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SCHAUM'S
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Theory and Problems of Basic Mathematics with

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Theory and Problems of Basic Mathematics with Applications to Science and Technology

Second Edition

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Dedicated to the Lion of Judah for his many blessings, opportunities and
the wonderful family that he has given me.
RAMT

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Contents

7.11 NEGATIVE SLOPE

The slope of the graph of a first-degree equation can be negative.

EXAMPLE 7.7. The equation $5y + 10x + 5 = 0$, when solved for y, is in the slope–intercept form

$$y = -2x - 1$$

The slope is -2 and the y-intercept is -1. Since the equation is of the first degree, we know that it defines a linear function and only two coordinate points are needed for a graph of the function. One of the points is the y-intercept point $(0, -1)$; an additional point is readily obtained by substituting for x some small integer (for $x = -2$, $y = 3$). The graph is shown in Fig. 7-3 as line B.

7.12 ZERO SLOPE

When the slope of a straight line is zero, the value of y is independent of x; that is, the value of y is constant.

EXAMPLE 7.8. The graph of the constant function defined by $y = -3$ is a straight line parallel to the x-axis and 3 units below it; it is represented by line C in Fig. 7-3.

7.13 APPLICATIONS

Many relationships in the physical sciences have linear graphs, as for example: $v = v_0 + at$; $F = (9/5)C + 32$; $E = V + Ir$; $L_t = L_0(1 + \beta t)$; $T = 273 + t$.

7.14 INTERCEPTS OF A LINE

An intercept is the point where a line crosses an axis. Intercepts are useful because they can help us to sketch quickly the graph of a linear equation. Since every straight line is completely defined by any two of its points, if we know these points we can sketch its graph.

A straight line may have up to two intercepts. The x-intercept represents the point where the line crosses the horizontal axis (x-axis). Since any point a on the x-axis is of the form $(a, 0)$ we can calculate this intercept by letting y be equal to zero in the equation $Ax + By + C = 0$ and solving for x.

To find the y-intercept, notice that any point on the vertical axis (y-axis) is of the form $(0, b)$. Therefore, we can find this intercept by letting x be equal to 0 in the equation $Ax + By + C = 0$ and solving the resulting equation for y.

EXAMPLE 7.9. What are the intercepts of the straight line defined by $2x + y - 6 = 0$?

To find the x-intercept, set $y = 0$ and solve for x. In this case, if we set $y = 0$, then $x = 3$. To find the y-intercept, set $x = 0$ and solve for y. In this case, if we set $x = 0$, then $y = 6$. Therefore, the line crosses the axis at $(3, 0)$ and $(0, 6)$. Since a straight line is completely defined by two of its points, we can plot the graph by $2x + y - 6 = 0$ by drawing the line that goes through these two points.

Application to Empirical Data-Graphing

7.15 EMPIRICAL EQUATIONS

An equation which can be obtained from experimental data is called *empirical*. The use of graph paper provides a convenient method for determining empirical equations for the representation of functions. It is recommended that the data be graphed on full-size sheets of graph paper.

7.16 GOOD GRAPHING PRACTICES

It is customary to plot the independent variable along the horizontal axis. In experimental work, the independent variable usually refers to the physical quantity which is readily controlled or which can be measured with the highest precision.

One of the most important requirements of a good graph is the choice of scales for the two coordinate axes. The "mathematical" functions like $y = 5x^2$ are usually graphed so that both variables are plotted to the same scale; this is seldom possible with experimental data. Also, in the case of mathematical graphs, letters such as y and x identify the axes; the "pure" numbers mark the scale subdivisions. However, to be meaningful, a graph of laboratory data must have each of its axes labeled to indicate the quantity being measured and the measurement units.

The following are suggestions for plotting a graph.

1. Use a sharp pencil.
2. Choose scales which make both plotting and reading of points easy. Some suggested scales of this kind are:

   One large scale division on the graph paper to represent

   $$\left.\begin{array}{llllllll} (a) & 0.1, 0.01, & \text{or} & 0.001\ldots; & 1, 10, & \text{or} & 100\ldots\text{units} \\ (b) & 0.2, 0.02, & \text{or} & 0.002\ldots; & 2, 20, & \text{or} & 200\ldots\text{units} \\ (c) & 0.5, 0.05, & \text{or} & 0.005\ldots; & 5, 50, & \text{or} & 500\ldots\text{units} \end{array}\right\} \begin{array}{c}\text{of the} \\ \text{plotted quantity}\end{array}$$

3. Choose scales so that the graph occupies most of the graph sheet.
4. Leave wide margins (1/2 to 1 in.) unless the graph paper already has wide margins.
5. It is not necessary to have both quantities plotted to the same scale.
6. It is conventional to plot the quantity which you choose or vary (independent variable) along the horizontal axis and the quantity which results or which you observe (dependent variable) along the vertical axis.
7. Number the major scale divisions on the graph paper from left to right below the horizontal axis, and from the base line upward at the left of the vertical axis.
8. Name the quantity plotted and the units in which it is expressed along each axis, below the horizontal axis (extending to the right) and to the left of the vertical axis (extending upward).
9. Print all symbols and words.
10. Plot all the observed data. Mark experimental points clearly; a point surrounded by a small circle, $\odot$, a small triangle, $\triangle$ or some similar symbol is suggested. Connect the points by a smooth curve if warranted by the nature of the data.
11. Place a title in the upper part of the graph paper. The title should not cross the curve. Examples of suitable working titles are: "Relationship between Distance and Time,"

"Distance versus Velocity," "Velocity as a Function of Distance." The first quantity stated is usually the dependent variable.

12. Place your name or initials and the date in the lower right-hand corner of the graph.
13. In drawing a curve, do not try to make it go through every plotted point. The graphing of experimental data is an averaging process. If the plotted points do not fall on a smooth curve, the best or most probable smooth curve should be drawn in such a way that there are, if possible, as many randomly plotted points above the curve as below it. If the graph is not a straight line, then a special device called a French curve may be used for the construction of the graph.
14. A graph is a picture of a relationship between variables. It ought to be neat and legible. It should tell a complete story.

7.17 GRAPHING THE SPECIAL LINEAR FUNCTION $y = mx$

If the graph of data on an ordinary sheet of graph paper is a straight line through the origin then its equation is of the form $y = mx$. The slope m is obtained directly from the graph.

EXAMPLE 7.10. The length L and weight W of a number of wooden cylinders with identical cross sections were measured as indicated:

L, cm	0	2.5	5.1	7.5	10	12.5	15.0
W, g-wt	0	7.0	13.7	20.6	27.7	34.7	42.5

To determine the empirical equation connecting L and W from the data in the table above, the data is graphed on cartesian (ordinary) graph paper. The graph is shown in Fig. 7-4. The length L was chosen as the independent variable.

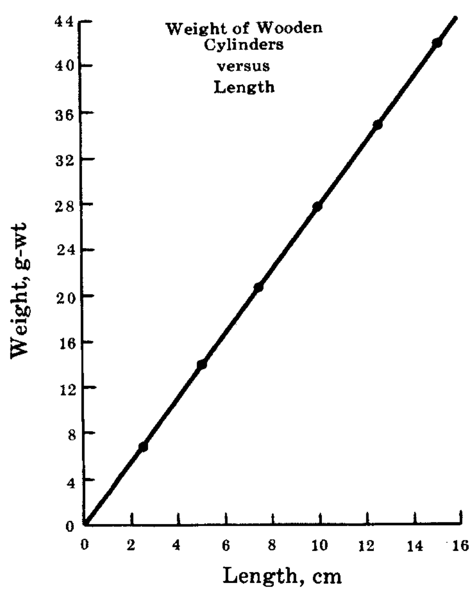

Fig. 7-4. A linear function through the origin

Since the graph goes through the origin, the W-intercept is zero. Therefore the equation is of the form $W = mL$. To find the slope of the line, divide any ordinate by the corresponding abscissa. In this case, the 10-cm abscissa is used because it simplifies division. The ordinate as read from the graph is 28 g-wt. Therefore, the slope

m is $28/10 = 2.8$ g-wt/cm. This is a *physical slope* because units are involved and its value was calculated by using the scales of measured physical quantities.

