Code Appendix

Nov 5, 2024

[17]:
```
import pandas as pd
import numpy as np
from gurobipy import Model, GRB, quicksum
from haversine import haversine, Unit
import warnings
warnings.filterwarnings("ignore")
```

1 Data Preprocessing

[18]:
```
#Data preprocessing
childcare = pd.read_csv(r'project1_new_datasets\new_child_care.csv')
employment = pd.read_csv(r'project1_new_datasets\new_employment.csv')
income = pd.read_csv(r'project1_new_datasets\new_income.csv')
population = pd.read_csv(r'project1_new_datasets\new_population.csv')
potential_loc = pd.read_csv(r'project1_new_datasets\new_potential_loc.csv')


population = population.iloc[:, :5].drop(['Total'], axis=1)
population['2w-12yrs'] = np.floor(population.iloc[:, 1:3].sum(axis=1)+3/
↪5*population.iloc[:, 3]).astype(int)
print(population)
demand_desert = pd.merge(population, employment, on='zip_code', how = 'outer')
demand_desert = pd.merge(demand_desert, income, on='zip_code', how = 'outer')
demand_desert['high_demand'] = (demand_desert['employment rate'] >= 0.
↪6)|(demand_desert['average income'] <= 60000)
demand_desert['high_demand'] = demand_desert['high_demand'].astype(int)
#Data cleaning
childcare=childcare[childcare['total_capacity']>0].reset_index(drop=True)
childcare_capacity = childcare.
↪groupby('zip_code')[['infant_capacity','toddler_capacity',
                                                        ␣
↪'preschool_capacity','school_age_capacity'
                                                        ,'children_capacity']].sum().
↪reset_index()
childcare_capacity['2w_5yr_cap'] = np.floor(childcare_capacity.iloc[:, 1:4].
↪sum(axis=1)+childcare_capacity['children_capacity']*5/12).astype(int)
```

```
childcare_capacity['2w_12yr_cap'] = np.floor(childcare_capacity.iloc[:, 1:6].
 ↪sum(axis=1)).astype(int)

demand_desert = pd.merge(demand_desert, childcare_capacity, on='zip_code', how␣
 ↪= 'outer')

demand_desert.reset_index(drop=True, inplace=True)

#def classify_desert(row):
#    if row['high_demand'] == 1:
#        return row['2w_12yr_cap'] <= row['2w-12yrs']*0.5
#    else:
#        return row['2w_12yr_cap'] <= row['2w-12yrs']*1/3
#
#demand_desert['desert'] = demand_desert.apply(classify_desert, axis=1).
 ↪astype(int)

demand_desert.to_csv(r'project1_new_datasets\demand_desert.csv', index=False)
```

```
      zip_code    -5   5-9  10-14  2w-12yrs
0        10001   744   784    942      2093
1        10002  2142  3046   3198      7106
2        10003  1440  1034    953      3045
3        10004   433   182    161       711
4        10005   484   204    229       825
…          …  …    …     …      …
1018     14767   101   219    168       420
1019     14770   137   197    223       467
1020     14772   256   253    224       643
1021     14805    31    16     29        64
1022     14806   127   111    154       330

[1023 rows x 5 columns]
```

[19]:
```
childcare.head()
```

[19]:
```
   zip_code  facility_id program_type facility_status  \
0     10001       837597         SACC    Registration
1     10001       661697         GFDC         License
2     10001       837329         SACC    Registration
3     10001       350076          FDC    Registration
4     10001       292419         SACC    Registration

                    facility_name      city school_district_name  \
0        I Have a Dream Foundation  New York          Manhattan 2
1     Chelsea Little Angels Day Care  New York          Manhattan 2
2   Bright Horizons at Hudson Yards  New York          Manhattan 2
```

```
3                    GRAMMAS HANDS  New York          Manhattan 2
4   The Hudson Guild @26th Street  New York          Manhattan 2

   infant_capacity  toddler_capacity  preschool_capacity  school_age_capacity  \
0                0                 0                   0                   84
1                0                 0                   0                    4
2                0                 0                   0                   17
3                0                 0                   0                    2
4                0                 0                   0                   79

   children_capacity  total_capacity   latitude  longitude
0                  0              84  40.748836 -73.999810
1                 12              16  40.748911 -74.001546
2                  0              17  40.752093 -74.002588
3                  6               8  40.748296 -74.001263
4                  0              79  40.749247 -74.001598
```

[20]:
```python
demand_desert.head()
```

[20]:
```
   zip_code    -5   5-9  10-14  2w-12yrs  employment rate  average income  \
0     10001   744   784    942      2093         0.595097   102878.033603
1     10002  2142  3046   3198      7106         0.520662    59604.041165
2     10003  1440  1034    953      3045         0.497244   114273.049645
3     10004   433   182    161       711         0.506661   132004.310345
4     10005   484   204    229       825         0.665833   121437.713311

   high_demand  infant_capacity  toddler_capacity  preschool_capacity  \
0            0                0                 0                   0
1            1                0                 0                  18
2            0                0                 0                   0
3            0                0                 0                   0
4            1                0                 0                   0

   school_age_capacity  children_capacity  2w_5yr_cap  2w_12yr_cap
0                  585                 24          10          609
1                 4508                203         102         4729
2                 1995                  0           0         1995
3                  263                  0           0          263
4                   39                  0           0           39
```

