MAT3007 Assignment 5

Due at noon (12pm), March 23rd, Wednesday

Problem 1 (30pts). Farmer Clink owns a big farm with many acres of land. He is going to plant some acres with wheat, others with corn, or alfalfa. Each acre planted with wheat yields \$200 profit and requires 3 workers and 2 tons of fertilizer. Each acre planted with corn yields \$300 profit and requires 2 workers and 4 tons of fertilizer. Each acre planted with alfalfa yields \$100 profit and requires 6 workers and 8 tons of fertilizer. One hundred workers and 120 tons of fertilizer are available. Farmer Clink would like to maximize the profit by planning the correct amount of crops to plant.

1. Formulate a linear program to solve the problem above. Solve it using MATLAB or Python and report the optimal value and the optimal solution.
2. Formulate the dual problem of the linear programming model in part 1. Solve it using MATLAB or Python and report the optimal value and the optimal dual solution.
3. How much more profit can Farmer Clink get if he obtains one more worker?
4. How much more profit can Farmer Clink get if he obtains one more ton of fertilizer?
5. Suppose a policy is established by the government to encourage alfalfa planting. Will it be protable to plant alfalfa if the subsidy from the government for alfalfa is \$500 per acre in addition to the \$100 profit per acre?
6. Suppose Farmer Clink can also grow soybeans, which yield a profit of \$250 per acre and require 3 workers and 5 tons of fertilizer. Should Farmer Clink consider?

Problem 2 (20pts). Consider the following linear program:

$$\begin{aligned} \max \quad & 4x_1 + x_2 \\ \text{s.t.} \quad & 3x_1 + x_2 \leq 6 \\ & 5x_1 + 3x_2 \leq 15 \\ & x_1, x_2 \geq 0. \end{aligned}$$

1. Use the simplex method to solve the linear program. Report the optimal value and the optimal solution.
2. Formulate the dual problem of the linear program above. Use the optimality conditions and the optimal primal solution to obtain the optimal dual solution.

3. Determine the range of values of c_2 (the coefficient of x_2 in the primal objective function) for which the current basis remains optimal.

4. Determine the range of values of b_2 (the right-hand side parameter of the second constraint in the primal linear program) for which the current basis remains optimal.