium), and 300 (regular), again all in gallons. Each compartment in the truck can only carry one type of gasoline.

1. Formulate a mixed-integer program that will determine how the truck should be loaded to minimize the shortage cost.
2. Implement and solve your model using CVX. Report your solution "by hand" in a way that someone without optimization background can understand.

Problem 2 (50pts). Solve the following integer program via the branch-and-bound algorithm using an exact tolerance ($\epsilon = 0$). Form the branch-and-bound tree and indicate the solution associated with each node, and why each node is fathomed (pruned). You can use the simplex method or a solver to solve the linear programming relaxation. If you choose to solve the LP relaxation using the simplex method, please include your calculation in the answer. If you choose to solve it using CVX, please include your codes and the solution output.

$$
\begin{aligned}
\max_{x} \quad & 2x_1 + 3x_2 + 4x_3 + 7x_4 \\
\text{s.t.} \quad & 4x_1 + 6x_2 - 2x_3 + 8x_4 = 20 \\
& x_1 + 2x_2 - 6x_3 + 7x_4 = 10 \\
& x_1, x_2, x_3, x_4 \geq 0 \text{ and integers.}
\end{aligned}
$$

Problem 3. A region wants to plan its electric power capacity for the next T years. The region has a forecast of d_t (megawatts) for the required capacity in each year $t = 1, 2, \ldots, T$. There is a certain quantity of existing capacity [megawatts] available in year t, denoted by

e_t, $t = 1, 2, \ldots, T$, which consists of coal power plants. Power can be further supplied via a combination of two new sources: natural gas and nuclear power plants, with corresponding costs per megawatt of c_t^g and c_t^n, respectively. Natural gas plants run for 15 years, while nuclear plants run for 25 years. We must ensure that demand is met, and that at least 20% of the total capacity is nuclear in any given year. Furthermore, the magnitude (absolute value) of change in available natural gas and nuclear power capacity from one year to the next may differ by no more than Δg_t and Δn_t megawatts, respectively, where $\Delta g_t > 0$ and $\Delta n_t > 0$.

1. Formulate a linear program to minimize total costs while obeying the constraints outlined above. Use as decision variables x_t and y_t to denote the amount of natural gas and nuclear capacity, respectively, brought on-line in year t (megawatts). Define any additional decision variables you need.

2. We are considering the possibility of not incorporating nuclear capacity as part of our set of generation capability. If we do include nuclear capacity then it must obey the "20% rule," and if not, the 20% rule can be neglected. Formulate mixed integer programming constraints to capture this situation. Be sure the resulting model is a (single) *linear* integer program. You do not need to rewrite all of your model from part (a). Just indicate what changes and/or additions are needed. If you require one or more "big M" constraints, specify a reasonably small value for big M.

3. Let z_a^* denote the optimal cost from part (a) and z_b^* denote the optimal cost from part (b). Is $z_a^* \leq z_b^*$, $z_a^* \geq z_b^*$, $z_a^* = z_b^*$, or is it impossible to say? (Answer this question, even if your answers to parts (a) and (b) might not be fully correct!)

Problem 4. Figure 1 contains an incomplete branch-and-bound tree for an integer program, whose objective function is being maximized. The branch-and-bound algorithm selects subproblems (nodes) upon which to branch, using the best-first search rule; i.e., when there are multiple nodes that can be explored, we select the one with the most promising LP relaxation objective value.

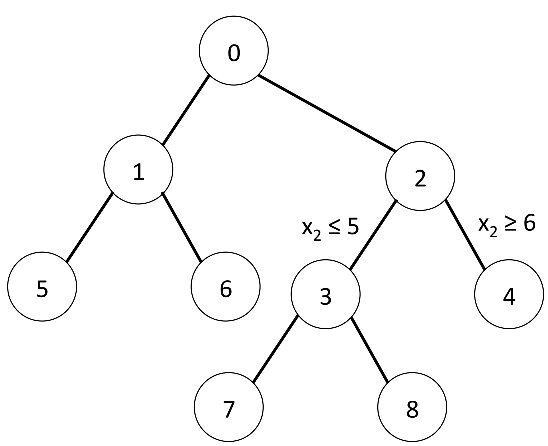

Figure 1: The figure depicts a partial branch-and-bound tree for an integer program.

1. Let $z_0^{LP}, z_1^{LP}, \ldots, z_8^{LP}$ denote the optimal objective function value of the linear programming relaxation associated with each of the nodes, and assume that the subproblem associated with each node has a feasible linear programming relaxation. *The numbers on the nodes denote the order in which the subproblems were solved in the branch-and-bound algorithm, using the best-first search rule.* Write all possible inequalities that relate $z_0^{LP}, z_1^{LP}, \ldots, z_8^{LP}$. For example, if you can infer that $z_5^{LP} \geq z_8^{LP}$ must hold then write that inequality. Briefly explain your reasoning.

2. Figure 1 indicates the branch from subproblem #2 involves decision variable x_2. What can you infer about the numerical value of x_2^{LP} in the solution to the linear programming relaxation of subproblem #2? Be as specific as possible.