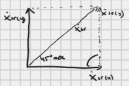
3.3) $\vec{r} = [4cm + (2{,}5cm/s^2)t^2]\vec{i} + (5{,}0cm/s)t\,\vec{j}$



a) $\dot{x}_{sr}(0-2s) = ?$, $\Delta t = 2s$

b) $\dot{x}(0) = ?$, $\dot{x}(1) = ?$, $\dot{x}(2) = ?$

a) $\vec{r}(2) = (4cm + 10cm)\vec{i} + 10cm\,\vec{j}$ $\vec{r}(0) = 4cm\,\vec{i}$

$\vec{r}(2) = 14cm\,\vec{i} + 10cm\,\vec{j}$

$\sin\alpha = \frac{nasp.}{hip.}$

$\cos\alpha = \frac{nal.}{hip.}$

$tg\alpha = \frac{nasp.}{nal.}$

$\dot{x}_{sr} = \frac{\Delta\vec{r}}{\Delta t}$

$\dot{x}_{sr(x)} = \frac{\Delta x}{\Delta t} = \frac{14cm - 4cm}{2s} = 5cm/s \Rightarrow \boxed{\dot{x}_{sr(x)} = 5cm/s}$

$\dot{x}_{sr(y)} = \frac{\Delta y}{\Delta t} = \frac{10cm}{2s} = 5cm/s \Rightarrow \boxed{\dot{x}_{sr(y)} = 5cm/s}$ $\boxed{\alpha = 45°}$

$\dot{x}_{sr} = \sqrt{\dot{x}_{sr(x)}^2 + \dot{x}_{sr(y)}^2} = \sqrt{50\,cm^2/s^2} = 7{,}07cm/s \Rightarrow \boxed{\dot{x}_{sr} = 7{,}07cm/s}$

b) $\dot{x} = \frac{d\vec{r}}{dt} = (5cm \cdot t)\vec{i} + (5cm/s)\vec{j}$

$\dot{x}_x(0) = 0$, $\dot{x}_y(0) = 5cm/s$, $\boxed{\dot{x} = 5cm/s}$, $\alpha = 90°$ (samo po y)

$\dot{x}_x(1) = 5cm/s$, $\dot{x}_y(1) = 5cm/s$, $\dot{x} = 7{,}1cm/s$, $\alpha = 45°$

$\dot{x}_x(2) = 10cm/s$, $\dot{x}_y(2) = 5cm/s$, $\dot{x} = 11{,}18cm/s$, $tg\alpha = \frac{5}{10} = \frac{1}{2} \Rightarrow x = arctg(\frac{1}{2}) = 26{,}6°$ $\boxed{\alpha = 26{,}6°}$

3.15) sto $\xrightarrow{z}$ loptica $= D = x_z(t)$

$\dot{x}_z(0) = \dot{x}_x(0) = \dot{x}(0)$

sto $\xrightarrow{x}$ loptica $= 2{,}76D = x_x(t)$

$\ddot{y}_x = ?$

$\ddot{y}_z = -g$ $\frac{d\dot{y}}{dt} = -g$ $d\dot{y} = -g\,dt$ $\dot{y} = -gt + C_1$ $(C_1 = \dot{x}(0)) \Rightarrow \boxed{\dot{y}(t) = -gt + \dot{x}(0)}$

$\frac{dy}{dt} = -gt + \dot{x}(0)$ $dy = dt(-gt + \dot{x}(0)) /\int$ $y = -\frac{1}{2}gt^2 + \dot{x}(0)t + C_2$ $(C_2 = h)$

$\boxed{y(t) = -\frac{1}{2}gt^2 + \dot{x}(0)t + h}$

$\frac{dx}{dt} = \dot{x}(0)$ $dx = \dot{x}(0)dt /\int$ $x = \dot{x}(0)t + C_1$ $(C_1 = 0)$ $x_z(t) = \dot{x}(0)t = D$

$\boxed{t = \frac{D}{\dot{x}(0)}}$

$y_z(t) = -\frac{1}{2}g\left(\frac{D^2}{\dot{x}(0)^2}\right) + \dot{x}(0) \cdot \frac{D}{\dot{x}(0)} + h = -\frac{1}{2}g\left(\frac{D^2}{\dot{x}(0)^2}\right) + D + h$

