Design and Analysis of Algorithms

Algorithm Design Techniques

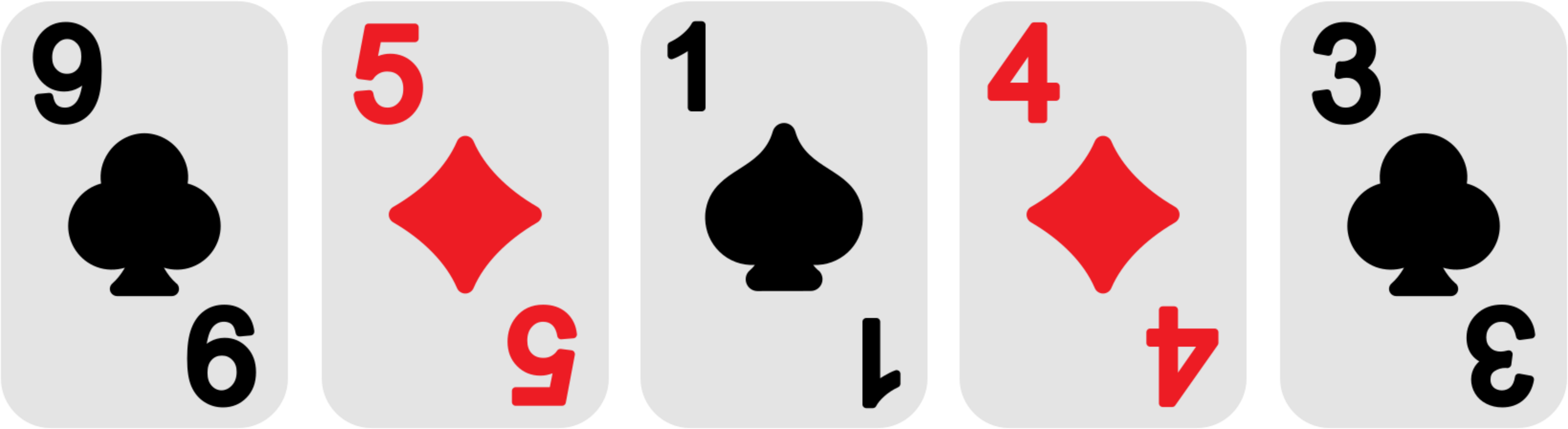
- **Brute Force and Exhaustive Search**
- Divide-and-Conquer
- **Decrease-and-Conquer**
- Transform-and-Conquer
- Space and Time Trade-Offs
- Dynamic Programming
- Greedy Technique
- Iterative Improvement
- Backtracking
- Branch-and-Bound

Sorting Algorithm

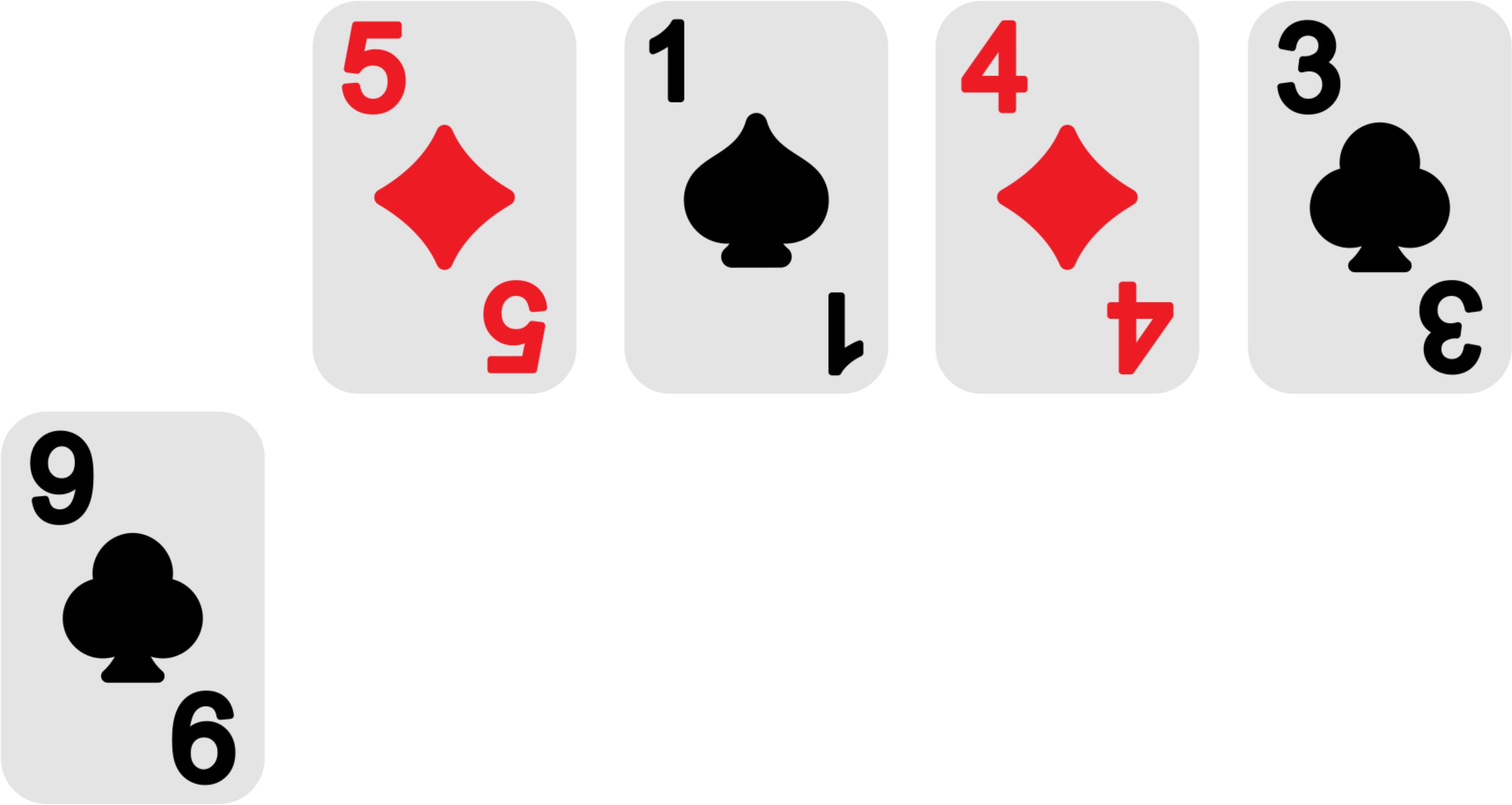
Brute Force

- Brute force is a straightforward approach to solving a problem, usually directly based on the problem statement and definitions of the concepts involved.
- In this section, we consider the application of the brute-force approach to the problem of sorting.
- As we mentioned, many algorithms have been developed for solving this very important problem.
- Now, ask yourself a question: "What would be the most straightforward method for solving the sorting problem?"

Sorting Playing Cards



Sorting Playing Cards



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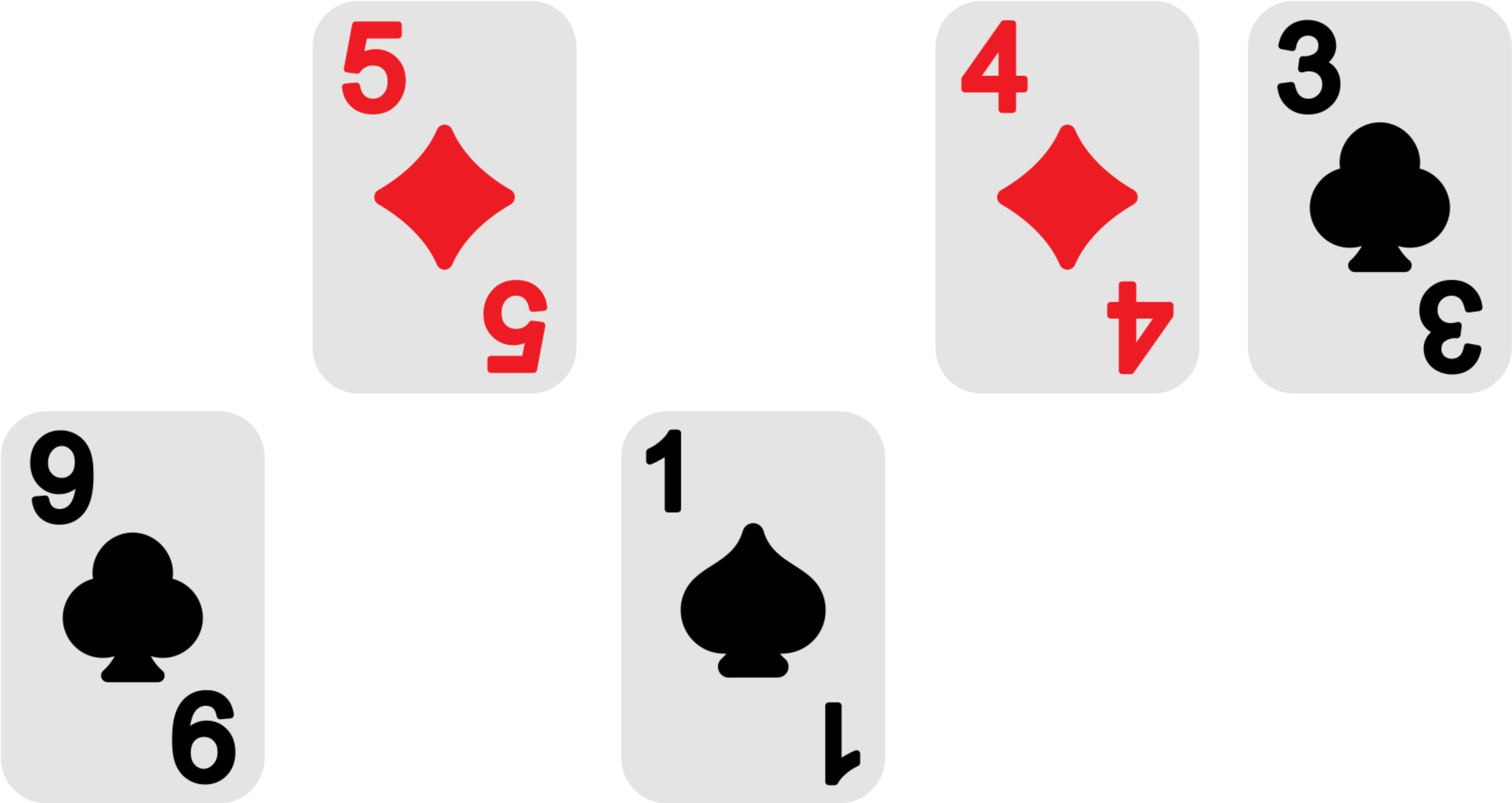

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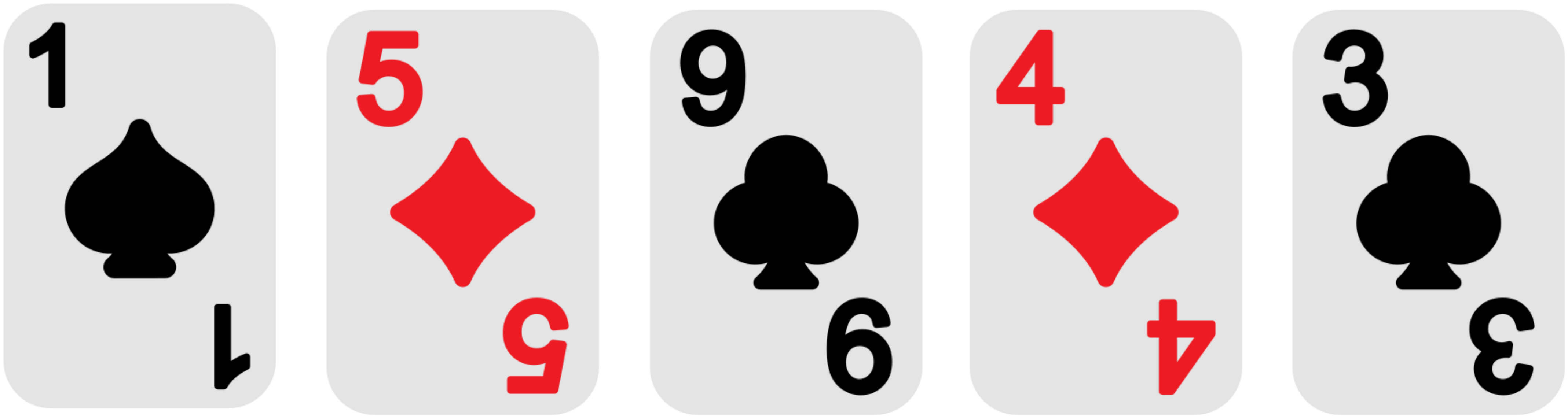

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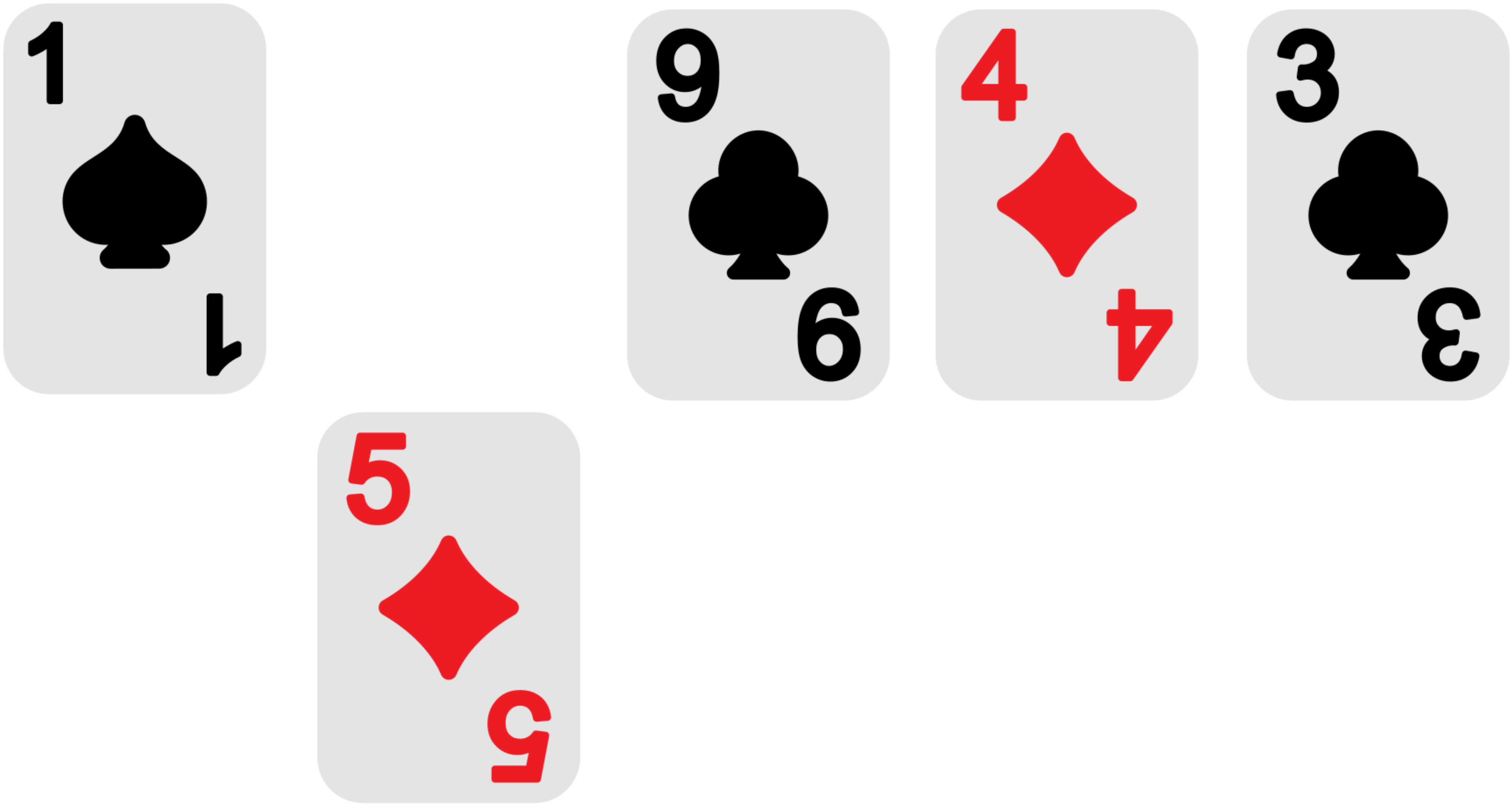

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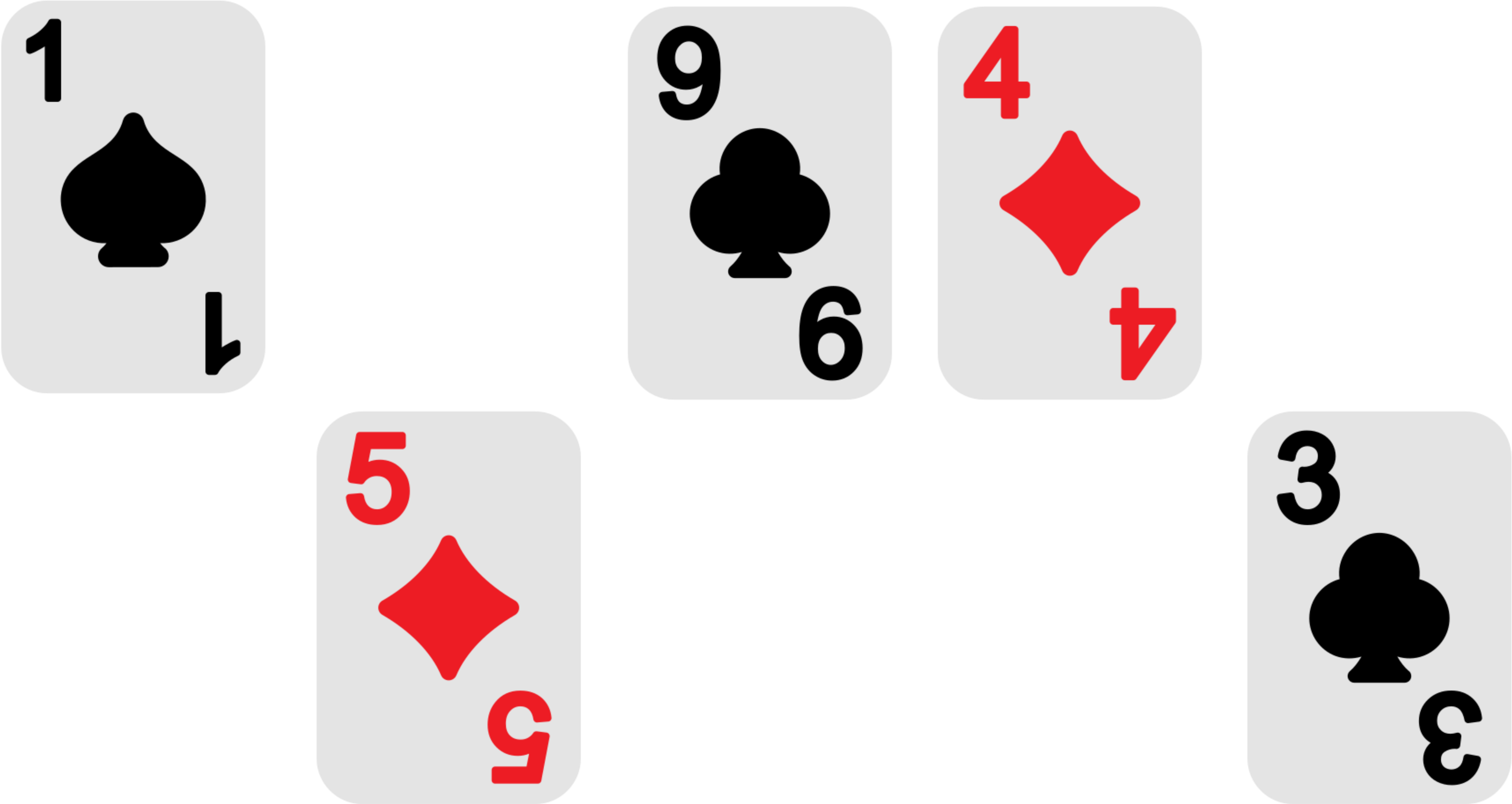

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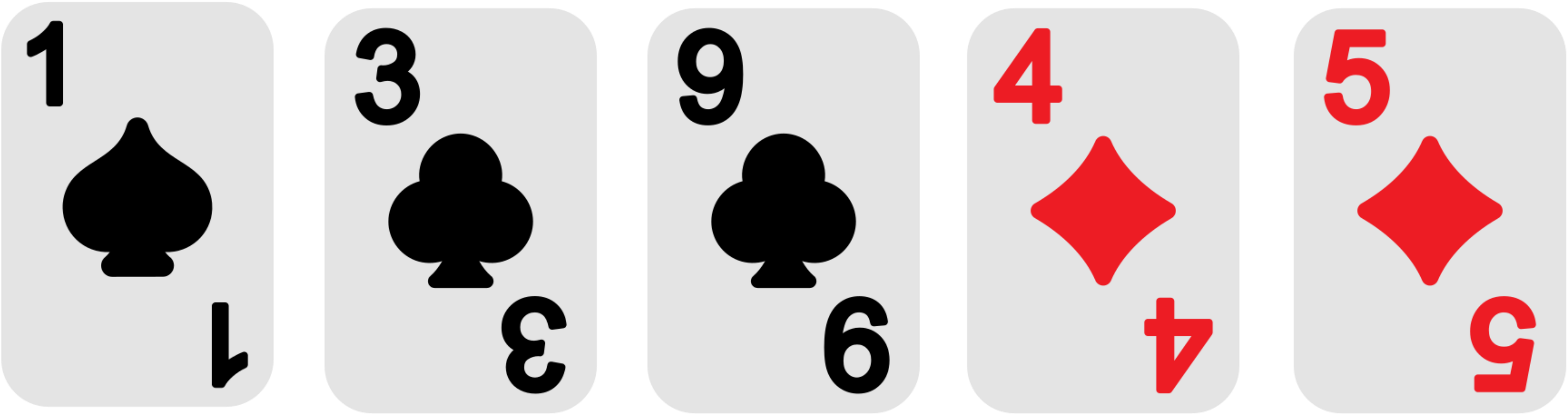
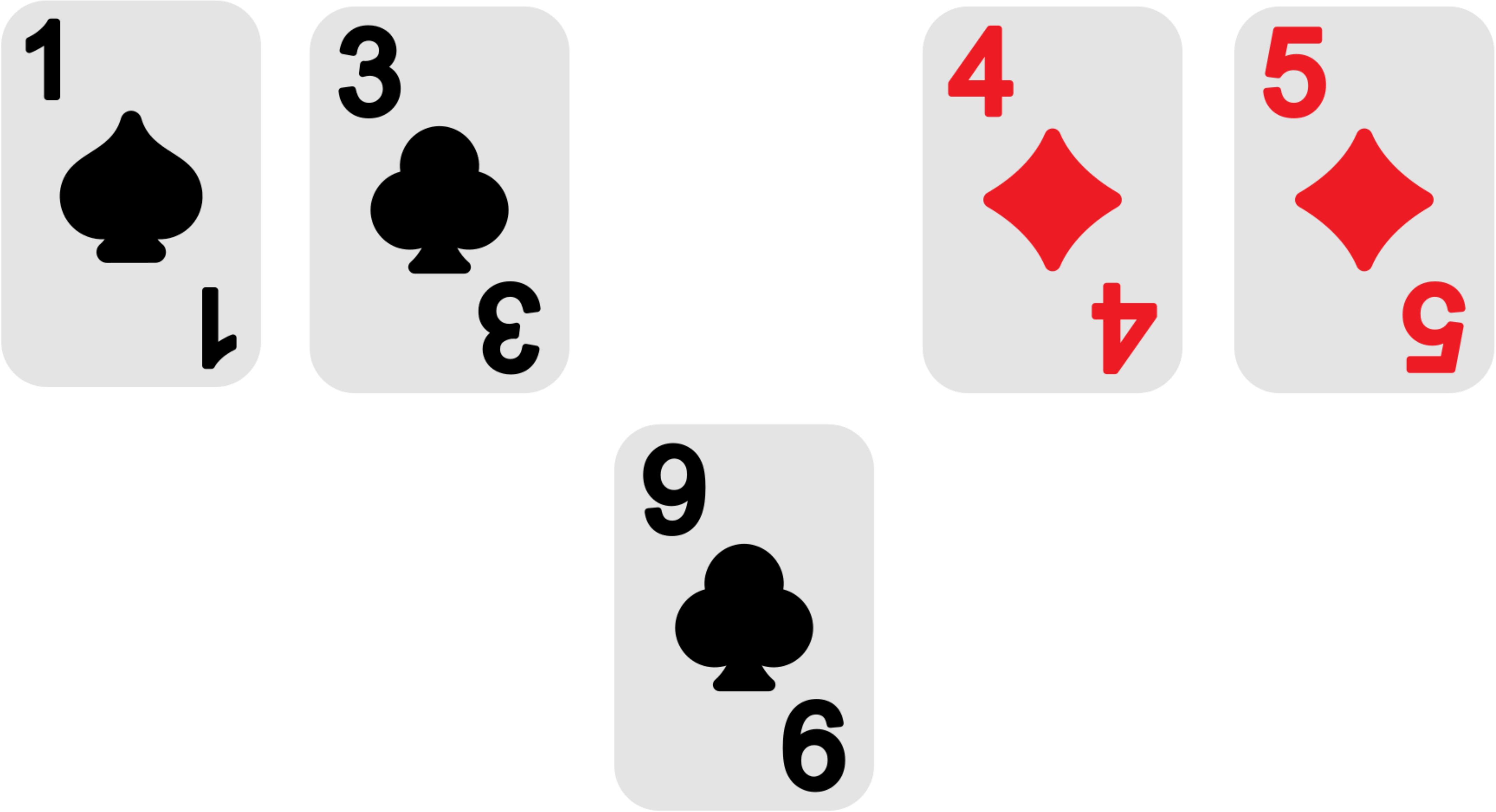
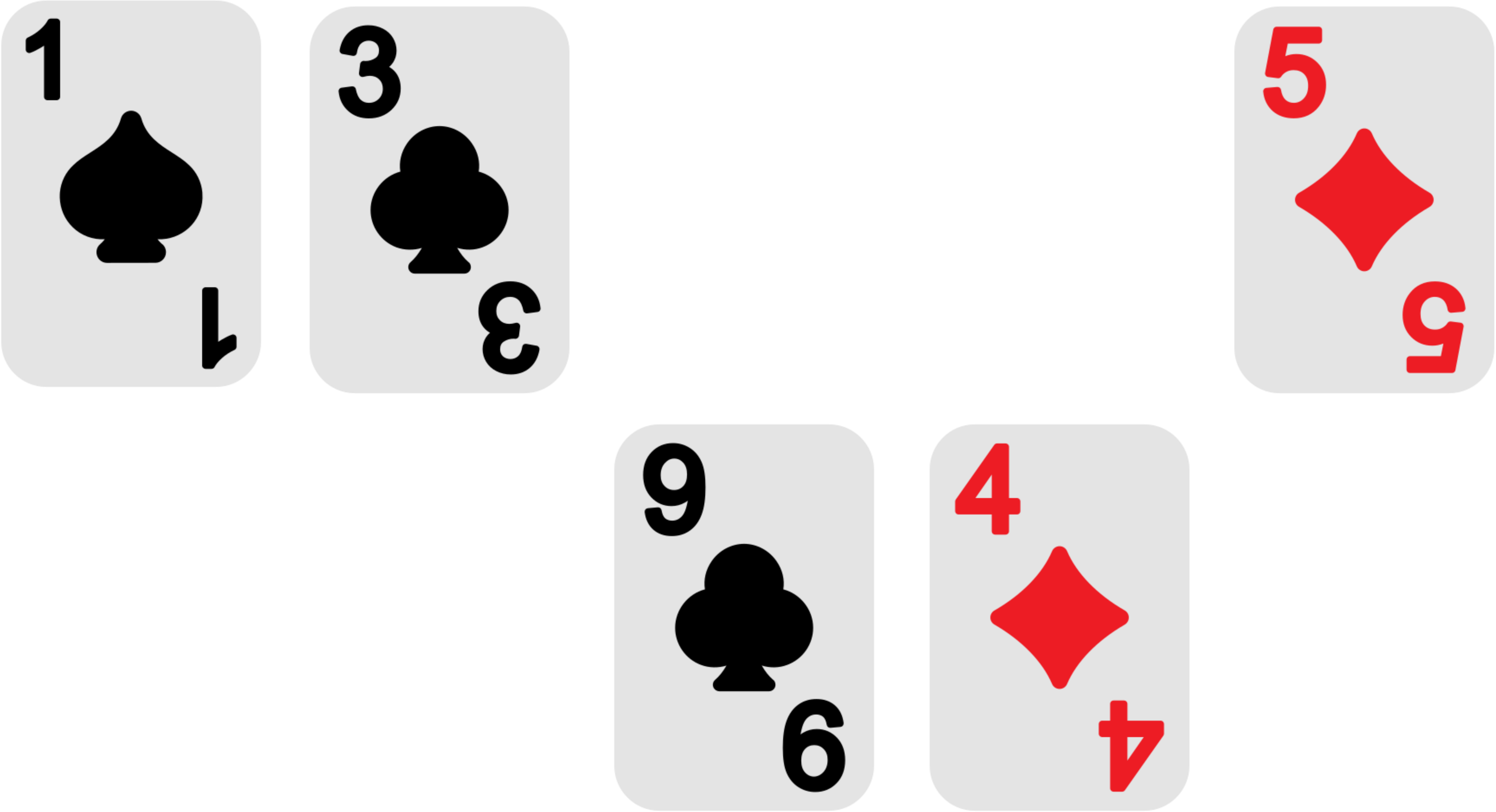


Sorting Playing Cards

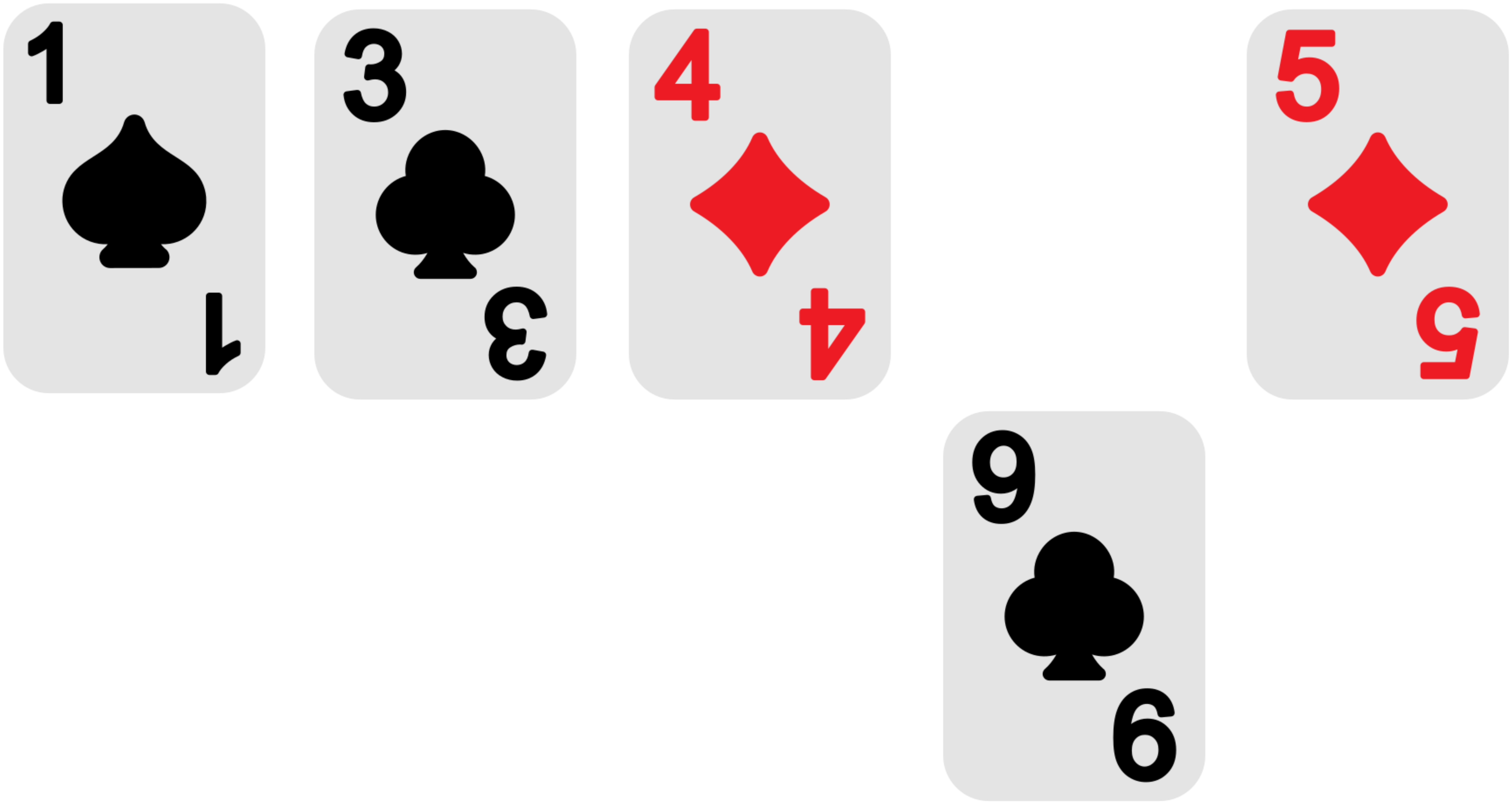


Sorting Playing Cards

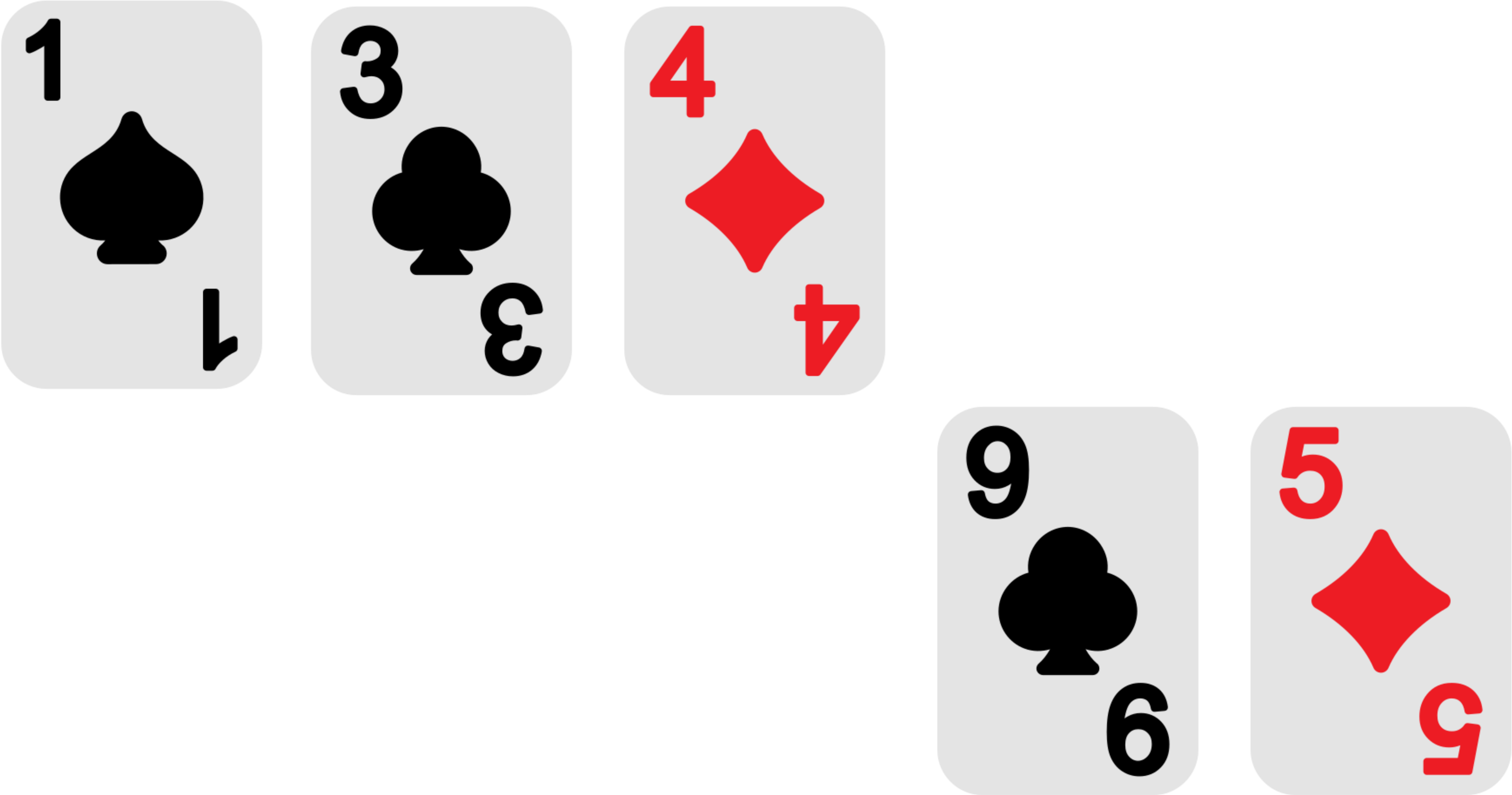


Sorting Playing Cards

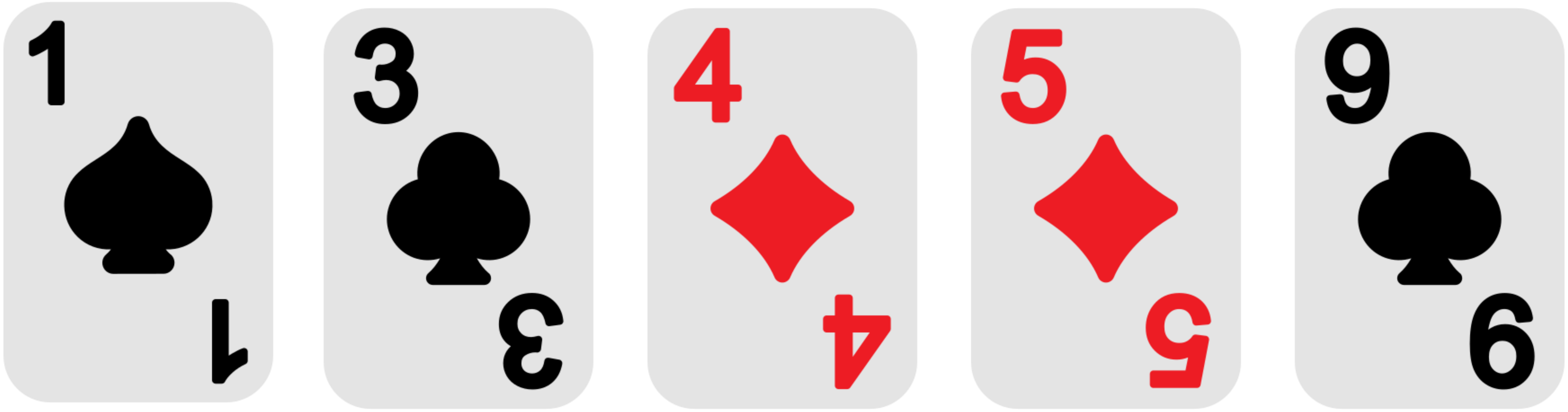
Sorting Playing Cards



Sorting Playing Cards



Sorting Playing Cards



Selection Sort: Pseudocode

ALGORITHM *SelectionSort*(A[0..$n-1$])

//Sorts a given array by selection sort
//Input: An array A[0..$n-1$] of orderable elements
//Output: Array A[0..$n-1$] sorted in nondecreasing order

```
for i ← 0 to n − 2 do
    min ← i
    for j ← i + 1 to n − 1 do
        if A[j] < A[min]
            min ← j
    swap(A[i], A[min])
```

Selection Sort: Analysis

- The analysis of selection sort is straightforward.
- The input size is given by the number of elements *n*.
- The basic operation is the key comparison.
- The number of times it is executed depends only on the array size and is given by the following sum:

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n - 1 - i) = \frac{(n-1)n}{2}$$

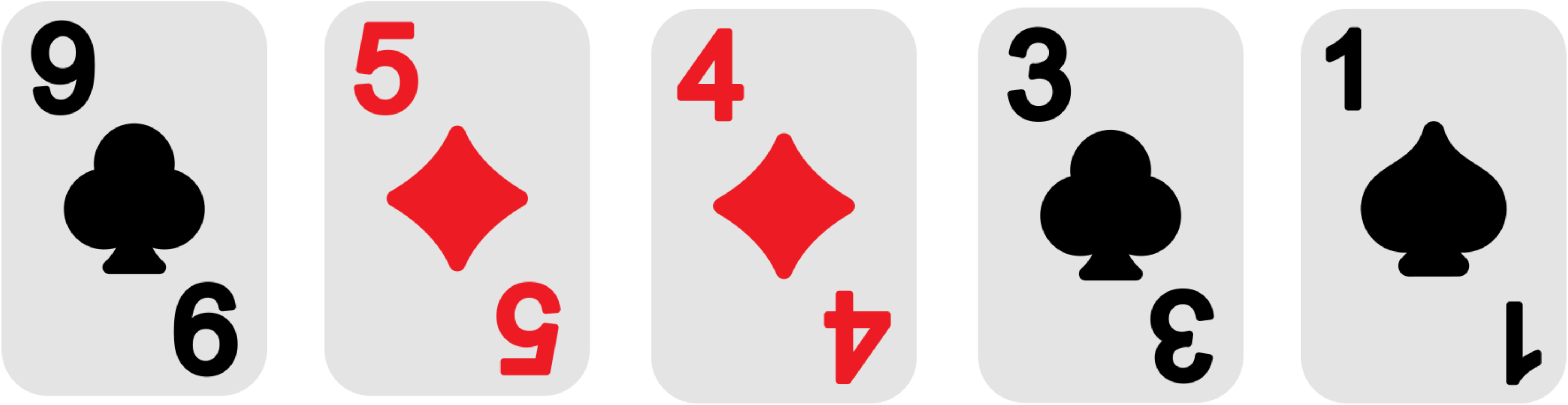
Working of Selection Sort

Sort 6, 4, 1, 8, 5

6		4		**1**		8		5
1	|	**4**		6		8		5
1		4	|	**6**		8		**5**
1		4		5	|	**8**		**6**
1		4		5		6	|	8

Selection Sort: Best Case

Selection Sort: Worst Case



Selection Sort Complexity

Time Complexity	
Best	$O(n^2)$
Worst	$O(n^2)$
Average	$O(n^2)$
Space Complexity	$O(1)$
Stability	No

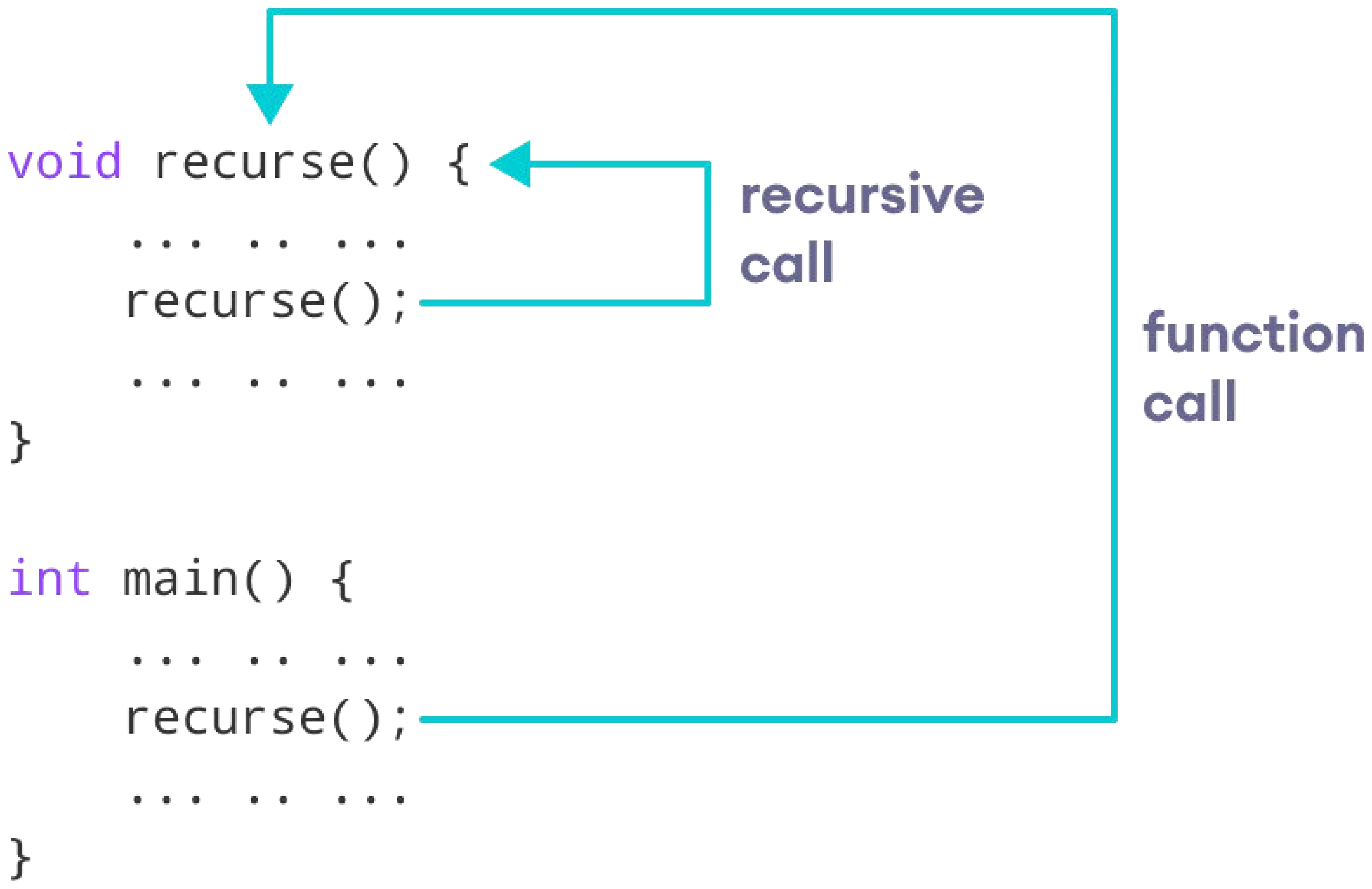
Sorting Algorithms

- Bubble Sort
- **Selection Sort**
- **Insertion Sort**
- Merge Sort
- Quicksort
- Counting Sort
- Radix Sort
- Bucket Sort
- Heap Sort
- Shell Sort

Recursion

- Recursion is a coding technique used in many algorithms.
- A recursive function is a function that calls itself.
- A problem can be solved with recursion if it can be broken down into successive smaller problems that are identical to the overall problem.

Recursion


void recurse() {
... .
recurse();
... .
}
int main() {
... .
recurse();
... .
}
recursive call
function call

Base Case and Recursive Case

- Because a recursive function calls itself, it’s easy to write a function incorrectly that ends up in an infinite loop.
- When you write a recursive function, you have to tell it when to stop recursing.
- That’s why every recursive function has two parts: the base case, and the recursive case.
- The recursive case is when the function calls itself.
- The base case is when the function doesn’t call itself again, so it doesn’t go into an infinite loop.

Sum of Integers

ALGORITHM $Sum(n)$

//Computes the sum of integers from 1 to n recursively

//Input: A nonnegative integer n

//Output: The sum of integers from 1 to n

if $n = 0$

  return 0

else

  return $Sum(n - 1) + n$

Factorial

- In mathematics, the notation $n!$ represents the factorial of the nonnegative integer n.
- The factorial of n is the product of all the integers from 1 to n.

$$n! = n \times (n - 1) \times (n - 2) \times (n - 3) \times \ldots \times 2 \times 1$$

$$n! = n \times (n - 1)!$$

- For example,

  $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

  $4! = 4 \times 3 \times 2 \times 1 = 24$

  $0! = 1$

Factorial

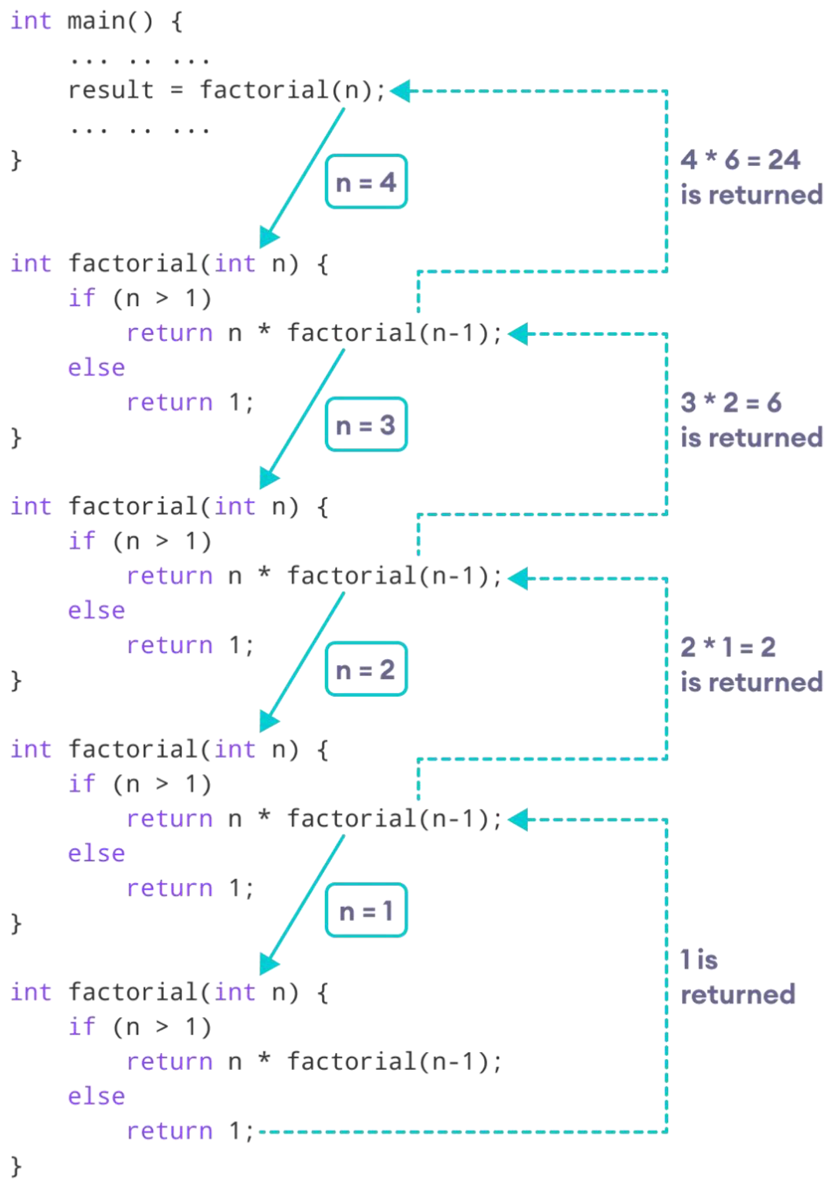
By definition, we can compute $F(n) = F(n-1).n$ with the following recursive algorithm.

ALGORITHM F(n)

//Computes n! recursively
//Input: A nonnegative integer n
//Output: The value of n!

if $n = 0$
    return 1 // 0! = 1
else
    return $F(n-1) * n$ // $n! = n \times (n-1)!$

Factorial


int main() {
... .
result = factorial(n);
... .
}
n = 4
4 * 6 = 24
is returned
int factorial(int n) {
if (n > 1)
return n * factorial(n-1);
else
return 1;
}
n = 3
3 * 2 = 6
is returned
int factorial(int n) {
if (n > 1)
return n * factorial(n-1);
else
return 1;
}
n = 2
2 * 1 = 2
is returned
int factorial(int n) {
if (n > 1)
return n * factorial(n-1);
else
return 1;
}
n = 1
1 is
returned
int factorial(int n) {
if (n > 1)
return n * factorial(n-1);
else
return 1;
}