Keypoint detection and description

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Reference Books:

- *Computer Vision: Algorithms and Applications*. Richard Szeliski. Springer. 2010.

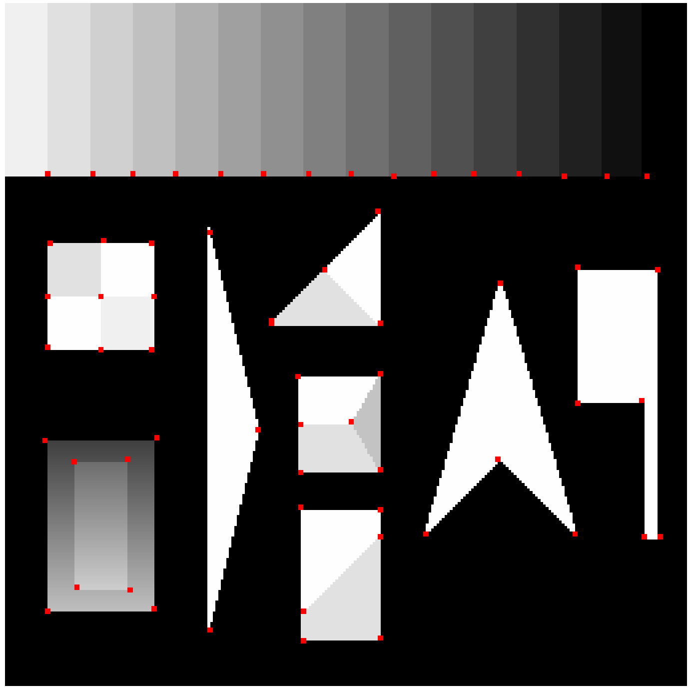
http://szeliski.org/Book

Content

1. Introduction

What are *keypoints* (also, *interest* or *feature points*)?

Distinctive pixels in the image that can likely be projections on 3D entities.



Synthetic image

Real image

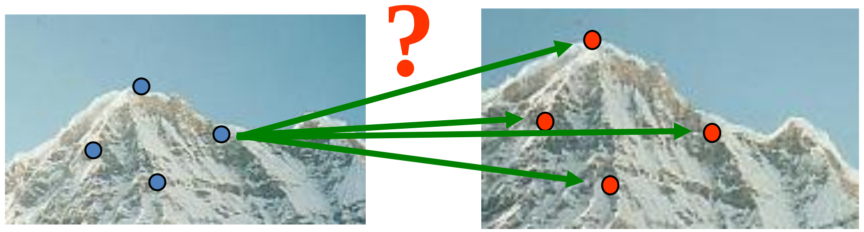
1. Introduction

Two problems involved: Detection and description

1. Detection

The operator must provide a reliable and repeatable response

2. Description: To match to its correspondence in other images



Invariant and discriminative description needed

Harris is just a detector (not a descriptor). The keypoint is described with a surrounding image patch
SIFT is a detector and also provides its own descriptor

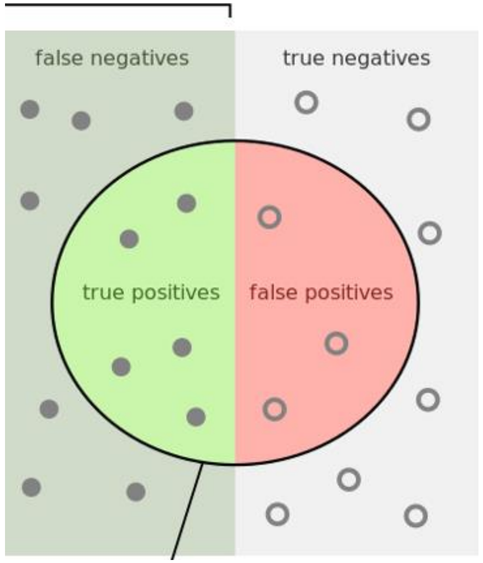
1. Introduction

Desired properties:

Detector:

- Accurate (subpixel accuracy, if possible)
- Detect all the keypoints in the image (few FALSE NEGATIVES)
- NOT detect irrelevant points (few FALSE POSITIVES)

Keypoints in the image
(Ground truth)



Detected keypoints
(Prediction)

few FALSE NEGATIVES → Wanted: High RECALL (to improve the SENSITIVITY of the detector)

$$= \frac{TP}{\boxed{TP + FN}}$$

actually positive

Among all matches that are actually positive, what percentage is predicted positive

few FALSE POSITIVES → Wanted: High PRECISION (to reduce the false alert rate)

$$= \frac{TP}{\boxed{TP + FP}}$$

predicted positive

Among all matches that were predicted positive, what percentage is predicted positive

Desired properties:

Descriptor should have *invariance* to:

✓ Illumination (changes in brightness and contrast)

✓ View point (scale, rotación, projective distortion)

Detector and descriptor: Computationally efficient and robust to image noise

1. Introduction

Applications

- Panoramic images (image sticking)
- 3D Reconstruction
- Object tracking
- Object recognition
- Image retrieval and indexing in database
- Robot navigation: mapping, localization, obstacle detection
- …. and many others

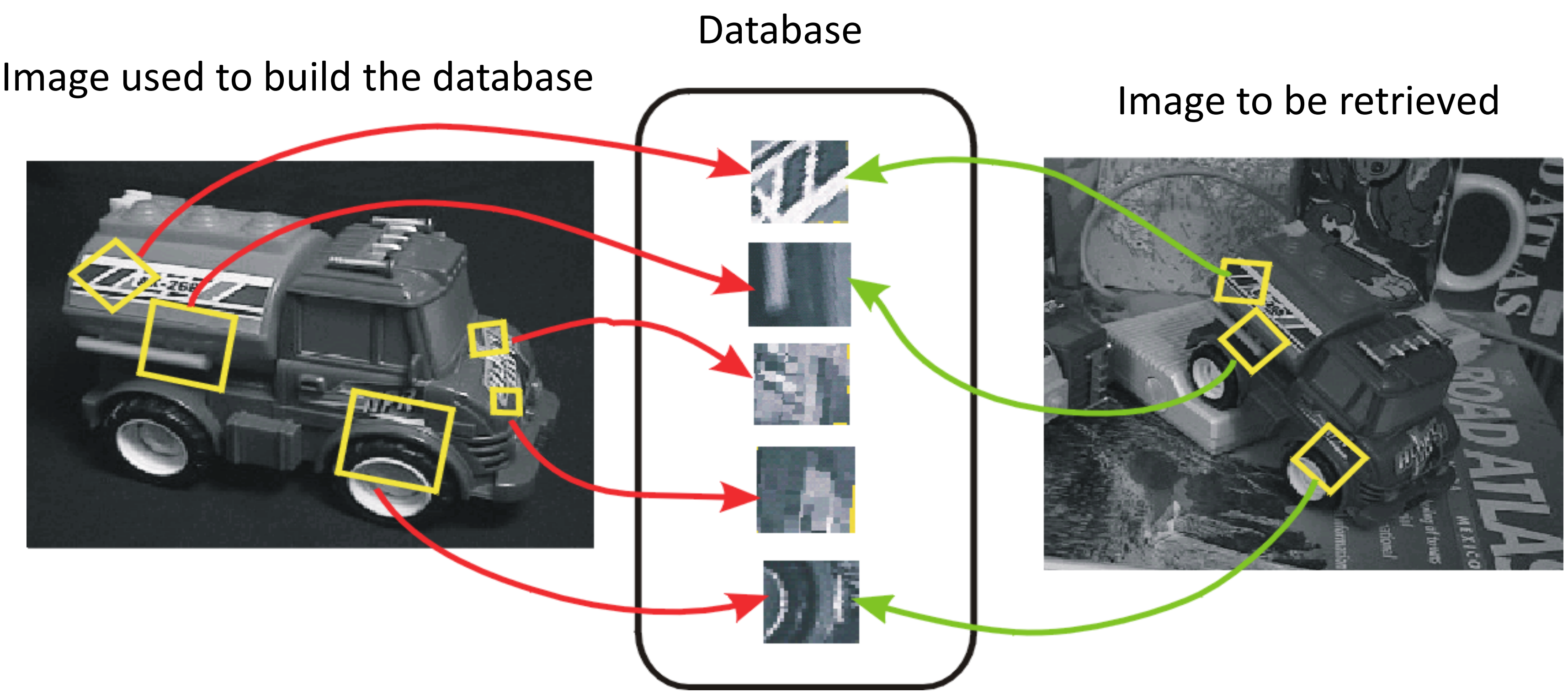
1. Introduction

Applications: Panoramic images (image stitching)

M. Brown and D. G. Lowe. Recognising Panoramas. ICCV 2003

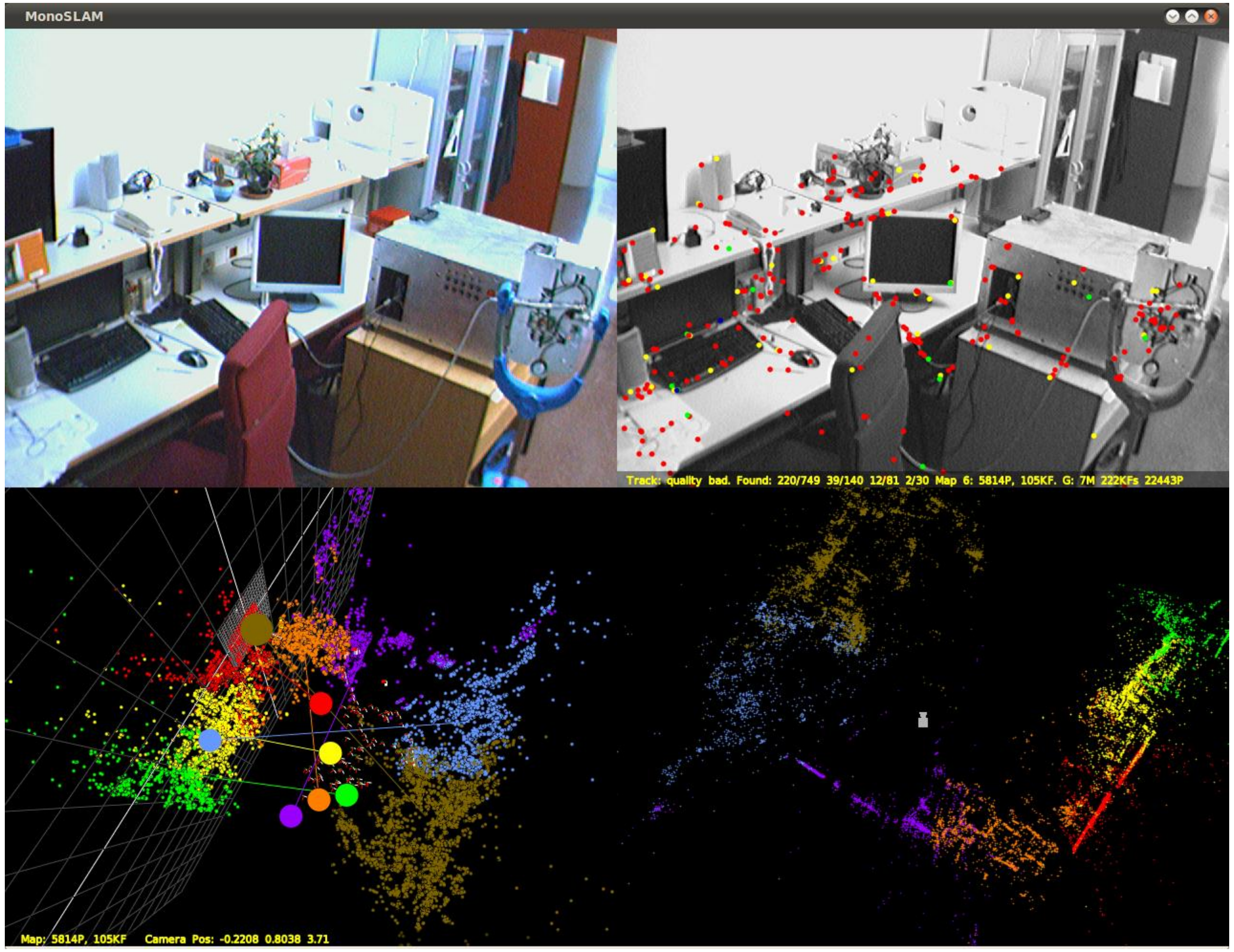
1. Introduction

Applications: Image retrieval and indexing in database



1. Introduction

Applications: Localization and 3D Reconstruction



2. Harris detector

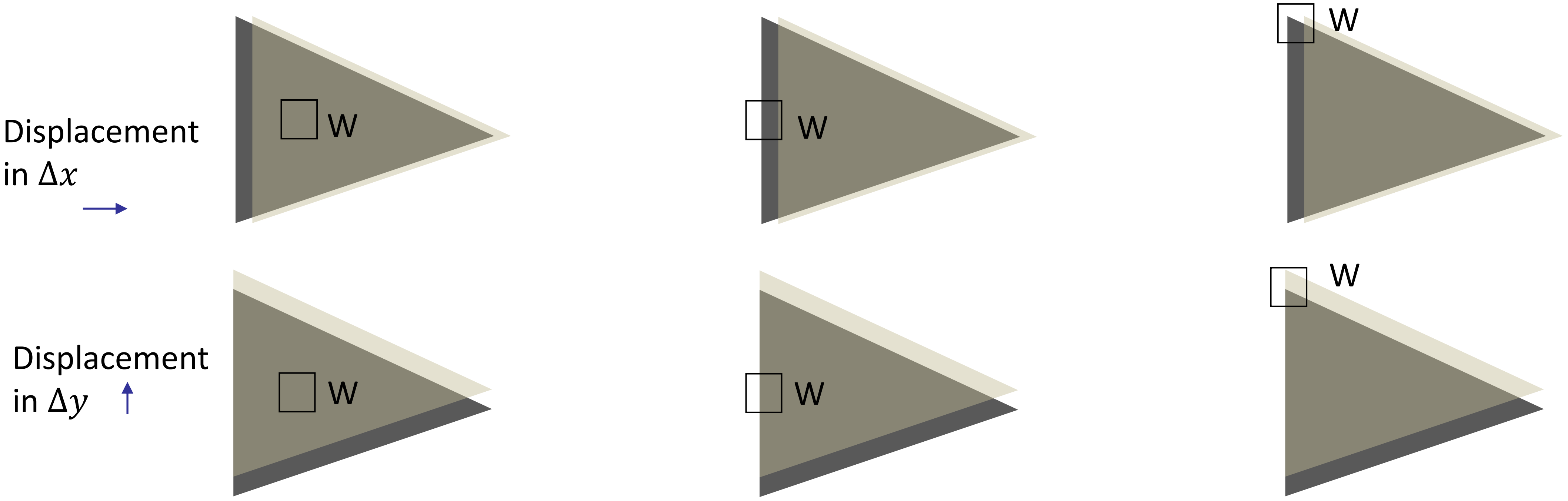
- Detect corners
- Simple and efficient implementation
- Robustness to noise (apply smoothing)
- Invariance to
  - Rotation: uses eigenvectors
  - Brightness (partially to contrast): uses derivatives ☺
- **Not invariance to scale** ☹

C.Harris, M.Stephens. "A Combined Corner and Edge Detector".
Proceedings of The Fourth Alvey Vision Conference, *1988*

2. Harris detector

- A keypoint is a corner (“esquina”)
- A corner is a point with high variation of intensity in 2 spatial directions

Basic idea: look in a small window W around a pixel if the displacements of the image in two directions provoke changes



“flat” region:
No change in W

“edge”:
change in W only if motion in the Δx direction

“corner”:
change in W if motion in both the Δx and Δy directions

Detecting the local change in intensity due to a shift $(\Delta x, \Delta y)$:

Sum-of-square weighted difference at a pixel $[x_0 y_0]$ when a window image I is shifted $(\Delta x, \Delta y)$:

$$E_{x_o y_0}(\Delta x, \Delta y) = \sum_{x,y} w(x,y)[I(x+\Delta x, y+\Delta y) - I(x,y)]^2$$

