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ORIGINAL ARTICLE

TODIM with XGBOOST and MVO metaheuristic approach for portfolio optimization

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Abstract This paper presents an innovative and comprehensive approach to portfolio optimization that integrates TODIM, a multi criteria decision making method, with the proposed forecasting model for asset selection and return prediction. The optimization process utilizes the multi-verse optimizer, a powerful metaheuristic algorithm. TODIM is initially employed to select the top 10 companies from the Nifty 100 index based on various financial indicators. Daily return predictions for the selected portfolio are enhanced through an ensemble forecasting approach, BN-XGBoost, which combines BiLSTM and N-BEATS models, with XGBoost serving as a meta-learner to improve accuracy. The portfolio optimization problem, framed as a higher-order asymmetric risk model, incorporates semi-variance, higher order moments of return, and entropy. The proposed optimization framework is compared against a variance-based model, demonstrating superior portfolio performance. This research provides valuable insights for investors seeking advanced strategies to navigate complex financial markets and achieve optimal portfolio outcomes.

Keywords Ensemble forecasting · MCDM · Multi-verse optimizer · Portfolio optimization · TODIM

1 Introduction and literature review

In modern finance, where economic landscapes are increasingly intertwined, making educated investment decisions is imperative. Investors and financial analysts face synthesizing vast amounts of data, leveraging advanced forecasting methodologies, and deploying innovative portfolio optimization algorithms.

1.1 MCDM for portfolio selection

This study addresses the growing complexities of the financial world by proposing an integrative strategy for the entire investment process, beginning with MCDM using the TOmada de Decisão Interativa e Multicritério, an acronym in Portuguese for "Interactive and Multicriteria Decision Making" (TODIM) technique which was introduced by Gomes & Lima in 1992. It aims to reduce subjectivity and bias by considering multiple criteria, making investment asset selection more comprehensive and informed (Taherdoost & Madanchian 2023).

MCDM is crucial for analyzing investment possibilities, allowing decision-makers to examine multiple criteria simultaneously while accounting for interdependencies and uncertainties. Several authors (Ehrgott et al. 2004; Srivastava 2021) have utilized MCDM in their studies to build optimal portfolios. Well-established methodologies, such as TODIM, use the dominance principle for decision-making and can be complemented by principles from prospect theory. TODIM was used in portfolio allocation techniques by Alali and Tolga (2019). Inspired by their work, TODIM was

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adapted to handle financial indicators as decision-making criteria for portfolio selection in this study. The TODIM method was chosen for its ability to incorporate both quantitative criteria and behavioral aspects of decision-making (Gomes & González, 2012). This method has been widely applied by various authors in their studies (Ju et al. 2021; Leoneti & Gomes 2021; Wang et al. 2023).

1.2 Ensemble methods for forecasting

In this research paper, predictive modelling employing an ensemble approach that combines Neural Basis Expansion Analysis for Time Series Forecasting (N-BEATS) and Bidirectional Long Short-Term Memory (BiLSTM) models, has been used. This combination allows for more accurate and reliable predictions, leveraging the strengths of each model while mitigating their weaknesses. The study utilizes Extreme Gradient Boosting (XGBoost) as a meta-learner to improve prediction accuracy. This combination of prediction methods has not been previously applied in the existing literature, presenting a novel approach to forecasting financial data.

N-BEATS and BiLSTM are two significant deep-learning algorithms in financial forecasting. N-BEATS excels in time series forecasting by directly modeling trends and seasonality, while BiLSTM captures complex sequential dependencies by considering both past and future context in the data. Studies have shown their effectiveness in financial domains (Cheng et al. 2024; Juairiah et al. 2022; Li et al. 2021; Liu et al. 2023; Singhal et al. 2022). Ensemble approaches, such as those introduced by Zhou et al. 2023, improve prediction precision and robustness by combining models. XGBoost has also been effectively used for various financial modeling and prediction problems (Cawood & Van Zyl 2022; Naik & Albuquerque 2022).

1.3 Portfolio optimization using metaheuristic method

Markowitz's seminal work (1952) laid the foundation for modern portfolio theory, introducing diversification, risk-return trade-off, and the efficient frontier. Researchers such as Konno et al. (1993) and Lai (1991) has emphasized the importance of considering higher-order moments in portfolio optimization. Subsequent researchers further expanded on Markowitz's model, incorporating advanced objectives like skewness, kurtosis, semi-variance, and Gini entropy for a more comprehensive assessment (Estrada 2004; Lai 2012; Maiti 2021).

Recent studies have shown a growing interest in hybrid methods and metaheuristics for portfolio optimization, highlighting their effectiveness in practical applications (Gunjan & Bhattacharyya 2023; Loke et al. 2023; Almufti et al. 2023). Multi-Verse Optimizer (MVO) is a nature-inspired algorithm that outperforms other algorithms in solving real problems with unknown search spaces (Mirjalili et al. 2016). MVO is the best choice for optimizing portfolios due to its simple construction, less controlling parameters, and reduced computation time (Fathy & Rezk 2018).

Constraints on weights, boundary limits, and minimum return objectives are considered to construct well-structured portfolios. The study is notable for using the MVO, which provides a versatile and efficient solution to the proposed portfolio optimization problems.

Our study in this paper, takes a well-rounded approach by combining different methods into a better-unified strategy. It integrates MCDM, predictive modeling, and advanced portfolio optimization techniques to empower investors with tools for making informed decisions and constructing resilient portfolios. The methodology aims to optimize portfolios by focusing on increasing returns while effectively managing risk, making it relevant for individuals and institutions navigating the intricacies of the financial world.

1.4 Selective comparative studies

Table 1 compares key studies relevant to the research's methodologies and objectives, focusing on representative works in portfolio optimization with higher moments.

1.5 Motivation and contribution

This research is motivated by the complexities of modern financial markets and the need for data-driven solutions, using advanced optimization techniques to help investors improve decision-making and portfolio performance. The main contribution of this study can be summarized below:

i. The TODIM method has been used for stock selection based on specific financial indicators. TODIM was chosen for portfolio selection in this study due to its ability to incorporate investors' behavioral aspects, handle trade-offs between conflicting criteria, and provide clear rankings based on dominance (Puppo et al. 2022).
ii. A novel ensemble methodology has been proposed, combining N-BEATS, BiLSTM, and XGBoost to predict the returns of the selected stocks. The proposed ensemble approach has given good results for the studied problem.
iii. The present study goes beyond typical risk and performance measures like mean return and variance, including skewness, kurtosis, semi-variance, and Gini entropy. Although higher moments, such as skewness and kurtosis, have been extensively studied in portfolio optimization since the 1990s, the specific combination of higher-order moments (skewness and kurto-

Table 1 Selective literature comparison

Feature	Jain et al. (2024)	Gupta et al. (2019)	Aksaraylı & Pala (2018)	Rahimi & Kumar (2019)	Vercher & Bermudez (2013)	Chen et al. (2020)	Zhai et al. (2018)	Our Study
MCDM for asset Selection	✗	✗	✗	✗	✗	✗	✗	✓
Forecasting	✓	✗	✗	✗	✗	✓	✗	✓
Higher-order Moments	✓	✓	✓	✓	✓	✓	✓	✓
Entropy	✓	✓	✓	✓	✗	✗	✗	✓
Semi-variance as risk measure	✗	✗	✗	✓	✓	✗	✗	✓
Metaheuristic Algorithm	✓	✗	✗	✓	✓	✓	✓	✓
Application to real-world financial market	✓	✓	✗	✗	✓	✓	✗	✓

sis) along with asymmetric risk (semi-variance) and Gini-Simpson entropy has not been explored in the literature. Mandal & Thakur's (2024) comprehensive review of higher-order moments in portfolio selection problems further supports this statement, highlighting existing research gaps.