The empirical equation giving the relationship between the weights and lengths is $W = 2.8$ L. This equation tells us that the weight of a wooden cylinder of given cross section is directly proportional to its length. The proportionality constant is 2.8; it means that the weight of a cylinder 1 cm long is 2.8 g-wt. The proportionality constant will depend on the material of the cylinder and on the units used. For example, if pounds and inches were used, the constant would be 0.016 lb/in. Had aluminum cylinders been used, the constant would have been 7.5 g-wt/cm.

If the scales were greater and/or the divisions were more evident, the values could be determined more accurately with approximately values of $m = 2.78$ g-wt/cm and $W = 2.78$ L.

7.18 GRAPHING THE GENERAL LINEAR FUNCTION $y = mx + b$

If the graph of the experimental data is a straight line *not* through the origin, then the associated empirical equation is of the form $y = mx + b$ with $b \neq 0$. The slope m and the y-intercept b are obtained directly from the graph.

EXAMPLE 7.11. Data for the relationship between the electrical resistance R of a coil of wire and its temperature t are shown in the table below. Temperature t is the independent variable since it can be varied easily and continuously.

The graph of the data is shown in Fig. 7-5. Since it is a straight line, its equation is of the form $R = mt + b$. The vertical scale need not begin at zero. However, in order to obtain the empirical equation directly from the graph, the temperature scale must begin at zero. The physical slope of the line is $1.7/100 = 0.017$ milliohm/degree. The y-intercept is 4.95 milliohm. Therefore, the empirical equation is $R = 0.017t + 4.95$. Thus, the resistance is related to the temperature in a linear way. However, these two physical equantities are *not* directly proportional to each other.

t, °C	27.5	40.5	56.0	62.0	69.5	83.0
R, milliohms	5.38	5.64	5.88	6.00	6.12	6.34

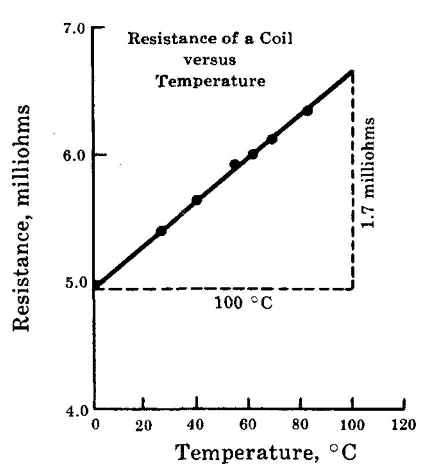

Fig. 7-5. Graph of a linear relationship

7.19 GRAPHING LINEAR FUNCTIONS WITH A CALCULATOR

To sketch a linear equation using a graphing calculator, you can use the following general guidelines:

1. Solve the equation for y in terms of x.
2. Set the calculator mode to rectangular (this step does not apply to all graphing calculators).
3. Set the size of the viewing window, the range, by defining the maximum and minimum x-values of the abscissa and the maximum and minimum y-values of the ordinate.
4. Always use parentheses if you are unsure of the order in which the calculator processes mathematical operations.

EXAMPLE 7.12. Sketch the graph of $y = 0.017x + 4.95$ using the graphing calculators shown below.

(NOTE): Function keys are enclosed in curly brackets in CAPITALS, for example, {ENTER}. Options on screen are shown underlined, for example, **EDIT**. Comments about the function of a key or option are enclosed in quotes.

Using an HP-38G

(1) Define the function.
{LIB}
Function {ENTER}
EDIT

0.017{*}{X,T,θ}{+}4.95{ENTER}

(2) Set the range.

{SHIFT}{SETUP-PLOT} "Press the shift key (the turquoise key) first and then press the PLOT key in the SETUP Menu."

XRNG: 0 {ENTER} "Minimum horizontal value" 120 {ENTER} "Maximum horizontal value"
YRNG: 0 {ENTER} "Minimum vertical value" 7 {ENTER} "Maximum vertical value"
XTICK: 20 {ENTER}"Horizontal tick spacing" YTICK: 1 {ENTER} "Vertical tick spacing"

(3) Graph the function.

{PLOT}.

Using a TI-8

(1) Define the function.
{Y=}

0.017{x}{X,T,θ}{+}4.95{ENTER} "Use {X,T,θ} to write the independent variable X"

(2) Set the range.

{WINDOW}{ENTER}

Xmin = 0 {ENTER} Ymin = 4 {ENTER}
Xmax = 120 {ENTER} Ymax = 7 {ENTER}
Xscl = 20 {ENTER} Yscl = 1 {ENTER}

(3) Graph the function.

{GRAPH}

Figure 7-6 shows the graph of this function.

EXAMPLE 7.13. In adult humans, the height of a person can be estimated as a linear function of the length of some of his bones. The following table shows some of the values relating the height of a person and the length of his tibia (connects the knee and the ankle). What is the linear relationship between these two quantities? What is the height of a man whose tibia is about 48 cm long?

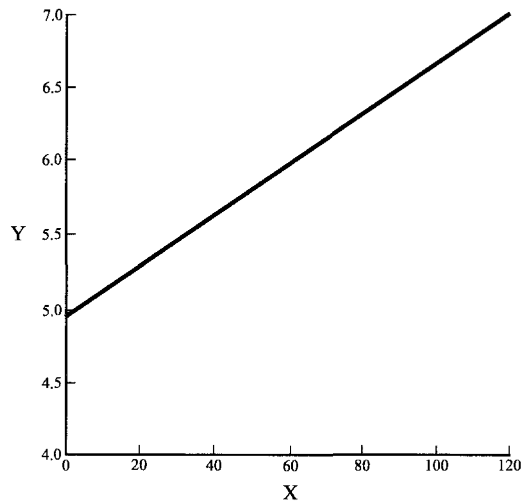

Fig. 7-6 Graph of $y = 0.017x + 4.95$

Height of a person, cm	173	188
Length of the tibia bone, cm	38	44

Let us assume that $(x_1, y_1) = (38, 173)$ and $(x_2, y_2) = (44, 188)$. The slope can be calculated as follows: $m = (y_2 - y_1)/(x_2 - x_1)$. That is, $m = (188 - 173)/(44 - 38) = 2.5$.

To calculate the equation of this straight line we can use the point–slope equation:

$$y = y_1 + m(x - x_1)$$

Replacing the value of the slope and any of the given points we obtain:

$y = y_1 + m(x - x_1)$

$y = 173 + 2.5(x - 38)$ $\leftarrow$ Notice that we have replaced x_1 by 38 and y_1 by 173

$y = 173 + 2.5x - 95$

$y = 2.5x + 78$

Figure 7-7 shows the graph of this line. From the figure, we can observe that the height of a person with a tibia of 48 cm is approximately 1.90 m.

To sketch this graph using a graphing calculator follow the steps indicated below.

Using an HP-38G

(1) Define the function.

{LIB}
<u>Function</u> {ENTER}
<u>EDIT</u>

2.5{*}{X,T,θ}{+}78{ENTER}

(2) Set the range.

{SHIFT}{SETUP-PLOT} "Press the shift key (the turquoise key) first and then press the PLOT key in the SETUP Menu"

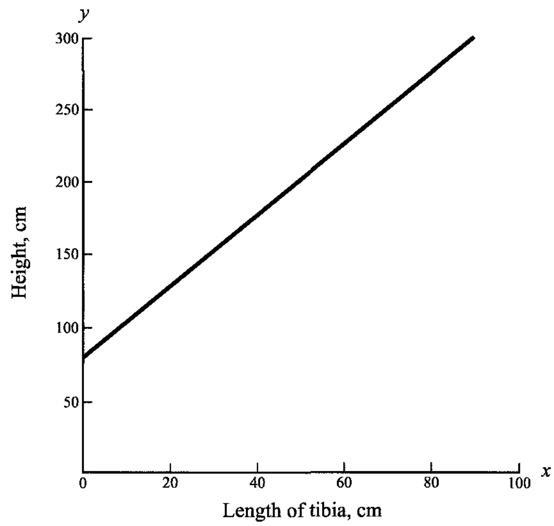

Fig. 7-7 Graph of $y = 2.5x \times 78$

XRNG: 0 {ENTER} "Minimum horizontal value" 100 {ENTER} "Maximum horizontal value"
YRNG: 0 {ENTER} "Minimum vertical value" 300 {ENTER} "Maximum vertical value"
XTICK: 20 {ENTER} "Horizontal tick spacing" YTICK: 50 {ENTER} "Vertical tick spacing"

(3) Plot the graph.

{PLOT}

Using a TI-82

(1) Define the function.

