

# Volatility Forecasting of TSLA stock by GARCH model

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## 1. INTRODUCTION

In this working paper we model TSLA returns with respect to other popular NASDAQ30 stocks by showing multicollinearity mitigation step by step for TSLA returns with respect to other NASDAQ30 stocks and we find which NASDAQ30 stock has most effect on TSLA returns via correlation matrix analysis and OLS performance scoring. We try to reduce the regression model so that multicollinearity is minimum via running OLS command in Python checking for the p-values of each stock parameters. We demonstrate skewness of TSLA stock returns within the download period and discuss the effects on the volatility prediction. We demonstrate overfitting model for the same data returns and discuss about the potential damage effects to predict volatility. We finally define GARCH model and show its performance with respect to OLS regression and discuss the results. As TSLA is one of the most volatile assets of NASDAQ in last 5 years we selected TSLA as our target asset for visualizing, modeling and forecasting the volatility via GARCH(1,1).

## 2. MULTICOLLINEARITY

### 2.1. DEFINITION:

This is a term to refer to high correlation between two or more independent variables that are chosen in modelling a given dependent variable.

### 2.2. CAUSES

Co-integration - This is a tendency of economic variables to move together over time. This apparently is the main cause of multicollinearity in stock volatilities.

Use of lagged variables - Despite the fact that lagged variables give satisfactory results in many researches , they however have a higher risk of introducing multicollinearity when they are used.

Statistical Procedures- statistical practices such as data smoothing, sampling procedures can also lead to multicollinearity.

### 2.3. DATA FOR MULTICOLLINEARITY

Real World data for log returns is as follows for the selected NASDAQ30 stocks.

	AMGN	MAR	ATVI	SBUX	DLTR	PCAR	MDLZ	SIRI	KDP	HON	TSLA
Date											
2017-09-07 00:00:00	0.007977	-0.005408	0.023269	-0.020402	0.001026	0.019089	0.004030	-0.014756	-0.005000	-0.001118	0.012264
2017-09-08 00:00:00	-0.005385	0.011277	-0.018876	-0.004793	-0.009818	0.006474	-0.013206	-0.016834	-0.011002	-0.001469	-0.025753
2017-09-11 00:00:00	0.026829	0.006466	0.013874	0.004883	0.001858	-0.003575	0.008910	0.004872	0.001002	0.005352	0.052716
2017-09-12 00:00:00	0.018892	-0.000647	-0.006091	-0.013849	0.001930	0.010192	-0.010160	0.004775	-0.003505	-0.007245	-0.007891
2017-09-13 00:00:00	-0.011821	0.010961	-0.015506	0.008958	0.000107	0.000594	-0.001738	-0.036623	-0.004253	-0.010677	0.004877
...	...	...	...	...	...	...	...	...	...	...	...
2022-08-29 00:00:00	-0.010291	-0.011210	-0.003475	-0.004286	-0.014417	-0.017189	-0.008993	-0.013137	-0.000350	-0.008321	-0.016416
2022-08-30 00:00:00	-0.006087	-0.020402	-0.006652	-0.013476	-0.000788	-0.014382	-0.015134	-0.006635	-0.014842	-0.013979	-0.030316
2022-08-31 00:00:00	-0.000077	-0.002395	-0.006655	0.002882	-0.021810	-0.011152	-0.004838	-0.008279	-0.012839	-0.012104	-0.012555
2022-09-01 00:00:00	0.016409	-0.003635	-0.004236	0.010696	0.007452	-0.005457	-0.003223	0.001547	0.000494	0.002262	0.000608
2022-09-02 00:00:00	-0.017831	-0.003183	-0.018070	-0.034229	-0.010840	-0.016498	-0.025213	-0.014836	-0.018130	-0.025339	-0.030396

Figure 1. Data of returns for selected NASDAQ30 stocks

## 2.4. DIAGNOSING AND MITIGATING MULTICOLLINEARITY



Figure 2. Correlation matrix of returns for selected NASDAQ30 stocks

A correlation matrix is one of the most effective tools to investigate multicollinearity. In the above correlation matrix, the coefficients are rather low indicating minimum presence of multicollinearity in the data. High correlation that exceeds 0.8 between regressors leads to the challenge of multicollinearity and its ripple adverse effects on the model. However, other metrics like p-values, R-squared etc will be employed in selecting the fitting regressors for the model.