2 The Problem of Budgeting

[21]:
```python
#Problem 1
m1=Model("Budgeting")
x={}
y={}
```

```
#Decision variables
for i in range(len(childcare)):
    x[1,i]=m1.addVar(vtype=GRB.INTEGER,name=f"new slots at facility {i}")
    x[2,i]=m1.addVar(vtype=GRB.INTEGER,name=f"new slots for children under 5 at␣
 ↪facility {i}")
for j in range(len(demand_desert)):
    y[1,j]=m1.addVar(vtype=GRB.INTEGER,name=f"newly built small facilities in␣
 ↪{j}")
    y[2,j]=m1.addVar(vtype=GRB.INTEGER,name=f"newly built medium facilities in␣
 ↪{j}")
    y[3,j]=m1.addVar(vtype=GRB.INTEGER,name=f"newly built large facilities in␣
 ↪{j}")
#Objective function
m1.setObjective(
    quicksum(65000*y[1,j]+95000*y[2,j]+115000*y[3,j] for j in␣
 ↪range(len(demand_desert)))+
        quicksum(200*x[1,i]+100*x[2,i]+20000*(x[1,i]/
 ↪childcare["total_capacity"][i])
            for i in range(len(childcare))),GRB.MINIMIZE
)
#Constraints
for i in range(len(childcare)):
    m1.addConstr(x[1,i]<=0.2*childcare["total_capacity"][i],f"Maximum expansion␣
 ↪rate {i}")
    m1.addConstr(x[2,i]-x[1,i]<=0)
    m1.addConstr(childcare["total_capacity"][i]+x[1,i]<=
                 max(childcare["total_capacity"][i],500),f"Maximum slots {i}")
    m1.addConstr(x[1,i]>=0,f"non-negativity {i}_1")
    m1.addConstr(x[2,i]>=0,f"non-negativity {i}_2")

care_reg=childcare.groupby("zip_code")
for j in range(len(demand_desert)):
    m1.addConstr(y[1,j]>=0,f"non-negativity {j}_y_1")
    m1.addConstr(y[2,j]>=0,f"non-negativity {j}_y_2")
    m1.addConstr(y[3,j]>=0,f"non-negativity {j}_y_3")
    jcare=care_reg.get_group(demand_desert.iloc[j,0]).index
    #high demand or not(changing greater than to greater than or equal)
    if demand_desert["high_demand"][j]==1:
        m1.
 ↪addConstr(demand_desert["2w_12yr_cap"][j]+100*y[1,j]+200*y[2,j]+400*y[3,j]+
                         quicksum(x[1,l] for l in jcare)>=int(1/
 ↪2*(demand_desert["2w-12yrs"][j]))+1)
    else:
        m1.
 ↪addConstr(demand_desert["2w_12yr_cap"][j]+100*y[1,j]+200*y[2,j]+400*y[3,j]+
```

```
                          quicksum(x[1,l] for l in jcare)>=int(1/
↪3*(demand_desert["2w-12yrs"][j]))+1)
    m1.addConstr(demand_desert["2w_5yr_cap"][j]+50*y[1,j]+100*y[2,j]+200*y[3,j]+
                          quicksum(x[2,l] for l in jcare)>=2/
↪3*(demand_desert["-5"][j]))
m1.optimize()
```

```
Gurobi Optimizer version 11.0.3 build v11.0.3rc0 (win64 - Windows 10.0
(19045.2))

CPU model: Intel(R) Core(TM) i7-10875H CPU @ 2.30GHz, instruction set
[SSE2|AVX|AVX2]
Thread count: 8 physical cores, 16 logical processors, using up to 16 threads

Optimize a model with 78890 rows, 32579 columns and 127247 nonzeros
Model fingerprint: 0xc733106a
Variable types: 0 continuous, 32579 integer (0 binary)
Coefficient statistics:
  Matrix range     [1e+00, 4e+02]
  Objective range  [1e+02, 1e+05]
  Bounds range     [0e+00, 0e+00]
  RHS range        [6e-01, 1e+04]
Found heuristic solution: objective 3.605250e+08
Presolve removed 78600 rows and 32000 columns
Presolve time: 0.84s
Presolved: 290 rows, 579 columns, 1158 nonzeros
Found heuristic solution: objective 3.159789e+08
Variable types: 0 continuous, 579 integer (88 binary)
Found heuristic solution: objective 3.159489e+08

Root relaxation: objective 3.157883e+08, 289 iterations, 0.00 seconds (0.00 work
units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 3.1579e+08    0    1 3.1595e+08 3.1579e+08  0.05%     -    0s
H    0     0                    3.158804e+08 3.1579e+08  0.03%     -    0s
H    0     0                    3.158466e+08 3.1579e+08  0.02%     -    0s
H    0     0                    3.157883e+08 3.1579e+08  0.00%     -    0s
     0     0 3.1579e+08    0    1 3.1579e+08 3.1579e+08  0.00%     -    0s

Explored 1 nodes (289 simplex iterations) in 0.87 seconds (0.22 work units)
Thread count was 16 (of 16 available processors)

Solution count 6: 3.15788e+08 3.15847e+08 3.1588e+08 … 3.60525e+08
```

```
Optimal solution found (tolerance 1.00e-04)
Best objective 3.157883173882e+08, best bound 3.157883173882e+08, gap 0.0000%
```

[22]:

```python
# Initialize lists to store results for CSV
facility_expansion_data = []
new_facilities_data = []

# Assuming the optimization model has been solved, let's collect the data
if m1.status == GRB.OPTIMAL:
    # Gather facility expansion data
    for i in range(len(childcare)):
        facility_expansion_data.append({
            "Facility ID": i,
            "Expanded Slots": x[1, i].x,
            "Slots for Children Under 5": x[2, i].x
        })

    # Gather data on new facilities
    for j in range(len(demand_desert)):
        new_facilities_data.append({
            "Region Zip Code": demand_desert['zip_code'][j],
            "Small Facilities": y[1, j].x,
            "Medium Facilities": y[2, j].x,
            "Large Facilities": y[3, j].x
        })

    # Create DataFrames
    facility_expansion_df = pd.DataFrame(facility_expansion_data)
    new_facilities_df = pd.DataFrame(new_facilities_data)

    # Save to CSV files
    facility_expansion_df.to_csv("facility_expansion_p1.csv", index=False)
    new_facilities_df.to_csv("new_facilities_p1.csv", index=False)
else:
    print("No optimal solution found")
```

[23]:

```python
facility_expansion_df
```

[23]:

	Facility ID	Expanded Slots	Slots for Children Under 5
0	0	11.0	11.0
1	1	-0.0	0.0
2	2	-0.0	0.0
3	3	-0.0	0.0
4	4	0.0	-0.0
…	…	…	…
14750	14750	0.0	-0.0
14751	14751	0.0	0.0

```
14752        14752          0.0                        0.0
14753        14753         -0.0                        0.0
14754        14754          0.0                       -0.0

[14755 rows x 3 columns]
```

[24]:
```python
new_facilities_df
```

[24]:
```
      Region Zip Code  Small Facilities  Medium Facilities  Large Facilities
0               10001              -0.0               -0.0               2.0
1               10002              -0.0               -0.0               3.0
2               10003              -0.0               -0.0               4.0
3               10004               1.0               -0.0               1.0
4               10005               0.0                0.0               2.0
…                 …                 …                 …                 …
1018            14767               0.0                1.0              -0.0
1019            14770               0.0                1.0              -0.0
1020            14772               0.0                0.0               1.0
1021            14805               1.0               -0.0              -0.0
1022            14806               0.0                1.0              -0.0

[1023 rows x 4 columns]
```

