Understand P/E Ratio

- Why do P/E ratios differ from nations, industries, companies?
  good Company → high P/E Ratio
- What factors decide P/E ratio?
- How to know whether the current P/E ratio sustains?

Back to DCF Model——Gordon Growth Model

- Gordon model, also known as constant growth model, which assumes dividends grow at a constant rate g:
  - （r 〉 g）

$$P_0 = \frac{D_0(1+g)}{(1+r)} + \frac{D_0(1+g)^2}{(1+r)^2} + \ldots = \frac{D_0(1+g)}{r-g} = \frac{D_1}{r-g}$$

$$\frac{P_0}{E_0} = \frac{\frac{D_0}{E_0}(1+g)}{r-g} = \frac{z(1+g)}{r-g}$$

Dividend Payout Ratio

Life Cycle and Growth Rate at Different Stages

TABLE 18.2

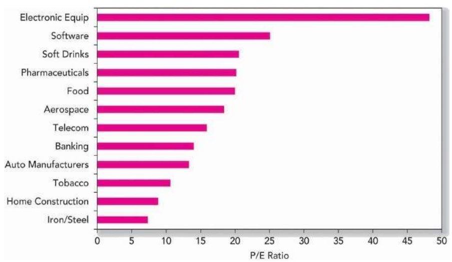
Financial ratios in two industries

	Return on Assets	Payout Ratio	Growth Rate 2005–2008
Computer Software			
Adobe Systems	21.5%	1.0%	8.2%
Cognizant	19.0	0.0	22.8
Compuware	10.5	0.0	17.6
Intuit	19.0	0.0	8.0
Microsoft	31.5	35.0	15.4
Novell	8.5	0.0	51.8
Oracle	33.0	0.0	18.6
Red Hat	17.0	0.0	17.6
Parametric Tech	20.0	0.0	33.9
SAP	22.5	18.0	13.8
Median	19.5%	0.0	17.6%
Electric Utilities			
Central Hudson G&E	6.0%	78.0%	5.1%
Central Vermont	7.5	60.0	8.0
Consolidated Edison	5.0	75.0	1.0
Duquesne Light	8.0	85.0	7.7
Energy East	6.0	74.0	4.1
Northeast Utilities	5.0	59.0	14.0
Nstar	8.5	61.0	3.2
Pennsylvania Power	11.0	52.0	9.3
Public Services Enter.	7.0	62.0	1.7
United Illuminating	5.0	113.0	1.3
Median	6.5%	68.0%	4.6%

Source: *Value Line Investment Survey*, 2006. Reprinted with permission of Value Line Investment Survey. © 2006 Value Line Publishing, Inc. All rights reserved.

Growth Opportunity and P/E Ratio

- the more growth opportunities, the higher P/E



P/E Ratio

- From the definition, P/E ratio is affected by
  - growth rate
  - discounted rate
  - dividend payout ratio (or reinvestment ratio)

$$P_0 = \frac{D_0(1+g)}{(1+r)} + \frac{D_0(1+g)^2}{(1+r)^2} + \ldots = \frac{D_0(1+g)}{r-g} = \frac{D_1}{r-g}$$

$$\frac{P_0}{E_0} = \frac{\frac{D_0}{E_0}(1+g)}{r-g} = \frac{z(1+g)}{r-g}$$

g and z are relevant

profitability affect r, g

What kind of Chinese companies have high P/E ratio?

- PE=24.08-7.50*beta+2.62*payratio+6.95*exp_5y_growth_in_EPS-1.19*std
  Std →（7.23） CAPM （2.58） （3.43）expected （0.45） standard Deviation

  R-square：20%

- Conclusion：the Chinese companies with high P/E 不正常的
  - weak cyclicality
  - relatively high dividend payout ratio → lower growth rate 正常公司
  - high expected growth rate on profit
  - but have no close relationship with
    - » volatility of stocks

What can explain the P/E ratio of a country?

- PE = 16.16 - 7.94 Interest Rates + 154.40 Growth in GDP - 0.1116 Country Risk
  R Squared = 73%
- Conclusion
  - a country would have a high P/E if
    - » it has low interest rates
    - » it has high GDP growth rate
    - » it has low risks

If Predicted PE > Actual PE, we can jump in to get profit.

PEG

- Is a company having a high P/E ratio due to its high price or high growth?
- The definition of PEG
  - =P/E ratio/ expected growth rate

P/B Ratio (Can be applied when P/E is inaccurate)

near 0/negative

- PB=price/book value of asset per share
  - Pros Problems: No relationship between earning and book value.

    We need to generate the profit in

  - Cons

Factors Affecting P/B Ratio

$$P_0=\frac{D_1}{(k-g_n)}$$

$$\frac{P}{BV}=\frac{ROE\times z}{k-g_n}$$

$$\frac{P}{BV}=\frac{ROE-g_n}{k-g_n}$$

k = discounted rate; z = dividend payout ratio; P_0 = price;

g_n = the growth rate of earning and dividends

D_1 = dividend next year, BV=book value

Select Stocks through P/B

PBV = 2.21 + 8.22 ROE – 2.63 Standard deviation $R^2 = 52\%$
[2.16] [2.92] [1.21]

Good oppotunity or Trick?

Company Name	Price-to-Book Ratio	Predicted PBV	Under- or Overvalued
Crown Central Petroleum "A"	0.29	–0.56	NMF
Giant Industries	0.54	1.80	–69.74%
Harken Energy Corp.	0.64	0.24	166.59%
Getty Petroleum Mktg.	0.95	1.19	–19.67%
Pennzoil–Quaker State	0.95	1.19	–19.93%
Ashland Inc.	1.13	2.48	–54.28%
Shell Transport	1.45	2.48	–41.56%
USX–Marathon Group	1.59	2.12	–25.11%
Lakehead Pipe Line	1.72	2.78	–38.03%
Amerada Hess	1.77	2.87	–38.33%

maybe other factors that you don't consider into the model

ROE may look high because of a high Accounts Receivable

tend to be a bad signal

Price/Sales Ratio

- Pros
  - less likely to be manipulated, compared the earnings and book value
  - earnings may be negative or close to zero
  - more stable than P/E
  - more suitable to price mature, cyclical company with close zero profit

Price/Sales Ratio

- Cons
  - large sales is not equal to high profit
  - stock price should not tightly related to sales
- If there are two stocks having the same P/E, which one would have higher price/sales ratio?

luxury goods: P/S small ⇒ Higher P/S

Grocery: P/S big ⇒ Lower P/S

Price/Sales Model：1-stage Model

$$P_0=\frac{D_1}{(k-g_n)} \Rightarrow \frac{P}{S}=\frac{PM\times z\times(1+g_n)}{k-g_n}$$

k = discounted rate; z = dividend payout ratio; P_0 = price;

g_n = the growth rate of earning and dividends

D_1 = dividend next year; S= sales; PM = net profit margin=E/S

$\frac{\text{Earnings}}{\text{Sales}}$

2-Stage Model

$$\frac{P_0}{S_0}=PM\times\left\{\frac{z_n\times(1+g)\times\left[1-\frac{(1+g)^n}{(1+k_{hg})^n}\right]}{(k_{hg}-g)}+\frac{z\times(1+g)^n\times(1+g_n)}{(k_{st}-g_n)\times(1+k_{hg})^n}\right\}$$

