Modul 8 - Zadaci

Tuesday, December 5, 2023 11:50 PM

8.7)

$m = 0{,}045\,kg$

$v_0 = 0\,m/s$

$v_1 = 25\,m/s$

$\Delta t = 2\,ms$

$F_{x_{sr}} = ?$

$J_x = \Delta p_x$ → impuls jednak promeni količine kretanja (momentuma)

$J_x = F_x \Delta t$

$\Delta p_x = m v_1 - m v_0 \quad (m \Delta v)$

$\Delta p_x = m(v_1 - v_0)$

$$F_x = \frac{m(v_1 - v_0)}{\Delta t}$$

$\boxed{F_x = 562{,}5\,N}$

$Q = mg$

$\boxed{Q = 0{,}441\,N}$

$\Rightarrow Q$ je mnogo manje od F_x tako da možemo reći da težina loptice nije imala značajan uticaj na promenu brzine

8.12)

$m = 0{,}145\,kg$

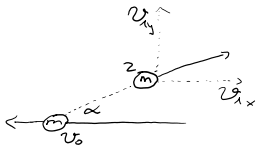
$v_0 = 50\,m/s$

$v_1 = 65\,m/s$

$\alpha = 30°$

$\Delta t = 1{,}75 \cdot 10^{-3}\,s$

$F_{x_{sr}} = ?\quad F_{y_{sr}} = ?$



$v_{1x} = v_1 \cos\alpha$

$v_{1y} = v_1 \sin\alpha$

$J_x = \Delta p_x, \quad J_y = \Delta p_y$

$J_x = F_{x_{sr}} \Delta t, \quad J_y = F_{y_{sr}} \Delta t$

$\Delta p_x = m(-v_{1x} - v_{0x})$

$\Delta p_x = m(-v_1 \cos\alpha - v_0)$

$\Delta p_y = m(-v_1 \sin\alpha - 0)$

$$F_{x_{sr}} = \frac{m(-v_1 \cos\alpha - v_0)}{\Delta t}$$

$\boxed{F_{x_{sr}} = -8807\,N}$

$$F_{y_{sr}} = \frac{m v_1 \sin\alpha}{\Delta t}$$

$\boxed{F_{y_{sr}} = 2692{,}85\,N}$

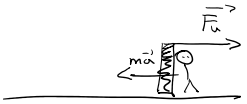
8.14) $F_{max} = -1{,}03 \cdot 10^8\,N/m^2$

$m = 70\,kg$

$S = 1{,}5\,cm^2 = 1{,}5 \cdot 10^{-4}\,m^2$

$\Delta t = 10 \cdot 10^{-3}\,s = 10^{-2}\,s$

$v_{max} = ?\quad v_{max} = v_0,\ v_1 = 0$ (zaustavio se)



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1) $J_x = F_u \Delta t$

$J_x = \Delta p_x$

2) $\Delta p_x = m(\cancel{v_1}^{\,0} - v_0)$

$F_u = F_{max} S$

1)+2) $\Rightarrow F_{max} S \Delta t = -m v_0 \Rightarrow v_0 = -\frac{F_{max} S \Delta t}{m}$ $\boxed{v_0 = 2{,}2\,m/s}$

$t_u = t_{max}$ ✓

1) + 2) $\Rightarrow$ $F_{max} \cdot \Delta t = -m v_0 \Rightarrow v_0 = -\frac{F_{max} \Delta t}{m}$ $\boxed{v_0 = 2{,}12\ m/s}$

8.19) $m = 6{,}5\ kg$

a) $m_r = 1{,}75\ kg$

$v_r = 2{,}50\ m/s$

b) $\Delta E_k = ?$

$m_0 = 6{,}5\ kg \quad v_0 = 0\ m/s$

$m_1 = 4{,}75\ kg \quad v_1 = -2{,}50\ m/s$

$P_{0x} = 0, \quad P_{1x} = m_1 v_1 + m_v v_v$

$P_{0x} = P_{1x}$

$m_1 v_1 + m_v v_v = 0$

$v_v = -\frac{m_1 v_1}{m_v}$

$\boxed{v_v = 6{,}7857\ m/s}$

$\Delta E_k = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_v v_v^2$

$\boxed{\Delta E_k = 55{,}13\ J}$

8.22) $m_A = 1750\ kg$

$v_A = 1{,}50\ m/s$

$m_B = 1150\ kg$

$v_B = -1{,}10\ m/s$

$v_{A2} = 0{,}250\ m/s$

$P_{0x} = m_A v_A + m_B v_B$

$P_{1x} = m_A v_{A2} + m_B v_{B2}$

$P_{0x} = P_{1x}$

$m_A v_A + m_B v_B = m_A v_{A2} + m_B v_{B2}$

$v_{B2} = \frac{m_A v_A + m_B v_B - m_A v_{A2}}{m_B}$ $\boxed{v_{B2} = 0{,}4038\ m/s}$