3 The Problem of Realistic Capacity Expansion and Distance

[25]:
```python
#Problem 2
m2=Model("Realistic Capacity Expansion and Distance")
x={}
y={}
a={}
#Decision variables
#a:at the upper bound of section 1 and 2
for i in range(len(childcare)):
    x[1,i]=m2.addVar(vtype=GRB.INTEGER,name=f"new slots at facility {i} between␣
 ↪0% and 10%")
    x[2,i]=m2.addVar(vtype=GRB.INTEGER,name=f"new slots at facility {i} between␣
 ↪10% and 15%")
    x[3,i]=m2.addVar(vtype=GRB.INTEGER,name=f"new slots at facility {i} between␣
 ↪15% and 20%")
    x[4,i]=m2.addVar(vtype=GRB.INTEGER,name=f"new slots for children under 5 at␣
 ↪facility {i}")
    a[1,i]=m2.addVar(vtype=GRB.BINARY,name=f"10%")
    a[2,i]=m2.addVar(vtype=GRB.BINARY,name=f"15%")
for j in range(len(potential_loc)):
    y[1,j]=m2.addVar(vtype=GRB.BINARY,name=f"build a small facility in {j}")
    y[2,j]=m2.addVar(vtype=GRB.BINARY,name=f"build a medium facility in {j}")
```

```
    y[3,j]=m2.addVar(vtype=GRB.BINARY,name=f"build a large facility in {j}")
#Objective function
m2.setObjective(
    quicksum(65000*y[1,j]+95000*y[2,j]+115000*y[3,j] for j in␣
 ↪range(len(potential_loc)))+
       ␣
 ↪quicksum(200*x[1,i]+400*x[2,i]+1000*x[3,i]+100*x[4,i]+20000*((x[1,i]+x[2,i]+x[3,i])/
 ↪childcare["total_capacity"][i])
            for i in range(len(childcare))),GRB.MINIMIZE
)
#Constraints
for i in range(len(childcare)):
    m2.addConstr(x[1,i]+x[2,i]+x[3,i]<=0.
 ↪2*childcare["total_capacity"][i],f"Maximum expansion rate {i}")
    m2.addConstr(x[4,i]-x[1,i]-x[2,i]-x[3,i]<=0)
    m2.addConstr(childcare["total_capacity"][i]+x[1,i]+x[2,i]+x[3,i]<=
                 max(childcare["total_capacity"][i],500),f"Maximum slots {i}")

care_reg=childcare.groupby("zip_code")
new_pos=potential_loc.groupby("zip_code")
for j in range(len(demand_desert)):
    jcare=care_reg.get_group(demand_desert.iloc[j,0]).index
    jpos=new_pos.get_group(demand_desert.iloc[j,0]).index
    #high demand or not(changing greater than to greater than or equal)
    if demand_desert["high_demand"][j]==1:
        m2.
 ↪addConstr(demand_desert["2w_12yr_cap"][j]+quicksum(100*y[1,k]+200*y[2,k]+400*y[3,k]␣
 ↪for k in jpos)+
                     quicksum(x[1,l]+x[2,l]+x[3,l] for l in jcare)>=int(1/
 ↪2*(demand_desert["2w-12yrs"][j]))+1)
    else:
        m2.
 ↪addConstr(demand_desert["2w_12yr_cap"][j]+quicksum(100*y[1,k]+200*y[2,k]+400*y[3,k]␣
 ↪for k in jpos)+
                     quicksum(x[1,l]+x[2,l]+x[3,l] for l in jcare)>=int(1/
 ↪3*(demand_desert["2w-12yrs"][j]))+1)
    m2.
 ↪addConstr(demand_desert["2w_5yr_cap"][j]+quicksum(50*y[1,k]+100*y[2,k]+200*y[3,k]␣
 ↪for k in jpos)+
                     quicksum(x[4,l] for l in jcare)>=2/
 ↪3*(demand_desert["-5"][j]))

#Only one type at most
for j in range(len(potential_loc)):
    m2.addConstr(y[1,j]+y[2,j]+y[3,j]<=1,f"One type for {j}")
```

```
#piecewise
for i in range(len(childcare)):
    # Constraints for x[1,i]
    m2.addConstr(x[1,i]>=a[1,i]*int(0.1*childcare["total_capacity"][i]),␣
↪f"200_1")
    m2.addConstr(x[1,i]<=int(0.1*childcare["total_capacity"][i]),f"200_2")
    m2.addConstr(x[2,i]>=(int(0.15*childcare["total_capacity"][i])-int(0.
↪1*childcare["total_capacity"][i]))*a[2,i],f"400_1")
    m2.addConstr(x[2,i]<=(int(0.15*childcare["total_capacity"][i])-int(0.
↪1*childcare["total_capacity"][i]))*a[1,i],f"400_2")
    m2.addConstr(x[3,i]>=0,f"1000_1")
    m2.addConstr(x[3,i]<=(int(0.2*childcare["total_capacity"][i])-int(0.
↪15*childcare["total_capacity"][i]))*a[2,i],f"1000_2")

#Distance
for j in range(len(demand_desert)):
    jcare=care_reg.get_group(demand_desert.iloc[j,0]).index
    jpos=new_pos.get_group(demand_desert.iloc[j,0]).index
    #between original and new facilities
    for k in jcare:
        if np.isnan(childcare["latitude"][k]) or np.
↪isnan(childcare["longitude"][k]):
            continue
        for l in jpos:
            ␣
↪dist0=haversine((childcare["latitude"][k],childcare["longitude"][k]),
                                ␣
↪(potential_loc["latitude"][l],potential_loc["longitude"][l]),
                             unit=Unit.MILES)
            if dist0<0.06:
                m2.addConstr(y[1,l]+y[2,l]+y[3,l]==0)
    #between new facilities
    for k in jpos:
        for l in jpos:
            if l>k:
                ␣
↪dist1=haversine((potential_loc["latitude"][k],potential_loc["longitude"][k]),
                                    ␣
↪(potential_loc["latitude"][l],potential_loc["longitude"][l]),
                                 unit=Unit.MILES)
                #Can't be chosen at the same time if dist1<0.06
                if dist1<0.06:
                    m2.addConstr(y[1,k]+y[2,k]+y[3,k]+y[1,l]+y[2,l]+y[3,l]<=1)
m2.optimize()
```

```
Gurobi Optimizer version 11.0.3 build v11.0.3rc0 (win64 - Windows 10.0
(19045.2))
```

```
CPU model: Intel(R) Core(TM) i7-10875H CPU @ 2.30GHz, instruction set
[SSE2|AVX|AVX2]
Thread count: 8 physical cores, 16 logical processors, using up to 16 threads

Optimize a model with 282106 rows, 395430 columns and 1519042 nonzeros
Model fingerprint: 0x40b707a9
Variable types: 0 continuous, 395430 integer (336410 binary)
Coefficient statistics:
  Matrix range     [1e+00, 4e+02]
  Objective range  [1e+02, 1e+05]
  Bounds range     [1e+00, 1e+00]
  RHS range        [6e-01, 1e+04]
Presolve removed 277075 rows and 387646 columns (presolve time = 50s) …
Presolve removed 277070 rows and 387643 columns
Presolve time: 49.96s
Presolved: 5036 rows, 7787 columns, 21580 nonzeros
Variable types: 0 continuous, 7787 integer (5686 binary)
Found heuristic solution: objective 3.320767e+08
Found heuristic solution: objective 3.311750e+08
Found heuristic solution: objective 3.293641e+08

Root simplex log…

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    3.0628423e+08   4.117375e+03   0.000000e+00     50s
    1943    3.2023925e+08   0.000000e+00   0.000000e+00     50s

Root relaxation: objective 3.202392e+08, 1943 iterations, 0.02 seconds (0.01
work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 3.2024e+08    0    8 3.2936e+08 3.2024e+08  2.77%     -   50s
H    0     0                    3.205571e+08 3.2024e+08  0.10%     -   50s
H    0     0                    3.203026e+08 3.2024e+08  0.02%     -   50s
H    0     0                    3.202693e+08 3.2024e+08  0.01%     -   50s

Explored 1 nodes (1943 simplex iterations) in 50.34 seconds (20.49 work units)
Thread count was 16 (of 16 available processors)

Solution count 6: 3.20269e+08 3.20303e+08 3.20557e+08 … 3.32077e+08

Optimal solution found (tolerance 1.00e-04)
Best objective 3.202693370780e+08, best bound 3.202392497221e+08, gap 0.0094%
```