{Y=}

2.5{x}{X,T,θ}{+}78{ENTER} "Use {X,T,θ} to write the variable X"

(2) Set the range.

{WINDOW}{ENTER}

Xmin = 0 {ENTER} Ymin = 0 {ENTER}
Xmax = 100 {ENTER} Ymax = 300 {ENTER}
Xscl = 20 {ENTER} Yscl = 50 {ENTER}

(3) Graph the function.

{GRAPH}

EXAMPLE 7.14. The Celsius and Fahrenheit scales are widely used to measure temperature. The following table shows the relationship between these two scales. What is the linear equation that allows us to calculate a temperature in Celsius degrees as a function of a given measure expressed in Fahrenheit degrees? What is the physical meaning of the slope?

Fahrenheit	32	212
Celsius	0	100

Let $(x_1, y_1) = (32, 0)$ and $(x_2, y_2) = (212, 100)$. Then, the slope m can be calculated as follows:

$$m = (y_2 - y_1)/(x_2 - x_1) = (100 - 0)/(212 - 32) = 100/180 = 5/9$$

Replacing the value of the slope and any of the given points in the point–slope equation we obtain:

$$y = y_1 + m(x - x_1)$$
$$y = 32 + (5/9)(x - 0) \leftarrow \text{Replacing } (x_1, y_1) \text{ with } (0, 32)$$
$$y = (5/9)x + 32$$

The slope 5/9 may be interpreted to mean that for every 9-degree increase in the Fahrenheit temperature, there is an increase of 5 degrees Celsius. Figure 7-8 shows the graph of this function.

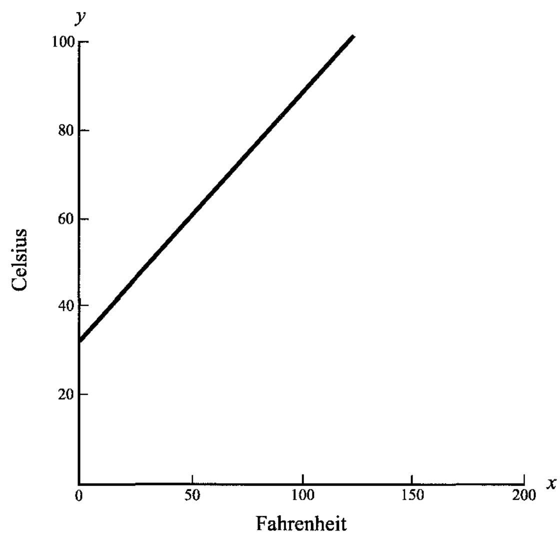

Fig. 7-8 Graph of $y = (\frac{5}{9})x + 32$

To sketch this graph using a graphing calculator follow the steps indicated below.

Using an HP-38G

(1) Define the function.

{LIB}
<u>Function</u> {ENTER}
<u>EDIT</u>

{(}5{/}9{)}{*}{X,T,θ}{+}32{ENTER}

(2) Set the range.

{SHIFT}{SETUP-PLOT} "Press the shift key (the turquoise key) first and then press the PLOT key in the SETUP Menu"

XRNG: 0 {ENTER} "Minimum horizontal value" 200 {ENTER} "Maximum horizontal value"
YRNG: 0 {ENTER} "Minimum vertical value" 100 {ENTER} "Maximum vertical value"
XTICK: 50 {ENTER} "Horizontal tick spacing" YTICK: 20 {ENTER} "Vertical tick spacing"

(3) Graph the function.

{PLOT}

Using a TI-82

(1) Define the function.

{Y=}

{(}5{÷}9{)}{x}{X,T,θ}{+}32{ENTER} "Use {X,T,θ} to write the variable X"

(2) Set the range.

{WINDOW}{ENTER}

Xmin = 0 {ENTER}	Ymin = 0 {ENTER}
Xmax = 200 {ENTER}	Ymax = 100 {ENTER}
Xscl = 50 {ENTER}	Yscl = 20 {ENTER}

(3) Graph the function.

{GRAPH}

EXAMPLE 7.15. Assume that a chemist mixes x ounces of a 20% alcohol solution with y ounces of another alcohol solution at 35%. If the final mixture contains 10 ounces of alcohol: (*a*) What is the linear equation relating both solutions and the total number of ounces of the final mixture? (*b*) How many ounces of the 35% solution need to be added to 5 ounces of the 20% solution to obtain 12 ounces of alcohol in the final mixture?

(*a*) If we have x ounces of an alcohol solution at 20%, this implies that only 20% of the x ounces are alcohol. Likewise, only 35% of the y ounces are alcohol. Since there are 10 ounces of alcohol in the final mixture, the relationship can be defined by the equation $0.2x + 0.35y = 10$.

(*b*) In this case, we know that there are 5 ounces of the 20% solution and 12 ounces of the final solution. Therefore, if we substitute these values in the previous equation and solve for y, we have that

$$y = (10 - 0.2*5)/0.35$$
$$y = 25.7 \text{ ounces}$$

Therefore, add 25.7 ounces (of the solution at 35%) to the 5 ounces (of the solution at 20%) to obtain 10 ounces of alcohol in the final mixture. Figure 7-9 shows the graph of this function.

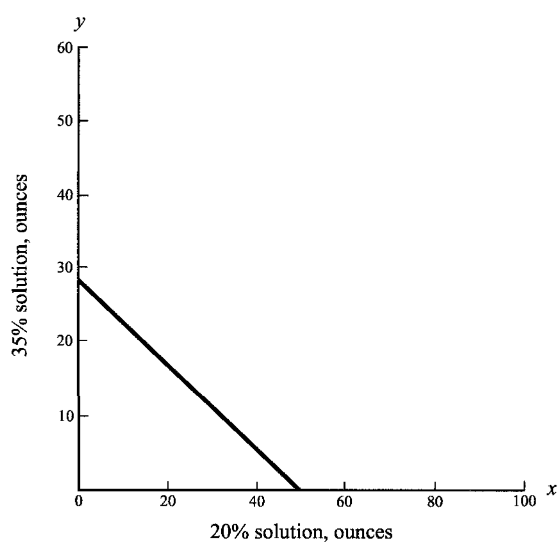

Fig. 7-9 Graph of $y = \frac{1}{0.35}(10 - 0.2x)$

This expression is in the form of a proportion, so that if any three of the quantities are known, the fourth one can be obtained. For instance, part (*e*) of the above example could have been obtained by setting up the proportion

$$\frac{G_1}{G_2} = \frac{d_1}{d_2}$$

Substituting the given values,

$$\frac{18}{50} = \frac{225}{d_2} \qquad \text{or} \qquad 18d_2 = (225)(50) \qquad \text{or} \qquad d_2 = \frac{(225)(50)}{18} = 625 \text{ miles}$$

Thus, the three expressions $\frac{G}{d} = k$, $G = kd$, $\frac{G_1}{G_2} = \frac{d_1}{d_2}$ are equivalent; each can be translated into the same statement of variation or proportionality.

8.6. At constant pressure and when well above the liquefaction temperature, the volume of a gas varies directly with its absolute temperature.

(*a*) Express the above relationship between the two quantities by an equivalent verbal statement.

(*b*) Write three mathematical expressions equivalent to the given statement.

(*c*) Explain in simple words the statement requested in (*a*).

(*a*) The volume of a gas is directly proportional to its absolute temperature.

(*b*) Let V = measure of volume of the gas, T = absolute temperature of the gas, k = constant. Then

$$\frac{V}{T} = k \qquad V = kT \qquad \frac{V_1}{V_2} = \frac{T_1}{T_2}$$

(*c*) If the absolute temperature is multiplied (or divided) by some number, the measure of the volume of the gas will be multiplied (or divided) by the same number.

DIRECT VARIATION, $n = 2$

8.7. The distance that a ball rolls along an inclined plane varies directly as the square of the time. The ball rolls 13.5 ft in 3.0 sec.

(*a*) Find the equation relating distance and time.

(*b*) Plot a graph of the function defined by (*a*).

(*c*) How far will the ball roll in 6.0 sec?

(*d*) How long will it take for the ball to roll 24 ft?

(*a*) Let d = distance in feet and t = time in seconds. Then the given statement means that $d/t^2 = k$ or $d = kt^2$, where k is a constant. Substituting the given values in the equation,

$$13.5 = k(3.0)^2 \qquad \text{or} \qquad \frac{13.5}{3^2} = \frac{13.5}{9} = 1.5$$

The exact functional equation is then $d = 1.5t^2$.