OLS Regression Results						
Dep. Variable:		TSLA		R-squared:		0.630
Model:		OLS		Adj. R-squared:		0.573
Method:		Least Squares		F-statistic:		10.96
Date:		Mon, 05 Sep 2022		Prob (F-statistic):		3.96e-20
Time:		22:59:53		Log-Likelihood:		354.11
No. Observations:		157		AIC:		-664.2
Df Residuals:		135		BIC:		-597.0
Df Model:		21				
Covariance Type:		nonrobust				
		coef	std err	t	P> t	[0.025 0.975]
	Intercept	0.0030	0.003	1.044	0.298	-0.003 0.009
	AMGN	-0.1395	0.215	-0.650	0.517	-0.564 0.285
	MAR	0.0305	0.142	0.215	0.830	-0.249 0.310
	ATVI	0.3460	0.381	0.907	0.366	-0.408 1.100
	SBUX	0.0471	0.167	0.282	0.778	-0.284 0.378
	DLTR	0.0391	0.088	0.442	0.659	-0.136 0.214
	PCAR	0.1658	0.217	0.764	0.446	-0.264 0.595
	MDLZ	-0.2915	0.256	-1.141	0.256	-0.797 0.214
	SIRI	0.0884	0.197	0.450	0.654	-0.300 0.477
	KDP	0.1154	0.232	0.497	0.620	-0.344 0.575
	HON	0.1781	0.217	0.822	0.412	-0.250 0.606
	FISV	-0.2056	0.180	-1.139	0.257	-0.562 0.151
	VRSK	0.2980	0.212	1.406	0.162	-0.121 0.717
	VRSN	0.0156	0.163	0.096	0.924	-0.307 0.338
	AZN	-0.1657	0.145	-1.142	0.255	-0.453 0.121
	BIDU	0.1790	0.062	2.883	0.005	0.056 0.302
	AMAT	0.3800	0.129	2.944	0.004	0.125 0.635
	MRNA	-0.0080	0.068	-0.117	0.907	-0.143 0.127
	FTNT	-0.0070	0.092	-0.076	0.939	-0.190 0.176
	CEG	0.0235	0.077	0.304	0.762	-0.130 0.177
	ZS	0.1692	0.085	1.988	0.049	0.001 0.338
	ZM	0.1053	0.076	1.385	0.168	-0.045 0.256
Omnibus:		13.255		Durbin-Watson:		2.044
Prob(Omnibus):		0.001		Jarque-Bera (JB):		18.738
Skew:		-0.490		Prob(JB):		8.53e-05
Kurtosis:		4.380		Cond. No.		177.

Figure 3. Iteration-1 for OLS regression of selected NASDAQ30 stocks

OLS Regression Results						
Dep. Variable:		TSLA			R-squared:	0.267
Model:		OLS			Adj. R-squared:	0.265
Method:		Least Squares			F-statistic:	136.4
Date:		Mon, 05 Sep 2022			Prob (F-statistic):	2.57e-75
Time:		22:59:54			Log-Likelihood:	2152.8
No. Observations:		1125			AIC:	-4298.
Df Residuals:		1121			BIC:	-4277.
Df Model:		3				
Covariance Type:		nonrobust				
	coef	std err	t	P> t	[0.025	0.975]
Intercept	0.0014	0.001	1.242	0.214	-0.001	0.003
BIDU	0.2541	0.038	6.702	0.000	0.180	0.329
AMAT	0.4521	0.041	11.000	0.000	0.371	0.533
ZS	0.1685	0.030	5.692	0.000	0.110	0.227
Omnibus:	105.656	Durbin-Watson:			2.031	
Prob(Omnibus):	0.000	Jarque-Bera (JB):			691.167	
Skew:	0.001	Prob(JB):			8.22e-151	
Kurtosis:	6.840	Cond. No.			43.5	

Figure 4. Iteration-2 for OLS regression after deleting stocks with low relevance.

