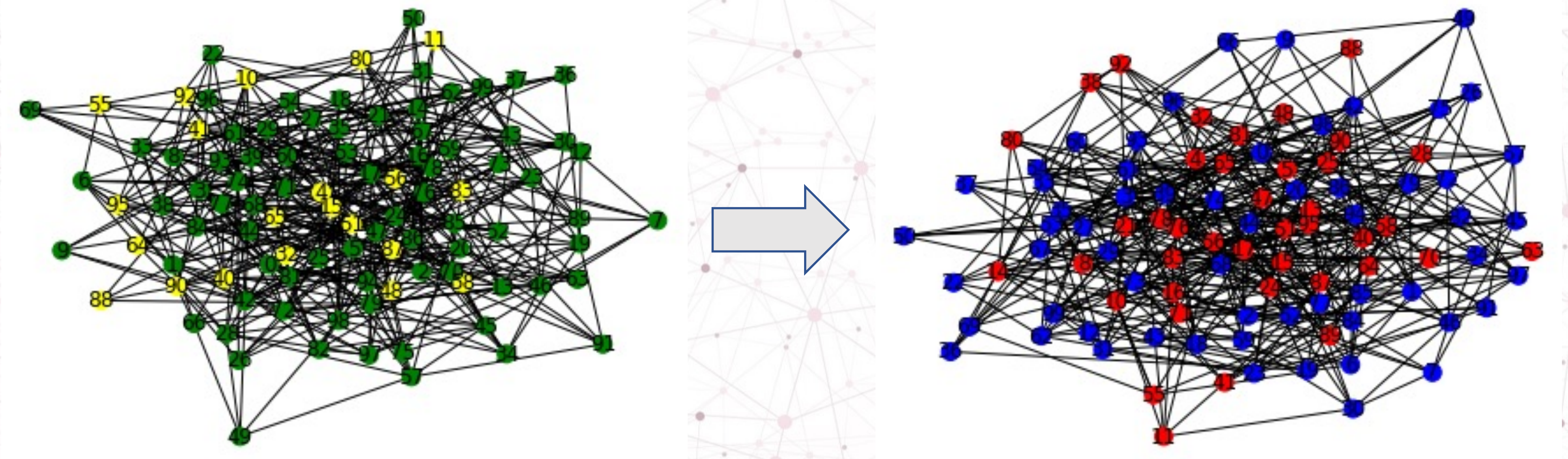
Diffusion Dynamics Under the T-LISA Model



Marko S. Suchy

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Presentation Outline

- Background

- Innovation Diffusion
- The LISA Model
- Limitations of the LISA model

- T-LISA, Mean-Field

- Coupled ODE Rules
- Mean-Field Results
- Phase Plot

- T-LISA, Graphical Methods

- What is a Graph?
- Agent Based Rules & Monte Carlo Methods
- Rescaling Mean-Field to fit ER Graphs

- Analysis of Results

- Total Average Error
- Local Environment Effects
- Global Environment Effects

Background - Innovation Diffusion:

Innovation

Durable consumer goods...

https://judithsalecich.com/wp-content/uploads/2019/09/Early-model-Hoovermatic-Twin-Tub-Washing-Machine.jpg

Social policy...

https://www.google.com/url?sa=i&url=https%3A%2F%2Fpsmag.com%2Fnews%2Fi-pledge-to-vote-gets-people-to-the-polls&psig=AOvVaw1gCOHm34qFLHcvPzOgMAEn&ust=1712536033418000&source=images&cd=vfe&opi=89978449&ved=0CBIQjRxqFwoTCOjQ0ILsroUDFQAAAAAdAAAAABAE

Diffusion

Spreading through a system.

https://www.thoughtco.com/thmb/TqVJtnnpcSaW38uJldPSqfi9ncM=/1500x0/filters:no_upscale():max_bytes(150000):strip_icc()/diffusion-in-water-530463502-5766aaec3df78ca6e4a98185.jpg

‘When new ideas are invented, diffused, and are adopted or rejected, leading to certain consequences, social change occurs’ [1].

- E. M. Rogers

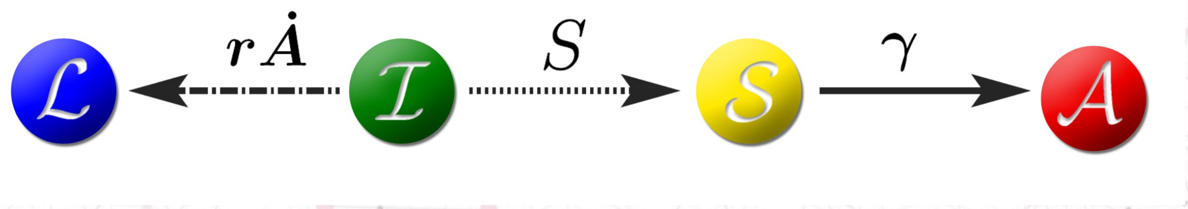
Background - Innovation Diffusion:

Behavior Innovation?

