MAT3007 Assignment 1

Due by noon (12pm), Feb 11th

For those questions that ask you to write MATLAB/Python codes to solve the problem, please attach the code to the homework. You also need to clearly state (write or type) the optimal solution and the optimal value you obtained. However, you do not need to attach the outputs in the command window of MATLAB/Python.

Problem 1 (20pts). A company produces two kinds of products. A product of the first type requires 1/4 hours of assembly labor, 1/8 hours of testing, and \$1.2 worth of raw materials. A product of the second type requires 1/3 hours of assembly, 1/3 hours of testing, and \$0.9 worth of raw materials. Given the current personnel of the company, there can be at most 90 hours of assembly labor and 80 hours of testing each day. Products of the first and second type have a market value of \$9 and \$8 respectively.

(a) Formulate a linear optimization that maximizes the daily profit of the company.

(b) Write the standard form of the LP you formulated in part (a)

(c) Consider the following modification to the original problem: Suppose that up to 50 hours of overtime assembly labor can be scheduled, at a cost of \$7 per hour. Can it be easily incorporated into the linear optimization formulation and how?

(d) Solve the LP using software (for the original problem).

Problem 2 (20pts). Reformulate the problem:

$$\begin{aligned} \text{minimize} \quad & 2x_2 + |x_1 - x_3| \\ \text{subject to} \quad & |x_1 + 2| + |x_2| \leq 5 \\ & x_3^2 \leq 1 \end{aligned}$$

as a linear optimization problem.

Problem 3 (20pts). Consider a school district with I neighborhoods, J schools, and G grades at each school. Each school j has a capacity of C_{jg} for grade g. In each neighborhood i, the student population of grade g is S_{ig}. Finally, the distance of school j from neighborhood i is d_{ij}. Formulate a linear optimization problem whose objective is to assign all students to schools, while minimizing the total distance traveled by all students.

Problem 4 (20pts). The China Railroad Ministry is in the process of planning relocations of freight cars among 5 regions of the country to get ready for the fall harvest. Table 1 shows the cost of moving a car between each pair of regions. Table 2 shows the current number of cars in each region and the number needed for harvest shipping.

From/To	1	2	3	4	5
1	-	10	12	17	34
2	10	-	18	8	46
3	12	18	-	9	27
4	17	8	9	-	20
5	34	46	27	20	-

Table 1: Costs of moving a car

	1	2	3	4	5
Present	115	385	410	480	610
Need	200	500	800	200	300

Table 2: Number of current and needed cars

Write down a linear optimization to compute the least costly way to move the cars such us the need is met. Solve the problem using software.

Problem 5 (20pts). Write a MATLAB/Python code to solve the vertex cover problem discussed in Lecture slides 1 (the graph is in Figure 1). When you solve it, use constraints $0 \leq x_i \leq 1$ rather than $x_i \in \{0, 1\}$. What are the optimal solution and optimal value returned by CVX (suppose we label the variables as $x_a, ..., x_j$)? What is the optimal value of the true problem? So whether one can remove the integrality constraint when solving this problem?