Close analysis and examination of the standard errors, t-ratios, R-squared statistic and the Durbin Watson statistics suggest that the explanatory variables other than AMAT, BIDU and ZS are superfluous. In the case of the volatility of TSLA, these three stocks are showing considerable significance in influencing TSLA stock price and can give a better model for TSLA since they have p-values < 0.05. A heuristic volatility model is therefore given by the model:

$$\text{TSLA (RETURNS)} = 0.0014 + 0.2541 (\text{BIDU}) + 0.4521 (\text{AMAT}) + 0.1685 (\text{ZS}) + e,$$

where, e = error term

The very low R-squared statistic on these statistically significant regressors could indicate that a non-linear regression model could best in explaining the variability in the TESLA stock. Thus we should always consider difficulty of finding the correct exogenous variable for modelling.

## 2.5. DAMAGE

Multicollinearity causes wide confidence intervals for the model coefficient estimates hence compromising its precision. The collinearity issue makes it hard to separate the individual impact from correlated independent variables in the model.

## 2.6. DIRECTIONS TO ADDRESS MULTICOLLINEARITY

1. **Use of different regressor:** This technique is only valid as long as mis-specification is avoided or there will be biased estimates.
2. **Increasing sample size:** The efficacy of this procedure in addressing multicollinearity lies in the model's ability to yield estimates that converge to their true values as sample size increases to infinity.
3. **Principal Component Analysis:** This procedure uses subspace of the sample information and thus reduces the information-set's dimensions by excluding all but the significantly important components from entering the estimation process.

### 3. OVERFITTING

#### 3.1. DEFINITIONS AND DESCRIPTIONS

According to Williams and Rodriguez, Overfitting is a situation where a model is overly complex, to the extent that the model has learned noise in the (training) data and its predictions is therefore generally poor to a test data (2).

Overfitting usually occurs when a higher dimensional data is used in model. That is to say if there are many exogenous variables used to build the model; for example, predicting volatility of stock A (endogenous variable) using the historical data of 300 other firms stock prices.

As Twin puts it, such a model appears to predict the exogenous variable with close accuracy but when subjected to an outside dataset, then a chance occurrence is realized.

The diagram below shows theoretical difference between overfitted model and good fitted model.

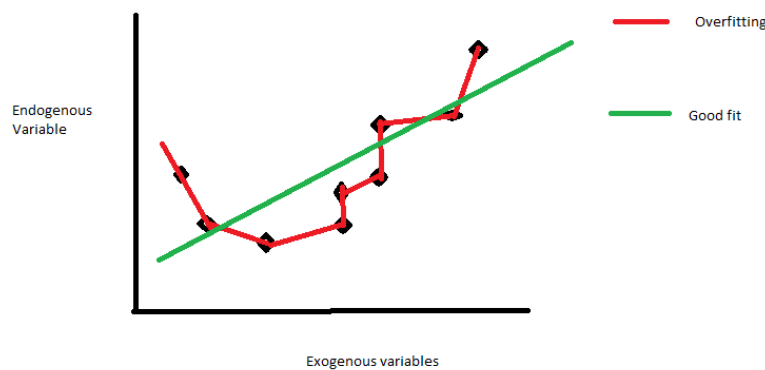


Figure 5. Overfitting vs a good fit

#### 3.2. DEMONSTRATION

We are going to illustrate overfitting using 25 stock prices from 2021 Jan 1, to 2021 June 1. The dataset has been downloaded from Yahoo and saved locally. We are going to build an **Ordinary Least Square (OLS)** regression, in which we shall predict **Kosmos Energy Ltd (KOS)**'s stock prices using the 24 stock prices of other firms.

