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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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Team member 2	Alper Ülkü
Team member 3	

Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. Note: You may be required to provide proof of your outreach to non-contributing members upon request.
Team member 3 was withdrawn by system

STEP 1: PRICING ASIAN CALL 20 DAYS

a. Calibrating Heston (1993) model

The call and put options with 15 days (the closest to the required) of market availability were used to calibrate the Heston (1993) model. Put-Call parity was used to derive the related call prices from the put prices, giving us a single set of call option prices to calibrate to. 1.5% is the yearly constant risk-free rate. Mean Squared Error is a type of error parameter used. The calibration successfully reached the Heston (1993) parameters listed below:

Results: The calibration converged successfully to the following Heston (1993) parameters:

a. $\kappa_v = 29.2038$ (mean-reversion factor)
b. $\theta_v = 0.0905$ (long-run mean of variance)
c. $\sigma_v = 0.0026$ (volatility of variance)
d. $\rho = -0.0036$ (correlation between variance and stock/index level)
e. $v_0 = 0.0973$ (initial level of variance)

The call option prices produced by the calibrated model based on the market's call options are shown in red on the graph below, while the put option prices are shown in green. The model offers a decent fit visually.

We first obtain data on option prices with a 15-day maturity in order to calibrate the model because the client is seeking a relatively short maturity for her derivative. The Heston model's valuation function is then developed. After that, we compute the discrepancy between model prices and actual market prices using an error function. By determining the parameter value that results in the least gap between the model price and the market price, we then optimize the error function.

c. Pricing under the calibrated model

We evaluated the potential price pathways for the SM stock in the risk-neutral environment in order to determine the price of the at-the-money (ATM) Asian option with 20 days before maturity. This indicates that we made the assumption that prices follow the Heston (1993) model and that market-accessible option prices reflect all available information. Asian call option payouts are calculated for each path, and the results are averaged across all paths, giving us:

- Fair price for Asian Call 20d: 4.74
- Fee: 4%
- The cost of the option for a client to purchase is $4.93 (4% fee included).

We price an Asian call option with 20 days maturity using Monte-Carlo methods under Heston model.

Heston model is a model that assumes volatility is not constant but rather fluctuates over time while Monte Carlo simulation is a statistical technique that involves generating a large number of random scenarios to estimate the expected value of the option.

To price the option, we first calibrate the Heston model based on historical data to ensure it accurately reflects the behavior of the stock price and volatility. As we do not have historical data of option price with 20 days maturity, we do the calibration using price with 15 days maturity instead.

After calibrating the Heston model, we may produce a huge number of hypothetical future scenarios for the stock price and volatility pathways. We determine the Asian option's payment for each scenario based on the underlying asset's average price over a given time frame. To calculate the expected value of the option, we then average all of the payoffs.

We discover that the price converges during the simulations when there are 100 time steps and 100000 simulations. In order to price the option, use these values. Call price converged as follows following our simulation after 100,000 simulations:

For N = 1000, the call price is 4.98
For N = 5000, the call price is 4.72
For N = 10000, the call price is 4.72
For N = 50000, the call price is 4.74
For N = 75000, the call price is 4.72
For N = 100000, the call price is 4.73

After our pricing, the price the client end up paying would be 4.93.