PM = net profit margin

g = growth rate in the previous n years

k_{hg} = discounted rate during high growth period

k_{st} = discounted rate during stable growth period

z = dividend payout ratio during high growth period

g_n = growth rate after the first n years

z_n = dividend payout after the first n years

Price/Sales Model：1-stage Model

$$V_0=\frac{EBIT\times(1-t)\times z}{(k-g_n)}$$

$$\frac{V_0}{S_0}=\frac{\text{ATOM}\times z}{k-g_n}$$

k = discounted rate; t = tax rate; S = Sales

ATOM=EBIT*（1-t）/S;

z = dividend payout ratio;

V_0 = enterprise value; g_n = earning growth rate

整个企业

2-stage Model

$$\frac{V_0}{S_0}=ATPM\times\left\{\frac{z_n\times(1+g)\times\left[1-\frac{(1+g)^n}{(1+k_{hg})^n}\right]}{(k_{c,hg}-g)}+\frac{z\times(1+g)^n\times(1+g_n)}{(k_{c,st}-g_n)\times(1+k_{c,hg})^n}\right\}$$

$ATPM = after-tax$ profit margin

g = growth rate in the previous n years

$k_{c,hg}$ = discounted rate during high growth period

$k_{c,st}$ = discounted rate during stable growth period

z = dividend payout ratio during high growth period

g_n = growth rate after the first n years

z_n = dividend payout after the first n years

How much does the brand worth?

Brand value = $(VS_b - VS_g)\times S$

VS_b = VS of company with impressing brands

VS_g = VS of company without impressing brands

VS: Enterprise Value / Sales

What is the brand value of Coca-Cola?

May 2001：Coca-Cola

	High-Growth Period	*Stable-Growth Period*
Length	10 years	Forever after year 10
Growth rate	8.92%	5%
After-tax operating margin	16.31%	16.31%
Cost of capital	9.71%	8.85%
Reinvestment rate	40%	31.25%

$$VS = 0.1631 \times \left\{ \frac{0.60 \times (1.0892) \times \left[1 - \frac{(1.0892)^{10}}{(1.0971)^{10}}\right]}{(.0971 - .0892)} + \frac{0.6875 \times (1.0892)^{10} \times (1.05)}{(.0885 - .05)(1.0971)^{10}} \right\} = 3.79$$

What is the brand value of Coca-Cola?

Cott, an ordinary baverage producing company in Canada

$$VS = 0.0482 \times \left\{ \frac{0.60 \times (1.0397) \times \left[1 - \frac{(1.0397)^{10}}{(1.0971)^{10}}\right]}{(.0971 - .0397)} + \frac{0.4958 \times (1.0397)^{10} \times (1.05)}{(.0885 - .05)(1.0971)^{10}} \right\} = 0.60$$

The sales of Coca-Cola in 2000 was \$20.458 B

FV of Coca-Cola=3.79*20.458=\$77.535B

FV of Ordinary company Cott=0.6*20.458=12.274B

Brand value=\$65.261 billion

What is the brand value of Coca-Cola?

$$VS = 0.0482 \times \left\{ \frac{0.60 \times (1.0397) \times \left[1 - \frac{(1.0397)^{10}}{(1.0971)^{10}}\right]}{(.0971 - .0397)} + \frac{0.4958 \times (1.0397)^{10} \times (1.05)}{(.0885 - .05)(1.0971)^{10}} \right\} = 0.60$$

The sales of Coca-Cola in 2000 was \$20.458 billion

Coca-Cola price=3.79*20458=77535

Ordinary company price=0.6*20458=12274

Brand value=\$65.261 billion

What is the brand value of Coca-Cola?

- Conclusion
  - The most value of Coca-Cola comes from the brand value
  - Brand could help the firm operate at a high margin for a long period of time

Specific Ratio in Different Industries

Commodity Company（oil refining, gold mining companies）

$$\text{unit commodity value} = \frac{\text{Stock value + debt value - cash}}{\text{commodity inventory}}$$ = Enterprise Value

商品、货物

If manufacturer produce products with identical quality：

$$\text{unit product value} = \frac{\text{stock value + debt value - cash}}{\text{product units（or capacity）}}$$

Specific Ratio in Different Industries

Internet company、information provider、power cable company, etc：

$$\text{Unit user value} = \frac{\text{stock value + debt value - cash}}{\text{number of users}}$$

Amazon：

$$\text{unit customer value} = \frac{\text{stock value + debt value- cash}}{\text{number of effective custormers}}$$

$$\text{unit visitor value} = \frac{\text{stock value + debt value - cash}}{\text{number of visitors}}$$

Company with patents: the price when Google bought Motorola

Examples of Valuation Application

苏宁

Price (RMB)	10.95
Shares	6,996,211,900
RMB'000	
Equity	76,608,520
Debt	35,348
Cash	21,568,655
EV	55,075,213
Store	962
Area (Sqm)	4,058,300
RMB care about area	
EV/Store	57,250,741
Sales/Store	61,955,525
EV/Sqm → care about location	13,571
Sales/Sqm	14,602

国美

Price (RMB)	2.00
Shares	15,055,332,000
RMB'000	
Equity	30,110,664
Debt	3,524,909
Cash	6,029,059
EV	27,606,514
Store	726
Area (Sqm)	2,675,000
RMB	
EV/Store	38,025,501
Sales/Store	58,771,350
EV/Sqm	10,320
Sales/Sqm	15,951

EV can be manipulated.（贪污等）.

Which one had higher estimated value in May 2010?

We should use EV/Sales → 苏宁 is more expensive

Examples of Valuation Application

沃尔玛

Price (USD)	50.02
Shares(Million)	3,786
USD Million	
Equity	189,376
Debt	33,754
Cash	7,907
EV	215,223
Store	8,416
Area (Sqm)	
RMB	
EV/Store	174,689,987
Sales/Store	328,677,519

Best Buy

Price (USD)	40.56
Shares(Million)	419
USD Million	
Equity	16,995
Debt	1,802
Cash	1,826
EV	16,971
Store	4,027
Area (Sqm)	
RMB	
EV/Store	28,787,385
Sales/Store	84,283,591

物美

Price (RMB)	16.37
Shares	1,251,274,116
RMB'000	
Equity	20,483,357
Debt	456,086
Cash	1,171,575
EV	19,767,868
Store	472
Area (Sqm)	542,241
RMB	
EV/Store	41,881,077
Sales/Store	24,961,883

The most expensive one

Which one is the most expensive?

Also use EV/Sales

Growth Opportunity and Return

- The growth rate of American computer software companies is 17.6%, while that of electricity companies is only 4.6%. Hence should we buy stocks of computer software companies?