##### 3.2.1. OLS REGRESSION

We first create **X** and **y** variables. **X** includes all our exogenous variables and **y** includes our endogenous variable (KOS). Then we split our dataset into two categories: **training dataset** and **testing dataset**. Training datasets are used to train the model while the testing datasets are used to test the model. Therefore, we get **X\_train** and **y\_train** as the training **exogenous variables** and **endogenous variable** respectively. While **X\_test** and **y\_test** are the testing **exogenous variables** and **endogenous variable** respectively.

OLS Regression Results			
Dep. Variable:	KOS	R-squared:	0.951
Model:	OLS	Adj. R-squared:	0.910
Method:	Least Squares	F-statistic:	22.88
Date:	Mon, 05 Sep 2022	Prob (F-statistic):	2.73e-12
Time:	08:04:35	Log-Likelihood:	67.413
No. Observations:	51	AIC:	-86.83
Df Residuals:	27	BIC:	-40.46
Df Model:	23		
Covariance Type:	nonrobust		

From the summaries above, we can see that R-Squared = 0.951 and Adjusted R-squared = 0.910. Which shows that the model that we just build fits closely when we used X\_train and y\_train, since the R-squared and Adjusted R-squared are high. One should also note that both R-squared and Adjusted R-squared range from 0 to 1. However, we have to test our model by using the testing dataset that we split earlier: X\_test and y\_test.

```
OLS_R^2 0.7422858324895707
OLS_SME 0.14272851556622107
```

From the results from our test dataset, we can see that the R-squared has now moved from 0.951 to 0.7422858 which shows that there was some overfitting. In the next section we are going to look at how to handle this overfitting.

### 3.3. HOW TO HANDLE OVERFITTING

According to Ying, there are several strategies for handling overfitting and these can be grouped into the following: early stopping, network reduction, data-expansion and regularization. We shall use regularization to handle the overfitting in our model above. It's important to know that regularization strategies employ the penalized regressions.

#### 3.3.1. PENALIZED REGRESSIONS

Penalized regressions add penalty to each added exogenous variable in the model as shown in the equation below.

$$\sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_{1i} - \dots - \beta_p X_{pi})^2 + \lambda \sum_{j=1}^p f(\beta_j)$$

From the equation above,  $f(\beta_j)$  is called a regularization function and it pulls the value of a too large coefficient close to 0. And  $\lambda$  is called a regularization factor and it controls the tradeoff between variance (brought by too many exogenous variables) and bias (brought by the regularization function). The two common penalized regression are Ridge and Least absolute Shrinkage and selection operation (Lasso) regressions. We are going to use Lasso to handle the overfitting issue in our model above.

##### 3.3.1.1. LASSO REGRESSION

According to Ying, Lasso uses L1 regularization function and this is shown in the equation below:-

$$f() = \sum_{j=1}^p |\beta_j| = \|\beta\|_1$$

Below we use lasso to train a new model and compare the results with the OLS model.

```
LASSO_R^2: 0.7698454166858153
LASSO_SME: 0.1348811983944432
```

From the results from lasso model, we can see that there has been an improvement in R-Squared with the test dataset: from 0.7422858324895735 (for the case of OLS) to 0.769845416685815 (for the case of Lasso).

What this means that Lasso has tried to minimize the effect of overfitting by penalizing the exogenous variables that have to large coefficients.

Note that we could also use Lasso for feature selection in this case by dropping off the exogenous variables with zero (or so close to zero) coefficients, and rerun the OLS model and see whether the results improve with the test dataset.

### 3.3.1.2. USING LASSO FOR EXOGENOUS VARIABLE SELECTION

Below we list all the exogenous variables with their coefficients. We will drop all the exogenous variables with zero (or so close to zero) coefficients since they are not contributing much to the model and rerun with OLS model.