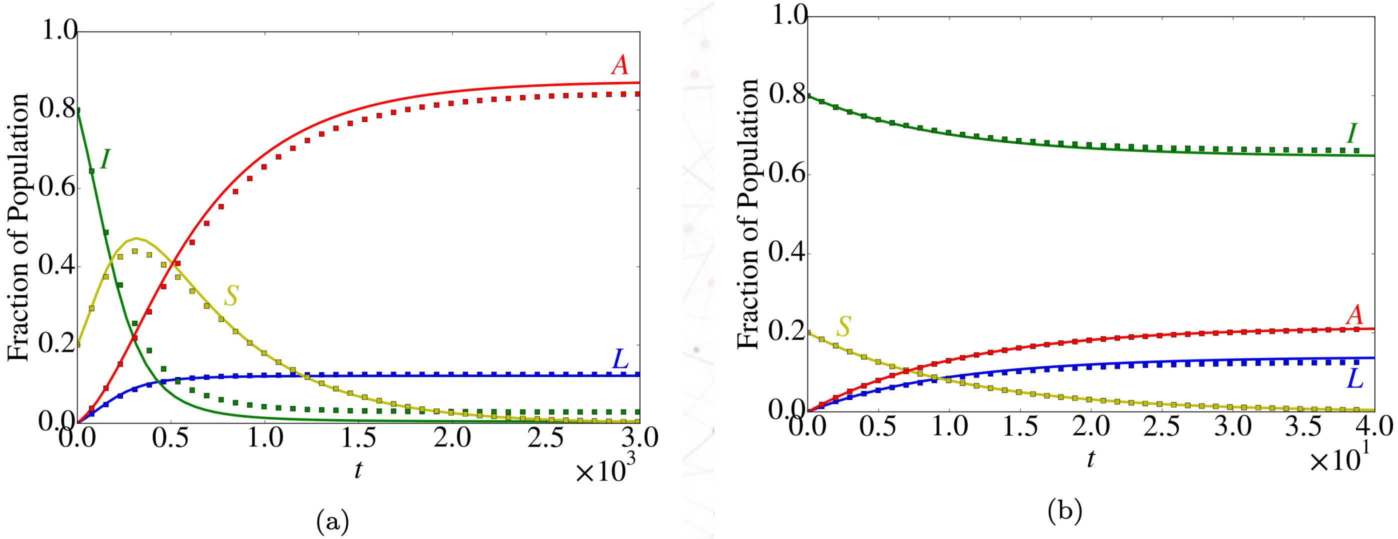
https://i.ytimg.com/vi/gC0QUfbfYy4/sddefault.jpg

Pulling out your smartphone to take a photo of something.

Background – LISA Model:



- Melor et al. (2015) [2]
- Implemented via both mean-field, and stochastic methods
- Showed 2 types of adoption:
  - Slow and widespread
  - Rapid but sparse



"FIG. 4: The evolution, averaged over 100 realizations, of the LISA model on an ER graph with $N = 10$^3 nodes, $k = 10$, and $I_0 = 0.8$. (a) $\gamma = 0.002$, such that $\gamma < \left(\frac{k}{N}\right) I_0$ and (b) $\gamma = 0.1$ such that $\gamma > \left(\frac{k}{N}\right) I_0$. Shown are the evenly distributed samples of the stochastic simulation (2) and the solution of Eq. (8) (solid line). The Luddism parameter $r = 0.9$." [2]

Background – Limitation of the LISA Model:

Sometimes, Ignorants are left in the steady state!

$$L' = rA'I = r\gamma SI$$

$$I' = -(1+\gamma r)SI$$

$$S' = S(I-\gamma)$$

$$A' = \gamma S$$

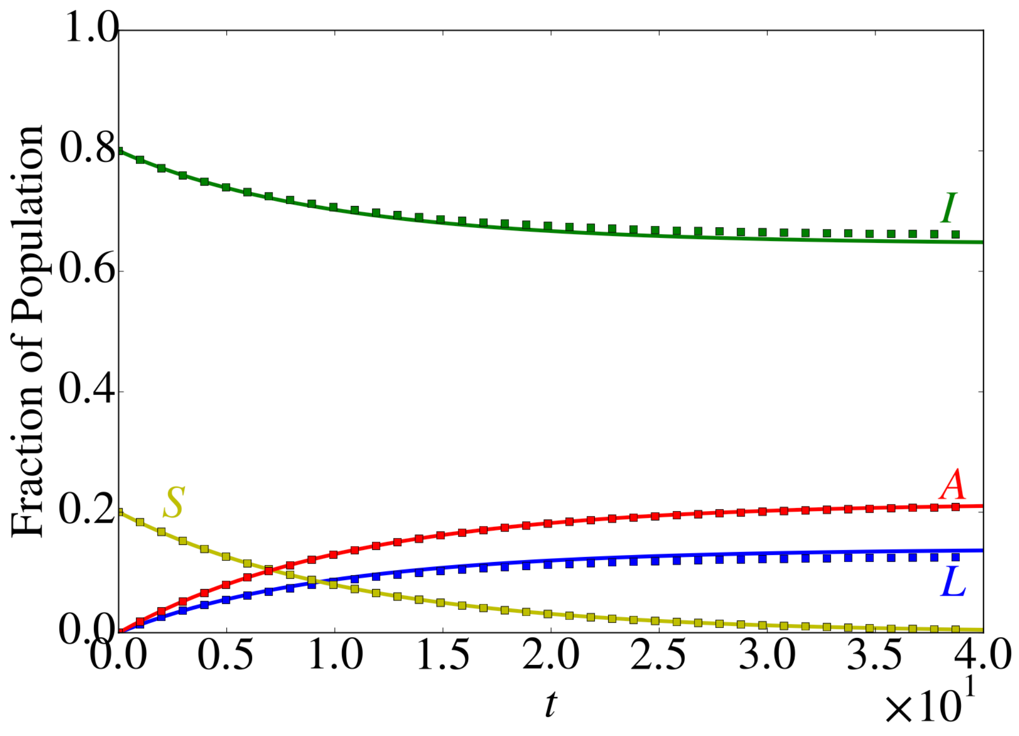
Adapted from [2]

Where:

γ = adoption rate

r = luddism parameter

L, I, S, A are population proportions



(b)

From [2]

T-LISA, Mean Field – Coupled ODEs :

$$L' = \gamma r S I + \omega r A I$$

$$I' = -(S + A)I - \gamma r S I - \omega r A I$$

$$S' = (S + A)I - \gamma S$$

$$A' = \gamma S$$

Where:
$\gamma =$ adoption rate
$r =$ luddism parameter
$\omega =$ anti-establishment tendency
L, I, S, A are population proportions

