MAT3007 Assignment 4

Due at noon (12pm), March 18th, Friday

Problem 1 (20pts). Consider the following linear program:

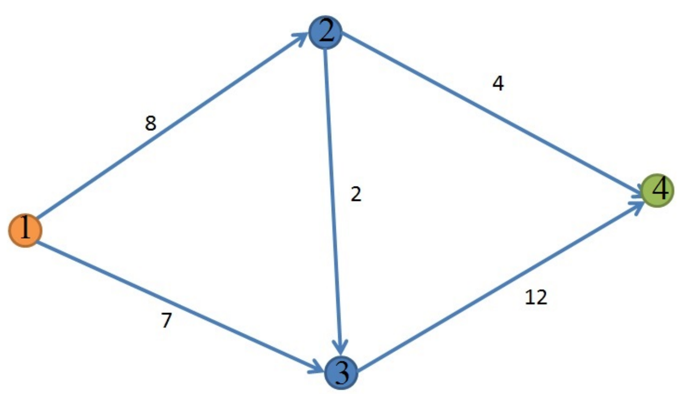
$$\begin{aligned} \max \quad & 5x_1 + 2x_2 + 5x_3 \\ \text{s.t.} \quad & 2x_1 + 3x_2 + x_3 \le 4 \\ & x_1 + 2x_2 + 3x_3 \le 7 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

1. What is the corresponding dual problem?

2. Solve the dual problem graphically.

3. Use complementarity slackness to solve the primal problem.

Problem 2 (25pts).



Consider the max flow problem on the graph below with the orange node (node 1) being the source node and the green node (node 4) being the terminal node. The number on each edge is its capacity.

1. Formulate the maximum flow problem as a linear program and write down its formulation. Solve it using MATLAB or Python and report the optimal solution.

2. Formulate the dual of the maximum flow problem and write down the dual problem's formulation. Solve it using MATLAB or Python and report dual problem's optimal solution.

3. Find the corresponding maximum flow and minimum cut based on the optimal solutions in part 1 and 2.

Problem 3 (20pts). Consider the linear program:

$$\begin{array}{ll} \max_x & c^\top x \\ \text{s.t.} & Ax \le b \\ & x \ge 0. \end{array} \tag{1}$$

Suppose linear program (1) is unbounded. Now consider the linear program:

$$\begin{array}{ll} \max_x & c^\top x \\ \text{s.t.} & Ax \le b' \\ & x \ge 0. \end{array} \tag{2}$$

Assume that linear program (2), under the modified right-hand side b', is feasible. Thus, model (2) either has a finite optimal solution or is unbounded. Are both situations possible? Explain your response in detail.

Problem 4 (20pts). Use linear program duality to show that exactly one of the following systems has a solution

1. $A\boldsymbol{x} \le \boldsymbol{b}$
2. $\boldsymbol{y}^\top A = 0$, $\boldsymbol{b}^\top \boldsymbol{y} < 0$, $\boldsymbol{y} \ge 0$

Hint: You can first show that it can't both have solutions. Then you show that if the second one is infeasible, the first one must be feasible.

Problem 5 (15pts). Consider the following linear program:

$$\begin{array}{lrrrll} \max\limits_x & -x_1 & -x_2 & +4x_3 & & \\ \text{s.t.} & x_1 & +x_2 & +2x_3 & \le 9 & : \pi_1 \\ & x_1 & +x_2 & -x_3 & \le 2 & : \pi_2 \\ & -x_1 & +x_2 & +x_3 & \le 4 & : \pi_3 \\ & & \multicolumn{2}{r}{x_1, x_2, x_3 \ge 0} & & \end{array}$$

An optimal solution to the dual of this linear program is $(\pi_1^*, \pi_2^*, \pi_3^*) = (1, 0, 2)$. *Use this information* to derive an optimal solution to the primal. Explain your reasoning. (You won't receive credit for solving the LP from scratch.)