[Check the Notebook]

From the results we will drop LYG, PUMSY, META, BEAM, PSMY, PTOM, QCOM, BYDDY and see how the model will perform.

```

=====
                        OLS Regression Results
=====
Dep. Variable:          OLS_R^2 0.8132095505245255          0.911
Model:                  OLS_SME 0.12151193465671764          0.873
Method:                                     23.84
Date:      Mol., ...                               5.30e-14
Time:      08:04:35   Log-Likelihood:          52.047
No. Observations:      51   AIC:          -72.09
Df Residuals:          35   BIC:          -41.19
Df Model:              15
Covariance Type:      nonrobust
=====
```

We can see that the result with the test dataset after feature selection now has improved with R-squared=0.8132095505245249

## 4. SKEWNESS

### 4.1. DESCRIPTION:

Skewness refers to a distortion or asymmetry that deviates from the symmetrical bell curve, or normal distribution, in a set of data.

The degree of asymmetry present in a probability distribution is known as skewness in statistics. Different degrees of right (positive) or left (negative) skewness can be seen in distributions. Zero skewness is present in a bell-shaped distribution, which is normal. In Figure X, we can see negative and positive and zero skewness resembling the normal distribution.

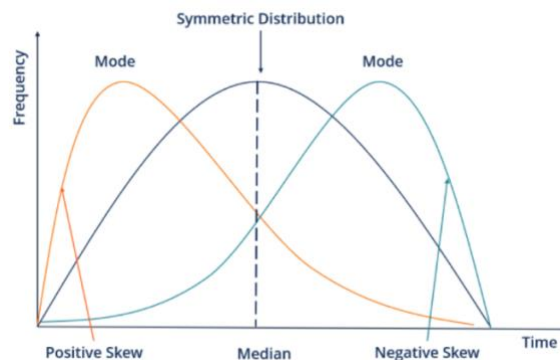


Figure 6. Skewness explained

Positive skewness refers to a distribution that is displaced to the left and has

its tail on the right. The distribution is also referred to as being right-skewed. The tapering of the curve that differs from the data points on the other side is referred to as a tail. A positively skewed distribution requires a skewness value greater than zero, as the name suggests. The given distribution has a right-to-left skewness, which causes the mean value to be bigger than the median and migrate to the right. The mode also occurs with the highest frequency in the distribution.

A negatively skewed distribution is a distribution pushed to the right and has its tail on the left. Another name for it is a left-skewed distribution. Any distribution with a negative skew has a skewness value that is less than zero. The left-handed skewness of the given distribution leads the mean value to deviate from the median and move to the left. The distribution also shows the mode the most commonly.

### 4.2. DEMONSTRATION OF SKEWNESS ON REAL DATA:

```
1 # 504 day averaged skewness of TSLA returns
2 log_returns['TSLA'].rolling(504).mean().skew()
-0.5635065821269012

[24] 1 # 252 day averaged skewness of TSLA returns
2 log_returns['TSLA'].rolling(252).mean().skew()
0.5718067350301886

[25] 1 # 50 day averaged skewness of TSLA returns
2 log_returns['TSLA'].rolling(50).mean().skew()
0.6731984827778821

[26] 1 # 14 day averaged skewness of TSLA returns
2 log_returns['TSLA'].rolling(14).mean().skew()
0.1787975322594338
```

Figure 7. Mean skewness of Returns of TSLA stock for different windows.



Here TSLA's log returns has a averaged skewness of -0.567, 0.57 and 0.673 and 0.171 in the periods of last 504 workdays (2 years), last 252 workdays (1 year), last 50 workdays and last 14 workdays respectively, which can be explained as follows: The returns distribution started with a negative skewed to a more positively skewed and ended up with a moderate 0.171; in other words the TSLA stock was once in downtrend and nowadays day by day making more positive returns rather than negative, getting more and more profitable however caution should be exercised as this skewness is getting less from 0.67 to 0.171. Thus a downtrend in skewness can be viewed as a downtrend in returns.