The price of high-growth company is higher, so it won't be much profit for the late incomers.

Growth Opportunity and Return

There is no obvious pattern for the growth and return rates in industries of China. The industry with high growth does not mean high return. Take agricultural and real estate industries as examples, the average annual growth rate of real estate is 23.13%, and that of agriculture is 6.69%. If you invest in real estate and agriculture in 1997, you should have returns of 6.27% and 8.58% separately in 2010. The similar things also appear in IT, retail, mining and construction industries. The returns of low growth industry are higher than that of high growth industry.

CUHK (Shenzhen) Jinfan Zhang

Growth Opportunity and Return

If we take a feasible strategy, which rebalances the portfolio according to the growth rate of the industry last year, no return pattern could be discovered.

High Growth ≠ High return (除非因增速过快造成股价重新计算而上涨的股价)

CUHK (Shenzhen) Jinfan Zhang

Growth Opportunity and Return

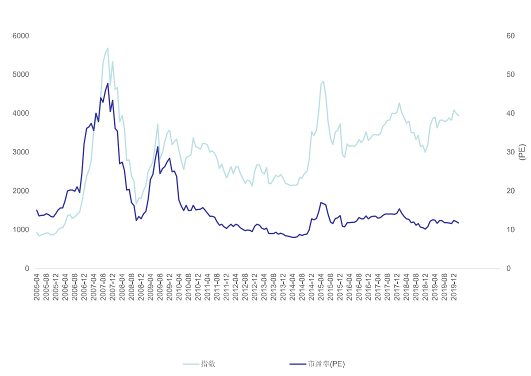
We do not see the solid relationship between high growth and high return. Why? Two factors may hinder the high growth transform into high return:

1. competition could dilute the profit.
2. market may over expect the growth

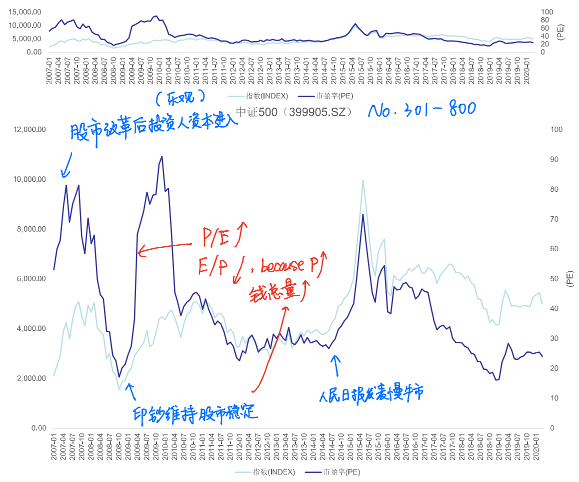
CUHK (Shenzhen) Jinfan Zhang

2005：股权分置改革

沪深300（000300.SH） No. 1 - 300



中证500（399905.SZ）



Can the return of China stock market be estimated?

Time Series Estimation vs Cross Sectional Estimation

Private Placement：定向增发（一般给机构增发）

IPO & SEO (Seasoned Equity offerings)

相比 IPO 更多资本收获

Can the return of China stock market be estimated?

Stock return consists of two parts, dividends and capital gains

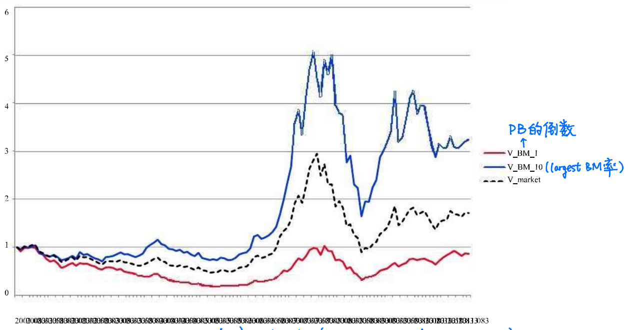
$$R = \frac{P_1 + D_1}{P_0} \approx 1 + g_{pb} + bROE + \frac{B_0}{P_0}(1-b)ROE \quad (1)$$

Obviously, valuation level (PB), earning ability (ROE) and dividend payout ratio (1-b) are factors that affect returns

P_i: ith period stock price, D_i: ith period dividends, B_i: ith period book value of assets, b retention ratio, g_{pb} is the growth of PB ratio

张劲帆

Value Stocks Won the Market: China (2001-2010)



越便宜的公司回报率高（可能有较大风险，from CAPM）

（也有可能是定价低了，后价格回到真值）

张劲帆

Dividend yield Predicts Return

Can the dividend payout ratio predict the return?

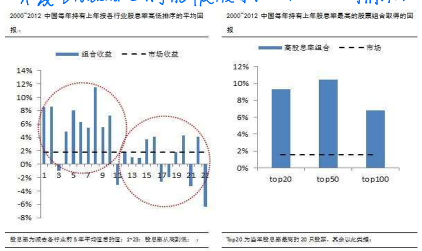
At the end of each year, we rank the industries or individual stocks with their dividend yields, calculate the average return from 2000 to 2012, and compare it with the market. High dividend yield stocks beat the market.

能发 Dividend 说明能赚钱，股东人好，(ROE 饱和了)

不发 Dividend 也有可能被股东私吞，不为公司前景，为自己谋利.

在中国适用，

国外不适用



张劲帆

Stock Market Fluctuation and Market Expectation

(price)

◆In the long run, stock must reflect a firm's profitability (the fundamentals of the firm)

◆If the stock price deviates from its fundamental, it must return

波动

– the larger the price fluctuation, the easier to estimate

– the stock price looks like a randomly flying pollen, but in fact subject to gravity and will fall eventually

花粉

Final Words on PE ： PE很大（P很大或E很小）

E can be different: Static PE, Forward/dynamic PE, and Trailing PE

过去 预测 过去12个月

Pay attention to the quality of E: One time items must be adjusted: hidden tricks include depreciation; research cost capitalization , cyclicality of earning (e.g 石油行业，高收益不持续)

PEG: Some practitioners think that PEG>1 is overvalued; PEG=1 is fairly valued. It is not entirely correct, but provides an angle to understand that high PE ratio is driven by high growth or not

The CAPM

Jinfan Zhang
Chinese University of Hong Kong (Shenzhen)

Overview

- The "fundamental value", "fair value", of asset is given by

$$FV_t = \frac{E[CF_{t+1}]}{1+r} + \frac{E[CF_{t+2}]}{(1+r)^2} + \frac{E[CF_{t+3}]}{(1+r)^3} + \dots$$

  - use all available information to make the best possible cash-flow forecast
  - use a discount rate r that is appropriate for the risk

The discount rate

老师推荐：10年国债无风险利率（risk-free rate）

- This lecture note is about the discount rate, *r*

反映通货膨胀率、GDP 增速 增速大，r↑

$$r = riskless\ rate + risk\ premium$$

E(r) 基本利率（无风险） 风险利率

- The CAPM measures the risk of an asset
  - provides the fair discount rate *r* for that risk
- Other applications include
  - Capital budgeting
  - Evaluating investment performance

Why do we hold the market portfolio?

