Modul 4 - Zadaci

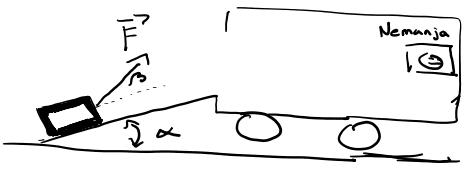
Sunday, November 19, 2023 10:48 PM

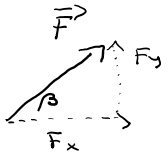
4.4) $\alpha = 20°$

$\beta = 30°$

a) $\vec{F} = ?$, $F_x = 60N$

b) $F_y = ?$





a) $\cos\beta = \frac{F_x}{\vec{F}}$ $\quad \vec{F} = \frac{F_x}{\cos\beta}$

$\cos\beta = 0{,}866$ $\quad \boxed{\vec{F} = 69{,}28N}$

b) $\sin\beta = \frac{F_y}{\vec{F}}$ $\quad F_y = \sin\beta\, \vec{F}$ $\quad \boxed{F_y = 34{,}64N}$

4.10) $F_x = 80N$

$x(t_1) = 11m$

$t_1 = 5s$

a) $F_x = ma$ $\quad (t=0 \Rightarrow \dot{x}(t)=0)$

$\ddot{x} = a$ $\quad \frac{d\dot{x}}{dt} = a$ $\quad d\dot{x} = a\,dt \,/\int$ $\quad \dot{x} = at + C_1$ $(C_1 = 0)$

$\dot{x}(t) = at$ $\quad \frac{dx}{dt} = at$ $\quad dx = at\,dt \,/\int$ $\quad x = \frac{1}{2}at^2 + C_2$ $(C_2 = 0)$

$x(t) = \frac{1}{2}at^2$

$x(t_1) = \frac{1}{2}a t_1^2$ $\quad 11 = \frac{1}{2}a \cdot 25$ $\quad 22 = a \cdot 25$ $\quad a = \frac{22}{25}$ $\quad \boxed{a = 0{,}88 m/s^2}$

$F_x = ma$ $\quad m = \frac{F_x}{a}$ $\quad \boxed{m = 90{,}9kg}$

b) $t_2 = 5s$ $\quad (t_1 \to t_2 = 5s)$

$\dot{x}(t_1) = at_1 = \boxed{4{,}4 m/s}$ $\Rightarrow$ posle t_1, a je const a a je 0 jer više niko ne gura kutiju

$x(t_2) = ?$

$\ddot{x} = a$ $\quad \frac{d\dot{x}}{dt} = a$ $\quad d\dot{x} = a\,dt \,/\int$ $\quad \dot{x} = at + C_1$ $(C_1 = \dot{x}(t_1))$

$\ddot{x} = a \quad \frac{d\dot{x}}{dt} = a \quad d\dot{x} = a\,dt \;/\int \quad \dot{x} = at + C_1 \quad (C_1 = \dot{x}(t_1))$

$\boxed{\dot{x}(t) = at + \dot{x}(t_1)}$

$\frac{dx}{dt} = at + \dot{x}(t_1) \quad dx = dt\,(at + \dot{x}(t_1)) \;/\int$

$x = \frac{1}{2}at^2 + \dot{x}(t_1)t + C_2 \quad (C_2 = 0)$

$\boxed{x(t) = \frac{1}{2}a\cancel{t^2}^{0} + \dot{x}(t_1)t}$

$x(t) = \dot{x}(t_1)\,t$

$x(t_2) = 4{,}4 \cdot 5 \quad \boxed{\dot{x}(t_2) = 22\,m}$

4.14) $m = 2{,}75\,kg$

a) $F_{max} = ? \quad F_{max}(t) = ? \quad t = 0 \quad t_1 = 2$

$F = ma \quad a = \frac{v(t - t_1)}{t_1 - t} = \frac{8\,m/s}{2\,s} = 4\,m/s^2$

$\boxed{F_{max} = 11\,N} \quad \boxed{a = 4\,m/s^2}$

b) $t_1 = 2\,s, \; t_2 = 6\,s \quad \boxed{\Delta t_{1-2} = 4\,s}$

od $t_1 \to t_2$, a je 0 pa $F = \cancel{ma}^{0}$

c) $t_3 = 8{,}5\,s \quad F(t_3) = ? \quad t_4 = 6\,s \quad t_5 = 10\,s \quad \boxed{\Delta t_{4-5} = 4\,s}$