Using skewness**,** kurtosis, and semi-variance together in portfolio optimization captures asymmetry, tail risk, and downside risk, offering better risk management for non-normal distributions and minimizing significant losses while maximizing returns.

iv. The metaheuristic technique, MVO, has been used in this study for its simplicity and efficiency (Fathy & Rezk 2018). MVO adopted by (Jain et al. 2024) in their work to solve a portfolio optimization problem with higher moments has shown promising results. A closer exploration of the existing literature suggests that this technique has yet to be fully utilized.

v. The combination of TODIM, the proposed ensemble method of forecasting, and the MVO has been experimented by authors to study a portfolio optimization problem.

The subsequent sections of this paper are meticulously structured. Section 2 explores this study's theoretical foundations, detailing the methods employed, including TODIM, N-BEATS, BiLSTM, XG Boost, and MVO. Section 3 explains the problem formulation, nomenclature, and assumptions that support the research. Section 4 gives an overview of the Research methodology. Section 5 is dedicated to the data, its analysis, and the findings that arise from it. Section 6 summarises the key findings and sheds light on future research potential and the study's limitations, allowing for a comprehensive understanding of the research's scope and implications.

2 Methods description

2.1 TODIM

The TODIM technique was developed in its current version in the early 1990s by Gomes & Lima (1992a, b). It is a discrete multicriteria approach based on Prospect Theory (Daniel Kahneman & Amos Tversky, 1979).

The input values of this method are in the form of a decision matrix such as $X = \left[x_{ij}\right], i = 1, \dots, m, j = 1, \dots, n$ where i's are the number of alternatives and j are the number of criteria. $x_{ij}\prime s$ are numerical values that represent the value of i^{th} alternative for j^{th} criteria.

The values of the weights of attributes are $\left[y_1, y_2, \dots, y_n\right]$, and θ is the parameter used to adjust the degree of dominance.

Following are the steps to find the dominance value of each alternative:

Step 1: Normalize the decision matrix values using the min-max formula as described by Alali & Tolga (2019), and the normalized decision matrix is represented as $X\prime = \left[x_{ij}\prime\right]$:

$$x_{ij}\prime = \frac{x_{ij} - minx_{ij}}{\max x_{ij} - \min x_{ij}},$$

for criteria where higher values are preferable

$$x_{ij}\prime = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}},$$

for criteria where lower values are preferable

Step 2: Find criteria weights using Shannon entropy: $H\left(x_j\right) = -\sum_{j=1}^{n}(p_{ij}log2(p_{ij}))$, where $p_{ij}\prime s$ are probability distributions for each criterion, but in the context of MCDM, they represent each criterion's relative importance or contribution. They are calculated using the value in the normalized decision matrix corresponding to a specific criterion, divided by the sum of all values in the normalized decision matrix for that same criterion, i.e., $p_{ij} = \frac{x_{ij}\prime}{\sum_{j=1}^{n} x_{ij}\prime}$. $H\left(x_j\right)$ represents the information or uncertainty associated with each criterion.

Weights are calculated using $y_j = 1 - H\left(x_j\right)\forall j \forall [1, n]$.

Weights need to be normalized to ensure they sum up to 1. Normalized weights are represented by $\widetilde{y_j}$ and can be calculated using $\widetilde{y_j} = \frac{y_j}{\sum_{j=1}^{n} y_j}$.

Step 3: Calculating the degree of dominance δ of alternatives using:

$$\delta_i = \delta\left(a_i, a_k\right) = \sum_{j=1}^{n} \psi_j\left(a_i, a_k\right); i, k \in \{1, \ldots, m\}$$

where $a's$ are different alternatives and $\psi_j\left(a_i, a_k\right)$ denotes the preference index and is calculated using:

$$\psi_j\left(a_i, a_k\right) = -\frac{1}{\theta}\sqrt{\frac{\left(\sum_{j=1}^{n} \tilde{y}_j\right)\left(x_{ij}\prime - x_{ji}\prime\right)}{\tilde{y}_j}} \quad \text{if}\left(x_{ij}\prime - x_{ji}\prime\right) < 0$$

$$\psi_j\left(a_i, a_k\right) = 0 \quad \text{if}\left(x_{ij}\prime - x_{ji}\prime\right) = 0$$

$$\psi_j\left(a_i, a_k\right) = \sqrt{\frac{\tilde{y}_j\left(x'_{ij} - x'_{ji}\right)}{\sum_{j=1}^{n} \tilde{y}_j}} \quad \text{if}\left(x_{ij}\prime - x_{ji}\prime\right) > 0$$

Step 4: The total degree of dominance of each alternative is calculated using:

$$\zeta_i = \frac{\delta_i - \min\delta_i}{\max\delta_i - \min\delta_i} \ ;i = 1, \ldots, m$$

Finally, the total degree of dominance of all the alternatives is organized in descending order to get the final ordering of the alternatives. These values represent the global dominance or overall ranking of alternatives based on the dominance matrix and the chosen theta value.

2.2 BN-XGBoost (BiLSTM and N-BEATS ensembled with XGBoost)

2.2.1 N-BEATS

N-BEATS is a deep-learning model designed for time series forecasting. It was introduced by Oreshkin et al. (2019). The architecture of this model is based on blocks and stacks, where each block describes a different aspect of the time series, as shown in Fig. 1. There are primarily two types of blocks:

- Trend block—It captures the underlying trend or pattern in the data, which involves a simple feedforward neural network structure. Its input is a sequence of historical data points (lookback period/back horizon). The length of the back horizon is typically nH, where $n = 2to7$ and H is the length of the forecast horizon. The input data passes through four fully connected layers within each block with rectified linear unit (ReLU) activation functions. After passing through these dense layers, the trend block produces two sets of coefficients, $\theta_b, and\theta_f$, which describe the back and forecasted horizon. These coefficients are then used to compute predictions for the back and forecasted horizon by applying them to predefined functions (e.g., Fourier basis functions).
- Seasonality block—It captures the periodic or seasonal patterns in the data and follows a similar structure as the trend block.

Stacks are composed of multiple layers of blocks. Each stack contains one or more trend and seasonality blocks. Combining stacks allows the model to capture different time-series features at multiple levels. N-BEATS is designed to be interpretable, i.e., one can understand how the model arrives at its predictions. Each block can be considered a simple forecasting model, and their outputs are combined to make predictions.

2.2.2 BiLSTM

BiLSTM (Graves & Schmidhuber 2005; Schuster & Paliwal 1997) is a type of recurrent neural network (RNN) type. It extends the traditional Long Short-Term Memory (LSTM) model by simultaneously processing data in both forward and backward directions; its basic architecture is shown in Fig. 2. This bidirectional processing allows the model to capture dependencies in the input data from past and future contexts, making it particularly effective when understanding the context is essential, e.g., time series analysis.

2.2.3 Ensemble methods

Ensemble techniques enhance machine learning predictions by combining multiple models to leverage strengths and offset weaknesses (Mohammed & Kora 2023). Stacking uses a meta-learner trained on base model predictions to improve accuracy, while averaging integrates forecasts to reduce biases and increase overall performance (Fig. 3).

2.2.4 XGBoost

XG Boost is a method of ensemble learning that combines the predictions of numerous weak models to produce a

