

# Longest Common Subsequence

- Given two sequences

$$X = \langle x_1, x_2, \dots, x_m \rangle$$

$$Y = \langle y_1, y_2, \dots, y_n \rangle$$

find a **maximum length common subsequence** (LCS) of X and Y

- e.g.: If  $X = \langle A, B, C, B, D, A, B \rangle$

Subsequences of X:

A subset of elements in the sequence taken in order

$\langle A, B, D \rangle, \langle B, C, D, B \rangle, \langle B, C, D, A, B \rangle$  etc.

## Example

$X = \langle A, B, C, B, D, A, B \rangle$

$X = \langle A, B, C, B, D, A, B \rangle$

$Y = \langle B, D, C, A, B, A \rangle$

$Y = \langle B, D, C, A, B, A \rangle$

## Example

$$X = \langle A, B, C, B, D, A, B \rangle$$
$$Y = \langle \textcolor{blue}{B}, D, C, A, B, A \rangle$$
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$$Y = \langle B, D, C, A, B, A \rangle$$

## Example

$$X = \langle A, B, C, B, D, A, B \rangle$$
$$Y = \langle B, D, C, A, B, A \rangle$$

The diagram illustrates the sequences X and Y. Sequence X is represented by the string  $\langle A, B, C, B, D, A, B \rangle$ . Sequence Y is represented by the string  $\langle B, D, C, A, B, A \rangle$ . Lines connect the first element of X to the first element of Y, the second element of X to the third element of Y, the third element of X to the second element of Y, the fourth element of X to the fifth element of Y, the fifth element of X to the sixth element of Y, and the sixth element of X to the fourth element of Y.

$$X = \langle A, B, C, B, D, A, B \rangle$$
$$Y = \langle B, D, C, A, B, A \rangle$$

## Example

$$X = \langle A, B, C, B, D, A, B \rangle$$
$$Y = \langle B, D, C, A, B, A \rangle$$

The diagram illustrates a longest common subsequence (LCS) between two sequences, X and Y. Sequence X is  $\langle A, B, C, B, D, A, B \rangle$  and sequence Y is  $\langle B, D, C, A, B, A \rangle$ . Arrows show the mapping from X to Y: (A,B) to (B,D), (B,C) to (D,C), (C,B) to (A,B), and (D,A) to (B,A). The letter 'B' in 'A,B' is highlighted with a yellow circle.

$$X = \langle A, B, C, B, D, A, B \rangle$$
$$Y = \langle B, D, C, A, B, A \rangle$$

The diagram illustrates another longest common subsequence (LCS) between two sequences, X and Y. Sequence X is  $\langle A, B, C, B, D, A, B \rangle$  and sequence Y is  $\langle B, D, C, A, B, A \rangle$ . Arrows show the mapping from X to Y: (A,B) to (B,D), (B,C) to (D,C), and (C,B) to (A,B).

- $\langle B, C, B, A \rangle$  and  $\langle B, D, A, B \rangle$  are **longest common subsequences** of X and Y (*length* = 4)
- $\langle B, C, A \rangle$ , however, is not a LCS of X and Y

## Brute-Force Solution

- For every subsequence of X, check whether it's a subsequence of Y
- There are  $2^m$  subsequences of X to check
- Each subsequence takes  $\Theta(n)$  time to check
  - scan Y for first letter, from there scan for second, and so on
- **Running time:**  $\Theta(n2^m)$



# A Recursive Solution

**Case 1:**  $x_i = y_j$

e.g.:  $X_i = \langle A, B, D, G, E \rangle$

$Y_j = \langle Z, B, D, E \rangle$

$$c[i, j] = c[i - 1, j - 1] + 1$$

- Append  $x_i = y_j$  to the LCS of  $X_{i-1}$  and  $Y_{j-1}$
- Must find a LCS of  $X_{i-1}$  and  $Y_{j-1}$

# A Recursive Solution

**Case 2:**  $x_i \neq y_j$

*e.g.:*

$$X_i = \langle A, B, D, G \rangle$$

$$Y_j = \langle Z, B, D \rangle$$

- Must solve two problems
  - find a LCS of  $X_{i-1}$  and  $Y_j$ :  $X_{i-1} = \langle A, B, D \rangle$  and  $Y_j = \langle Z, B, D \rangle$
  - find a LCS of  $X_i$  and  $Y_{j-1}$ :  $X_i = \langle A, B, D, G \rangle$  and  $Y_{j-1} = \langle Z, B \rangle$

$$c[i, j] = \max \{ c[i - 1, j], c[i, j-1] \}$$

# Computing the Length of the LCS

$$c[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ c[i-1, j-1] + 1 & \text{if } x_i = y_j \\ \max(c[i, j-1], c[i-1, j]) & \text{if } x_i \neq y_j \end{cases}$$