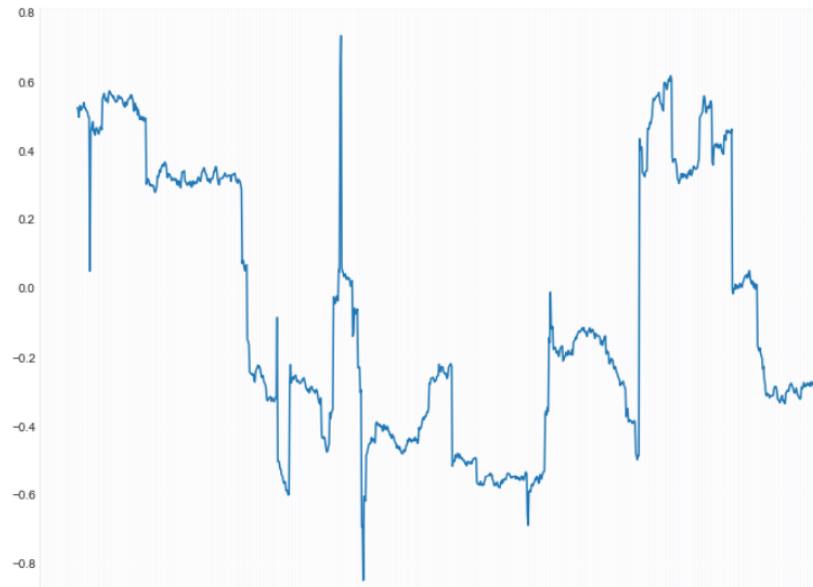


Figure 8. Yearly averaged Skewness of Returns of TSLA stock after Python command:  
`plt.plot(log_returns['TSLA'].rolling(252).skew())`

#### 4.3. DIAGRAM:

Figure 2 shows TSLA's 252 day average returns skewness. Nowadays traders may have noticed that TSLA is losing ground and has not been so profitable. This can be detected when last one year average skewness is observed around -0.3. This means we have negative returns distribution more prevalent and in dominance. Our python code shows in a glance how TSLA behaves for mean variance and skewness as below:

```
1 print("5 year Mean of TSLA log_returns", log_returns['TSLA'].mean())
2 print("5 year standard deviation of TSLA log_returns", log_returns['TSLA'].std())
3 print("5 year skewness of TSLA log_returns", log_returns['TSLA'].skew())
4 plt.hist(log_returns['TSLA'], bins=100)
```

5 year Mean of TSLA log\_returns -0.0030522122564259997  
 5 year standard deviation of TSLA log\_returns 0.04014546491862166  
 5 year skewness of TSLA log\_returns -0.13495251310509948

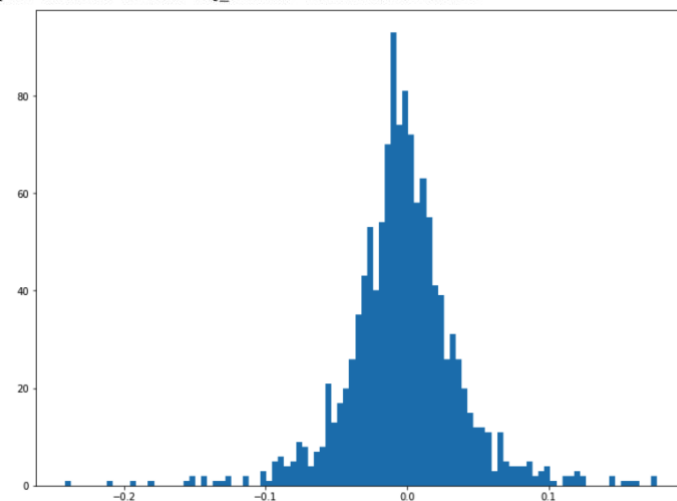


Figure 9. Histogram of TSLA log returns

We can observe from Figure 9 that TSLA has negative skewness meaning, mean value of returns is negative and tendency of negative return generation is greater than that of positive returns.

#### 4.4. DIAGNOSIS of Skewness in TSLA returns data

Formula for Pearson's Skewness

$$Sk_1 = \frac{X - Mo}{s}$$

---


$$Sk_2 = \frac{3\bar{X} - Md}{s}$$

where:

$Sk_1$  = Pearson's first coefficient of skewness and  $Sk_2$   
the second

$s$  = the standard deviation for the sample

$\bar{X}$  = is the mean value

$Mo$  = the modal (mode) value

$Md$  = is the median value

Figure 10. Skewness Formulas

We use above formula for skewness. But in python *skew()* and *hist()* commands are used extensively for returns data to detect the value of skewness.