[26]:
```
facility_expansion_data_p2 = []
new_facilities_data_p2 = []

# Assuming the optimization model has been solved, let's collect the data for␣
↪Problem 2
if m2.status == GRB.OPTIMAL:
    # Gather facility expansion data
    for i in range(len(childcare)):
        facility_expansion_data_p2.append({
            "Facility ID": i,
            "Expanded Slots": x[1, i].x + x[2, i].x + x[3, i].x,
            "Slots for Children Under 5": x[4, i].x
        })

    # Gather data on new facilities
    for j in range(len(potential_loc)):
        small_facility = int(y[1, j].x)
        medium_facility = int(y[2, j].x)
        large_facility = int(y[3, j].x)

        if small_facility == 1 or medium_facility == 1 or large_facility == 1:
            new_facilities_data_p2.append({
                "Region Zip Code": potential_loc['zip_code'][j],
                "Longitude": potential_loc['longitude'][j],
                "Latitude": potential_loc['latitude'][j],
                "Small Facility": small_facility,
                "Medium Facility": medium_facility,
                "Large Facility": large_facility
            })

    # Create DataFrames
    facility_expansion_df_p2 = pd.DataFrame(facility_expansion_data_p2)
    new_facilities_df_p2 = pd.DataFrame(new_facilities_data_p2)

    # Save to CSV files
    facility_expansion_df_p2.to_csv("facility_expansion_p2.csv", index=False)
    new_facilities_df_p2.to_csv("new_facilities_p2.csv", index=False)
else:
    print("No optimal solution found")
```

[27]:
```
facility_expansion_df_p2
```

[27]:

	Facility ID	Expanded Slots	Slots for Children Under 5
0	0	12.0	12.0
1	1	0.0	-0.0
2	2	0.0	-0.0
3	3	0.0	0.0

```
4                4          11.0                         11.0
…             …          …                          …
14750        14750           0.0                          0.0
14751        14751           0.0                          0.0
14752        14752           0.0                          0.0
14753        14753           0.0                         -0.0
14754        14754           0.0                         -0.0

[14755 rows x 3 columns]
```

[28]:
```python
new_facilities_df_p2
```

[28]:
```
      Region Zip Code  Longitude   Latitude  Small Facility  Medium Facility  \
0                10001 -74.004994  40.740011               0                0
1                10001 -73.995710  40.744408               0                0
2                10002 -73.993451  40.707449               0                0
3                10002 -73.987162  40.720105               0                0
4                10002 -73.983491  40.723590               0                0
…                  …        …         …             …               …
2751             14767 -79.482863  42.064277               0                1
2752             14770 -78.327679  42.022032               0                1
2753             14772 -78.966816  42.169988               0                0
2754             14805 -76.724885  42.358652               1                0
2755             14806 -77.796393  42.150087               0                1

      Large Facility
0                  1
1                  1
2                  1
3                  1
4                  1
…               …
2751               0
2752               0
2753               1
2754               0
2755               0

[2756 rows x 6 columns]
```