$\Delta E_k = E_2 - E_{k1} = \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2 - \frac{1}{2} m_A v_A^2 - \frac{1}{2} m_B v_B^2$

$= \frac{1}{2}\left(m_A (v_{A2}^2 - v_A^2) + m_B (v_{B2}^2 - v_B^2)\right)$

$\boxed{\Delta E_k = -2670{,}6261\ J}$

8.24) $m_A = 1{,}00\ kg$

$m_B = 3{,}00\ kg$

• nema trenja

$v_{A0} = v_{B0} = 0\ m/s$

$v_{B1} = 1{,}20\ m/s$

$v_{A1} = ? \quad \Pi_0 = ?$

$P_{0x} = 0$

$P_{1x} = m_A v_{A1} + m_B v_{B1}$

$P_{0x} = P_{1x}$

$m_A v_{A1} = -m_B v_{B1}$

$v_{A1} = -\frac{m_B v_{B1}}{m_A}$

$\boxed{v_{A1} = -3{,}60\ m/s}$

$E_0 = E_1$

$E_{k0} + \Pi_0 = E_{k1} + \Pi_1$

$\Pi_1 = 0$

$E_{k0} = 0$

$\Pi_0 = E_{k1}$

$\Pi_0 = \frac{1}{2} m_A v_{A1}^2 + \frac{1}{2} m_B v_{B1}^2$

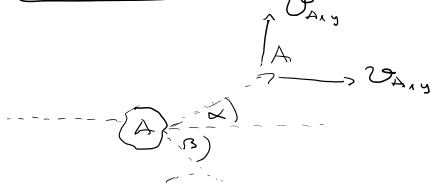
$\Pi_0 = \frac{1}{2}\left(m_A v_{A1}^2 + m_B v_{B1}^2\right)$

$\boxed{\Pi_0 = 8{,}64\ J}$

8.31) $m_A = m_B = m$

$v_{A0} = 40\ m/s$

$\alpha = 30° \quad \beta = 45°$



$v_{A_0} = 40\ m/s$

$\alpha = 30°$ $\beta = 45°$

$v_{B_0} = 0\ m/s$

$v_{A_1} = ?$, $v_{B_1} = ?$

$\sin\alpha = \frac{v_{A_1 y}}{v_{A_1}}$ $\sin\beta = \frac{v_{B_1 y}}{v_{B_1}}$

$\cos\alpha = \frac{v_{A_1 x}}{v_{A_1}}$ $\cos\beta = \frac{v_{B_1 x}}{v_{B_1}}$

$P_{0x} = m v_{A_{0x}} + m v_{B_{0x}}$

$P_{1x} = m v_{A_1 x} + m v_{B_1 x}$

$P_{1x} = m\cos\alpha\, v_{A_1} + m\cos\beta\, v_{B_1}$

$P_{0x} = P_{1x} \Rightarrow m v_{A_{0x}} + \cancel{m v_{B_{0x}}}^{\,=0} = m\cos\alpha\, v_{A_1} + m\cos\beta\, v_{B_1}$

$\Rightarrow \boxed{v_{A_{0x}} = \cos\alpha\, v_{A_1} + \cos\beta\, v_{B_1}}$

$P_{0y} = m v_{A_{0y}} + m v_{B_{0y}}$

$P_{1y} = m\sin\alpha\, v_{A_1} + m\sin\beta\, v_{A_1}$

$P_{0y} = P_{1y} \Rightarrow \cancel{m v_{A_{0y}}}^{\,=0} + \cancel{m v_{B_{0y}}}^{\,=0} = m\sin\alpha\, v_{A_1} + m\sin\beta\, v_{B_1}$

$\Rightarrow \boxed{\sin\alpha\, v_{A_1} - \sin\beta\, v_{B_1} = 0}$

$\sin\alpha\, v_{A_1} = \sin\beta\, v_{B_1}$

$v_{B_1} = \frac{\sin\alpha\, v_{A_1}}{\sin\beta}$ → $\boxed{v_{B_1} = 20{,}7\ m/s}$