- Suppose:
  - Everyone knows the same set of information. All have the same estimates of expected returns, variances, covariances, etc
  - Everyone has the same preference. All use risk-reward analysis (mean-variance analysis)

Why is covariance the right measure?

假设市场只有一个人

- Suppose you are holding a portfolio with return r_p
- Add a small amount δ of a new security with return r_i, how does this affect the riskiness of your portfolio?

$var((1-\delta)r_p + \delta r_i)$
$= (1-\delta)^2 var(r_p) + \delta^2 var(r_i) + 2\delta(1-\delta)cov(r_p, r_i)$
$\approx var(r_p) + 2\delta cov(r_p, r_i)$

- We should measure the risk of a security by how much its inclusion in your portfolio affects your *portfolio's* risk
- A security's risk is therefore driven by its *covariance* with your existing portfolio

The CAPM argument

- A security's risk is its covariance with the rest of the portfolio
  - Variance is a good measure of overall *portfolio* risk
  - For an individual security, *covariance* is the right measure of risk
- The rest of your portfolio is the market portfolio (under some assumptions)
- Risk is covariance with the market portfolio
- **A security's fair average return is determined by its covariance with the market**

Summary of the CAPM argument

- Finance theory says a security's fair average return is determined by its risk
  - We determined a security's "risk" is measured by its covariance with our portfolio
  - And we determined that our portfolio was the market portfolio
  - A security's fair average return is determined by its covariance with the market portfolio, because this measures risk

More details on the CAPM argument

- A security's fair average return is determined by its covariance with the market
  - This relationship is linear, that is for any asset, i, we have
    $$\underbrace{E[r_i] - r_f}_{\text{risk premium(补偿)}} = a + b\,cov(r_i, r_m)$$
- To find a and b, let's apply this formula to:
  - The risk-free asset, which gives a=0
  - The market portfolio itself
    $$E[r_m] - r_f = b cov(r_m, r_m) = b var(r_m)$$
    $$b = \frac{E[r_m] - r_f}{var(r_m)}$$

More details on the CAPM argument

- Putting this all together,
  $$E[r_i] - r_f = \frac{E[r_m] - r_f}{var(r_m)} cov(r_i, r_m)$$
- Written slightly differently 风险度量
  $$E[r_i] = r_f + \beta_i(E[r_m] - r_f)$$ 风险单位价格
  where $\beta_i = cov(r_i, r_m)/var(r_m)$

The CAPM Equation

$\beta_i > 0$: 涨时也涨，亏时也亏，investor 不喜欢，E(r)大，价格低 (锦上添花)

$\beta_i < 0$: 保险，E(r) 小，出高价 (雪中送炭)

- The CAPM says: for any asset, i, the expected return of that asset is
  $$\boldsymbol{E[r_i] - r_f = \beta_i(E[r_m] - r_f)}$$
  局限性：e.g. 彩票 (不符合 CAPM)
  - r_f is the riskless rate
  - $\beta_i = cov(r_i, r_m)/var(r_m)$ tells you about the assets co-variance with the market, our measure of risk
  - $E[r_m] - r_f$ is the *market risk premium* or *equity premium*

The CAPM and discount rate

- The expected return (aka discount rate, required return, fair average return) is:
  $$E[r] = \text{riskless rate} + \text{risk premium}$$
- The CAPM formalizes this:
  $$\overbrace{E[r_i]}^{\text{Expected return}} = \underbrace{r_f}_{\text{Riskless rate}} + \overbrace{\underbrace{\beta_i}_{\text{(Amount of risk)}} \underbrace{\left(E[r_m] - r_f\right)}_{\times\text{ (Price of risk)}}}^{\text{Risk premium}}$$

(股票给market增加的risk)

cov 必·而不是自己的 var

CAPM Intuition

- A high beta stock is riskier, and must therefore offer higher returns to compensate for risk. Why?
- Because by moving in the same direction as the rest of your portfolio, it adds variance to your portfolio
- A stock with a beta of 2 means if the market falls by 10% the stock on average falls by 20%, amplifying your losses during **bad time**
- Beta is also known as "systematic risk"

CAPM Intuition

- Systematic vs Idiosyncratic:
- Suppose an asset with only firm specific uncertainty
  - i.e., zero covariance with the market
- CAPM says a fair average return is the risk free rate
- Only uncertainty related to the overall market deserves compensation because you can diversify the rest away

Implementing the CAPM Equation

CAPM 预计：$\alpha = 0$: $E(r_i) - r_f = \beta_i(E(r_m) - r_f)$

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f)$$

- r_f is the riskless rate (historically ≈ 4.5%)
- $E[r_m] - r_f$ is the equity premium (≈ 6.5%)
- β_i needs to be *estimated*, typically $0 \le \beta_i \le 2$, but in principle can be anything
- You estimate β_i and the CAPM tells you what the fair expected return, or discount rate, should be for the asset
- We then use this discount rate to discount future cash flows in the present value formula (next lecture)

Where does beta come from?

- Typically we estimate beta using a regression:
  $$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \varepsilon_{i,t}$$
- The coefficient β_i is the stocks beta
- We use "excess returns" in the regression
  - Accounts for fact that risk free rate changes over time
- Let's do an example with actual data from GM
  - We will use 12 months of data, in practice, it is more common to use 5 years (60 months)

What is "alpha" (α)?

- Our regression: $r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \varepsilon_t$
- Take averages of both sides
  $$\overline{r_i} - \overline{r_f} = \alpha_i + \beta_i(\overline{r_m} - \overline{r_f})$$
  - The error term ε_t is zero on average
- Rearranging,
  $$\alpha_i = \underbrace{\overline{r_i}}_{\text{Actual average return}} - \underbrace{\left(\overline{r_f} + \beta_i(\overline{r_m} - \overline{r_f})\right)}_{\text{CAPM implied expected return}}$$
  - α = the *actual* average historical return - the *fair* expected return implied by the CAPM

missing risk factors

What is "alpha"?

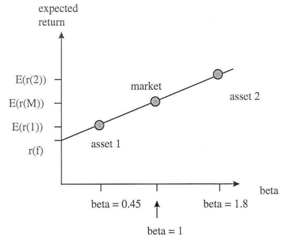
- The CAPM says **alpha should be zero**
- If a stock or asset has positive alpha (α) it means its average return *historically* has exceeded the fair return implied by the CAPM

窗口时期长：公司变化，对现状估计有偏差
短：数据不够，估计不准确

- The security may have been mispriced historically
  - Over- or under-priced?
- It may have also just been luck

Does the CAPM work in practice?

- The CAPM says expected returns should line up perfectly with betas

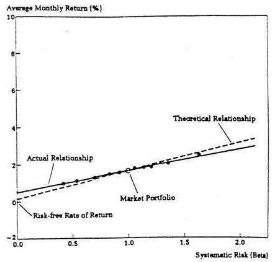


- Let's see if this is true…

Does the CAPM work in practice?