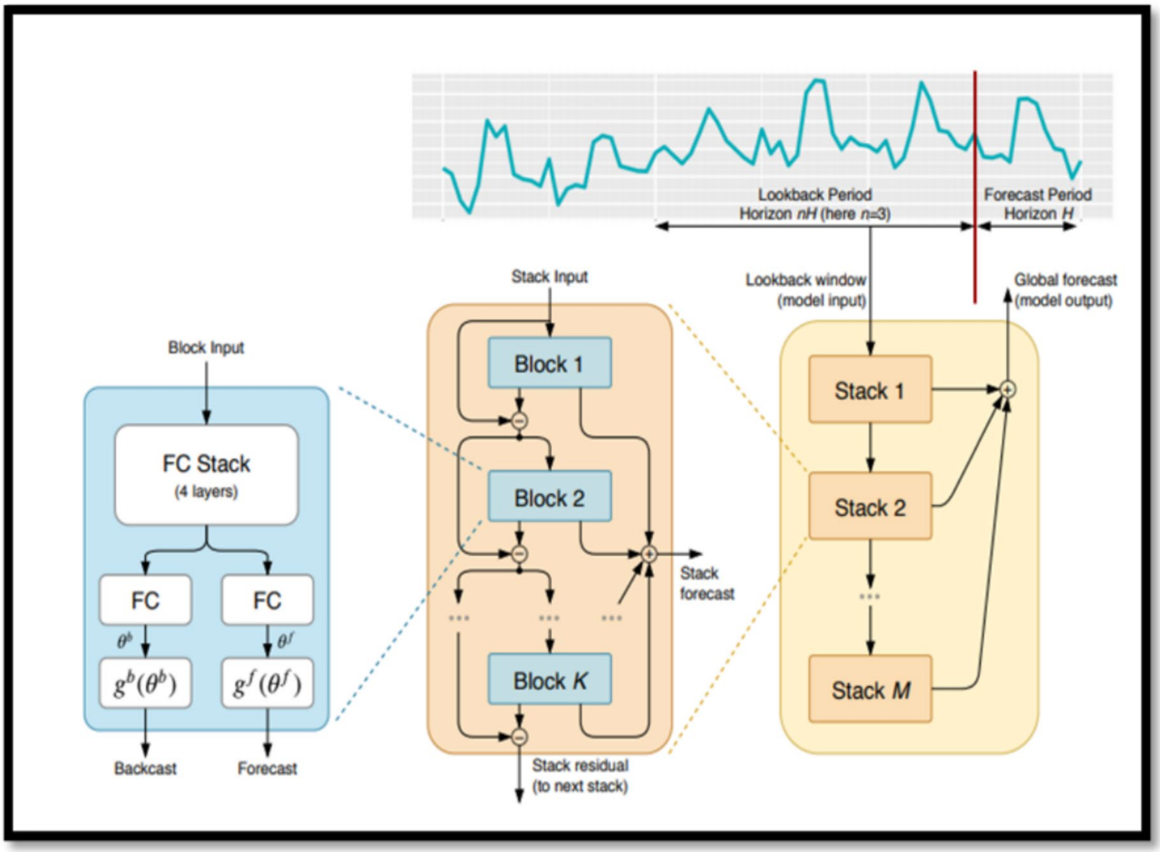
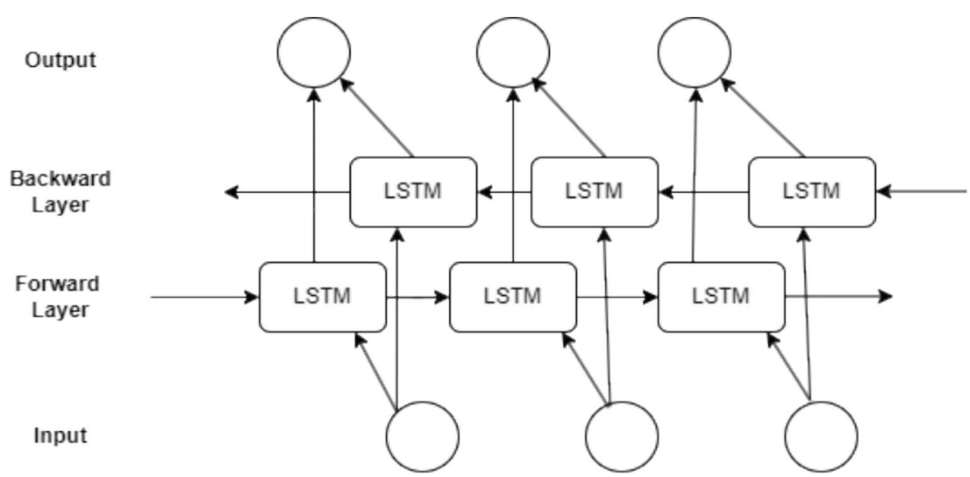


Fig. 1 N-BEATS architecture (*Source*: (Oreshkin et al. 2019))

Fig. 2 BiLSTM architecture



robust predictive model. It utilizes a boosting approach that iteratively corrects prior models' faults, enhancing forecast accuracy.

In the proposed BN-XGBoost model for forecasting, predictions are first obtained separately using the N-BEATS and BiLSTM methods. These predictions are then averaged, and the resulting values are fed into the XGBoost meta-learner to capture patterns missed by N-BEATS and BiLSTM, thereby improving prediction accuracy through stacking.

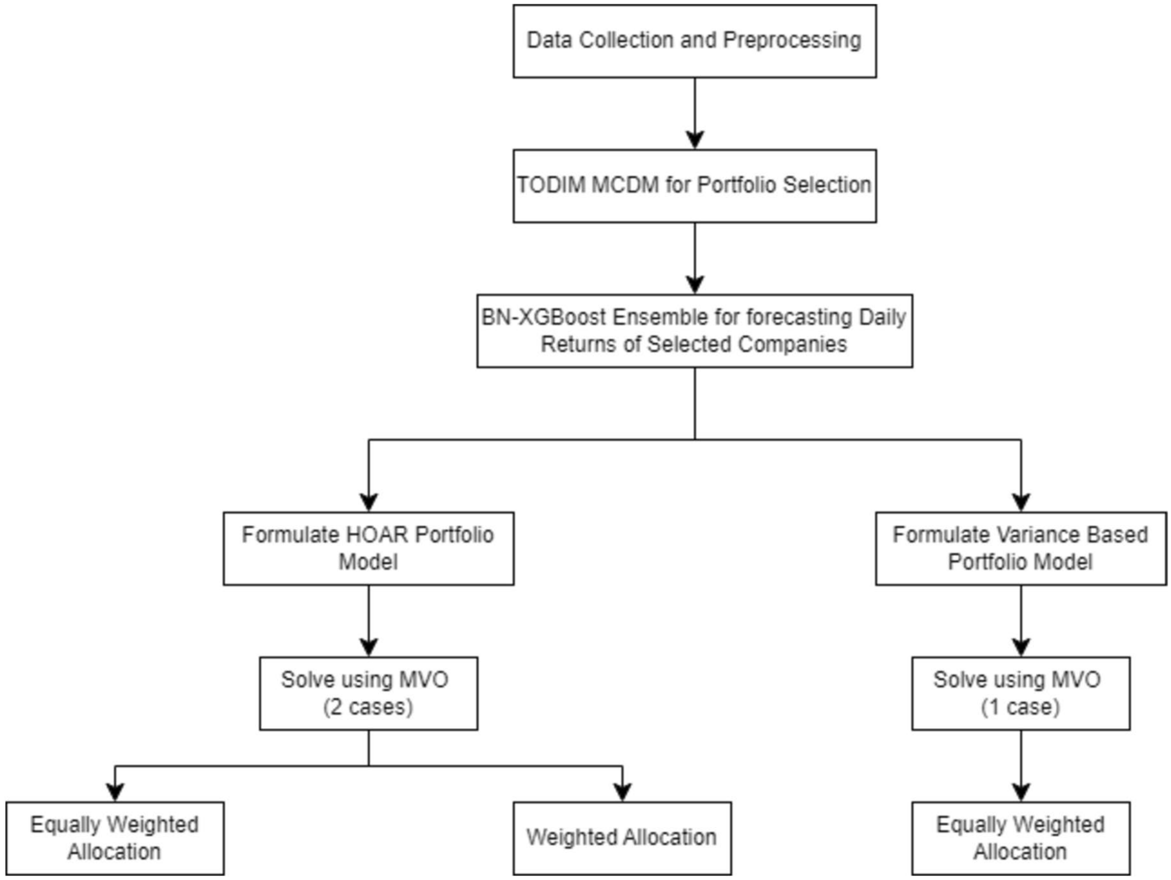


Fig. 3 Flowchart of methodology

2.3 Multi-verse optimizer (MVO)

MVO is a nature-inspired optimization algorithm based on multiverse theory, introduced by Mirjalili et al. (2016). It models universes, objects, and the interactions between them.

A brief explanation of MVO:

1. Universe Representation

- Each solution in MVO represents a universe.
- Variables within the universe are treated as objects.
- An inflation rate is associated with each universe and is proportional to the fitness value.

