Modul 6 - Zadaci

Thursday, November 23, 2023 3:29 PM

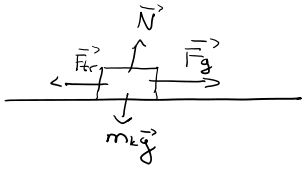
$\vec{A}_{12} = \int (F\cos\theta\,\vec{i} + F\sin\theta\,\vec{j})\,dx\,\vec{i}$

6.3) $v = \text{const.} \Rightarrow a = 0$

$m_k = 30\,kg$

$\Delta l = 4{,}5\,m$

$\mu_D = 0{,}25$



$A_{1,2} = F\cos\theta\,\Delta l$

$A_{1,2} = F\sin\theta\,\Delta l$

a) $F_g = ?$

$\vec{F} = m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_g + \vec{F}_{tr} \quad / \cdot \vec{i}$

$ma = F_g - F_{tr}$

$F_{tr} = F_g$

$F_g = \mu_D \cdot m_k g$ $\quad \boxed{F_g = 73{,}575\,N}$

b) $A = ?$

$A_{\Delta l} = F_g \cos(0°)\,\Delta l$

$\boxed{A_{\Delta l} = 331{,}08\,J}$

c) $A_{\Delta l}^{\vec{F}_{tr}} = F_{tr}\cos(\theta°)\,\Delta l$

$\boxed{A_{\Delta l}^{\vec{F}_{tr}} = -331{,}08\,J}$

d) $A_{\Delta l'}^{m\vec{g}} = A_{\Delta l'}^{\vec{N}}$

$\Delta l' = 0$

$\Rightarrow$ nemamo kretanje po y-osi

$\Rightarrow \boxed{A_{\Delta l'}^{m\vec{g}} = A_{\Delta l'}^{\vec{N}} = 0\,J}$

e) $\boxed{A = 0\,J}$

$\Rightarrow A = A_{\Delta l'}^{m\vec{g}} + A_{\Delta l'}^{\vec{N}} + A_{\Delta l}^{\vec{F}_g} + A_{\Delta l}^{\vec{F}_{tr}}$

6.18) $m_k = 4{,}8\,kg$

$h = 25\,m$ $\quad \boxed{\Delta h = 25\,m}$ $\quad \theta = 90°$ (y-osa)

a) $A_{\Delta h}^{m\vec{g}} = m\vec{g}\sin\theta\,\Delta h = mg\,\Delta h = 1177{,}2\,J$

$\boxed{A_{\Delta h}^{m\vec{g}} = 1177{,}2\,J}$ $\quad$ (0,0 u mestu odakle pada) $+\downarrow +$

b) $E_k(t_1) = ?$ $\quad \dot{y}(t_1) = ?$ $\quad y(t_1) = 25\,m$ $\quad \ddot{y}(t) = g$

$\frac{d\dot{y}}{dt} = g \quad d\dot{y} = g\,dt \int \quad \dot{y}(t) = gt + C_1 \quad (C_1 = 0) \Rightarrow \dot{y}(t) = gt$

$\frac{dy}{dt} = gt \quad dy = g\,t\,dt \,/\int \quad y = \frac{1}{2}gt^2 + C_2 \quad (C_2 = 0) \quad y(t) = \frac{1}{2}gt^2 \Rightarrow t^2 = \frac{2\,y(t)}{g}$

$t_1 = \sqrt{\frac{2y(t_1)}{g}} = 2{,}257\,s$ $\quad \boxed{t_1 = 2{,}257\,s}$ $\quad \dot{y}(t_1) = g\,t_1$ $\quad \boxed{\dot{y}(t_1) = 22{,}147\,m/s}$

$E_k(t_1) = m\,\dot{x}(t_1)^2\,\frac{1}{2} = 1177\,J \Rightarrow \boxed{E_k(t_1) = 1177\,J}$

c) (sila otpora vazduha više nije zanemarljiva)

$\Rightarrow$ a) rad se neće promeniti

b) brzina bi bila manja zbog otpora pa bi i kinetička energija bila manja

6.29) v_0 – početna brzina

a) $\Delta l = ?$, μ_d, v_0, g

$A_{\Delta l}^{\vec{F}_{tr}} = -\mu_d mg \Delta l$ $A_{\Delta l}^{\vec{F}_n} = 0$ (auto klizi dok se ne zaustavi)

$A_{\Delta l}^{\vec{F}_{tr}} = Ek_2 - Ek_1$ $Ek_2 = 0$ $Ek_1 = \frac{1}{2} m v_0^2$

$A_{\Delta l}^{\vec{F}_{tr}} = -Ek_1 \Rightarrow -\mu_d mg \Delta l = -\frac{1}{2} m v_0^2$

$$\boxed{\Delta l = \frac{v_0^2}{\mu_d g 2}}$$

b) 1° $\mu_1 = 3\mu_d$

$$\boxed{\Delta l_1 = \frac{v_0^2}{2\mu_d g \cdot 2} = \frac{v_0^2}{4\mu_d g}}$$

$$\Rightarrow \frac{\Delta l}{\Delta l_1} = \frac{1}{2}$$

2° $v_{0_2} = 2v_0$

$$\boxed{\Delta l_2 = \frac{2^2 v_0^2}{2\mu_d g} = \frac{2v_0^2}{\mu_d g}}$$

$$\frac{\Delta l}{\Delta l_2} = \frac{4}{1}$$

3° $v_{0_3} = 2v_0$ $\mu_{d_3} = 2\mu_d$

$$\Delta l_3 = \frac{4v_0^2}{4\mu_d g} = \frac{v_0^2}{\mu_d g} \quad \boxed{\frac{\Delta l}{\Delta l_3} = \frac{2}{1}}$$

6.40) $m_L = 4\,kg$

$c = 200\,N/m$

$l_1 = -0{,}025\,m$ $l_2 = 0$ $\Delta l = 0{,}025$

a) $\boxed{A_l^{\vec{F}_0} = \frac{1}{2} c l_1^2}$ → sila opruge kada nam je data krutost opruge

$A_{\Delta l}^{\vec{F}_0} = \frac{1}{2} c l_2^{\,2\,0} - \frac{1}{2} c l_1^2 = -\frac{1}{2} c l_1^2 = -0{,}0625\,J$

$\boxed{A_{\Delta l}^{\vec{F}_0} = -0{,}0625\,J}$

b) $Ek = m v^2 \frac{1}{2}$ $v = ?$ $Ek = A_{0l}^{\vec{F}_0}$

$2Ek = m v^2$

$v = \sqrt{\frac{2Ek}{m}}$ $\boxed{v = 0{,}1767\,m/s}$

6.42) $m = 2kg$ $\vec{F}/x$ -> grafik

$v_0 = 0$

$x(0) = 0$

$\vec{F}$

(rad je jednak površini na grafiku)

b) od 3 do 4 se ne izvrši rad

=> brzina ostaje ista ($x = 4m$)

$\boxed{v_4(4m) = 2m/s}$

a) $v_0 = ?$ $x_2 = 3m$ $Ek_{1-2} = ?$ $Ek_1 = 0$ (početna sila je 0)

$A^{\vec{F}_x(W)}_{1,2} = Ek_2 - Ek_1$ $A^{\vec{F}_x(W)}_{1,2} = Ek_2 = \frac{1}{2} m v_2^2$ $(v_0 = v_2)$

$v_2 = \sqrt{\frac{2A^{\vec{F}_x(W)}_{1,2}}{m}} = \sqrt{A^{\vec{F}_x(W)}_{1,2}}$

$A^{\vec{F}_x(W)}_{1,2} = \int_0^3 \vec{F}(x)\,dx$

$A^{\vec{F}_x(W)}_{1,2} = 0,5J + 1,5J + 2J = 4J$

=> $v_2 = \sqrt{4}$

$\boxed{v_2 = 2m/s}$

c) $x = 7m$

$A^{\vec{F}_x}_{0,7} = 4J - \frac{2 \cdot 1}{2} J$

$A^{\vec{F}_x}_{0,7} = 4J - 1J$

$\boxed{A^{\vec{F}}_{0,7} = 3J}$

=> $v_7 = \sqrt{3}$

$\boxed{v_7 = 1,73 m/s}$

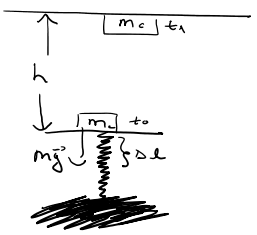
6.46) $c = 450 N/m$ m - zanemarljiva

$m_c = 1,8kg$

$h = 3,6m$

(0,0) – u mestu opruženе opruge)

$\boxed{\Delta l = ?}$ -> koliko treba sabiti podlogu



$x(t_1) = h = 3,6m$

$\vec{F} = m\vec{a} = \vec{F}_0 + m\vec{g}$ / $\vec{j}$

$F = F_0 - mg$

$ma = F_0 - mg$

$A = Ek_2 - Ek_1$

$A = \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2$ $v_1 = v_0 = 0$ -> $\boxed{A = 0}$

-> zigla miruje i na početku i na kraju njenog kretanja

$A^{m\vec{g}}_{1,0} = mgh(-1) = -mgh$

$\boxed{A^{\vec{F}_0}_{-1,0} = \frac{1}{2} c \Delta l^2}$

$A = A^{\vec{F}_0}_{-1,0} + A^{m\vec{g}}_{1,0} = \frac{1}{2} c \Delta l^2 + (-mgh)$

$A = \frac{1}{2} c \Delta l^2 - mgh$

$mgh = \frac{1}{2} c \Delta l^2$ $\Delta l = \sqrt{\frac{2mgh}{c}}$ $\boxed{\Delta l = 0,53m}$

6.56) $m_l = 600kg$

$h = 20m$

v - const. $a = 0$

$h = 20m$

v - const. $a = 0$

$t = 16s$

$P = 40ks = 30kW$ $\boxed{P = 30kW}$

$n = ?$ $m_p = 65kg$

$P = \frac{\Delta Ek}{\Delta t}$ $\Delta Ek = Ek_2 - Ek_1 = A_{12}$

$\Delta t = 16s$

$A_{12} = F \cdot \sin\theta \cdot \Delta l = mgh$ $h = \frac{m - 600}{65}$ $\boxed{h = 28,4 \simeq 28}$

$\Rightarrow 28$ osoba

$P = \frac{mgh}{t}$ $Pt = mgh$ $\boxed{m = \frac{Pt}{gh}}$ $\boxed{m = 2446kg}$

6.60) $x_0 = 0$ $x_1 = 6,9m$

$F_x = -[20N + (3N/m)x]$

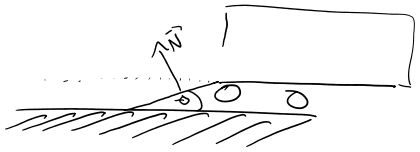
$$A_{12}^{F_x} = \int_{x_0}^{x_1} F_x(x)\,dx = \int_{x_0}^{x_1} (-20N - 3N/m\,x)\,dx = -\int_{x_0}^{x_1} 20\,dx - \int_{x_0}^{x_1} 3x\,dx$$

$$= -20x\Big|_{x_0}^{x_1} - \left(\frac{1}{2}x^2 \cdot 3\right)\Big|_{x_0}^{x_1} = -(20x)\Big|_{x_0}^{x_1} - \left(\frac{3}{2}x^2\right)\Big|_{x_0}^{x_1}$$

$= -(20 \cdot 6,9) - \left(\frac{3}{2} \cdot 6,9^2\right) = -209,415$ J

$\boxed{A_{12}^{F_x} = -209,415\,J}$

6.68) $m = 5kg$, $\Delta l = 1,5m$



$\mu_D = 0,310$ $\theta = 21°$

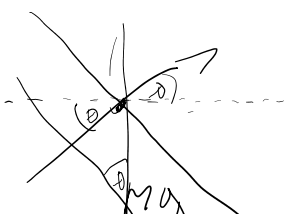
a) $A_{12}^{\vec{F}_{tr}} = -F_{tr}\cos\theta\,\Delta l$

$A_{12}^{\vec{F}_{tr}} = -mg\mu_D\cos\theta\,\Delta l = 20,836$ J

$\boxed{A_{12}^{\vec{F}_{tr}} = -20,836\,J}$

b) $A_{12}^{m\vec{g}} = mg \cdot \sin(\theta) \cdot \Delta l_y$

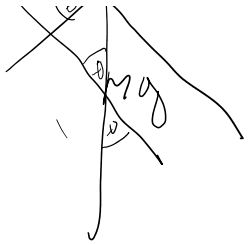
$\boxed{A_{12}^{m\vec{g}} = 29,92\,J}$



c) $\boxed{A^{\vec{N}} = 0\,J}$

$\boxed{A_{12}^{\vec{F_{tr}}} = -20{,}836\,J}$

c) $\boxed{A_{1,2}^{\vec{N}} = 0\,J}$



d) $A = A_{12}^{\vec{F_{tr}}} + A_{12}^{m\vec{g}}$

$\boxed{A = 9{,}084\,J}$

e) $v_0 = 2{,}2\,m/s$, $v_1 = ?$

$A = \Delta E_k$

$A = E_{k_2} - E_{k_1}$

$A = m v_1^2 \frac{1}{2} - m v_0^2 \frac{1}{2}$ $\quad 2A = m v_1^2 - m v_0^2$ $\quad m v_1^2 = 2A + m v_0^2$ $\quad v_1 = \sqrt{\frac{2A + m v_0^2}{m}}$

$\boxed{v_1 = 2{,}91\,m/s}$

6.76) $m_p = 1{,}67 \cdot 10^{-27}\,kg$

$v_{0_p} = 3 \cdot 10^5\,m/s$

$\Delta l = 5\,m$

$F_0 = \frac{\alpha}{x^2}$, $\alpha = 2{,}12 \cdot 10^{-26}\,Nm^2$

a) $\Delta l' = 8 \cdot 10^{-10}\,m$ $\quad v_2 = ?$ $\quad (1-2)$

$\boxed{E_{k_1} = m_p v_{0_p}^2 \cdot \frac{1}{2}}$

$A_{12} = \int_{\Delta l}^{\Delta l'} F_0(x)\,dx = \int_{\Delta l}^{\Delta l'} \frac{\alpha}{x^2}\,dx = \int_{\Delta l}^{\Delta l'} \alpha x^{-2}\,dx = -\alpha x^{-1} \Big|_{\Delta l}^{\Delta l'}$

$A_{12} = E_{k_2} - E_{k_1}$

$\boxed{= -2{,}65 \cdot 10^{-17}\,J}$

$A_{12} = m_p v_2^2 \frac{1}{2} - E_{k_1}$ $\quad 2(A_{12} + E_{k_1}) = m_p v_2^2$ $\quad v_2 = \sqrt{\frac{2(A_{12} + E_{k_1})}{m_p}}$

$\boxed{v_2 = 2{,}41 \cdot 10^5\,m/s}$

b) $v_3 = 0$ $\quad \Delta l_3 = ?$

$A_{13} = \cancel{E_{k_3}} - E_{k_1}$

$A_{1,3} = \int_{\Delta l}^{\Delta l_3} F_0(x)\,dx = \int_{\Delta l}^{\Delta l_3} \frac{\alpha}{x^2}\,dx = -\alpha x^{-1} \Big|_{\Delta l}^{\Delta l_3} = -\frac{\alpha}{\Delta l_3} + \frac{\alpha}{\Delta l}$

$\frac{\alpha}{\Delta l} - \frac{\alpha}{\Delta l_3} + E_{k_1} = 0$

$\frac{\alpha}{\Delta l} + E_{k_1} = \frac{\alpha}{\Delta l_3}$ $\quad \Delta l_3 = \frac{\alpha}{\frac{\alpha}{\Delta l} + E_{k_1}}$

$\boxed{\Delta l_3 = 2{,}82 \cdot 10^{-10}\,m}$

$\Delta l_3 = 2{,}82 \cdot 10^{-10}\ m$

c) $v_4 = v_0 \Rightarrow$ za isti pretez put Δl_3 vratiće se u početnu brzinu (ali sa suprotnim znakom ubrzanja)

6.77) $m = 6\,kg$

$\vec{F}$

$x(t) = \alpha t^2 + \beta t^3$, $\alpha = 0{,}2\,m/s^2$, $\beta = 0{,}02\,m/s^3$

a) $t = 4\,s$

$\dot{x}(t) = 2\alpha t + 3\beta t^2$ $\quad \boxed{\dot{x}(t) = 2{,}56\,m/s}$

b) $\ddot{x}(t) = 2\alpha + 6\beta t$ $\quad \vec{F} = m\vec{a} \,/\, \vec{j}$ $\quad F = ma$

$\boxed{\ddot{x}(t) = 0{,}88\,m/s^2}$ $\quad \boxed{F = 5{,}28\,N}$

c) $A_{12} = Ek_2 - Ek_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}m\cancel{v_1^2}^{=0} = \frac{1}{2}mv_2^2$

$\boxed{A_{12} = 19{,}66\,J}$

6.78) $m_b + m_v = 80\,kg$

$h_{max} = 5{,}2\,m$

$v_0 = 5\,m/s$ $\quad v_2 = 1{,}5\,m/s$

a) $A_{12} = Ek_2 - Ek_1$ $\quad A_{12} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$\boxed{A_{12} = -910\,J}$

b) $A_{12} = A_{12}^{m\vec{g}} + A_{12}^{\vec{F}}$

$A_{12}^{\vec{F}} = A_{12} - A_{12}^{m\vec{g}}$

$A^{\vec{F}} = A_{12} + mgh$

$\boxed{A^{\vec{F}} = 3170{,}96\,J}$

6.85) $m_t = 5\,kg$

$v_0 = 6\,m/s$

$k = 500\,N/m$

$A_{12} = \frac{1}{2}c\Delta l^2$ $\quad 90 = \frac{1}{2}k\Delta l_{max}^2 \,/\,\cdot 2$ $\quad 180 = k\Delta l_{max}^2$

$\Delta l_{max} = \sqrt{\frac{180}{500}}$

$k = 500 N/m$ $\quad 90 = \frac{1}{2} k \Delta l_{max}^2 \quad | \cdot 2 \quad \Delta l_{max} = \sqrt{\frac{180}{500}}$

a) $\Delta l_{max} = ?$ $\quad \boxed{v_2 = 0} \rightarrow E_{k2}$ nemam $\quad \boxed{\Delta l_{max} = 0{,}6 m}$

$A_{12} = E_{k1} = \frac{1}{2} m v_0^2 \quad \boxed{A_{12} = 90 J}$

b) $\Delta l' = 0{,}15 m$

$v_0' = ?$

$\frac{1}{2} m v_0'^2 = \frac{1}{2} k \Delta l'^2$

$v_0' = \sqrt{\frac{k \Delta l'^2}{m}} \quad \boxed{v_0' = 15 m/s}$