IEOR E4004: Optimization Models and Methods
Instructor: Dr. Yaren Bilge Kaya
Due: 11/3/2024, 11:59 pm

Assignment 3

October 17, 2024

Question 1. *(40 points) Tosla needs to schedule its workforce for the upcoming week in order to maintain efficient operation across all its departments, including battery, body, assembly, paint, and quality control. Each department has specific minimum and maximum staffing needs for three shifts: morning, afternoon, and night for each day of the week (Mon-Sun). Tosla's workforce is flexible, but workers have specific availabilities, preferences for working certain shifts, and different effectiveness scores that measure their productivity in certain roles (on a scale of 1-10, with 10 being the most preferred or effective).*

The goal is to maximize the total preference-adjusted effectiveness of the workers scheduled for each shift, subject to the following constraints:

- *Each worker can only be assigned to work a maximum of one shift per day.*
- *Each worker can only work a maximum of 5 days a week.*
- *Each worker can only be assigned to shifts that they are available for.*
- *The total number of workers assigned to a shift in a department must meet the department's specific minimum and maximum staffing requirements.*

Your task is to formulate this scheduling problem as a linear/integer programming problem and solve it using an appropriate optimization software (preferably Gurobi).

Use the randomly generated data given below to solve the model where we assume there are 100 skilled workers waiting to be assigned to a shift and department. Keep in mind that, since this is a randomly generated dataset, some of you might be working with infeasible problems.

(a) *(10 points)Define the parameters, sets, decision variables, constraints, and the objective you are using to model.*

(b) *(20 points)Formulate the problem algebraically.*

(c) *(10 points) Solve the problem using Python & Gurobi.*

```
# Define Workers, Departments, Shifts, and Days
workers = [i for i in range(1, 101)]
departments = ['Battery', 'Body', 'Assembly', 'Paint', 'Quality']
shifts = ['Morning', 'Afternoon', 'Night']
days = ['Mon', 'Tue', 'Wed', 'Thur', 'Fri', 'Sat', 'Sun']

# Create Workers DataFrame
workers_df = pd.DataFrame({
    'Worker_ID': np.repeat(workers, len(departments)*len(shifts)*len(days)),
    'Department': np.tile(np.repeat(departments, len(shifts)*len(days)), len(workers)),
    'Shift': np.tile(np.repeat(shifts, len(days)), len(workers)*len(departments)),
    'Day': np.tile(days, len(workers)*len(departments)*len(shifts)),
    'Availability': np.random.choice([0, 1], len(workers)*len(departments)*len(shifts)*len(days)),
    'Preference_Score': np.random.randint(1, 10, len(workers)*len(departments)*len(shifts)*len(days)),
    'Effectiveness_Score': np.random.randint(1, 10, len(workers)*len(departments)*len(shifts)*len(days))
})

# Create Department DataFrame
dept_df = pd.DataFrame({
    'Department': np.repeat(departments, len(shifts)*len(days)),
    'Shift': np.tile(np.repeat(shifts, len(days)), len(departments)),
    'Day': np.tile(days, len(departments)*len(shifts)),
    'Min_Workers': np.random.randint(1, 5, len(departments)*len(shifts)*len(days)),
    'Max_Workers': np.random.randint(5, 10, len(departments)*len(shifts)*len(days))
})
```

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Question 2. *(10 points) Suppose that $x_1, \ldots, x_n$ are unrestricted but bounded continuous variables, and the following two constraints are given:*

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n \geq b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n \geq b_2$$

How can the satisfaction of at least one of these constraints be modeled?

Hint: *Consider a larger variable space, as well as a "big enough" positive number M.*

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Question 3. *(10 points) Consider the following optimization problem featuring non-negative general integer variables:*

$$
\begin{aligned}
\max \quad & z = 4x_1 - x_2 \\
\textit{subject to} \quad & 7x_1 - 2x_2 \leq 14 \\
& x_2 \leq 3 \\
& 2x_1 - 2x_2 \leq 3 \\
& x_j \in Z_+^2.
\end{aligned}
$$

Draw the feasible region of the integer program (possible as there are just two variables).

Question 4. *(40 points) Consider the IP in Question 3 and solve the problem using branch-and-bound. Draw out the tree, as well as the changes to the feasible region. Use the following criterion for node selection. For the first branch, choose the left-most branch. After this, always choose the right-most branch (if one exists), and prefer higher branches to lower branches.*