STEP 2: PRICING ASIAN CALL 60 DAYS

a. Calibrating Bates (1996) model using Lewis 2001 approach

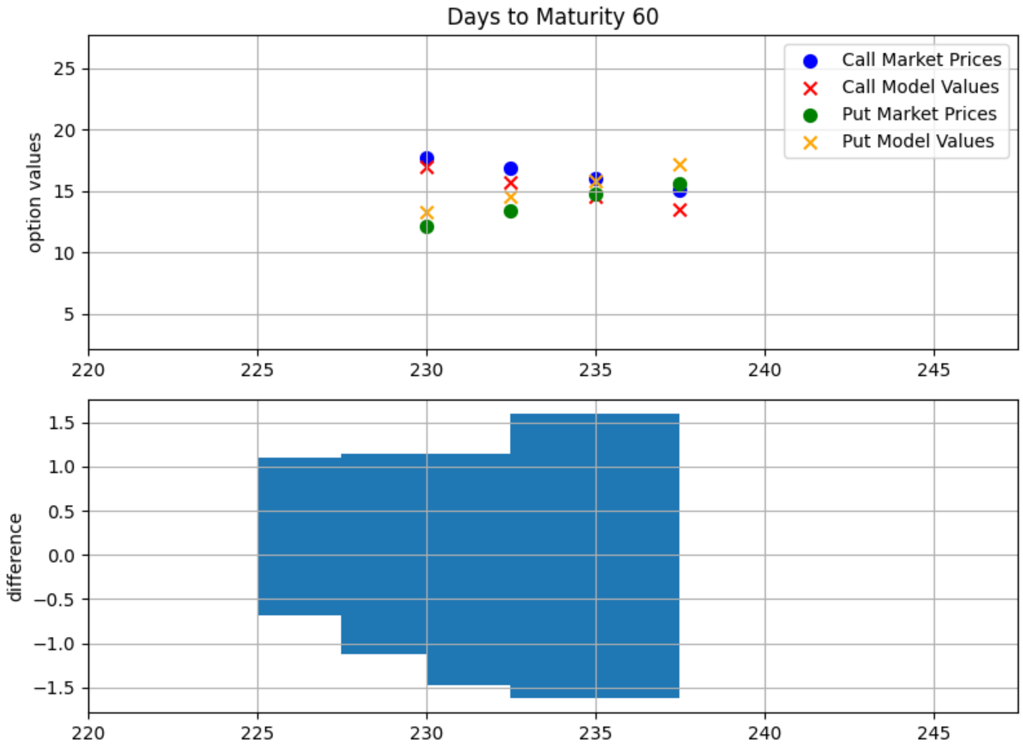
The Bates (1996) model (i.e., Heston model with jumps) was calibrated to replicate call and put options with 60 days (the closest to the required) available on the market. We calculated the related put prices from the call prices using Put-Call parity and obtained the united set of call option prices to calibrate to. The annual constant risk-free rate is 1.5%. Error function is Mean Squared Error.

Results: The calibration converged successfully to the following Bates (1996) parameters:

a. $\kappa_\nu = 13.83$ (mean-reversion factor)
b. $\theta_\nu = 0.1377$ (long-run mean of variance)
c. $\sigma_\nu = 5.7864\ x\ 10^{-6}$ (volatility of variance)
d. $\rho = -5.4375x10^{-3}$(correlation between variance and stock/index level)
e. $\nu_0 = 0.0458$ (initial level of variance)
f. $\lambda\ =\ 0$ (jump intensity)
g. $\mu\ =\ -0.5011$ (expected jump size)
h. $\delta\ =\ 0$ (standard deviation of jump)

The call option prices produced by the calibrated model based on the market's call options are shown in red on the graph below, while the put option prices are shown in green.

Lambda and delta are calibrated as zero, which is unusual but not surprising given that the model converges to a very smooth curve devoid of leaps, as is expected.