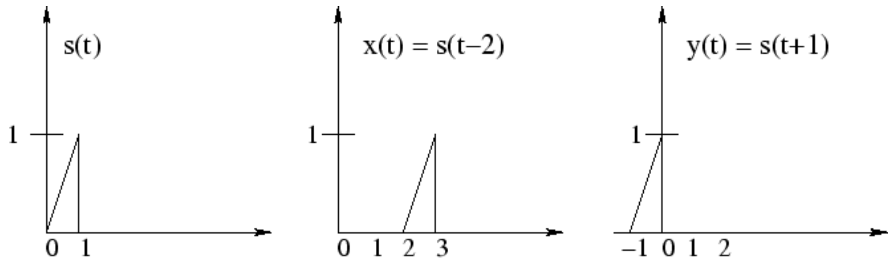
Weighting window centered at $[x_0 y_0]$

Image shifted $(\Delta x, \Delta y)$

Image

RECALL:

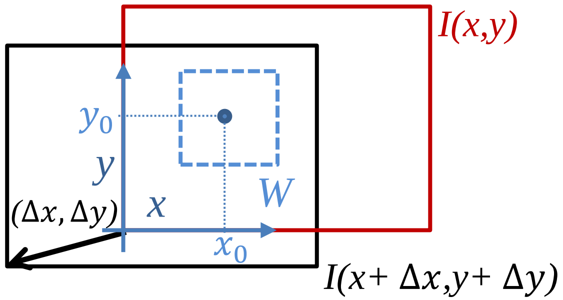
1D signal shifting



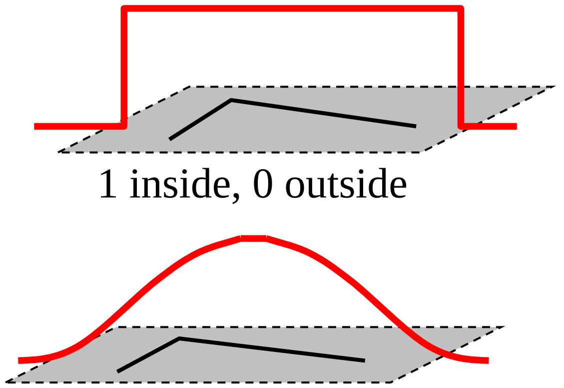
Right shifting

Left shifting

2D Left-down image shifting



Weighting function *w(x,y)* may be:



Boolean

1 inside, 0 outside

Gaussian

Gaussian weighting

Let's make more practical the computation of $E_{x_o y_0}(\Delta x, \Delta y)$

$$E_{x_o y_0}(\Delta x, \Delta y) = \sum_{x,y} w(x,y)[I(x+\Delta x, y+\Delta y) - I(x,y)]^2 = \sum_{(x_i,y_i)\in W} [\underline{I(x_i+\Delta x, y_i+\Delta y)} - I(x_i,y_i)]^2$$

Sum only over a boolean window W

First order Taylor approximation

Image derivative at (x_i, y_i)along the y axis

$$E(\Delta x, \Delta y) \approx \sum_{(x_i,y_i)\in W} \left[\cancel{I(x_i,y_i)} + [I_x(x_i,y_i) \quad I_y(x_i,y_i)] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} - \cancel{I(x_i,y_i)} \right]^2$$

$$= [\Delta x \ \Delta y] \sum_{(x_i,y_i)\in W} \begin{bmatrix} \left(I_x(x_i,y_i)\right)^2 & I_x(x_i,y_i) I_y(x_i,y_i) \\ I_x(x_i,y_i) I_y(x_i,y_i) & \left(I_y(x_i,y_i)\right)^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$(A^T B)^2 = (A^T B)^T (A^T B)$
$= B^T A A^T B = B^T M B$

$$E_{x_o y_0}(\Delta x, \Delta y) \approx [\Delta x \ \Delta y] \underbrace{\begin{bmatrix} \sum_W \left(I_x(x_i,y_i)\right)^2 & \sum_W I_x(x_i,y_i) I_y(x_i,y_i) \\ \sum_W I_x(x_i,y_i) I_y(x_i,y_i) & \sum_W \left(I_y(x_i,y_i)\right)^2 \end{bmatrix}}_{M} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = [\Delta x \ \Delta y] M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

M is computed from the **first derivatives at each image point** $(x_o y_0)$

$E_{x_o y_0}(\Delta x, \Delta y)$ is a **quadratic polynomial** which coefficients are the entries of M

Understanding M

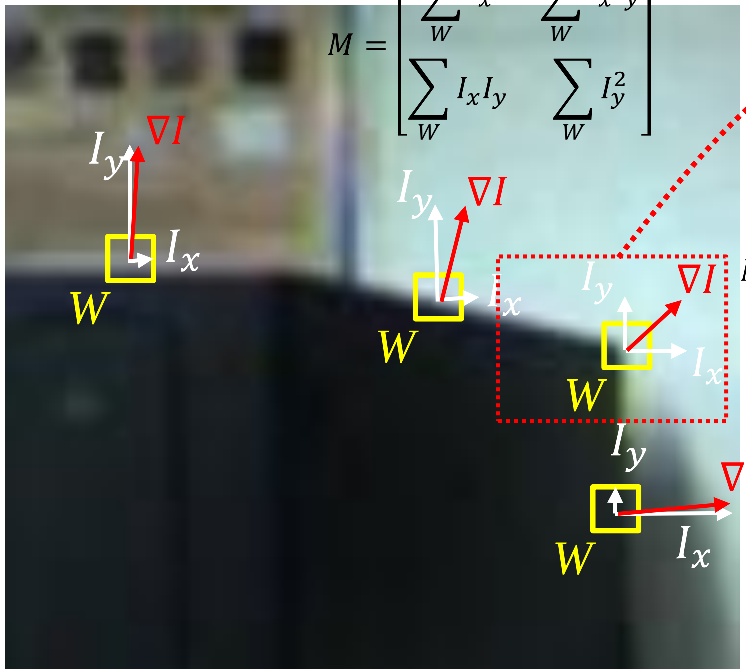
Summation of all the square derivatives **along the x-axis** of the points inside W

$$M = \begin{bmatrix} \sum_W I_x^2 & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y^2 \end{bmatrix}$$

M is computed from the **first derivatives at each image point** $(x_o y_0)$

$$M = \begin{bmatrix} \sim 0 & \sim 0 \\ \sim 0 & \sum_W I_y^2 \end{bmatrix}$$

$$M = \begin{bmatrix} \sum_W I_x^2 & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y^2 \end{bmatrix}$$

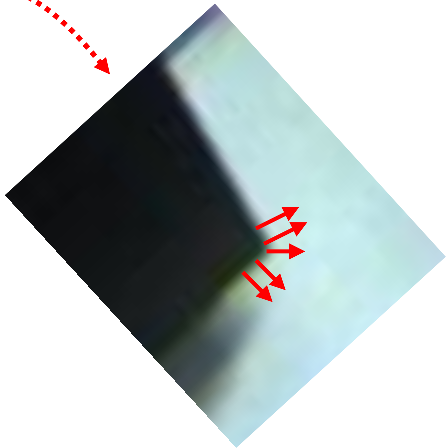


$$M = \begin{bmatrix} \sum_W I_x^2 & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y^2 \end{bmatrix}$$

$$M = \begin{bmatrix} \sum_W I_x^2 & \sim 0 \\ \sim 0 & \sim 0 \end{bmatrix}$$

Derivative vector at different pixels (always perpendicular to the border)

Same corner rotated

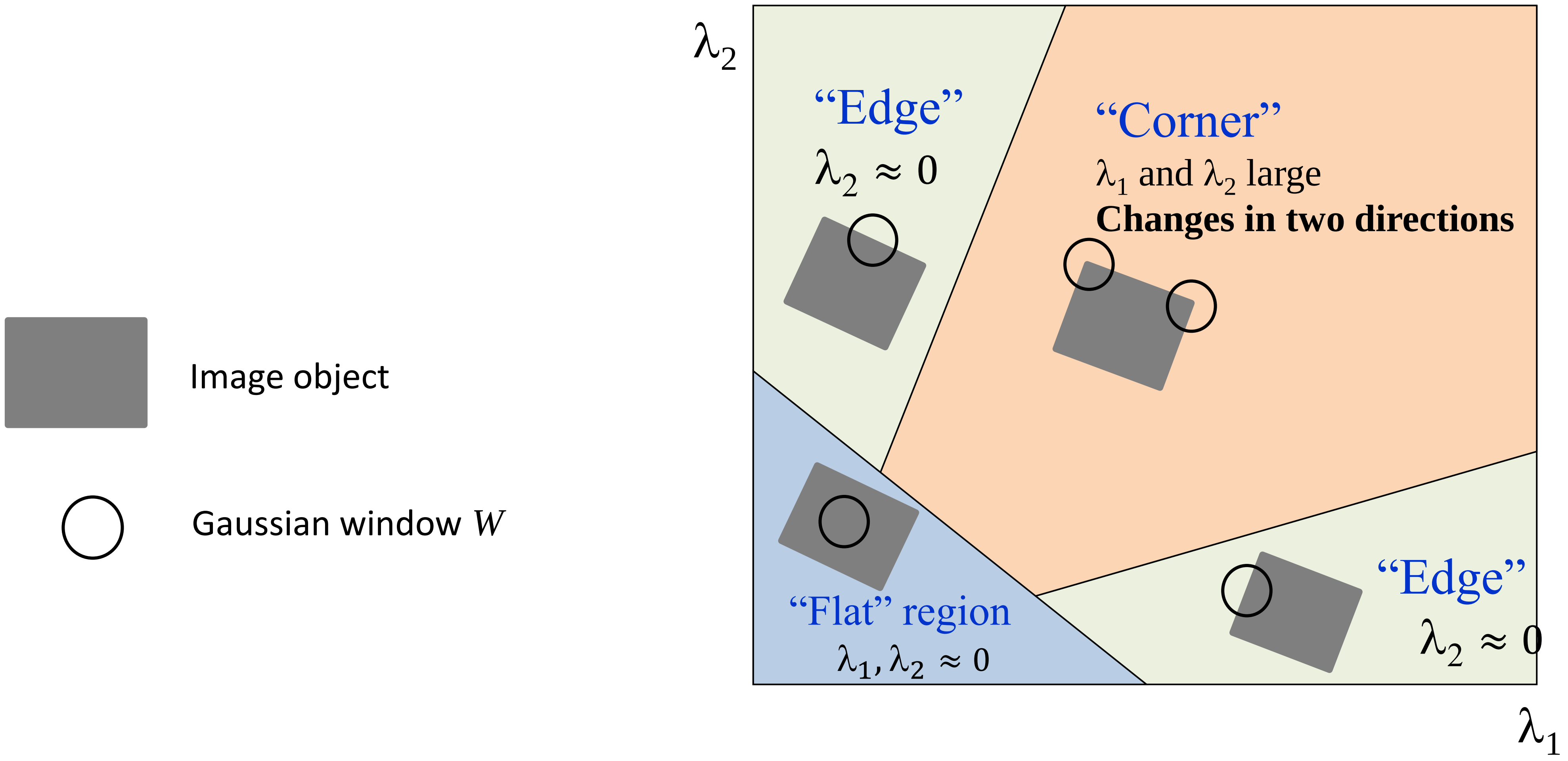


$$M = \begin{bmatrix} \sum_W I_x^2 & \sim 0 \\ \sim 0 & \sum_W I_y^2 \end{bmatrix}$$

M changes when the corners and edges appear rotated in the image

2. Harris detector

The image points are now classified according to the eigenvalues



But computing the eigenvalues of a 2x2 matrix (M) at each pixel is costly!

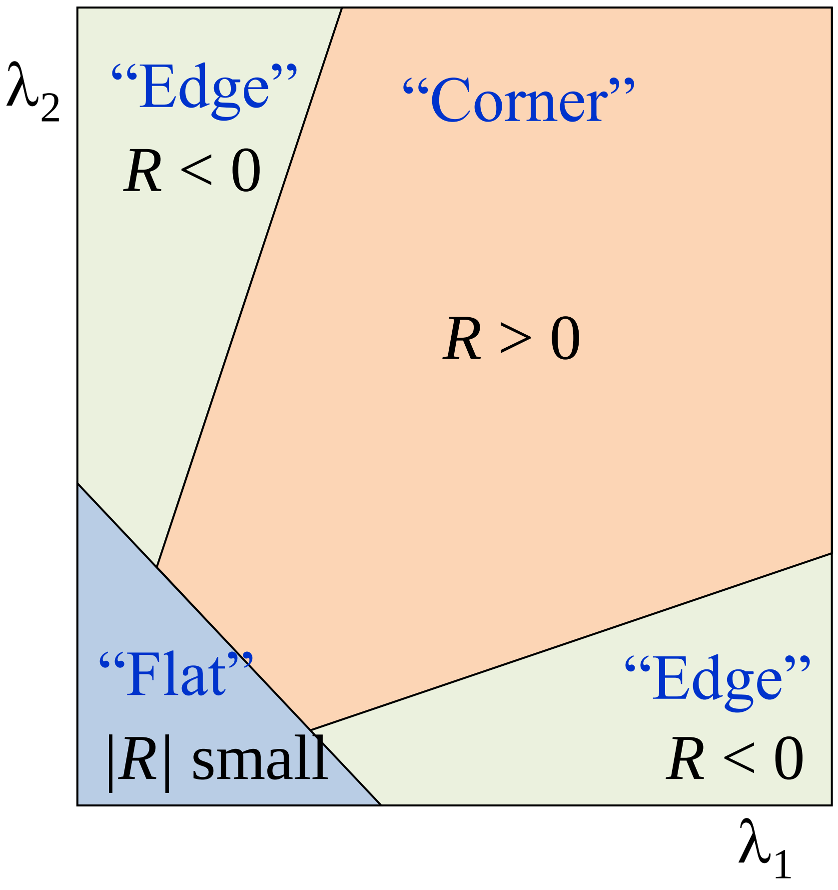
2. Harris detector

Let's define a scalar variable *R* that indexes the same domain:

$$R = \lambda_1 \lambda_2 - k\left(\lambda_1 + \lambda_2\right)^2$$

Determinant Trace

($k = 0.04\text{-}0.06$ is an empiric constant)



- R is large and positive at corners
- R is negative at edges
- $|R|$ is small at flat regions

Trace and determinant of *M* and $D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ are the same:

$$\det M = \lambda_1 \lambda_2$$

$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

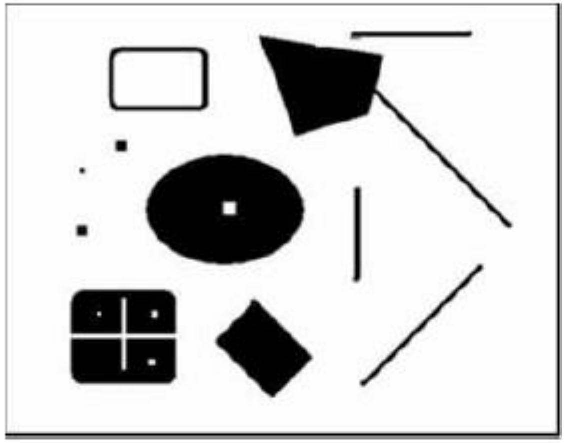
$$\boxed{R = \det M - k\left(\operatorname{trace} M\right)^2}$$

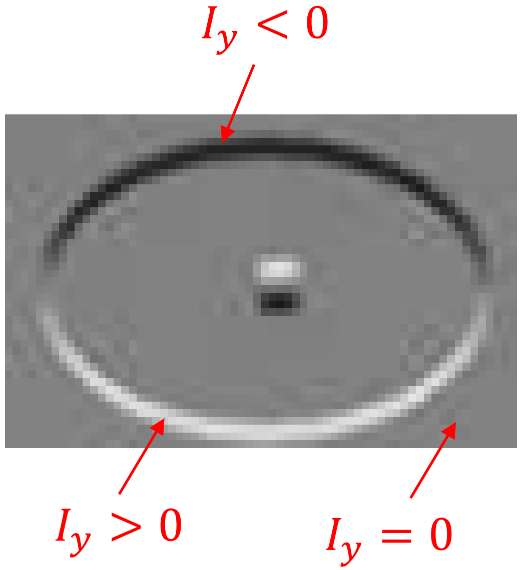
No need to compute the eigenvalues!!

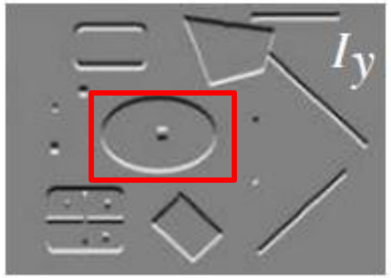
2. Harris detector

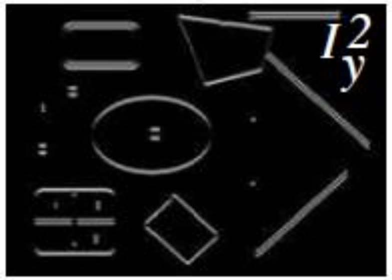
Summary:

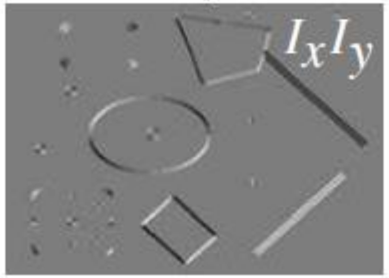
Original image







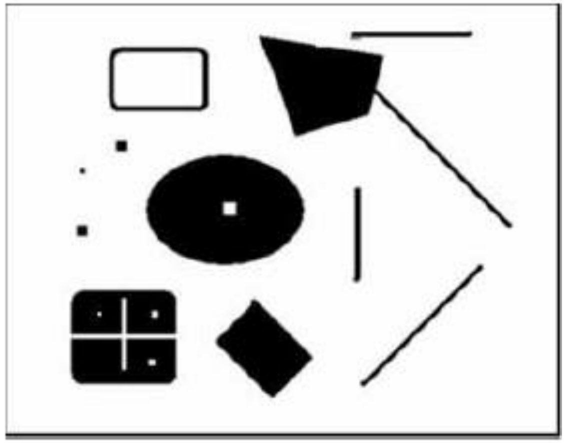


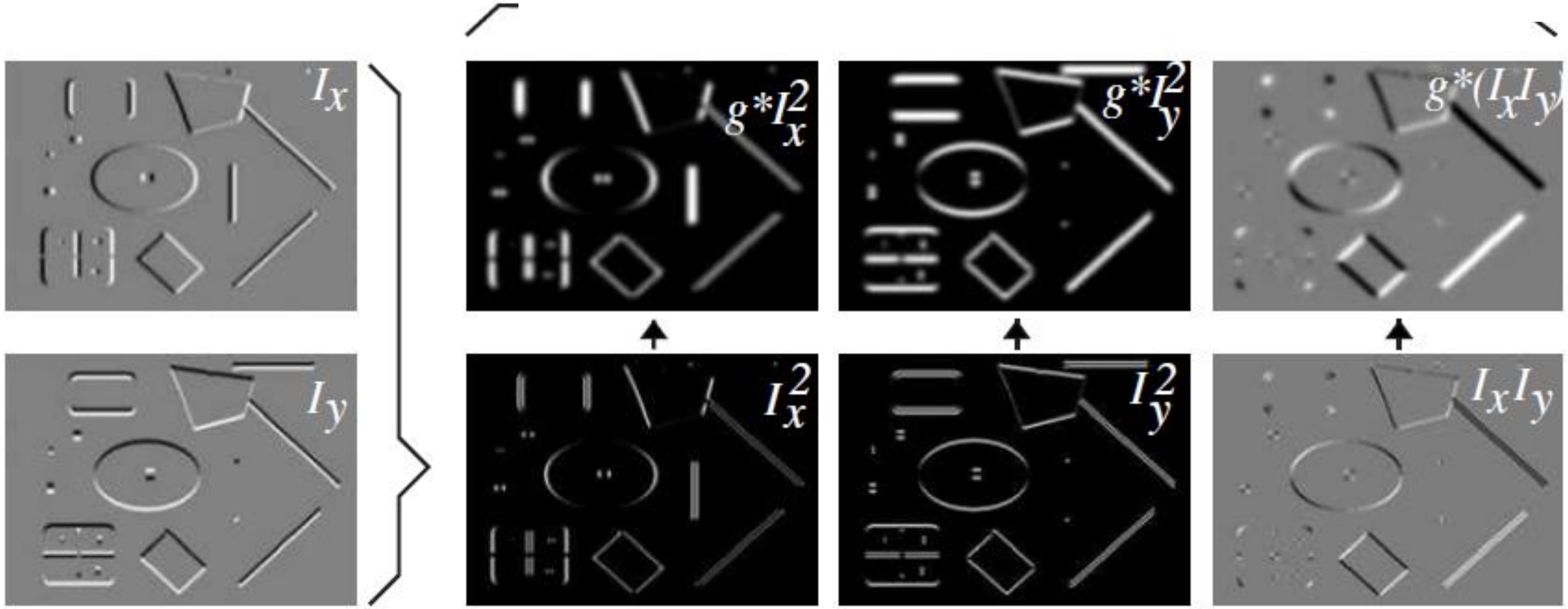


2. Harris detector

Summary:

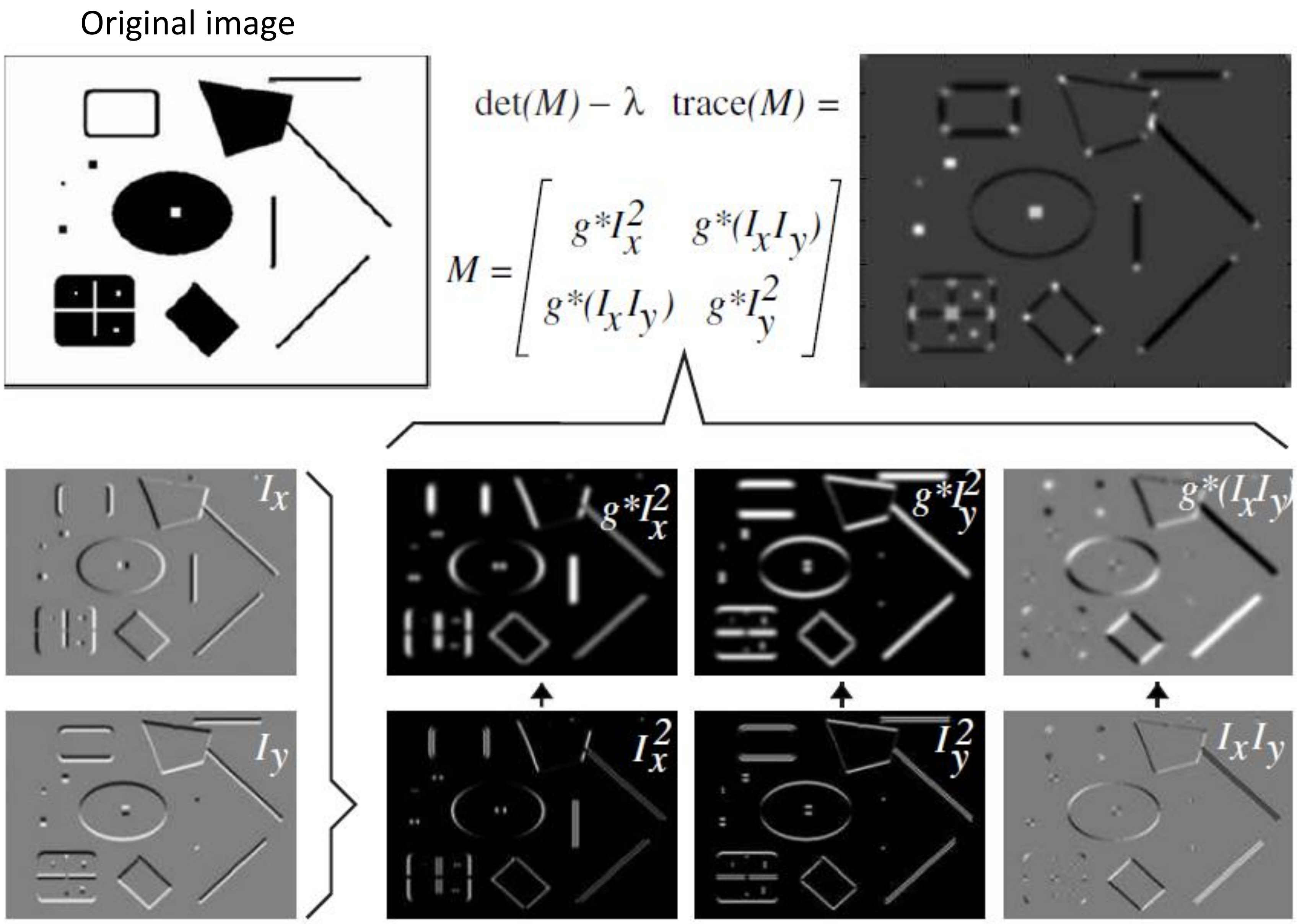
Original image





2. Harris detector

Summary:



2. Harris detector

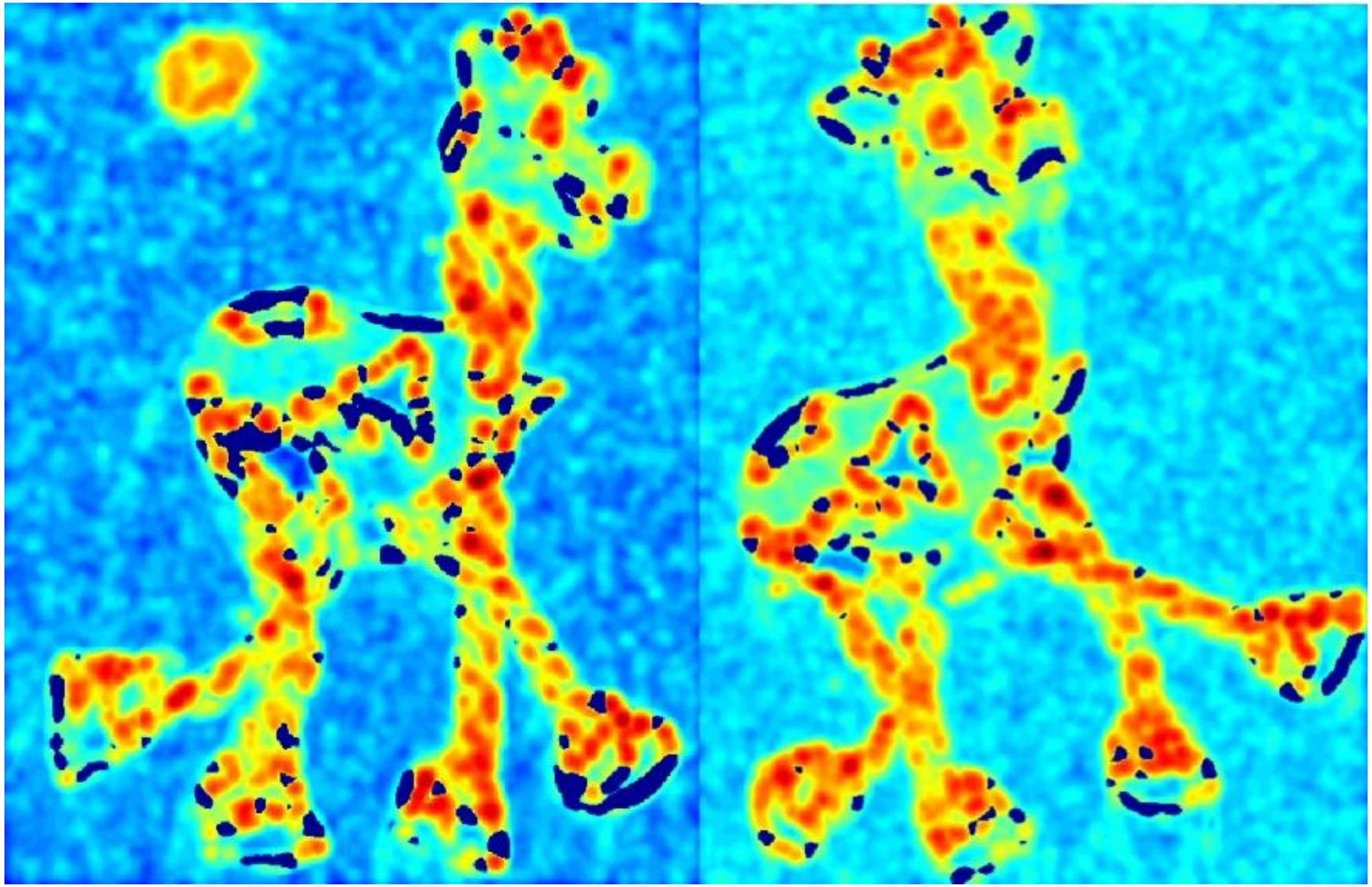
Algorithm:

- Compute the image derivatives I_x, I_y (i.e. Sobel)
- Gaussian smoothing of the 3 images: $(I_x)^2$,$(I_y)^2$, I_xI_y
- Compute the image *R* from the formula (trace and determinant)
- Find pixels where R is high (*R* > threshold)
- Select local maxima

2. Harris detector

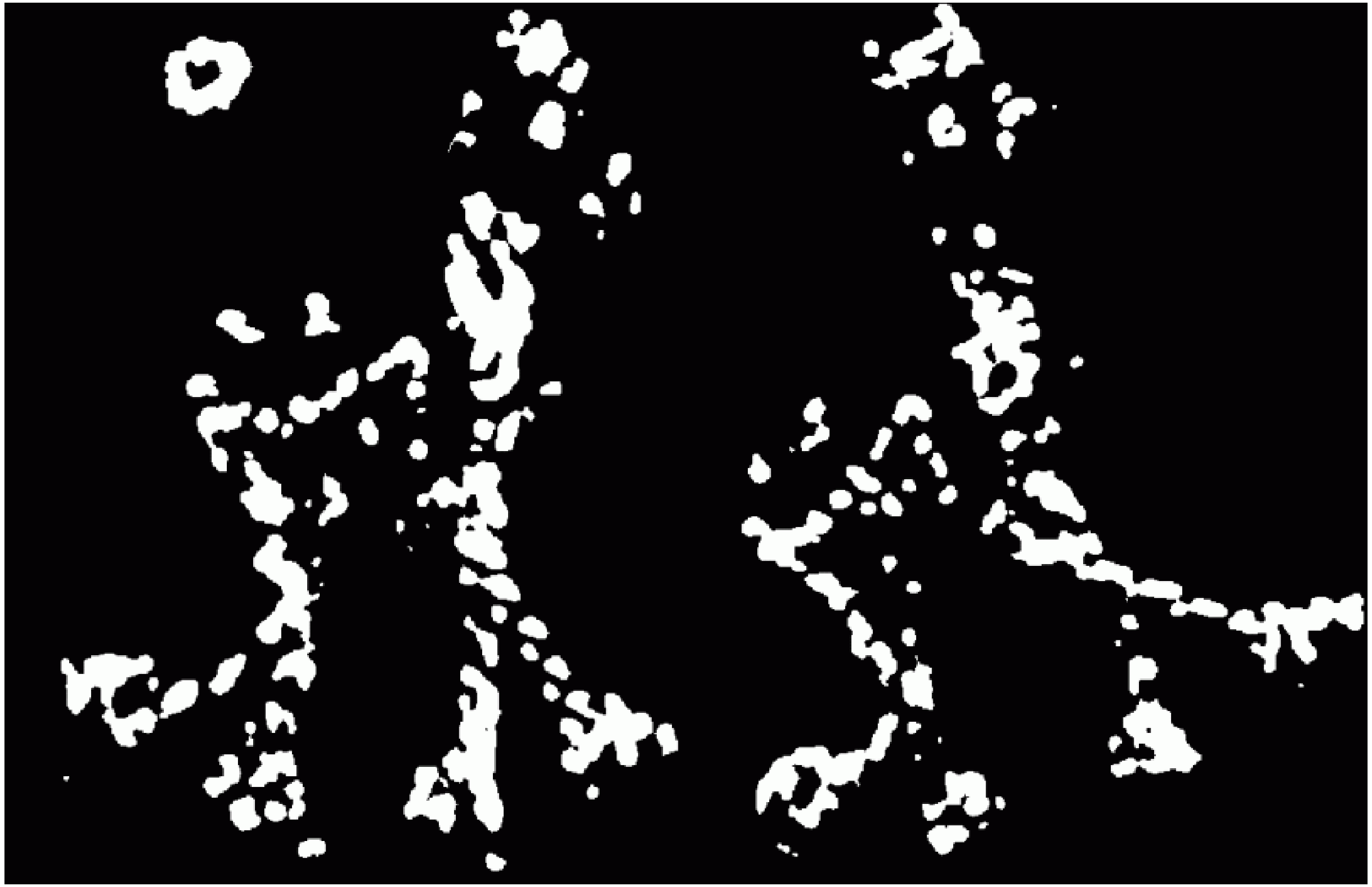
2. Harris detector

R image



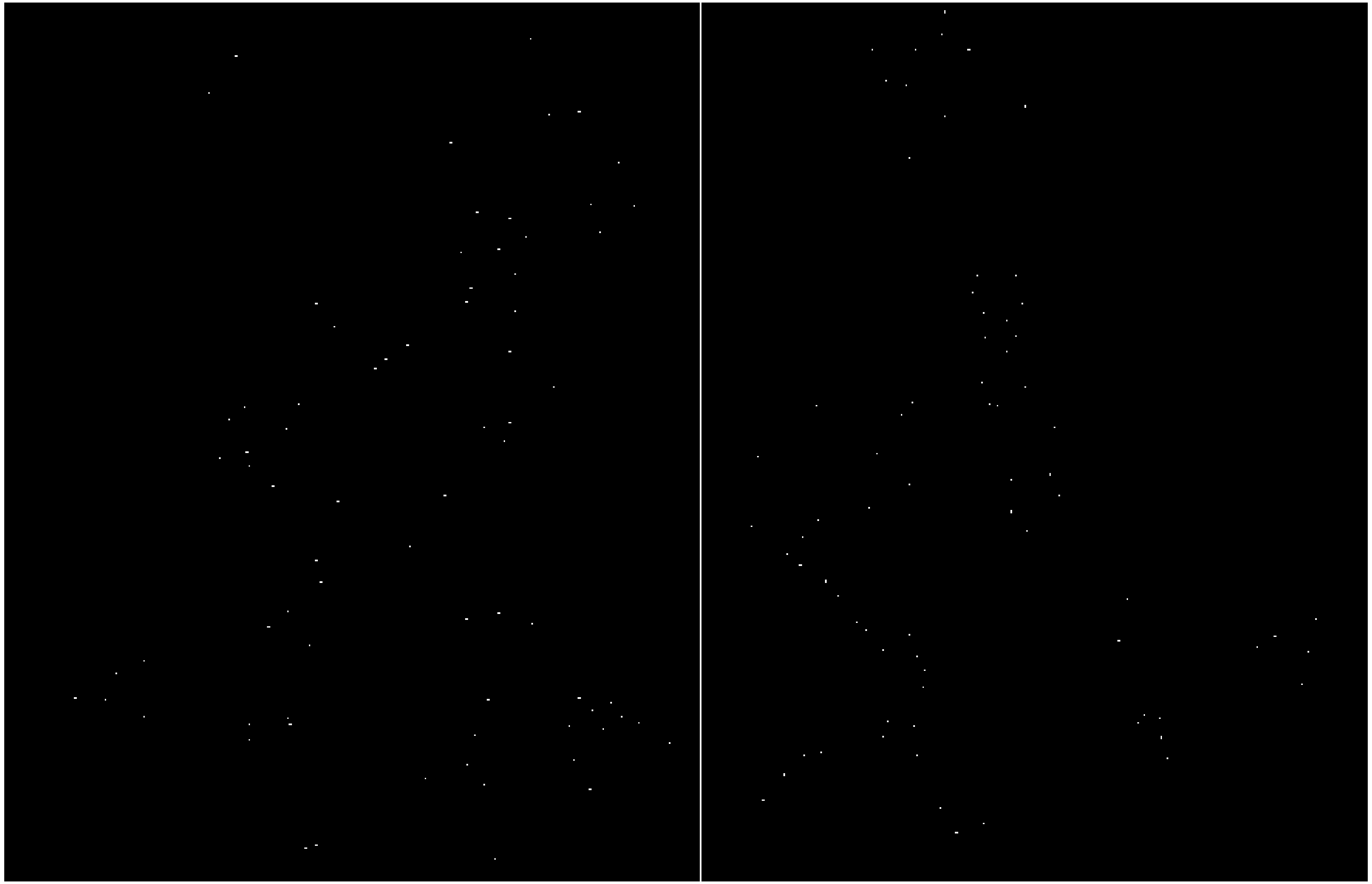
2. Harris detector

R>threshold



2. Harris detector

Local maxima of R



2. Harris detector

3. KLT operator (Kanade-Lucas-Tomasi)

Objective: Detect distintive points, suitable to be tracked in a image sequence

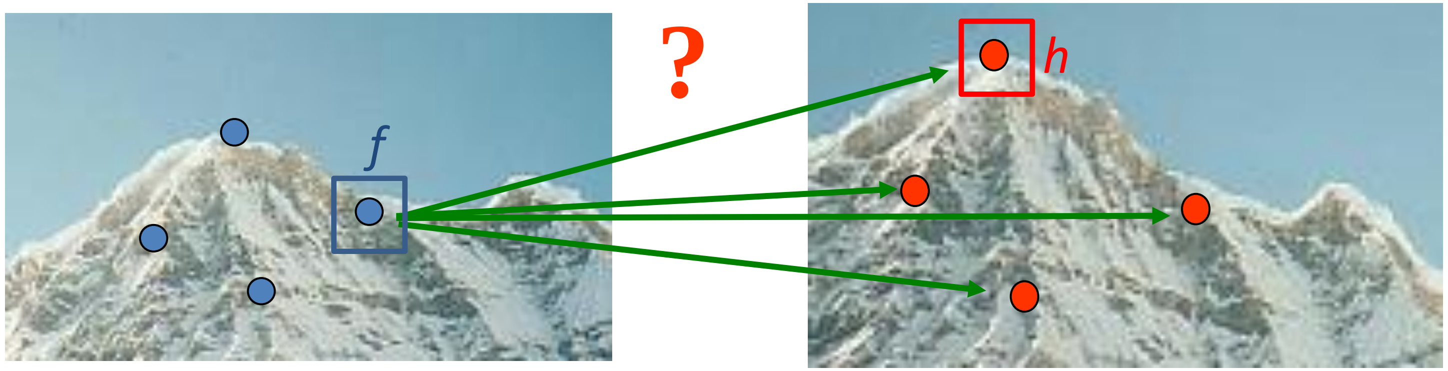
- Similar principle to Harris: "A good keypoint is that with a high intensity derivative in two directions" → **Min (λ_1, λ_2) > threshold**
- Also based on the first derivative matrix:

$$M = \begin{bmatrix} \sum_W I_x^2 & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y^2 \end{bmatrix} \longrightarrow D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

- But now, the eigenvalues are computed (no approximation with *R*) → better behavior under affine image deformation

4. Keypoint matching through correlation

Which is the correspondence of each point in other image?



Sum of squared differences (SSD): $$SSD(f,h)=\sum\left[f(i,j)-h(i,j)\right]^2$$

SSD is approximated by the SAD (more efficient computationally):

Sum of absolute differences (SAD): $$\sum\left|f(i,j)-h(i,j)\right|$$

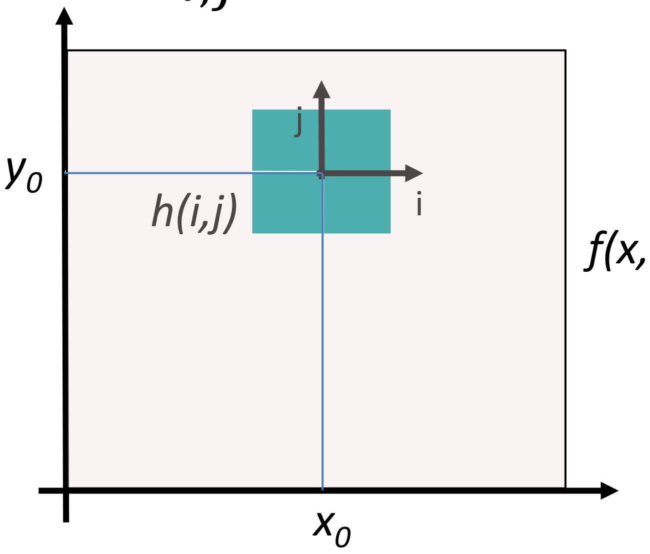
<u>Problem</u>: SSD and SAD are not invariant to brightness or contrast changes

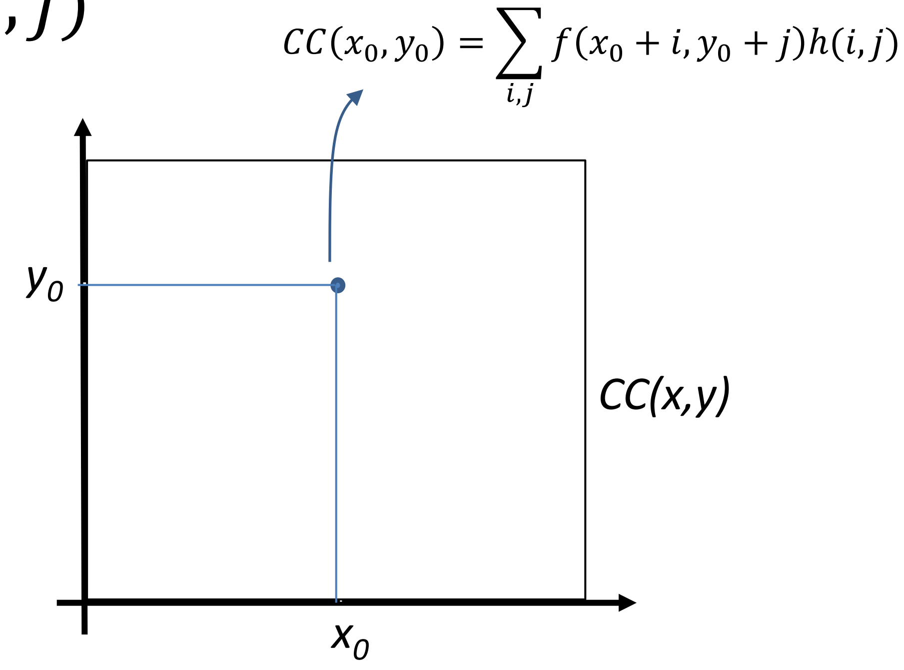
Still, SAD is employed in keypoint tracking along an image sequence (where image brightness and contrast do not change very much.

4. Keypoint matching through correlation

Cross-Correlation:

$$CC(x,y) = \sum_{i,j} f(x+i, y+j)h(i,j)$$



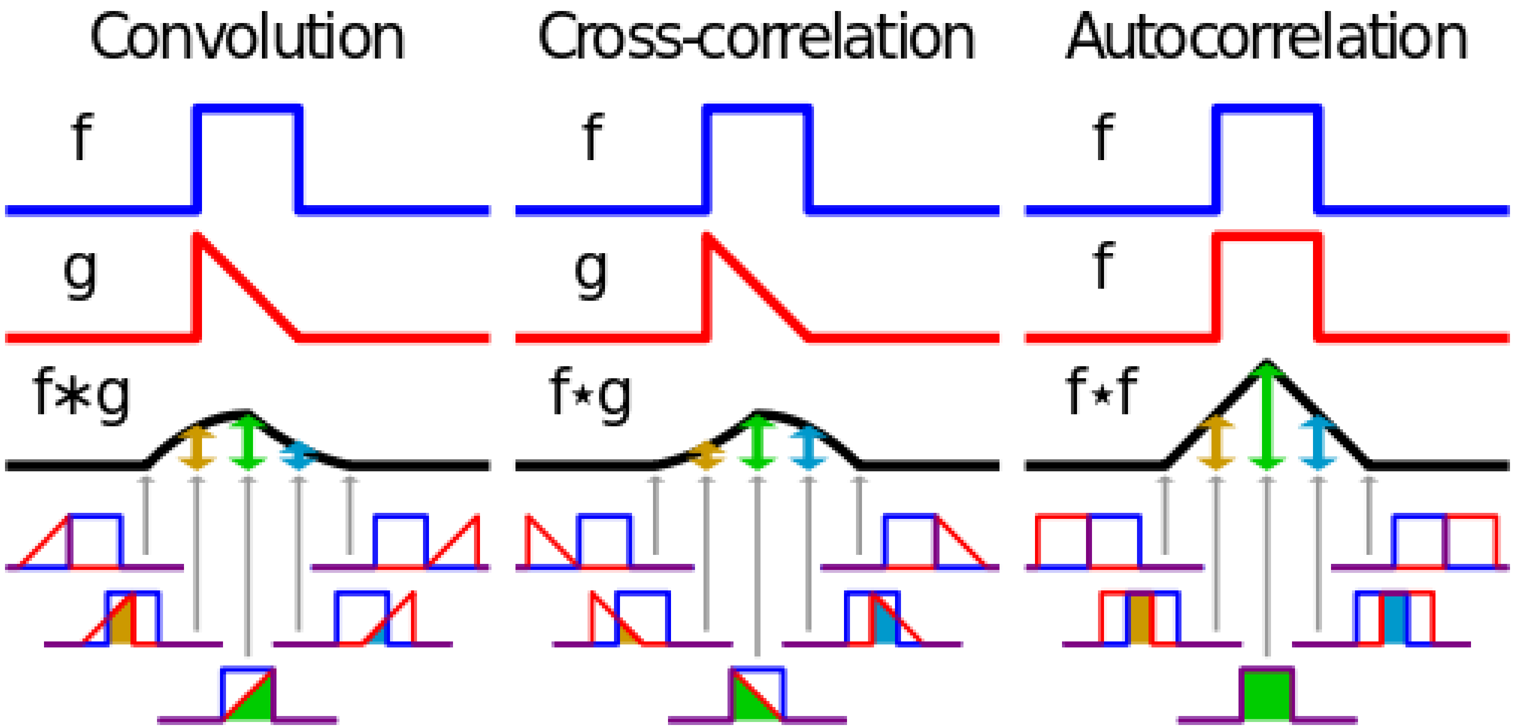


CC is similar to the convolution but without flipping the kernel

$$f \otimes h = \sum_{x,y} f(x-i, y-j)h(i,j)$$

equivalent to the cross-correlation of $f(-i,-j)$ and $h(i,j)$

Correlation vs. Convolution:



The convolution f * g is equivalent to the cross-correlation of f(t) and g(−t)

Normalized Cross-Correlation (NCC):

Cross correlation is not invariant to change in brightness and contrast of f y h
→ Normalization required

$$NCC(x,y) = \sum_{i,j} \hat{f}(x+i, y+j)\hat{h}(i,j)$$

Normalization: $\hat{f}$ and $\hat{h}$ have zero mean and contrast one

$$\hat{f}(x+i, y+j) = \frac{f(x+i, y+j) - \bar{f}}{\left\| f - \bar{f} \right\|_{W_m(x,y)}}$$

brightness

$$\hat{h}(x,y) = \frac{h(x,y) - \bar{h}}{\left\| h - \bar{h} \right\|_{W_m(x,y)}}$$

brightness

Mean brightness (intensity) of f and h in the window W_m

$$\bar{f} = \frac{1}{|W_m(x,y)|} \sum_{(i,j)\in W_m(x,y)} f(i,j)$$

$$\bar{h} = \frac{1}{|W_m(x,y)|} \sum_{(i,j)\in W_m(x,y)} h(i,j)$$

Contrast (norm) of brigntness in the window W_m

$$\left\| f - \bar{f} \right\|_{W_m(x,y)} = \sqrt{\sum_{(i,j)\in W_m(x,y)} \left[f(i,j) - \bar{f}(i,j) \right]^2}$$

$$\left\| h - \bar{h} \right\|_{W_m(x,y)} = \sqrt{\sum_{(i,j)\in W_m(x,y)} \left[h(i,j) - \bar{h}(i,j) \right]^2}$$

Why does NCC measure similarity between two image patches?

Let's consider an image patch as a vector

Image patch $f - \bar{f}$ (has zero mean)

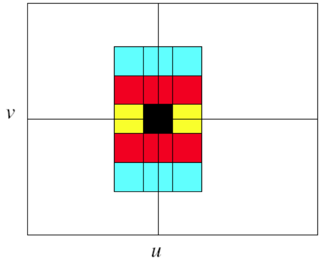
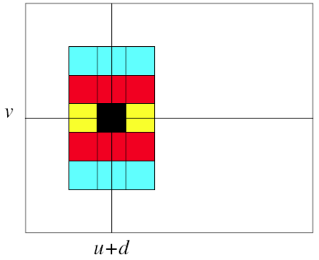


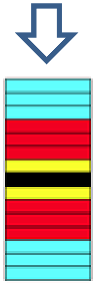
Image patch $h - \bar{h}$

Average brightness

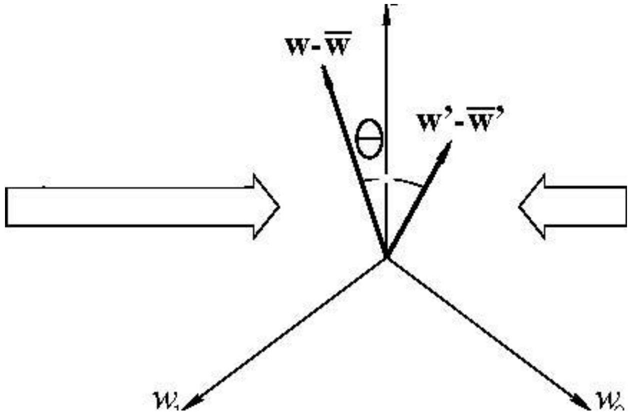


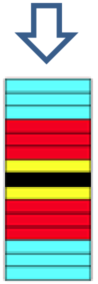
Space dimension = number of pixels in the vector (image patch)

Image patch put as a vector



$w - \bar{w}$





$w' - \bar{w}'$ Average Brightness

$$\frac{w' - \bar{w}'}{\boxed{|w' - \bar{w}'|}}$$ Unity vector

Contrast

$$C(d) = \frac{1}{|w - \bar{w}|} \frac{1}{|w' - \bar{w}'|} (w - \bar{w}) \cdot (w' - \bar{w}') = \cos\theta$$

The similarity between two unity vectors is given by the angle (or *cos*) between them

4. Keypoint matching through correlation

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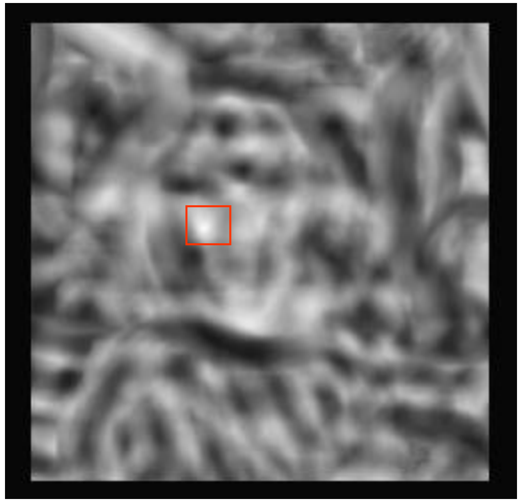

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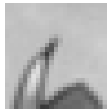

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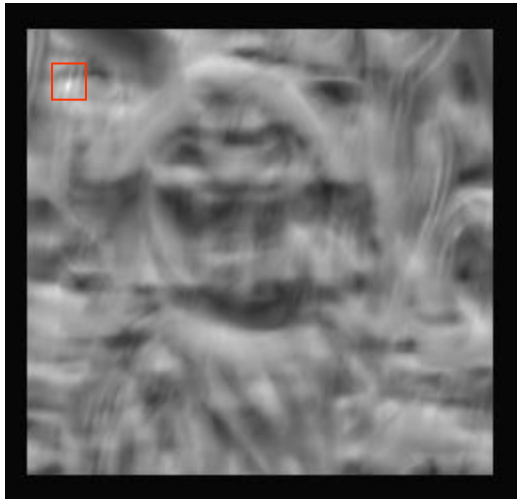

Correlation output

DEMO IN MATLAB

```
%Read and show the image
flowers = imread('flowers.tif'); figure, imshow(flowers)
% select a template from the image with the mouse
[sub_flowers,rect_flowers] = imcrop(flowers);
% Show the selected template
figure, imshow(sub_flowers)
% Do NCC with the blue channel and display the result
c = normxcorr2(sub_flowers(:,:,1),flowers(:,:,1));
figure, surf(c), shading flat
```

Notice: Output not invariant to the rotation of the patch

Content

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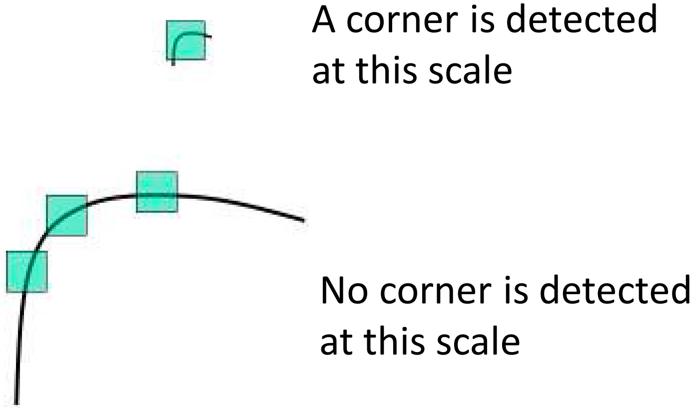
2. Harris detector
   - Idea
   - Formulation
   - Implementation
3. KLT operator
4. Keypoint matching through correlation
5. SIFT operator
   - Scale Space
   - Detector
   - Descriptor

5. The SIFT (**S**cale **I**nvariant **F**eature **T**ransform) operator

- **Objective**: Find in image projections of **distintive** 3D points (not necessary corners!) that are invariant to scale

Harris is not invariant to scale



- **Provides** both: Detector y descriptor of the detected keypoints

Proposed (and patented) by **David Lowe**: **"Distinctive image features from scale-invariant keypoints,"** *International Journal of Computer Vision,* 60, 2 (2004).

Some slides borrowed from: Francisco A. Moreno Dueñas

5. The SIFT operator

- Both, detector and descriptor have invariance to:
  - Scale
  - Rotation
  - Ilumination [constrast+brightness]
  - Affine transformation [parcially]

Important difference against Harris

Important difference against NCC

- Principle
  - Search for extrema in the scale space **[Detector]**
  - Normalized histogram of orientation [**Descriptor**]
- Descriptor is a vector up to 128 dimensions

5. The SIFT operator

Overview of the method

Build scale space

Invariance to Scale

Difference of images

Search for extrema

Filter out
- Low contrast
- At image borders

Descriptor computation

Magnitude of the gradiente

Dominant orientation

Invariance to Orientation

Histogram of orientation

Contrast and brightness normalization

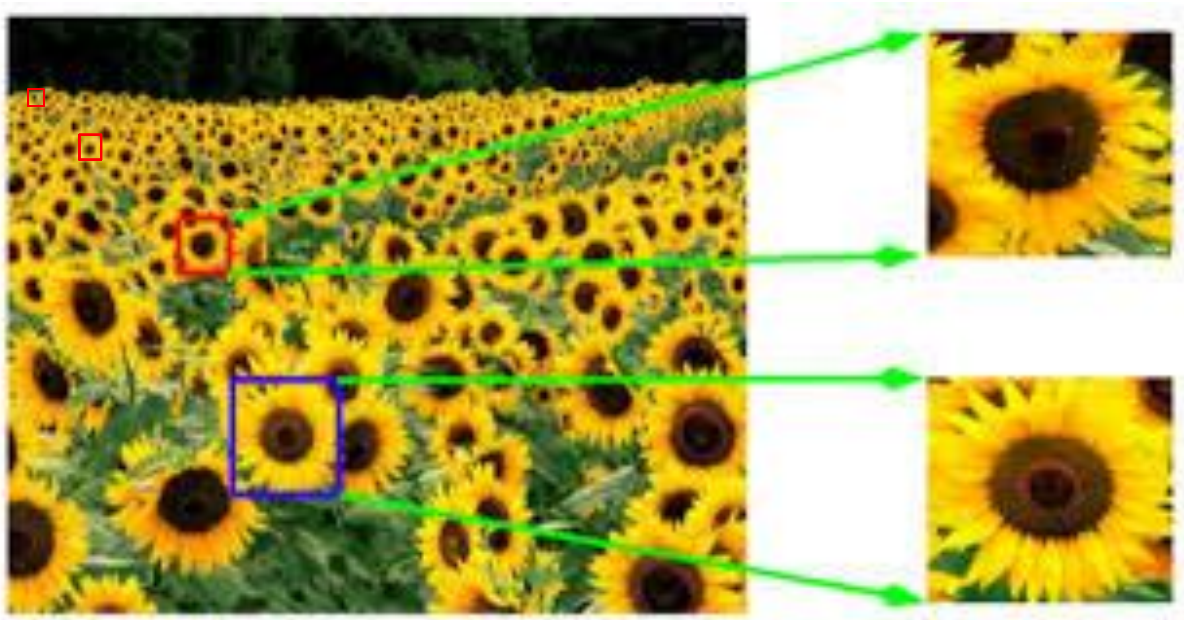
Invariance to Illumination

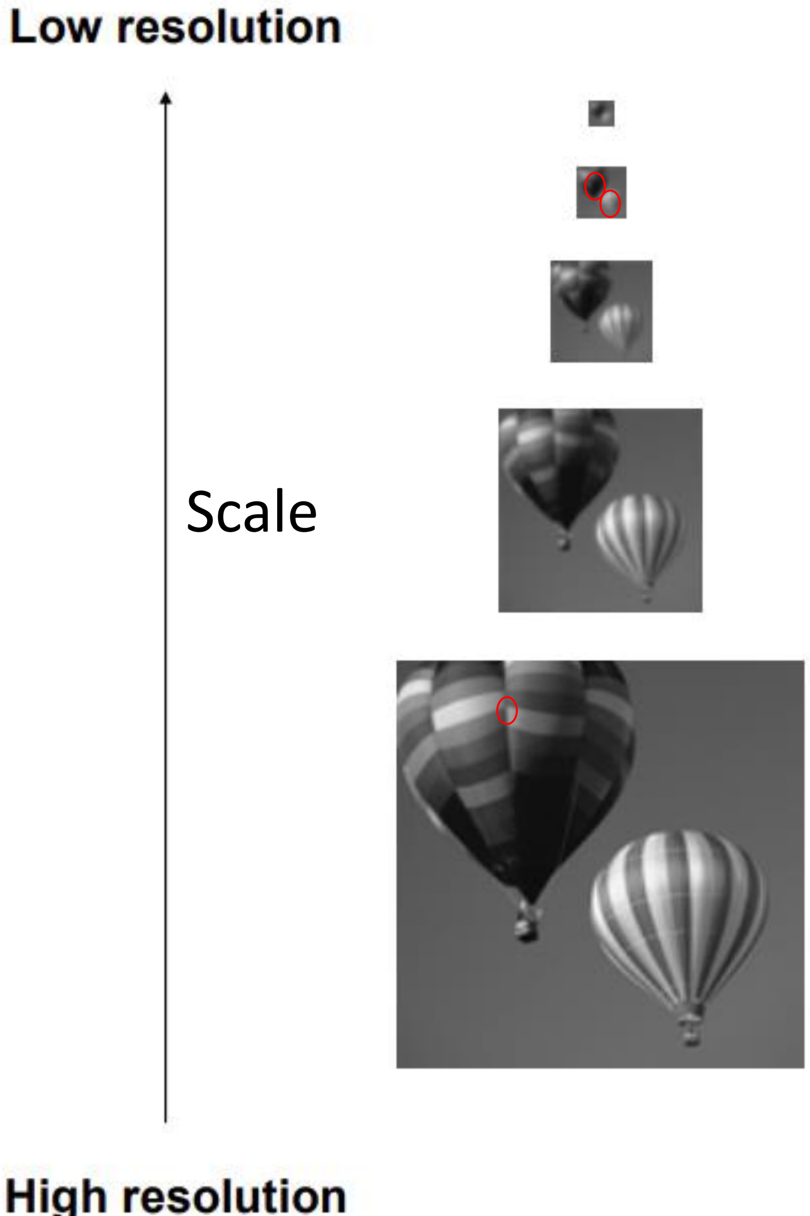
5. The SIFT operator

Changing the scale (size) shows up different features

Scale Space

In images, features emerge at different scales



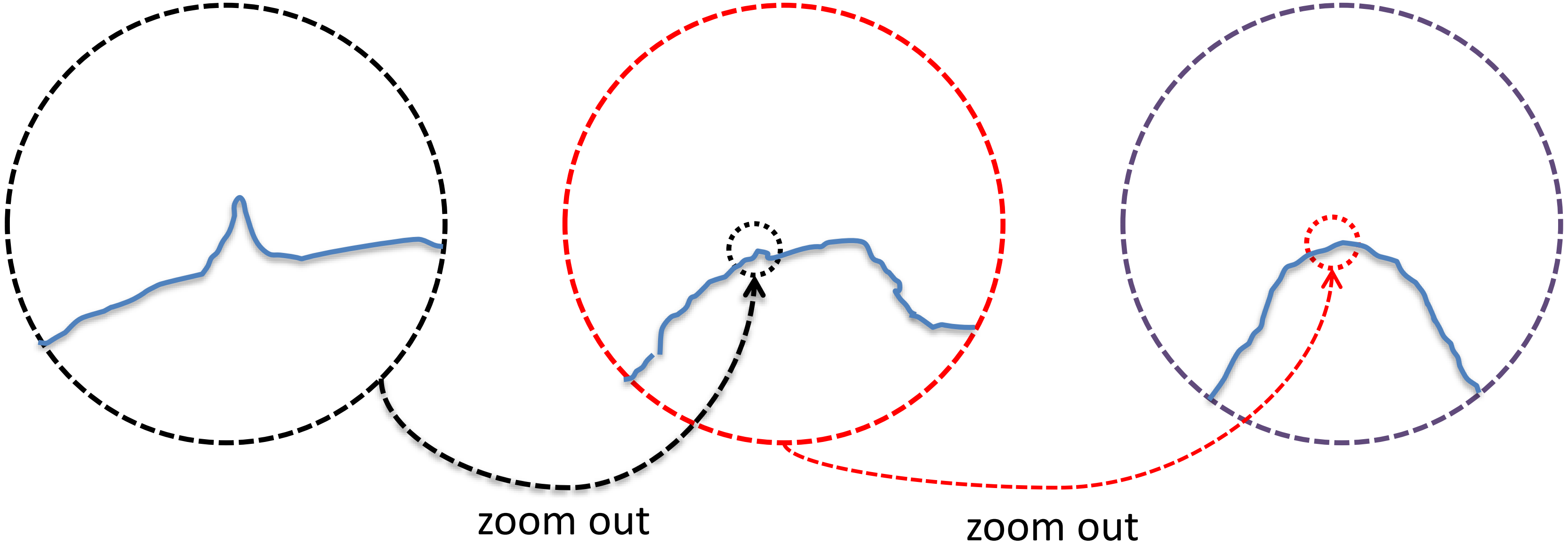


5. The SIFT operator

Scale Space

Features show up at a given scale

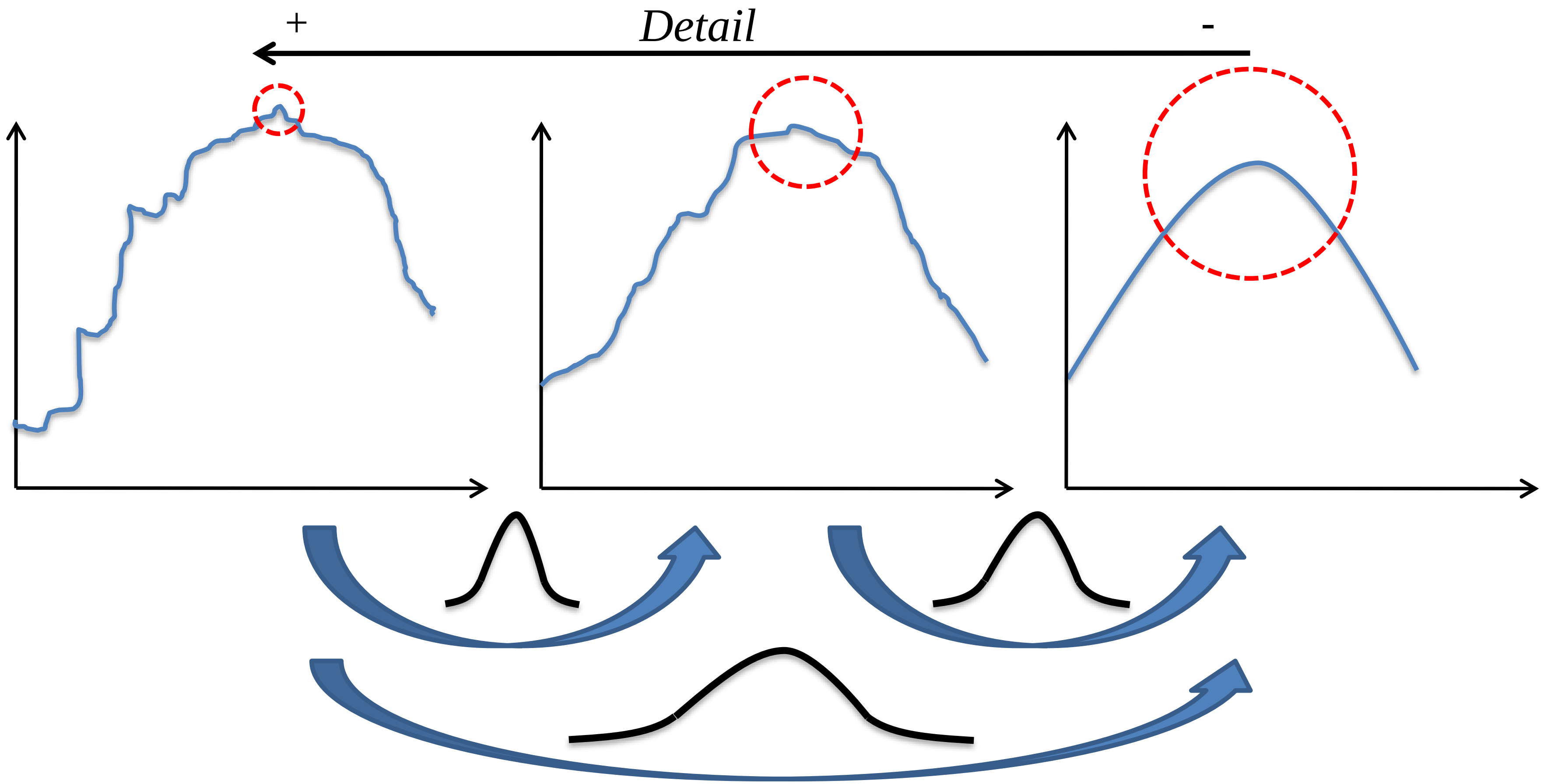
Example in one-dimension



5. The SIFT operator

Scale Space

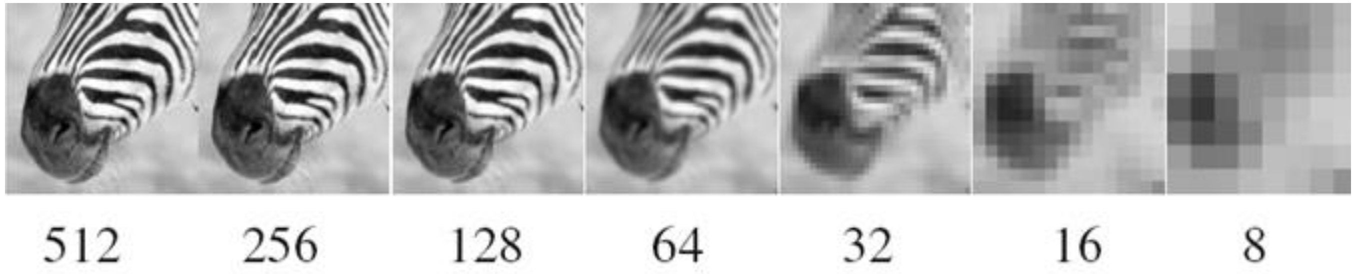
We can change the scale by smoothing the signal with a Gaussian



5. The SIFT operator

Scale Space

Changing the scale of an image by smoothing with a Gaussian



Scale Space

Changing the scale of an image by smoothing with a Gaussian

Gaussian operator: $$G(x,y,t) = \frac{1}{2\pi t} e^{-\frac{(x^2+y^2)}{2t}} \qquad t = \sigma^2 \Rightarrow \sigma = \sqrt{t}$$

Smoothed image: $$L(x,y,t) = I(x,y) * G(x,y,t)$$

+ *Detail* -

The starting image is always the original image (t=0)

$L(x,y,0) = I(x,y)$ | $L(x,y,1)$ | $L(x,y,4)$

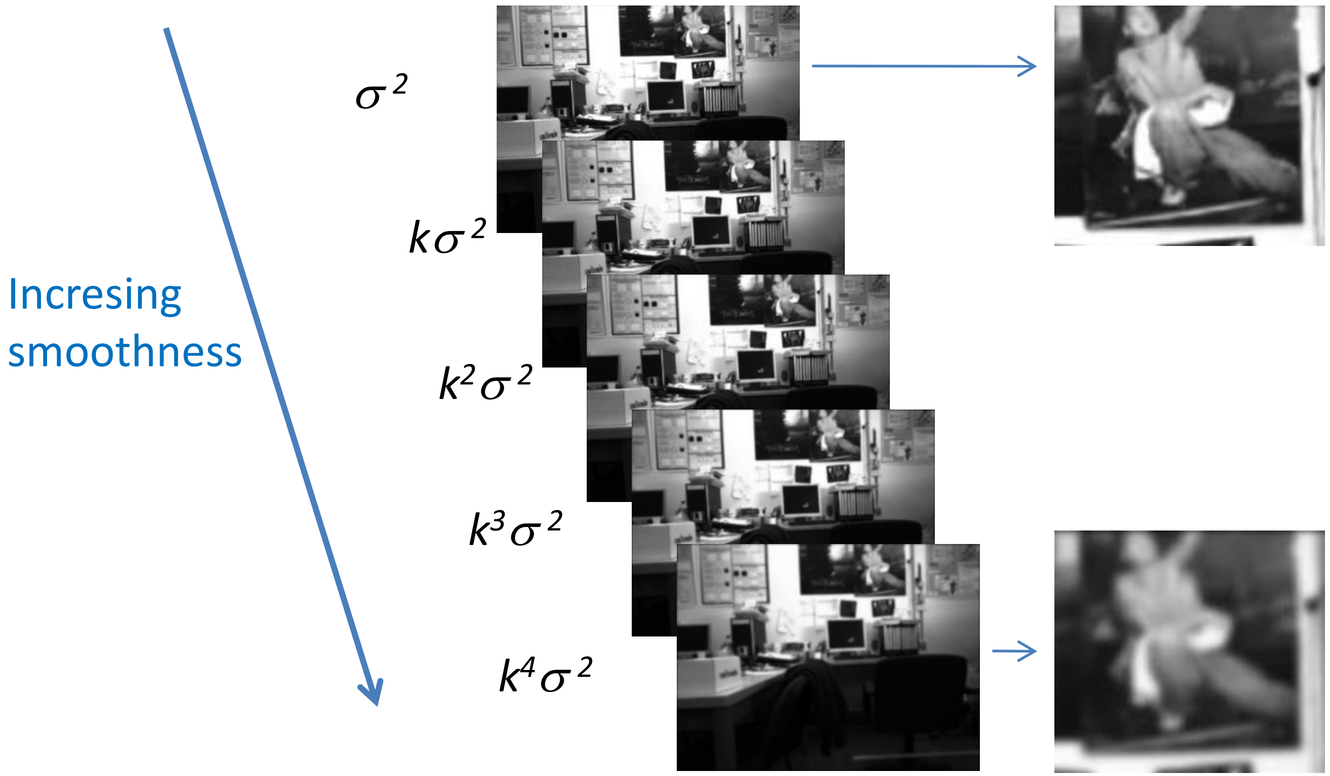
starting image t=0 | σ=1 | σ=2

So, the scale space stores samples of the function $L(x,y,\sigma^2)$

5. The SIFT operator

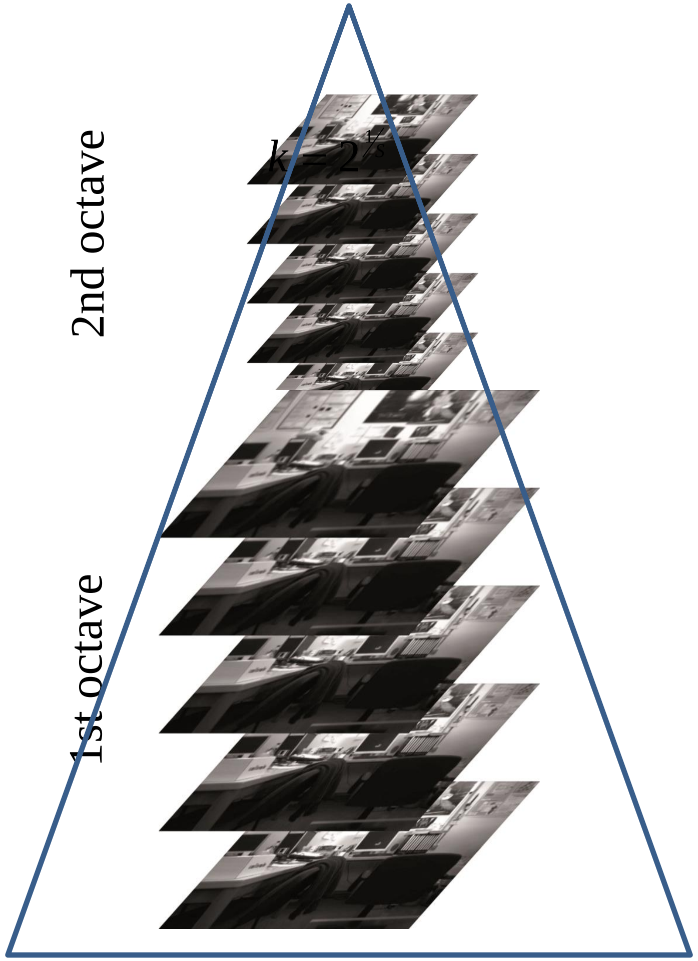
Objetive: “continuous” scale space

Progressive convolution of the input image with a Gaussian controlled with a constant factor K



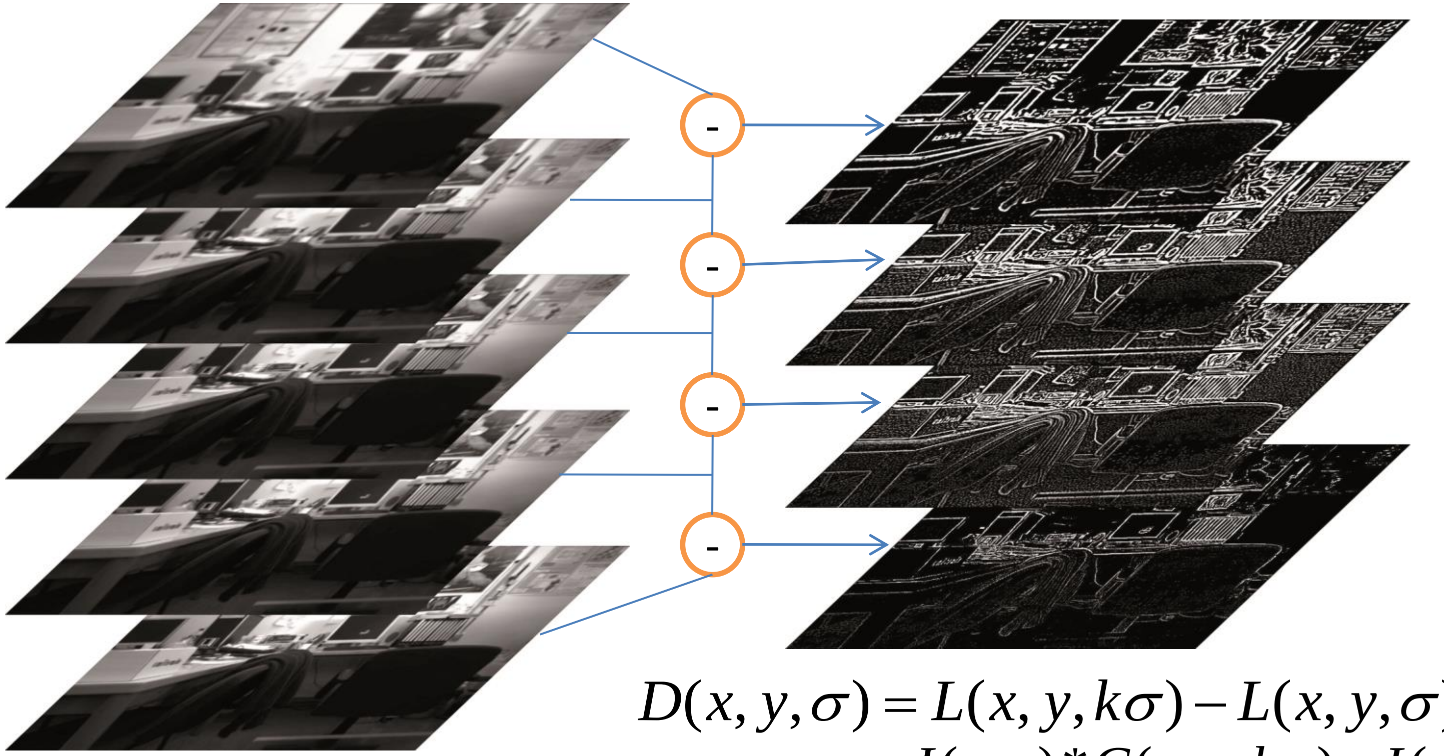
5. The SIFT operator

- The scale space has the structure of a ***pyramid***: a collection of digital images sampled at progressively coarser spatial resolution and hence of progressively smaller size.
- The pyramid is built upon a number of ***Octaves.***
- Each ***Octave (o)*** consists of s+2 images of the same size (resolution) with increasing smoothness.
- In the **following *Octave*** the image has half the resolution (size) since it does not make sense to keep the resolution when small details have been removed.



5. The SIFT operator

From smoothed images to **Difference of Gaussians** (DoG)



The DoG images at the ends are also in the neighbors octaves. See next slide.

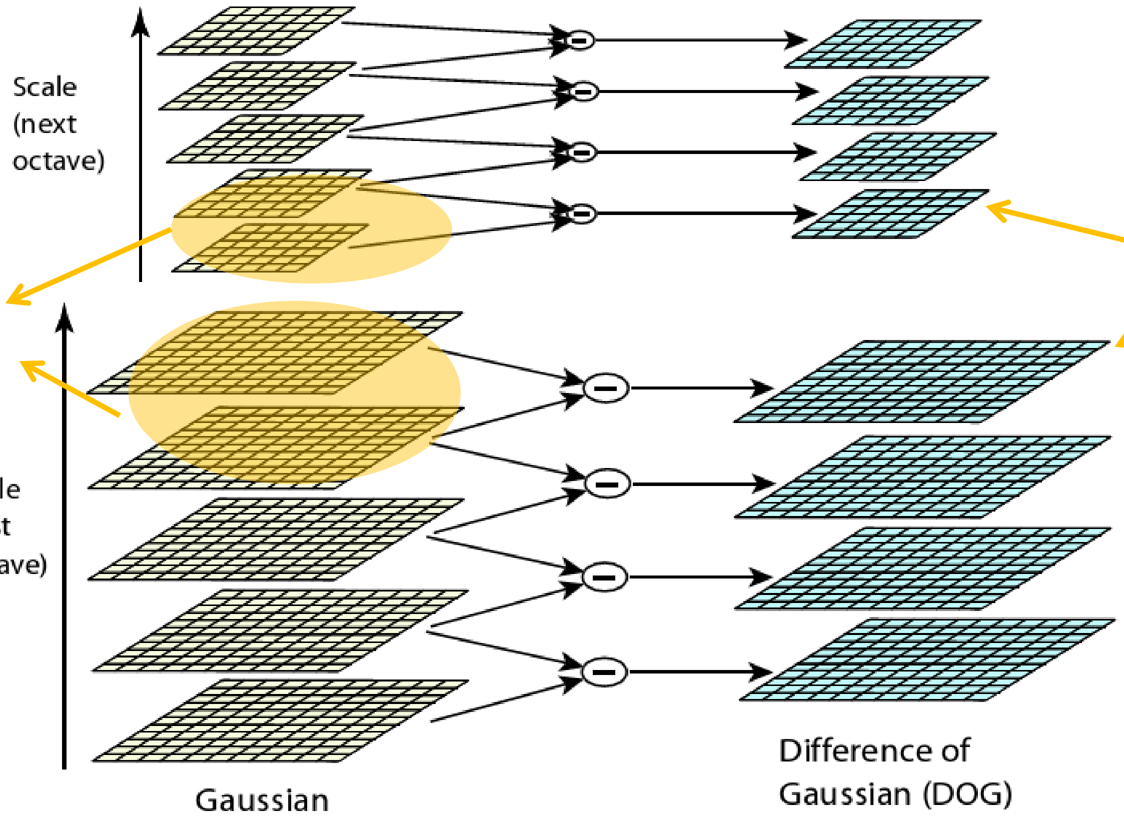
$$
\begin{aligned}
D(x,y,\sigma) &= L(x,y,k\sigma) - L(x,y,\sigma) \\
&= I(x,y) * G(x,y,k\sigma) - I(x,y) G(x,y,\sigma) \\
&= I(x,y) * \left[G(x,y,k\sigma) - G(x,y,\sigma) \right] \\
&= I(x,y) * DoG(x,y,\sigma)
\end{aligned}
$$

Diference of smoothed images = Image convolved with a DoG

5. The SIFT operator

Construction of the scale space through “octaves”

These two images are the same but with half resolution



Same DOG images but half resolution

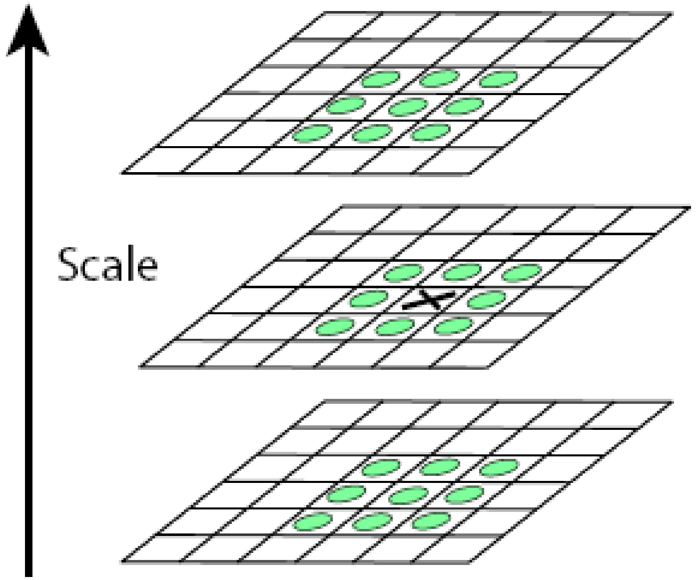
DoG are used here to detect BLOBS (not edged)!!

5. The SIFT operator

Search for extreme points

Each pixel value is compared to its 26 neighbors along the full scale:

- **8** in ithe same scale,
- **9** in the upper scale and
- **9** in the lower scale

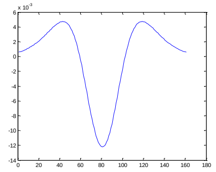


A extreme point in the scale gives us a distintive point in (x,y) and in the pyramid

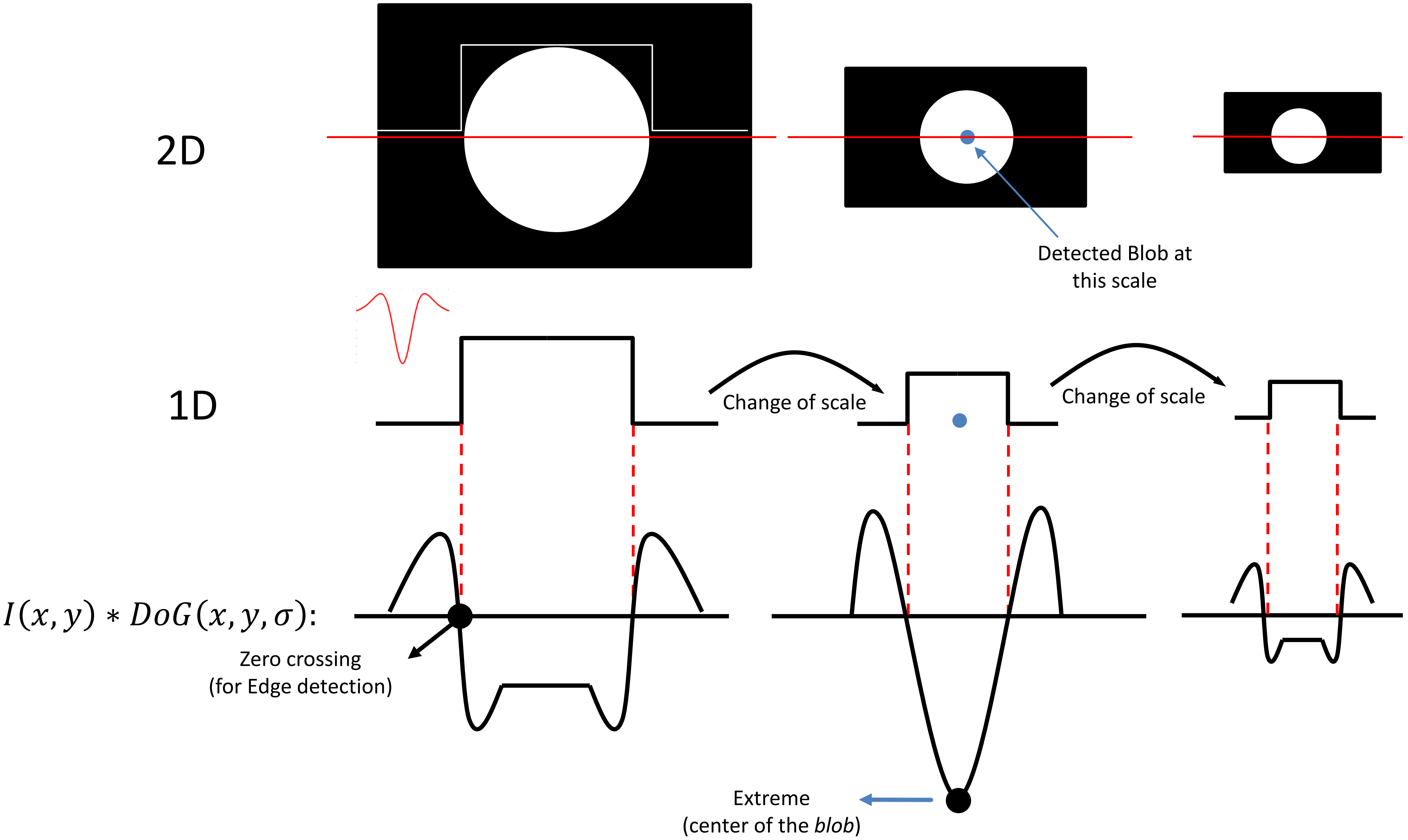
A extreme point in the scale gives us a distintive point in (x,y) and in the pyramid (k)

WHY?

Recall:

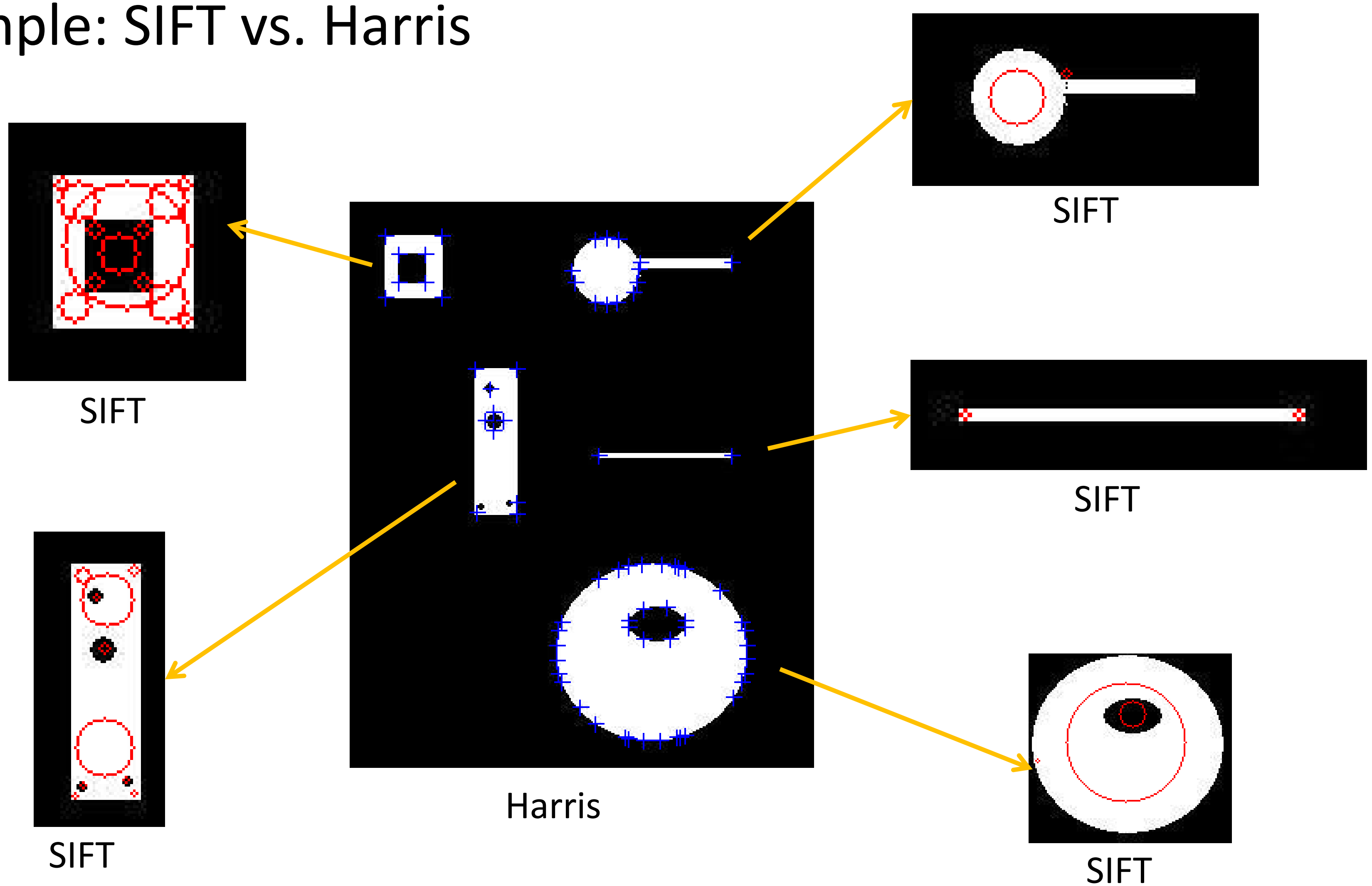


DoG operator (1D)



5. The SIFT operator

Example: SIFT vs. Harris



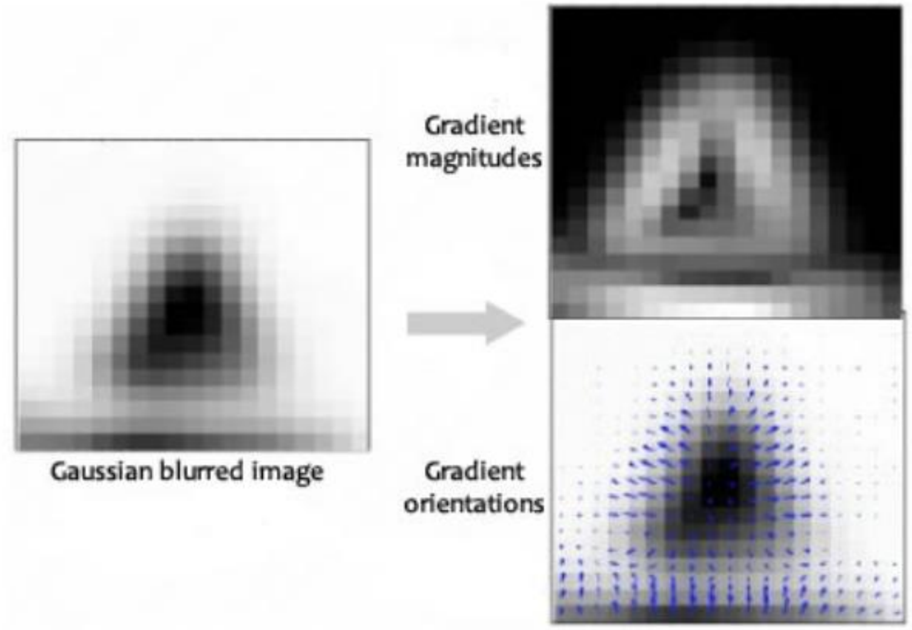
SIFT Descriptor: Histogram of orientations around the extreme point

Obtaining the descriptor orientation:

The magnitude (m) and orientation (θ) of the gradient is calculated for all pixels around the keypoint.

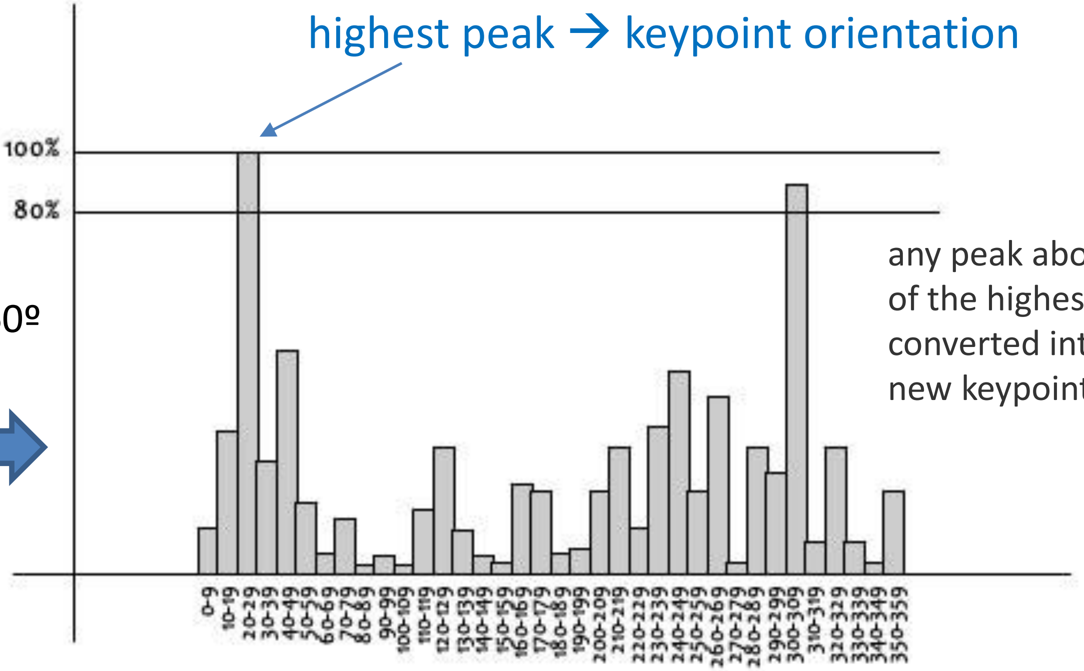
$$m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2}$$

$$\theta(x,y) = \tan^{-1}((L(x,y+1) - L(x,y-1))/(L(x+1,y) - L(x-1,y)))$$



16 bins for 360º (every 10º)

highest peak → keypoint orientation



any peak above 80% of the highest is converted into a new keypoint

A histogram is created for these orientations, weighted by the magnitude of the gradient

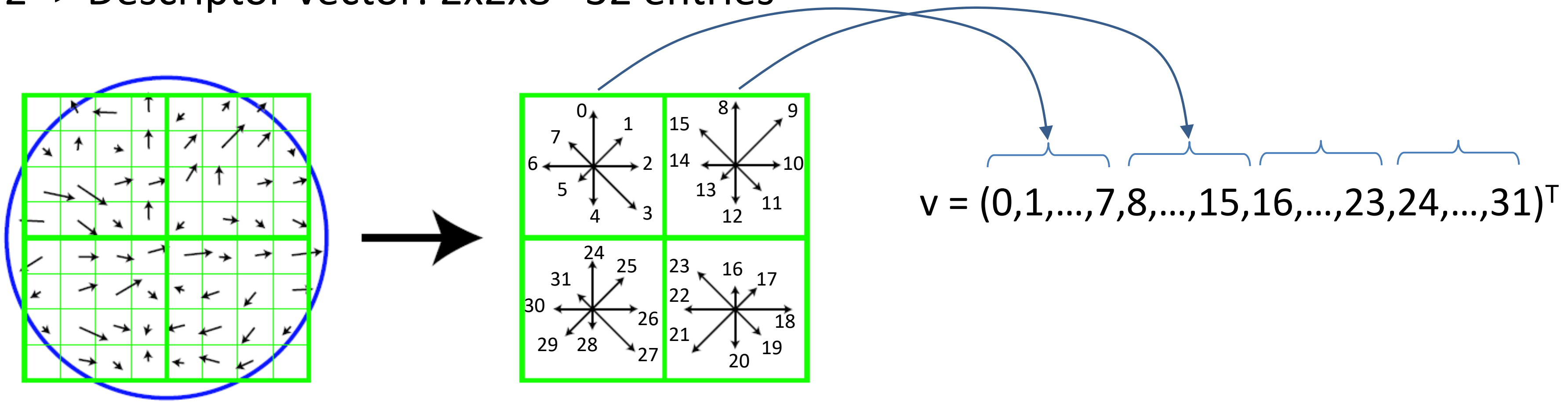
SIFT Descriptor: Histogram of orientations around the extreme point (EP)