$\frac{d\dot{y}_x}{dt} = \ddot{y}_x$ $d\dot{y}_x = \ddot{y}_x dt /\int$ $\dot{y}_x = \ddot{y}_x t + C_1$ $(C_1 = 0)$

$\boxed{\dot{y}_x(t) = \ddot{y}_x t}$ $\frac{dy_x}{dt} = \ddot{y}_x t$ $dy_x = dt(\ddot{y}_x t)$ $y_x = -\frac{1}{2}\ddot{y}_x t^2 + C_2$ $(C_2 = h)$

$\boxed{y_x(t) = -\frac{1}{2}\ddot{y}_x t^2 + h}$

$\frac{dx_x}{dt} = \dot{x}(0)$ $dx_x = \dot{x}(0)dt /\int$ $x_x = \dot{x}(0)t + C_1$ $(C_1 = 0)$

$x_x(t) = \dot{x}(0)t$ $\boxed{t = \frac{x_x(t)}{\dot{x}(0)}}$

$y_x(t) = -\frac{1}{2}\ddot{y}_x\left(\frac{2{,}76^2D^2}{\dot{x}_x(0)^2}\right) + h$

$-\frac{1}{2}\ddot{y}_x\left(\frac{2{,}76^2D^2}{\dot{x}_x(0)^2}\right) + h = 0 \quad / \cdot 2\dot{x}(0)^2$

$-\ddot{y}_x(2{,}76D)^2 + 2h\dot{x}_x(0)^2 = 0$

$\dot{x}_x(0)^2\, 2h = \ddot{y}_x(2{,}76D)^2$

$\dot{x}_x(0)^2 = \frac{\ddot{y}_x(2{,}76D)^2}{2h} \;/\; \dot{x}_z(0)^2 = \frac{gD^2}{2h}$

$\frac{\ddot{y}_x(2{,}76D)^2}{2h} = \frac{gD^2}{2h}$

$\ddot{y}_x = \frac{g}{2{,}76^2}$

$\boxed{\ddot{y}_x = 1{,}28m/s^2}$

3.22) $\dot{x}(0) = 25m/s$

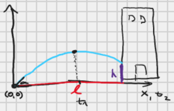
$l = 45m$

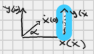
$t = 3s$

a) $\alpha = ?$

b) $\ddot{x}(t_1) = ?$, $v(t_1) = ?$

c) $h = ?$, $\dot{x}(t_2) = ?$





a) $\ddot{x}(0) = 0$, $\ddot{y}(0) = -g$

$\sin\alpha = \frac{\dot{x}(0)_y}{\dot{x}(0)}$, $\cos\alpha = \frac{\dot{x}(0)_x}{\dot{x}(0)}$ $\dot{x}(0)_x = \cos\alpha\,\dot{x}(0)$,

$\ddot{x}_x(t) = a_x$

$\frac{d\dot{x}_x(t)}{dt} = a_x$ $d\dot{x}_x(t) = a_x dt /\int$ $\dot{x}_x(t) = a_x t + C_1$ $(C_1 = \dot{x}(0)_x)$

$\dot{x}_x(t) = a_x t + \dot{x}(0)_x$

$\frac{dx_x}{dt} = a_x t + \dot{x}(0)_x$ $dx_x = (a_x t + \dot{x}(0)_x)dt /\int$ $x_x = \frac{1}{2}a_x t^2 + \dot{x}(0)_x t + C_2$ $(C_2 = 0)$

$\boxed{x_x(t) = \frac{1}{2}a_x t^2 + \dot{x}(0)_x t}$

$x_x(t) = \frac{1}{2}a_x t^2 + \cos\alpha\,\dot{x}(0)t \Rightarrow \cos\alpha\,\dot{x}(0)t = x_x(t) - \frac{a_x t^2}{2} = \frac{2x_x(t) - a_x t^2}{2}$

$x_x(t) = 45$

$\cos\alpha = \frac{2x_x(t) - a_x t^2}{2\dot{x}(0)t}$ ($a_x t^2 \to 0$)

