

★ PRIRODNA DEDUKCIJA ★

1. U sistemu prirodne dedukcije dokazati $\vdash A \Rightarrow A$.

Rešenje:

$$\frac{[A]^1}{A \Rightarrow A} \Rightarrow I, 1$$

2. U sistemu prirodne dedukcije dokazati $\neg A, A \vdash B$.

Rešenje:

1. $\neg A$ premise
2. A premise
3. \perp $\neg E$ 1,2
4. B efq 3

$$\frac{\neg A \quad A}{\perp} \neg E$$

$$\frac{\perp}{B} efq$$

3. U sistemu prirodne dedukcije dokazati

$$\vdash (A \vee (B \wedge C)) \Rightarrow ((A \vee B) \wedge (A \vee C)).$$

Rešenje:

1.	$A \vee (B \wedge C)$	pret p.
2.	A	pret p
3.	$A \vee B$	$\vee I_1$ 2
4.	$A \vee C$	$\vee I_1$ 2
5.	$(A \vee B) \wedge (A \vee C)$	$\wedge I$ 3,4
6.	$B \wedge C$	pret p.
7.	B	$\wedge E_1$ 6
8.	C	$\wedge E_2$ 6
9.	$A \vee B$	$\vee I_2$ 7
10.	$A \vee C$	$\vee I_2$ 8
11.	$(A \vee B) \wedge (A \vee C)$	$\wedge I$ 9,10
12.	$(A \vee B) \wedge (A \vee C)$	$\vee E$ 1, 2-5, 6-11

$$13. A \vee (B \wedge C) \Rightarrow (A \vee B) \wedge (A \vee C) \Rightarrow I \quad 1-12$$

$$\begin{array}{c}
 \begin{array}{c}
 [A]^2 \quad [A]^2 \\
 \frac{A \vee B}{\vee I_1} \quad \frac{A \vee C}{\vee I_2} \\
 \hline
 (A \vee B) \wedge (A \vee C) \quad \wedge I
 \end{array}
 \quad
 \begin{array}{c}
 [B \wedge C]^3 \quad [B \wedge C]^3 \\
 \frac{B}{\wedge E_1} \quad \frac{C}{\wedge E_2} \\
 \hline
 \frac{A \vee B}{\vee I_2} \quad \frac{A \vee C}{\vee I_2} \\
 \hline
 (A \vee B) \wedge (A \vee C) \quad \wedge I
 \end{array}
 \end{array}$$

$$\begin{array}{c}
 (A \vee B) \wedge (A \vee C) \\
 \hline
 A \vee (B \wedge C) \Rightarrow (A \vee B) \wedge (A \vee C) \Rightarrow I, 1
 \end{array}$$

4. U sistemu prirodne dedukcije dokazati
 $\vdash ((A \wedge B) \vee (A \wedge C)) \Rightarrow (A \wedge (B \vee C)).$

Rešenje:

1.	$(A \wedge B) \vee (A \wedge C)$	pretp.
2.	$A \wedge B$	pretp.
3.	A	$\wedge E_1$ 2
4.	B	$\wedge E_2$ 2
5.	$B \vee C$	$\vee I_1$ 4
6.	$A \wedge (B \vee C)$	$\wedge I$ 3,5
7.	$A \wedge C$	pretp.
8.	A	$\wedge E_1$ 7
9.	C	$\wedge E_2$ 7
10.	$B \vee C$	$\vee I_2$ 9
11.	$A \wedge (B \vee C)$	$\wedge I$ 8,10
12.	$A \wedge (B \vee C)$	$\vee E$ 1,2-6,7-11