(*b*) To graph the function, substitute a few simple numbers for t in the equation and find the corresponding values of d. Tabulate the results as shown in the table below. Plot t horizontally, as it is the independent variable. Connect the points which we have plotted with a smooth curve. The resulting graph shown in Fig. 8-13 is a portion of a curve called a *parabola*.

t, sec	0	1	2	3	4	500
d, ft	0	1.5	6	13.5	24	37.5

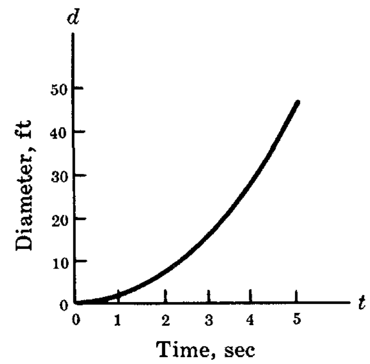

Fig. 8-13. Distance–time graph

(*c*) Substitute 6.0 sec for t and solve for d:

$$d = 1.5(6^2) = (1.5)(36) = 54 \text{ ft}$$

(*d*) Substitute 24 ft for d and solve for t:

$$24 = (1.5)t^2 \qquad \text{or} \qquad t^2 = \frac{24}{1.5} = 16$$

$$t = \pm\sqrt{16} = \pm 4 \text{ sec}$$

Although there are two possible solutions, we reject $t = -4$ because negative time has no physical meaning in this problem, and conclude that $t = 4$.

8.8. Given $P = 3I^2$, where P = rate of heat generation in an electrical conductor and I = current:

(*a*) Write two equivalent forms of the given equation.

(*b*) Translate the equation into statements of variation and proportionality.

(*c*) Explain the meaning of the equation in simple words.

(*d*) What happens to the value of P when I is increased 5 times?

(*e*) What happens to the value of I if P is divided by 16?

(*a*) $\dfrac{P}{I^2} = 3 \qquad \dfrac{P_1}{P_2} = \dfrac{I_1^2}{I_2^2}$

(*b*) The rate of heat generation in an electrical conductor varies directly as the square of the current; or, the rate of heat generation in an electrical conductor is directly proportional to the square of the current.

(*c*) When the current is doubled, the rate of heat generation is increased four times. If the current is reduced to one-third, the rate of heat generation is one-ninth of its former value. This can be worked out by substituting simple numbers for I in the equation as shown in the table below.

I	0	1	2	3
P	0	$3 \cdot 1^2 = 3$	$3 \cdot 2^2 = 12$	$3 \cdot 3^2 = 27$

(*d*) P is increased 25 times.

(*e*) I is divided by 4 or reduced to one-fourth of its previous value.

DIRECT VARIATION, $n = 1/2$

The power function $y = kx^{1/2}$ may be written also as $y = k\sqrt{x}$.

8.9. The velocity of a transverse wave in a stretched wire varies directly as the square root of the tension in the wire. The velocity of a transverse wave in a certain wire is 300 m/sec when the tension is 900 newtons.

(*a*) Represent the above variation by four equivalent equations.

(*b*) Find the equation constant and write the functional equation.

(*c*) Graph the function defined in (*b*).

(*d*) What will be the wave velocity when the tension is 3600 newtons?

(*e*) What should be the tension in the wire for a wave velocity of 250 m/sec?

(*f*) Translate the variation into a statement of proportionality.

(*g*) Explain the variation using small integers.

Let v = velocity of the transverse wave, t = tension in the wire, and k = constant.

(*a*) $\frac{v}{\sqrt{t}} = k \qquad v = k\sqrt{t} \qquad v = kt^{1/2} \qquad \frac{v_1}{v_2} = \frac{\sqrt{t_1}}{\sqrt{t_2}}$

(*b*) Substitute the given values of v and t into the first equation in (*a*) to obtain

$$\frac{300}{\sqrt{900}} = k \qquad \text{or} \qquad k = \frac{300}{30} = 10$$

The exact functional equation connecting v and t is $v = 10\sqrt{t}$, or $v = 10t^{1/2}$.

(*c*) Make a table of values by substituting a few integral squares for t and calculating v. Plot the values, with t along the horizontal axis, as shown in Fig. 8-14. The graph is of the parabolic type.

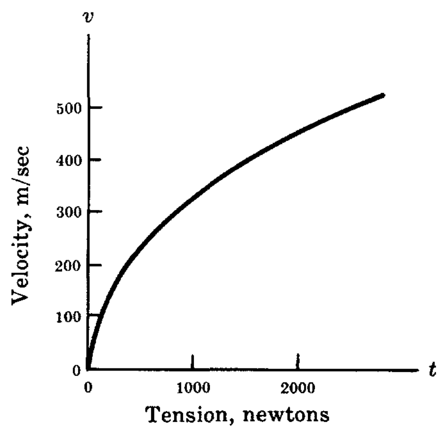

Fig. 8-14. Graph of $v = 10\sqrt{t}$

t, newtons	0	100	400	900	1600	2500
v, m/sec	0	100	200	300	400	500

(*d*) Substitute $t = 3600$ newton into the equation in (*b*):

$$v = 10\sqrt{3600} = 10 \cdot 60 = 600 \text{ m/sec}$$

(*e*) Substitute $v = 250$ m/sec into the equation in (*b*):

$$250 = 10\sqrt{t} \qquad \frac{250}{10} = \sqrt{t} \qquad 25 = \sqrt{t}$$

Squaring both sides

$$25^2 = t \qquad \text{or} \qquad 625 = t$$

Therefore, the required tension is 625 newtons.

(*f*) The velocity of the wave is directly proportional to the square root of the tension.

(*g*) If the tension is multiplied by 4, the velocity is doubled; if the tension is divided by 9, the velocity is divided by 3.

8.10. The period of a loaded oscillating coiled spring is directly proportional to the square root of the vibrating mass. The period is 1.33 second when the mass is 0.350 kilogram.

(*a*) Write the variation in mathematical symbols.

(*b*) Write three equivalent equations for the above variation.

(*c*) Determine the equation constant and write the functional equation.

(*d*) Graph the function defined in (*c*).

(*e*) What would be the value of the period for a mass of 0.150 kilogram?

(*f*) What should be the value of the mass so that the period is 1.15 second?

Let P = period of the spring; M = suspended mass; C = constant.

(*a*) $P \propto \sqrt{M}$ or $P \propto M^{1/2}$

(*b*)
$$\frac{P}{\sqrt{M}} = C \qquad P = C\sqrt{M} \qquad \frac{P_1}{P_2} = \frac{\sqrt{M_1}}{\sqrt{M_2}}$$

(*c*) Substituting the given values for P and M into the second equation of (*b*) we obtain:

$$1.33 = C\sqrt{0.350}$$

Solving for C,

$$C = \frac{1.33}{\sqrt{0.350}} \approx \frac{1.33}{0.592} \approx 2.25$$

Therefore the functional equation is $P = 2.25\sqrt{M}$ or $P = 2.25M^{1/2}$.

(*d*) The results of substituting a few integral squares for M and calculating P are shown in the table below.

M, kilograms	0	1	4	9	16
P, seconds	0	2.25	4.50	6.75	9.00

The graph of P versus M is shown in Fig. 8-15.

(*e*) Substitute 0.150 kilogram into the functional equation:

$$P = 2.25\sqrt{0.150} \approx (2.25)(0.387) \approx 0.871 \text{ sec}$$

(*f*) Substitute 1.15 second for P in the functional equation:

$$1.15 = 2.25\sqrt{M}$$

$$\frac{1.15}{2.25} = \sqrt{M} \qquad 0.511 = \sqrt{M}$$

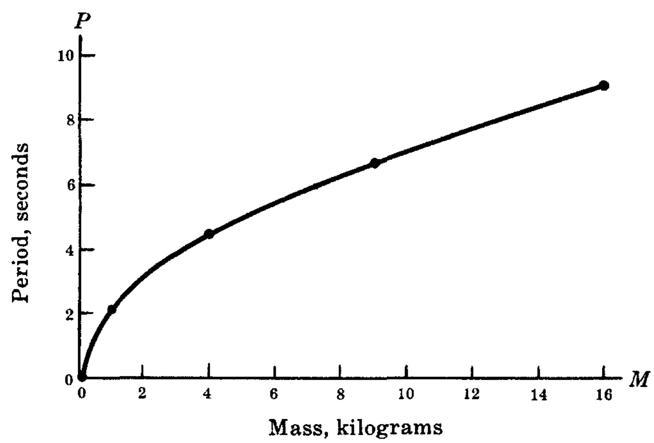

Fig. 8-15. Period versus mass

Square both sides to obtain

$$M = (0.511)^2 \qquad M \approx 0.261 \text{ kilogram}$$

INVERSE VARIATION, $n = 1$

8.11. The pressure of a gas at constant temperature varies inversely as the volume. The pressure of 50 ft^3 of the gas is 12 lb/in^2.