$F(t_3) = m\,a(t_3) \quad v(t_4) = 8\,m/s, \quad v(t_5) = 12\,m/s$

$\boxed{F(t_3) = 2{,}75\,N} \quad \boxed{a_{sr} = \frac{4\,m/s}{4\,s} = 1\,m/s^2}$

4.15) $m = 8\,kg \quad F = A + Bt^2$

$t = 0, \; F = 100\,N$

$t_1 = 2\,s, \; F(t_1) = 150\,N$

a) $100 = A$

$150 = 100 + 4B$

$4B = 50 \quad \boxed{B = 12{,}5}$

b) $F_r = F - mg$

a) $t = 0$

$F_r = 100 - 78{,}48$

$\boxed{F_r = 21{,}52\,N}$

b) $t_3 = 3\,s$

$F(t_3) = 212{,}5\,N$

$F_r = F(t_3) - mg$

$\boxed{F_r = 134{,}02\,N}$

c) $a = \frac{F(t_3)}{m}$

$\boxed{a = 26{,}56\,m/s^2}$

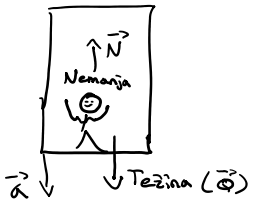
$F_r = 134,02\,N$

$F_r = ma$

$a(3s) = \frac{F_r}{m} = 16,75\,m/s^2$

4.24) $\vec{Q} = 650N$

$\vec{N} = 620N$



$\vec{Q} = m\vec{g}$

$m = \frac{\vec{Q}}{\vec{g}} = 66,25\,kg$

$\vec{F_y} = \vec{Q} + \vec{N} \,/\cdot \vec{j}$ $\quad F_y = ma$

$F_y = Q - N$

$a = \frac{F_y}{m} = 0,452\,m/s^2$

→ lift se spušta

4.27) A, B m_A, m_B

4.37) $\vec{F_1}, \vec{F_2}$

a) $F_{1y} + F_{2y} + F_{3y} = 0$

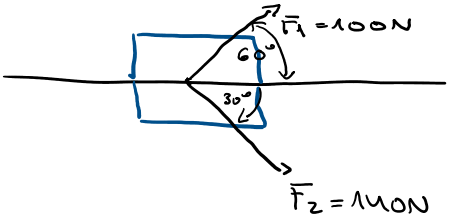
$F_{3y} = -(F_{1y} + F_{2y})$

$F_{1y} = 86,6N$ $\quad (F_1 \sin(60°))$

$F_{2y} = F_2 \sin(-30°) = -70N$

$F_{3y} = -(-70 + 86,6) = -16,6N$

$F_{3y} = -16,6N$



b) $\vec{F} = m\vec{a} = \vec{F_{1x}} + \vec{F_{2x}} \,/\cdot \vec{i}$ $\quad ma = F_{1x} + F_{2x}$