		0	1	2	n		
		$y_0$ : $y_1$	$y_1$	$y_2$	$y_n$		
0		$x_0$ :	0	0	0	0	0
1		$x_1$	0				
2		$x_2$	0				
		0					
		0					
m		$x_m$	0				

i

j

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$$c[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ c[i-1, j-1] + 1 & \text{if } x_i = y_j \\ \max(c[i, j-1], c[i-1, j]) & \text{if } x_i \neq y_j \end{cases}$$

		0	1	2	n	
		$y_0$	$y_1$	$y_2$	$y_n$	
		0	0	0	0	0
0	$x_0$	0	0	0	0	0
1	$x_1$	0				
2	$x_2$	0				
		0			⋮	
		0			⋮	
m	$x_m$	0				

j

first

second

i

# Additional Information

$$c[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ c[i-1, j-1] + 1 & \text{if } x_i = y_j \\ \max(c[i, j-1], c[i-1, j]) & \text{if } x_i \neq y_j \end{cases}$$

b & c:

	0	1	2	3	n
y <sub>j</sub> :	A	C	D	F	
0 x <sub>i</sub>	0	0	0	0	0
1 A	0				
2 B	0		c[i-1, j]		
3 C	0	c[i, j-1]			
m D	0				

i

j

A matrix b[i, j]:

- For a subproblem [i, j] it tells us what choice was made to obtain the optimal value
- If  $x_i = y_j$   
 $b[i, j] = "↖"$
- Else, if  $c[i - 1, j] \geq c[i, j-1]$   
 $b[i, j] = "↑"$
- else,  
 $b[i, j] = "←"$

# Example

$$c[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ c[i-1, j-1] + 1 & \text{if } x_i = y_j \\ \max(c[i, j-1], c[i-1, j]) & \text{if } x_i \neq y_j \end{cases}$$

X = ⟨A, B, C, B, D, A, B⟩

Y = ⟨B, D, C, A, B, A⟩

If  $x_i = y_j$

b[i, j] = "↖"

else if  $c[i - 1, j] \geq c[i, j - 1]$

b[i, j] = "↑"

else

b[i, j] = "←"

	0	1	2	3	4	5	6
0	Y <sub>j</sub>	B	D	C	A	B	A
1	A	0					
2	B	0					
3	C	0					
4	B	0					
5	D	0					
6	A	0					
7	B	0					

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	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	X <sub>i</sub>	0	0	0	0	0	0
2	A	0	0				
3	B	0					
4	C	0					
5	D	0					
6	A	0					
7	B	0					

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	0	1	2	3	4	5	6
Y <sub>j</sub>	B	D	C	A	B	A	
0	x <sub>i</sub>	0	0	0	0	0	0
1	A	0	0	0			
2	B	0					
3	C	0					
4	B	0					
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	0	1	2	3	4	5	6
Y <sub>j</sub>	B	D	C	A	B	A	
0	x <sub>i</sub>	0	0	0	0	0	0
1	A	0	0	0	0	0	
2	B	0					
3	C	0					
4	B	0					
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 else  
 $b[i, j] = "←"$

	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	$x_i$	0	0	0	0	0	0
2	A	0	0	0	0	1	
3	B	0					
4	C	0					
5	D	0					
6	A	0					
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 $b[i, j] = "↑"$   
else  
 $b[i, j] = "←"$

	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	A	0	0	0	0	1	←1
2	B	0					
3	C	0					
4	B	0					
5	D	0					
6	A	0					
7	B	0					

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	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	A	0	0	0	1	-1	1
2	B	0					
3	C	0					
4	B	0					
5	D	0					
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else

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		0	1	2	3	4	5	6
	$x_i$	0	0	0	0	0	0	0
0	$y_j$	0	0	0	0	1	-1	1
1	A	0	0	0	0	1	-1	1
2	B	0	1					
3	C	0						
4	B	0						
5	D	0						
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else  
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	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	X <sub>i</sub>	0	0	0	0	0	0
2	A	0	0	0	0	1	-1
3	B	0	1	-1			
4	C						
5	B						
6	D						
7	A						
8	B						

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 $b[i, j] = "↑"$   
else  
 $b[i, j] = "←"$

	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	A	0	0	0	0	1	-1
2	B	0	1	-1	-1		
3	C	0					
4	B	0					
5	D	0					
6	A	0					
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else  
 $b[i, j] = "←"$

	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	A	0	0	0	0	1	←1
2	B	0	↖1	←1	←1	1	↑
3	C	0					
4	B	0					
5	D	0					
6	A	0					
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else  
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	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	A	0	0	0	0	1	1
2	B	0	1	-1	-1	1	2
3	C	0					
4	B	0					
5	D	0					
6	A	0					
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	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	A	0	0	0	0	1	-1
2	B	0	-1	-1	-1	1	-2
3	C	0					
4	B	0					
5	D	0					
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 else  
 $b[i, j] = "←"$