(*a*) Write the statement in terms of proportionality.

(*b*) Write four equivalent forms of the equation representing the statement.

(*c*) Determine the equation constant and write the exact functional equation.

(*d*) Graph the function defined in (*c*).

(*e*) Determine the pressure of the gas if the volume is reduced to 20 ft^3.

(*f*) Determine the volume of the gas if the pressure is reduced to 10 lb.

(*g*) Without making calculations on paper, predict what will happen to the volume if the pressure is (1) doubled, (2) tripled, (3) halved, (4) quartered.

Let p = pressure of the gas, v = volume of the gas, and k = constant.

(*a*) The pressure of a gas at constant temperature is inversely proportional to its volume.

(*b*) $$pv = k \qquad p = \frac{k}{v} \qquad p_1v_1 = p_2v_2 \qquad p = kv^{-1}$$

(*c*) Substitute the given values of p and v into the first equation in (*b*):

$$(12)(50) = k \qquad \text{or} \qquad k = 600$$

The exact functional equation connecting p and v is

$$pv = 600 \quad \text{or} \quad p = \frac{600}{v} \qquad \text{or} \qquad p = 600v^{-1}$$

(*d*) Make a table of values by substituting a few simple values for v and calculating p. Plot the values with v along the horizontal axis and connect with a smooth curve as shown in Fig. 8-16.

v	6	10	20	30	60	100	120
p	100	60	30	20	10	6	5

The graph is called a *rectangular hyperbola*.

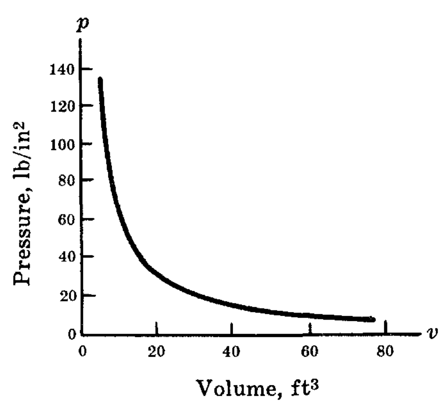

Fig. 8-16. Pressure–volume graph

(*e*) Substitute 20 for v into the equation:

$$p = \frac{600}{20} = 30 \text{ lb/in}^2$$

(*f*) Substitute 10 for p into the equation:

$$10 = \frac{600}{v} \qquad \text{or} \qquad v = \frac{600}{10} = 60 \text{ ft}^3$$

(*g*) (1) Volume will be reduced to one-half of its former value.

(2) Volume will be reduced to one-third of its former value.

(3) Volume will be doubled.

(4) Volume will be quadrupled.

8.12. When drilling brass at high speed, the drill diameter in inches is inversely proportional to the drill's speed in revolutions per minute. When the drill diameter is 0.50 inch, the drill speed is 2300 rpm.

(*a*) Write the statement of proportionality in mathematical symbols.

(*b*) Express the relationship by means of four equivalent equations.

(*c*) Determine the equation constant and write the functional equation.

(*d*) Graph the equation in (*c*).

(*e*) Calculate the drill speed for a diameter of 0.75 inch.

(*f*) Making only metal calculations, by what factor should the drill diameter be changed if the speed is quadrupled?

Let $s=$ the drill speed, $d=$ drill diameter, $c=$ constant.

(*a*) $d \propto \frac{1}{s}$ or $d \propto s^{-1}$

(*b*) $ds = c \qquad d = \frac{c}{s} \qquad d = cs^{-1} \qquad d_1 s_1 = d_2 s_2$

(*c*) Substitute the given values of d and s in the first equation of (*b*):

$$(0.50)(2300) = c$$

$$1150 = c \qquad \text{or} \qquad c = 1150$$

The functional equation is

$$ds = 1150 \qquad \text{or} \qquad d = \frac{1150}{s}$$

(*d*) Substitute a few values for s and calculate corresponding values of d as shown in the table below.

s, rpm	575	1150	2300	4600
d, inches	2	1	0.5	0.25

The graph of d versus s is shown in Fig. 8-17.

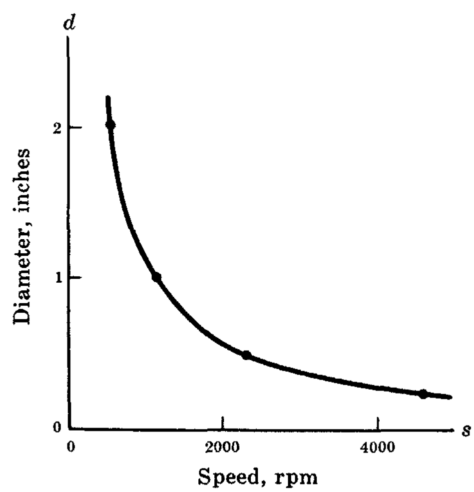

Fig. 8-17. Diameter versus speed

(*e*) Substitute 0.75 for d into the equation:

$$0.75 = \frac{1150}{s} \qquad \text{or} \qquad s = \frac{1150}{0.75} \approx 1530 \text{ rpm}$$

(*f*) The drill diameter should be quartered or reduced to one-fourth of its former value.

INVERSE VARIATION, $n = 2$

8.13. The dose rate from a radioactive point source in air is inversely proportional to the square of the distance from the source. For a distance of 2 meters the dose rate is 80 milliroentgens per hour.

(*a*) State in words the relationship between the variables as a power variation.

(*b*) Write five equivalent forms of the equations representing the relationship.

(*c*) Determine the proportionality constant and write the exact functional equation.

(*d*) Graph the equation in (*c*).

(*e*) Determine the dose rate at a distance of 5 meters.

(*f*) Determine the distance at which the dose rate is 0.5 milliroentgen/hour.

(*g*) Without paper calculations predict the effect on dose rate if the distance is (*a*) halved, (*b*) multiplied by 4.

(*h*) Without paper calculations predict the required change in distance if the dose rate is to (1) increase by a factor of 9, (2) be reduced by a factor of 25.

Let I = dose rate, r = distance, c = constant.

(*a*) The dose rate from a radioactive source varies inversely as the second power of the distance from the source (or inversely as the square of the distance).

(*b*) $$I = \frac{c}{r^2} \qquad Ir^2 = c \qquad I = cr^{-2} \qquad I_1 r_1^2 = I_2 r_2^2 \qquad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

(*c*) Substitute the given values of I and r in the second equation of (*b*)

$$(80)(4) = c \qquad c = 320$$

The functional equation is

$$Ir^2 = 320 \qquad \text{or} \qquad I = \frac{320}{r^2}$$

(*d*) Substitute a few values for r and calculate the corresponding values of I as shown in the table below.

r, meters	1	2	4
I, mr/hr	320	80	20

The graph of dose rate versus distance is shown in Fig. 8-18.

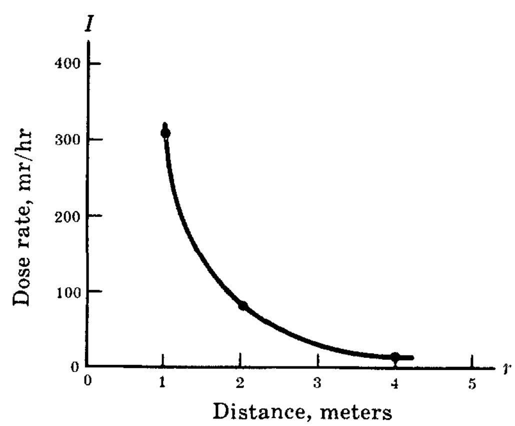

Fig. 8-18. Dose rate versus distance

(*e*) Substitute 5 for r in the equation.

$$I = \frac{320}{5^2} = \frac{320}{25}$$

$$I = 12.8 \text{ milliroentgen/hour}$$

(*f*) Substitute 0.5 for I in the equation.

$$0.5 = \frac{320}{r^2} \qquad 0.5r^2 = 320 \qquad r^2 = \frac{320}{0.5} = 640$$

$$r = \sqrt{640} \approx 25.3 \text{ meter}$$

(*g*) (1) The dose rate will be multiplied by 4.