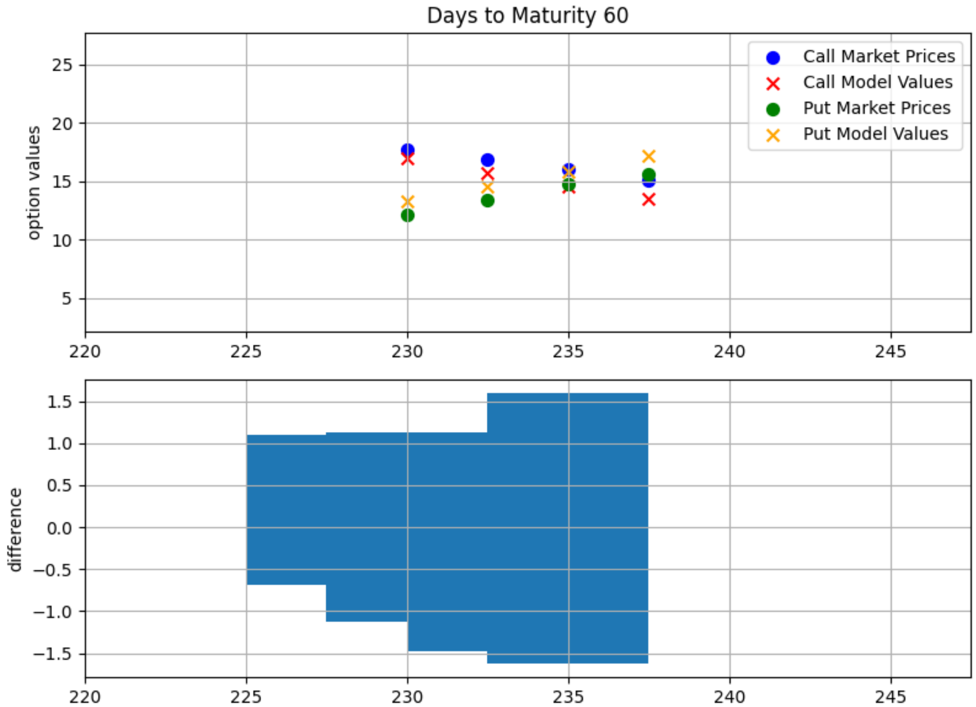
b. Calibrating Bates (1996) model using Carr-Madan (1999) approach

Carr-Madan (1999) was used to calibrate the Bates (1996) model to reproduce call and put options with 60 days (the closest to the necessary) of market availability. Put-Call parity was used to derive the related call prices from the put prices, giving us a single set of call option prices to calibrate to. 1.5% is the yearly constant risk-free rate. Mean Squared Error is a type of error. The calibration successfully converged to the parameters listed by Bates (1996):

a. $\kappa_\nu = 13.83$ (mean-reversion factor)
b. $\theta_\nu = 0.1377$ (long-run mean of variance)
c. $\sigma_\nu = 6.0166\ x\ 10^{-6}$ (volatility of variance)
d. $\rho = -5.4498x10^{-3}$(correlation between variance and stock/index level)
e. $\nu_0 = 0.0458$ (initial level of variance)
f. $\lambda\ =\ 5.76\ x\ 10^{-6}$ (jump intensity)
g. $\mu\ =\ -0.5011$ (expected jump size)
h. $\delta\ =\ 1.13x10^{-5}$ (standard deviation of jump)

λ and δ are calibrated extremely close to zero, much like in the continuous mode, as the model converges to a very smooth curve with no abnormal jumps. Also we observe, without any surprise that, volatility of the variance of the call price is so low since the expected curve is a curvilinear character with no hint of randomness. Both procedures give call option prices that are equivalent to the seventh decimal place when the parameters calibrated and call prices calculated are compared. The call option prices produced by the calibrated model based on the market's call options are shown in red on the graph below, while the put option prices are shown in green.

Visually, the model produces outcomes that are strikingly comparable to those of Step 2a, offering a good fit.