#### 4.5. DAMAGE caused by skewness of TSLA returns data

We cannot consider skewness of TSLA returns (or any such statistical data) cause any harm to modeling of volatility on the contrary, it gave useful information on detection of trend of the stock. However skewness tool is inadequate for any kind of modelling of volatility. Skewness can not be used alone in any kind of regression method. Thus we propose the model in next section for the purpose of volatility modeling.

#### 4.6. DIRECTIONS: GARCH FOR MODELING VOLATILITY

Each modelling tool like OLS and MLS shown above has its own deficiencies. One thing in common is they do not have an auto-regressive component. Here is how we handle the issue of modeling volatility or returns by introduction of autoregression into the modelling.

Generalized Autoregressive Conditional Heteroskedasticity, sometimes known as GARCH, is a development of the ARCH model (Autoregressive Conditional Heteroskedasticity). The conventional econometric method for predicting volatility of financial time series is GARCH, which includes lag variance terms with lag residual errors from a mean process.

GARCH can be modeled mathematically as follows:

$$\sigma_t^2 = \omega + \sum_i^q \alpha_i \epsilon_{t-i}^2 + \sum_1^p \beta_i \sigma_{t-i}^2$$

where epsilon  $\epsilon_{t-i}^2$  is the model residuals at time step t-i and  $\sigma_t^2$  is variance at time step t. Only first-order lagged terms are included in GARCH(1,1), and its mathematical equation is:

$$\sigma_t^2 = \omega + \alpha \epsilon_{(t-1)}^2 + \beta \sigma_{(t-1)}^2$$

where omega is the long-term variance and alpha, beta, and omega add up to 1. Although GARCH is generally regarded as an insightful improvement over naively assuming future volatility will be

like the past, some experts in the field of volatility also believe it to be vastly overrated as a predictor. The key elements of volatility are captured by GARCH models: volatility will cluster (be near to what it is today) tomorrow and will likely mean revert (be close to whatever the historical long-term average has been) over the long term.

Constant Mean - GARCH Model Results

Dep. Variable: TSLA

R-squared: 0.000

Mean Model: Constant Mean

Adj. R-squared: 0.000

Vol Model: GARCH

Log-Likelihood: 1393.98

Distribution: Normal

AIC: -2779.96

Method: Maximum Likelihood

BIC: -2761.52

No. Observations: 743

Date: Sun, Sep 04 2022

Df Residuals: 742

Time: 21:08:41

Df Model: 1

Mean Model

coefstd errtP>|t|95.0% Conf. Int.

mu-3.5558e-031.353e-03-2.6288.583e-03[-6.207e-03,-9.041e-04]

Volatility Model

coefstd errtP>|t|95.0% Conf. Int.

omega5.7719e-052.056e-052.8074.997e-03[1.742e-05,9.802e-05]

alpha[1]0.05502.377e-022.3152.059e-02[8.447e-03,0.102]

beta[1]0.90783.427e-0226.4921.201e-154[0.841,0.975]

Covariance estimator: robust

Figure 11. Performance parameters of GARCH modelling of volatility

Now we can rewrite our volatility model for TSLA as follows:

$$\sigma_t^2 = 5.7 * 10^{-5} + 0.055 * \epsilon_{(t-1)}^2 + 0.9078 * \sigma_{(t-1)}^2$$

And depiction of volatility real values and predicted values are in Figure X.