2. Selection of universe:

- MVO employs a roulette wheel selection process based on the fitness value.
- Mathematical model for universe selection:

$$y_i^j = \begin{cases} y_k^j, R_1 < NI(U_i) \\ y_i^j, R_1 \geq NI(U_i) \end{cases}$$

where y_k^j is the j^{th} parameter of k^{th} universe, and U_i is the i^{th} universe,

$R_1 \in [0,1]$ is a random value, NI is the normalized rate of inflation.

3. Wormhole Mechanism:

- Wormholes exist in each universe, enabling the random interchange of objects without considering their inflation rates.
- Coefficients in MVO:

– WEP (Wormhole Existence Probability) determines the likelihood of wormholes in other universes based on an iterative formula.
– TDR (Traveling Distance Rate) influences the distance at which a wormhole can transfer objects.

– $WEP = min + e \times \left(\frac{\max - \min}{E}\right)$-, where e is the current iteration, and E is the maximum iteration.
– $TDR = 1 - \frac{e^{\frac{1}{p}}}{E^{\frac{1}{p}}}$-, where p is the exploitation accuracy.

- Mathematical model for the wormhole mechanism:

$$y_i^j = \begin{cases} \begin{cases} Y_j + TDR \times \left(\left(UB_j - LB_j \right) \times R_4 + LB_j \right), R_3 < 0.5 \\ Y_j + TDR \times \left(\left(UB_j - LB_j \right) \times R_4 + LB_j \right), R_3 < 0.5 \end{cases} , R_2 < WEP \\ y_i^j, R_2 \geq WEP \end{cases}$$

Where Y_j is the j^{th} parameter of the fitted universe,

UB_j and LB_j are the upper and lower bounds of the j^{th} parameter,

$\{R_2, R_3, R_4\} \in [0,1]$ are random values

These principles and mathematical models constitute the core components of the MVO algorithm, which applies the concept of multiverse theory to solve numerical optimization problems (Mirjalili et al. 2017).

3 Proposed higher-order asymmetric risk (HOAR) portfolio optimization model

This section discusses the optimization model used to compute the optimal proportion of investments to be made in all assets in the portfolio, as well as the preliminaries, assumptions and notations used in the problem.

3.1 Preliminaries

3.1.1 Higher moments (Skewness and kurtosis)

Skewness and kurtosis assess return distribution shapes, providing insights into asymmetry and fat-tail risks. They help account for non-normal, asymmetric returns and extreme events in portfolio optimization, leading to more robust, diversified portfolios.

Let R_p represent the portfolio return as a random variable. The return vector $R = \left(R_1, R_2, \ldots, R_N\right)$ denotes the return of N assets, where R_i represents the return of the i^{th} asset. Further, let $X = \left(x_1, x_2, \ldots, x_N\right)$ be the weight vector where $x_i's$ represents the proportion of investment in the i^{th} asset.

Then, the first four moments (Kemalbay, Murat ÖZKUT and Franko, 2011;Aksaraylı and Pala 2018) of portfolio return, R_p can be calculated as follows:

$$\text{Mean} = E\left(R_p\right) = E\left[X^T R\right] = \sum_{i=1}^{n} x_i \mu_i = X^T \mu = X^T M_1$$

$$\begin{aligned} \text{Variance} = V\left(R_p\right) &= E\left[X^T R - E\left[X^T R\right]\right]^2 \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} = X^T V X = X^T M_2 X \end{aligned}$$

$$\begin{aligned} \text{Skewness} S\left(R_p\right) &= E\left[X^T R - E\left[X^T R\right]\right]^3 \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} x_i x_j x_k s_{ijk} \\ &= X^T S (X \otimes X) = X^T M_3 \end{aligned}$$

$$\begin{aligned} \text{Kurtosis } K\left(R_p\right) &= E\left[X^T R - E\left[X^T R\right]\right]^4 \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} x_i x_j x_k x_l k_{ijkl} \\ &= X^T K (X \otimes X \otimes X) = X^T M_4 \end{aligned}$$

Here, $\mu = E[R] = (\mu_1, \mu_2, \cdots, \mu_n)$ are the mean return of each asset vector,

$V = E[R - E[R]]^2$ is $n \times n$ variance–covariance matrix consisting of values like $\sigma_{ij}'s \forall (i,j) \forall [1, \cdots, n]$ and $\sigma_{ij} = E\left[\left(R_i - E\left[R_i\right]\right)\left(R_j - E\left[R_j\right]\right)\right]$,

$S = E[R - E[R]]^3$ is $n \times n^2$ skewness coskewness matrix consisting of values like $s_{ijk}' s \forall (i,j,k) \forall [1, \cdots, n]$ and $s_{ijk} = E\left[\left(R_i - E\left[R_i\right]\right)\left(R_j - E\left[R_j\right]\right)\left(R_k - E\left[R_k\right]\right)\right]$,

$K = E[R - E[R]]^4$ is $n \times n^3$ kurtosis cokurtosis matrix consisting of values like $k_{ijkl}' s \forall (i,j,k,l) \forall [1, \cdots, n]$ and $k_{ijkl} = E\left[\left(R_i - E\left[R_i\right]\right)\left(R_j - E\left[R_j\right]\right)\left(R_k - E\left[R_k\right]\right)\left(R_l - E\left[R_l\right]\right)\right]$. $M_1, M_2, M_3, and M_4$ denote these moments. $\otimes$ denotes Kronecker product.

These metrics provide insights into the shape of the distribution of returns. High kurtosis and skewness suggest non-normality in returns, which can be important in understanding the risk-return trade-off.

3.1.2 Gini-simpson entropy

The following expression describes Gini-Simpson entropy as proposed by Aksaraylı and Pala (2018):

$$\text{Gini - Simpson Entropy} : 1 - \sum_{i=1}^{N} x_i^2 = 1 - X^T X$$

3.1.3 Semi-variance

Semi-variance measures the risk associated with negative deviations from the mean. The formula for semi-variance (Macedo et al. 2017) is:

$$\text{Semi\_Var}(x) = E\left([\min\left(R_{it} - B_t, 0\right)]^2\right) = \frac{1}{T}\sum_{i=1}^{T}[\min\left(R_{it} - B_t, 0\right)]^2$$

where R_{it} is the return of asset i in period t, B_t is the benchmark return chosen, and T is the number of periods.

Semi-variance focuses on downside risk by considering only the negative returns. It quantifies the variability of returns for values less than the mean. It represents the risk of experiencing losses below a certain threshold. The endogenous structure of the portfolio semi-covariance matrix, dependent on the weights assigned to each asset, makes semi-variance calculation challenging since it influences the times when the portfolio underperforms the benchmark (Macedo et al. 2017). Estrada (2007) presented a streamlined solution to overcome endogeneity and offered a heuristic method for obtaining an exogenous, symmetric semi-covariance matrix that would approximate portfolio semi-variance. This method lowers computing costs and streamlines computations.

3.2 Variables used in the problem

$w_i's$: weights assigned to different goals in the objective function, $i = 1,2,\dots,m$ (m is the number of goals).

$x_i's$: proportion of investment in each i^{th} asset , $i = 1,2,\dots,N$ (N is the number of assets).

$\mu_i's$: average return of an i^{th} asset, $i = 1,2,\dots,N$

$k_{ijkl} and s_{ijk}$: cokurtosis and coskewness matrix , $i,j,k,l = 1,2,\dots,N$.

MinRet : minimum value of return aspired by the investor.

LB&UB : lower and upper bounds on the investment proportion of assets.

3.3 Assumptions considered in the proposed problem

i. An investor distributes his wealth among N assets with different average rates of return.
ii. The investor is risk-averse and is interested in maximizing the utility of the investment.
iii. The investor wants to invest in a diversified and efficient portfolio that minimizes his risk at a given level of return.
iv. Investors are rational and make all their decisions based on risk and return, where the return of every asset is assumed to follow a probability distribution.
v. Investors are ready to bear more risk for more return since risk increases with increased return.
vi. All the available funds ought to be invested.
vii. Each asset's capital investment is assumed to be constrained between a lower and an upper bound.
viii. The probability distribution of returns is not normally distributed.
ix. Investors are particularly concerned about the downside risk or volatility of negative returns. Hence, semivariance is considered as one of the risk measures.