$F_{1x} = 100 \cdot \cos(60°) = 50N$

$F_{2x} = 140 \cdot \cos(-30°) = 121,24\,N$

$ma = 171,24$

$F_{2x} = 140 \cdot \cos(-30°) = 121{,}24\ N$

$ma = 171{,}24$

$\boxed{m = 85{,}62\ kg}$ $\qquad$ $\boxed{Q = mg = 839{,}93\ N}$

4.38) $\dot{x} = 1{,}5\ m/s = const.$

$l = 500\ m$

$m_t = 3{,}6 \cdot 10^7\ kg$

$F_x = 8 \cdot 10^4\ N$

$\boxed{\dot{x} < 0{,}2\ m/s}$

$\vec{F} = m\vec{a} = \vec{F_x} + m\vec{g} \;/\; \vec{i}$

$-ma = F_x$

$a = -\frac{F_x}{m} = -0{,}0022\ m/s^2$

$\boxed{a = -0{,}0022\ m/s^2}$

$\dot{x}(0) = 1{,}5\ m/s$

$\ddot{x} = a$

$\frac{d\dot{x}}{dt} = a \quad d\dot{x} = a\,dt \;/\int \quad \dot{x} = at + C_1 \quad (C_1 = \dot{x}(0))$

$\dot{x}(t) = at + \dot{x}(0)$

$\frac{dx}{dt} = at + \dot{x}(0) \quad dx = (at + \dot{x}(0))\,dt \;/\int$

$x = \frac{1}{2}at^2 + \dot{x}(0)t + C_2 \quad (C_2 = 0)$

$\boxed{x(t) = \frac{1}{2}at^2 + \dot{x}(0)t}$

a) $\dot{x}(t) = 0$ pps.

$0 = at + \dot{x}(0) \quad \Rightarrow \boxed{\text{dolće do sudara!}}$

$at = -\dot{x}(0)$

$\boxed{t = \frac{\dot{x}(0)}{a} = 681\ s}$ $\qquad$ $\boxed{x(t) = 511{,}36\ m}$

b) $\dot{x}(t_1) = 0{,}2\ m/s$ $\qquad$ $\boxed{x(t_1) = 502{,}27\ m}$

$0{,}2 = at + \dot{x}(0)$

$at = 0{,}2 - \dot{x}(0)$

$t = \frac{0{,}2 - \dot{x}(0)}{a}$

$\boxed{t = 590{,}9\ s}$

$\Rightarrow$ neće biti bezbedan (biće zvezdiran)

4.51) $m = 75\ kg$

$h = 3{,}1\ m$

$h_s = 0{,}6\ m$

t_1 - udar o tlo

t_2 - kraj savijanja

a) $\dot{x}(t_1) = ?$

$F = m\vec{a} = m\vec{g} \;/\; \vec{i}$

$ma = -mg$

$\boxed{a = -g}$

$\ddot{y} = -g$

$\frac{d\dot{y}}{dt} = -g \quad d\dot{y} = -g\,dt \;/\int \quad \dot{y} = -gt + C_1 \quad (C_1 = 0)$

$\boxed{\dot{y}(t) = -gt}$ $\qquad$ $\frac{dy}{dt} = -gt \quad dy = -gt\,dt \;/\int$

$y(t) = -\frac{1}{2}gt^2 + C_2 \quad (C_2 = h)$

$y(t) = -\frac{1}{2}gt^2 + h \quad y(t) = 0$

$\Rightarrow h = +\frac{1}{2}gt^2 \quad 2h = gt^2 \quad t^2 = \frac{2h}{g} \quad \boxed{t = \sqrt{\frac{2h}{g}}}$

$\boxed{t = 0{,}79\ s} = t_1$

$t = 0{,}79\,s = t_1$

$\dot{y}(t_1) = -gt = -7{,}74\,m/s$

$\dot{y}(t_1) = -7{,}74\,m/s$

b) $a = \text{const}$

$\ddot{y}' = a' \quad \frac{d\dot{y}'}{dt} = a' \quad d\dot{y}' = a'\,dt \;/\int \quad \dot{y}' = a't + C_1 \;(\dot{y}(t_1))$

$\dot{y}'(t) = a't + \dot{y}(t_1) \quad \frac{dy}{dt} = a't + \dot{y}(t_1) \quad dy' = (a't + \dot{y}(t_1))\,dt \;/\int$