4 The Problem of Fairness

[29]:
```python
# Problem 3
m3=Model("Fairness")
x={}
y={}
a={}
```

```python
# Decision variables
for i in range(len(childcare)):
    x[1, i] = m3.addVar(vtype=GRB.INTEGER, name=f"new slots at facility {i} between 0% and 10%")
    x[2, i] = m3.addVar(vtype=GRB.INTEGER, name=f"new slots at facility {i} between 10% and 15%")
    x[3, i] = m3.addVar(vtype=GRB.INTEGER, name=f"new slots at facility {i} between 15% and 20%")
    x[4, i] = m3.addVar(vtype=GRB.INTEGER, name=f"new slots for children under 5 at facility {i}")
    a[1, i] = m3.addVar(vtype=GRB.BINARY, name=f"10%")
    a[2, i] = m3.addVar(vtype=GRB.BINARY, name=f"15%")

# Decision Variables for New Facilities
for j in range(len(potential_loc)):
    y[1, j] = m3.addVar(vtype=GRB.BINARY, name=f"build a small facility in {j}")
    y[2, j] = m3.addVar(vtype=GRB.BINARY, name=f"build a medium facility in {j}")
    y[3, j] = m3.addVar(vtype=GRB.BINARY, name=f"build a large facility in {j}")

# Define the minimum and maximum slot ratio variables
min_ratio = m3.addVar(lb=0, vtype=GRB.CONTINUOUS, name="min_slot_ratio")
max_ratio = m3.addVar(lb=0, vtype=GRB.CONTINUOUS, name="max_slot_ratio")

# Objective Function: Maximize the Social Coverage Index
m3.setObjective(
        2 * (
            (demand_desert['2w_5yr_cap'].sum()+
            (quicksum(x[4, i] for i in range(len(childcare))) + quicksum(50 * y[1, j] + 100 * y[2, j] + 200 * y[3, j] for j in range(len(potential_loc))))) / demand_desert["-5"].sum()
        ) +
        (
            (demand_desert['2w_12yr_cap'].sum()+
            (quicksum(x[1, i] + x[2, i] + x[3, i] for i in range(len(childcare))) + quicksum(100 * y[1, j] + 200 * y[2, j] + 400 * y[3, j] for j in range(len(potential_loc))))) / demand_desert["2w-12yrs"].sum()
        ),
    GRB.MAXIMIZE
)

# Fairness Constraint: Calculate slot ratio dynamically using decision variables
care_reg=childcare.groupby("zip_code")
new_pos=potential_loc.groupby("zip_code")
# Calculate the slot ratios for each zip code and constrain them between min_ratio and max_ratio
```

```
for i in range(len(demand_desert)):
    # Calculate the slot ratio for zip code i
    if demand_desert["2w-12yrs"][i] > 0:
        slots_ratio_i = (
            quicksum(x[1, l] + x[2, l] + x[3, l] for l in care_reg.
 ↪get_group(demand_desert.iloc[i, 0]).index) +
            quicksum(100 * y[1, k] + 200 * y[2, k] + 400 * y[3, k] for k in␣
 ↪new_pos.get_group(demand_desert.iloc[i, 0]).index)
        ) / demand_desert["2w-12yrs"][i]

        # Constrain each slot ratio to be within min_ratio and max_ratio
        m3.addConstr(slots_ratio_i >= min_ratio, f"Min ratio constraint for zip␣
 ↪{demand_desert.iloc[i,0]}")
        m3.addConstr(slots_ratio_i <= max_ratio, f"Max ratio constraint for zip␣
 ↪{demand_desert.iloc[i,0]}")
    else:
        print(f"Skipping zip code {demand_desert.iloc[i,0]} due to zero␣
 ↪population.")

# Enforce fairness by limiting the difference between max_ratio and min_ratio␣
 ↪to 0.1
m3.addConstr(max_ratio - min_ratio <= 0.1, "Fairness constraint")

# Budget Constraint
m3.addConstr(
    quicksum(65000 * y[1, j] + 95000 * y[2, j] + 115000 * y[3, j] for j in␣
 ↪range(len(potential_loc))) +
    quicksum(200 * x[1, i] + 400 * x[2, i] + 1000 * x[3, i] + 100 * x[4, i] +␣
 ↪20000 * ((x[1, i] + x[2, i] + x[3, i]) / childcare["total_capacity"][i]) for␣
 ↪i in range(len(childcare))) <= 1000000000,
    "Budget constraint"
)

# Constraint from Problem 2
for i in range(len(childcare)):
    m3.addConstr(x[1,i]+x[2,i]+x[3,i]<=0.
 ↪2*childcare["total_capacity"][i],f"Maximum expansion rate {i}")
    m3.addConstr(x[4,i]-x[1,i]-x[2,i]-x[3,i]<=0)
    m3.addConstr(childcare["total_capacity"][i]+x[1,i]+x[2,i]+x[3,i]<=
                 max(childcare["total_capacity"][i],500),f"Maximum slots {i}")

care_reg=childcare.groupby("zip_code")
new_pos=potential_loc.groupby("zip_code")
for j in range(len(demand_desert)):