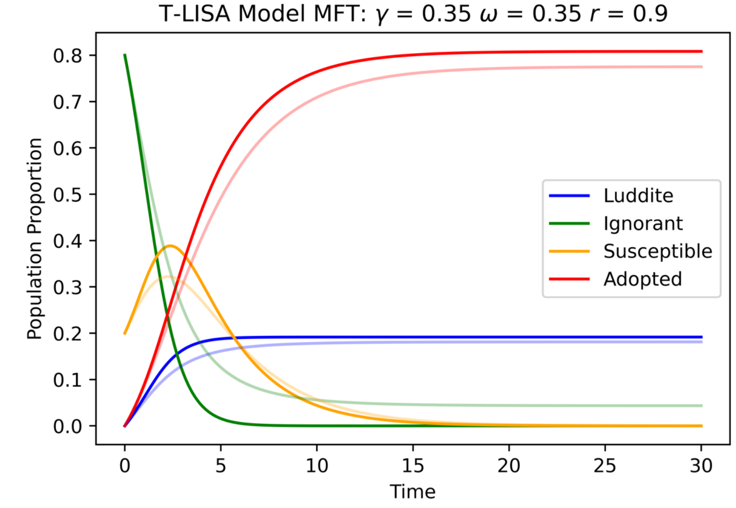
Now our rules:

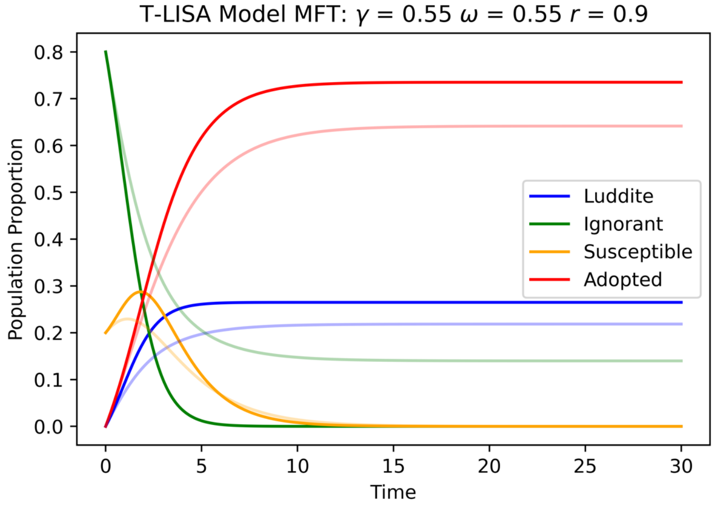
- includes pressure from Adopters to adopt and reject.
- always result in Adopter-Luddite dichotomy.

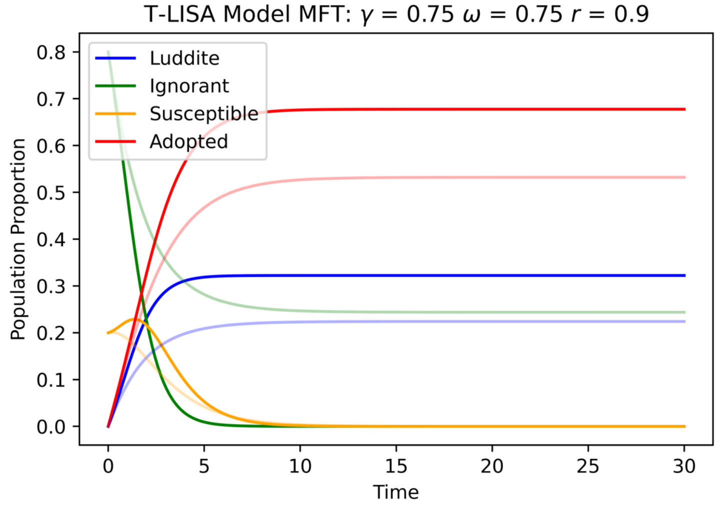
T-LISA, Mean Field – Results:

γ_{small} $\longleftrightarrow$ γ_{big}

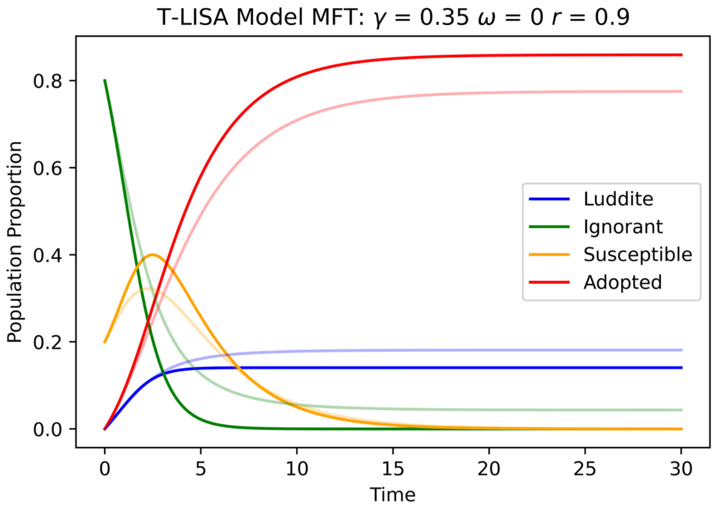
$\omega = \gamma$:

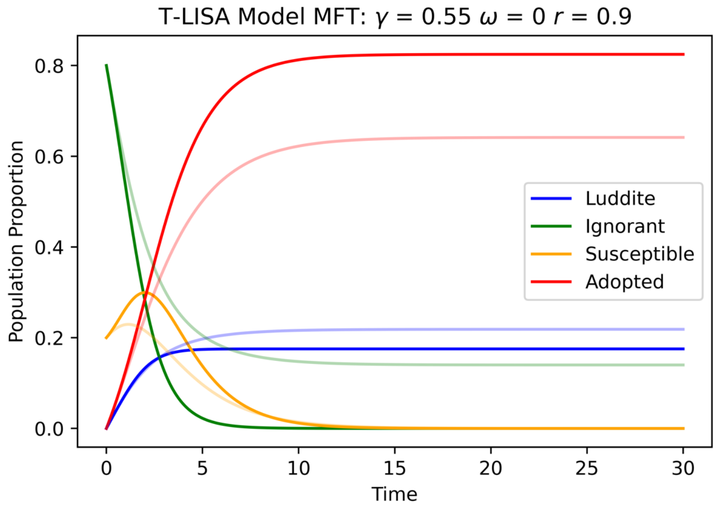


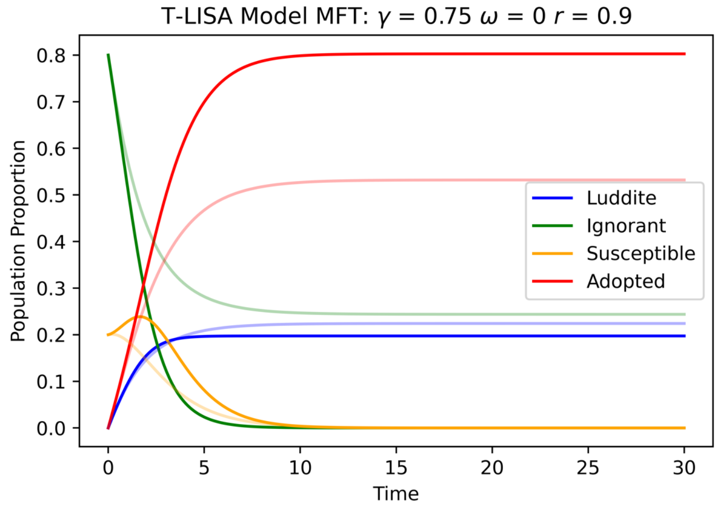




$\omega = 0$:

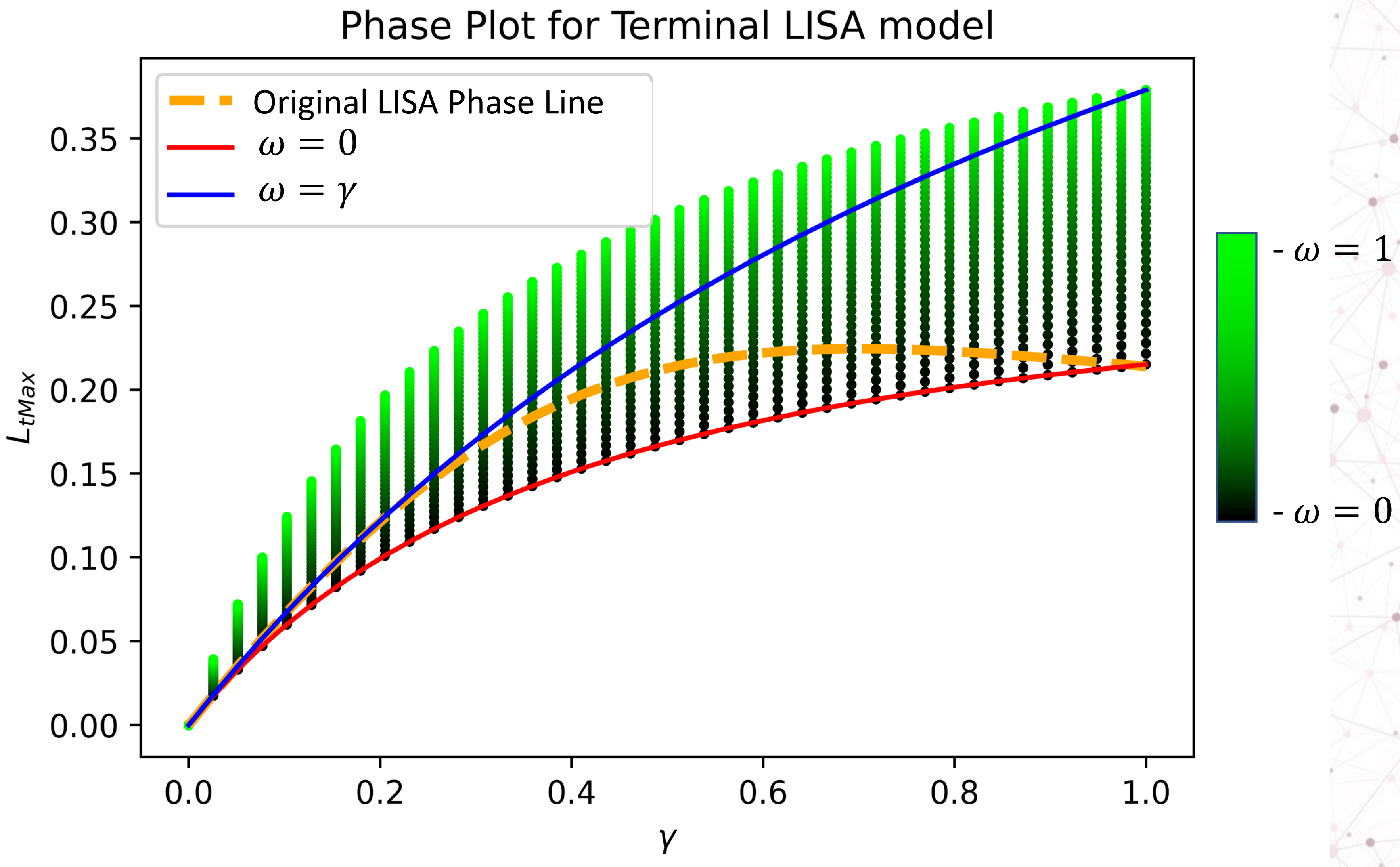




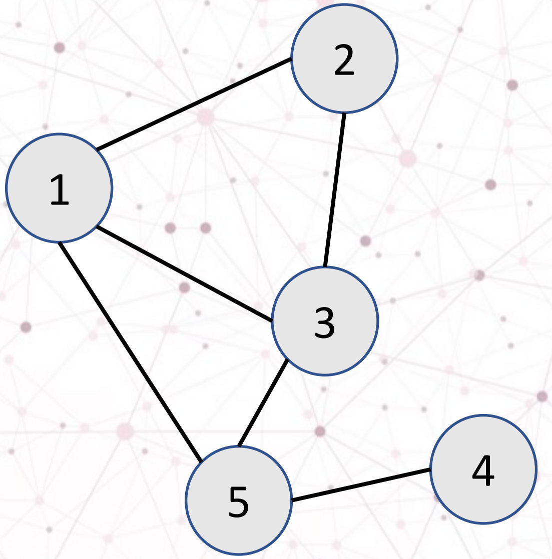


T-LISA, Mean Field – Results:

Phase diagram for the Terminal LISA model with $r = 0.9$, $S_0 = 0.2$. The range of black and green dots coming off the $\omega = 0$ line represent the possible results for L_{tMax} (final luddite proportion) when introducing an anti-establishment pressure parameter ranging from 0 to 1.