- Black Jensen Scholes 1972 get mixed results
- The fit is reasonable but not perfect

提高β：
① 借钱 X
② 买高β stock
⇓
price ↑
⇓
Actual r < theoretical r



Does the CAPM work in practice?

- Higher beta does correspond to higher return
- But…
  - **low beta** stocks have **higher average returns** than the CAPM predicts
  - High beta stocks have lower average returns than the CAPM predicts

Does the CAPM work in practice?

- Other work by Fama and French 1992
  - Find no relationship between beta and average returns!
  - Declare "**beta is dead**"
- We will use it when we compute fundamental values in the next lecture note

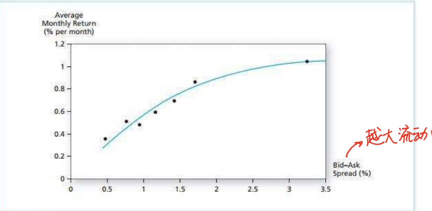
Assumptions of the CAPM

- Individuals
  - Mean-variance optimizers
  - Homogeneous expectations
  - All assets are publicly traded
- Markets
  - All assets are publicly held
  - All information is available
  - No taxes
  - No transaction costs

Liquidity and the CAPM

- Liquidity: The ease and speed with which an asset can be sold at fair market value
- Illiquidity Premium: Discount from fair market value the seller must accept to obtain a quick sale. 非流动性资产
  - Measured partly by bid-asked spread
  - As trading costs are higher, the illiquidity discount will be greater.

The Relationship Between Illiquidity and Average Returns



越大流动性越差

Figure 9.4 The relationship between illiquidity and average returns

Source: Derived from Yakov Amihud and Haim Mendelson, "Asset Pricing and the Bid-Ask Spread," *Journal of Financial Economics* 17 (1986), pp. 223–49. Copyright 1986, with permission from Elsevier.

Liquidity Risk

- In a financial crisis, liquidity can unexpectedly dry up.
- When liquidity in one stock decreases, it tends to decrease in other stocks at the same time.
  - This is systemic
- Investors demand compensation for liquidity risk
  - Liquidity betas

Multifactor Models and the Arbitrage Pricing Theory

Jinfan Zhang
Chinese University of Hong Kong (Shenzhen)

Systematic versus Non-systematic Risk

- Asset returns can be decomposed into two sources:
  1. A small number of common factors which proxy for economic events that affect a large number of assets.
     - Example: changes in interest rates, inflation, and productivity.
     - These represent **Systematic risk**, which cannot be diversified away.
  2. A risk component that is unique to the asset.
     - Example – new product innovations, changes in management, lawsuits, labor strikes, etc.
     - These are **Non-systematic idiosyncratic, or firm-specific risk**, which typically is diversifiable.

The One Factor Model

- Sometimes convenient to think of the one factor as the market, hence, we can refer to this model as the **market model**.
- However, the market model and the CAPM are NOT the same.

Market Model

- To illustrate this model, let's regress the returns of a stock, say Microsoft, on the market index.
- The regression equation is the following:

将后面两项减去各自mean，
α可代表平均收益率

systematic idiosyncratic

$$\tilde{r}_{Micro} = \alpha_{Micro} + \beta_{Micro}\tilde{R}_{S\&P} + \tilde{\epsilon}_{Micro}$$
$$\text{dependent var.} = \text{intercept} + (\text{slope})\text{independent var.} + \text{residual}$$

- By the properties of regression, the residuals have a mean of zero, and are uncorrelated with the returns on the S&P 500.

Market Model

- Thus, the return on Microsoft is decomposed into:
  1. A constant term.
  2. Movements in the S&P 500 index return.
  3. Movements in a component unrelated to market movements.
- Decomposing the variance of Microsoft

$$\begin{aligned} var(\tilde{r}_{Micro}) &= var(\beta_{Micro}\tilde{R}_{S\&P}) + var(\tilde{\epsilon}_{Micro}) \\ &\quad + 2cov(\beta_{Micro}\tilde{R}_{S\&P}, \tilde{\epsilon}_{Micro}) = 0 \\ &= var(\beta_{Micro}\tilde{R}_{S\&P}) + var(\tilde{\epsilon}_{Micro}) + 0 \end{aligned}$$

Market Model

- Variance decomposition

we can cancel it through making
the sums 0.

$$\begin{aligned} var(\tilde{r}_{Micro}) &= \beta^2_{Micro} var(\tilde{R}_{S\&P}) + var(\tilde{\epsilon}_{Micro}) \\ &= \text{systematic risk} + \text{non-systematic risk} \\ &= \text{non-diversifiable risk} + \text{diversifiable risk} \\ &= \text{market risk} + \text{unique or firm-specific risk} \end{aligned}$$

Market Model vs. CAPM

- The difference between the Market Model and the CAPM:
  - The CAPM describes **expected** or **mean returns.**
  - The market model describes **realized returns.**
- If the market model is true, then

$$\tilde{r}_i = \alpha_i + \beta_i \tilde{R}_M + \tilde{\epsilon}_i \qquad \forall i$$

- Key assumption: The residuals, $\tilde{\epsilon}_i$, are independent across stocks. Hence, they are firm specific, and diversifiable

Multi-factor Model

- However, aren't there other sources of risk that affect a group of assets, which are not picked up by an asset's sensitivity to the market?
- Example, interest rate changes.
  - Changes in interest rates affect the return on Microsoft, as well as almost all assets in the economy.
  - Some of these effects will be accounted for by the market factor, but not all.
  - The remaining effects show up in the residual of Microsoft.
  - But, because interest rate changes are not "firm specific"

Multi-factor Model

- Hence, residuals cannot be diversified away. We can no longer view them as diversifiable risk
- How can we account for this common factor?
- **Multifactor Models**: Add additional common factors to the return process:

$$\tilde{r}_i = \alpha_i + \beta_{i1}\tilde{F}_1 + \beta_{i2}\tilde{F}_2 + \ldots + \beta_{iK}\tilde{F}_K + \tilde{\epsilon}_i$$
$$\tilde{r}_i = \alpha_i + \sum_{k=1}^{K} \beta_{ik}\tilde{F}_k + \tilde{\epsilon}_i$$

  - $\tilde{F}$'s are a small number of **common factors** (1...K), for which different assets have different sensitivities, or loadings, β_{ik}

Multi-factor Model (cont'd)

- With enough common factors, the residuals are now truly firm-specific.
- We can think of the common factors as representing new information about macroeconomic variables.
  - Example: interest rates, industrial production, inflation, market volatility.
- Think of these as **surprises** in the macroeconomy (i.e., they cannot be predicted).
- Therefore, the means of the factors must be **zero.**
- This means, that the α's are the **expected returns** of the asset.