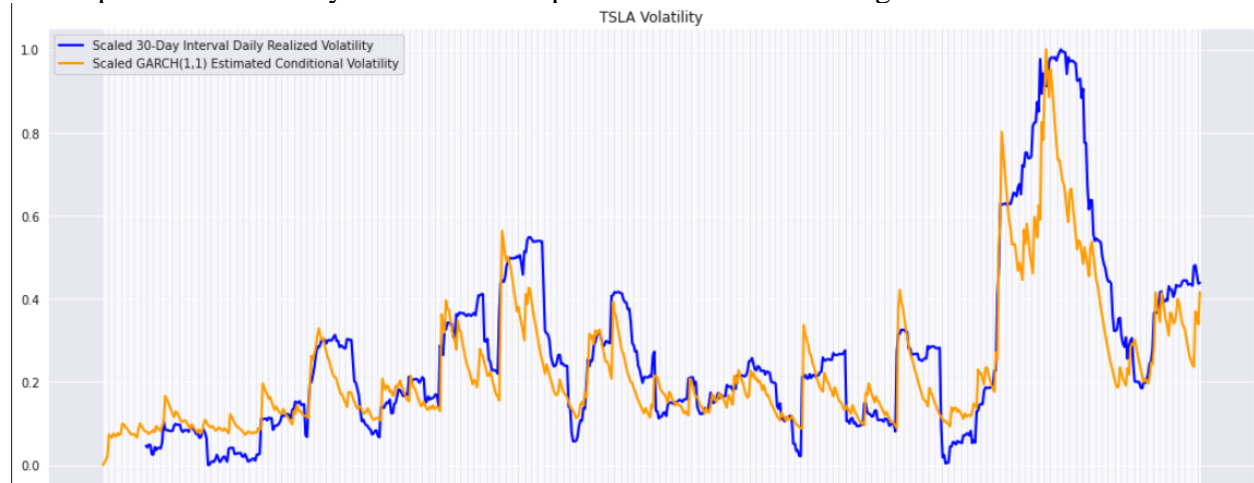


Figure 12. Realized TSLA Volatility against the GARCH estimated TSLA volatility

As seen clearly performance of GARCH(1,1) in peak value detection of local volatility is significant but estimation of steady state volatility values can be improved more.

## 5. REFERENCES

Twin, Alexandra. "How Overfitting Works." Investopedia, Investopedia, 28 June 2022, <https://www.investopedia.com/terms/o/overfitting.asp>.

Williams, Donald Ray, and Josue E. Rodriguez. "Why Overfitting Is Not (Usually) a Problem in Partial Correlation Networks." 2020, <https://doi.org/10.31234/osf.io/8pr9b>

Ying, Xue. "An Overview of Overfitting and Its Solutions." Journal of Physics: Conference Series, vol. 1168, 2019, p. 022022., <https://doi.org/10.1088/1742-6596/1168/2/022022>

*Skewness - Overview, Types, How to Measure and Interpret.* (2022, May 7). Corporate Finance Institute. <https://corporatefinanceinstitute.com/resources/knowledge/other/skewness/>

Ozdemir, O. (n.d.). *ARCH-GARCH Tutorial with rugarch package*. ARCH-GARCH Tutorial with Rugarch Package. Retrieved September 4, 2022, from <http://users.metu.edu.tr/ozancan/ARCHGARCHTutorial.html>

C. (2021, September 11). *GitHub - chibui191/bitcoin\_volatility\_forecasting: GARCH and Multivariate LSTM forecasting models for Bitcoin realized volatility with potential applications in crypto options trading, hedging, portfolio management, and risk management*. GitHub. [https://github.com/chibui191/bitcoin\\_volatility\\_forecasting](https://github.com/chibui191/bitcoin_volatility_forecasting)

*Positional Option Trading: An Advanced Guide (Wiley Trading): Sinclair, Euan: 9781119583516: Amazon.com: Books.* (2020, September 15). Positional Option Trading: An Advanced Guide (Wiley Trading): Sinclair, Euan: 9781119583516: Amazon.Com: Books. <https://www.amazon.com/Positional-Option-Trading-Wiley/dp/1119583519>

*Skewness Definition.* (2022, June 15). Investopedia. <https://www.investopedia.com/terms/s/skewness.asp>

---

Christ, C.F (1966), *Econometric Models and Methods*, John Wiley, New York

Johnston J. (1984), *Econometric Methods*, McGraw-Hill Book Co. , Singapore

Koutsoyiannis A. (1991) , *Theory of Econometrics: An Introductory Exposition of Econometric Methods*, Macmillan, Hongkong

---