Laplasova transformacija

Funkcija prenosa

LAPLASOVA TRANSFORMACIJA

- Prelazak iz vremenskog u frekventni (kompleksni) domen
- Ideja: pretvoriti diferencijalne jednačine u algebarske
- Osnovni izraz:

$$L\{\mathrm{f(t)}\} = \mathrm{F(s)} = \int_0^\infty f(t)\mathrm{e}^{-\mathrm{st}}dt$$

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$$L\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}dt$$

f(t) – funkcija u vremenskom domenu
F(s) - funkcija u kompleksnom domenu
s - kompleksna promenljiva

$$s = \sigma \pm j\omega$$

PRIMER 2

- Odrediti rešenje diferencijalne jednačine y(t), ako je u(t) = δ(t).

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2. Rastaviti složeni izraz na sabirke (pronaći potrebne koeficijente)
3. Nad svakim sabirkom primeniti inverznu Laplasovu transformaciju

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REŠENJE:

$y(t) = (e^{-2t} - e^{-3t}) * h(t)$

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Kao u prošlom zadatku, samo bez LT
(ovde već imamo izraz u kompleksnom domenu)

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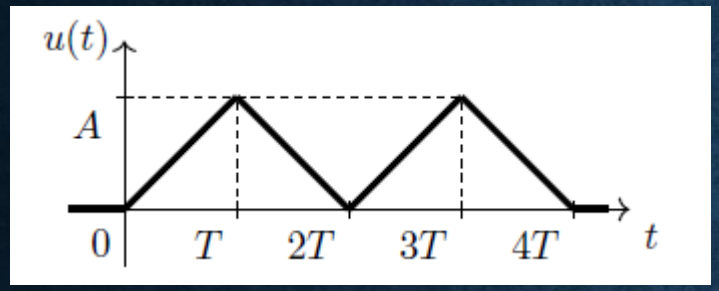
t zbog $(s+1)^2$

PRIMER 5

- Za signal sa slike odrediti u(t) i U(s).



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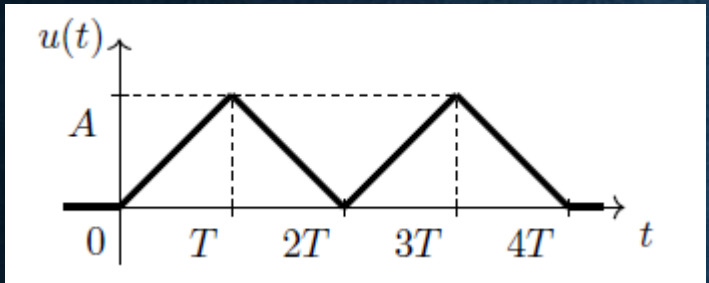

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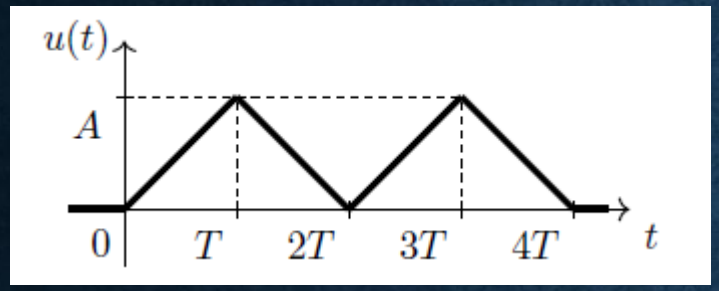

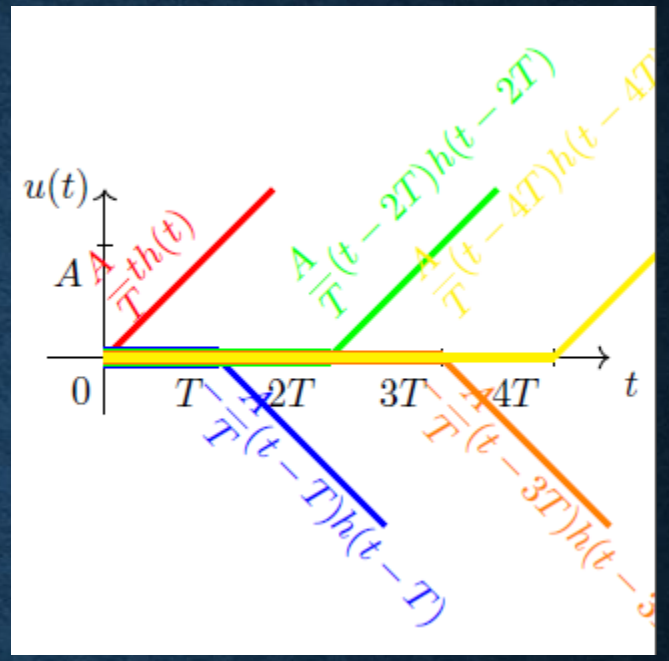

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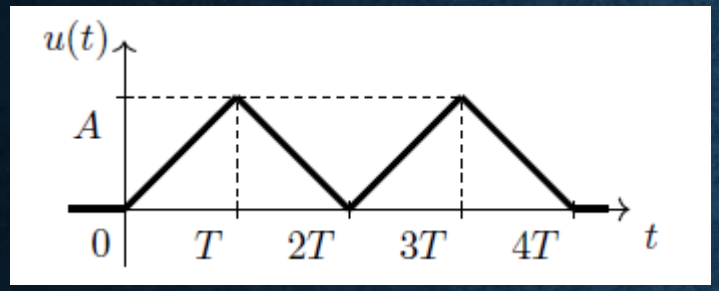

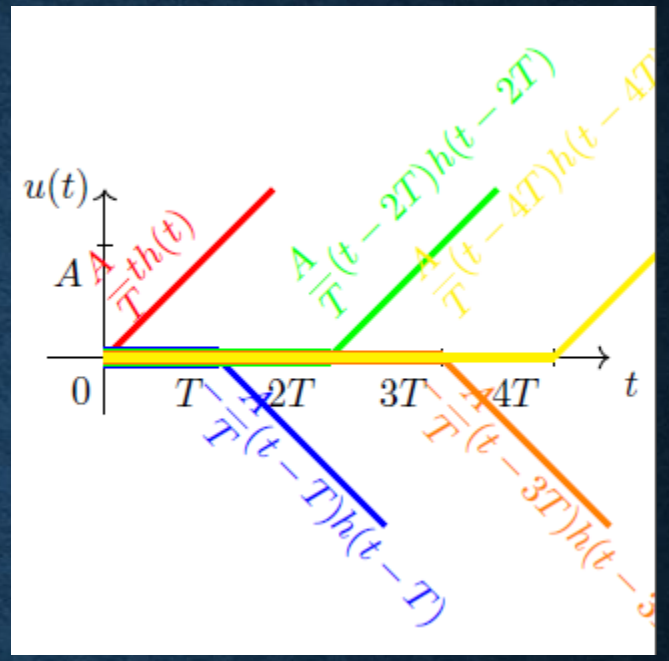

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PRIMER 5

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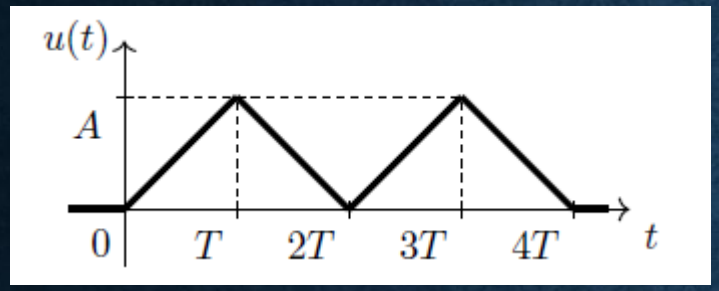

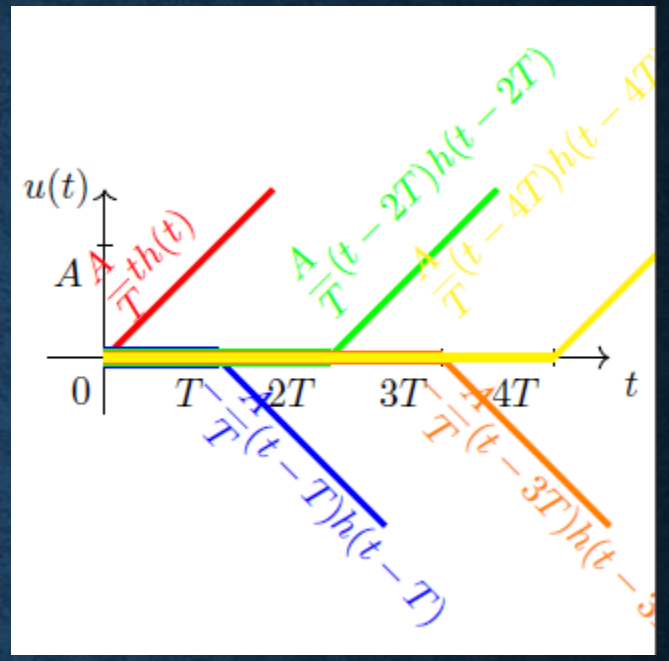

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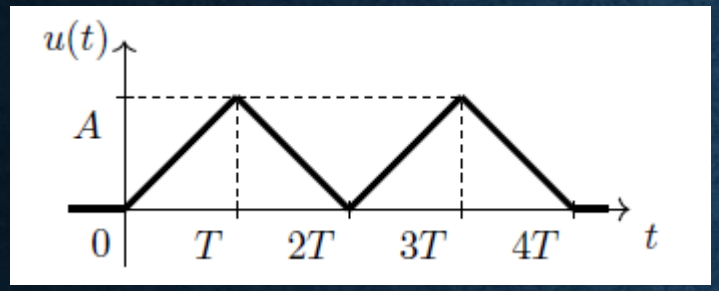

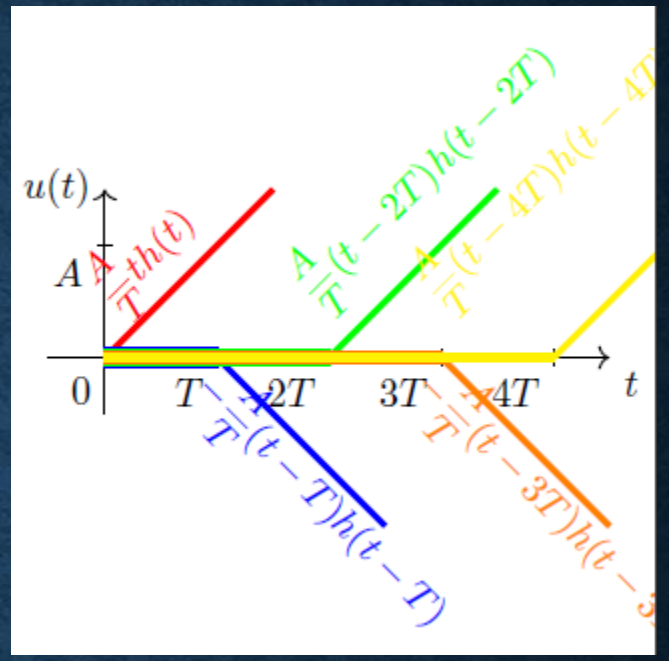

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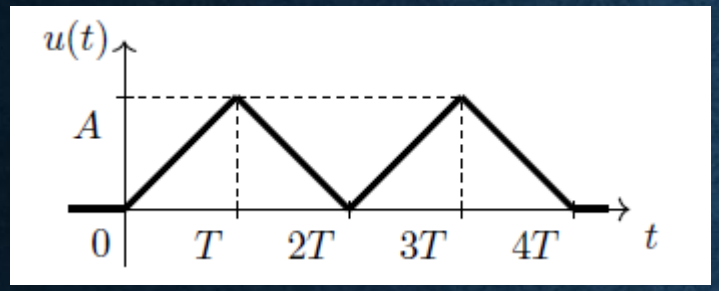

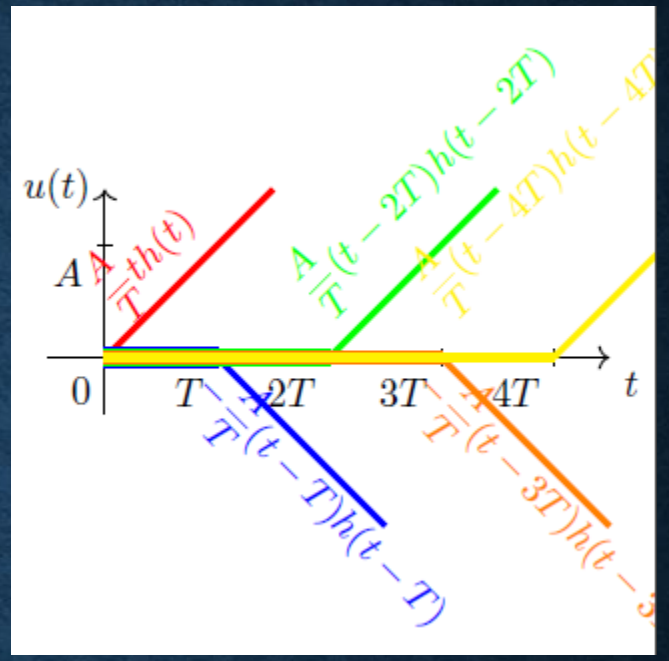

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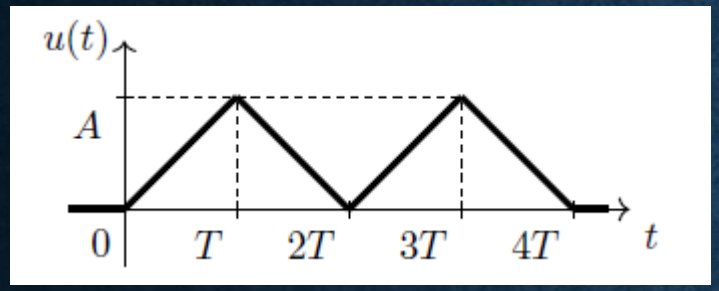

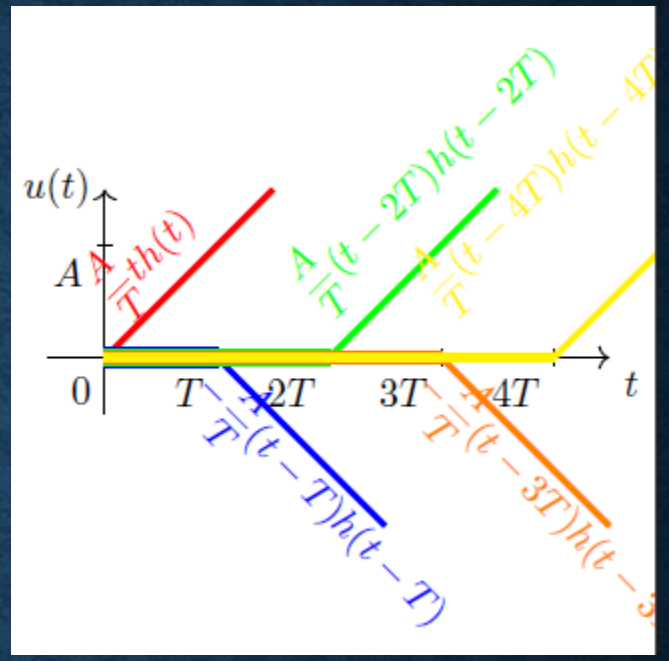

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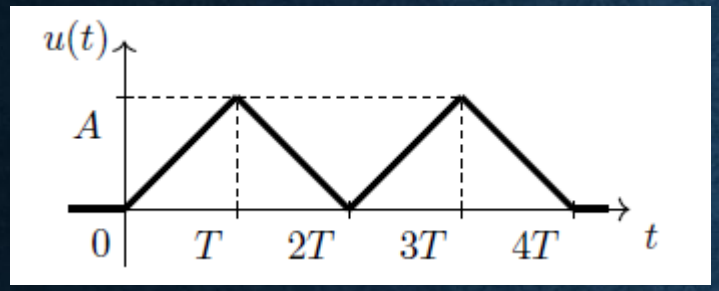

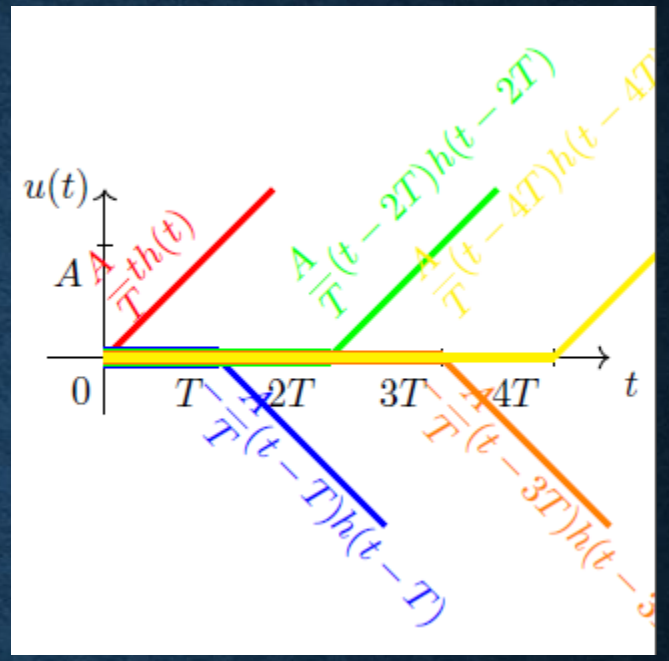

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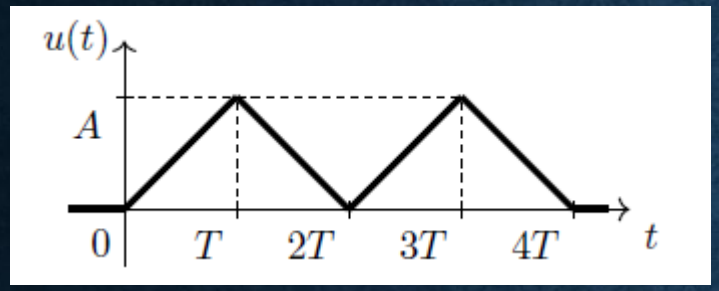

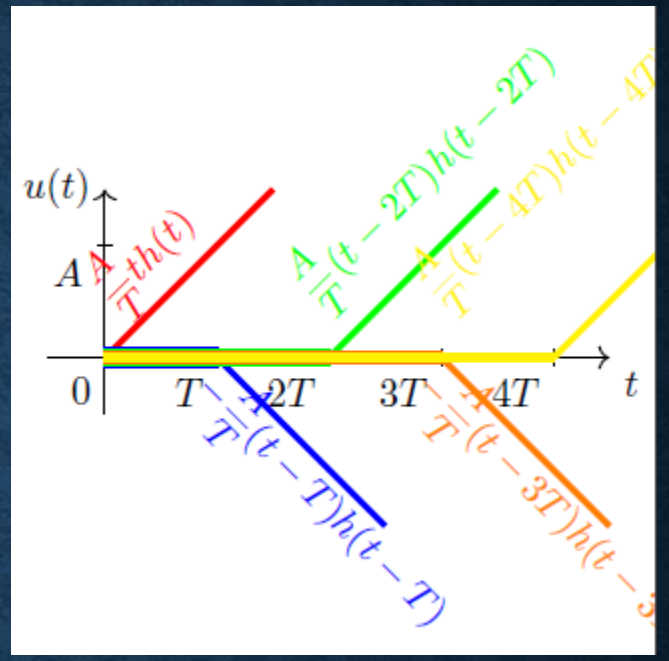

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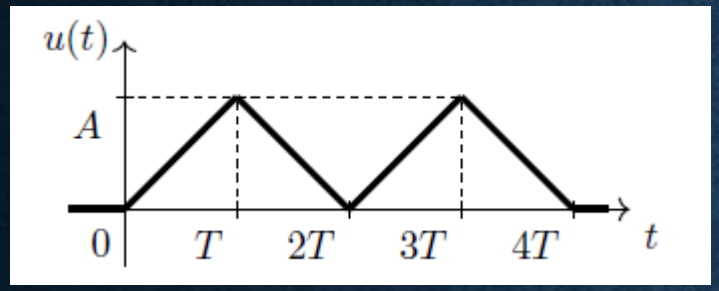

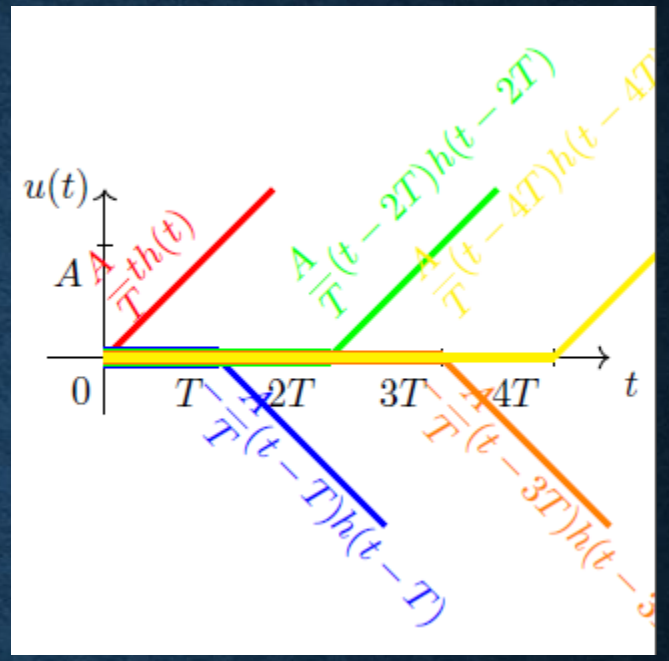

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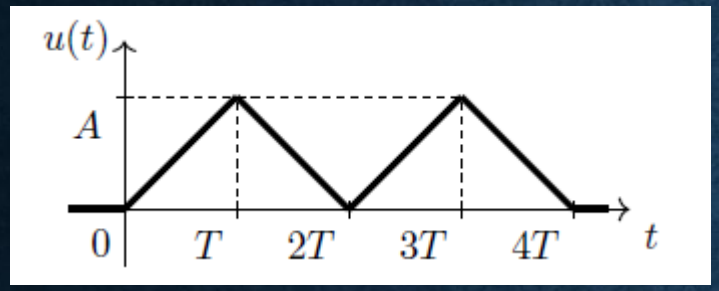

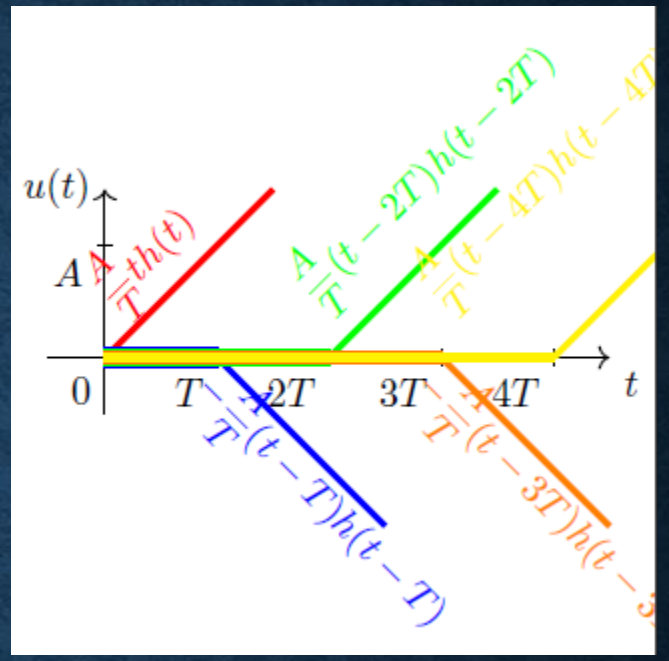

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Veliko T!

FUNKCIJA PRENOSA

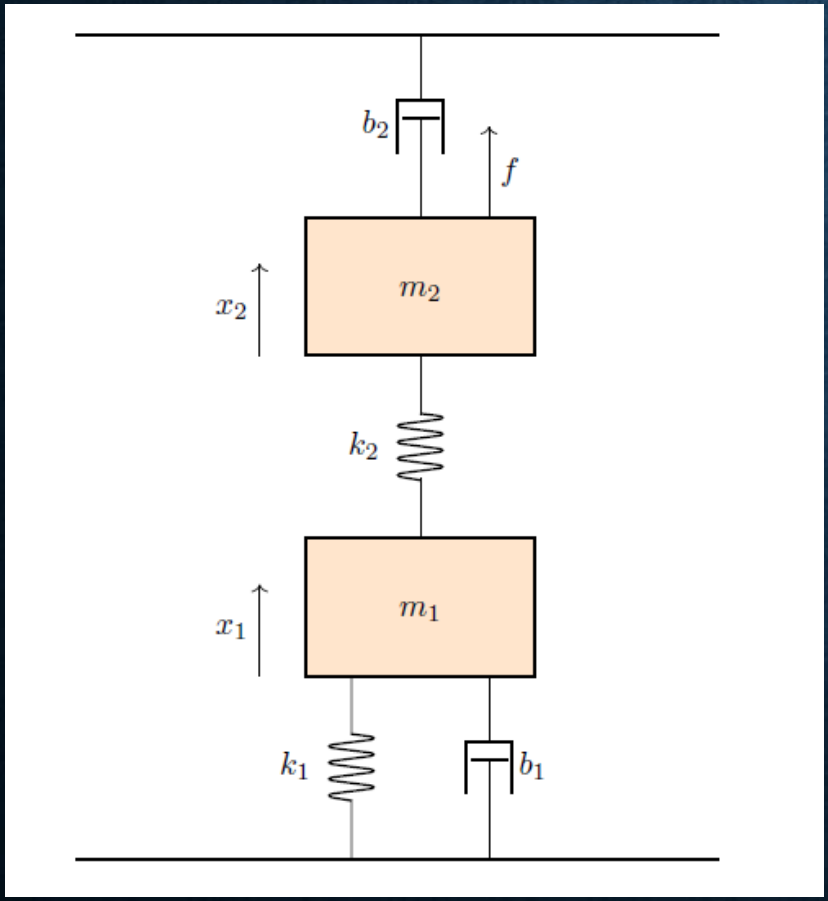
- Odnos kompleksnih likova izlaza i ulaza
- Osnovni izraz:

$$G(s) = \frac{Y(s)}{U(s)}$$

PRIMER 3

- Odrediti funkciju prenosa mehaničkog translatornog sistema sa slike ako je ulaz pobudna sila f(t), a izlaz pozicija $x_2(t)$. Parametri modela su:

  m1 = 1, m2 = 1, b1 = 10, b2 = 8, k1 = 9, k2 = 16. Zanemariti gravitaciju.



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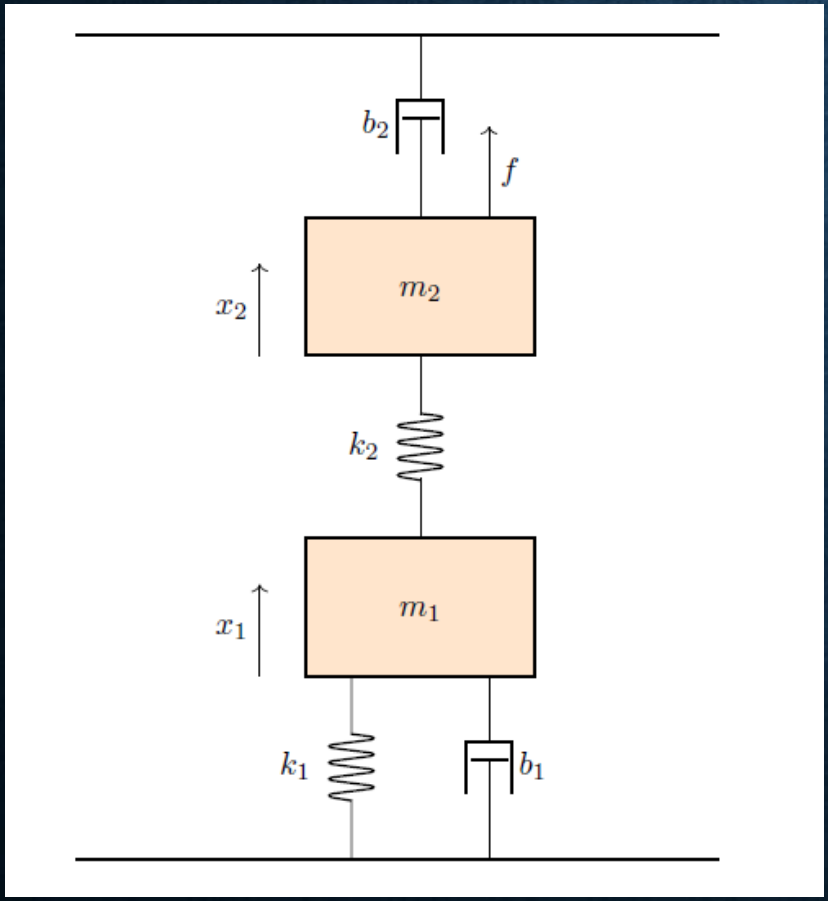

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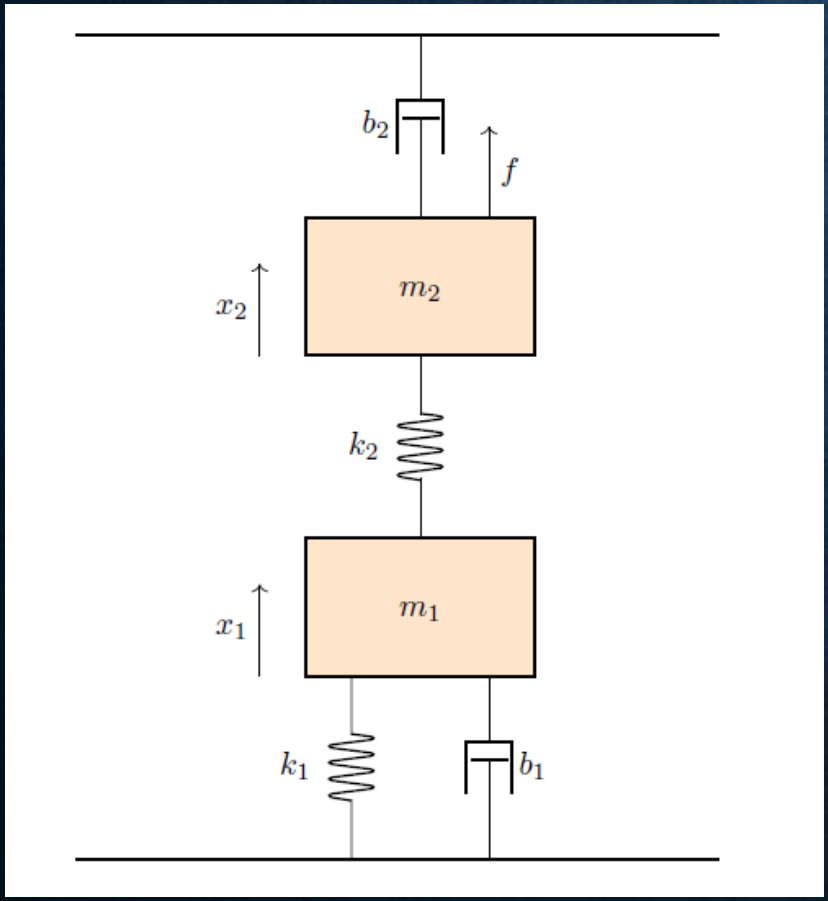

- IDEJA:

  1. Formirati FBD i dif. jednačine

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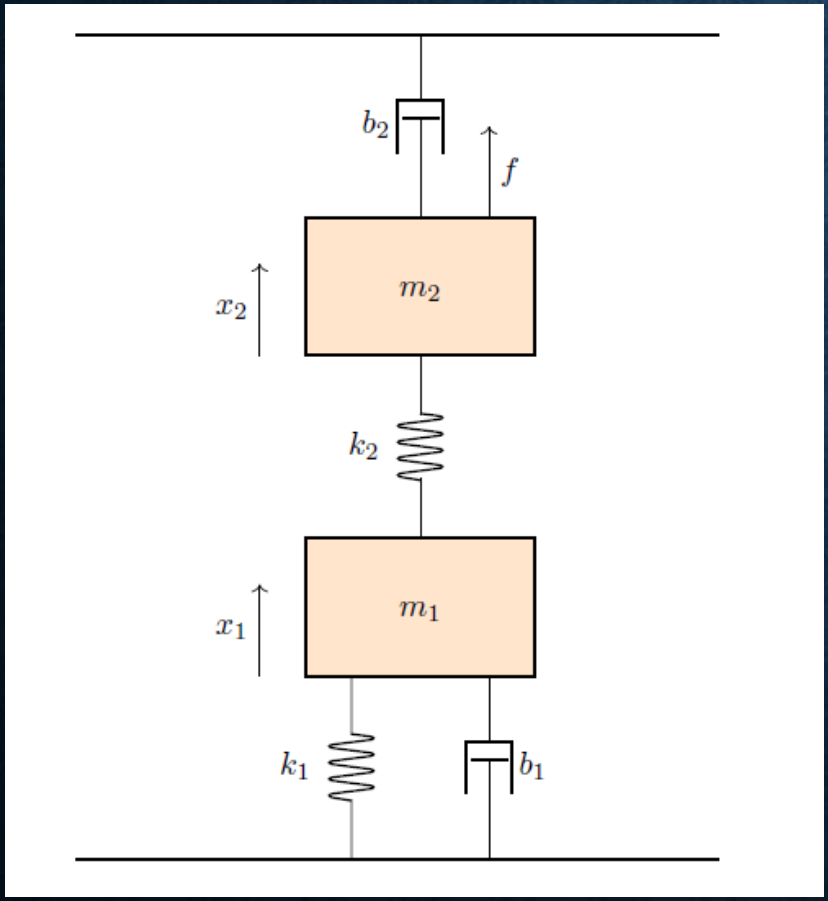

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  2. Primeniti LT nad dobijenim jednačinama

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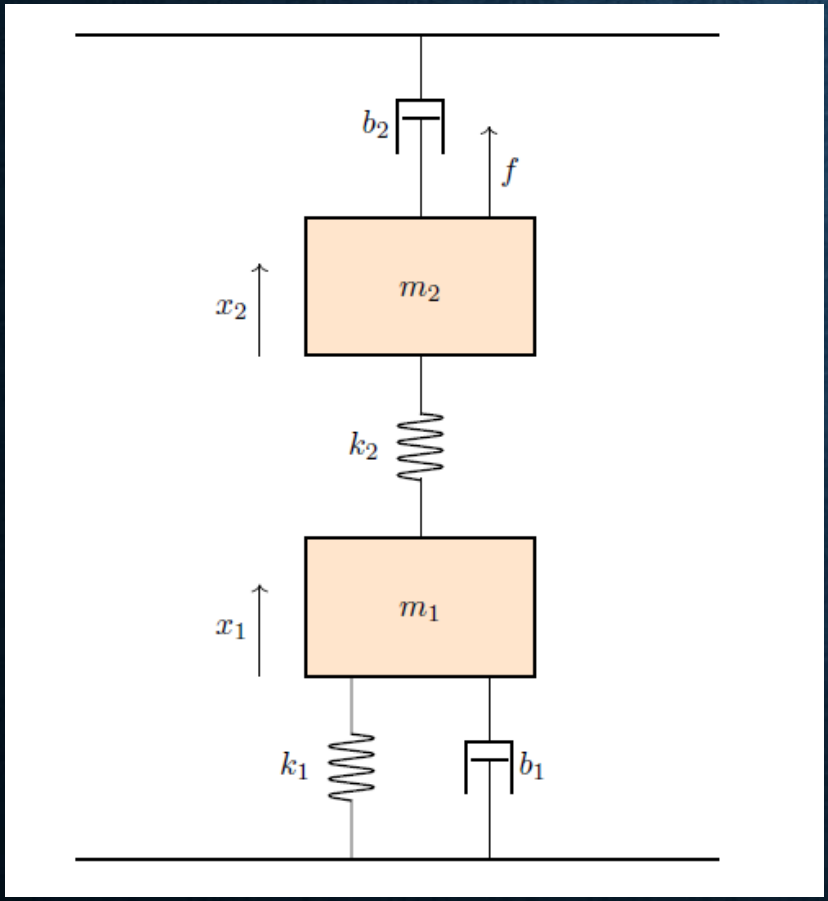

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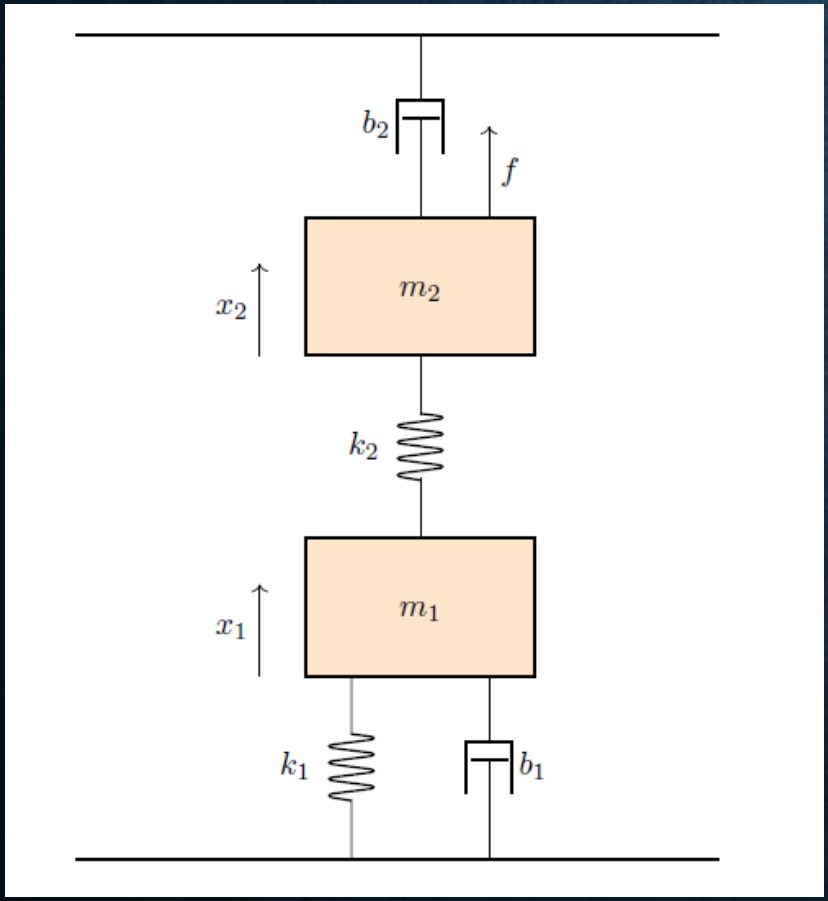

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  4. Pronaći funkciju prenosa

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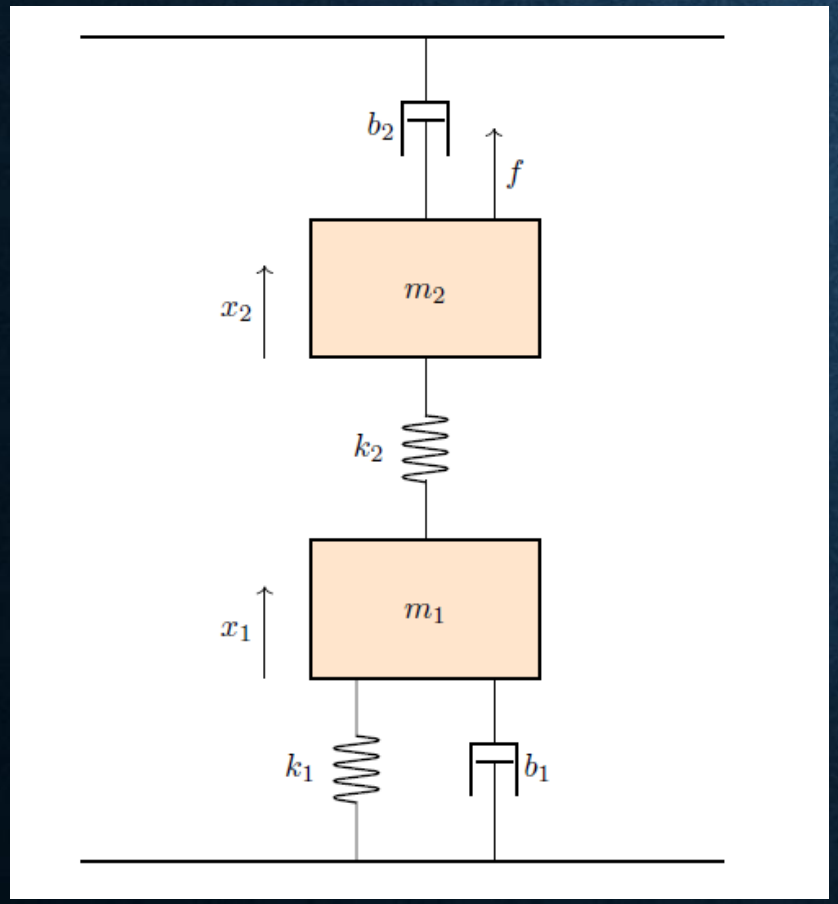

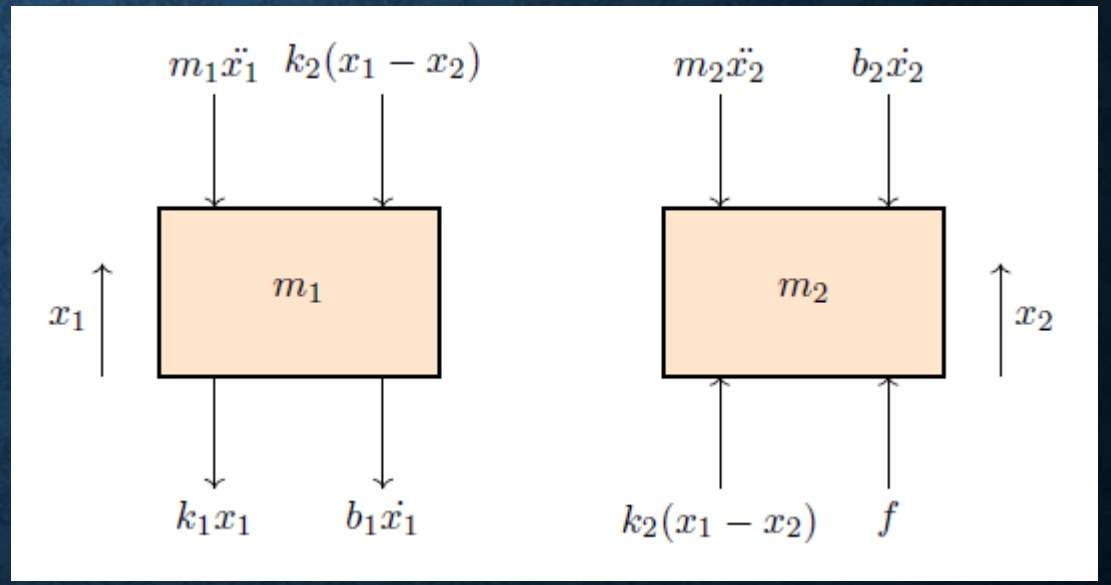

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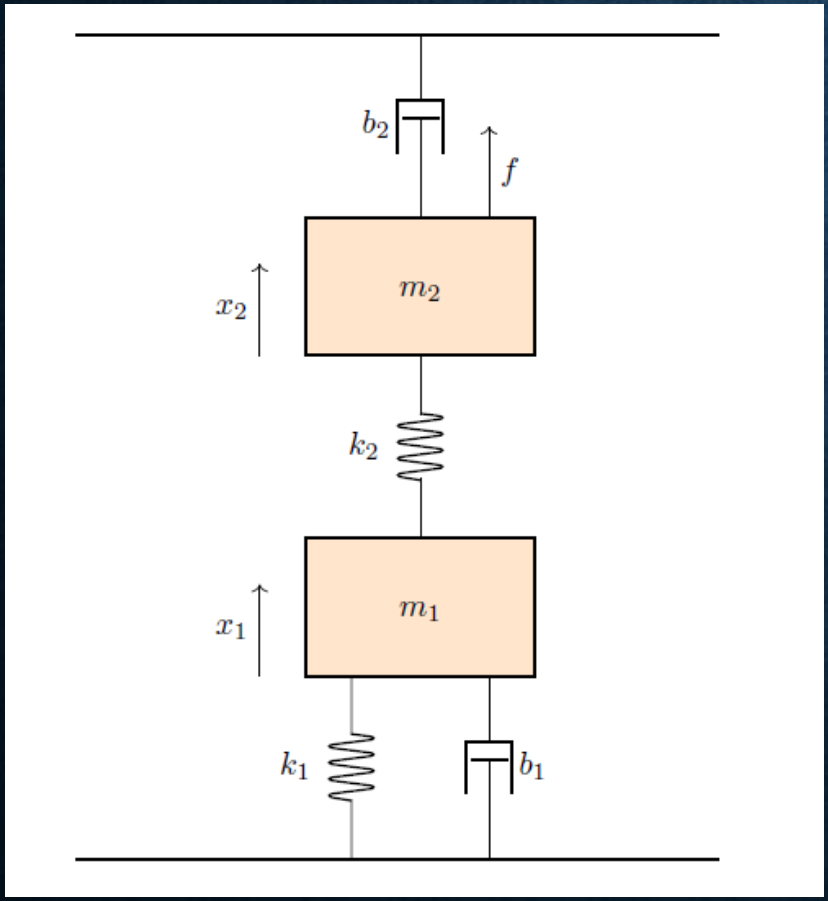

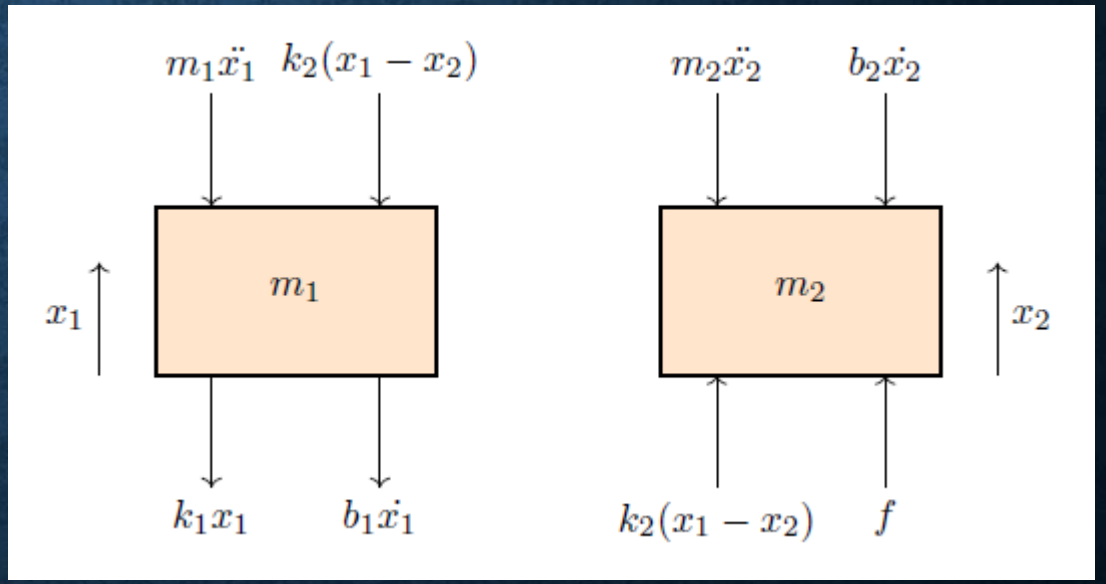

$$m_1\ddot{x}_1 + b_1\dot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = 0$$

$$m_2\ddot{x}_2 + b_2\dot{x}_2 - k_2(x_1 - x_2) = f(t)$$

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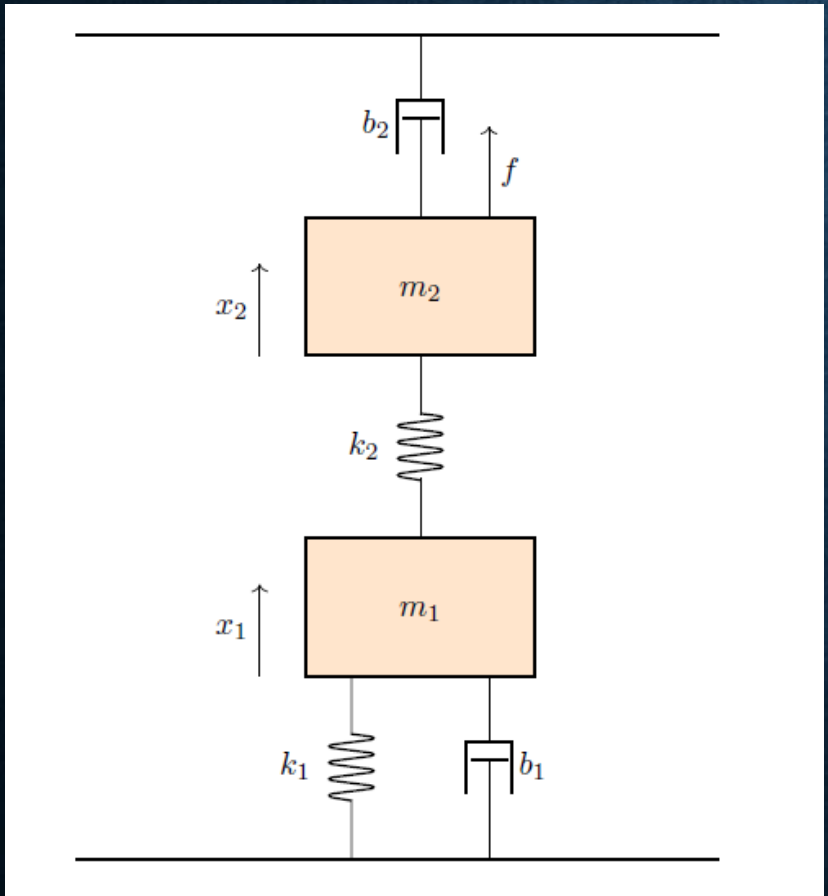

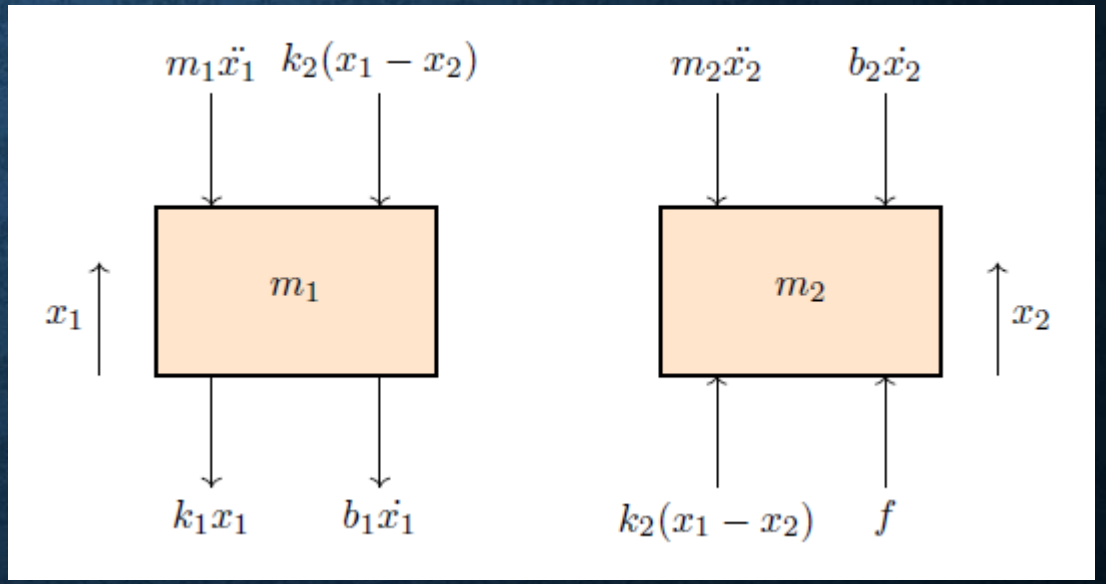

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/ ℒ

$$m_1 s^2 X_1(s) + b_1 s X_1(s) + k_1 X_1(s) + k_2 X_1(s) - k_2 X_2(s) = 0$$

$$m_2 s^2 X_2(s) + b_2 s X_2(s) - k_2 X_1(s) + k_2 X_2(s) = F(s)$$

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Na osnovu prve jednačine, izrazimo X_1 u zavisnosti od X_2 (izlaza) i dobijeni izraz uvrstimo u drugu jednačinu

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$$(m_1 s^2 + b_1 s + k_1 + k_2) * X_1(s) = k_2 X_2(s) \quad \Rightarrow \quad X_1(s) = \frac{k_2}{m_1 s^2 + b_1 s + k_1 + k_2} X_2(s)$$

$$(m_2 s^2 + b_2 s + k_2) * X_2(s) - k_2 X_1(s) = F(s)$$

$$(m_2 s^2 + b_2 s + k_2) * X_2(s) - \frac{k_2^2}{m_1 s^2 + b_1 s + k_1 + k_2} X_2(s) = F(s)$$

$$m_1 s^2 X_1(s) + b_1 s X_1(s) + k_1 X_1(s) + k_2 X_1(s) - k_2 X_2(s) = 0$$

$$m_2 s^2 X_2(s) + b_2 s X_2(s) - k_2 X_1(s) + k_2 X_2(s) = F(s)$$

Na osnovu prve jednačine, izrazimo X_1 u zavisnosti od X_2 (izlaza) i dobijeni izraz uvrstimo u drugu jednačinu

$$(m_1 s^2 + b_1 s + k_1 + k_2) * X_1(s) = k_2 X_2(s) \quad \Rightarrow \quad X_1(s) = \frac{k_2}{m_1 s^2 + b_1 s + k_1 + k_2} X_2(s)$$

$$(m_2 s^2 + b_2 s + k_2) * X_2(s) - k_2 X_1(s) = F(s)$$

$$(m_2 s^2 + b_2 s + k_2) * X_2(s) - \frac{k_2^2}{m_1 s^2 + b_1 s + k_1 + k_2} X_2(s) = F(s)$$

Uvrstimo zadate vrednosti

$$m_1 s^2 X_1(s) + b_1 s X_1(s) + k_1 X_1(s) + k_2 X_1(s) - k_2 X_2(s) = 0$$

$$m_2 s^2 X_2(s) + b_2 s X_2(s) - k_2 X_1(s) + k_2 X_2(s) = F(s)$$

Na osnovu prve jednačine, izrazimo X_1 u zavisnosti od X_2 (izlaza) i dobijeni izraz uvrstimo u drugu jednačinu

$$(m_1 s^2 + b_1 s + k_1 + k_2) * X_1(s) = k_2 X_2(s) \quad \Rightarrow \quad X_1(s) = \frac{k_2}{m_1 s^2 + b_1 s + k_1 + k_2} X_2(s)$$

$$(m_2 s^2 + b_2 s + k_2) * X_2(s) - k_2 X_1(s) = F(s)$$

$$(m_2 s^2 + b_2 s + k_2) * X_2(s) - \frac{k_2^2}{m_1 s^2 + b_1 s + k_1 + k_2} X_2(s) = F(s)$$

Uvrstimo zadate vrednosti

$$(s^2 + 8s + 16 - \frac{256}{s^2 + 10s + 25}) * X_2(s) = F(s)$$

$$m_1 s^2 X_1(s) + b_1 s X_1(s) + k_1 X_1(s) + k_2 X_1(s) - k_2 X_2(s) = 0$$

$$m_2 s^2 X_2(s) + b_2 s X_2(s) - k_2 X_1(s) + k_2 X_2(s) = F(s)$$

Na osnovu prve jednačine, izrazimo X_1 u zavisnosti od X_2 (izlaza) i dobijeni izraz uvrstimo u drugu jednačinu

$$(m_1 s^2 + b_1 s + k_1 + k_2) * X_1(s) = k_2 X_2(s) \quad \Rightarrow \quad X_1(s) = \frac{k_2}{m_1 s^2 + b_1 s + k_1 + k_2} X_2(s)$$

$$(m_2 s^2 + b_2 s + k_2) * X_2(s) - k_2 X_1(s) = F(s)$$

$$(m_2 s^2 + b_2 s + k_2) * X_2(s) - \frac{k_2^2}{m_1 s^2 + b_1 s + k_1 + k_2} X_2(s) = F(s)$$

Uvrstimo zadate vrednosti

$$(s^2 + 8s + 16 - \frac{256}{s^2 + 10s + 25}) * X_2(s) = F(s)$$

$$\frac{S^4+18s^3+121s^2+360s+144}{S^2+10s+25} * X_2(s) = F(s)$$

$$m_1 s^2 X_1(s) + b_1 s X_1(s) + k_1 X_1(s) + k_2 X_1(s) - k_2 X_2(s) = 0$$

$$m_2 s^2 X_2(s) + b_2 s X_2(s) - k_2 X_1(s) + k_2 X_2(s) = F(s)$$

Na osnovu prve jednačine, izrazimo X_1 u zavisnosti od X_2 (izlaza) i dobijeni izraz uvrstimo u drugu jednačinu

$$(m_1 s^2 + b_1 s + k_1 + k_2) * X_1(s) = k_2 X_2(s) \quad \Rightarrow \quad X_1(s) = \frac{k_2}{m_1 s^2 + b_1 s + k_1 + k_2} X_2(s)$$

$$(m_2 s^2 + b_2 s + k_2) * X_2(s) - k_2 X_1(s) = F(s)$$

$$(m_2 s^2 + b_2 s + k_2) * X_2(s) - \frac{k_2^2}{m_1 s^2 + b_1 s + k_1 + k_2} X_2(s) = F(s)$$

Uvrstimo zadate vrednosti

$$\left(s^2 + 8s + 16 - \frac{256}{s^2 + 10s + 25}\right) * X_2(s) = F(s)$$

$$\frac{S^4+18s^3+121s^2+360s+144}{S^2+10s+25} * X_2(s) = F(s) \quad \Rightarrow \quad G(s) = \frac{X_2(s)}{F(s)} = \frac{S^2+10s+25}{S^4+18s^3+121s^2+360s+144}$$

$$m_1s^2X_1(s) + b_1sX_1(s) + k_1X_1(s) + k_2X_1(s) - k_2X_2(s) = 0$$

$$m_2s^2X_2(s) + b_2sX_2(s) - k_2X_1(s) + k_2X_2(s) = F(s)$$

Na osnovu prve jednačine, izrazimo X_1 u zavisnosti od X_2 (izlaza) i dobijeni izraz uvrstimo u drugu jednačinu

$$(m_1s^2 + b_1s + k_1 + k_2) * X_1(s) = k_2X_2(s) \quad \Rightarrow \quad X_1(s) = \frac{k_2}{m_1s^2 + b_1s + k_1 + k_2} X_2(s)$$

$$(m_2s^2 + b_2s + k_2) * X_2(s) - k_2X_1(s) = F(s)$$

$$(m_2s^2 + b_2s + k_2) * X_2(s) - \frac{k_2^2}{m_1s^2 + b_1s + k_1 + k_2} X_2(s) = F(s)$$

Uvrstimo zadate vrednosti

$$(s^2 + 8s + 16 - \frac{256}{s^2 + 10s + 25}) * X_2(s) = F(s)$$

$$\frac{S^4+18s^3+121s^2+360s+144}{S^2+10s+25} * X_2(s) = F(s) \quad \Rightarrow$$

$$G(s) = \frac{X_2(s)}{F(s)} = \frac{S^2+10s+25}{S^4+18s^3+121s^2+360s+144}$$