$(1+1)-D$ QCA Algorithm To Simulate Quantum TASEP Dynamics Using Matrix Product States

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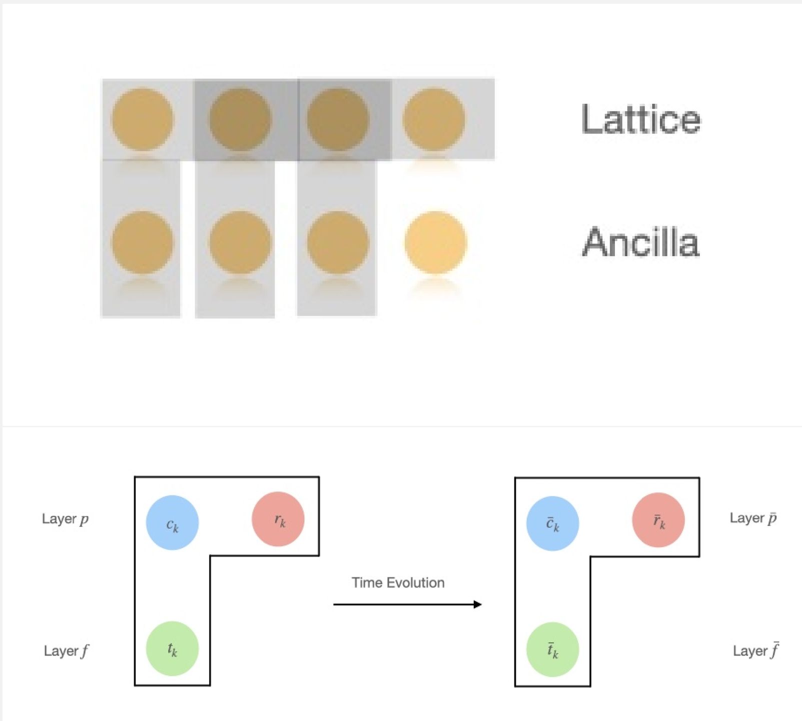
The System

- The **Totally Asymmetric Simple Exclusion Process (TASEP)** is a fundamental stochastic model used to describe **non-equilibrium transport phenomena**[1].
- It consists of **particles hopping in one direction** (totally asymmetric) on a **discrete lattice**, subject to the **exclusion principle**—each site can be occupied by at most one particle.
- We start with nearest neighbour interactions given by the Hamiltonian $\mathcal{H} = -\sum_{<\mu,\nu>} J_{\mu\nu}\sigma^{\dagger}_{\mu}\sigma^{-}_{\nu} + h.c.$
- The jump operators for the Quantum TASEP are given by $L_{pump} = \sigma^{\dagger}$, $L_{drain} = \sigma^{-}$, $L_{bulk} = \sigma^{-}\sigma^{\dagger}$.

