★ Prirodna dedukcija ★

1. **U sistemu prirodne dedukcije dokazati** $\vdash A \Rightarrow A$**.**

   Rešenje:

$$\frac{[A]^1}{A \Rightarrow A} \Rightarrow I, 1$$

2. **U sistemu prirodne dedukcije dokazati** $\neg A, A \vdash B$**.**

   Rešenje:

1. $\neg A$ premise
2. A premise
3. $\bot$ $\neg E$ 1,2
4. B efq 3

$$\frac{\dfrac{\neg A \quad A}{\bot}\,\neg E}{B}\,\text{efq}$$

3. **U sistemu prirodne dedukcije dokazati**
$\vdash (A \vee (B \wedge C)) \Rightarrow ((A \vee B) \wedge (A \vee C))$.

Rešenje:

1.	$A \vee (B \wedge C)$	pret p.
2.	A	pret p
3.	$A \vee B$	$\vee I_1$ 2
4.	$A \vee C$	$\vee I_1$ 2
5.	$(A \vee B) \wedge (A \vee C)$	$\wedge I$ 3,4
6.	$B \wedge C$	pret p.
7.	B	$\wedge E_1$ 6
8.	C	$\wedge E_2$ 6
9.	$A \vee B$	$\vee I_2$ 7
10.	$A \vee C$	$\vee I_2$ 8
11.	$(A \vee B) \wedge (A \vee C)$	$\wedge I$ 9,10
12.	$(A \vee B) \wedge (A \vee C)$	$\vee E$ 1, 2-5, 6-11

13. $A \vee (B \wedge C) \Rightarrow (A \vee B) \wedge (A \vee C)$ $\Rightarrow I$ 1–12

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vee (B \wedge C)}^{1} \quad \dfrac{\dfrac{[A]^{2}}{A \vee B}\vee I_1 \quad \dfrac{[A]^{2}}{A \vee C}\vee I_2}{(A \vee B) \wedge (A \vee C)}\wedge I \quad \dfrac{\dfrac{\dfrac{[B \wedge C]^{3}}{B}\wedge E_1}{A \vee B}\vee I_2 \quad \dfrac{\dfrac{[B \wedge C]^{3}}{C}\wedge E_2}{A \vee C}\vee I_2}{(A \vee B) \wedge (A \vee C)}\wedge I}{(A \vee B) \wedge (A \vee C)}\vee E_{1,2,3}}{(A \vee B) \wedge (A \vee C)}\Rightarrow I_{1,1}}{A \vee (B \wedge C) \Rightarrow (A \vee B) \wedge (A \vee C)}$$

4. **U sistemu prirodne dedukcije dokazati** $\vdash ((A \wedge B) \vee (A \wedge C)) \Rightarrow (A \wedge (B \vee C))$.

Rešenje:

1.	$(A \wedge B) \vee (A \wedge C)$	pretp.
2.	$A \wedge B$	pretp.
3.	A	$\wedge E_1$ 2
4.	B	$\wedge E_2$ 2
5.	$B \vee C$	$\vee I_1$ 4
6.	$A \wedge (B \vee C)$	$\wedge I$ 3,5
7.	$A \wedge C$	pretp
8.	A	$\wedge E_1$ 7
9.	C	$\wedge E_2$ 7
10.	$B \vee C$	$\vee I_2$ 9
11.	$A \wedge (B \vee C)$	$\wedge I$ 8,10
12.	$A \wedge (B \vee C)$	$\vee E$ 1, 2-6, 7-11

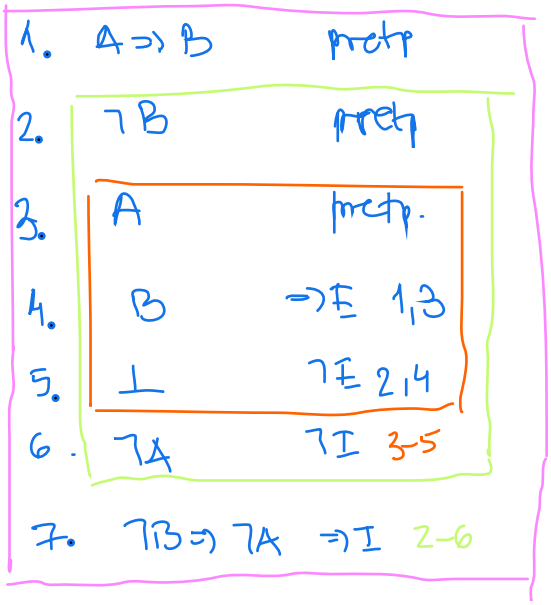
13. $(A \wedge B) \vee (A \wedge C) \Rightarrow A \wedge (B \vee C)$ $\Rightarrow I$ 1-12