Obtaining the descriptor vector:

- The neighborhood of the EP is divided in **m x m cells** (a cell is 4x4 pixels)
- Histogram of 8 orientations for each cell → Size of descriptor vector: **mxmx8**

Example: m = 2 -> Descriptor vector: 2x2x8= 32 entries

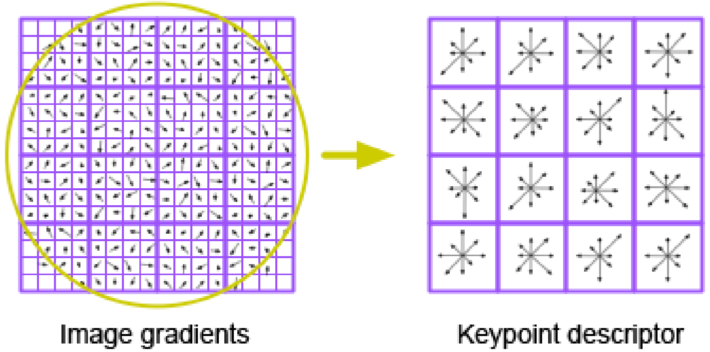
m = 2



$v = (0,1,\ldots,7,8,\ldots,15,16,\ldots,23,24,\ldots,31)^T$

In the Lowe's original paper (D. Lowe): **m = 4 ->** Descriptor: 4x4x8 = **128D**

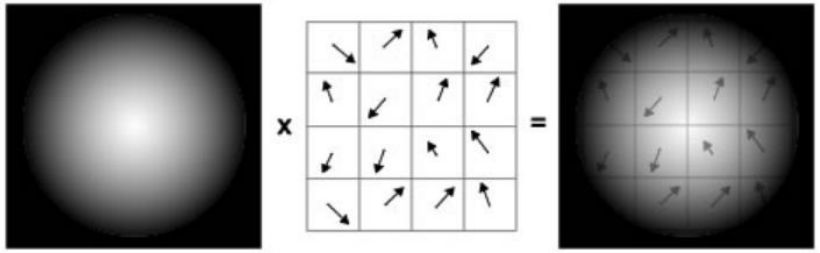
m = 4



Obtaining the descriptor vector:

The histogram of orientations is weighted by

- Magnitude of the gradient: more importance to strong gradients
- Gaussian centered at the extreme point: more importance to close pixels



Descriptor: 4x4x8 = **128D**

SIFT Invariances

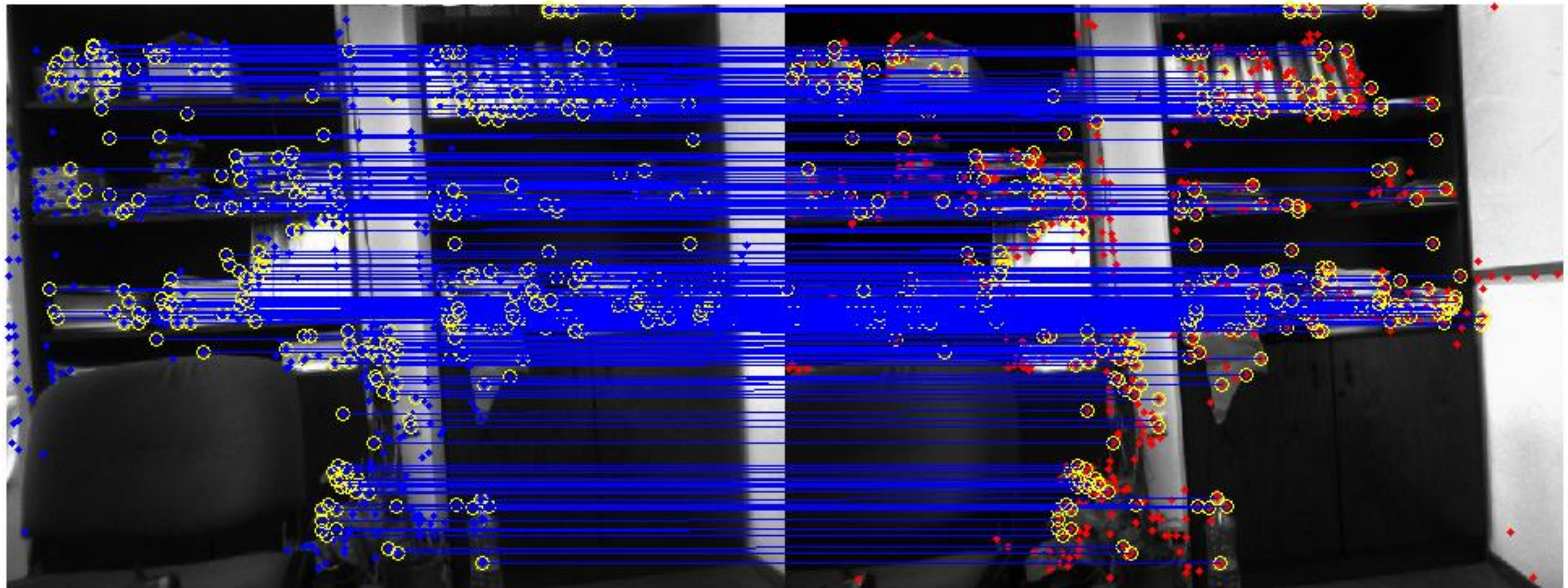
- **Scale**: Window size based on the scale at which the extreme was found
- **Orientation:** Histogram rotated along the keypoint orientation: the keypoint orientation is subtracted from each orientation of the vector

5. The SIFT operator

Example: Stereo matches from Euclidean distance

A keypoint of the lest image is matched to the one in the left with the closest descriptor

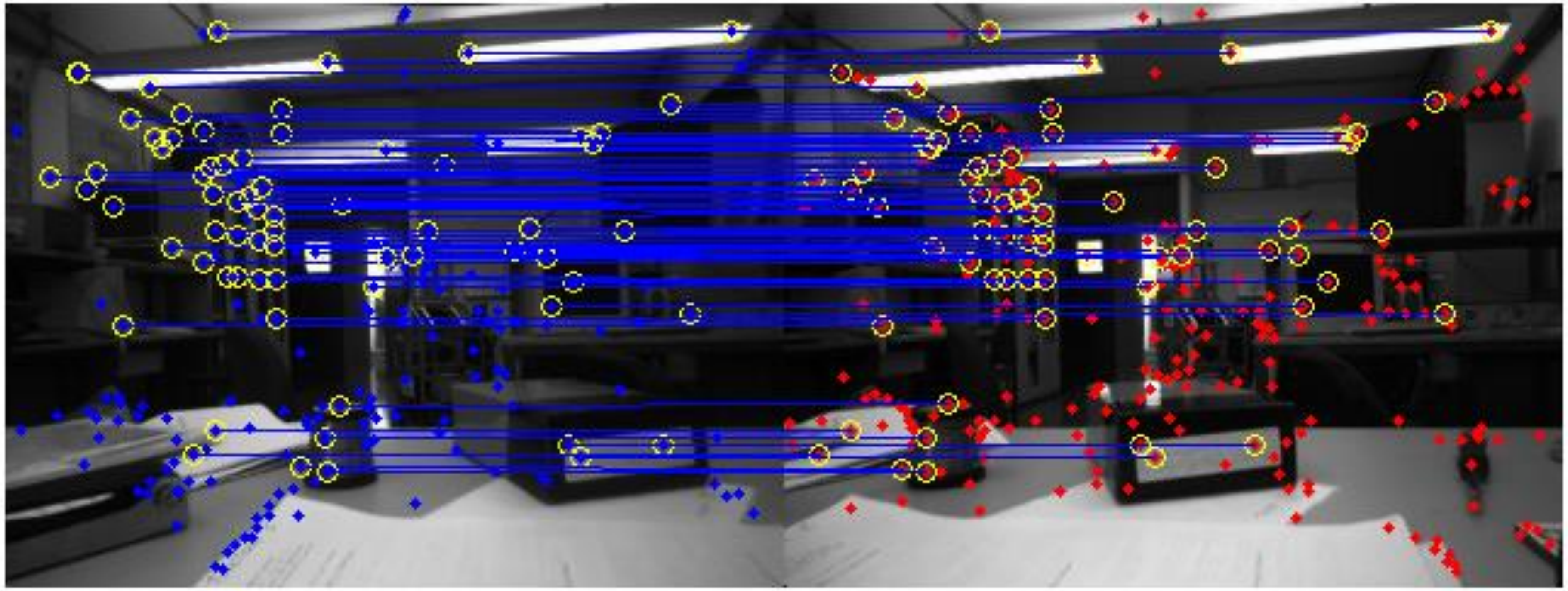
Points: 965/994 Matches: 359 37.20%



5. The SIFT operator

Example: Stereo matches

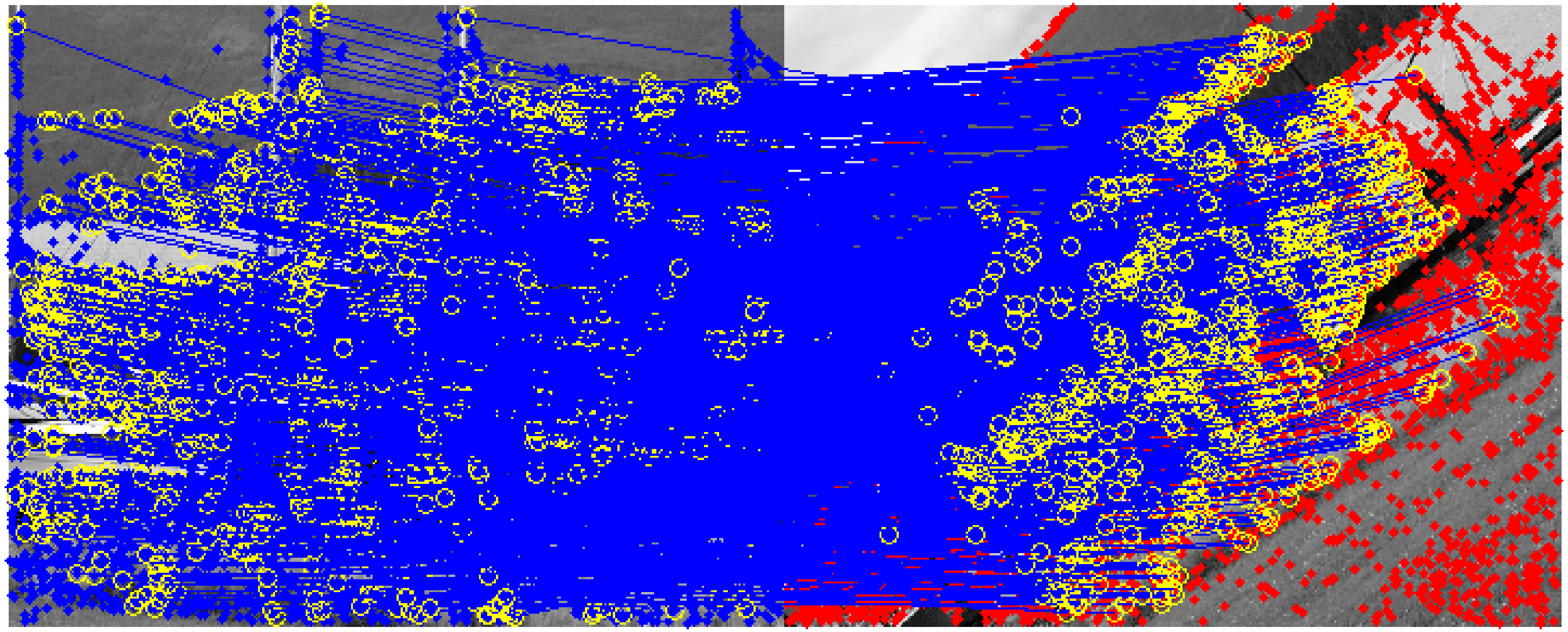
Points: 245/326 Matches: 89 36.33%



5. The SIFT operator

Example: Stereo matches (with rotation)

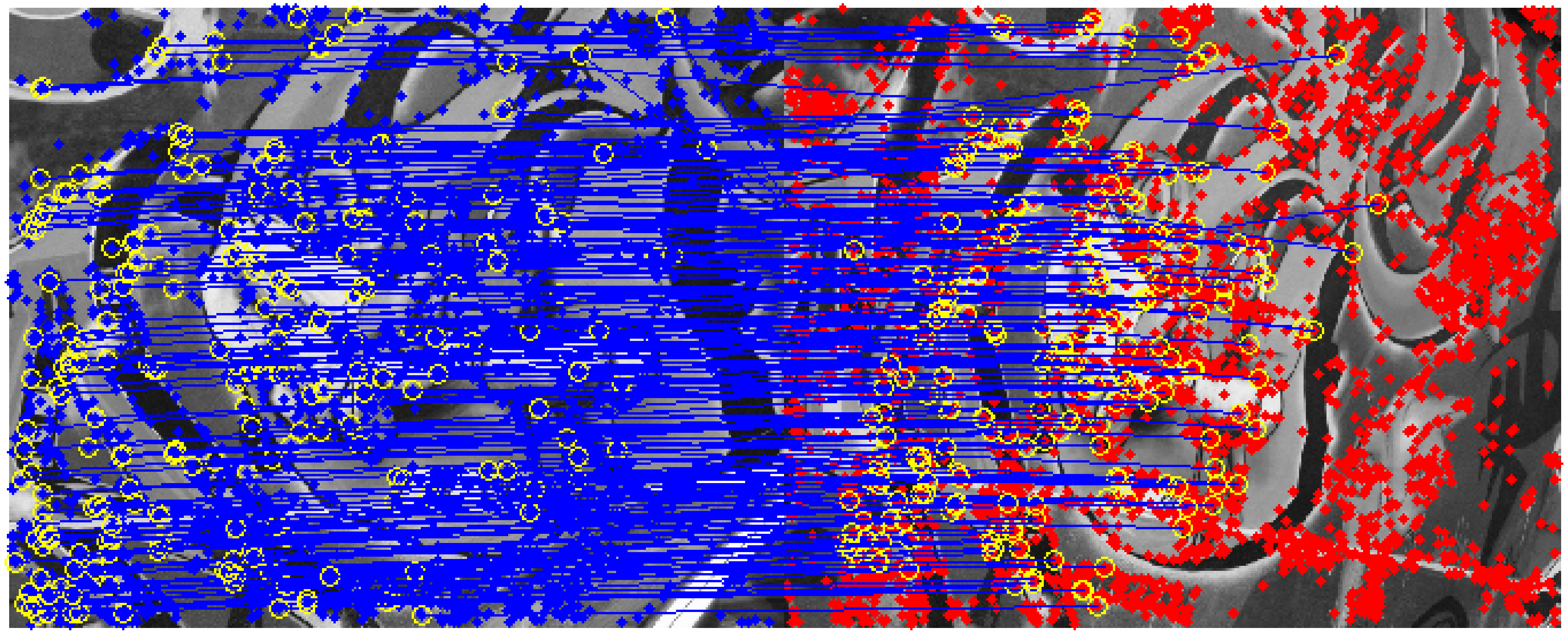
Points: 9687/7113   Matches: 1621   22.79%



5. The SIFT operator

Example: Stereo matches (with rotation)

Points: 3105/3990 Matches: 242 7,79%



Summary

- Harris operator is …
  - a corner detector which is combined with NCC for matching in other images
  - Based on first-order image derivatives
  - invariant to rotation (because derivatives along the eigenvectors)
  - NOT invariant to scale
  - Invariant to brightness (pixel intensities are not directly considered but derivatives)
  - Robust to noise (because of gaussian smoothing)
- KLT operator
  - Same idea as Harris but the two eigenvalues are used
- SIFT operator
  - Detect Blobs at different scales based on the DoG operator
  - Provides a descriptor with the information of the gradient around the detected keypoint at its scale