$v_{A_{0x}} = \cos\alpha\, v_{A_1} + \cos\beta\, \frac{\sin\alpha\, v_{A_1}}{\sin\beta}$

$v_{A_{0x}} = v_{A_1}(\cos\alpha + \mathrm{ctg}\beta\sin\alpha)$

$v_{A_1} = \frac{v_{A_{0x}}}{\cos\alpha + \mathrm{ctg}\beta\sin\alpha}$ → $\boxed{v_{A_1} = 29{,}28\ m/s}$

b) $\Delta E_{k_A} = ?$

$\frac{E_{k_1}}{E_{k_0}} = \frac{\frac{1}{2}m(v_{A_1}^2 + v_{B_1}^2)}{\frac{1}{2}m(v_{A_0}^2 + \cancel{v_{B_0}^2}^{\,=0})} = \frac{v_{A_1}^2 + v_{B_1}^2}{v_{A_0}^2}$

$\frac{E_{k_1}}{E_{k_0}} = 0{,}80363$

$\frac{\Delta E_k}{E_{k_0}} = \frac{E_{k_1} - E_{k_0}}{E_{k_0}} = \frac{E_{k_1}}{E_{k_0}} - 1 = -0{,}196$

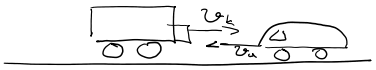
$\Rightarrow$ Kinetička energija se smanjila za 19%

8.36) $m_a = 1050\,kg$

$v_{a_0} = 15\,m/s = -15\,m/s$

$m_k = 6320\,kg$

$v_{k_0} = 10\,m/s$



$v_{a_1} = ?$, $v_{k_1} = ?$, $v_{a_1} = v_{k_1} = v_1$

$P_{0x} = m_a v_{a_0} + m_k v_{k_0}$

$P_{1x} = v_1 (m_a + m_k)$ $\rightsquigarrow$ $m_a v_{a_0} + m_k v_{k_0} = v_1 (m_a + m_k)$

$$v_1 = \frac{m_a v_{a_0} + m_k v_{k_0}}{m_a + m_k}$$

$\boxed{v_1 = 6{,}438\,m/s}$

b) $v_1 = 0$

$v_{a_0} = -15\,m/s$ $v_{k_0}' = ?$

$$\frac{m_a v_{a_0} + m_k v_{k_0}'}{m_a + m_k} = 0 \quad \Rightarrow \quad m_a v_{a_0} = -m_k v_{k_0}'$$

$$v_{k_0}' = -\frac{m_a v_{a_0}}{m_k}$$

$\boxed{v_{k_0}' = 2{,}492\,m/s}$

c) 1° $E_{k_0} = \frac{1}{2}(m_a v_{a_0}^2 + m_k v_{k_0}^2) = 444125\,J$

$E_{k_1} = \frac{1}{2} v_1^2 (m_a + m_k) = 156880$

$\boxed{\Delta E_k = 287245\,J}$

2° $E_{k_0}' = \frac{1}{2}(m_a v_{a_0}^2 + m_k v_{k_0}'^2) = 138369\,J$

$E_{k_1}' = 0$

$\boxed{\Delta E_k' = 138369\,J}$

$\boxed{1° > 2°}$

8.43) $m_z = 12{,}0\,g = 12 \cdot 10^{-3}\,kg$

$v_0 = 380\,m/s$

$m_k = 6{,}00\,kg$

$l = 70{,}0\,cm = 0{,}7\,m$

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a) $\Delta l = ?$

$P_{0x} = m_z v_0 + m_k v_{k_0}$

$P_{1x} = m_z v_1 + m_k v_1 = v_1 (m_z + m_k)$

$P_{0x} = P_{1x} \Rightarrow m_z v_0 + \cancel{m_k v_{k_0}^{\rightarrow 0}} = v_1 (m_z + m_k)$

$p_{1x} = m_2 v_1 + m_k v_1 = v_1 (m_2 + m_k)$

$p_{0x} = p_{1x} \Rightarrow m_2 v_0 + \cancel{m_k v_{k0}}^{=0} = v_1 (m_2 + m_k)$

$$v_1 = \frac{m_2 v_0}{m_2 + m_k} \qquad \boxed{v_1 = 0{,}75843 \, m/s}$$

$$\frac{1}{2} m_2 v_1^2 = m_k g l \qquad l = \frac{m_2 v_1^2}{2 m_k g} = \frac{v_1^2}{2g} \qquad \boxed{l = 0{,}02893 \, m}$$

b) $E_{k_{02}} = \frac{1}{2} m_2 v_0^2$ $\boxed{E_{k_{02}} = 866{,}4 \, J}$

c) $E_{k_1} = \frac{1}{2} (m_2 + m_k) v_1^2$ $\boxed{E_{k_1} = 1{,}7291 \, J}$

8.48)