13. $(A \wedge B) \vee (A \wedge C) \Rightarrow A \wedge (B \vee C) \Rightarrow \text{I } 1-12$

$$\begin{array}{c}
 \frac{\frac{\frac{[A \wedge B]^2}{A} \wedge E_1 \quad \frac{\frac{[A \wedge B]^2}{B} \wedge E_2}{B \vee C} \vee I_1}{A \wedge (B \vee C)} \wedge I \quad \frac{\frac{[A \wedge C]^3}{A} \wedge E_1 \quad \frac{\frac{[A \wedge C]^3}{C} \wedge E_2}{B \vee C} \vee I_2}{A \wedge (B \vee C)} \wedge I}{A \wedge (B \vee C)} \vee E, 2, 3 \\
 \hline
 A \wedge (B \vee C) \Rightarrow \text{I}, \wedge \\
 \hline
 (A \wedge B) \vee (A \wedge C) \Rightarrow (A \wedge (B \vee C))
 \end{array}$$

5. U sistemu prirodne dedukcije dokazati:

$$\vdash (A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$$

Rešenje:

1.	$A \Rightarrow B$	pretp
2.	$\neg B$	pretp
3.	A	pretp.
4.	B	$\Rightarrow E$ 1,3
5.	\perp	$\neg E$ 2,4
6.	$\neg A$	$\neg I$ 3-5
7.	$\neg B \Rightarrow \neg A$	$\Rightarrow I$ 2-6

$$8. (A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A) \Rightarrow I \text{ 1-7}$$

$$\begin{array}{c}
 \frac{[A \Rightarrow B]^1 \quad [A]^2}{B} \Rightarrow E \\
 \frac{\quad}{\neg A} \neg I, 2 \\
 \frac{\quad}{\neg B \Rightarrow \neg A} \Rightarrow I, 3 \\
 \frac{\quad}{(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)} \Rightarrow I, 1
 \end{array}$$

6. U sistemu prirodne dedukcije dokazati:

(a) $\vdash ((A \wedge B) \Rightarrow C) \Rightarrow (A \Rightarrow (B \Rightarrow C))$

(b) $\vdash (A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \wedge B) \Rightarrow C)$

Rešenje:

(a)

1.	$(A \wedge B) \Rightarrow C$	pretp.
2.	A	pretp.
3.	B	pretp.
4.	$A \wedge B$	$\wedge I$ 2,3
5.	C	$\Rightarrow E$ 1,4
6.	$B \Rightarrow C$	$\Rightarrow I$ 3-5
7.	$A \Rightarrow (B \Rightarrow C)$	$\Rightarrow I$ 2-6
8.	$((A \wedge B) \Rightarrow C) \Rightarrow (A \Rightarrow (B \Rightarrow C))$	

$$\begin{array}{c}
 \frac{[A]^2 \quad [B]^3}{A \wedge B} \wedge I \\
 \frac{[A \wedge B \Rightarrow C]^1}{C} \Rightarrow E \\
 \frac{C}{B \Rightarrow C} \Rightarrow I, 3 \\
 \frac{B \Rightarrow C}{A \Rightarrow (B \Rightarrow C)} \Rightarrow I, 2 \\
 \frac{A \Rightarrow (B \Rightarrow C)}{((A \wedge B) \Rightarrow C) \Rightarrow (A \Rightarrow (B \Rightarrow C))} \Rightarrow I, 1
 \end{array}$$

(b)

1.	$A \Rightarrow (B \Rightarrow C)$	pretp.
2.	$A \wedge B$	pretp.
3.	A	$\wedge E_1, 2$
4.	B	$\wedge E_2, 2$
5.	$B \Rightarrow C$	$\Rightarrow E$ 1,4
6.	C	$\Rightarrow E$ 4,5
7.	$(A \wedge B) \Rightarrow C$	$\Rightarrow I$ 2-6
8.	$(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \wedge B) \Rightarrow C)$	$\Rightarrow I$ 1-7

$$\begin{array}{c}
 \frac{[A \wedge B]^2}{A} \wedge E_1 \\
 \frac{[A \Rightarrow (B \Rightarrow C)]^1}{B \Rightarrow C} \Rightarrow E \\
 \frac{B \Rightarrow C}{C} \Rightarrow E \\
 \frac{C}{A \wedge B \Rightarrow C} \Rightarrow I, 2 \\
 \frac{A \wedge B \Rightarrow C}{(A \Rightarrow (B \Rightarrow C)) \Rightarrow (A \wedge B \Rightarrow C)} \Rightarrow I, 1
 \end{array}$$

