Normalne forme

★ Preneks normalna forma ★

1. **Odrediti preneks normalnu formu za formulu**

$$F = (\forall x)P(x) \wedge (\forall x)(\exists y)(\forall z)\Big(Q(y,z) \Rightarrow R\big(g(x),y\big)\Big).$$

Rešenje:

$F \equiv (\forall x)P(x) \wedge (\forall x)(\exists y)(\forall z)\big(\neg Q(y,z) \vee R(g(x),y)\big)$

$\equiv (\forall x)\big(P(x) \wedge (\forall x)(\exists y)(\forall z)(\neg Q(y,z) \vee R(g(x),y))\big)$

x je slobodna

$\equiv (\forall x)\big(P(x) \wedge (\forall u)(\exists y)(\forall z)(\neg Q(y,z) \vee R(g(u),y))\big)$

$\equiv (\forall x)(\forall u)\big(P(x) \wedge (\exists y)(\forall z)(\neg Q(y,z) \vee R(g(u),y))\big)$

$\equiv (\forall x)(\forall u)(\exists y)(\forall z)\big(P(x) \wedge (\neg Q(y,z) \vee R(g(u),y))\big)$

2. **Odrediti preneks normalnu formu za formulu**

$$F = \quad (\forall x)(\exists y)P\big(f(x,a),y\big) \vee (\exists x)(\forall y)(\forall z)\Big(\neg Q\big(g(x),f(y,z)\big)\Big).$$

Rešenje:

U potformuli $(\exists x)(\forall y)(\forall z)\big(\neg Q(g(x),f(y,z))\big)$

preimenujemo promenljive $x \mapsto u$, $y \mapsto v$

$$F \equiv (\forall x)(\exists y)P(f(x,a),y) \vee (\exists u)(\forall v)(\forall z)\big(\neg Q(g(u),f(v,z))\big)$$

$$\equiv (\forall x)\big((\exists y)P(f(x,a),y) \vee (\exists u)(\forall v)(\forall z)\big(\neg Q(g(u),f(v,z))\big)\big)$$

$$\equiv (\forall x)\big((\exists y)\big(P(f(x,a),y) \vee (\exists u)(\forall v)(\forall z)\big(\neg Q(g(u),f(v,z))\big)\big)\big)$$

$$\equiv (\forall x)(\exists y)\big((\exists u)\big(P(f(x,a),y) \vee (\forall v)(\forall z)\big(\neg Q(g(u),f(v,z))\big)\big)\big)$$

$$\equiv (\forall x)(\exists y)(\exists u)\big((\forall v)\big(P(f(x,a),y) \vee (\forall z)\big(\neg Q(g(u),f(v,z))\big)\big)\big)$$

$$\equiv (\forall x)(\exists y)(\exists u)(\forall v)(\forall z)\big(P(f(x,a),y) \vee \neg Q(g(u),f(v,z))\big)$$

3. **Odrediti preneks normalnu formu za formulu**

$$F = (\forall x)(\exists y)\big(P(x,y) \vee Q(x,a)\big) \Rightarrow (\exists z)(\forall y)R(y,z).$$

Rešenje:

$$F \equiv \neg(\forall x)(\exists y)\big(P(x,y) \vee Q(x,a)\big) \vee (\exists z)(\forall y)R(y,z)$$

$$\equiv (\exists x)\neg(\exists y)\big(P(x,y) \vee Q(x,a)\big) \vee (\exists z)(\forall y)R(y,z)$$

$$\equiv (\exists x)(\forall y)\neg\big(P(x,y) \vee Q(x,a)\big) \vee (\exists z)(\forall y)R(y,z)$$

$$\equiv (\exists x)(\forall y)\big(\neg P(x,y) \wedge \neg Q(x,a)\big) \vee (\exists z)(\forall y)R(y,z)$$

$[y \mapsto u]$

$$\equiv (\exists x)(\forall y)\big(\neg P(x,y) \wedge \neg Q(x,a)\big) \vee (\exists z)(\forall u)R(u,z)$$

$$\equiv (\exists x)(\forall y)\Big(\big(\neg P(x,y) \wedge \neg Q(x,a)\big) \vee (\exists z)(\forall u)R(u,z)\Big)$$

$$\equiv (\exists x)(\forall y)(\exists z)(\forall u)\Big(\big(\neg P(x,y) \wedge \neg Q(x,a)\big) \vee R(u,z)\Big)$$

★ Klauzalna normalna forma ★

4. **Formulu**

$$F = \big((\exists x)P(x) \wedge (\exists x)Q(x)\big) \Rightarrow (\exists x)\big(P(x) \wedge Q(x)\big)$$

transformisati u klauzalno normalnu formu.