$$\dfrac{\dfrac{[(A \wedge B) \vee (A \wedge C)]^1 \qquad \dfrac{\dfrac{[A \wedge B]^2}{A}\wedge E_1 \quad \dfrac{[A \wedge B]^2}{B}\wedge E_2}{B \vee C}\vee I_1}{A \wedge (B \vee C)}\wedge I \qquad \dfrac{\dfrac{[A \wedge C]^3}{A}\wedge E_1 \quad \dfrac{[A \wedge C]^3}{C}\wedge E_2}{B \vee C}\vee I_2}{A \wedge (B \vee C)}\wedge I}{A \wedge (B \vee C)}\vee E_{1,2,3}$$

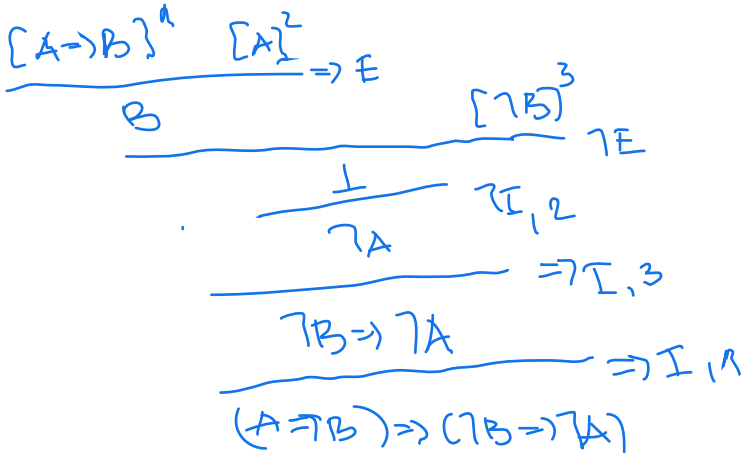
$$\dfrac{A \wedge (B \vee C)}{(A \wedge B) \vee (A \wedge C) \Rightarrow (A \wedge (B \vee C))} \Rightarrow I, 1$$

5. **U sistemu prirodne dedukcije dokazati:**
$\vdash (A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$

Rešenje:



8. $(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$ ⇒I 1-7



6. **U sistemu prirodne dedukcije dokazati:**

(a) $\vdash ((A \wedge B) \Rightarrow C) \Rightarrow (A \Rightarrow (B \Rightarrow C))$
(b) $\vdash (A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \wedge B) \Rightarrow C)$

Rešenje:

(a)

1. $(A \wedge B) \Rightarrow C$ — pretp.
2. A — pretp.
3. B — pretp.
4. $A \wedge B$ — $\wedge I$ 2,3
5. C — $\Rightarrow E$ 1,4
6. $B \Rightarrow C$ — $\Rightarrow I$ 3-5
7. $A \Rightarrow (B \Rightarrow C)$ — $\Rightarrow I$ 2-6
8. $((A \wedge B) \Rightarrow C) \Rightarrow (A \Rightarrow (B \Rightarrow C))$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[(A \wedge B) \Rightarrow C]^1 \quad \dfrac{[A]^2 \quad [B]^3}{A \wedge B}\wedge I}{C}\Rightarrow E}{B \Rightarrow C}\Rightarrow I,3}{A \Rightarrow (B \Rightarrow C)}\Rightarrow I,2}{((A \wedge B) \Rightarrow C) \Rightarrow (A \Rightarrow (B \Rightarrow C))}\Rightarrow I,1$$

(b)