7. U sistemu prirodne dedukcije dokazati

$$\vdash (A \wedge B) \Rightarrow ((A \Rightarrow C) \wedge (B \Rightarrow D)) \Rightarrow (C \wedge D).$$

Rešenje:

$$1. A \wedge B \quad \text{pretp.}$$

$$2. (A \Rightarrow C) \wedge (B \Rightarrow D) \quad \text{pretp.}$$

$$3. A \quad \wedge E_1 1$$

$$4. B \quad \wedge E_2 1$$

$$5. A \Rightarrow C \quad \wedge E_1 2$$

$$6. B \Rightarrow D \quad \wedge E_2 2$$

$$7. C \quad \Rightarrow E 3, 5$$

$$8. D \quad \Rightarrow E 4, 6$$

$$9. C \wedge D \quad \wedge I 7, 8$$

$$10. ((A \Rightarrow C) \wedge (B \Rightarrow D)) \Rightarrow (C \wedge D) \quad \Rightarrow I 2-9$$

$$11. (A \wedge B) \Rightarrow ((A \Rightarrow C) \wedge (B \Rightarrow D)) \Rightarrow (C \wedge D) \quad \Rightarrow I 1-10$$

$$\begin{array}{c}
 \frac{[A \wedge B]^1}{A} \wedge E_1 \quad \frac{[(A \Rightarrow C) \wedge (B \Rightarrow D)]^2}{A \Rightarrow C} \wedge E_1 \quad \Rightarrow E \\
 \hline
 C \\
 \\
 \frac{[A \wedge B]^1}{B} \wedge E_2 \quad \frac{[(A \Rightarrow C) \wedge (B \Rightarrow D)]^2}{B \Rightarrow D} \wedge E_2 \quad \Rightarrow E \\
 \hline
 D \\
 \\
 \hline
 C \wedge D \quad \wedge I \\
 \\
 \hline
 ((A \Rightarrow C) \wedge (B \Rightarrow D)) \Rightarrow (C \wedge D) \quad \Rightarrow I, 2 \\
 \\
 \hline
 (A \wedge B) \Rightarrow ((A \Rightarrow C) \wedge (B \Rightarrow D)) \Rightarrow (C \wedge D) \quad \Rightarrow I, 1
 \end{array}$$

★ RAČUN SEKVENATA ★

1. U računu sekvenata za iskaznu logiku dokazati da je formula $A \Rightarrow A$ teorema.

Rešenje:

$$\frac{\overline{A \vdash A}^{Ax}}{\vdash A \Rightarrow A} \Rightarrow R$$

2. U računu sekvenata dokazati da je formula $(A \Rightarrow B) \Rightarrow ((B \Rightarrow C) \Rightarrow (A \Rightarrow C))$ teorema.

Rešenje:

$$\frac{\Gamma \vdash A \quad B, \Delta \vdash C}{\Delta \vdash B, \Gamma, \Delta \vdash C} \Rightarrow L$$

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{}{B \vdash B} \text{Ax}}{B \vdash B} \text{Ax}}{B \vdash B} \text{Ax} \quad \frac{}{C \vdash C} \text{Ax}}{B \vdash C} \Rightarrow L \\
 \frac{\frac{}{B \Rightarrow C} \text{Ax} \quad \frac{}{B \vdash C} \text{Ax}}{B \Rightarrow C, B \vdash C} P_L \\
 \frac{\frac{}{A \vdash A} \text{Ax} \quad \frac{}{B, B \Rightarrow C \vdash C} \Rightarrow L}{A \Rightarrow B, A, B \Rightarrow C \vdash C} \Rightarrow L \\
 \frac{}{A \Rightarrow B, A, B \Rightarrow C \vdash C} P_L \\
 \frac{}{A, B \Rightarrow C, A \Rightarrow B \vdash C} \Rightarrow R \\
 \frac{}{B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow R \\
 \frac{}{A \Rightarrow B \vdash (B \Rightarrow C) \Rightarrow (A \Rightarrow C)} \Rightarrow R \\
 \frac{}{\vdash (A \Rightarrow B) \Rightarrow ((B \Rightarrow C) \Rightarrow (A \Rightarrow C))} \Rightarrow R
 \end{array}$$