	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	$x_i$	0	0	0	0	0	0
2	A	0	0	0	1	-1	1
3	B	0	1	-1	-1	1	2
4	C	0	1	1	2	-2	2
5	B	0					
6	D	0					
7	A	0					
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	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	A	0	0	0	1	-1	1
2	B	0	1	-1	-1	1	2
3	C	0	1	1	2	-2	2
4	B	0	1	1	2	2	3
5	D	0					
6	A	0					
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	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	A	0	0	0	1	-1	1
2	B	0	1	-1	-1	1	2
3	C	0	1	1	2	-2	2
4	B	0	1	1	2	2	3
5	D	0	1	2	2	3	3
6	A	0					
7	B	0					

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 else if  $c[i - 1, j] \geq c[i, j-1]$   
 $b[i, j] = "↑"$   
 else  
 $b[i, j] = "←"$

	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	$x_i$	0	0	0	0	0	0
2	A	0	0	0	0	1	←1
3	B	0	1	←1	←1	1	2
4	C	0	1	1	2	←2	2
5	D	0	1	1	2	2	3
6	A	0	1	2	2	3	3
7	B	0					

# Example

$$c[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ c[i-1, j-1] + 1 & \text{if } x_i = y_j \\ \max(c[i, j-1], c[i-1, j]) & \text{if } x_i \neq y_j \end{cases}$$

$X = \langle A, B, C, B, D, A, B \rangle$   
 $Y = \langle B, D, C, A, B, A \rangle$

If  $x_i = y_j$   
 $b[i, j] = "↖"$   
 else if  $c[i - 1, j] \geq c[i, j-1]$   
 $b[i, j] = "↑"$   
 else  
 $b[i, j] = "←"$

	0	1	2	3	4	5	6
0	$y_j$	B	D	C	A	B	A
1	A	0	0	0	1	-1	1
2	B	0	1	-1	-1	1	2
3	C	0	1	1	2	-2	2
4	B	0	1	1	2	2	3
5	D	0	1	2	2	2	3
6	A	0	1	2	2	3	3
7	B	0	1	2	2	3	4

# Constructing a LCS

- Start at  $b[m, n]$  and follow the arrows
- When we encounter a “ $\nwarrow$ ” in  $b[i, j] \Rightarrow x_i = y_j$  is an element of the LCS

	0	1	2	3	4	5	6
	$y_i$	B	D	C	A	B	A
0	$x_i$	0	0	0	0	0	0
1	A	0	↑	0	↑	1	↖1
2	B	0	↑	1	↖1	↑1	↖2
3	C	0	↑	1	↑2	↑2	↑2
4	B	0	↑	1	↑1	↑2	↑3
5	D	0	↑	1	↑2	↑2	↑3
6	A	0	↑	1	↑2	↑3	↑4
7	B	0	↑	1	↑2	↑3	↑4



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	0	1	2	3	4	5	6
	$y_i$	B	D	C	A	B	A
0	$x_i$	0	0	0	0	0	0
1	A	0	↑	0	↑	↑1	↖1
2	B	0	↖1	←1	←1	↑1	↖2
3	C	0	↑1	↑1	↖2	↑2	↑2
4	B	0	↖1	↑1	↑2	↑2	↖3
5	D	0	↑1	↖2	↑2	↑2	↑3
6	A	0	↑1	↑2	↑2	↖3	↑3
7	B	0	↖1	↑2	↑2	↑3	↖4

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	$y_i$	B	D	C	A	B	A
0	$x_i$	0	0	0	0	0	0
1	A	0	↑	0	↑	1	↖1
2	B	0	1	↖1	↖1	↑1	↖2
3	C	0	1	1	↖2	↑2	↑2
4	B	0	1	1	↑2	↑2	↖3
5	D	0	1	2	↑2	↑3	↑3
6	A	0	1	2	2	3	↖4
7	B	0	1	2	2	3	4

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	0	1	2	3	4	5	6
	$y_i$	B	D	C	A	B	A
0	$x_i$	0	0	0	0	0	0
1	A	0	↑	0	↑	1	↖1
2	B	0	↖1	←1	←1	↑1	↖2
3	C	0	↑1	↑1	↖2	↑2	↑2
4	B	0	↖1	↑1	↑2	↑2	↖3
5	D	0	↑1	↖2	↑2	↑2	↑3
6	A	0	↑1	↑2	↑2	↖3	↖4
7	B	0	↖1	↑2	↑2	↑3	↑4