$\cos\alpha = \frac{x_x(t)}{\dot{x}(0)t} = \frac{3}{5} \Rightarrow \boxed{\alpha = 53{,}1°}$

b) $\ddot{x}(t_1) = 0$, $\ddot{y}(t_1) = -g$, $\boxed{\dot{y}(t_1) = 0}$

$\dot{x}(t_1) = \dot{x}(0) \cdot \cos(53{,}1°)$

$\boxed{\dot{x}(t_1) = 15{,}01m/s^2}$

$v(t_1) = \sqrt{\dot{x}(t_1)^2 + \dot{y}_y(t_1)^2} = 15{,}01m/s^2$

$\boxed{v(t_1) = 15{,}01m/s^2}$

c) $\ddot{y}(t_2) = -g$

$\frac{d\dot{y}}{dt} = -g$ $d\dot{y} = -g\,dt /\int$ $\dot{y} = -gt + C_1$ $(C_1 = \dot{x}(0)_y)$

$\dot{y}(t) = -gt + \dot{x}(0)_y$

$\frac{dy}{dt} = -gt + \dot{x}(0)_y$ $dy = dt(-gt + \dot{x}(0)_y) /\int$

$y = -\frac{1}{2}gt^2 + \dot{x}(0)_y t + C_2$ $(C_2 = 0)$

$\boxed{y(t) = -\frac{1}{2}gt^2 + \dot{x}(0)_y t}$

$\boxed{y(t) = -\frac{1}{2}gt^2 + \sin\alpha\,\dot{x}(0)t}$

$\boxed{h = y(3) = 15{,}83m}$

$\dot{x}(t)_x = a_x t + \cos\alpha \cdot \dot{x}(0)$ ($a_x t \to 0$)

$\boxed{\dot{x}(t)_x = 15m/s}$

$\dot{x}(t)_y = -gt + \sin\alpha \cdot \dot{x}(0)$

$\boxed{\dot{x}(t)_y = -9{,}43m/s}$

$\dot{x}(t) = \sqrt{\dot{x}(t)_x^2 + \dot{x}(t)_y^2}$

$\boxed{\dot{x}(t) = 17{,}71m/s}$

3.26) $l = 3{,}4m$

$\boxed{a_k = \frac{v^2}{R}}$ $\boxed{T = \frac{1}{obt/s}}$

$R = l = 3{,}4m$

$550\,obt/min = \frac{550}{60}\,obt/s = \boxed{9{,}166\,obt/s} \Rightarrow T = \frac{1}{9{,}166} = \boxed{0{,}109s}$

a) $v = ?$ $\boxed{v = \frac{2\pi R}{T}} \to$ brzina tačke u kružnici $\boxed{v = 195{,}98m/s}$

b) $a_k = ?$ $a_k = 11296\,m/s^2 = \boxed{1151g}$

3.27) $R = 350m$

$a_{rad} = 5{,}5g$ $a_{rad} = \frac{v^2}{R}$ $v^2 = a_{rad}R$ $v = \sqrt{a_{rad}R}$ $\boxed{v = 137{,}4m/s}$

$v = ?$

3.33) $v_{ji} = 0{,}4m/s$

$v_r = 0{,}5m/s$ (i)

$\vec{v}_{ji} = \vec{v}_i + \vec{v}_j$

$\vec{v}_j = \frac{0{,}4}{\sqrt{2}} = 0{,}28m/s$

$\vec{v}_i = -v_r + \frac{0{,}4}{\sqrt{2}} = -0{,}22m/s$

$\boxed{v = \sqrt{v_j^2 + v_i^2} = 0{,}356m/s}$

Modul 3 - Zadaci

Saturday, November 18, 2023 10:59 AM

3.49) $l = 50\,m = R$

$v_{max} = ?$

$a_{rad} = g$

$a_{rad} = \frac{v^2}{R}$ $\quad v^2 = a_{rad} \cdot R$ $\quad v = \sqrt{a_{rad} R}$

$\boxed{v = 22{,}1\,m/s}$

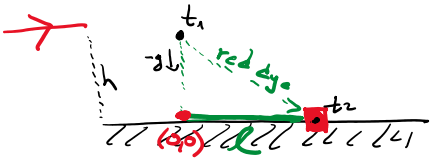
$$\boxed{R = \frac{v_0^2 \sin 2\alpha}{g}}$$

3.53) $h = 90\,m$

$\dot{x}(t) = 64\,m/s$

$g = 10\,m/s^2$

$l = ?$



$\dot{x}_r(t_1) = \dot{x}(t)$ $\quad \frac{dx_r}{dt} = \dot{x}(t_1)$ $\quad dx_r = \dot{x}(t_1)dt \,/\int$ $\quad x_r = \dot{x}(t_1)t + C_1$ $\quad (C_1 = 0)$

$\boxed{x_r(t) = \dot{x}(t_1)t}$

$\boxed{x_r(t_2) = l,\ x_r(t_1) = 0}$

$\dot{y}_r(t_1) = 0$

$\ddot{x}_r(t_1) = 0$

$\ddot{y}_r(t_1) = -g$

$x_r(t_2) = \dot{x}_r(t_1)t_2$

$\boxed{l = \dot{x}_r(t_1)t_2}$

$\frac{d\dot{y}}{dt} = -g$ $\quad d\dot{y} = -g\,dt \,/\int$ $\quad \dot{y} = -gt + C_1$ $\quad (C_1 = 0)$

$\dot{y}(t) = -gt$ $\quad \frac{dy}{dt} = -gt$ $\quad dy = -gt\,dt \,/\int$ $\quad y = -\frac{1}{2}gt^2 + C_2$ $\quad (C_2 = h)$

$\boxed{y(t) = -\frac{1}{2}gt^2 + h}$ $\quad y(t_2) = 0,\ y(t_1) = h$

$y(t_2) = -\frac{1}{2}gt_2^2 + h \,/^{-2}$ $\quad gt_2^2 = 2h$ $\quad \boxed{t_2 = \sqrt{\frac{2h}{g}} = 4{,}24\,s}$

$\boxed{l = 271{,}36\,m}$

3.60) $2R = D$

$h = 2D$

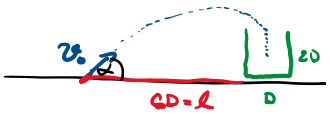
$\alpha = 45^\circ$

$l = ?D$

$\alpha = 45^\circ$

$l = 6D$

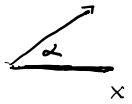
$v_0 = ?$ (da ubacuje ☺)



$\ddot{x}(0) = 0$, $\ddot{y}(t) = -g$

$\cos\alpha = \frac{\dot{x}(0)}{v_0} = \frac{\text{nalegla}}{\text{hipot.}}$ $\quad \dot{x}(0) = v_0\cos\alpha$

$\dot{y}(0) = v_0 \sin\alpha$



$\sin\alpha = \frac{\dot{y}(0)}{v_0}$

$\frac{d\dot{y}(t)}{dt} = -g \quad d\dot{y}(t) = -g\,dt \;/\int \quad \dot{y}(t) = -gt + C_1 \;(C_1 = v_0\sin\alpha)$

$$\boxed{\dot{y}(t) = -gt + v_0\sin\alpha}$$

$\frac{dy(t)}{dt} = -gt + v_0\sin\alpha \quad dy(t) = (-gt + v_0\sin\alpha)\,dt \;/\int \quad y(t) = -\frac{1}{2}gt^2 + v_0\sin\alpha\, t + C_2 \;(C_2 = 0)$

$$\boxed{y(t) = -\frac{1}{2}gt^2 + v_0\sin\alpha\, t} \qquad y(t_1) = 2D$$

$\frac{d\dot{x}(t)}{dt} = \ddot{x}(0) \quad d\dot{x}(t) = \ddot{x}(0)\,dt \;/\int \quad \dot{x}(t) = \ddot{x}_0 t + C_1 \;(C_1 = v_0\cos\alpha)$

$$\boxed{\dot{x}(t) = \ddot{x}(0)\,t + v_0\cos\alpha}$$

$\frac{dx}{dt} = \ddot{x}(0)t + v_0\cos\alpha \quad dx = dt(\ddot{x}(0)t + v_0\cos\alpha) \;/\int$