1. $A \Rightarrow (B \Rightarrow C)$ — pretp.
2. $A \wedge B$ — pretp.
3. A — $\wedge E_1$ 1,2
4. B — $\wedge E_2$ 1,2
5. $B \Rightarrow C$ — $\Rightarrow E$ 1,3
6. C — $\Rightarrow E$ 4,5
7. $(A \wedge B) \Rightarrow C$ — $\Rightarrow I$ 2-6
8. $(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \wedge B) \Rightarrow C)$ — $\Rightarrow I$ 1-7

$$\dfrac{\dfrac{\dfrac{\dfrac{[A \Rightarrow (B \Rightarrow C)]^1 \quad \dfrac{[A \wedge B]^2}{A}\wedge E_1}{B \Rightarrow C}\Rightarrow E \quad \dfrac{[A \wedge B]^2}{B}\wedge E_2}{C}\Rightarrow E}{A \wedge B \Rightarrow C}\Rightarrow I,2}{(A \Rightarrow (B \Rightarrow C)) \Rightarrow (A \wedge B \Rightarrow C)}\Rightarrow I,1$$

7. **U sistemu prirodne dedukcije dokazati**
$\vdash (A \wedge B) \Rightarrow \Big(\big((A \Rightarrow C) \wedge (B \Rightarrow D)\big) \Rightarrow (C \wedge D)\Big)$.

Rešenje:

1. $A \wedge B$ pretp.
2. $(A \Rightarrow C) \wedge (B \Rightarrow D)$ pretp.
3. A $\wedge E_1$ 1
4. B $\wedge E_2$ 1
5. $A \Rightarrow C$ $\wedge E_1$ 2
6. $B \Rightarrow D$ $\wedge E_2$ 2
7. C $\Rightarrow E$ 3,5
8. D $\Rightarrow E$ 4,6
9. $C \wedge D$ $\wedge I$ 7,8
10. $\big((A \Rightarrow C) \wedge (B \Rightarrow D)\big) \Rightarrow (C \wedge D)$ $\Rightarrow I$ 2-9

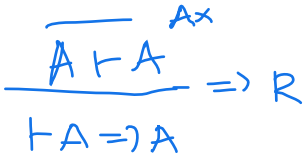
11. $(A \wedge B) \Rightarrow \big((A \Rightarrow C) \wedge (B \Rightarrow D) \Rightarrow C \wedge D\big)$ $\Rightarrow I$ 1-10

$$\dfrac{\dfrac{\dfrac{[A \wedge B]^1}{A}\wedge E_1 \quad \dfrac{[(A \Rightarrow C) \wedge (B \Rightarrow D)]^2}{A \Rightarrow C}\wedge E_1}{C}\Rightarrow E \qquad \dfrac{\dfrac{[A \wedge B]^1}{B}\wedge E_2 \quad \dfrac{[(A \Rightarrow C) \wedge (B \Rightarrow D)]^2}{B \Rightarrow D}\wedge E_2}{D}\Rightarrow E}{C \wedge D}\wedge I}{\big((A \Rightarrow C) \wedge (B \Rightarrow D)\big) \Rightarrow (C \wedge D)}\Rightarrow I, 2}{(A \wedge B) \Rightarrow \big((A \Rightarrow C) \wedge (B \Rightarrow D) \Rightarrow C \wedge D\big)}\Rightarrow I, 1$$

★ Račun sekvenata ★

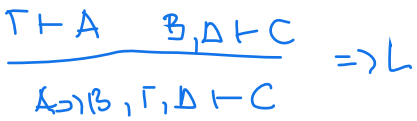
1. **U računu sekvenata za iskaznu logiku dokazati da je formula $A \Rightarrow A$ teorema.**

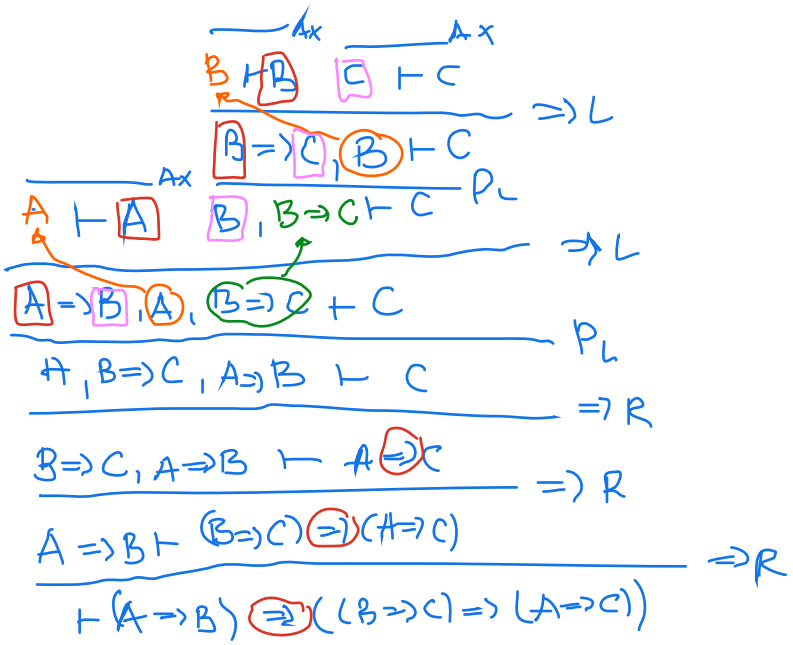
   Rešenje:



2. **U računu sekvenata dokazati da je formula $(A \Rightarrow B) \Rightarrow ((B \Rightarrow C) \Rightarrow (A \Rightarrow C))$ teorema.**

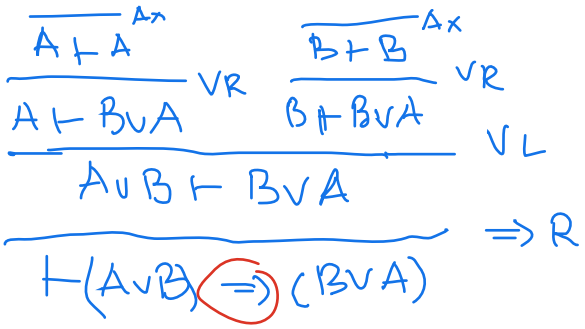
   Rešenje:





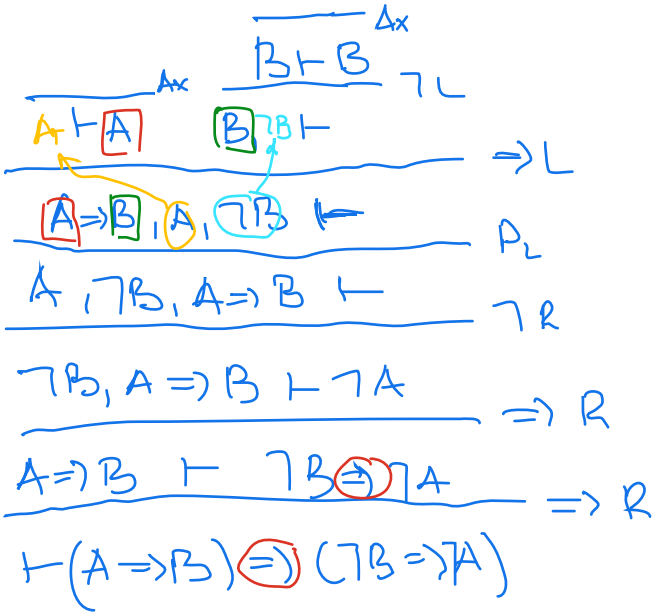
3. **U računu sekvenata dokazati da je formula $(A \vee B) \Rightarrow (B \vee A)$ teorema.**

   Rešenje:



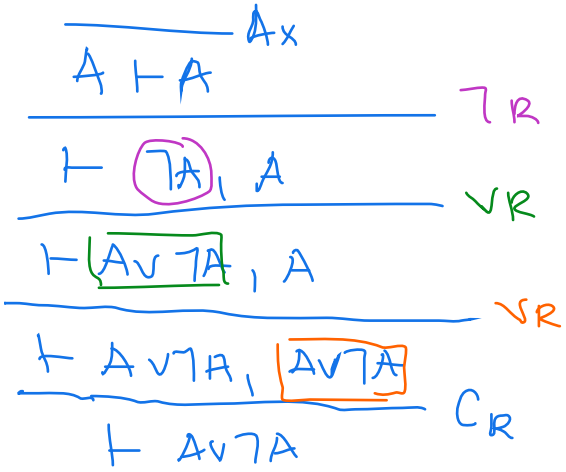
4. **U računu sekvenata dokazati da je formula $(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$ teorema.**

   Rešenje:



5. **Dokazati da je formula $A \vee \neg A$ teorema u računu sekvenata za klasičnu iskaznu logiku.**

Rešenje:



6. **U računu sekvenata dokazati da je formula** $\vdash ((A \vee B) \wedge \neg A) \Rightarrow B$ **teorema.**

Rešenje: