Modul 13 - Zadaci

Wednesday, January 10, 2024 12:06 AM

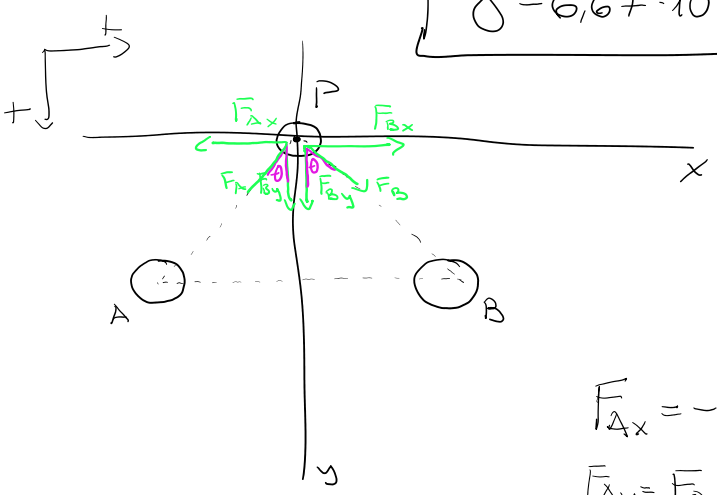
$\gamma = 6{,}67 \cdot 10^{-11}\ Nm^2/kg^2$

13.5) $m_1 = m_2 = 0{,}260\,kg$

$m = 0{,}010\,kg$

$\sin\theta = 0{,}8$

$\cos\theta = 0{,}6$



a) $\vec{F}_A = \gamma \frac{m_A m}{r^2} = \gamma \frac{m_A m}{r^2}$

$r = 0{,}1$

$F_A = 1{,}7342 \cdot 10^{-11}$

$F_A = F_B$

$F_{Ax} = -F_A \sin\alpha$

$F_{Ay} = F_A \sin\alpha$

$F_{Bx} = F_B \sin\alpha$

$F_{By} = F_B \cos\alpha$

$a_x = \frac{\sin\alpha(\vec{F}_B - \vec{F}_A)}{m} = 0$

$m a_x = -F_A \sin\alpha + F_B \sin\alpha$

$m a_y = F_B \cos\alpha + F_A \cos\alpha$

$a_y = \frac{\cos\alpha\,(F_A + F_B)}{m}$

$a_y = 2{,}08 \cdot 10^{-9}$ u smeru ↓

13.18) $M = 629\,kg$

$t = 10d = 10 \cdot 24 \cdot 60 \cdot 60 = 8{,}64 \cdot 10^5\,s$

$l = 2{,}87 \cdot 10^6\,km = 2{,}87 \cdot 10^9\,m$

$v = 1{,}20 \cdot 10^4\,km/h = 43200\,m/s$

$E_k = \frac{1}{2} M v^2$

$E_k = 5{,}86 \cdot 10^{11}\,J$

$\Pi = -\gamma \frac{M m_2}{l}$

$\Pi = -8{,}727 \cdot 10^7\,J$

13.22) $t = 1d = 24 \cdot 60 \cdot 60 = 86400\,s$

15,65 obrtaja na dan

$T = \frac{t}{15,65}$ $\boxed{T = 5520,76\,s}$ $T = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{\gamma m_z}}$ $r = r_z + h$

$r^{\frac{3}{2}} = \frac{T\sqrt{\gamma m_z}}{2\pi}$ $r = \sqrt[3]{\frac{T^2 \gamma m_z}{4\pi^2}}$ $\boxed{r = 6,748 \cdot 10^6}$

$h = r - r_z$ $\boxed{h = 3,68 \cdot 10^5\,m}$

13,28) $l_{pz} = \frac{1}{9} l_{ms}$, $r_{ms} = 5,79 \cdot 10^{10}\,m$

1 obrt / $T = 3,09 \cdot 24 \cdot 60^2 = 266976\,s$

$m_{zv} = ?$

a) $l_{pz} = r = \frac{1}{9} \cdot r_{ms}$

$l_{pz} = 643 \cdot 10^7$

$T = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{\gamma m_{zv}}} \quad /^2$ $T^2 = \frac{4\pi^2 r^3}{\gamma m_{zv}}$ $m_{zv} = \frac{4\pi^2 r^3}{\gamma T^2}$

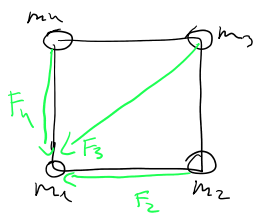
$\boxed{m_{zv} = 2,207 \cdot 10^{30}\,kg}$

$\boxed{m_{zv} = 1,109 m_s}$

b) $T = \frac{2\pi r}{v}$

$v = \frac{2\pi r}{T}$ $\boxed{v = 1,51 \cdot 10^5\,m/s}$

13.40)



$F = F_2 + F_3 + F_4$

$F_2 = F_4 = \gamma \frac{m^2}{d^2} \cos 45°$

$F_3 = \gamma \frac{m^2}{(d\sqrt{2})^2}$

$d = 10{,}0\,cm = 0{,}10\,m$

$m_1 = m_2 = m_3 = m_u = 800\,kg$

$F = \gamma m^2 \left(\frac{2\cos 45°}{d^2} + \frac{1}{d^2 2} \right)$

$\boxed{F = 8{,}17 \cdot 10^{-3}\,N}$

13.42) $l = 4{,}25\,m$

$\omega = ?$

$\mathcal{E} = \mathcal{E}_{Europe}$

$m_E = 4{,}8 \cdot 10^{22}$

$R_E = 3138\,m$

$\boxed{r_E = 1569\,m}$

$\mathcal{E} = l\omega^2 \qquad \mathcal{E} = g$

$\omega = \sqrt{\frac{g}{l}} = 0{,}553 \cdot \frac{60\,s}{2\pi\,rad}$

$\boxed{\omega = 5{,}28\,rad/s}$

$g = \gamma \frac{m_E}{R_E^2} \qquad \boxed{g = 1{,}3 \cdot 10^{6}\,m/s^2}$

13.47) $r_T = 5{,}00\,km = 5000\,m$

$\rho = 5{,}50\,g/cm^3$

a) $m_T = ? \qquad g_T = ?$

$\rho = \frac{m_T}{V_T} \qquad \boxed{V_T = \frac{4}{3} r_T^3 \pi}$

$m_T = \rho V_T \qquad m_T = \frac{4}{3} \rho r_T^3 \pi \qquad \boxed{m_T = 2{,}879 \cdot 10^{15}\,kg}$

$g_T = \gamma \frac{m_T}{r_T^2} \qquad \boxed{g_T = 7{,}68 \cdot 10^{-3}\,m/s^2}$

b) $mg_T = m\omega_T \qquad mg_T = m\frac{v^2}{r_T}$

$\omega_T = \frac{v^2}{r_T} \qquad v = \sqrt{g_T r_T} \qquad \boxed{v = 6{,}196\,m/s}$

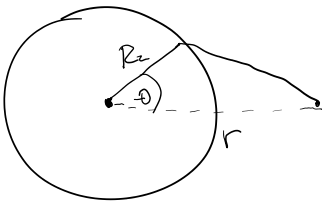
13.51) $T = 1d = 24 \cdot 60^2 = 8{,}64 \cdot 10^4$

$r = ?$

$r = ?$

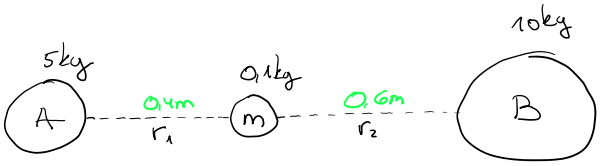
a) $T = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{\gamma m_2}}$ $\quad r^{\frac{3}{2}} = \frac{T\sqrt{\gamma m_2}}{2\pi}$ $\quad r = \sqrt[3]{\frac{T^2 \gamma m_2}{4\pi^2}}$ $\quad \boxed{r = 4{,}222 \cdot 10^7 m}$

b)



$\theta = 81{,}3°$

13.62)



$E_{k1} + \Pi_1 = E_{k2} + \Pi_2$

$E_{kx} = m v^2 \frac{1}{2}$

$\Pi_x = -\gamma \frac{m_1 m_2}{r}$

$m = 0{,}1 kg$

$m_A = 5 kg$

$m_B = 10 kg$

$v_2 = ?$

$E_{k1} = 0$

$E_{k2} = m v_2^2 \frac{1}{2}$

$\Pi_1 = -\gamma \frac{m m_A}{r_1} - \gamma \frac{m m_B}{r_2}$ $\quad \Pi_1 = -1{,}945 \cdot 10^{-10}$

$\Pi_2 = -\gamma \frac{m m_A}{0{,}8} - \gamma \frac{m m_B}{0{,}2}$ $\quad \Pi_2 = -3{,}751 \cdot 10^{-10}$

$E_{k1} + \Pi_1 = E_{k2} + \Pi_2$ ($E_{k1} = 0$)

$E_{k2} = \Pi_1 - \Pi_2$

$\boxed{E_{k2} = 1{,}806 \cdot 10^{-10}}$

$E_{k2} = \frac{1}{2} m v_2^2$

$v_2 = \sqrt{\frac{2 E_{k2}}{m}}$

$\boxed{v_2 = 6{,}01 \cdot 10^{-5} m/s}$

13,65) $m_{\ddot{o}}, h, m_2$

$h_{\ddot{o}} = R_2 + h$

$v = ?$

$E_{k1} + \Pi_1 = E_{k2} + \Pi_2$

$E_{k1} = 0$

$\Pi_1 = -\gamma \frac{m_2 m_{\ddot{o}}}{R_2 + h}$

$E_{k2} = \frac{1}{2} m_{\ddot{o}} v^2$

$\Pi_2 = -\gamma \frac{m_2 m_{\ddot{o}}}{R_2}$

$-\gamma \frac{m_2 m_{\ddot{o}}}{R_2 + h} = \frac{1}{2} m_{\ddot{o}} v^2 - \gamma \frac{m_2 m_{\ddot{o}}}{R_2}$