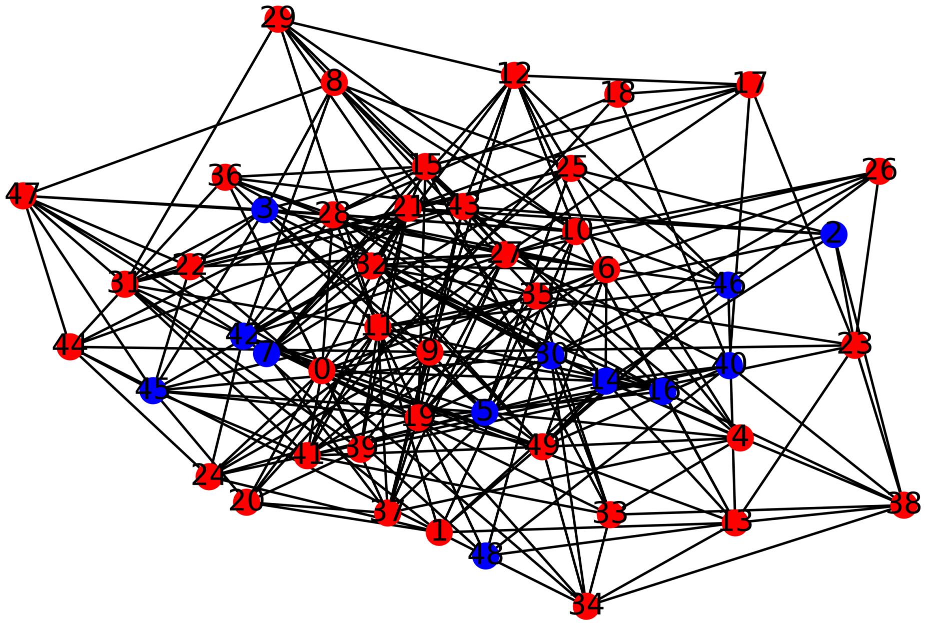


T-LISA, Graphical Methods – What is a Graph:



$$A_{i,j} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

A simple example of an unweighted, undirected network with 5 nodes, and it's adjacency matrix.



An Erdos-Renyi (ER) graph, G, with $N = 50$, $k = 10$ in the stable state of the T-LISA model.

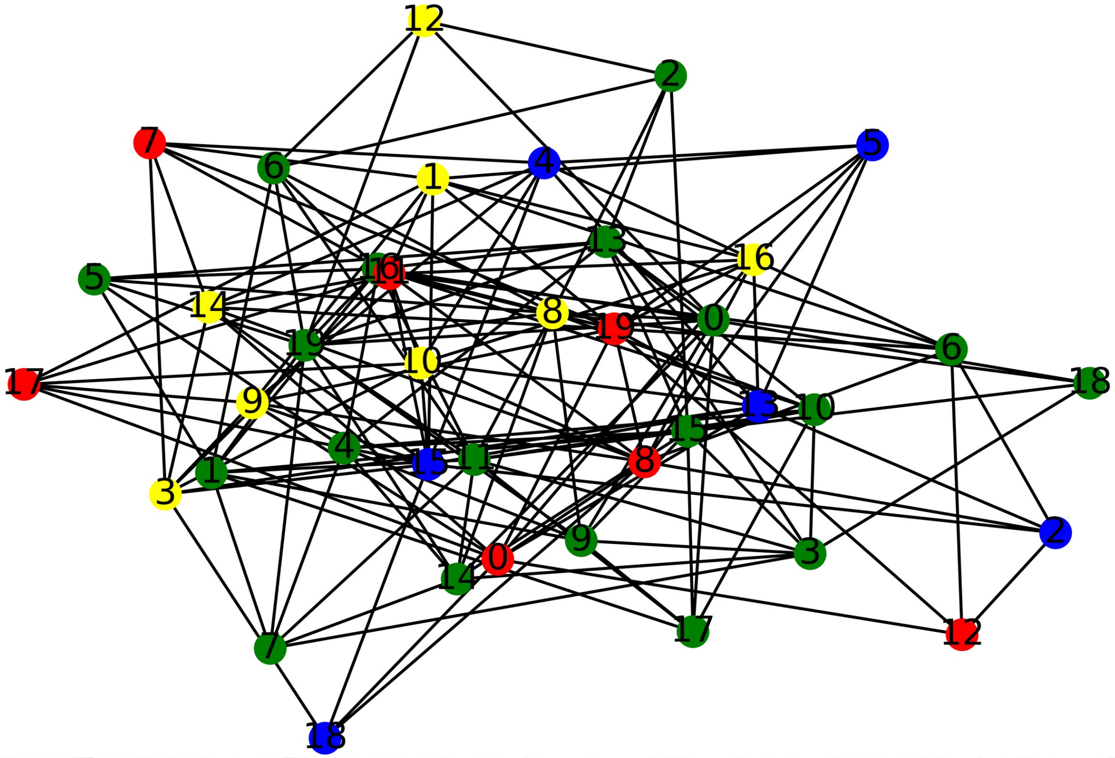
'a pattern of interconnections among a set of things' [3]

T-LISA, Graphical Methods – Agent Based Rules:

$$I \rightarrow L : r(\gamma \frac{s_i}{k_i} + \omega \frac{a_i}{k_i})$$

$$I \rightarrow S : \frac{s_i + a_i}{N}$$

$$S \rightarrow A : \gamma$$



Where:

s_i = Susceptible neighbors of node i

a_i = Adopter neighbors of node i

k_i = Number of neighbors (degree) of node i

N = number of nodes

γ = adoption rate

r = luddism parameter

T-LISA, Graphical Methods – Rescaling MFT:

2 body contagion processes must be rescaled by $\frac{k}{N}$

$$L' = \gamma r S I + \omega r A I$$

$$I' = -\frac{k}{N}(S + A)I - \gamma r S I - \omega r A I$$

$$S' = \frac{k}{N}(S + A)I - \gamma S$$

$$A' = \gamma S$$

Where:

$\gamma =$ adoption rate

$r =$ luddism parameter

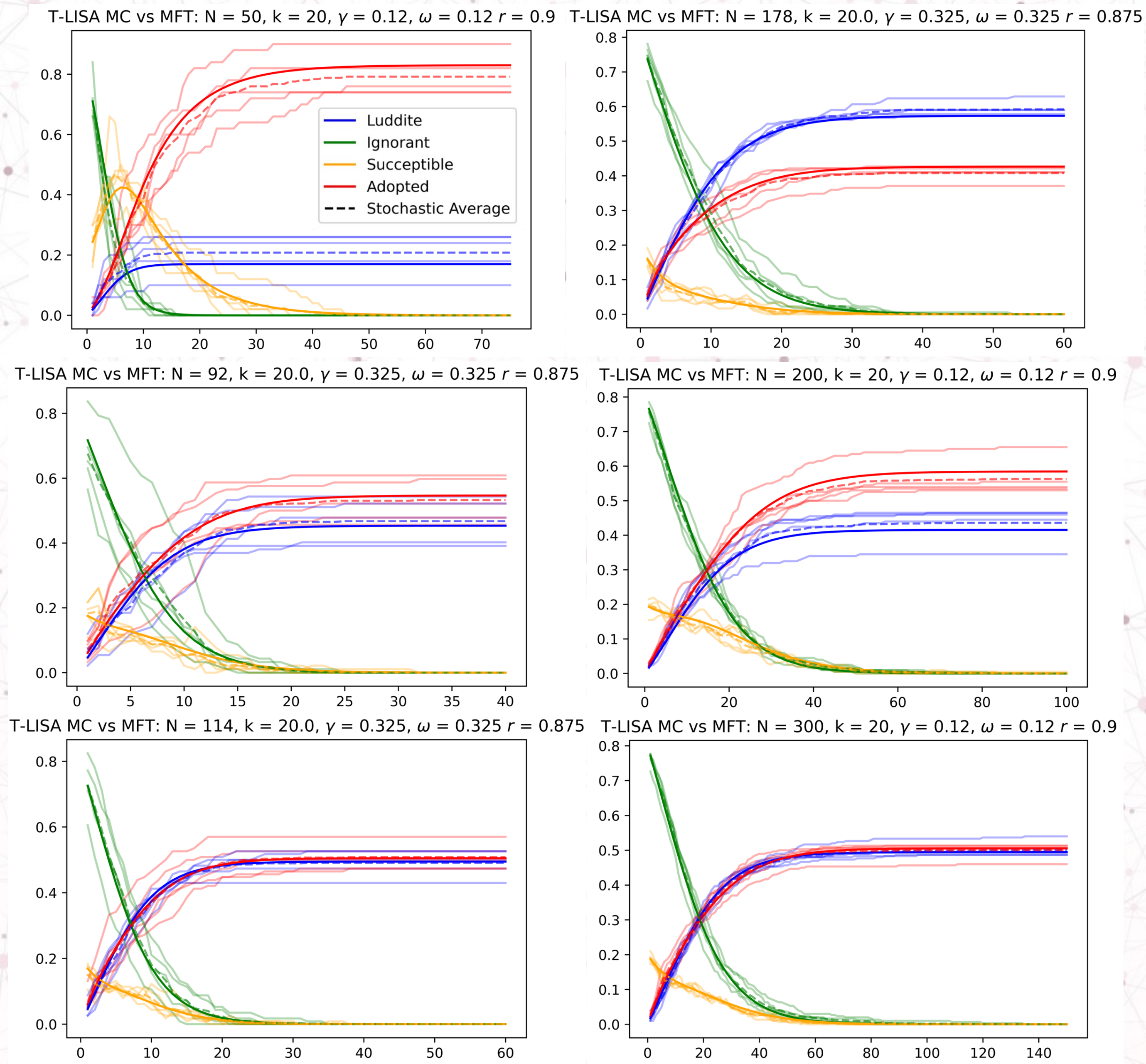
$\omega =$ anti-establishment tendency

L, I, S, A are population proportions

$N =$Number of nodes in G

$k =$ average degree of G

Analysis of Results – Total Average Error:



Quantifying Error:

$$D = MC_{sol} - MFT_{sol}$$

Where:

MC_{sol} is the MC solutions matrix

MFT_{sol} is the MFT solutions matrix

D is the $m \times n$ difference matrix

$$error_n = \frac{1}{m}\sum_{i=1}^{m} d_{in}$$

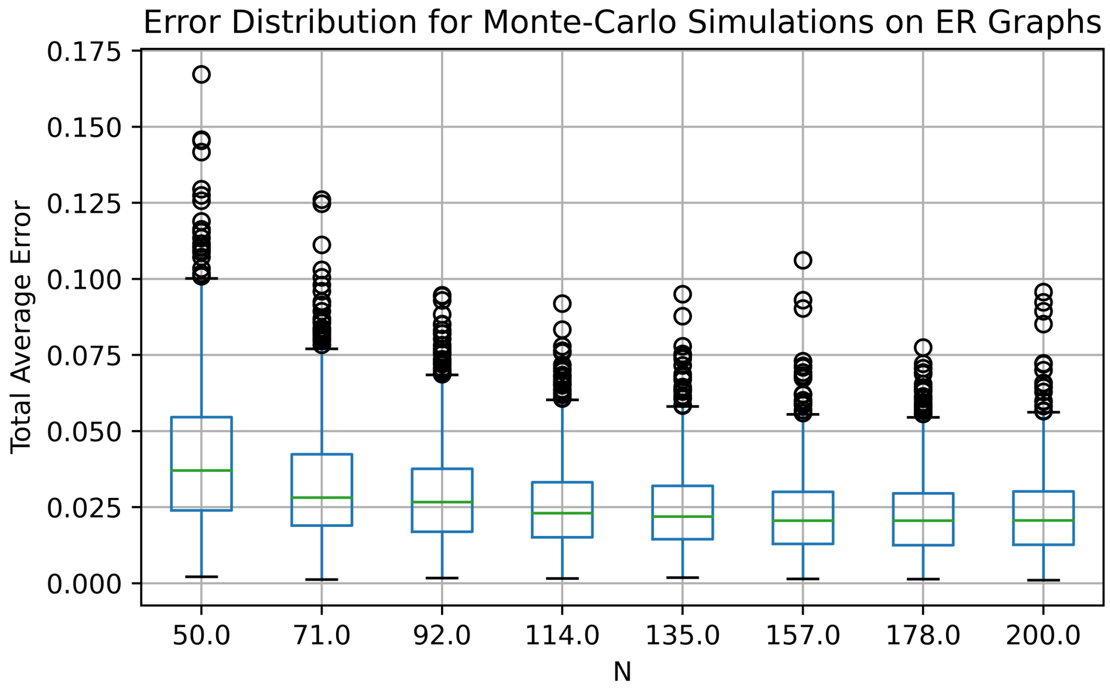
Where:

d is an element of D

$error_n$ is an element of $\boldsymbol{error}$

$$Total\ Average\ Error = \frac{1}{n}\sum_{i=1}^{n} |error_i|$$

Analysis of Results – Total Average Error:



Distribution of total average error across N for 9000 stochastic runs. Parameters r, k, γ, and ω varied on the intervals [0.5, 1], [10, 30], [0.1, 1] and [0.1, 1] respectively.

OLS Regression Analysis:

	coef	std err	t	P> |t|	[0.025	0.975]
const	0.0445	0.001	71.073	0.000	0.043	0.046
N	-0.0001	3.48e-06	-32.079	0.000	-0.000	-0.000
k	-0.0002	2.09e-05	-7.256	0.000	-0.000	-0.000

OLS regression with total average error as response variable. Regressions performed on data with $r \in [0.5, 1]$, $k \in [10, 30]$, $\gamma \in [0.1, 1]$, and $\omega \in [0.1, 1]$, $R^2 = 0.107$.

	coef	std err	t	P> |t|	[0.025	0.975]
const	0.0391	0.001	37.582	0.000	0.037	0.041
N	-0.0001	3.46e-06	-32.207	0.000	-0.000	-0.000
r	0.0029	0.001	3.006	0.003	0.001	0.005
gamma	0.0020	0.001	3.762	0.000	0.001	0.003
omega	0.0038	0.001	7.190	0.000	0.003	0.005
k	-0.0002	2.08e-05	-7.285	0.000	-0.000	-0.000

OLS regression with total average error as response variable Regressions performed on data with $r \in [0.5, 1]$, $k \in [10, 30]$, $\gamma \in [0.1, 1]$, and $\omega \in [0.1, 1]$. $R^2 = 0.115$.

Analysis of Results – Local Environment Effects:

Metric	Definition
Degree	The number of edges connected to a node.
Closeness Centrality	How close a node is to all other nodes in the network, based on the shortest paths between them. A high closeness centrality indicates that a node is close with many other nodes.
Betweenness Centrality	The proportion of all shortest paths in the network that pass through a given node, indicating the node's role as a bridge.
Eigenvector Centrality	A measure of the influence of a node in a network, taking into account the centrality of its neighbors.
Clustering Coefficient	The degree to which nodes in a graph tend to cluster together, measured as the ratio of existing links connecting a node's neighbors to each other to the maximum possible number of such links.
Triangles	The number of triangles that include the node as one vertex, indicating the node's tendency to form tightly knit groups.

Node-level structural metrics used as independent variables to predict $A_{tMax}(i)$, the proportion of adoption for node i after several simulations

LPM Regression Analysis:

	coef	std err	t	P> |t|	[0.025	0.975]
const	0.4656	0.073	6.421	0.000	0.323	0.608
degree_normalized	0.0748	0.386	0.194	0.846	-0.682	0.831
closeness_normalized	0.1627	0.144	1.126	0.260	-0.121	0.446
betweenness_normalized	-0.1818	0.184	-0.989	0.323	-0.542	0.178
eigenvector_normalized	0.2443	0.210	1.161	0.246	-0.168	0.657
clustering_normalized	-0.0586	0.093	-0.632	0.527	-0.240	0.123
triangles_normalized	0.0196	0.190	0.103	0.918	-0.352	0.391

Regression results for a system with N = 300, k = 30, r = 0.9, γ = .08, and ω = .08, with explanatory variables forced on the interval [0, 1].

	coef	std err	t	P> |t|	[0.025	0.975]
const	0.4744	0.057	8.388	0.000	0.364	0.585
degree_normalized	0.7503	0.102	7.367	0.000	0.551	0.950
closeness_normalized	-0.0880	0.108	-0.812	0.417	-0.301	0.125
betweenness_normalized	-0.2222	0.070	-3.161	0.002	-0.360	-0.084
eigenvector_normalized	0.0527	0.068	0.777	0.437	-0.080	0.186
clustering_normalized	-0.0033	0.037	-0.090	0.928	-0.075	0.068
triangles_normalized	-0.1556	0.071	-2.190	0.029	-0.295	-0.016

Regression results for a system with N = 50, k = 5, r = 0.9, γ = .08, and ω = .08, with explanatory variables forced on the interval [0, 1].

Analysis of Results – Global Environment Effects:

Metric	Definition
N	The number of nodes within the graph.
k	The average degree of nodes within the graph.
Attribute Assortativity Coefficient	Measures the similarity of connections in the graph with respect to the node attribute 'state'. For a network where nodes have attributes, this coefficient quantifies how likely nodes with similar attributes are to be connected.
Average Clustering	Represents the overall level of clustering in the network. It is the average of the local clustering coefficients of all the nodes, reflecting the degree to which nodes in a graph tend to cluster together.
Average Shortest Path Length	The average number of steps along the shortest paths for all possible pairs of network nodes. It gives a measure of the efficiency of information or connectivity spread on the network.
Transitivity	A global measure of clustering that quantifies the fraction of all possible triangles in the network. This metric provides an indication of the probability that two neighbors of a node are neighbors themselves.