Rešenje:

$F \equiv \neg\big((\exists x)P(x) \wedge (\exists x)Q(x)\big) \vee (\exists x)\big(P(x) \wedge Q(x)\big)$

$\equiv \neg(\exists x)P(x) \vee \neg(\exists x)Q(x) \vee (\exists x)\big(P(x) \wedge Q(x)\big)$

$\equiv \underbrace{(\forall x)\neg P(x)}_{[x \mapsto y]} \vee \underbrace{(\forall x)\neg Q(x)}_{[x \mapsto z]} \vee (\exists x)\big(P(x) \wedge Q(x)\big)$

$\equiv (\forall y)\neg P(y) \vee (\forall z)\neg Q(z) \vee (\exists x)\big(P(x) \wedge Q(x)\big)$

$\equiv (\exists x)\big((\forall y)\neg P(y) \vee (\forall z)\neg Q(z) \vee (P(x) \wedge Q(x))\big)$

$\equiv (\exists x)(\forall y)\big(\neg P(y) \vee (\forall z)\neg Q(z) \vee (P(x) \wedge Q(x))\big)$

$\equiv (\exists x)(\forall y)(\forall z)\big(\neg P(y) \vee \neg Q(z) \vee (P(x) \wedge Q(x))\big)$ prenex

$\equiv (\exists x)(\forall y)(\forall z)\underbrace{\big((\neg P(y) \vee \neg Q(z) \vee P(x)) \wedge (\neg P(y) \vee \neg Q(z) \vee Q(x))\big)}_{\text{KNF}}$

$[x \mapsto c]$

$\underbrace{(\forall y)(\forall z)\big((\neg P(y) \vee \neg Q(z) \vee P(c)) \wedge (\neg P(y) \vee \neg Q(z) \vee Q(c))\big)}_{\text{klauzalna forma}}$

5. **Transformisati formulu**

$$F = (\exists x)P(x) \wedge (\forall y)(\exists x)(\exists z)\Big(P\big(f(z)\big) \vee Q(y,x) \Rightarrow Q\big(f(y),a\big)\Big)$$

u klauzalno normalnu formu. $[x \mapsto u]$

Rešenje:

$F \equiv (\exists x)P(x) \wedge (\forall y)(\exists u)(\exists z)\big(P(f(z)) \vee Q(y,u) \Rightarrow Q(f(y),a)\big)$

$\equiv (\exists x)\big(P(x) \wedge (\forall y)(\exists u)(\exists z)\big(P(f(z)) \vee Q(y,u) \Rightarrow Q(f(y),a)\big)$

$\equiv (\exists x)(\forall y)(\exists u)(\exists z)\big(P(x) \wedge \big(P(f(z)) \vee Q(y,u) \Rightarrow Q(f(y),a)\big)\big)$

$\equiv (\exists x)(\forall y)(\exists u)(\exists z)\big(P(x) \wedge \big[\neg\big(P(f(z)) \vee Q(y,u)\big) \vee Q(f(y),a)\big]\big)$

$\equiv (\exists x)(\forall y)(\exists u)(\exists z)\big(P(x) \wedge \big(\big(\neg P(f(z)) \wedge \neg Q(y,u)\big) \vee Q(f(y),a)\big)\big)$

$\equiv (\exists x)(\forall y)(\exists u)(\exists z)\big(P(x) \wedge \big(\neg P(f(z)) \vee Q(f(y),a)\big) \wedge \big(\neg Q(y,u) \vee Q(f(y),a)\big)\big)$

$[x \mapsto c]$

$(\forall y)(\exists u)(\exists z)\big(P(c) \wedge \big(\neg P(f(z)) \vee Q(f(y),a)\big) \wedge \big(\neg Q(y,u) \vee Q(f(y),a)\big)\big)$

$[u \mapsto g(y)]\ [z \mapsto h(y)]$

$(\forall y)\big(P(c) \wedge \big(\neg P(f(h(y))) \vee Q(f(y),a)\big) \wedge \big(\neg Q(y,g(y)) \vee Q(f(y),a)\big)\big)$

6. **Transformisati formulu**

$$(\forall x)(\exists y)P(x,y) \Rightarrow \neg\big((\exists x)Q(x) \vee (\forall x)(\forall y)\neg P(x,y)\big)$$

u klauzalno normalnu formu.

Rešenje: