Region description

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Reference Books:

- *Computer Vision: Algorithms and Applications*. Richard Szeliski. Springer. 2010.

http://szeliski.org/Book

Content

- Introduction
- Shape description
  - Simple descriptors
  - Fourier descriptor
- Region-content description
  - Moments
  - Texture
  - Bag-of-words (not included)

1. Introduction

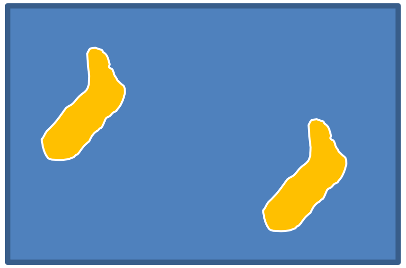
Aim:: obtain a mathematical description of a segmented region of the image with a feature vector

Example:

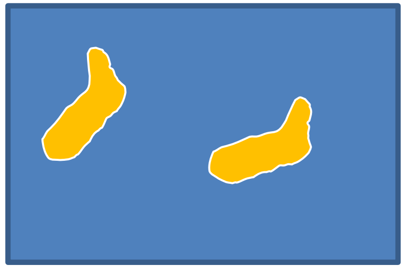
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

This vector is a compact representation of the region

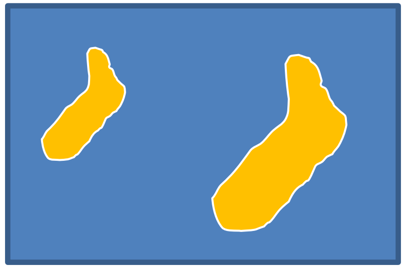
Sometimes we wish the descriptor to be invariant to **position**, **orientation** and **size (scale)** of the region in the image.



Position invariance



Orientation invariance (+position)



Scale invariance (+position)

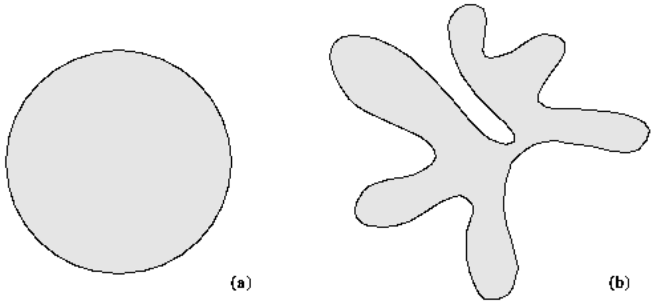
2. Shape description

Simple descriptors

How big is the area for a given perimeter

$$\boldsymbol{Compactness} = \frac{\text{area}}{\text{perimeter}^2}$$

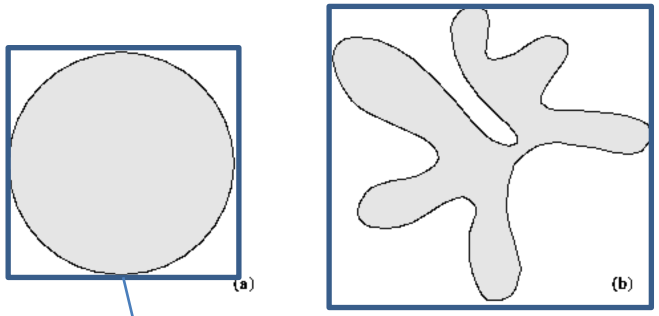
$square$ to keep compactness dimensionless



Compact Non-Compact

The most compact shape is the circle (*compactness*=1/4π)

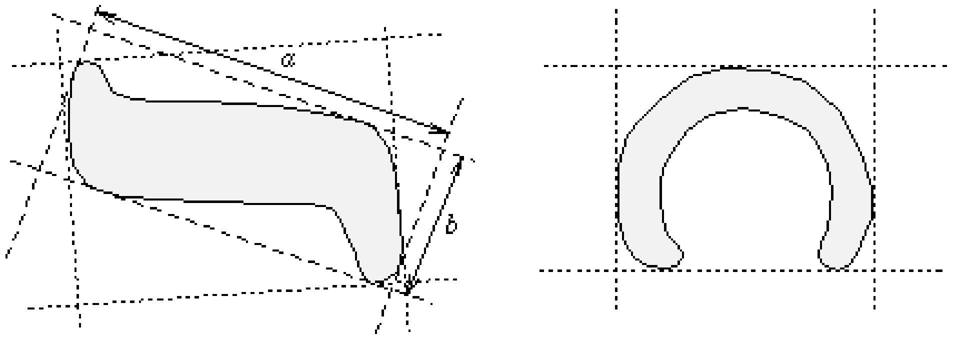
$$\boldsymbol{Extent} = \frac{\text{area}}{\text{bounding box}}$$



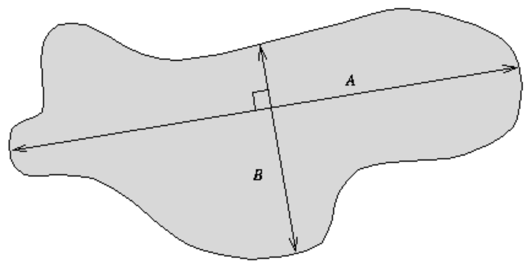
Bounding box: minimum rectangle that contains all the pixels of a region whose bottom edge is horizontal and its left edge is vertical.

Simple descriptors

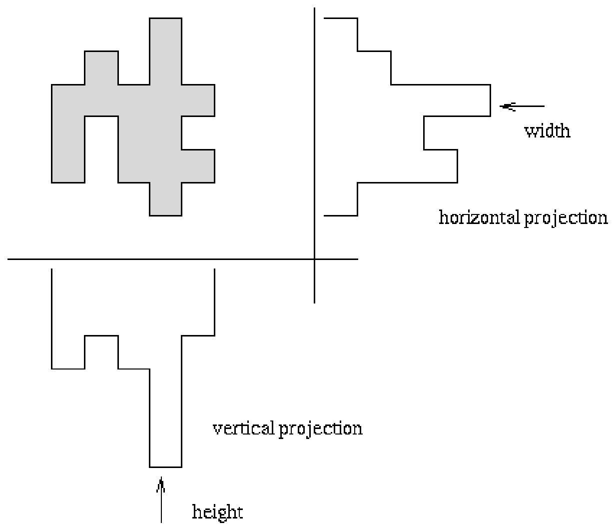
Elongation: Ratio of the height and width of a rotated minimal bounding box



Eccentricity: Ratio of the longest chord and longest perpendicular chord



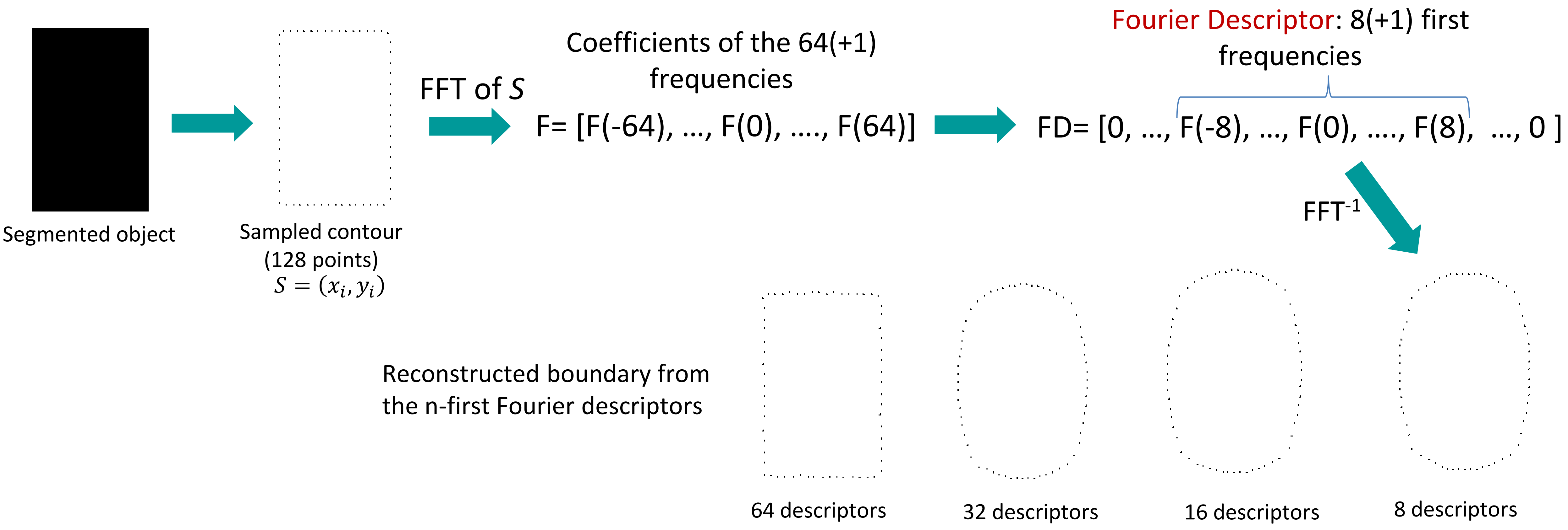
Lateral projection: histogram of the pixels within the region projected on a given axis



2. Shape description

Fourier Descriptor

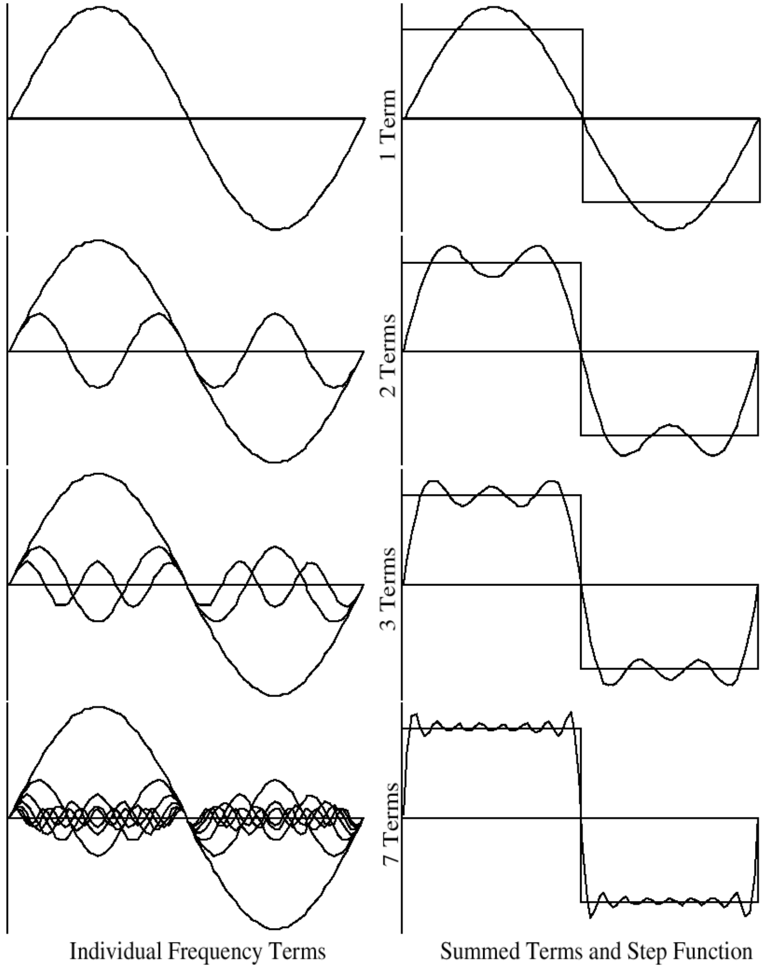
We want to represent the boundary points in the frequency domain



Easy to get invariance to: translation, scale, rotation, starting point.

Fourier Descriptor

Example of Fourier decomposition in 1D: decomposition of a step function



Fourier Descriptor

Fourier Descriptor algorithm

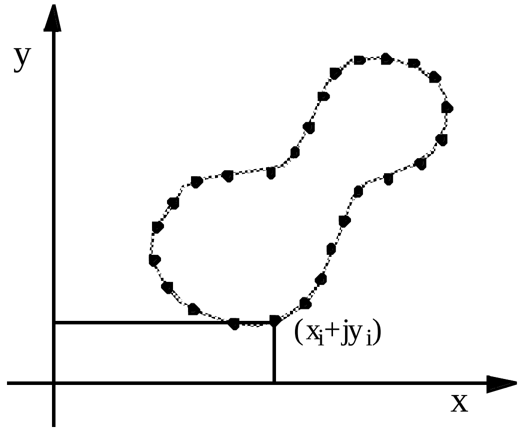
- Take the set $S = \{(x_i, y_i), i = 1, \ldots, N\}$ of boundary points
- Express S as complex points: $S = \{s(i) = x_i + jy_i, i = 1, \ldots, N\}$
- Convert S to the discrete Fourier domain: DF = fft(S)

  Get invariance to *translation, scale, rotation and starting point* if needed
- Keep the first M centered components of FD

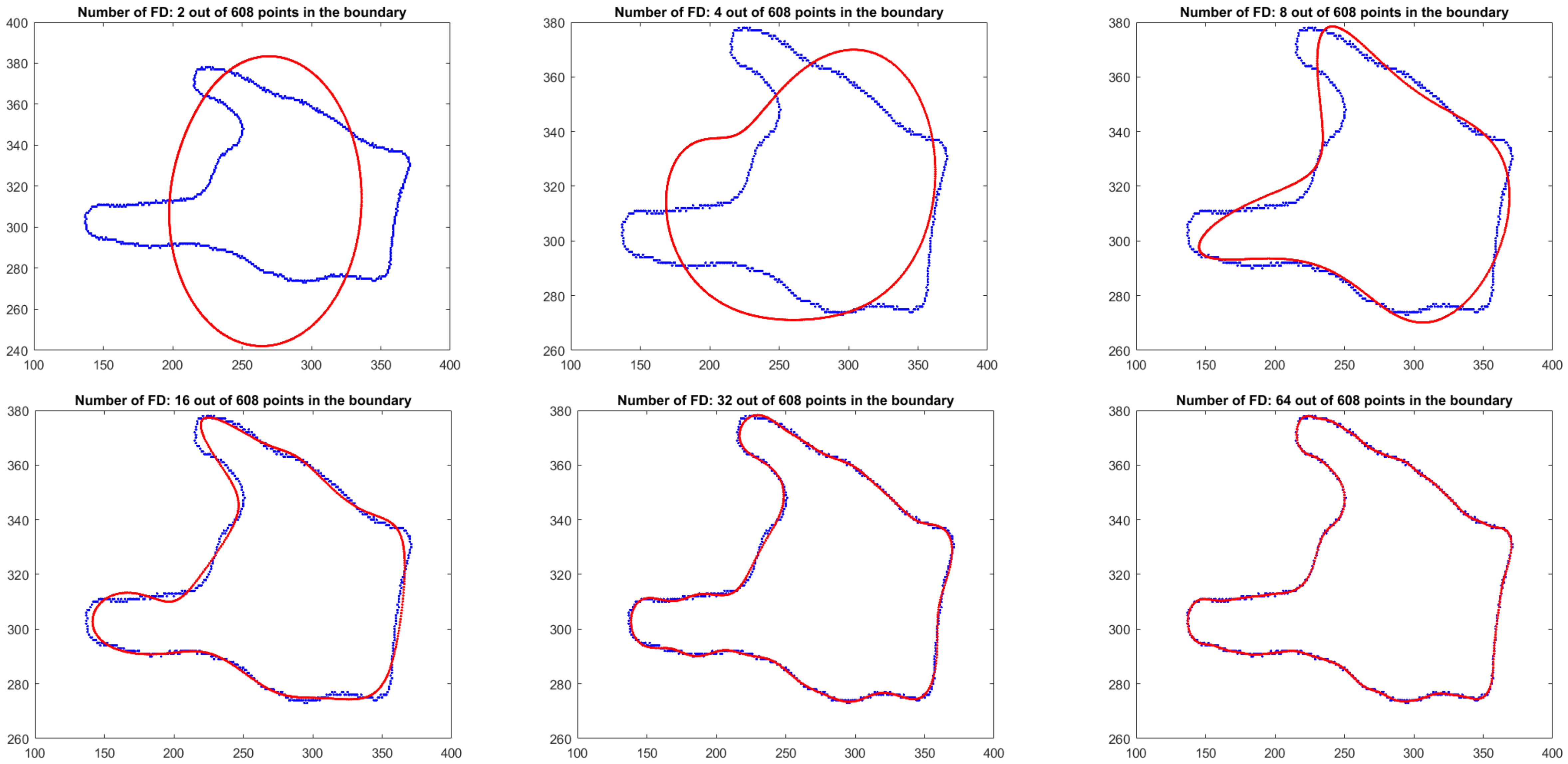
  Center the FD: FD = fftshift(FD)

  FD = FD[0, …, 0, FD(N/2-M/2:N/2+M/2), 0, …, 0]

  The shape represented by this vector can be recovered by the inverse FFT



Fourier Descriptor



3. Region description

Moments

No central

Central

1 dimension:

$$m_n = \int_{-\infty}^{\infty} x^n \ f(x)\, dx \qquad \mu_n = \int_{-\infty}^{\infty} (x - \overline{x})^n \ f(x)\, dx$$

$$m_n = \sum_{x=1}^{N} x^n \ f(x) \qquad \mu_n = \sum_{x=1}^{N} (x - \overline{x})^n \ f(x)$$

Applicable to histograms (i:intensity, f(i): #pixels for intensity i)

$$m_0 = \sum_{i=0}^{L-1} f(i) = N$$

Number of pixels in the image

$$\overline{i} = \frac{m_1}{m_0} = \frac{1}{N} \sum_{i=0}^{L-1} i.f(i)$$

Average intensity/brightness of the image

$$\sigma^2 = \frac{1}{N} \sum_{i=0}^{L-1} (i - \overline{i})^2 f(i) = \frac{\mu_2}{m_0}$$

Contrast of the image

3. Region description

Moments: 2 dimensions (e.g. image)

No central: $$m_{ij} = \sum_{y=1}^{N} \sum_{x=1}^{M} x^i y^j \; f(x,y)$$

Central: $$\mu_{ij} = \sum_{y=1}^{N} \sum_{x=1}^{M} (x-\bar{x})^i (y-\bar{y})^j \; f(x,y)$$

$$\mu_{00} = m_{00} \equiv \mu$$
$$\mu_{01} = 0$$
$$\mu_{10} = 0$$
$$\mu_{20} = m_{20} - \mu\bar{x}^2$$
$$\mu_{11} = m_{11} - \mu\bar{x}\bar{y}$$
$$\mu_{02} = m_{02} - \mu\bar{y}^2$$
$$\mu_{30} = m_{30} - 3m_{20}\bar{x} + 2\mu\bar{x}^3$$
$$\mu_{21} = m_{21} - m_{20}\bar{y} - 2m_{11}\bar{x} + 2\mu\bar{x}^2 y$$
$$\mu_{12} = m_{12} - m_{02}\bar{x} - 2m_{11}\bar{y} + 2\mu\bar{x}y^2$$
$$\mu_{03} = m_{03} - 3m_{02}\bar{y} + 2\mu\bar{y}^3$$

Relation between central and no-central moment:

Centroid (mean): $$(\bar{x},\bar{y}) = \left(\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}}\right)$$

3. Region description

Moments

HU Moments. Invariants to scale, position and rotation

$$v_1 = u_{20} + u_{02}$$

$$u_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}} \quad \gamma = \frac{p+q}{2} + 1$$

$$v_2 = (u_{20} - u_{02})^2 + 4u_{11}^2$$

$$v_3 = (u_{30} - 3u_{12})^2 + (3u_{21} - u_{03})^2$$

$$v_4 = (u_{30} + u_{12})^2 + (u_{21} + u_{03})^2$$

$$\begin{aligned} v_5 = {} & (u_{30} - 3u_{12})(u_{30} + u_{12})\left[(u_{30} + u_{12})^2 - 3(u_{21} + u_{03})^2\right] \\ & + (3u_{21} - u_{03})(u_{21} + u_{03})\left[3(u_{30} + u_{12})^2 - (u_{21} + u_{03})^2\right] \end{aligned}$$

$$\begin{aligned} v_6 = {} & (u_{20} - u_{02})\left[(u_{30} + u_{12})^2 - (u_{21} - u_{03})^2\right. \\ & \left. + 4u_{11}(u_{30} + u_{12})(u_{21} + u_{03})\right] \end{aligned}$$

$$\begin{aligned} v_7 = {} & (3u_{21} - u_{03})(u_{30} + u_{12})\left[(u_{30} + u_{12})^2 - 3(u_{30} + u_{12})^2\right] \\ & + (u_{30} - 3u_{12})(u_{21} + u_{03})\left[3(u_{30} + u_{12})^2 - (u_{21} + u_{03})^2\right] \end{aligned}$$

3. Region description

Moments

<u>Example of use</u>: Centroid and orientation of the region (not necessary binary)

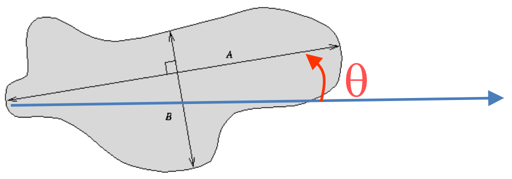
Centroid: $(\bar{x}, \bar{y}) = \left(\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}} \right)$

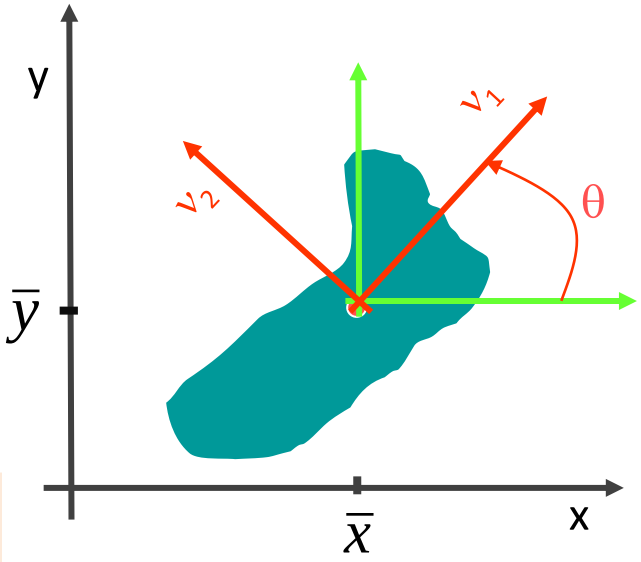
Covariance matrix: $\frac{1}{\mu_{00}} \begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix}$ In the eigenvector base (v_1, v_2) $\Rightarrow \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

(matrix of moments of order 2)

Orientation: $\theta = \frac{1}{2} \tan^{-1} \frac{v_{1y}}{v_{1x}}$

Eccentricity: $\frac{\lambda_2}{\lambda_1}$



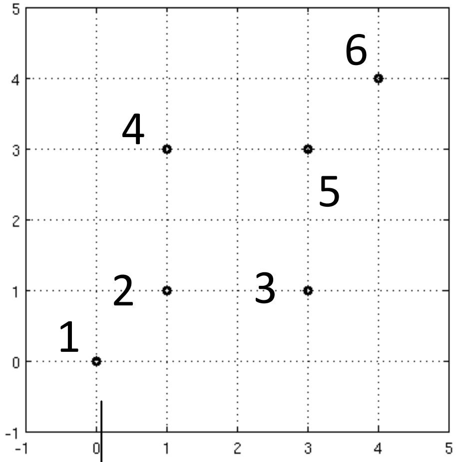


For binary regions, orientation and eccentricity make more sense

Computing the Centroid and Orientation of a region

$$m_{ij} = \sum_{x}^{N} \sum_{y}^{M} x^i y^j f(x,y)$$

$$\mu_{ij} = \sum_{x}^{N} \sum_{y}^{M} (x-\bar{x})^i (y-\bar{y})^j f(x,y)$$



$$(\bar{x},\bar{y}) = \left(\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}}\right)$$

$$\Sigma = \frac{1}{\mu_{00}} \begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix}$$

$$P = \{p_i\} = \begin{bmatrix} 0 & 1 & 3 & 1 & 3 & 4 \\ 0 & 1 & 1 & 3 & 3 & 4 \end{bmatrix}$$

$$M_{00} = m_{00} = \sum f x^0 y^0 = 6$$

$$f = 1$$

$$P = \begin{bmatrix} 0 & 1 & 3 & 1 & 3 & 4 \\ 0 & 1 & 1 & 3 & 3 & 4 \end{bmatrix} \rightarrow \Delta p_i = p_i - \bar{p} = \begin{bmatrix} x_i - \bar{x} \\ y_i - \bar{y} \end{bmatrix}_{i=1,6} = \begin{bmatrix} -2 & -1 & +1 & -1 & 1 & 2 \\ -2 & -1 & -1 & 1 & 1 & 2 \end{bmatrix}$$

$$\bar{p} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \end{bmatrix} \frac{1}{6} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\mu_{20} = \sum_{i=1}^{6} (x_i - \bar{x})^2 \underbrace{f}_{=1} = 4+1+1+1+1+4 = 12$$

$$\mu_{02} = 12$$

$$\mu_{11} = 4 + 1 \cancel{+} 1 - 1 + 1 + 4 = 8$$

$$\rightarrow \quad \Sigma = \frac{1}{6} \begin{bmatrix} 12 & 8 \\ 8 & 12 \end{bmatrix}$$

3. Region description

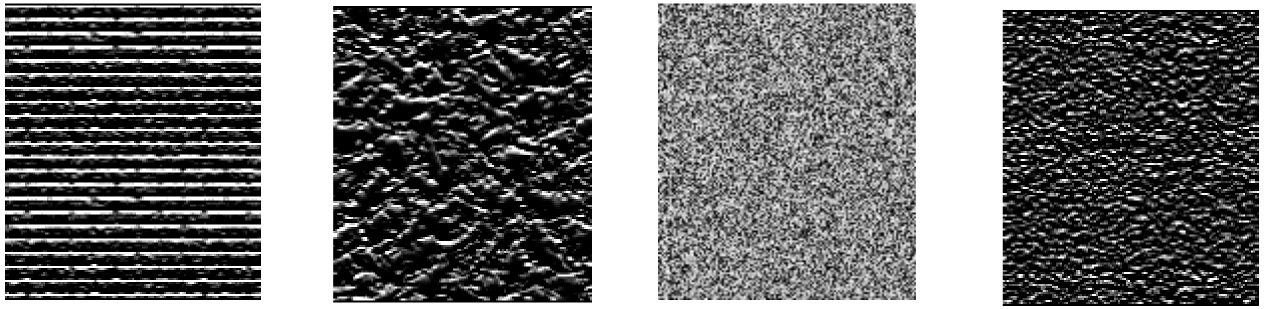
Texture

- **Definition**: measures the *spatial arrangement* of the colours/intensities in an image

  describes properties such as smoothness, coarseness, and regularity

  For example, if a region does not present changes in intensity we say that is a untextured region

EXAMPLE OF DIFERENT TEXTURES



- Usually, texture descriptos have spatial (position, orientation, scale), and radiometric (contrast and brightness) invariance.

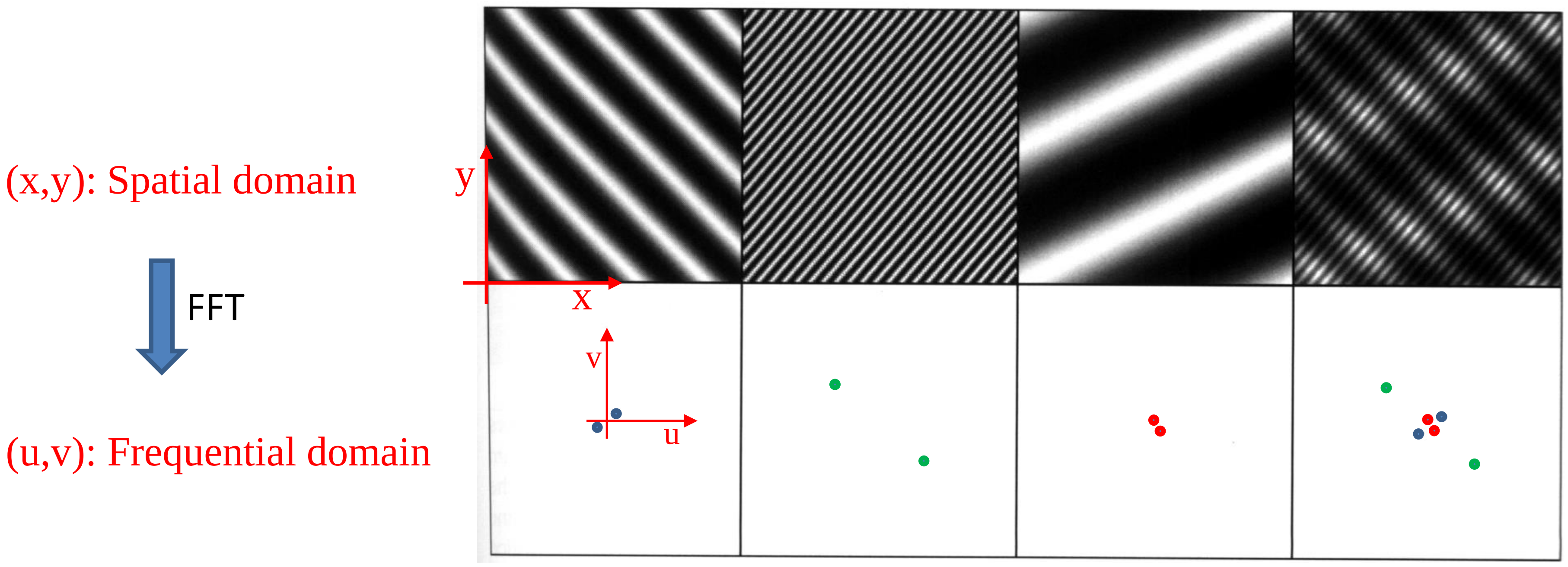
- Useful for region classification and image segmentation

3. Region description

Texture: SPECTRAL APPROACH

Based on identifying narrow peaks in the Fourier spectrum

<u>Example</u>: Pure sinusoidal images and their frequency spectrum



3. Region description

Texture: STATISTICAL APPROACH

Central moments of the Histogram (histogram: $h(z_i)$)

$$\mu_n = \sum_{i=1}^{N} (z_i - \bar{z})^n \; h(z_i)$$

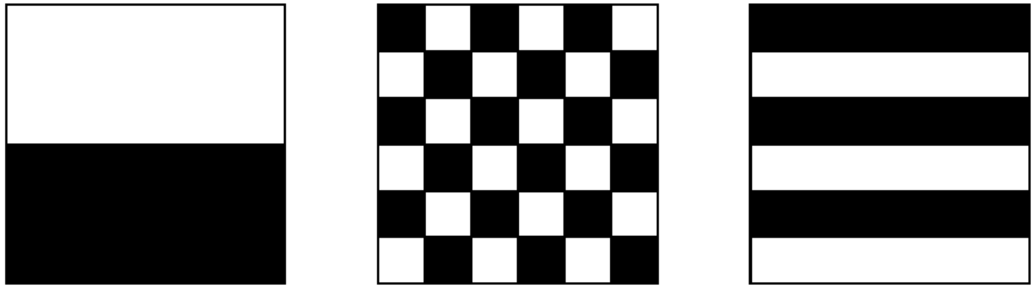
$\mu_0 = \mu_1 = 0$

μ_2: variance (contrast)

μ_3: histogram skew

μ_4: histogram uniformity

Capture frequency of intensities but no pattern structures.



block pattern **checkerboard** **striped pattern**

All these patterns have the same histogram → same moments

Texture: STATISTICAL APPROACH: Co-occurrence matrix

Co-occurrence matrix: is a square matrix A(i,j) in which

- i and j represent intensity values (e.g.: 0 to 255)
- the entry a_{ij} indicates how many times the intensity i co-occurs with intensity j in some designated *spatial relationships P* (**texture pattern**).
- *P* is given by a displacement vector $d = [d_c, d_R]$, where d_c and d_R are the displacement in columns (i) and rows (j), respectively.

<u>Example</u>: *P= "below and to the right 1 pixel"* → $d = [1,1]$

$$\begin{array}{c|ccccc} & \multicolumn{5}{c}{i \rightarrow} \\ \hline & 0 & 0 & 0 & 1 & 2 \\ j \downarrow & 1 & 1 & 0 & 1 & 1 \\ & 2 & 2 & 1 & 0 & 0 \\ & 1 & 1 & 0 & 2 & 0 \\ & 0 & 0 & 1 & 0 & 1 \end{array}$$

Image with 3 intensity level {0, 1, 2}

value 0 four times at [1,1] from values 0

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix} \xrightarrow{\text{normalized}} C = \frac{1}{16}\begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

value 1 four times at [1,-1] from values 2

$$\sum_{ij} a_{ij}$$

Texture: STATISTICAL APPROACH : Co-occurrence matrix

Features from the co-ocurrence matrix

Maximum probability: gives us the strongest response to the texture pattern P

$$\max_{ij} c_{ij}$$

Energy: mínimum when all the entries c_{ij} are identical (máximum uniformity)

$$\sum_{i=1}^{L}\sum_{j=1}^{L} c_{ij}^{2}$$

Entropy: measure randomness. Maximum value when all the entries c_{ij} are identical (máximum entropy → mínimum mínima energía)

$$-\sum_{i=1}^{L}\sum_{j=1}^{L} c_{ij} \log c_{ij}$$

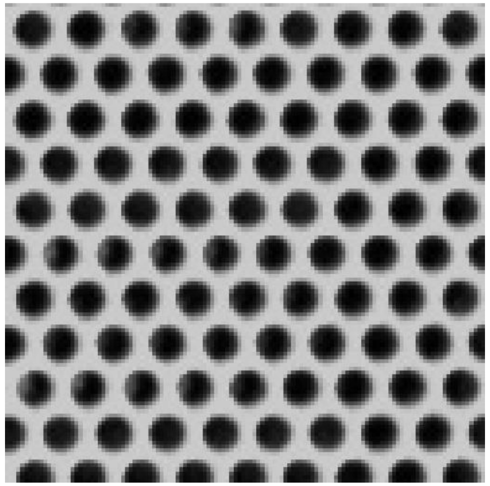
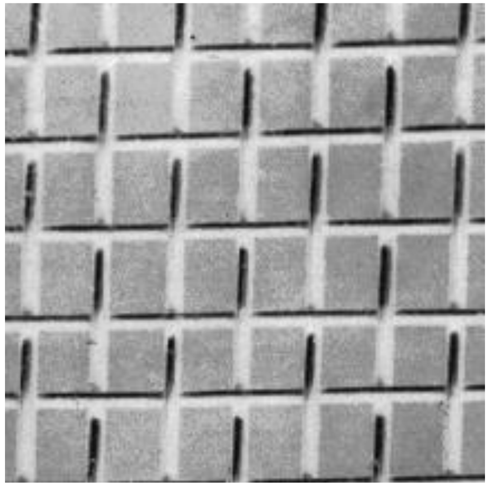
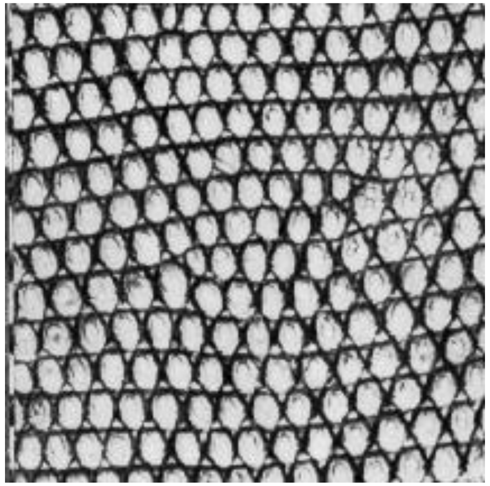
Order k central moment

$$\sum_{i=1}^{L}\sum_{j=1}^{L} (i-j)^{k} c_{ij}$$

One **big issue** with this approach (co-ocurrence matrix) is how to select the appropriate displacement d

Texture: STATISTICAL APPROACH: Histogram of textons

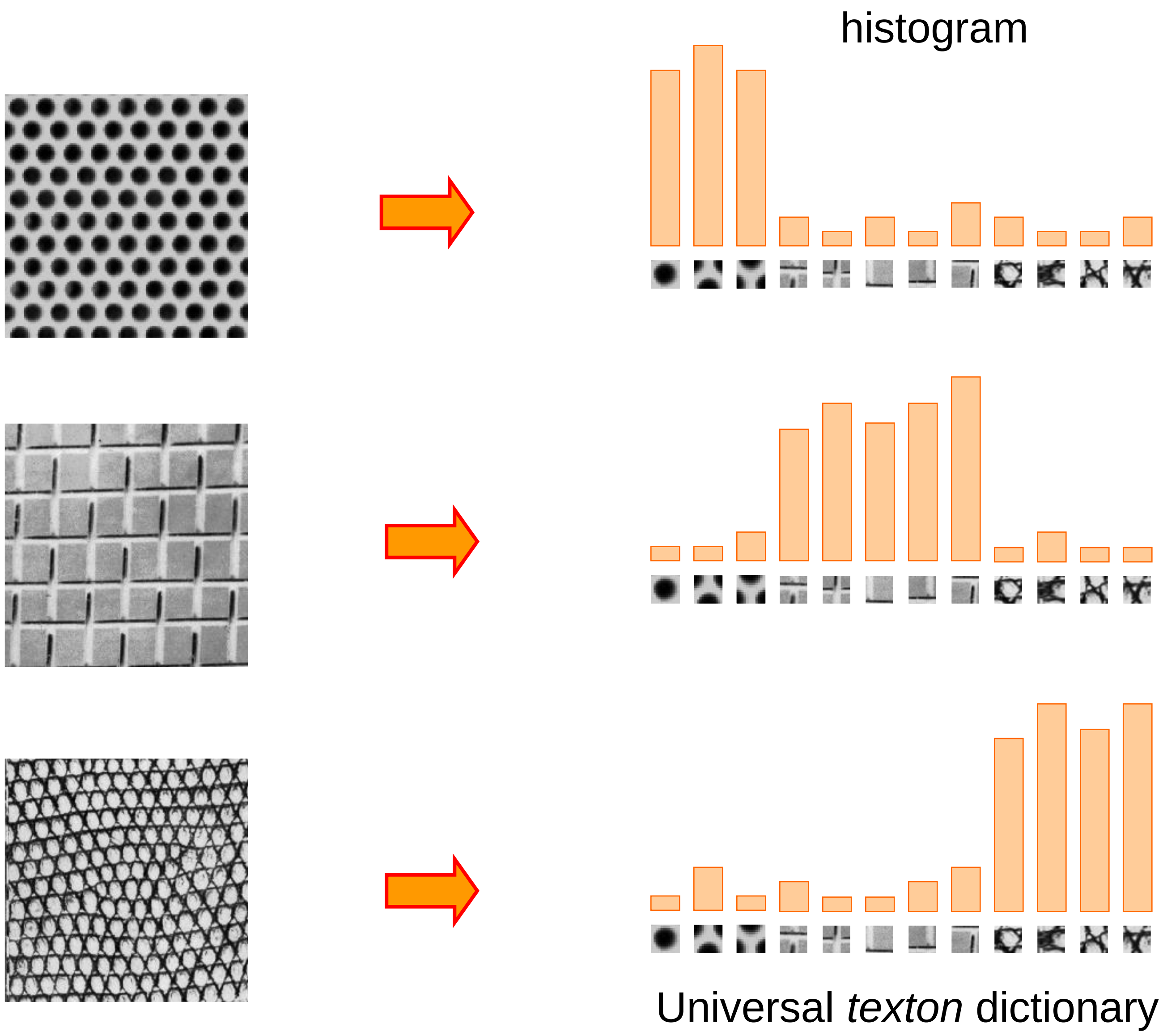
- Texture is characterized by the repetition of basic elements or *textons*
- It is the number of the *textons* what matters, not where they are (i.e. no need to define a displacement d)

textons



Texture: STATISTICAL APPROACH : Histogram of textons



Summary

- Segmented objects are represented by a **feature vector**
- Feature vector described either the **shape** or the **full region content**
- **Shape descriptors**: compactness, eccentricity, … and Fourier descriptor (much powerful)
- **Moments** are simple and powerful region content descriptor
- Moment Invariance achieved by **Hu Moments**
- Object **centroid and orientation** computed from moments
- Texture descriptors capture pattern repetitions in the region: **co-occurrence matrix** and **histogram of textons**