Tracking Portfolios

- A **tracking portfolio** is a portfolio that is designed to "track" the movements of an asset or investment.
- Under a factor model, this portfolio is designed to **load** on all the factors in the same way as the asset.
- That is , the tracking portfolio and the asset have the same β_{ik}

Tracking Portfolios

- Applications
  1. Hedging – to track exposures to interest rates, currencies, etc.
  2. Benchmarking – to minimize deviations from a benchmark such as the S&P 500 index.
  3. Arbitrage – to exploit mispricing among securities or investments.

Designing tracking portfolios

1. Determine the number of factors and their identity (very difficult!)
2. Compute factor betas or loadings
3. Form a portfolio which has the same factor betas as the investment you wish to track.
   - This can be done by setting up one equation for each factor beta, and then solving the equations simultaneously.

One example

- Imagine a two factor model.
- Goal: Track an asset, which has a loading of 2 on the first factor and a loading of 1 on the second factor
- Using the following three securities.

E(r_i)

$$\begin{aligned}\tilde{r}_A &= 0.04 + 2\tilde{F}_1 - 2\tilde{F}_2 + \tilde{\epsilon}_A \\ \tilde{r}_B &= 0.03 - 1\tilde{F}_1 + 2\tilde{F}_2 + \tilde{\epsilon}_B \\ \tilde{r}_C &= 0.11 + 3\tilde{F}_1 + \tilde{\epsilon}_C\end{aligned}$$

One example (cont'd)

- We need to find a portfolio with weights that sum to one such that the portfolio weighted average of the betas equals 2 for factor one and 1 for factor two.

$$\begin{aligned} 2w_A - 1w_B + 3w_C &= 2 \\ -2w_A + 2w_B &= 1 \\ w_A + w_B + w_C &= 1 \end{aligned}$$

→ $w_A = -0.2$; $w_B = 0.3$; $w_C = 0.9$

shortsell 做空

- Note: we need K+1 securities to track an investment under a K factor model.

Pure factor portfolios

- Tracking portfolios designed to track a factor.
- Portfolio has a loading of 1 on the factor, and loadings of zero on all other factors.
- Note: these are well-diversified portfolios. Hence, they have no firm-specific risk.

Factor 1 portfolios

- Portfolio construction

$$\begin{aligned} 2w_A - 1w_B + 3w_C &= 1 \\ -2w_A + 2w_B &= 0 \\ w_A + w_B + w_C &= 1 \end{aligned}$$

→ $w_A = 0.4$; $w_B = 0.4$; $w_C = 0.2$

- Expected return
  - 0.4*0.04 + 0.4*0.03 + 0.2*0.11=5%

Factor 2 portfolios

- Portfolio construction

$$\begin{aligned} 2w_A - 1w_B + 3w_C &= 0 \\ -2w_A + 2w_B &= 1 \\ w_A + w_B + w_C &= 1 \end{aligned}$$

→ $w_A = 0.2$; $w_B = 0.7$; $w_C = 0.1$

- Expected return
  - 0.2*0.04 + 0.7*0.03 + 0.1*0.11=4%

Risk free asset

- Portfolio construction

  $2\,w_A - 1\,w_B + 3\,w_C = 0$

  $-2\,w_A + 2\,w_B = 0$

  $w_A + w_B + w_C = 1$

→ $w_A = 0.6$; $w_B = 0.6$; $w_C = -0.2$

- Asset return (risk free rate)
  - 0.6*0.04 + 0.6*0.03 - 0.2*0.11=2%

Interpretation

- The risk premiums of factor portfolios are usually denoted as λ_k for factor k.
- Premium depends on aggregate supply of the factor in the economy and the tastes of investors.

Interpretation (cont'd)

- We can use the factor portfolios to track any investment.
- To track the asset in the earlier example, we would simply buy two of factor portfolio one and buy one of factor portfolio two.
- Since pure factor portfolios are themselves portfolios of individual assets, we can further break down the tracking portfolio for the asset.

Interpretation (cont'd)

- We can create tracking portfolios from factor portfolios or from individual securities to track any investment.

	Portfolio Weights			
Tracking Portfolio	Security A	Security B	Security C	Sum
(2) Pure Factor 1	0.40	0.40	0.20	1
(1) Pure Factor 2	0.20	0.70	0.10	1
(-2) Risk-Free Port.	0.60	0.60	-0.20	1
(1) Asset (= $2F_1 + F_2$)	-0.20	0.30	0.90	1

- The position in the risk-free asset is 1- β1 – β2

  weight

Interpretation (cont'd)

- If the tracking portfolio's expected return differs from the investment's, then there is **arbitrage**.
- We can buy the one with the higher expected return, and sell the one with the lower, and make money with no risk and no initial cash expenditure!

Arbitrage and Arbitrage Pricing

- Previously, we tracked the asset by forming a portfolio with the same factor beta configuration. This implied,

做多 → $\tilde{R}_{asset} = \alpha_{asset} + \overset{\beta_1}{2}\tilde{F}_1 + \overset{\beta_2}{1}\tilde{F}_2$ 当 β_1、β_2 确定时, α_{asset} 被确定

做空 → $\tilde{R}_P = \alpha_P + 2\tilde{F}_1 + 1\tilde{F}_2$

⇒ 获得 $\alpha_{asset} - \alpha_P$

  - where portfolio P is the tracking portfolio
- Since both portfolios are **well-diversified**, idiosyncratic risk is eliminated.
- Thus, the only difference in the equations are the α's or expected returns.

Arbitrage and Arbitrage Pricing

- What happens if $\alpha_{asset} \neq \alpha_P$?
- Thus the expected return of the tracking portfolio is

risk premium

$$\alpha_P = (1-\beta_1-\beta_2)E[R_{f*}] + \beta_1 E[R_{FP1}] + \beta_2 E[R_{FP2}]$$
$$\alpha_P = (1-\beta_1-\beta_2)r_f + \beta_1(r_f+\lambda_1) + \beta_2(r_f+\lambda_2)$$
$$= r_f + \beta_1\lambda_1 + \boxed{\beta_2\lambda_2}$$

承担一单位风险所获得的补偿

- In our example, it is

pure factor portfolio returns

$$\alpha_P = 0.02 + 2(0.03) + 1(0.02) = 0.10$$

↳ rf portfolio return

Arbitrage and Arbitrage Pricing

- Suppose the expected return on the asset is 11%. Is there arbitrage? And, how do we exploit it?
- No matter what happens in the macroeconomy, we are perfectly hedged, since our arbitrage strategy takes exactly opposite positions on the factors, which are the sole source of risk for every investment.