    jcare=care_reg.get_group(demand_desert.iloc[j,0]).index
    jpos=new_pos.get_group(demand_desert.iloc[j,0]).index
```

```
    #high demand or not(changing greater than to greater than or equal)
    if demand_desert["high_demand"][j]==1:
        m3.
↪addConstr(demand_desert["2w_12yr_cap"][j]+quicksum(100*y[1,k]+200*y[2,k]+400*y[3,k]␣
↪for k in jpos)+
                                  quicksum(x[1,l]+x[2,l]+x[3,l] for l in jcare)>=int(1/
↪2*(demand_desert["2w-12yrs"][j]))+1)
    else:
        m3.
↪addConstr(demand_desert["2w_12yr_cap"][j]+quicksum(100*y[1,k]+200*y[2,k]+400*y[3,k]␣
↪for k in jpos)+
                                  quicksum(x[1,l]+x[2,l]+x[3,l] for l in jcare)>=int(1/
↪3*(demand_desert["2w-12yrs"][j]))+1)
    m3.
↪addConstr(demand_desert["2w_5yr_cap"][j]+quicksum(50*y[1,k]+100*y[2,k]+200*y[3,k]␣
↪for k in jpos)+
                                  quicksum(x[4,l] for l in jcare)>=2/
↪3*(demand_desert["-5"][j]))

#Only one type at most
for j in range(len(potential_loc)):
    m3.addConstr(y[1,j]+y[2,j]+y[3,j]<=1,f"One type for {j}")

#piecewise
for i in range(len(childcare)):
    # Constraints for x[1,i]
    m3.addConstr(x[1,i]>=a[1,i]*int(0.1*childcare["total_capacity"][i]),␣
↪f"200_1")
    m3.addConstr(x[1,i]<=int(0.1*childcare["total_capacity"][i]),f"200_2")
    m3.addConstr(x[2,i]>=(int(0.15*childcare["total_capacity"][i])-int(0.
↪1*childcare["total_capacity"][i]))*a[2,i],f"400_1")
    m3.addConstr(x[2,i]<=(int(0.15*childcare["total_capacity"][i])-int(0.
↪1*childcare["total_capacity"][i]))*a[1,i],f"400_2")
    m3.addConstr(x[3,i]>=0,f"1000_1")
    m3.addConstr(x[3,i]<=(int(0.2*childcare["total_capacity"][i])-int(0.
↪15*childcare["total_capacity"][i]))*a[2,i],f"1000_2")

#Distance
for j in range(len(demand_desert)):
    jcare=care_reg.get_group(demand_desert.iloc[j,0]).index
    jpos=new_pos.get_group(demand_desert.iloc[j,0]).index
    #between original and new facilities
    for k in jcare:
        if np.isnan(childcare["latitude"][k]) or np.
↪isnan(childcare["longitude"][k]):
            continue
```

```
        for l in jpos:
            dist0=haversine((childcare["latitude"][k],childcare["longitude"][k]),
                            (potential_loc["latitude"][l],potential_loc["longitude"][l]),
                            unit=Unit.MILES)
            if dist0<0.06:
                m3.addConstr(y[1,l]+y[2,l]+y[3,l]==0)
    #between new facilities
    for k in jpos:
        for l in jpos:
            if l>k:
                dist1=haversine((potential_loc["latitude"][k],potential_loc["longitude"][k]),
                                (potential_loc["latitude"][l],potential_loc["longitude"][l]),
                                unit=Unit.MILES)
                #Can't be chosen at the same time if dist1<0.06
                if dist1<0.06:
                    m3.addConstr(y[1,k]+y[2,k]+y[3,k]+y[1,l]+y[2,l]+y[3,l]<=1)
m3.optimize()
```

```
Skipping zip code 11042 due to zero population.
Skipping zip code 12742 due to zero population.
Skipping zip code 13441 due to zero population.
Gurobi Optimizer version 11.0.3 build v11.0.3rc0 (win64 - Windows 10.0
(19045.2))

CPU model: Intel(R) Core(TM) i7-10875H CPU @ 2.30GHz, instruction set
[SSE2|AVX|AVX2]
Thread count: 8 physical cores, 16 logical processors, using up to 16 threads

Optimize a model with 284148 rows, 395432 columns and 2587516 nonzeros
Model fingerprint: 0xca0490b6
Variable types: 2 continuous, 395430 integer (336410 binary)
Coefficient statistics:
  Matrix range     [4e-05, 1e+05]
  Objective range  [4e-07, 5e-04]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e-01, 1e+09]
Presolve removed 83711 rows and 24405 columns
Presolve time: 0.25s

Explored 0 nodes (0 simplex iterations) in 0.56 seconds (0.83 work units)
Thread count was 1 (of 16 available processors)

Solution count 0
```

```
Model is infeasible or unbounded
Best objective -, best bound -, gap -
```