$m_A = 10 \cdot 10^{-3} \, kg$

$v_{A_0} = -0{,}4 \, m/s$

$m_B = 30 \cdot 10^{-3} \, kg$

$v_{B_0} = 0{,}2 \, m/s$

a) $v_1 = ?$

$p_{0x} = m_A v_{A0} + m_B v_{B0}$

$p_{1x} = m_A v_{A_1} + m_B v_{B_1}$

$p_{0x} = p_{1x} \Rightarrow m_A v_{A_0} + m_B v_{B0} = m_A v_{A_1} + m_B v_{B_1}$

$v_{B_1} - v_{A_1} = -(v_{A_0} - v_{B0})$

?

$2 \cdot 10^{-3} = 10 \cdot 10^{-3} v_{A_1} + 30 \cdot 10^{-3} v_{B_1} \quad | \cdot 10^3$

$v_{B_1} - v_{A_1} = -(-0{,}4 - 0{,}2) = 0{,}6$

$\boxed{v_{B_1} = v_{A_1} + 0{,}6}$

$0{,}04 \cdot 10^3 = 10 v_{A_1} + 30 v_{B_1} \quad | :10$

$4 = v_{A_1} + 3 v_{B_1}$

?

8.68) $m = 2400 \, kg$

$3 m_A = m_B$

$m = m_A + m_B = 4 m_A$

$\boxed{m_A = \frac{m}{4} = 600 \qquad m_B = 1800 \, kg}$

$h = ?$

$v_{1_0} = v_{2_0} = v_1$

$a_{y_{1/2}} = -g$

$a_{x_{1/2}} = 0$

$l_1 = 318 \, m$

$\dot{x}_A = v_{A1}$

$\frac{dx_A}{dt} = v_{A1}$ $\quad dx_A = v_{A1} \, dt$

$\boxed{x_A = v_{A1} t}$

$\dot{x}_B = v_{B1}$

$\frac{dx_B}{dt} = v_{B1}$ $\quad \boxed{x_B = v_{B1} t}$

$p_{0x} = m_A v_0 + m_B v_0 = 0 \quad | v_0 = 0$

$p_{1x} = m_A v_{A1} + m_B v_{B1}$

$m_A v_{A1} + m_B v_{B1} = 0$

$m_A v_{A1} = -m_B v_{B1}$

$\frac{x_A}{x_B} = \frac{v_{A1} t}{v_{B1} t}$

$\frac{x_A}{x_B} = \frac{-3 v_{B1}}{v_{B1}}$

$$m_A v_{A1} + m_B v_{B1} = 0$$

$$m_A v_{A1} = -3 m_A v_{B1}$$

$$\boxed{v_{A1} = -3 v_{B1}}$$

$$\frac{X_A}{X_B} = \frac{-3 v_{B1}}{v_{B1}}$$

$$X_A = -3 X_B$$

$$X_B = -\frac{X_A}{3}$$

$$\boxed{X_B = -1{,}06\,m}$$

8.86) $m_A = m_B = m$, R

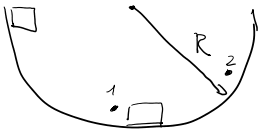
$$v_{A0} = v_{B0} = 0$$

$$P_1 = m v_{A1} + m v_{B1}$$

$$P_2 = 2m v_2 \quad (v_2 = v_{A2} = v_{B2})$$

$$m v_{A1} = 2m v_2$$

$$v_{A1} = 2 v_2 \Rightarrow \boxed{v_2 = \frac{v_{A1}}{2}}$$



$$\frac{1}{2} m v_{A1}^2 = m g R$$

$$v_{A1}^2 = 2gR$$

$$v_{A1} = \sqrt{2gR}$$

$$v_2 = \frac{\sqrt{2gR}}{2} \quad \boxed{v_2 = \sqrt{\frac{gR}{2}}}$$

$$\frac{1}{2}(2m) v_2^2 = 2mgh$$

$$\frac{gR}{2} = 2gh \quad \boxed{h = \frac{R}{4}}$$