(2) The dose rate will be reduced by a factor of 16.

(*h*) (1) The distance should be reduced by a factor of 3.

(2) The distance should be increased by a factor of 5.

8.14. The exposure time for photographing an object varies inversely as the square of the lens diameter. The exposure time is 1/400 sec when the lens diameter is 2 cm.

(*a*) Translate the above into a statement of proportionality.

(*b*) Write five equivalent forms of the equation representing the statement.

(*c*) Determine the equation constant and write the exact functional equation.

(*d*) Graph the function defined in (*c*).

(*e*) Find the exposure time for a lens diameter of 4 cm.

(*f*) Find the lens diameter needed to give an exposure time of 1/25 sec.

(*g*) Without making calculations on paper, predict what will happen to the exposure time if the lens diameter is: (1) halved; (2) tripled; (3) divided by 4.

(*h*) Without making calculations on paper, predict what should be the lens diameter if the exposure time is to be: (1) quadrupled; (2) divided by 4; (3) multiplied by 9.

Let t = exposure time, d = lens diameter, and k = constant.

(*a*) The exposure time for photographing an object is inversely proportional to the square of the lens diameter.

(*b*)
$$t = \frac{k}{d^2} \qquad td^2 = k \qquad t = kd^{-2}$$
$$t_1 d_1^2 = t_2 d_2^2 \qquad \frac{t_1}{t_2} = \frac{d_2^2}{d_1^2}$$

(*c*) Substitute the given values of t and d into the second equation in (*b*):

$$\frac{1}{400}(2^2) = k \qquad k = \frac{4}{400} = 0.01$$

The exact functional equation is

$$td^2 = 0.01 \qquad \text{or} \qquad t = \frac{0.01}{d^2}$$

(*d*) Make a table of values by substituting a few simple values for d and calculating t. Plot the values with d along the horizontal axis and connect with a smooth curve as shown in Fig. 8-19.

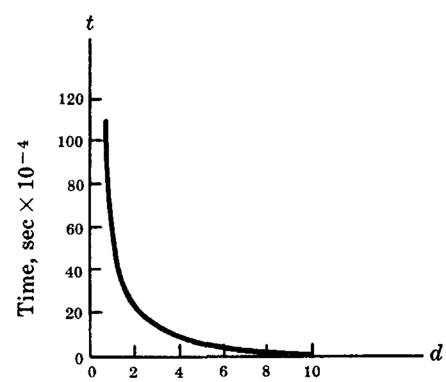

Fig. 8-19. Exposure time versus lens diameter

d, cm	1	2	3	4	5	10
t, sec	0.010	0.0025	0.0011	0.00063	0.0040	0.0001

The graph is of the hyperbolic type.

(*e*) Substitute 4 for d in the equation:

$$t = \frac{0.01}{4^2} = \frac{0.01}{16} = 0.00062 \approx 0.006 \text{ sec}$$

(*f*) Substitute $1/25$ for t in the equation:

$$\frac{1}{25} = \frac{0.01}{d^2}$$
$$d^2 = (0.01)(25) = 0.25$$
$$d = \pm\sqrt{0.25} = 0.5 \text{ cm}$$

The negative value of d is discarded because it has no physical meaning here.

(*g*) (1) multiplied by 4; (2) divided by 9; (3) multiplied by 16.

(*h*) (1) halved; (2) multiplied by 2; (3) divided by 3.

JOINT VARIATION

8.15. The centripetal force on a rotating object varies jointly as its mass and the square of its speed and inversely with the radius of rotation. For a mass of 90 g rotating at 140 cm/sec in a circle with a 60-cm radius the force is 29,400 dynes.

(*a*) Write equivalent forms of the equation representing the statement.

(*b*) Find the equation constant and write the exact functional equation.

(*c*) Determine the centripetal force when the mass is 200 g, the speed is 98 cm/sec, and the radius of the circular path is 40 cm.

(*d*) Without making calculations on paper, predict what the effect will be on the centripetal force if (1) the mass alone is doubled; (2) the mass alone is reduced to one-fourth of its former value; (3) the speed alone is tripled; (4) the speed alone is divided by 2; (5) the radius alone is multiplied by 6; (6) the radius alone is divided by 3; (7) the mass, the speed, and the radius are each doubled.

Use the symbols F, m, v, and r to represent measures of the centripetal force, the mass, the speed, and radius of rotation, respectively.

(*a*) Since F varies directly as the product of m and v^2 and inversely with r, we must have $Fr/mv^2 = k$, for some constant k. Solving the equation for F, we have $F = kmv^2/r$ as the equivalent form.

(*b*) Substitute the given values of m, v, and r in the first equation in (*a*):

$$k = \frac{(29{,}400)(60)}{90(140)^2} = 1$$

Therefore, the exact functional equation is

$$F = \frac{mv^2}{r}$$

(*c*)
$$F = \frac{200(98)^2}{40} \approx 48{,}000 \text{ dynes}$$

(*d*) Centripetal forces will be: (1) doubled; (2) divided by 4; (3) increased 9 times; (4) divided by 4; (5) divided by 6; (6) multiplied by 3; (7) multiplied by 4.

TRANSLATING EQUATIONS

Translate the equations in Problems 8.16 through 8.23 into statements of variation and proportionality.

(2) Plot the graph.

{PLOT} ← "Using the standard window settings".

Using a TI-82

(1) Define the function.

{Y =}

{LOG}{(}{X,T,θ}{)}{÷}{LOG}{(}2{)}{ENTER}

(2) Set the range

{WINDOW}{ENTER}

Xmin = −10 {ENTER}

Xmax = 10 {ENTER}

Xscl = 1 {ENTER}

Ymin = −10 {ENTER}

Ymax = 10 {ENTER}

Yscl = 1 {ENTER}

(3) Plot the graph.

{GRAPH}

9.15 LOGARITHMIC EQUATIONS

An equation is called *logarithmic* if it contains the logarithm of an unknown. The definition and laws of logarithms and the rules for solving ordinary equations are used in the solutions of logarithmic equations.

EXAMPLE 9.19. Solve for x: $2\log(x-2) = 6.6064$.

Dividing both sides of the equation by 2, first obtain $\log(x-2) = 3.3032$, and a glance at the log table shows that $\log 2010 = 3.3032$. Hence, $x - 2 = 2010$, so that $x = 2012$.

EXAMPLE 9.20. Solve for t: $2\log t + \log t = 6$.

Consider $\log t$ as the unknown of the equation. Then adding the two terms on the left side, we have

$$3\log t = 6$$

Dividing both sides by 3, $\log t = 2$ and

$$t = \text{antilog } 2 = 100$$

To solve logarithmic equations with a graphing calculator follow a similar process to the one used to solve exponential equations. You can rewrite the equation so that one of its sides is zero and consider the nonzero side to define a function of the unknown variable. Plot the graph and find its intercepts. Alternatively, you can consider each side of the equation to define a function of the unknown variable. Plot their graphs and find their intercepts. These intercepts represent the solution of the original equation. Both methods are illustrated below.

EXAMPLE 9.21. Solve $\log(x+3) = \log(7-3x)$ using a graphing calculator.

Using an HP-38G (method 1)

(1) Define the function.

{LIB}FUNCTION{ENTER}EDIT

{SHIFT}{LOG}{X,T,θ}{+}3{)}{−}{SHIFT}{LOG}7{−}3{*}{X,T,θ}{)}{ENTER}

(2) Plot the graph.

{PLOT}

(3) Find the x-intercept.

MENU FCN Root {ENTER}

The root value is 1.

Using a TI-82

(1) Define the functions.

{Y=}

{LOG}{(}{X,T,θ}{+}3{)}{ENTER} "First function. $y = \log(x+3)$"

{LOG}{(}7{−}3{×}{X,T,θ}{)}{ENTER} "Second function. $y = \log(7-3x)$"

(2) Plot the graph.

{GRAPH} "Set the range for the x and y values between −5 and 5 with scale of 1 for both axis"

(3) Find the intercept.

{2nd}{CALC}intersect {ENTER}

First curve? {ENTER}

Second curve? {ENTER}

Guess? {ENTER}

The intersection occurs at $x = 1$.