$\frac{1}{2} v^2 = -\gamma \frac{m_2}{R_2 + h} + \gamma \frac{m_2}{R_2}$

$v^2 = -2\gamma m_2 \left(\frac{1}{R_2 + h} - \frac{1}{R_2} \right)$

$$v^2 = -2\gamma m_2 \left(\frac{1}{R_2+h} - \frac{1}{R_2}\right)$$

$$v^2 = 2\gamma m_2 \left(\frac{1}{R_2} - \frac{1}{R_2+h}\right)$$

$$v^2 = 2\gamma m_2 \left(\frac{R_2+h-R_2}{R_2(R_2+h)}\right)$$

$$v^2 = \frac{2\gamma m_2 h}{R_2(R_2+h)} \Rightarrow \boxed{v = \sqrt{\frac{2\gamma m_2 h}{R_2(R_2+h)}}}$$

13.66) $A = E_2 - E_1$

$R_0 \approx R_2$

$E_1 = -\gamma \frac{m m_2}{R_2}$

$E_2 = -\gamma \frac{m m_2}{2R_2}$

a) $A = ?$

$$A = -\gamma \frac{m m_2}{2R_2} + \gamma \frac{m m_2}{R_2} = -\gamma \frac{m m_2}{2R_2} + 2\gamma \frac{m m_2}{2R_2}$$

$$\boxed{A = \gamma \frac{m m_2}{2R_2}}$$

b) $E_{k_\infty} = 0$

$\Pi_\infty = -\gamma \frac{m m_2}{r} \Rightarrow$ što je dalje $\rightarrow$ r jeveće $\rightarrow$ Π_∞ teži 0

$E_3 = 0 \Rightarrow E_3 - E_2 = A_2 \Rightarrow \boxed{A_2 = A}$

13.73) $v_1 = 20 \cdot 10^4 \, m/s$

$r_1 = 2{,}50 \cdot 10^{11} \, m$

$v_2 = ?$

$r_2 = 5{,}00 \cdot 10^{10} \, m$

$E_{k_1} + \Pi_1 = E_{k_2} + \Pi_2$

$E_{k_1} = \frac{1}{2} m v_1^2$

$E_{k_2} = \frac{1}{2} m v_2^2$

$\Pi_1 = -\gamma \frac{m m_s}{r_1}$

$\Pi_2 = -\gamma \frac{m m_s}{r_2}$

$$\frac{1}{2} m v_1^2 - \gamma \frac{m m_s}{r_1} = \frac{1}{2} m v_2^2 - \gamma \frac{m m_s}{r_2} \quad | \cdot 2$$

$$v_1^2 - v_2^2 = 2\gamma \left(\frac{m_s}{r_1} - \frac{m_s}{r_2}\right)$$

$$v_2 = \sqrt{v_1^2 - 2\gamma \left(\frac{m_s}{r_1} - \frac{m_s}{r_2}\right)}$$

$$\boxed{v_2 = 6{,}81 \cdot 10^4 \, m/s}$$

13.77) a) $2a = r_a + r_p = h_a + h_p + 2r_z$

$h_a = 4000 \cdot 10^3 m$

$h_p = 400 \cdot 10^3 m$

$T = ?$

r_a - apogeja

r_p - perigeja

$a = \frac{h_a + h_p}{2} + r_z \Rightarrow \boxed{a = 8,58 \cdot 10^6 m/s^2}$

$T = \frac{2\pi a^{\frac{3}{2}}}{\sqrt{\gamma m_z}} \quad \boxed{T = 7,451 \cdot 10^3 s}$

b) $\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{R_z + h_a}{R_z + h_p} = 1,153$

c) $v_p = 1,153 v_a$

$E_{kp} + \Pi_p = E_{ka} + \Pi_a$

$\frac{1}{2} m v_p^2 - \gamma \frac{m m_z}{r_p} = \frac{1}{2} m v_a^2 - \gamma \frac{m m_z}{r_a} \quad | \cdot 2$

$v_p^2 - \frac{2\gamma m_z}{r_z + h_p} = v_a^2 - \frac{2\gamma m_z}{r_z + h_a}$

$(1,153 v_a)^2 - v_a^2 = -\frac{2\gamma m_z}{r_z + h_a} + \frac{2\gamma m_z}{r_z + h_p}$

$2,34 v_a^2 - v_a^2 = 2\gamma m_z \left(\frac{1}{r_z + h_p} - \frac{1}{r_z + h_a} \right)$

$1,34 v_a^2 = 2\gamma m_z \left(\frac{1}{r_z + h_p} - \frac{1}{r_z + h_a} \right)$

$v_a = \sqrt{\frac{2\gamma m_z \left(\frac{1}{r_z + h_p} - \frac{1}{r_z + h_a} \right)}{1,34}}$

$\boxed{v_a = 5,455 \cdot 10^3 m/s}$

$\boxed{v_p = 8414 \cdot 10^3 m/s}$