Design and Analysis of Algorithms

Matrix Multiplication

- one condition to multiplicate two metrices (the #col of first metric is equal to #row of second metric)

$$\begin{matrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{matrix} * \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{matrix} = \begin{matrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 29 & 40 & 51 \end{matrix}$$

3*2 2*3 3*3

- time = 3*3*2 two mul operation for 3*3 elements
- to produce 9 we do two mul operation 1*1 + 2*4 the summation don't take many time so ignore it.
- two mul operation for one value and we need to produce 3*3 values so we need 2* 3*3 operation mul

Matrix Chain Multiplication

- if we need to multiply three metrices A1(3*100)*A2(100*5)*A3(5*5)
- we can (A1*A2)*A3   multiply A1*A2 and the result mul A3
  - A1*A2 = (3*5)   time = 3*5*100 = 1500
  - (3*5)*A3(5*5) = (3*5)   time = 3*5*5 = 75
  - Total time = 1500+75 = 1575
- or we can A1*(A2*A3)   multilpy A2*A3 and the A1 mul result
  - A2*A3 = (100*5)   time = 100*5*5=2500
  - A1(3*100)*(100*5) = (3*5)   time = 3*5*100=1500
  - Total time = 2500+1500 = 4000
- we cannot mul A1*A3 first because i don't know if the condition is true or not
- so i have two way to mul three metrices (matrix chain multiplication is choose the way that give minimum cost). Here the first way is minimum cost.

Matrix Chain Multiplication

- if we have five metrices and i need to mul with minimum cost
  - A1 * A2 * A3 * A4 * A5
  - 4*10 10*3 3*12 12*20 20*7
- the way is said to divide it (A1*A2) *(A3*A4*A5) or (A1*A2*A3)*(A4*A5)
- i will work on (A1*A2) *(A3*A4*A5)
  - A1*A2 will produce matrix 4*3 res1
  - A3*A4*A5 will produce matrix 3*7 res2
  - so the time to mul res1*res2 is 4*7 *3 4*7 element each with 3 mul operation
  - so the total time to mul the five metrices is
    - M[1,5] = M[1,2] + M[3,5] + 4*3*7

      time to mul A1*A2 time to mul A3*A4*A5 timetomulres1*res2

      res 4*3 res 3*7
  - M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk * Pj) M[i,i] = 0 no mul found only one matrix

Matrix Chain Multiplication

- now let know how the algorithm work(DP work by store the result in tubule).
- we want to start with i=j then i<j starting with a spread of 1 working our way up
- M[i,j] = {0 if i=j not need to mul the metrix with it self

  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}

  i<=k<j

  }
- i is the rows and j is the columns (this table will contain at the end with minimum cost)

i/j	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	x	

A2 can't multiply with A1 so Ai cannot mul Aj if i>j so put x in the table

Matrix Chain Multiplication

- بملي الجدول قطر قطر
- step 1: fill the table for i=j
- M[i,j] = {0 if i=j not need to mul the metrix with it self

  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}

  i<=k<j

  }

i/j	1	2	3	4	5
1	0				
2	X	0			
3	X	X	0		
4	X	X	X	0	
5	X	X	X	X	0

Matrix Chain Multiplication

- step 2: fill the table for
- A1 * A2 * A3 * A4 * A5

  4*10 10*3 3*12 12*20 20*7

  p1 p2 p2 p3 p3 p4 p4 p5 p5 p6

- M[i,j] = {0 if i=j not need to mul the metrix with it self

  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}

  i<=k<j

  }
- i=1 j=2
- M[1,2] = min(1<=k<2){ M[1,1] + M[1+1,2] + (P1 * P2 * P3)}
- = 0 + 0 + 4 * 10 * 3 = 120

i/j	1	2	3	4	5
1	0	120			
2	X	0			
3	X	X	0		
4	X	X	X	0	
5	X	X	X	X	0

Matrix Chain Multiplication

- step 2: fill the table for
- A1 * A2 * A3 * A4 * A5

  4*10 10*3 3*12 12*20 20*7

  p1 p2 p2 p3 p3 p4 p4 p5 p5 p6

- M[i,j] = {0 if i=j not need to mul the metrix with it self

  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}

  i<=k<j

  }

- i=2 j=3
- M[2,3] = min(2<=k<3){ M[2,2] + M[2+1,3] + (P2 * P3 * P4)}
- = 0 + 0 + 10 * 3 * 12 = 360

i/j	1	2	3	4	5
1	0	120			
2	X	0	360		
3	X	X	0		
4	X	X	X	0	
5	X	X	X	X	0

Matrix Chain Multiplication

- step 2: fill the table for
- A1 * A2 * A3 * A4 * A5

  4*10 10*3 3*12 12*20 20*7

  p1 p2 p2 p3 p3 p4 p4 p5 p5 p6

- M[i,j] = {0 if i=j not need to mul the metrix with it self

  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}

  i<=k<j

  }

- i=3 j=4
- M[3,4] = min(3<=k<4){ M[3,3] + M[3+1,4] + (P3 * P4 * P5)}
- = 0 + 0 + 3 * 12 * 20 =720

i/j	1	2	3	4	5
1	0	120			
2	X	0	360		
3	X	X	0	720	
4	X	X	X	0	
5	X	X	X	X	0

Matrix Chain Multiplication

- step 2: fill the table for
- A1 * A2 * A3 * A4 * A5

  4*10 10*3 3*12 12*20 20*7

  p1 p2 p2 p3 p3 p4 p4 p5 p5 p6

- M[i,j] = {0 if i=j not need to mul the metrix with it self

  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}

  i<=k<j

  }

- i=4 j=5
- M[4,5] = min(4<=k<5){ M[4,4] + M[4+1,5] + (P4 * P5 * P6)}
- = 0 + 0 + 12 * 20 * 7=1680

i/j	1	2	3	4	5
1	0	120			
2	X	0	360		
3	X	X	0	720	
4	X	X	X	0	1680
5	X	X	X	X	0

Matrix Chain Multiplication

- step 3: fill the table for
- A1 * A2 * A3 * A4 * A5

A1	A2	A3	A4	A5
4*10	10*3	3*12	12*20	20*7
p1 p2	p2 p3	p3 p4	p4 p5	p5 p6

- M[i,j] = {0 if i=j not need to mul the metrix with it self
  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}
  i<=k<j
  }
- i=1 j=3
- M[1,3] = min(1<=k<3) have two solution
- A1*(A2*A3) when k=1
  - { M[1,1] + M[1+1,3] + (P1 * P2 * P4)} = 0+360+4*10*12 = 840
- or (A1*A2)*A3 when k=2
  - { M[1,2] + M[2+1,3] + (P1 * P3 * P4)} = 120+0+4*3*12 = 264
- so 264<840

i/j	1	2	3	4	5
1	0	120	264		
2	X	0	360		
3	X	X	0	720	
4	X	X	X	0	1680
5	X	X	X	X	0

Matrix Chain Multiplication

- step 3: fill the table for
- A1 * A2 * A3 * A4 * A5

A1		A2		A3		A4		A5
4*10		10*3		3*12		12*20		20*7
p1 p2		p2 p3		p3 p4		p4 p5		p5 p6

- M[i,j] = {0 if i=j not need to mul the metrix with it self
  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}
  i<=k<j
  }
- i=2 j=4
- M[2,4] = min(2<=k<4) have two solution
- A2*(A3*A4) when k=2
  - { M[2,2] + M[2+1,4] + (P2 * P3 * P5)} = 0+720+10*3*20 = 1320
- or (A2*A3)*A4 when k=3
- { M[2,3] + M[3+1,4] + (P2 * P4 * P5)} = 360+0+10*12*20 = 2760
- so 1320<2760

i/j	1	2	3	4	5
1	0	120	264		
2	X	0	360	1320	
3	X	X	0	720	
4	X	X	X	0	1680
5	X	X	X	X	0

Matrix Chain Multiplication

- step 3: fill the table for
- A1 * A2 * A3 * A4 * A5

A1	A2	A3	A4	A5
4*10	10*3	3*12	12*20	20*7
p1 p2	p2 p3	p3 p4	p4 p5	p5 p6

- M[i,j] = {0 if i=j not need to mul the metrix with it self

  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}

  i<=k<j

  }
- i=3 j=5
- M[3,5] = min(3<=k<5) have two solution
- A3*(A4*A5) when k=3
- { M[3,3] + M[3+1,5] + (P2 * P3 * P5)} = 0+1680+3*12*7 = 1932
- or (A3*A4)*A5 when k=4
- { M[3,4] + M[4+1,5] + (P3 * P5 * P6)} = 720+0+3*20*7 = 1140
- so 1140<1932

i/j	1	2	3	4	5
1	0	120	264		
2	X	0	360	1320	
3	X	X	0	720	1140
4	X	X	X	0	1680
5	X	X	X	X	0

Matrix Chain Multiplication

- step 4: fill the table for
- A1 * A2 * A3 * A4 * A5

  4*10 10*3 3*12 12*20 20*7

  p1 p2 p2 p3 p3 p4 p4 p5 p5 p6

- M[i,j] = {0 if i=j not need to mul the metrix with it self

  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}

  i<=k<j

  }
- i=1 j=4
- M[1,4] = min(1<=k<4) have three solution
- A1*(A2*A3*A4) when k=1
- { M[1,1] + M[1+1,4] + (P1 * P2 * P5)} = 0+1320+4*10*20 = 2120
- or (A1*A2)*(A3*A4) when k=2
- { M[1,2] + M[2+1,4] + (P1 * P3 * P5)} = 120+720+4*3*20 = 1080
- or (A1*A2*A3)*A4 when k=3
- { M[1,3] + M[3+1,4] + (P1 * P4 * P5)} = 264+0+4*12*20 = 1224
- so 1080<1224<2120

i/j	1	2	3	4	5
1	0	120	264	1080	
2	X	0	360	1320	
3	X	X	0	720	1140
4	X	X	X	0	1680
5	X	X	X	X	0

Matrix Chain Multiplication

- step 4: fill the table for
- A1 * A2 * A3 * A4 * A5

  4*10 10*3 3*12 12*20 20*7

  p1 p2 p2 p3 p3 p4 p4 p5 p5 p6

- M[i,j] = {0 if i=j not need to mul the metrix with it self

  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}

  i<=k<j

  }
- i=2 j=5
- M[2,5] = min(2<=k<5) have three solution
- A2*(A3*A4*A5) when k=2
- { M[2,2] + M[2+1,5] + (P2 * P3 * P6)} = 0+1140+10*3*7 = 1350
- or (A2*A3)*(A4*A5) when k=3
- { M[2,3] + M[3+1,5] + (P2 * P4 * P6)} = 360+1680+10*12*7 = 2880
- or (A2*A3*A4)*A5 when k=4
- { M[2,4] + M[4+1,5] + (P2 * P5 * P6)} = 1320+0+10*20*7 = 2720
- so 1350<2720<2880

i/j	1	2	3	4	5
1	0	120	264	1080	
2	X	0	360	1320	1350
3	X	X	0	720	1140
4	X	X	X	0	1680
5	X	X	X	X	0

Matrix Chain Multiplication

- step 5: fill the table for
- A1 * A2 * A3 * A4 * A5

A1	A2	A3	A4	A5
4*10	10*3	3*12	12*20	20*7
p1 p2	p2 p3	p3 p4	p4 p5	p5 p6

- M[i,j] = {0 if i=j not need to mul the metrix with it self

  min {M[i,j] = M[i,k] + M[k+1,j] + (Pi * Pk+1 * Pj+1)}

  i<=k<j

  }
- i=1 j=5
- M[1,5] = min(1<=k<5) have four solution
- A1*(A2*A3*A4*A5) when k=1
- { M[1,1] + M[1+1,5] + (P1 * P2 * P6)} = 0+1350+4*10*7 = 1630
- or (A1*A2)*(A3*A4*A5) when k=2
- { M[1,2] + M[2+1,5] + (P1 * P3 * P6)} = 120+1140+4*3*7 = 1344
- or (A1*A2*A3)*(A4*A5) when k=3
- { M[1,3] + M[3+1,5] + (P1 * P4 * P6)} = 264+1680+4*12*7 = 2280
- or (A1*A2*A3*A4)*A5 when k=4
- { M[1,4] + M[4+1,5] + (P1 * P5 * P6)} = 1080+0+4*20*7 = 1640
- so 1344<1630<1640<2280

i/j	1	2	3	4	5
1	0	120	264	1080	1344
2	X	0	360	1320	1350
3	X	X	0	720	1140
4	X	X	X	0	1680
5	X	X	X	X	0

Matrix Chain Multiplication

- result = 1344 = (A1*A2) * (A3*A4*A5)
- (A3*A4*A5)=1140 = (A3*A4)*A5
- final result = (A1*A2) * ((A3*A4) *A5)

i/j	1	2	3	4	5
1	0	120	264	1080	1344
2	X	0	360	1320	1350
3	X	X	0	720	1140
4	X	X	X	0	1680
5	X	X	X	X	0