Arbitrage Pricing Theory

- The use of tracking portfolios and the possibility to exploit arbitrage is the basis for the Arbitrage Pricing Theory (APT)
- The APT only requires three assumptions: APT假设

  1. Returns follow a factor model 未限制 factor 数量
  2. There are no arbitrage opportunities
  3. **There are many securities, so that we can form well-diversified portfolios that eliminate firm-specific risk** $E(\varepsilon_i)=0$

Arbitrage Pricing Theory

- If we assume no arbitrage, and returns follow a factor model, then every asset must have the same expected return as its tracking portfolio.
- The expected return of the tracking portfolio, α_P, is

$$r_f + \beta_{i1}\lambda_1 + ... + \beta_{iK}\lambda_K$$

  - where the λ's are the risk premiums on the K factors,

APT equation:

- Then, we have the APT equation:

$$E[\tilde{r}_i] = \alpha_i = \alpha_P = r_f + \beta_{i1}\lambda_1 + ... + \beta_{iK}\lambda_K$$

碰巧，因为 Assumption 不同

  - Reminder: In CAPM: $\mathrm{E}[r_i] = r_f + \beta_i(\mathrm{E}[r_m] - r_f)$

一维的APT，且 risk premium 类型被固定

- Thus, we now have a statement about expected returns. In other words, we now have a pricing model.

CAPM vs. APT

- Note: unlike the CAPM, the only assumption here is that no arbitrage exists.
- Thus, the APT is much less restrictive than the CAPM.
- However, this comes at a cost, because the APT is also **less specific.**
  - the APT does not tell us how many factors
  - or what those factors are
  - the CAPM tells us there is one factor, and it is the market portfolio

General Ideas in Asset Pricing

- A security's expected return depends on when it makes its payments
- A security is attractive if it pays up when times are bad
  - that's exactly when you need the money!
  - you are prepared to accept a *low* expected return on these securities
- A security is not attractive if it pays up when times are good
  - you don't need any more money at these times
  - you require a high expected return on these securities

Another explanation of high returns, high risk

General Ideas in Asset Pricing

- In the CAPM, good and bad times are captured by the market return
  - Good times: market return is high, and vice-versa
- Securities with a high beta tend to pay up (i.e., earn high returns) when the market is doing well
  - What use is that?
  - The security is unattractive
  - It must earn a high expected return

Risk and Return Revisit

Jinfan Zhang
Chinese University of Hong Kong (Shenzhen)

Definition of return

- Return=(what you get tomorrow)/what you paid today or, R=(value at time t+1)/(value at time t)

$$R_{t+1} = \frac{P_{t+1} + CF_{t+1}}{P_t}$$

e.g. Dividend

- What you get tomorrow is the price of the asset plus any cash flow it pays off
  - Ex: for a stock CF=dividend, for a bond CF=coupon payment
- This is often called **gross return**

Gross vs. net return

- We will typically work with **net returns**, r

$$r_{t+1} = \frac{P_{t+1} + CF_{t+1}}{P_t} - 1$$

- Example, suppose you buy a stock at P=$100, tomorrow's price is $101 and it pays a $3 dividend
  - Gross return = (101+3)/100=1.04
  - Net return =1.04-1=0.04 or 4%
  - 4%, the net return, is often a more convenient number to think about than 1.04

Expected Return

- Actual returns are not known ahead of time, so we will often compute **expected returns**

$$E[r_{t+1}] = \frac{E[P_{t+1}] + E[CF_{t+1}]}{P_t} - 1$$

- This tells us what the return is likely to be *on average*
- **Realized returns** (what you actually get) are often very different from expected returns
  - Example: stock price today is $100, equal chance it will go up to either $150 or down to $50. Realized return will be either +50% or -50%. Expected return is E[r]=0.5*(50%)+0.5*(-50%)=0.

Multi-period returns and compounding

- $R_{t,t+1}$ gross return from time t to t+1
- $R_{t+1,t+2}$ gross return from time t+1 to t+2
- $R_{t,t+2}$ gross return from time t to t+2
- $r_{t,t+2} = R_{t,t+2} - 1$ net return from t to t+2

Multi-period returns and compounding

- Then $R_{t,t+2} = R_{t,t+1}R_{t+1,t+2}$

$$(1+r_{t,t+2}) = (1+r_{t,t+1})(1+r_{t+1,t+2})$$

- This is the standard compounding formula
  - Example: Suppose you earn **10%** in year 1 and **20%** in year 2.
  - Your 2-year net return is (1+0.1)(1+0.2)-1=**32%**

Multi-period returns and compounding

- For a stock

Assumption: Dividend is reinvested.

$$R_{t,t+2} = \frac{P_{t+2} + D_{t+2} + D_{t+1}\left(\frac{P_{t+2} + D_{t+2}}{P_{t+1}}\right)}{P_t}$$

$$R_{t,t+2} = \frac{P_{t+2} + D_{t+2}}{P_{t+1}} \frac{P_{t+1} + D_{t+1}}{P_t} = R_{t,t+1}R_{t+1,t+2}$$

Real vs. nominal returns

- So far we have been working with nominal returns

Investment: To increase the purchasing power (购买力)

- To convert a nominal return to a real return we divide by the rate of inflation, π 考虑通胀率

$$(1+r_{real}) = \frac{(1+r_{nom})}{(1+\pi)}$$

- Approximation $r_{real} \approx r_{nom} - \pi$

Historical Returns

- What have the real, annual, realized returns on securities been historically?

Period	T-Bills	T-Bonds	Stocks (S&P500)
1870-1925	2.6%	2.8%	6.5%
1926-2014	0.6%	2.2%	6.6%

- **Equity premium** is the average return of stocks minus the average return on T-Bills: It is around 6% historically.

Risk and Return

- Risk is uncertainty about the future
- Over the long run stocks do better on average, but year to year their annual returns are more volatile or uncertain

	T-Bills	T-Bonds	Stocks (S&P500)
Average return	0.6%	2.2%	6.6%
Volatility	3.0%	6.1%	19.1%
Worst year	-15.2%	-16.0%	-48.0%

- Stocks are “riskier”
- Example: the S&P500 dropped over 40% in 2008

Risk and Return

- Human beings are *mostly* **risk averse**, i.e. dislike risk
- To be willing to hold a riskier security, they must receive a higher average return as a reward for doing so
  - This is why stocks have higher average returns than bonds
  - Ex ante vs. ex post
- “Average return is compensation for risk” is a good way of thinking about long-term patterns in average returns
- We will discuss how to measure risk more precisely later in the course

Expected Return

- Expected returns

$$E(r) = \sum_s p(s)r(s)$$

  - p(s) = Probability of a state
  - r(s) = Return if a state occurs
  - s = State

Scenario Returns: Example

State	Prob. of State	r in State
Excellent	.25	0.3100
Good	.45	0.1400
Poor	.25	-0.0675
Crash	.05	-0.5200

E(*r*) = (.25)(.31) + (.45)(.14) + (.25)(−.0675) + (0.05)(− 0.52)
= .0976 or 9.76%

Standard Deviation

- Variance (VAR):

$$\sigma^2 = \sum_s p(s)[r(s) - E(r)]^2$$

- Standard Deviation (STD):

$$\text{STD} = \sqrt{\sigma^2}$$

VAR and STD: Example

- VAR calculation:

σ^2 = .25(.31 − 0.0976)^2 + .45(.14 − .0976)^2
+ .25(− 0.0675 − 0.0976)^2 + .05(−.52 − .0976)^2
= .038

- STD calculation:

$$\sigma = \sqrt{.038} = .1949$$

Time Series Analysis of Past Rates of Return

- You can only observe a time series of past returns
  - You cannot see all possible states
- True means and variances are unobservable because we don't actually know possible scenarios like the one in the examples
- So we must estimate the means and variances

Arithmetic vs. Geometric Average

- Arithmetic Average

$$E(r) = \frac{1}{n}\sum_{i=1}^{n} r_i$$

- Arithmetic Average is an unbiased estimate of the **expected future return**
- What is the **actual** performance of the **past** sample?