9.16 GROWTH AND DECAY

Exponential functions can be used to describe exponential growth and decay. Growth allows us to calculate how an amount will increase as a function of time given an initial amount or population. Decay allows us to calculate how an amount will decrease as a function of time from a given initial amount or population. Both, growth and decay, can be modeled by a function of the form $f(x) = ab^t$, however, they are generally written in some other form such as those listed below.

1. $A(t) = A_0(1+r)^t$ where A_0 is the existing amount at time $t=0$ and r is growth rate. If $r>0$, then the initial amount grows exponentially. If $-1<r<0$, then the initial amount decays exponentially. The quantity $1+r$ is called the growth factor.
2. $A(t) = A_0 b^{(t/k)}$ where k is the amount of time needed to multiply A_0 by b.

EXAMPLE 9.22. In 1996, the population of the town of New Virginia was 35,000. It has been determined that due to the new industries coming into the area, the population is expected to grow at the rate of 4%. (*a*) What will be the population of the town 5 years from now? (*b*) Draw the corresponding graph and determine when the population will reach 50,000?

Call $A(5)$ the population in 5 years. Then, $A(5) = 35{,}000*(1 + 0.04)^5 \approx 42{,}583$ individuals. Figure 9-6 shows the graph of this function.

From the graph we can observe that the population will reach the 50,000 mark in approximately 9 years. To graph this function using a graphing calculator follow the steps indicated below.

Using an HP-38G

(1) Define the function.

{LIB}<u>FUNCTION</u>{ENTER}<u>EDIT</u>

35000{*}{(}1{+}0.04{)}{x^y}{X,T,θ}{ENTER}

(2) Set the range.

{SHIFT}{SETUP-PLOT}

XRNG: 0 {ENTER}	14 {ENTER}
YRNG: 30000 {ENTER}	50000 {ENTER}
XTICK: 2 {ENTER}	YTICK: 5000 {ENTER}

(3) Plot the graph.

{PLOT}

Using a TI-82

(1) Define the function.

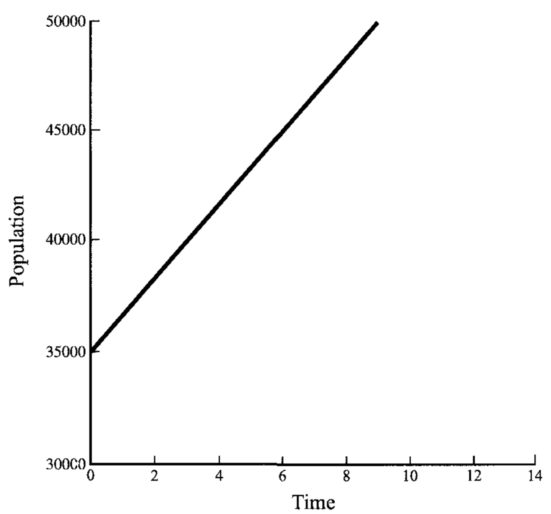

Fig. 9-6

{Y=}

35000{x}{(}1{+}0.04{)}{^}{X,T,θ}

(2) Set the range.

{WINDOW}{ENTER}

Xmin = 0 {ENTER}

Xmax = 14 {ENTER}

Xscl = 2 {ENTER}

Ymin = 30000 {ENTER}

Ymax = 50000 {ENTER}

Yscl = 5000 {ENTER}

(3) Plot the graph.

{GRAPH}

EXAMPLE 9.23. The atoms of a radioactive element are said to be unstable because they go spontaneously through a process called radioactive decay. In this process each atom emits some of its mass as radiation, the remainder of the atom reforms to make an atom of some new element. For example, radioactive carbon-14 decays into nitrogen; radium decays into lead. One method of expressing the stability of a radioactive element is known as the half-life of the element. That is, the time required for half of a given sample of a radioactive element to change to another element. Assume that a radioactive isotope has a half-life of 5 days, At what rate does the substance decay each day?

We will use the formula $A(t) = A_0 b^{(t/k)}$ where k is the amount of time needed to multiply A_0 by b. Since the half-life of this radioactive isotope is 5 days, this implies that after 5 days only half of the original sample remains. In consequence, $b = 1/2$ and $t = 5$. Replacing these values in the given formula, we have

$$A(5) = A_0(1/2)^{t/5} = A_0(0.5)^{t/5}$$

To find the decay rate, we need to rewrite the formula given above so that it looks like the formula $A(t) = A_0(1 + r)^t$.

Notice that we can rewrite $A(5) = A_0(0.5^{1/5})^t = A_0(0.87)^t = A_0(1 - 0.13)^t$. Therefore, the decay rate is approximately 13%.

EXAMPLE 9.24. The half-life of carbon-14 is 5750 years. Samples taken at an excavation site show that 1 pound of a charcoal sample has 65% of carbon-14 compared to a standard sample. What is the age of the sample?

$A_0 = 1$, $k = 5750$ and $b = 1/2$. Therefore,

$$\begin{aligned} A(t) &= A_0 b^{t/k} \\ 0.65 &= A_0(1/2)^{t/5750} \end{aligned}$$

To solve for t proceed as follows:

$$\begin{aligned} \log(0.65) &= \log[(0.5)^{t/5750}] \\ \log(0.65) &= (t/5750)\log(0.5) \\ t &= \log(0.65)*5750/\log(0.5) \\ t &\approx 3574 \text{ years.} \end{aligned}$$

Logarithmic and Semilogarithmic Graph Paper

9.17 LOGARITHMIC GRAPH PAPER

Logarithmic or log–log paper is a graph paper on which the rulings are not spaced uniformly. The lines are laid off at distances which are proportional to the logarithms of numbers from 1 to 10. Logarithmic paper can have one or more logarithmic cycles along each axis; if there is just one it is called single logarithmic paper. Thus, logarithmic paper makes it possible to plot logarithms of numbers without the use of logarithmic tables.

Logarithmic paper is used:

(1) to graph a power function from a known equation.

(2) to determine the equation of a power function which satisfies empirical data.

Taking the logarithm of the power function $y = Cx^m$, we have

$$\log y = \log C + m \log x$$

Now, let $\log y = Y$; $\log C = b$; $\log x = X$. Then, by substitution $Y = b + mX$, which is an equation of a straight line, with slope m and Y-intercept b. Thus, if one plots the logarithm of one variable against the logarithm of the second variable and the graph is a straight line, then the equation connecting the two variables is of the form $y = Cx^m$. The geometrical slope m is measured as on a uniform scale; the coefficient C is read as the intercept on the vertical axis through $x = 1$.

It is important to note that since the logarithm of zero is not defined, there is no zero on either of the two scales of logarithmic paper.

EXAMPLE 9.25. Plot a graph of $T = 2\sqrt{m}$ on logarithmic paper.

Assume m is the independent variable. Assign a few values to m and calculate the corresponding values of T as shown in the table below.

m	1	4	9
T	2	4	6

Plot m along the horizontal scale of single logarithmic paper and T along the vertical scale. The graph is shown in Fig. 9-7.

The graph is a straight line. The measured geometrical slope is $m = \dfrac{h}{a} = 0.5$; the T-intercept is 2. These values agree with the constants of the equation:

$$T = 2\sqrt{m} = 2m^{1/2} = 2m^{0.5}$$

EXAMPLE 9.26. The breaking strength S of three-strand manila rope is a function of its diameter D. Determine the empirical equation from the data in the following table.

D, in.	1.0	1.25	1.50	1.75	2.0	2.25
S, lb-f	1700	2700	3700	5300	6200	8200

Use single logarithmic paper. Plot diameter as the independent variable along the horizontal axis. The vertical scale will have a range of 1000 to 10,000 lb. The graph of the data is shown in Fig. 9-8. Since the graph is a straight line, the relationship between the two variables can be expressed as a power function of the form $y = Cx^m$. The geometrical slope is 1.9; the S-intercept is 1700. Therefore, the equation is $S = 1700D^{1.9}$.