The Lattice

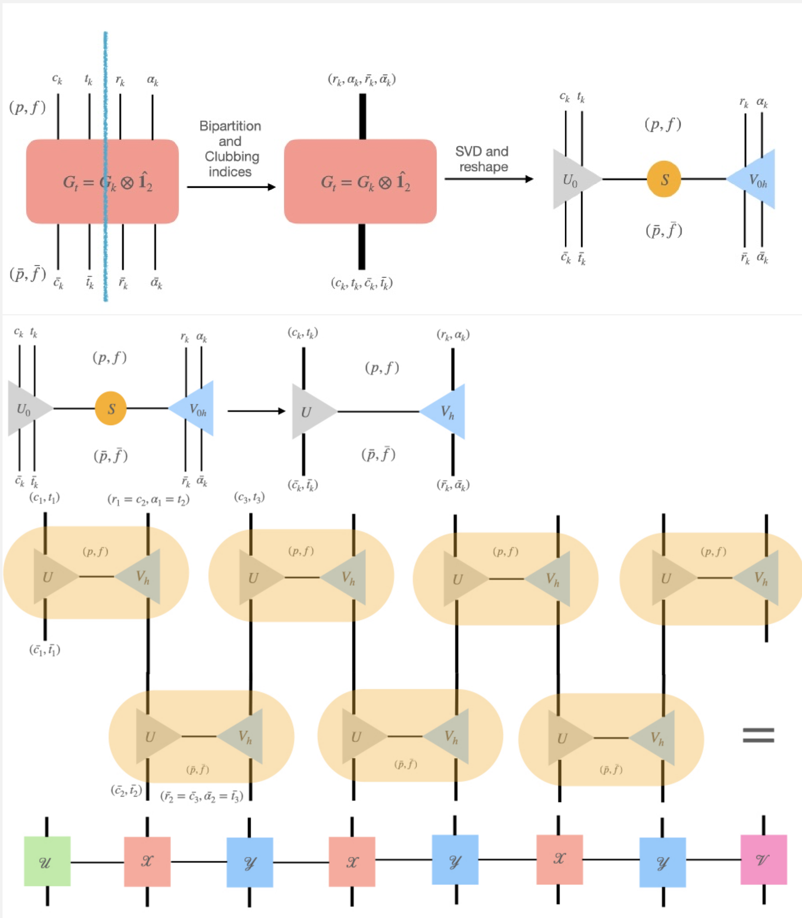


- Lattice Diagram for a $N = 4$ system.
- We create a reverse L-shaped perceptron structure and sweep through the lattice.
- The bulk perceptron $G_k = D_k U_k$ has U_k acting on $c_k - r_k$ and D_k on $c_k - t_k$ qubits respectively.
- The left and right gates $G_{pump/drain}$ are added to the left and right boundaries.

Quantum Cellular Automata

- QCA can be seen as an extension of **Classical Cellular Automata**; where each cells hold a **binary** state.
- QCA follows some fundamental quantum principles:
  1. **Superposition**
  2. **Entanglement**
  3. **Unitary Evolution**
  4. **Locality**
- They are Quantum **Turing Complete**[2]!
- Tensor Network methods like **TEBD** and **DMRG** are widely used to investigate many-body quantum systems[3].
- QCAs modeled with Tensor Networks have recently found increasing popularity to simulate similar systems out of equilibrium.
- They can also be easily extended to higher dimensions; on top of the simplicity of their structure.

Tensor Network Construction



- MPO for the L-shaped lattice.
- The alternating full lattice MPO from the ladder construction for a $N = 8$ system.

The Perceptron Dynamics

- The system evolves as

$$\rho(t+1) = tr_{\bar{p}}(\mathcal{G}(\rho(t) \otimes |0\rangle\langle 0|_{anc})\mathcal{G}^{\dagger}).$$

- More explicitly we have,

$$\rho(t+1) = tr_{\bar{p}}(D_{pump}\prod_{k}\mathcal{G}_k\, D_{drain}(\rho(t) \otimes |0\rangle\langle 0|_{anc})D^{\dagger}_{drain}\prod_{k}\mathcal{G}^{\dagger}_k\, D^{\dagger}_{pump}).$$

- Here we have the following operators;
  1. $U = e^{-idt\mathcal{H}} = e^{-idt\sum_k h_{k,k+1}}$
  2. $D_{pump} = e^{i\theta_p(\sigma^{\dagger}\otimes\sigma^{\dagger}+\sigma^{-}\otimes\sigma^{-})}$
  3. $D_{drain} = e^{i\theta_d(\sigma^{\dagger}\otimes\sigma^{-}+\sigma^{-}\otimes\sigma^{\dagger})}$
  4. $D_{bulk} = e^{i\theta\sum_k(\sigma^{-}_k\otimes\sigma^{\dagger}\otimes\sigma^{\dagger}_{k+1}+\sigma^{\dagger}_k\otimes\sigma^{-}\otimes\sigma^{-}_{k+1})}$
- $\mathcal{G}_k := D_k U_k$ is defined as the **bulk perceptron** of the QCA.
- Our choice of operators recovers the **Lindblad** Master equation with the choice of $\theta = \sqrt{\gamma dt}$ in $O(dt)$ limit[4],

$$\partial_t\rho = -i[\mathcal{H}, \rho] + \sum_{\mu} A_{\mu}\rho A^{\dagger}_{\mu} - \frac{1}{2}\{A^{\dagger}_{\mu}A_{\mu}, \rho\}$$

- We create a QCA perceptron with these equations and evolve the MPO-MPS system till we hopefully reach steady state!!

Plots

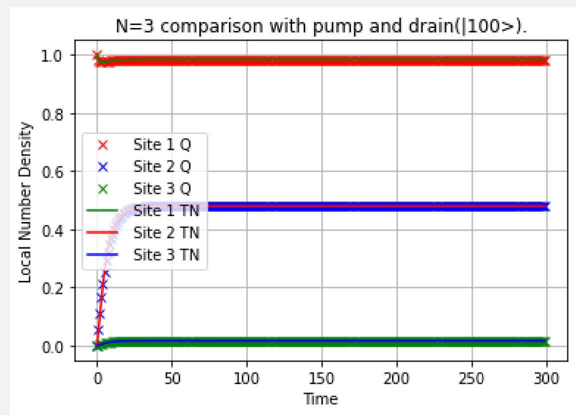


Figure: Verification of Q-TASEP with QuTIP.

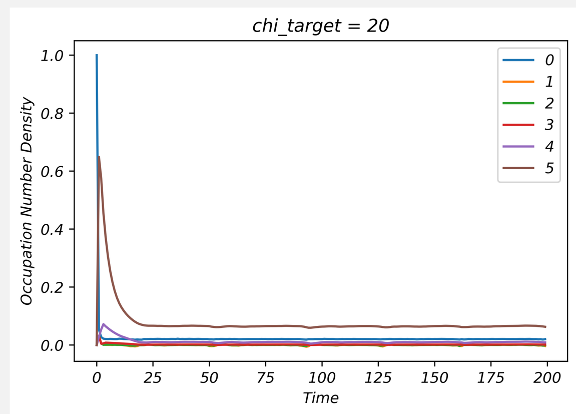


Figure: Individual Lattice Simulation of N=6 with $|100000\rangle$.

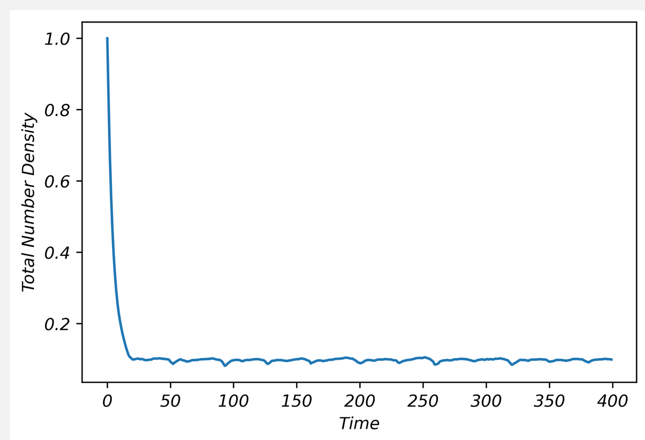


Figure: Full Lattice Simulation of N=6 with $|100000\rangle$.

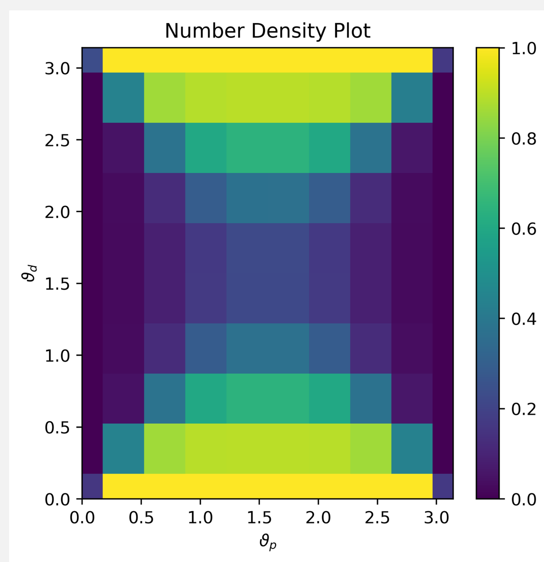


Figure: Full Lattice Occupation Number Density Plot For various In/Out Rates.

Conclusions And Future Work

- The Poster provides a framework to simulate a quantised version of TASEP with **open boundary conditions** with MPS.
- We are currently simulating large-scale systems on the HPC and looking for possible **phase transitions** for various order parameters like current density etc.
- This is an essential building block for **Quantum Active Matter** and looking for interesting kinds of quantum/classical correlations.

References

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