STEP 3: Cox-Ingersoll-Ross (1985) model

a. Calibrate Cox-Ingersoll-Ross (1985) model

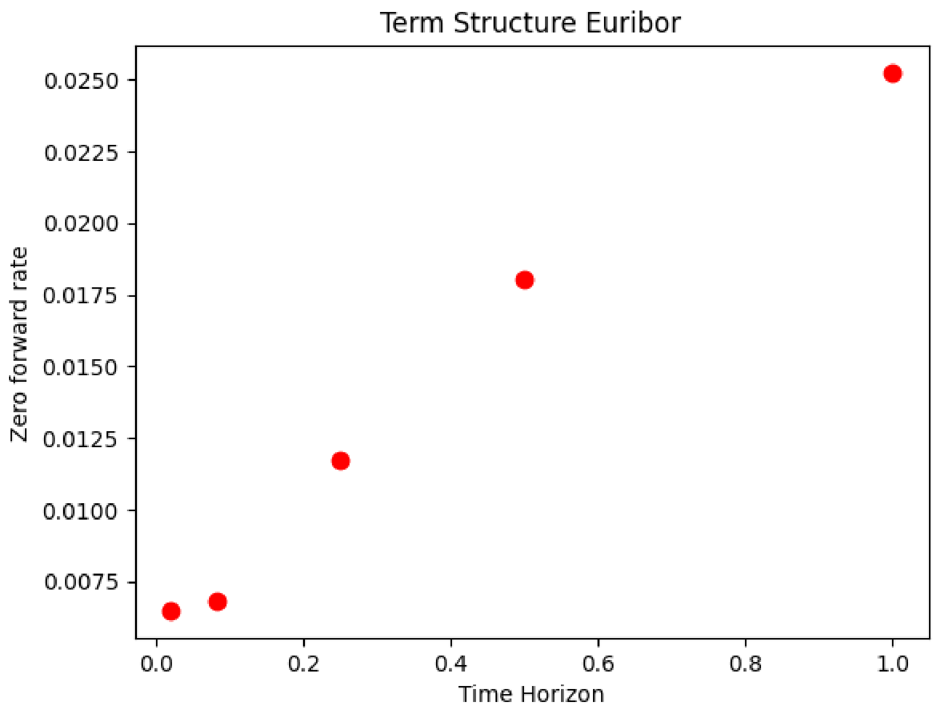
The Cox-Ingersoll-Ross (1985) model will be calibrated by using the following steps:

1. Importing market data
2. Creating valuation function according to the model
3. Creating error function (difference between model output and observed market prices)
4. Creating optimization function (minimizing error function)

We use the current Euribor rates and maturities as shown in the figure below:

Euribor 1 week	0.648 %
Euribor 1 month	0.679 %
Euribor 3 months	1.173 %
Euribor 6 months	1.809 %
Euribor 12 months	2.556 %

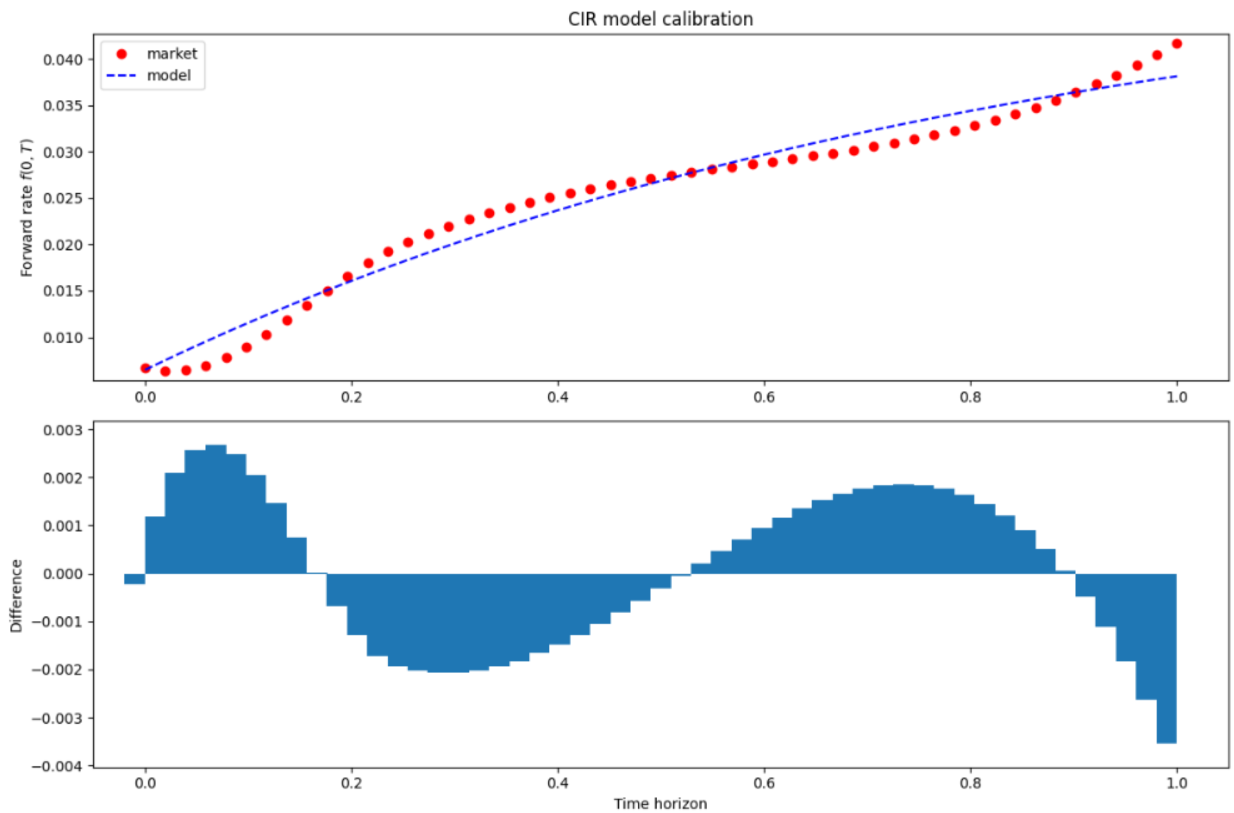
The illustration of the Euribor rates is shown below:



The calibration converged successfully to the following CIR (1985) parameters:

a. $\kappa_v = 0.9965$
b. $\theta_v = 0.1076$
c. $\sigma_v = 0.0498$

The illustration of the market and the calibrated forward rate curves is shown below.

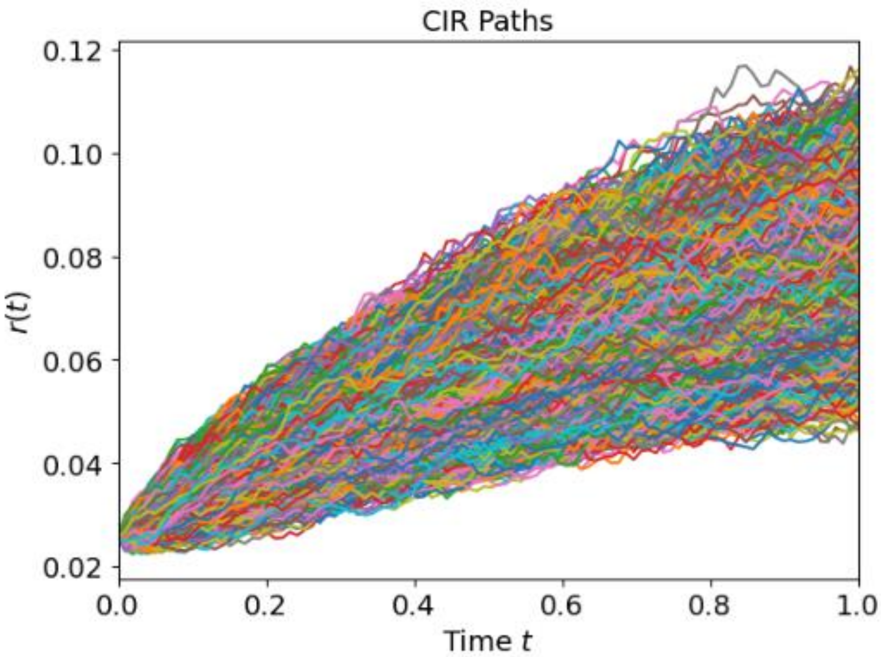


b. Monte-Carlo Simulations of Euribor 12-month rates

We will run 100,000 Monte-Carlo simulations to simulate the Euribor 12-month rates everyday for a year using the various CIR model parameters we acquired in Step 3a. As a result of Step 3a's calibration of the CIR (1985) parameters, we have:

a. $\kappa_v = 0.9965$
b. $\theta_v = 0.1076$
c. $\sigma_v = 0.0498$

According to the simulated outcomes, Euribor 12-month rates would range between 0.1761% and 2.5835% after a year. 1.0406% is what would be anticipated. The value of the short-term OTC instrument would rise if the forecasted figure turned out to be less than the present rate, which would indicate that the interest rate is anticipated to decline. This is due to the fact that investing in new securities with higher interest rates would not increase investors' yields. See below figure for the simulated variance of the interest rate according to



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