3.4 Constraints of the model

3.4.1 Return constraint: no less than a specified return is desired, i.e.,

$$\sum_{i=1}^{N} x_i\mu_i \geq \text{Min Ret} \tag{1}$$

3.4.2 Capital budget constraint: capital should be invested, i.e.,

$$\sum_{i=1}^{N} x_i = 1 \tag{2}$$

3.4.3 Bound constraint: a limit on the proportion of capital invested in each asset, i.e.,

$$\text{LB} \leq \text{x}_\text{i} \leq \text{UB} \tag{3}$$

LB represents the lowest fraction of investment, and UB represents each asset's maximum fraction of investment.

3.5 Problem formulation

The Mult objective nonlinear Higher-Order Asymmetric Risk (HOAR) portfolio optimization problem is formulated as follows:

$$\text{Max} \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} x_i x_j x_k s_{ijk}$$

$$\text{Min} \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} x_i x_j x_k x_l k_{ijkl}$$

$$\text{Max}1 - \sum_{i=1}^{N} x_i^2$$

$$\text{Min} x_i x_j \text{Semi\_Var}\left(R_i, R_j\right)$$

subject to Constraints (1)–(3).

The above problem is solved as a single objective problem using:

$$\mathrm{Min} w_1 \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} x_i x_j x_k x_l k_{ijkl} - w_2 \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} x_i x_j x_k s_{ijk} + w_3 \left(1 - \sum_{i=1}^{N} x_i^2\right) + w_4\left(x_i x_j \mathrm{Semi\_Var}\left(R_i, R_j\right)\right)$$

Subject to Constraints (1)–(3).

3.6 Comparative portfolio model with variance replacement

The proposed HOAR portfolio optimization model is compared to a variant model where semi-variance is replaced with variance while keeping the other objectives and constraints unchanged. The variance-based portfolio model is outlined below:

$$\mathrm{Min} w_1 \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} x_i x_j x_k x_l k_{ijkl} - w_2 \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} x_i x_j x_k s_{ijk} + w_3 \left(1 - \sum_{i=1}^{N} x_i^2\right) + w_4 \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}$$

Subject to Constraints (1)–(3).

Finding an optimal solution to the considered problems is challenging. To solve this, we used a metaheuristic method, MVO.

4 4. Research methodology

The methodology employed in this paper can be summarized in the following flowchart:

5 Data analysis and results

5.1 Data collection and preprocessing

The data for Nifty 100 index firms was collected for this study, covering the five-year period from January 1, 2018, to July 25, 2023. The data was sourced from the Prowess IQ database. We carefully gathered information on 12 financial indicators—Adjusted Closing Price, Adjusted High Price, Adjusted Low Price, Adjusted Opening Price, Market Capitalization, Total Returns, Earnings Per Share (EPS), Price-to-Earnings Ratio (P/E), Price-to-Book Ratio (P/B), Book Value Per Share (BVPS), Turnover, and Cash Earnings Per Share (Cash EPS).

5.2 TODIM MCDM criteria for ranking companies

Before applying TODIM, the raw data was pre-processed to handle missing values and ensure consistency. The averaged five-year dataset was then used for further analysis. We applied the TODIM to rank Nifty 100 companies based on financial criteria.

5.2.1 Normalization of the decision matrix

We constructed a decision matrix $\mathbf{X} = \left[\mathbf{x_{ij}}\right]$, where $\mathbf{i} = 1, \ldots, \mathbf{m}$ represents the alternatives (companies), and $\mathbf{j} = 1, \ldots, \mathbf{n}$ represents the criteria. In our case, $\mathbf{i} = 95\mathbf{andj} = 1$. Although Nifty 100 includes 100 equities, we selected only those for which there was complete data during the period covered in this study. We applied the min–max normalization method to bring the criteria to a comparable scale, explained in Step 1 in Sect. 2.1. The normalized values reflect the relative performance of each company on each criterion.

5.2.2 Entropy calculation for weights

Next, we calculated the weight of each criterion using Shannon Entropy to reflect the uncertainty or significance of the criteria. First, the probability distribution p_{ij} for each

Table 2 Weights of criteria

Normalized Weights	$\tilde{y}_j$
$\widetilde{y_1}$	0.088
$\widetilde{y_2}$	0.088
$\widetilde{y_3}$	0.088
$\widetilde{y_4}$	0.088
$\widetilde{y_5}$	0.073
$\widetilde{y_6}$	0.087
$\widetilde{y_7}$	0.083
$\widetilde{y_8}$	0.088
$\widetilde{y_9}$	0.088
$\widetilde{y_{10}}$	0.071
$\widetilde{y_{11}}$	0.088
$\widetilde{y_{12}}$	0.070

criterion was calculated. Then, the entropy $H(x_j)$ for each criterion. The weight of each criterion was determined using $y_j = 1 - H(x_j)$. Finally, the weights were normalized $\tilde{y}_j$. The resulting weights are displayed in Table 2, reflecting the relative importance of each criterion.

5.2.3 Calculation of the dominance degree

The dominance degree $\boldsymbol{\delta_i}$ of each alternative was calculated based on pairwise comparisons between alternatives. The preference index $\boldsymbol{\psi_j}(\mathbf{a_i}, \mathbf{a_k})$ for each criterion was computed using conditions explained in Step 3 of Sect. 2.1.

The overall dominance degree δ_i for each alternative was then computed by summing the preference indices across all criteria: $\delta_i = \sum_{j=1}^{n} \psi_j(a_i, a_k)$

5.2.4 Normalization of dominance degree

The alternatives were then ranked based on their normalized dominance degree ζ_i, as shown in Table 3 for various values of θ. We have presented the top 10 ranked companies based on our analysis.

5.2.5 Sensitivity analysis of $\boldsymbol{\theta}$

We performed a sensitivity analysis by testing different values of attenuation factor θ (0.1, 0.5, 1, 2, 5), which controls the degree of risk aversion in the decision-making process. A lower θ value in the TODIM method signifies reduced tolerance for variations between alternatives, enhancing sensitivity to minor deviations and aligning with a risk-averse strategy. In contrast, a higher θ accommodates more significant performance differences, favoring a more risk-tolerant approach. Hence, theta values of 0.1 and 0.5 are preferable,

Table 3 Result at different θ values

Company name	Rankings ($\theta = 0.1$)	Company name	Rankings ($\theta = 0.5$)	Company name	Rankings (θ = 1)	Company name	Rankings (θ = 2)	Company name	Rankings (θ = 5)
Shree Cement Limited	1.000	Shree Cement Limited	1.000	Shree Cement Limited	1.000	Shree Cement Limited	1.000	Shree Cement Limited	1.000
Bosch Limited	0.921	Bosch Limited	0.916	Bosch Limited	0.913	Bosch Limited	0.909	Bosch Limited	0.901
Eicher Motors Limited	0.660	Eicher Motors Limited	0.640	Eicher Motors Limited	0.624	Page Industries Limited	0.634	Page Industries Limited	0.679
Reliance Industries Limited	0.636	Reliance Industries Limited	0.618	Page Industries Limited	0.612	Eicher Motors Limited	0.604	Maruti Suzuki India Limited	0.589
Maruti Suzuki India Limited	0.620	Maruti Suzuki India Limited	0.612	Maruti Suzuki India Limited	0.606	Maruti Suzuki India Limited	0.599	Eicher Motors Limited	0.569
Page Industries Limited	0.572	Page Industries Limited	0.595	Reliance Industries Limited	0.604	Reliance Industries Limited	0.586	Reliance Industries Limited	0.556
Ultratech Cement Limited	0.561	Ultratech Cement Limited	0.545	Ultratech Cement Limited	0.535	Ultratech Cement Limited	0.520	Ultratech Cement Limited	0.496
Tata Consultancy Services Limited	0.531	Tata Consultancy Services Limited	0.515	Tata Consultancy Services Limited	0.504	Tata Consultancy Services Limited	0.489	Nestle India Limited	0.475
Tata Steel Limited	0.525	Tata Steel Limited	0.498	Tata Steel Limited	0.478	Tata Steel Limited	0.452	Tata Consultancy Services Limited	0.465
Bajaj Holdings & Invst. Limited	0.477	Bajaj Holdings & Invst. Limited	0.459	Bajaj Holdings & Invst. Limited	0.446	Nestle India Limited	0.446	Tata Steel Limited	0.407

and even results at both values are the same. By experimenting with different values of θ, the model allows us to explore how changing risk preferences affects the ranking of companies.