$x = \frac{1}{2}\ddot{x}(0)t^2 + v_0\cos\alpha\, t + C_2 \;(C_2 = 0)$

$x(t) = \frac{1}{2}\cancel{\ddot{x}(0)}^{\,0}\,t^2 + v_0\cos\alpha\, t$

$$\boxed{x(t) = v_0\cos\alpha\, t} \qquad x(t_1) = 6D$$

$x(t_1) = v_0\cos\alpha\, t_1 \quad \boxed{t_1 = \frac{6D}{v_0\cos\alpha}}$

$$y(t_1) = -\frac{1}{2}g t_1^2 + v_0\sin\alpha\, t_1 = -\frac{1}{2}g\frac{36D^2}{v_0^2\cos^2\alpha} + \cancel{v_0}\cdot\frac{6D}{\cancel{v_0}\cos\alpha}\sin\alpha = -\frac{18D^2 g}{v_0^2\cos^2\alpha} + 6D\,tg\alpha \;\Big|\cdot v_0^2$$

$$v_0^2\, y(t_1) = -\frac{18D^2 g}{\cos^2\alpha} + v_0^2\, 6D\, tg\alpha$$

$$\frac{18D^2 g}{\cos^2\alpha} = v_0^2\,(6D\,tg\alpha - y(t_1)) \qquad v_0^2 = \frac{18D^2 g}{(\cos^2\alpha)\,(6D\,tg\alpha - y(t_1))}$$

$$v_0^2 = \frac{18D^2 g}{\frac{2}{4}(6D - 2D)} = \frac{18D^2 g}{\frac{8D}{4}} = \frac{\frac{1}{4}\cdot\frac{9}{18}D^2 g}{\frac{1}{8}D} = 9gD \qquad \boxed{v_0 = \sqrt{9gD} = 3\sqrt{gD}}$$

$$v_s = \frac{\dots}{\frac{2}{4}(6D-2D)} = \frac{\dots}{\frac{8D}{4}} = \frac{\dots \cdot g}{2 \cdot 8D} = 9gD \qquad \boxed{v_s = \sqrt{9gD} = 3\sqrt{gD}}$$

3.74) $h, \dot{x}(0), \ddot{x}_v(t) = a, g$

$h = ?$

$\ddot{x}(t) = -a$

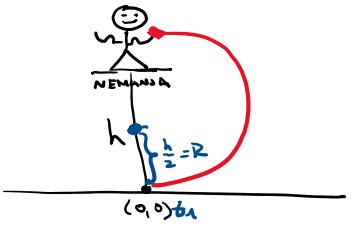
$\frac{d\dot{x}}{dt} = -a \quad d\dot{x} = -a\,dt \,/\int \quad \dot{x} = -at + C_1 \; (C_1 = \dot{x}(0))$

$$\boxed{\dot{x}(t) = -at + \dot{x}(0)}$$

$\frac{dx}{dt} = -at + \dot{x}(0) \qquad dx = dt(-at + \dot{x}(0)) \,/\int$

$x = -\frac{1}{2}at^2 + \dot{x}(0)t + C_2 \; (C_2 = 0)$

$$\boxed{x(t) = -\frac{1}{2}at^2 + \dot{x}(0)t}$$



$\ddot{y}(t) = -g$

$\frac{d\dot{y}}{dt} = -g \quad d\dot{y} = -g\,dt \,/\int \quad \dot{y} = -gt + C_3 \; (C_3 = 0)$

$$\boxed{\dot{y}(t) = -gt}$$

$\frac{dy}{dt} = -gt \qquad dy = -gt\,dt \,/\int$

$y = -\frac{1}{2}gt^2 + C_4 \; (C_4 = h)$

$$\boxed{y(t) = -\frac{1}{2}gt^2 + h}$$

$0 = -\frac{1}{2}at_1^2 + \dot{x}(0)t_1 \quad | \quad y(t_1) = -\frac{1}{2}gt_1^2 + h \qquad y(t_1) = 0 \quad x(t_1) = 0$

$\frac{1}{2}at_1^2 = \dot{x}(0)t_1 \,/^2$

$$\boxed{h = \frac{1}{2}gt_1^2}$$

$at_1 = 2\dot{x}(0)$

$$\boxed{t_1 = \frac{2\dot{x}(0)}{a}}$$

$t_1^2 g = 2h$

$t_1^2 = \frac{2h}{g}$

$$\boxed{t_1 = \sqrt{\frac{2h}{g}}}$$

$\frac{2\dot{x}(0)}{a} = \sqrt{\frac{2h}{g}} \,/^2$

$\frac{4\dot{x}(0)}{a^2} = \frac{2h}{g} \,/^{\cdot 2} \cdot g$

$$\boxed{h = \frac{2g\dot{x}(0)}{a^2}}$$

3.79) $x(t) = R(\omega t - \sin(\omega t))$ R-const, ω-const.