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	0	1	2	3	4	5	6
	$y_i$	B	D	C	A	B	A
0	$x_i$	0	0	0	0	0	0
1	A	0	↑	0	↑	1	↖1
2	B	0	↖1	←1	←1	↑1	↖2
3	C	0	↑1	↑1	↖2	←2	↑2
4	B	0	↖1	↑1	↑2	↑2	↖3
5	D	0	↑1	↖2	↑2	↑2	↑3
6	A	0	↑1	↑2	↑2	↖3	↖4
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- Start at  $b[m, n]$  and follow the arrows
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	0	1	2	3	4	5	6
	$y_i$	B	D	C	A	B	A
0	$x_i$	0	0	0	0	0	0
1	A	0	↑	0	↑	↑1	↖1
2	B	0	↖1	←1	←1	↑1	↖2
3	C	0	↑1	↑1	↖2	←2	↑2
4	B	0	↖1	↑1	↑2	↑2	↖3
5	D	0	↑1	↖2	↑2	↑2	↑3
6	A	0	↑1	↑2	↑2	↖3	↖4
7	B	0	↖1	↑2	↑2	↑3	↑4

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- Start at  $b[m, n]$  and follow the arrows
- When we encounter a “↖” in  $b[i, j] \Rightarrow x_i = y_j$  is an element of the LCS

	0	1	2	3	4	5	6
	$y_i$	B	D	C	A	B	A
0	$x_i$	0	0	0	0	0	0
1	A	0	↑	0	↑	1	↖1
2	B	0	↖1	←1	←1	↑1	↖2
3	C	0	↑1	↑1	↖2	←2	↑2
4	B	0	↖1	↑1	2	2	↖3
5	D	0	↑1	↖2	2	2	3
6	A	0	↑1	↑2	2	3	4
7	B	0	↖1	↑2	2	3	4

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- Start at  $b[m, n]$  and follow the arrows
- When we encounter a “↖” in  $b[i, j] \Rightarrow x_i = y_j$  is an element of the LCS

	0	1	2	3	4	5	6
	$y_i$	B	D	C	A	B	A
0	$x_i$	0	0	0	0	0	0
1	A	0	↑	0	↑	↑1	↖1
2	B	0	1	↖1	↖1	↑1	↖2
3	C	0	↑1	↑1	↖2	↖2	↑2
4	B	0	↖1	↑1	↑2	↑2	↖3
5	D	0	↑1	↖2	↑2	↑2	↑3
6	A	0	↑1	↑2	↑2	↖3	↖4
7	B	0	↖1	↑2	↑2	↑3	↑4

# Constructing a LCS

- Start at  $b[m, n]$  and follow the arrows
- When we encounter a “↖” in  $b[i, j] \Rightarrow x_i = y_j$  is an element of the LCS

	0	1	2	3	4	5	6
	$y_i$	B	D	C	A	B	A
0	$x_i$	0	0	0	0	0	0
1	A	0	↑	0	↑	↑1	↖1
2	B	0	↖1	←1	←1	↑1	↖2
3	C	0	↑1	↑1	↖2	←2	↑2
4	B	0	↖1	↑1	↑2	↑2	↖3
5	D	0	↑1	↖2	↑2	↑2	↑3
6	A	0	↑1	↑2	↑2	↖3	↖4
7	B	0	↖1	↑2	↑2	↑3	↑4

# Constructing a LCS

- Start at  $b[m, n]$  and follow the arrows
- When we encounter a “↖” in  $b[i, j] \Rightarrow x_i = y_j$  is an element of the LCS

	0	1	2	3	4	5	6
	$y_i$	B	D	C	A	B	A
0	$x_i$	0	0	0	0	0	0
1	A	0	0	0	0	1	1
2	B	0	1	-1	-1	1	2
3	C	0	1	1	2	2	2
4	B	0	1	1	2	2	3
5	D	0	1	2	2	2	3
6	A	0	1	2	2	3	4
7	B	0	1	2	2	3	4

Diagram illustrating the construction of the Longest Common Subsequence (LCS) between two sequences  $x$  and  $y$ . The sequences are:

- $x = A, B, C, B, D, A$
- $y = B, D, C, A, B, A$

The table shows the lengths of the common subsequences at each position. Arrows indicate the construction path:

- Upward arrow from  $(0, 0)$  to  $(1, 0)$ .
- Leftward arrow from  $(1, 0)$  to  $(2, -1)$ .
- Upward arrow from  $(2, -1)$  to  $(3, 1)$ .
- Leftward arrow from  $(3, 1)$  to  $(4, 1)$ .
- Upward arrow from  $(4, 1)$  to  $(5, 2)$ .
- Leftward arrow from  $(5, 2)$  to  $(6, 3)$ .
- Upward arrow from  $(6, 3)$  to  $(7, 4)$ .

The final length of the LCS is 4, corresponding to the sequence  $B, C, B, A$ .

# Constructing a LCS

$X = \langle A, B, C, B, D, A, B \rangle$

$Y = \langle B, D, C, A, B, A \rangle$

$\langle B, C, B, A \rangle$