Graph-level structural metrics used as independent variables to predict $A_{tMax}(G)$, the proportion of adoption for graph G after several simulations.

OLS Regression Analysis:

	coef	std err	t	P> |t|	[0.025	0.975]
const	0.8780	0.004	250.592	0.000	0.871	0.885
gamma	-0.1433	0.002	-65.026	0.000	-0.148	-0.139
omega	-0.2636	0.002	-119.644	0.000	-0.268	-0.259
r	-0.2124	0.004	-53.562	0.000	-0.220	-0.205

	coef	std err	t	P> |t|	[0.025	0.975]
const	0.8815	0.002	357.530	0.000	0.877	0.886
gamma	-0.1433	0.001	-113.122	0.000	-0.146	-0.141
omega	-0.2636	0.001	-208.083	0.000	-0.266	-0.261
r	-0.2125	0.002	-93.182	0.000	-0.217	-0.208
N	-0.0014	8.2e-06	-172.925	0.000	-0.001	-0.001
k	0.0087	4.94e-05	175.630	0.000	0.009	0.009

	coef	std err	z	P> |z|	[0.025	0.975]
const	1.0636	0.010	107.109	0.000	1.044	1.083
gamma	-0.1427	0.001	-109.540	0.000	-0.145	-0.140
omega	-0.2637	0.001	-211.886	0.000	-0.266	-0.261
r	-0.2122	0.002	-96.267	0.000	-0.216	-0.208
N	-0.0006	1.9e-05	-30.273	0.000	-0.001	-0.001
k	0.0028	0.000	19.167	0.000	0.002	0.003
atr_assort_coef	0.0461	0.013	3.423	0.001	0.020	0.073
transitivity	0.0871	0.179	0.486	0.627	-0.264	0.438
avg_clustering	0.1744	0.179	0.975	0.330	-0.176	0.525
avg_shortest_path_length	-0.1132	0.004	-26.818	0.000	-0.121	-0.105

Regression results with $A_{tMax}(G)$ as the response, for 30,000 simulation runs with $r \in [0.5,1]$, $\gamma \in [0.1,1]$, and $\omega \in [0.1,1]$. From top to bottom, $R^2 = .417$, $R^2 = .807$, $R^2 = .822$

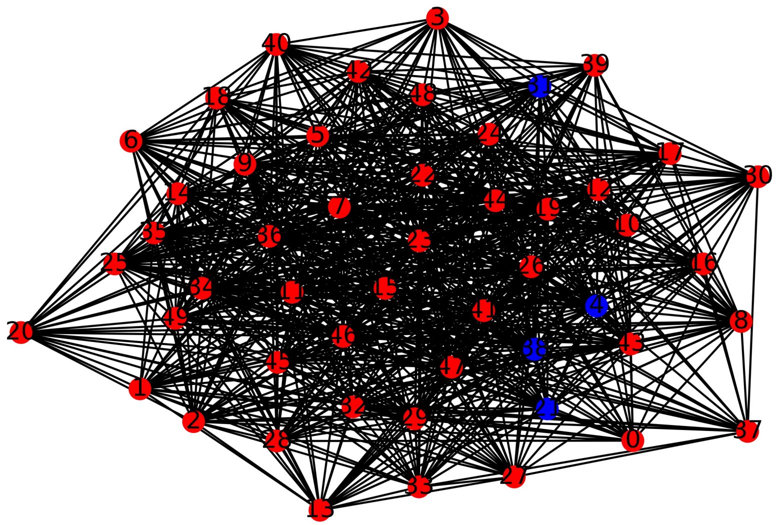
Conclusions:

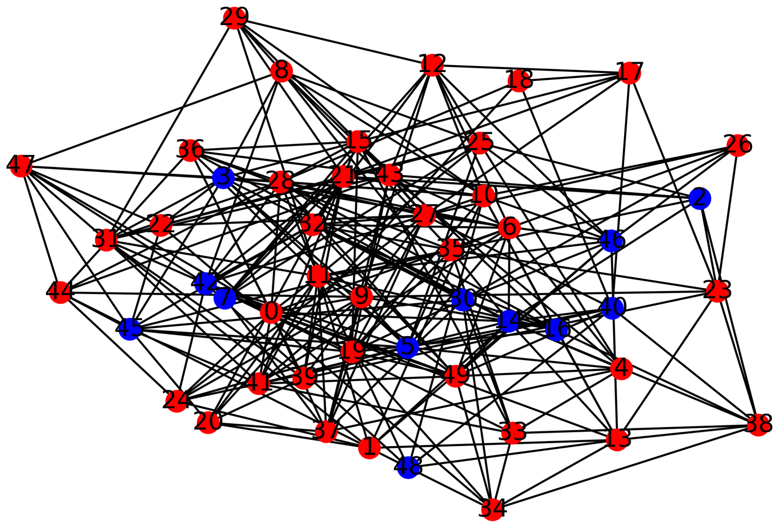
Under T-LISA dynamics:

- N and k are the biggest predictors of $A_{tMax}(G)$
  - Highly interconnected, small graphs, result in greater adoption.

- There are small 'synergistic' effects of adopters who are clustered together.

- $A_{tMax}(i)$is not well predicted by local structural variables, however they may have small effects.





Other Contributions, and Further Questions:

- Quantification of Error in Mean-Field vs. Monte-Carlo simulation.
- Interdisciplinary techniques and information sharing.
- Weighted and directed Graphs?
- Structural importance in non-ER graphs?
- Comparison of T-LISA simulation to actual time-series data?

References

[1] Everett M. Rogers. Diffusion of Innovations, 5th Ed. New York: The Free Press, 2003.

[2] Andrew Mellor, Mauro Mobilia, S. Redner, Alastair M. Rucklidge, and Jonathan A. Ward. Influence of luddism on innovation diffusion. Phys. Rev. E, 92:012806, Jul 2015.

[3] D. Easley and J. Kleinberg. Networks, Crowds, and Markets: Reasoning about a Highly Connected World. Cambridge University Press, 2010.

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Questions?