5.3 Selection of portfolio companies

We then selected the top 10 ranked companies for the portfolio, which are as follows: Shree Cement Limited, Bosch Limited, Eicher Motors Limited, Reliance Industries Limited, Maruti Suzuki India Limited, Page Industries Limited, Ultratech Cement Limited, Tata Consultancy Services Limited, Tata Steel Limited, Bajaj Holdings & Investment Limited.

We downloaded their five-year (November 2018-November 2023) adjusted closing price data from Yahoo Finance to calculate daily return prices by using the formula:

$$\text{Daily Return Price} = \frac{\text{New adjusted closing price} - \text{Old adjusted closing price}}{\text{Old adjusted closing price}}$$

5.4 Forecasting using BN-XGBoost

Next, we used BN-XGBoost technique for predicting daily returns. Using selected companies' five-year daily return data, we trained Interpretable N-BEATS and BiLSTM models separately, using the model setup detailed in Tables 4 and 5 below. We obtain their forecasts for future time points, i.e., 30 days. The arithmetic mean of the predictions is calculated at each time step, resulting in a new forecast series that reduces individual model errors and provides more stable, predictions. The averaged predictions are input to the XGBoost model to capture patterns missed by N-BEATS and BiLSTM. This stacking improves prediction accuracy. Figure 4 shows the architecture of the proposed ensemble technique.

5.4.1 *Validity of proposed ensemble method*

Table 6 presents the results of four evaluation metrics used to assess the performance of the proposed ensemble method: Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and R-squared ($\mathbf{R}^2$).

The results indicate that the proposed ensemble method performs well across all companies, demonstrating high predictive accuracy and a firm fit. Most companies achieve low MAE, MSE, and RMSE values, along with high R^2 scores,

Table 4 Configuration for interpretable N-BEATS model

Parameter	Description	Values/Settings
Training iterations	Number of iterations for training	20
Forecast horizon	Number of time steps forecasted	1 (single-time step)
Batch size	Batch size used during training for efficiency	64
Seasonality stack	Number of blocks and fully connected layers in the seasonality stack	S-blocks = 3, S-block-layers = 4
Trend stack	Number of blocks and fully connected layers in the trend stack	T-blocks = 3, T-block-layers = 4
T-degree	Trend stack polynomial degree (hyperparameter tuning)	Range: 1 to 3
S-width and T-width	Widths of fully connected layers in seasonality and trend stacks (hyperparameter tuning)	Range: 4 to 16
Lookback period	Duration of the history window (hyperparameter tuning)	Range: 1 to 7

Table 5 Key components and configuration of the BiLSTM model

Parameter	Description	Values/Settings
Bidirectional	Enables capturing information from both past and future at each time step	Yes
Window size	Number of previous time steps used as input features	60
BiLSTM layers	Number of BiLSTM layers and their respective units	Two layers (128 units, 64 units)
Dense layer	Number of units in the dense layer with ReLU activation	25 units
Output layer	A single unit providing a predicted stock return	1 unit
Loss function	The loss function used for model training	Mean Squared Error (MSE)
Optimizer	Optimizer used for model training	Adam

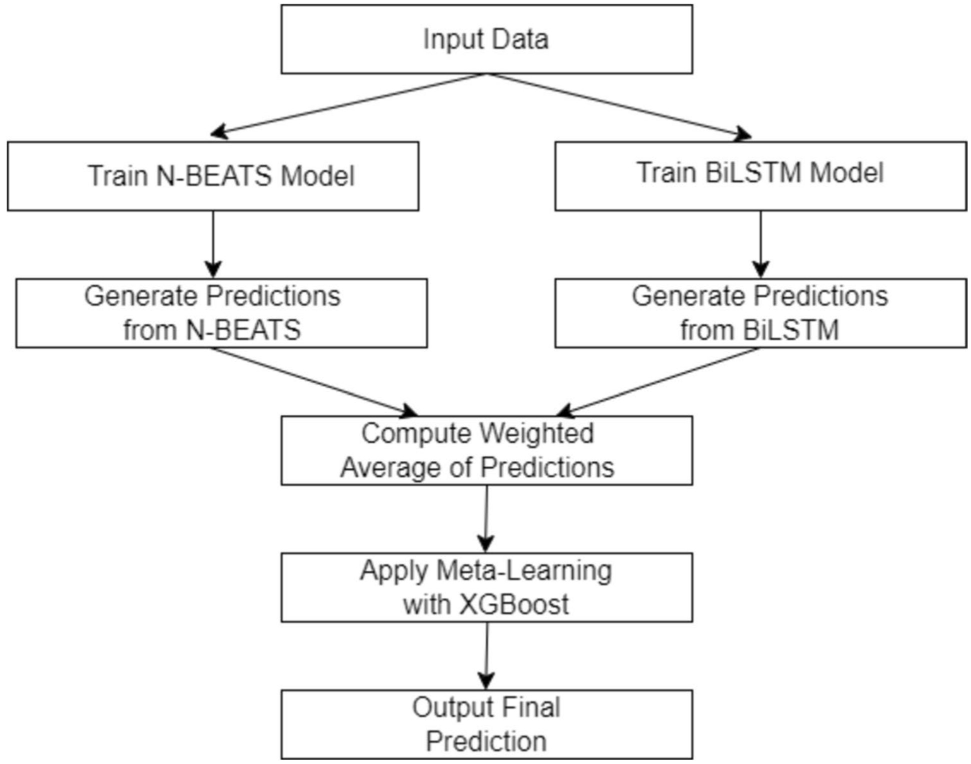


Fig. 4 Ensemble architecture

Table 6 Evaluation metrics of the ensemble method

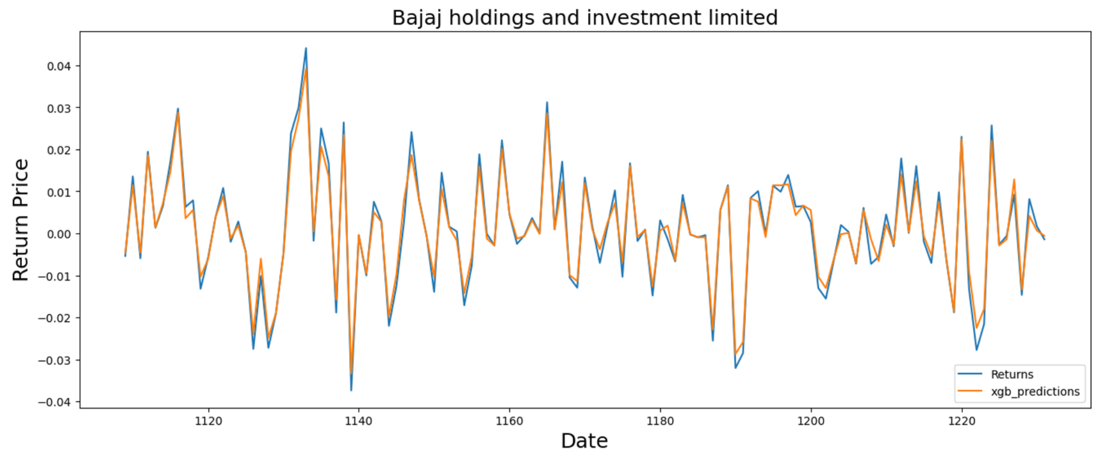
Company Name	MAE	MSE	RMSE	R^2
Bajaj holdings and investment Limited	0.00192	0.000006	0.00243	0.971
Shree cement limited	0.00184	0.000006	0.00237	0.971
Bosch limited	0.00157	0.000004	0.00204	0.964
Eicher motors limited	0.00188	0.000006	0.00238	0.969
Reliance industries limited	0.00182	0.000005	0.00233	0.965
Maruti suzuki India limited	0.00167	0.000004	0.00203	0.965
Page Industries limited	0.00182	0.000005	0.00218	0.974
Ultratech cement limited	0.00177	0.000005	0.00229	0.951
Tata consultancy services limited	0.00193	0.000006	0.00247	0.946
Tata steel limited	0.00165	0.000005	0.00214	0.970

reflecting that the models effectively capture the variance in the data. The ensemble method consistently provides reliable and accurate predictions, with only minor variations in company performance metrics.