[30]:
```
new_facilities_df_p2['Facility_total'] = new_facilities_df_p2.iloc[:, 3:6].
  ↪sum(axis=1)
new_facilities_df_p2
```

[30]:
```
      Region Zip Code  Longitude   Latitude  Small Facility  Medium Facility  \
0               10001 -74.004994  40.740011               0                0
1               10001 -73.995710  40.744408               0                0
2               10002 -73.993451  40.707449               0                0
3               10002 -73.987162  40.720105               0                0
4               10002 -73.983491  40.723590               0                0
…                 …         …          …              …                …
2751            14767 -79.482863  42.064277               0                1
2752            14770 -78.327679  42.022032               0                1
2753            14772 -78.966816  42.169988               0                0
2754            14805 -76.724885  42.358652               1                0
2755            14806 -77.796393  42.150087               0                1

      Large Facility  Facility_total
0                  1               1
1                  1               1
2                  1               1
3                  1               1
4                  1               1
…                …              …
2751               0               1
2752               0               1
2753               1               1
2754               0               1
2755               0               1

[2756 rows x 7 columns]
```

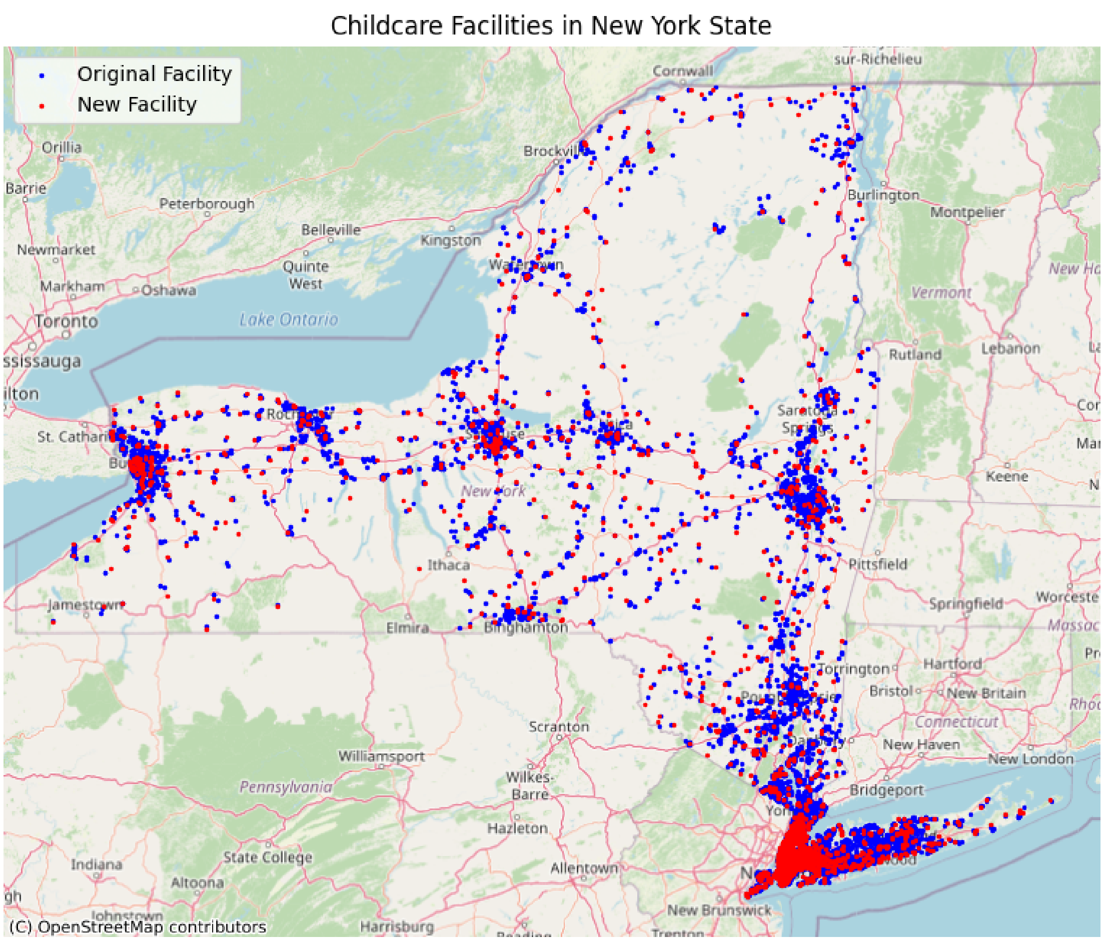
[31]:
```
import geopandas as gpd
import contextily as ctx
import matplotlib.pyplot as plt

# Convert the coordinates to a suitable CRS (usually EPSG:3857 for web maps)
gdf = gpd.GeoDataFrame(childcare, geometry=gpd.points_from_xy(childcare.
  ↪longitude, childcare.latitude), crs="EPSG:4326")
gdf_new = gpd.GeoDataFrame(new_facilities_df_p2, geometry=gpd.
  ↪points_from_xy(new_facilities_df_p2.Longitude, new_facilities_df_p2.
  ↪Latitude), crs="EPSG:4326")
gdf = gdf.to_crs(epsg=3857)  # Convert to web map projection
```

```python
gdf_new = gdf_new.to_crs(epsg=3857)  # Convert to web map projection

# Plot the facilities with contextily basemap
fig, ax = plt.subplots(figsize=(10, 8))
gdf.plot(ax=ax, marker='o', color='blue', markersize=2, label="Original␣
↪Facility")
gdf_new.plot(ax=ax, marker='o', color='red', markersize=2, label="New Facility")
ctx.add_basemap(ax, source=ctx.providers.OpenStreetMap.Mapnik)  # Google Maps␣
↪style basemap

ax.set_axis_off()
plt.title("Childcare Facilities in New York State")
plt.legend()
plt.show()
fig.savefig("childcare_facilities_map.png", dpi=500)
```



[32]:

```
import numpy as np
import matplotlib.pyplot as plt

# Calculate the availability ratio
demand_desert['availability_ratio'] = demand_desert['2w_12yr_cap'] /␣
  ↪demand_desert['2w-12yrs']

# Replace infinite values (where denominator is zero) and drop NaN values
demand_desert['availability_ratio'].replace([np.inf, -np.inf], np.nan,␣
  ↪inplace=True)
demand_desert.dropna(subset=['availability_ratio'], inplace=True)

# Plotting the fairness metric comparison
plt.hist(demand_desert['availability_ratio'], bins=20, edgecolor='black')
plt.title("Childcare Availability Ratio Across Regions")
plt.xlabel("Availability Ratio")
plt.ylabel("Number of Regions")
plt.xticks(np.arange(0, 5, 0.5))
plt.xlim(0, 5)
plt.show()
```