$y'(t) = \frac{1}{2}a't^2 + t\dot{y}(t_1)$

$\dot{y}'(t_2) = 0 \quad 0 = a't_2 + \dot{y}(t_1) \quad a't_2 = -\dot{y}(t_1) \quad \boxed{t_2 = \frac{-\dot{y}(t_1)}{a'}}$

$y'(t_2) = \frac{1}{2}a' \cdot \frac{\dot{y}(t_1)^2}{a'^2} + \frac{-\dot{y}(t_1)}{a'} \cdot \dot{y}(t_1) \;/\cdot a' \cdot 2$

$2a'\,y'(t_2) = \dot{y}(t_1)^2 - 2\dot{y}(t_1)^2$

$2a'\,y'(t_2) = -\dot{y}(t_1)^2$

$a' = \frac{-\dot{y}(t_1)^2}{2y'(t_2)} \qquad \boxed{a' = 50{,}57\,m/s^2}$

c) $\vec{F} = m\vec{a} = m\vec{g} + m\vec{a} \;/\vec{j}$

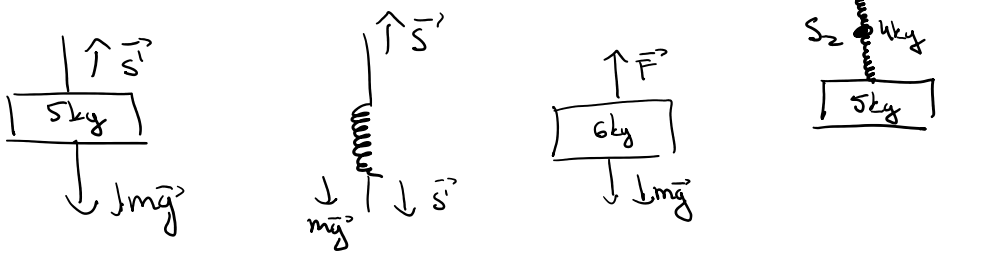
$F = mg + ma' \quad F = m(a' + g) \quad N = -F \quad N = -m(a' + g)$

$\boxed{N = -4{,}53 \cdot 10^3\,N}$

4.54) $m_k = 4\,kg$

$F = 200\,N$

a)



b) $a = ? \qquad m_s = (6 + 4 + 5)\,kg \qquad \boxed{m_s = 15\,kg}$

$\vec{F} = m\vec{a} = \vec{F}' + m_s\vec{g} \;/\vec{j} \qquad ma = F - mg \qquad \boxed{a = \frac{F - mg}{m_s} = 3{,}53\,m/s^2}$

c) $\vec{S}=?$ $\quad \vec{F}_y = m\vec{a} = \vec{F}+m\vec{g}+\vec{S} \,/\vec{j}$

$ma = F - mg - S$

$S = F - mg - ma$ $\quad \boxed{S = 119{,}96\,N}$

d) $\vec{F} = m\vec{a} = \vec{F}' + m\vec{g} + S_2 + \frac{mL}{2}\vec{g} \,/\vec{j}$

$ma = F - mg - S_2 - \frac{mL}{2}g$

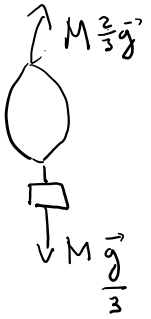
$S_2 = F - g(m + \frac{mL}{2}) - ma$

$\boxed{S_2 = 93{,}28\,N}$

4.56) M – ukupna masa

$a_s = g/3$

a)



b) $F_{gu2} = \frac{2}{3}Mg$

c) $a' = \frac{1}{2}g$

$F_y = m\vec{a} = \vec{F}_{gu2} + m\vec{g} \,/\vec{j}$

$m\cdot\frac{1}{2}g = F_{gu2} - mg$ $\quad m$ – nova masa

$mg = 2F_{gu2} - 2mg$

$3mg = 2F_{gu2}$

$3mg = 2\cdot\frac{3}{3}Mg$ $\quad \Rightarrow$ treba rasteretit $\frac{5}{9}M$

$3mg = \frac{4}{3}M$ $\quad \boxed{m = \frac{4}{9}M}$

4.58) $Q = 2{,}75\cdot 10^3\,kg$

$\vec{r}(t) = (0{,}02\,m/s^3)\,t^3\vec{i} + (2{,}2\,m/s)\,t\vec{j} - (0{,}06\,m/s^2)\,t^2\vec{k}$