$<$ Arithmetic , $\forall r$

- Geometric (Time-Weighted) Average

$$g = \left((1+r_1)(1+r_2)\ldots(1+r_n)\right)^{1/n} - 1$$

Arithmetic vs. Geometric Average

- Your returns in the last 2 years are 10%, -10%.
- What is your expected return next year?
  - **Arithmetic Average: 0%**
- What was your return over the last two years?
  - $1*(1+10%)*(1-10%)=0.99, that is -1% over 2 years,
  - Or, SQRT(0.99)-1=-0.5% per year
  - **The geometric average return is -0.5%**
- Asymmetry: negative return has a larger effect
  - You had a -50% return last year, what is the return you need this year to break even? Ans: 100%

Arithmetic vs. Geometric Average

- What determines the difference between the two?
- Case 1: returns are not volatile: 9%, 11%
  - Arithmetic average: 10% 短期持有
  - Geometric average: 9.995% 长期持有：累计回报率

Cumulative return：与 std 呈负相关

- Case 2: returns are volatile: -90%, 110%
  - Arithmetic average: 10%
  - Geometric average: -54%
- The difference is larger when returns are more volatile

Estimating Variance and Standard Deviation

- Estimated Variance

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}[r_i - \bar{r}]^2$$

- Unbiased estimated standard deviation

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}[r_i - \bar{r}]^2}$$

Sharpe Ratio

- Excess Return $r - r_f$
  - The difference in any particular period between the **actual** rate of return on a risky asset and the actual risk-free rate
- Risk Premium $E(r) - r_f$
  - The difference between the **expected** return on a risky asset and the risk-free rate
- Sharpe Ratio

越大越好
>1 已经很好

$$\frac{\text{Risk premium}}{\text{SD of excess returns}} = \frac{E(r) - r_f}{\sigma(r - r_f)}$$

The Normal Distribution

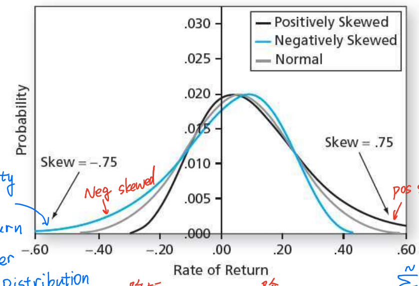
- **If returns are normally distributed**
  - Standard deviation is a good measure of risk
  - Portfolio returns are also normal
  - Future scenarios can be estimated using only the mean and the standard deviation
  - The dependence of returns across securities can be summarized using only the pairwise correlation coefficients

Normality and Risk Measures

- What if excess returns are not normally distributed? usual situation
  - Standard deviation is no longer a complete measure of risk
  - Sharpe ratio is not a complete measure of portfolio performance
  - Need to consider skewness and kurtosis

Normal and Skewed Distributions 偏斜分布

Skewness: 三阶矩 $(\ )^3$



The probability that gets a negative return is much larger than Normal Distribution

Mean = 6%, SD = 17%

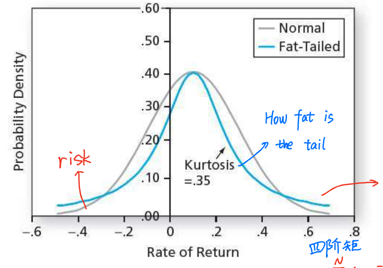
一阶矩 $(\ )^1$ 二阶 $(\ )^2$

$$g_1 = \frac{\sum_{i=1}^{N}(Y_i-\bar{Y})^3/N}{s^3}$$ (Stand Deviation)

For Gaussian Dist. $g_1 = 0$

Normal and Fat-Tailed Distributions

Kurtosis ↑
⇓
Fatter tail



Mean = .1, SD = .2

四阶矩 $$k = \frac{\sum_{i=1}^{N}(Y_i-\bar{Y})^4/N}{s^4} - 3$$

Normality and Risk Measures

- Value at Risk (VaR) 左侧尾部较小
  - Loss corresponding to a very low percentile of the entire return distribution, such as the fifth or first percentile return
  - Example: a 5% VaR of $1M: The chance of losing **more than** or equal to $1M is 5%.
    → At most 5% chance
- Expected Shortfall (ES)
  - Also called conditional tail expectation (CTE), focuses on the expected loss in the worst-case scenario (left tail of the distribution)
  - More conservative measure of downside risk than VaR

Normality and Risk Measures

- Lower Partial Standard Deviation (LPSD)
  - Similar to usual standard deviation, but uses only negative deviations from the risk-free return, thus, addressing the asymmetry in returns
    不对称 r's that $r < r_f$ 只考虑小于 r_f 的回报率的std.
- Sortino Ratio (replaces Sharpe Ratio)
  - The ratio of average excess returns to LPSD

Historic Returns on Risky Portfolios

- The second half of the 20th century, politically and economically the most stable sub-period, offered the highest average returns
- Firm capitalization is highly skewed to the right: Many small but a few gigantic firms 巨大的
- Average realized returns have generally been higher for stocks of small rather than large capitalization firms

Price: The Present Value Formula

- When investing in an asset, we have in mind an average rate of return that is fair for that asset
  - The "expected return" for an asset $r = E[r_{t+1}]$
  - r= riskless interest rate + risk premium
  - = real interest rate + expected inflation + risk premium
    ∠ impatience of people 印的钱多, inflation↑
    与GDP Growth 成正比, GDP Growth↑, 公司借钱多, interest rate↑
- r, the fair average return, is also called:
  - the "expected return"
  - the "discount rate" Alternative names
  - the "required return"

The Present Value Formula

- We can rearrange the definition of return to obtain an important pricing formula, the present value formula

$$1 + r_{t+1} = \frac{CF_{t+1} + P_{t+1}}{P_t}$$

$$1 + r = \frac{E[CF_{t+1}] + E[P_{t+1}]}{P_t}$$

$$\Downarrow$$

$$P_t = \frac{E[CF_{t+1}] + E[P_{t+1}]}{1+r}$$

$$P_{t+1} = \frac{E[CF_{t+2}] + E[P_{t+2}]}{1+r}$$

The Present Value Formula

- Substituting, we get

$$P_t = \frac{E[CF_{t+1}]}{1+r} + \frac{E[CF_{t+2}]}{(1+r)^2} + \frac{E[P_{t+2}]}{(1+r)^2}$$

- and so on, until

$$P_t = \frac{E[CF_{t+1}]}{1+r} + \frac{E[CF_{t+2}]}{(1+r)^2} + \frac{E[CF_{t+3}]}{(1+r)^3} + \ldots$$