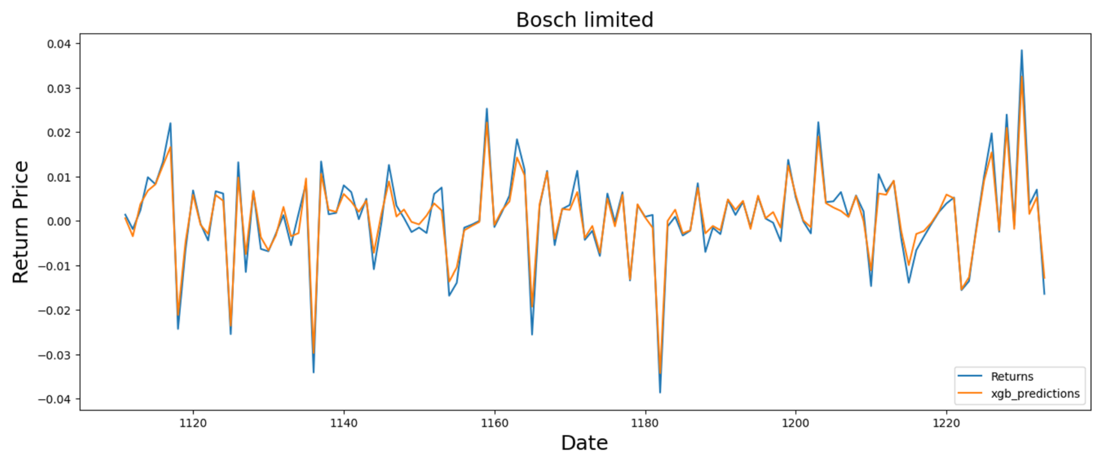
Figure 5 illustrates the model's performance for selected ten companies, where the close alignment of actual and predicted return prices demonstrates the proposed BN-XGBoost model's effectiveness. To maintain brevity, only three representative graphs of actual versus predicted values are presented; additional graphs for other companies exhibit similar results. The horizontal axis shows the timeline, while the vertical axis shows the return price values. Each subplot compares actual return prices (blue line) with predicted return prices (orange line). The alignment of the lines implies high accuracy. This analysis demonstrates the ensemble model's effectiveness in forecasting return prices.

5.5 Descriptive analysis of predicted returns

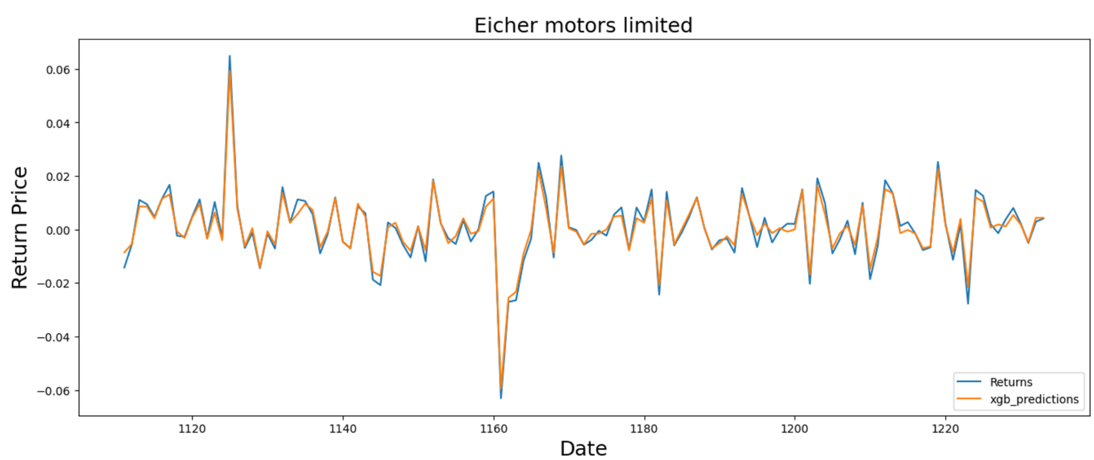
Table 7 presents the descriptive statistics for the predicted daily return values of the selected ten stocks. The table includes each stock's mean, variance, skewness, kurtosis, Shapiro–Wilk statistic, and p-value. The skewness and kurtosis provide insights into the distribution shape, while the Shapiro–Wilk test assesses normality.



(a)



(b)



(c)

Fig. 5 Graph of actual and predicted return using BN-XGBoost. (**a**) Bajaj Holdings and Investment Limited. (**b**) Bosch Limited. Eicher Motors Limited. (**c**) Eicher Motors Limited

Notably, Bajaj Holdings and Investment Limited, Shree Cement Limited, Reliance Industries Limited, Maruti Suzuki India Limited, Ultratech Cement Limited, Tata Consultancy Services Limited, and Tata Steel Limited exhibit return distributions that do not significantly deviate from normality, as indicated by their high Shapiro–Wilk (p-values > 0.05). Conversely, Bosch Limited, Eicher Motors Limited, and Page Industries Limited show significant deviations from normality (p-values < 0.05), reflected in their high skewness and kurtosis values. Specifically, Eicher Motors and Page Industries display extreme kurtosis and skewness, suggesting heavy tails and potential outliers in their return distributions.

Overall, the Shapiro–Wilk test confirms our assumption that the return values of all stocks do not follow a normal distribution, highlighting the importance of using skewness, kurtosis, and semi-variance in portfolio optimization problems.

Table 7 Descriptive statistics and Shapiro-Wilks test results for predicted return of selected companies

Company	Mean	Variance	Skewness	Kurtosis	Shapiro–wilk statistic	Shapiro–wilk p-value	Normality check
Bajaj holdings and investment limited	0.00407	0.00022	0.165	−0.296	0.983	0.90308	True
Shree cement limited	0.00454	0.00023	0.173	−0.244	0.984	0.91888	True
Bosch limited	−0.00018	0.00010	−1.249	2.224	0.904	0.01064	False
Eicher motors limited	0.00413	0.00015	3.058	13.513	0.706	0.0000019	False
Reliance industries limited	0.00119	0.00006	0.589	2.574	0.943	0.11191	True
Maruti suzuki India limited	0.00250	0.00005	−0.219	0.077	0.978	0.77719	True
Page industries limited	0.00045	0.00033	−3.140	14.607	0.690	0.0000011	False
Ultratech cement limited	0.00159	0.00006	−0.415	−0.460	0.964	0.39354	True
Tata consultancy services limited	0.00186	0.00005	0.655	1.211	0.957	0.25669	True
Tata steel limited	0.00043	0.00009	0.203	−0.248	0.986	0.95900	True

Table 8 Parameter setting for MVO

Parameter	Specification
Lower bound for portfolio weights	0.01
Upper bound for portfolio weights	0.99
Dimension	10 (number of assets)
Count of universes (solutions) generated per iteration	30
Maximum number of iterations	100
Minimum WEP	0.1
Maximum WEP	1
Minimum desired return	5%
Benchmark for semi-variance (3 different cases)	Mean return of the portfolio, 0, and the annual return of the Nifty 100 index (12.3%)

