21. 6. 2021.

① X и Y независне

$$F_X(x)=\begin{cases}0, & x\le 0\\ x/4, & 0<x\le 4\\ 1, & x>4\end{cases}\qquad F_Y(y)=\begin{cases}0, & y\le 0\\ 1-e^{-2y}, & y>0\end{cases}$$

$$W_t=t\cdot X+t^2\cdot Y,\quad t\ge 0$$

a) $m_W(t)=?$ $R_W(t,s)=?$ $D(W_t)=?$

$$\varphi_X(x)=\begin{cases}\frac{1}{4}, & x\in(0,4)\\ 0, & \text{иначе}\end{cases}\qquad Y:\mathcal{E}(2)\Rightarrow \varphi_Y(y)=\begin{cases}2\cdot e^{-2y}, & y>0\\ 0, & y\le 0\end{cases}$$

$$m_W(t)=E(W_t)=E(tX+t^2Y)=t\cdot E(X)+t^2\cdot E(Y)\quad (*)$$

$$E(X)=\int_0^4 x\cdot\frac{1}{4}\,dx=\frac{1}{4}\,\frac{x^2}{2}\Big|_0^4=\frac{16}{8}=2$$

НАПОМЕНА: $X:\mathcal{U}(0,4)\Rightarrow E(X)=\frac{0+4}{2}=2$

$$Y:\mathcal{E}(2)\Rightarrow E(Y)=\frac{1}{2}$$

$$m_W(t)\overset{(*)}{=}t\cdot 2+t^2\cdot\frac{1}{2}$$

$$D(W_t)=D(t\cdot X+t^2Y)\overset{X\text{ и }Y\text{ нез.}}{=}t^2\cdot D(X)+(t^2)^2D(Y)$$
$$=t^2\cdot\frac{(2-0)^2}{12}+t^4\cdot\frac{1}{2^2}=\frac{1}{3}t^2+\frac{1}{4}t^4$$

$$R_W(t,s)=E(W_t\cdot W_s)=E\big((\underline{tX}+t^2Y)\cdot(\underline{sX}+s^2Y)\big)$$
$$=tsE(X^2)+\underline{ts^2E(XY)}+\underline{t^2s\,E(XY)}+t^2s^2E(Y^2)\quad(**)$$
$$\overset{X\text{ и }Y\text{ нез.}}{=}tsE(X^2)+E(X)E(Y)ts(s+t)+t^2s^2E(Y^2)$$

$$E(X^2)=D(X)+E(X)^2=\frac{(2-0)^2}{12}+\left(\frac{0+4}{2}\right)^2=\frac{1}{3}+4=\frac{13}{3}$$

$$E(Y^2)=D(Y)+E(Y)^2=\frac{1}{2^2}+\left(\frac{1}{2}\right)^2=\frac{2}{4}=\frac{1}{2}$$

$$R_W(t,s)=\frac{13}{3}\cdot ts+2\cdot\frac{1}{2}ts(s+t)+\frac{1}{2}t^2s^2$$

b) $m_W(t)\ne\text{const}$ $\quad W_t$ није слабо стационаран

□

② $$P = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 2 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 3 & 0 & \frac{1}{2} & \frac{1}{2} \end{array}$$

$S = \{0, 1, 2\}$

$p(0) = [3x \quad 2x \quad x]$

$6x = 1$

$x = 1/6 \quad p(0) = [3/6 \quad 2/6 \quad 1/6]$

а) $p(2) = p(0) \cdot P^2$

$$P^2 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 5/12 & 5/12 & 2/12 \\ 5/18 & 8/18 & 5/18 \\ 2/12 & 5/12 & 5/12 \end{bmatrix}$$

$$p(2) = [3/6 \quad 2/6 \quad 1/6] \cdot \begin{bmatrix} 5/12 & 5/12 & 2/12 \\ 5/18 & 8/18 & 5/18 \\ 2/12 & 5/12 & 5/12 \end{bmatrix} = \left[\frac{71}{216} \quad \frac{92}{216} \quad \frac{53}{216}\right]$$

мах

$P_2(2) = \frac{92}{216}$ мах, највероватније је у С2.

б) $P(X_0 = 1, X_1 = 2, X_2 = 2) = p_1(0) \cdot p_{12}(1) \cdot p_{22}(1)$

$= \frac{3}{6} \cdot \frac{1}{2} \cdot \frac{1}{3} = \ldots$

г) $P(X_2 = 1, X_3 = 2 \mid X_0 = 1, X_1 = 1) = P(X_2 = 1, X_3 = 2 \mid X_1 = 1)$

$$= \frac{P(X_1 = 1, X_2 = 1, X_3 = 2)}{P(X_1 = 1)} = \frac{\cancel{p_1(1)} \cdot p_{11}(1) \cdot p_{12}(1)}{\cancel{p_1(1)}} = \frac{1}{2} \cdot \frac{1}{2}$$

д) $P^2 > 0$, постоје финалне вероб, $p^* = [x \quad y \quad z]$

$[x \quad y \quad z] \cdot P^* = [x \quad y \quad z]$

$x + y + z = 1$

$$\frac{1}{2}x + \frac{1}{3}y = x$$
$$\frac{1}{2}x + \frac{1}{3}y + \frac{1}{2}z = y$$
$$\frac{1}{3}y + \frac{1}{2}z = z$$
$$x + y + z = 1$$

решити систем.

$$[x \quad y \quad z] \cdot \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{bmatrix} = [x \quad y \quad z]$$

$x + y + z = 1$