8 август 2025.

(1) 100 - гађања

$P(+) = 0{,}8$

X - број погођених тројки

$X : \mathcal{B}(100, 0{,}8)$

награда = погоди бар 70 тројки

$P(X \geq 70) = ?$

$n \cdot p = 100 \cdot 0.8 = 80$ $\qquad X^* = \dfrac{X-80}{4} : \mathcal{N}(0,1)$

$\sqrt{npq} = \sqrt{100 \cdot 0.8 \cdot 0.2} = 4$

$$P(X \geq 70) = 1 - P(X < 70) = 1 - P\left(\frac{X-80}{4} < \frac{70-80}{4}\right)$$

$$= 1 - P(X^* < -2.5) = 1 - \phi(-2.5) = 1 - (1 - \phi(2.5))$$

$$= \phi(2.5) = 0{,}9938.$$

(2) (за Е1 и МиР) $X : \begin{pmatrix} -1 & 0 & 1 \\ \theta/3 & \theta/3 & 1 - 2\theta/3 \end{pmatrix}$

n - величина узорка

$(X_1, \dots, X_n)$ узорак

$(x_1, \dots, x_n)$ реализовани узорак

K = број -1 у узорку, k = број -1 у реализованом узорку

M = број 0 у узорку, m = број 0 у реал. уз.

$n-K-M$ = број 1 у узорку, $n-k-m$ = број 1 у реал. уз.

$$L(x_1, \dots, x_n; \theta) = p(-1)^k \cdot p(0)^m \cdot p(1)^{n-k-m}$$

$$L(\theta) = \left(\frac{\theta}{3}\right)^k \cdot \left(\frac{\theta}{3}\right)^m \cdot \left(1 - \frac{2\theta}{3}\right)^{n-k-m}$$

$$L(\theta) = \frac{\theta^{k+m}}{3^{k+m}} \cdot \frac{(3-2\theta)^{n-k-m}}{3^{n-k-m}}$$

$$L(\theta) = \frac{\theta^{k+m} (3-2\theta)^{n-k-m}}{3^n}$$

$$\ln L(\theta) = (k+m)\cdot \ln\theta + (n-k-m)\cdot \ln(3-2\theta) - n\cdot \ln 3$$

$$\frac{\partial \ln L(\theta)}{\partial\theta} = \frac{k+m}{\theta} + \frac{(n-k-m)\cdot(-2)}{3-2\theta}$$

$$\frac{\partial \ln L(\theta)}{\partial\theta} = 0 \qquad \frac{k+m}{\theta} + \frac{-2n+2k+2m}{3-2\theta} = 0 \;/\cdot\, \theta(3-2\theta)$$

$$(k+m)(3-2\theta) + \theta(-2n+2k+2m) = 0$$

$$3k - \cancel{2k\theta} + 3m - \cancel{2m\theta} - 2n\theta + \cancel{2k\theta} + \cancel{2m\theta} = 0$$

$$3k + 3m - 2n\theta = 0 \Rightarrow \boxed{\theta = \frac{3k+3m}{2n}}$$

$$\hat{\theta} = \frac{3}{2n}(K+M) \qquad K : B\left(n, \frac{\theta}{3}\right) \qquad M : B\left(n, \frac{\theta}{3}\right)$$

центрираност:

$$E(\hat{\theta}) = E\left(\frac{3}{2n}(K+M)\right) = \frac{3}{2n}\left(E(K)+E(M)\right)$$

$$= \frac{3}{2n}\left(n\cdot\frac{\theta}{3} + n\cdot\frac{\theta}{3}\right) = \frac{\cancel{3}}{\cancel{2n}}\cdot\frac{\cancel{2n}\theta}{\cancel{3}} = \theta \Rightarrow \text{центрирана оцена}$$

постојаност: Оцена је центрирана, користимо неједнакост Чебишева: $P(|X-E(X)| \geq \varepsilon) \leq \frac{D(X)}{\varepsilon^2}$

$$P(|\hat{\theta}-\theta| \geq \varepsilon) = P(|\hat{\theta} - E(\hat{\theta})| \geq \varepsilon) \leq \frac{D(\hat{\theta})}{\varepsilon^2} =$$

$$= \frac{D\left(\frac{3}{2n}(K+M)\right)}{\varepsilon^2} = \frac{9}{4n^2}\,\frac{D(K+M)}{\varepsilon^2} = \frac{\cancel{9}\,3}{\cancel{4}\,2\,n^2\varepsilon^2}\left(\cancel{2}\cdot\cancel{n}\cdot\frac{\theta}{\cancel{3}}\left(1-\frac{\theta}{3}\right)\right)$$

$$= \frac{3\theta}{2n\varepsilon^2}\left(1-\frac{\theta}{3}\right)\cdot\frac{1}{\varepsilon^2} \to 0, \quad n\to\infty,$$