$y(t) = R(1 - \cos(\omega t))$

a) skica trajektorije

b) $x(t) = R(\omega t - \sin(\omega t))$

$x(t) = R\omega t - R\sin(\omega t)$

$\dot{x}(t) = R\omega - \omega R\cos(\omega t)$

$(R\sin(\omega t))' = \cos(\omega t)\,\omega R$

a) skica trajektorije

b) projekcije u t $\dot{x}, \dot{y}, \ddot{x}, \ddot{y}$

c) $t_x = ?$ $\dot{x} = 0 \wedge \dot{y} = 0$ $(x, y) = ?$, $\ddot{x} = ?$, $\ddot{y} = ?$

d) da li $\sqrt{\ddot{x}^2 + \ddot{y}^2}$ zavisi od t

$\dot{x}(t) = R\omega - \omega R\cos(\omega t)$

$(R\sin(\omega t))' = \cos(\omega t)\,\omega R$

$(\omega t)' = \omega$

$$\boxed{\dot{x}(t) = R\omega\,(1 - \cos(\omega t))}$$

$y(t) = R - R\cos(\omega t)$

$$\boxed{\dot{y}(t) = \sin(\omega t) R\omega}$$

$(R\cos(\omega t))' = -\sin(\omega t) R\omega$

$(\omega t)' = \omega$

$\dot{x}(t) = R\omega - R\omega\cos(\omega t)$

$$\boxed{\ddot{x}(t) = \sin(\omega t)\,\omega^2 R}$$

$(R\omega\cos(\omega t))' = -\sin(\omega t)\,\omega^2 R$

$\dot{y}(t) = R\omega \sin(\omega t)$

$$\boxed{\ddot{y}(t) = \cos(\omega t)\,\omega^2 R}$$

c) $\dot{x}(t) = \dot{y}(t) = 0$

$\cancel{R\omega}(1 - \cos(\omega t)) = \sin(\omega t)\cancel{R\omega}$

$1 - \cos(\omega t) = \sin(\omega t)$

$\cos(\omega t) = 1$

$\omega t = 2\pi$

$t = \frac{2k\pi}{\omega}$ $\Rightarrow t_x = \{0, \frac{2\pi}{\omega}, \frac{4\pi}{\omega}, \frac{6\pi}{\omega} \dots\}$

$y(t_x) = 0$ (uvek) $x(t_x) = \{0, 2\pi R, 4\pi R, 6\pi R \dots\}$

$$\boxed{\ddot{x}(t_x) = 0} \quad \boxed{\ddot{y}(t_x) = \omega^2 R}$$

d) $a = \sqrt{0^2 + (\omega^2 R)^2} = \omega^2 R \Rightarrow$ **ne zavisi**

3.81) $\dot{x}_z(t) = 220\,km/h$ brz/vaz masu

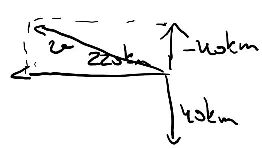
$t_1 = 0{,}5\,h$

a) $\dot{x}_v(t) = 20\,km/h$

$\dot{y}_v(t) = 40\,km/h$

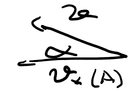
$v_v = \sqrt{20^2 + 40^2} = 44{,}72\,km/h$

b) $v = \sqrt{220^2 + 45^2}$



$v = 223{,}6\,km/h$

$\cos\alpha = \frac{220}{223{,}6}$



$\alpha = \arccos\left(\frac{220}{223{,}6}\right) = 10{,}30^\circ$

$$\boxed{\alpha = 10{,}30^\circ}$$ (severo-zapad)