Table 9 Optimization cases

Optimization cases	Equal weight allocation	Weighted allocation	Variance replacement
Description	All objectives (kurtosis, semi-variance, entropy, skewness) are assigned equal weights of 0.25 each	Kurtosis and semi-variance are weighted at 0.20 each, while entropy and skewness receive weights of 0.30 each	Equal weights are assigned to kurtosis, variance, entropy, and skewness, i.e., 0.25
Objective	Evaluate performance with equal importance given to all objectives	Assess the impact of prioritizing return and diversification over risk	Investigate the effect of replacing semi-variance with variance

Table 10 Proportion of investment in assets when equal weights are assigned to all objectives

Company	Benchmark = portfolio mean	Benchmark = 0	Benchmark = 12.3%
Bajaj holdings and investment limited	4.33%	14.59%	12.48%
Shree cement limited	6.12%	4.65%	5.97%
Bosch limited	7.72%	9.49%	16.54%
Eicher motors limited	8.44%	15.78%	27.74%
Reliance industries limited	1.00%	2.05%	1.80%
Maruti suzuki India limited	16.09%	11.24%	1.93%
Page industries limited	14.17%	15.17%	12.15%
Ultratech cement limited	14.96%	9.52%	4.02%
Tata consultancy services limited	26.17%	16.51%	13.09%
Tata steel limited	1.00%	1.00%	4.28%
Best fitness value	1000000.015	1000000.015	1000000.021
Portfolio return	24.14%	27.54%	28.38%
Portfolio risk	35.43%	34.22%	38.29%

Table 11 Proportion of investment in assets for weighted allocation

Company	Benchmark = portfolio mean	Benchmark = 0	Benchmark = 12.3%
Bajaj holdings and investment limited	7.24%	19.90%	6.40%
Shree cement limited	10.02%	1.35%	8.43%
Bosch limited	12.99%	13.90%	21.65%
Eicher motors limited	30.02%	14.54%	14.49%
Reliance industries limited	10.77%	5.68%	4.83%
Maruti suzuki India limited	1.02%	2.65%	1.00%
Page industries limited	12.36%	12.03%	15.62%
Ultratech cement limited	4.90%	11.67%	13.91%
Tata consultancy services limited	9.35%	16.78%	9.50%
Tata steel limited	1.34%	1.51%	4.16%
Best fitness value	1000000.012	1000000.01	1000000.012
Portfolio return	29.43%	25.87%	21.42%
Portfolio risk	40.97%	34.74%	36.27%

5.6 Portfolio optimization using MVO metaheuristic

We used the MVO method with the parameters specified in Table 8 to solve the proposed HOAR portfolio optimization problem.

5.7 Optimization cases and their analysis

We formulated and analyzed two distinct cases to investigate the impact of varying weight distributions on optimization outcomes. Additionally, we evaluated a third scenario where semi-variance is replaced with variance in the objective function while keeping the rest of the problem unchanged, for comparative purposes where equal weights were assigned to all objectives. The optimization scenarios are presented in Table 9 below.

The effects of the varied weight distributions on the optimization results are detailed in Tables 10 and 11.

With equal weights assigned to all objectives, the highest portfolio return (28.38%) was achieved with the benchmark of 12.3%, followed by the benchmark of 0% (27.54%). The portfolio mean benchmark produced the lowest return (24.14%). The lowest risk was associated with the benchmark of 0% (34.22%), while the highest risk was linked to the benchmark of 12.3% (38.29%). Results suggest that higher returns came with increased risk.

Tata Consultancy Services Limited showed consistently high performance, especially in the portfolio mean scenario with an allocation of 26.17%, indicating stability and strong return potential. Eicher Motors Limited had notable allocations across all scenarios, with the highest in the benchmark of 12.3% (27.74%), reflecting high return potential and higher risk. Maruti Suzuki India Limited had the largest

Table 12 Proportion of investment in assets for variance as objective instead of semi-variance

Company	Results
Bajaj holdings and investment limited	8.14%
Shree cement limited	4.93%
Bosch limited	19.37%
Eicher motors limited	11.24%
Reliance industries limited	1.99%
Maruti suzuki india limited	2.96%
Page industries limited	15.25%
Ultratech cement limited	14.61%
Tata consultancy services limited	14.26%
Tata steel limited	7.24%
Best fitness value	1000000.044
Portfolio return	20.31%
Portfolio risk	35.46%

allocation in the portfolio mean scenario (16.09%) but lower allocations in the other benchmarks, indicating return variability. Risk-averse investors might prefer the benchmark of 0% due to its lower risk profile, while those with higher risk tolerance may favor the benchmark of 12.3% for its higher returns.

With weighted allocation, the highest portfolio return (29.43%) was achieved with the portfolio mean benchmark, followed by the benchmark of 0% (25.87%). The lowest return was for the benchmark of 12.3% (21.42%). The risk was lowest for the benchmark of 0% (34.74%) and highest for the portfolio mean scenario (40.97%).

Eicher Motors Limited had the highest allocation in the portfolio mean scenario (30.02%), indicating significant return potential with associated risk. Bajaj Holdings & Investment Limited had a high allocation in the benchmark of 0% scenario (19.90%), suggesting better performance with lower semi-variance constraints. Bosch Limited had notable allocations across all benchmarks, particularly the benchmark of 12.3% (21.65%). Tata Consultancy Services Limited had significant allocations, notably in the benchmark of 0% scenario (16.78%). Maruti Suzuki India Limited had low overall allocations, indicating lower performance or more significant risk than other assets. Risk-averse investors might prefer the benchmark of 0% due to its lower risk profile, while those with higher risk tolerance may favor the benchmark of portfolio mean for its higher returns.

Replacing semi-variance with variance resulted in a lower portfolio return of 20.31% and a risk of 35.46%. Bosch Limited had a significant allocation (19.37%), reflecting its performance under the variance objective (Refer to Table 12).

The models used in this study were implemented in a Google Colab environment using Python programming.

Hence in our analysis, after evaluating different cases, we found that in the equal weight case, the highest return was achieved using a benchmark of 12.3% (Nifty Index Return). Conversely, in the unequal weight case, the maximum return was achieved using the portfolio return as the benchmark. However, a more balanced risk-return profile was achieved using zero as a benchmark for estimating semi-variance in both the cases. Therefore, based on our analysis, estimating semi-variance using 0 as the benchmark is appropriate.

Furthermore, our analysis shows that optimizing for semi-variance produces higher returns than variance, as indicated by the better model performance when semi-variance is used as an objective in place of variance. This suggests that using semi-variance with higher moments can improve portfolio performance.

6 Conclusion and future directions

In this study, we optimized a portfolio of 10 top ranked companies selected from Nifty 100 index using MCDM, deep learning, and metaheuristic techniques. We applied TODIM method for portfolio selection and integrated an ensemble approach with XGBoost, N-BEATS, and BiLSTM to enhance forecasting accuracy. Our approach incorporates higher order moments like skewness and kurtosis, along with mean, semi-variance, and Gini entropy, for addressing gaps in the existing literature and providing a more comprehensive strategy for managing risk and investment constraints. The proposed integrated methodology worked well in optimizing the proposed portfolio problem.

For future research following options are suggested:

1. Incorporate advanced TODIM techniques to better account for investor's behavior and enhance strategy adaptability.
2. Evaluate the proposed strategy against other market indexes to identify strengths and weaknesses.
3. Include transaction costs and liquidity constraints for a more realistic portfolio strategy.
4. Explore other metaheuristic algorithms for a broader perspective on optimization performance.

However, there are several limitations to consider. Financial data quality, particularly from emerging markets, can pose challenges. Deep learning models and metaheuristic optimization methods often require sensitive parameter tuning, which can be complex. Assumptions such as zero transaction costs may not reflect real-world conditions. Furthermore, focusing on the Nifty 100 index might limit the generalizability of findings to other market indexes. Lastly, using semi-variance as a risk measure remains an approximation, possibly introducing inaccuracies.

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Data and code availability The datasets and code used for data analysis and modeling in this study are available upon request by contacting the corresponding author via email.

Declarations

Conflict of interest There is no conflict of interest.

Human or animal rights No human or animal participation was involved in this research.

Informed consent All authors have approved the manuscript and agree with its submission to the journal for publication.

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