оцена јесте постојана

(3) $X_t = X \cdot \cos(t+Y)$ $\quad X : \mathcal{U}(0,1)$

X и Y независне $\quad Y : \mathcal{U}(0, 2\pi)$

испитати слабу стационарност

$X : \mathcal{U}(0,1) \Rightarrow E(X) = \frac{0+1}{2} = \frac{1}{2}$ $\quad E(X^2) = D(X) + E(X)^2$

$D(X) = \frac{(1-0)^2}{12} = \frac{1}{12}$ $\quad = \frac{1}{12} + \frac{1}{4} = \frac{1+3}{12}$

$E(X^2) = \frac{1}{3}$

$Y : \mathcal{U}(0, 2\pi) \quad \varphi_Y(y) = \begin{cases} \frac{1}{2\pi}, & y \in (0, 2\pi) \\ 0, & \text{иначе} \end{cases}$

1)

$m_X(t) = E(X_t) = E(X \cdot \cos(t+Y)) \overset{\text{нез.}}{=} E(X) \cdot \underset{\overset{\shortparallel}{S}}{E(\cos(t+Y))}$ (*)

$E(\cos(t+Y)) = \int_{-\infty}^{\infty} \cos(t+y) \cdot \varphi_Y(y)\, dy = \int_0^{2\pi} \cos(t+y) \cdot \frac{1}{2\pi}\, dy$

$= \frac{1}{2\pi} \cdot \sin(t+y) \Big|_0^{2\pi} = \frac{1}{2\pi} \cdot (\sin(t+2\pi) - \sin(t+0))$

$= \frac{1}{2\pi}(\sin t - \sin t) = 0$

$m_X(t) \overset{(*)}{=} \frac{1}{2} \cdot 0 = \frac{1}{2} = const$

2) $R_X(t,s) = E(X_t \cdot X_s) = E(X \cdot \cos(t+Y) \cdot X \cdot \cos(s+Y))$

$= E(X^2 \cdot \cos(t+Y)\cos(s+Y)) \overset{X \text{ и } Y \text{ нез.}}{=} E(X^2)\, E(\cos(t+Y)\cos(s+Y))$ (**)

$\cos\alpha \cdot \cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$

$E(\cos(t+Y) \cdot \cos(s+Y)) = E\left(\frac{1}{2}(\cos(t+s+2Y) + \cos(t-s))\right)$

$= \frac{1}{2}\left(E(\cos(t+s+2Y)) + \cos(t-s)\right)$ (***)

$E(\cos(t+s+2Y)) = \int_0^{2\pi} \cos(t+s+2y) \cdot \frac{1}{2\pi}\, dy$

$\shortparallel$

$u = t+s+2y$

$du = 2dy$

$$= \frac{1}{2\pi}\cdot\frac{1}{2}\cdot \sin(t+s+2y)\Big|_0^{2\pi} = \frac{1}{4\pi}\left(\sin(t+s+2\pi)-\sin(t+s)\right)$$

$$= \frac{1}{4\pi}\left(\sin(t+s)-\sin(t+s)\right) = 0$$

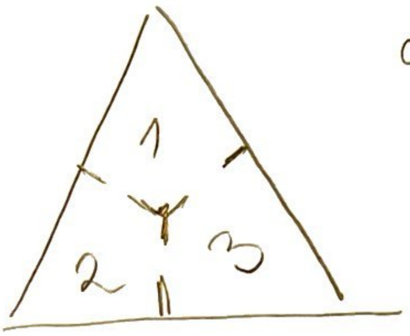
$$E(\cos(t+Y)\cos(s+Y)) \overset{(***)}{=} \frac{1}{2}(0+\cos(t-s)) = \frac{1}{2}\cos(t-s)$$

$$R_X(t,s) \overset{(**)}{=} \frac{1}{3}\cdot\frac{1}{2}\cos(t-s) = \frac{1}{6}\cos(t-s)$$

$R_X(t,s) \leftarrow$ ЗАВИСИ ОД РАЗЛИКЕ АРГУМЕНАТА

X_t јесте слабо стационаран јер је очекивање константно и корелациона функција зависи од разлике аргумената.

4)



а) $S=\{1,2,3\}$

$$P = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1/2 & 1/2 \\ 2 & 1/2 & 0 & 1/2 \\ 3 & 1/2 & 1/2 & 0 \end{array}$$

б) $$P^2 = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 2/4 & 1/4 & 1/4 \\ 1/4 & 2/4 & 1/4 \\ 1/4 & 1/4 & 2/4 \end{bmatrix} > 0$$

$P^2 > 0$ постоје финалне вероватноће

$p^* = [x \quad y \quad z]$

$$\left.\begin{array}{l} p^*\cdot P = p^* \\ x+y+z=1 \end{array}\right\}$$ решити систем

В-ТРАЖИМО

в) $p(3) = p(0)\cdot P^3 = [A \quad B \quad C]$

$p(0) = [1/3 \quad 1/3 \quad 1/3]$

израчунати P^3