3. U računu sekvenata dokazati da je formula $(A \vee B) \Rightarrow (B \vee A)$ teorema.

Rešenje:

$$\begin{array}{c}
 \frac{}{A \vdash A} \text{Ax} \quad \frac{}{B \vdash B} \text{Ax} \\
 \frac{}{A \vdash B \vee A} \vee R \quad \frac{}{B \vdash B \vee A} \vee R \\
 \hline
 A \vee B \vdash B \vee A \quad \vee L \\
 \hline
 \vdash (A \vee B) \Rightarrow (B \vee A) \quad \Rightarrow R
 \end{array}$$

4. U računu sekvenata dokazati da je formula $(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$ teorema.

Rešenje:

$$\begin{array}{c}
 \frac{}{B \vdash B} \text{Ax} \\
 \frac{}{A \vdash A} \text{Ax} \quad \frac{}{B \vdash B} \neg L \\
 \hline
 A \vdash A \quad B \vdash B \quad \Rightarrow L \\
 \hline
 A \Rightarrow B, A, \neg B \vdash \quad P_L \\
 \hline
 A, \neg B, A \Rightarrow B \vdash \quad \neg R \\
 \hline
 \neg B, A \Rightarrow B \vdash \neg A \quad \Rightarrow R \\
 \hline
 A \Rightarrow B \vdash \neg B \Rightarrow \neg A \quad \Rightarrow R \\
 \hline
 \vdash (A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)
 \end{array}$$

5. Dokazati da je formula $A \vee \neg A$ teorema u računu sekvenata za klasičnu iskaznu logiku.

Rešenje:

$$\frac{\vdash A}{\vdash A \vee \neg A} \vee R$$

ili

$$\frac{\vdash \neg A}{\vdash A \vee \neg A} \vee R$$

X

$$\begin{array}{l} \frac{}{A \vdash A} Ax \\ \hline \vdash (\neg A), A \quad \neg R \\ \hline \vdash (A \vee \neg A), A \quad \vee R \\ \hline \vdash A \vee \neg A, \boxed{A \vee \neg A} \quad \vee R \\ \hline \vdash A \vee \neg A \quad CR \end{array}$$

6. U računu sekvenata dokazati da je formula $\vdash ((A \vee B) \wedge \neg A) \Rightarrow B$ teorema.

Rešenje:

$$\begin{array}{c}
 \frac{\frac{A \vdash B \quad \overline{B \vdash B}^{Ax}}{A \vee B \vdash B} \vee L}{(A \wedge B) \wedge \neg A \vdash B} \wedge L \\
 \hline
 \vdash ((A \vee B) \wedge \neg A) \Rightarrow B
 \end{array}$$

X

$$\begin{array}{c}
 \frac{\overline{A \vdash A}^{Ax}}{A \vdash B, A} \vee R \quad \frac{\overline{B \vdash B}^{Ax}}{B \vdash B, A} \vee R \\
 \hline
 \frac{A \vdash B, A \quad B \vdash B, A}{A \vee B \vdash B, A} \vee L \\
 \hline
 \frac{(A \vee B) \wedge \neg A \vdash B, A}{\neg A, (A \vee B) \wedge \neg A \vdash B} \wedge L \\
 \hline
 \frac{\neg A, (A \vee B) \wedge \neg A \vdash B}{(A \vee B) \wedge \neg A, (A \vee B) \wedge \neg A \vdash B} \neg L \\
 \hline
 \frac{(A \vee B) \wedge \neg A, (A \vee B) \wedge \neg A \vdash B}{(A \vee B) \wedge \neg A \vdash B} \wedge L \\
 \hline
 \frac{(A \vee B) \wedge \neg A \vdash B}{\vdash ((A \vee B) \wedge \neg A) \Rightarrow B} \Rightarrow R
 \end{array}$$