$t = 5\,s$

$\dot{\vec{r}}(t) = 3(0{,}02)\,t^2\vec{i} + \cancel{2{,}2\vec{j}^{\,0}} - 0{,}12\,m/s^2\,t\vec{k}$

$\ddot{\vec{r}}(t) = 6(0{,}02)\,t\vec{i} - 0{,}12\vec{k}$

$\ddot{\vec{r}} = a \quad a(t) = 0{,}12\,t\vec{i} - 0{,}12\vec{k}$ $\quad (m/s^3,\ m/s^2)$

$\boxed{a(5) = [0{,}6\,m/s^3\,\vec{i} - 0{,}12\,m/s^2\,\vec{k}]}$

$\vec{F} = ma$

$m = \frac{Q}{g} = 280{,}32\,kg$

$\vec{F} = ma = \boxed{1684\,N\vec{i} - 3363\,N\vec{k}}$

$r = a$ $\quad a(t) = 0{,}12\,t\,\frac{m}{s^3}\,\vec{i} - 0{,}12\,\frac{m}{s^2}\,\vec{k}$

$\boxed{a(5) = \boxed{0{,}6\,\frac{m}{s^3}\,\vec{i} - 0{,}12\,\frac{m}{s^2}\,\vec{k}}}$

$\vec{F} = ma = \boxed{1681{,}9\,N\,\vec{i} - 3363\,N\,\vec{k}}$

4.62) m - masa tela u $(0,0)$

$t = 0$, $\vec{F}$

$F_x = k_1 + k_2 y \quad F_y = k_3 t \quad k_1, k_2, k_3$ - const

$\vec{F}(t) = ?$, $\vec{v}(t)$

$F_y = a_y m = \boxed{a_y = \frac{k_3}{m} t}$

$\frac{d\dot{y}}{dt} = \frac{k_3 t}{m} \quad \dot{y}(t) = dt\left(\frac{k_3 t}{m}\right) \quad \boxed{\dot{y}(t) = \frac{k_3 t^2}{2m}}$

$\frac{dy}{dt} = \frac{k_3 t^2}{2m} \quad dy = dt\left(\frac{k_3 t^2}{2m}\right) / S \quad \boxed{y(t) = \frac{k_3 t^3}{6m}}$

$a_x = \frac{k_1 + k_2 y}{m}$

$\frac{d\dot{x}}{dt} = \frac{k_1 + k_2 y}{m} = \frac{k_1 + k_2 k_3 t^3}{6m^2}$

$d\dot{x} = dt\left(\frac{tk_1 + k_2 k_3 t^3}{6m^2}\right) \quad \dot{x}(t) = \frac{k_1 t}{m} + \frac{k_2 k_3 t^4}{24 m^2}$

$\frac{dx}{dt} = \dot{x}(t) \quad \boxed{x(t) = \frac{k_1 t^2}{2m} + \frac{k_2 k_3 t^5}{120 m^2}}$

$\vec{F}(t) = \boxed{\left(\frac{k_1 t^2}{2m} + \frac{k_2 k_3 t^5}{120 m^2}\right)\vec{i} + \left(\frac{k_3 t^3}{6m}\right)\vec{j}}$

$\vec{v}(t) = \boxed{\left(\frac{k_1 t}{m} + \frac{k_2 k_3 t^4}{24 m^2}\right)\vec{i} + \left(\frac{k_3 t^2}{2m}\right)\vec{j}}$