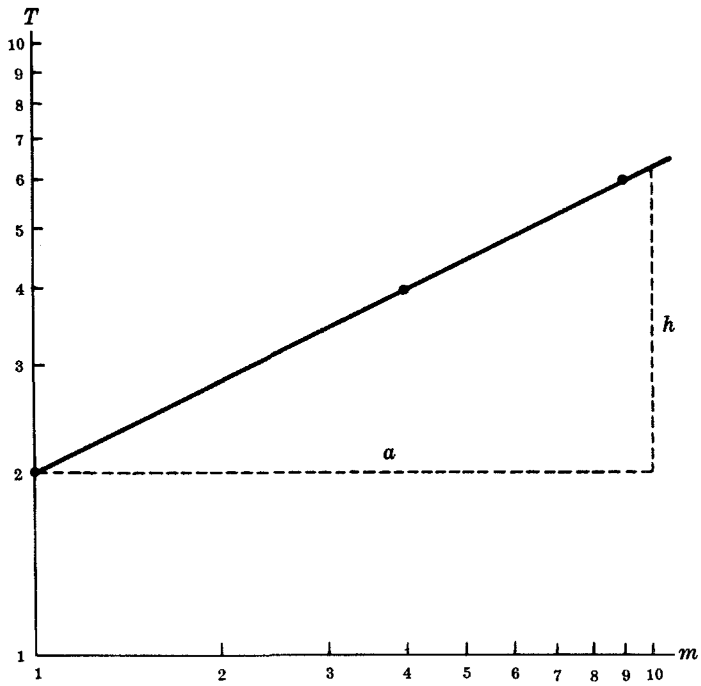

Fig. 9-7. Graph of $T = 2\sqrt{m}$

9.18 SEMILOGARITHMIC PAPER

Semilogarithmic paper is a graph paper on which one of the axes has uniform subdivisions, while those on the second axis are proportional to the logarithms of numbers.

Semilogarithmic paper is used:

(1) to graph an exponential function,

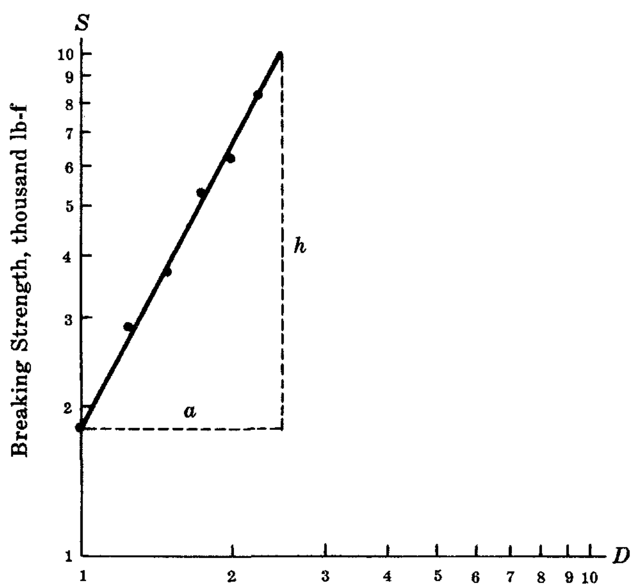

Fig. 9-8. The power function $S \approx 1700D^{1.9}$ (Slope $= h/a \approx 1.9$; S-intercept ≈ 1700)

(2) to determine the equation of an exponential function which satisfies empirical data.

Taking the logarithm of both terms of the equation $Y = C(10)^{mx}$, one gets

$$\log Y = \log C + mx$$

Now, let $\log Y = y$ and $\log C = b$. Then, by substitution $y = b + mc$, which is an equation of a straight line, with slope m and y-intercept b. Thus, if one plots the logarithm of one variable versus the values of the second variable and the graph is a straight line, then the equation is of the form $Y = C(10)^{mx}$. The geometric slope m is measured; the coefficient C is read as the intercept on the Y-axis. Semilogarithmic paper enables one to plot the logarithms of numbers without having to look them up in tables.

EXAMPLE 9.27. If a rope is wound around a wooden pole, then the frictional force between the pole and the rope depends on the number of turns. From the data in the table below determine the equation relating the force of friction F to the number of turns N.

N	0.25	0.5	0.75	1.0
F, lb-f	21	33	47	70

Use one-cycle semilogarithmic paper. Plot the number of turns N along the horizontal axis and the force F along the vertical axis as shown in Fig. 9-9. Since the graph is a straight line, its equation is of the form $F = C \cdot 10^{mN}$. The slope of the line is 0.70; the y-intercept is 14. Therefore, the equation is $F = 14 \cdot 10^{0.70N}$. The geometric slope is measured on uniform scales by means of a rule. Do *not* use the scales on the graph to determine geometric slope.

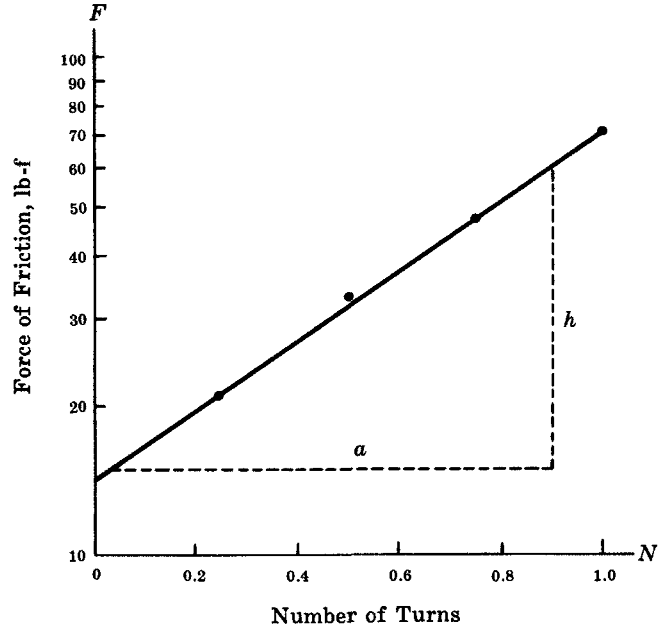

Fig. 9-9. The exponential function $F \approx 14 \cdot 10^{0.70N}$ (Geometrical slope $h/a \approx 0.70$; F-intercept ≈ 14)

Solved Problems

9.1. Write each of the following exponential equations in equivalent logarithmic form:

(*a*) $5^3 = 125$ (*b*) $16^{1/2} = 4$ (*c*) $\left(\frac{1}{3}\right)^r = k$ (*d*) $10^{-4} = 0.0001$

A direct application of the definition leads us to the following results:

(*a*) $\log_5 125 = 3$ (*b*) $\log_{16} 4 = 1/2$ (*c*) $\log_{1/3} k = r$ (*d*) $\log_{10} 0.0001 = -4$

9.2. Write each of the following logarithmic equations in equivalent exponential form:

(*a*) $\log_{10} 1000 = 3$ (*b*) $\log_4 2 = 1/2$ (*c*) $\log_b 1 = 0$ (*d*) $-2 = \log_3 1/9$

Again the definition leads us to the solutions:

(*a*) $10^3 = 1000$ (*b*) $4^{1/2} = 2$ (*c*) $b^0 = 1$ (*d*) $1/9 = 3^{-2}$

9.3. Determine the common logarithm of

(*a*) 10,000 (*b*) 100,000,000 (*c*) 0.001 (*d*) 0.0000001

Express the numbers in scientific notation. All of them will be in the form 10^p. The exponent p is the common logarithm.

(*a*) $\log 10{,}000 = \log 10^4 = 4$ (*c*) $\log 0.001 = \log 10^{-3} = -3$

(*b*) $\log 100{,}000{,}000 = \log 10^8 = 8$ (*d*) $\log 0.0000001 = \log 10^{-7} = -7$

With a calculator these logarithms can be calculated as follows.

Using an HP-38G:

(*a*) {SHIFT}{LOG}10000{ENTER} (*c*) {SHIFT}{LOG}0.001{ENTER}

Using a TI-82:

(*b*) {LOG}100000000{ENTER} (*d*) {LOG}0.0000001{ENTER}

9.4. Determine N if we know that $\log N$ is: (*a*) 5, (*b*) -2, (*c*) $1/3$, (*d*) 1.5.

The common logarithm is the exponent to base 10. Therefore, the values of N are:

(*a*) $N = 10^5 = 100{,}000$ (*c*) $N = 10^{1/3} = \sqrt[3]{10} \approx 2.154$

(*b*) $N = 10^{-2} = 0.01$ (*d*) $N = 10^{1.5} = 10^{3/2} = (10^3)^{1/2} = \sqrt{1000} = 10\sqrt{10} \approx 31.62$

The values of N can be calculated with the help of a calculator as follows:

Using an HP-38G:

(*a*) {SHIFT}{10^x}5{ENTER} (*c*) {SHIFT}{10^x}{(}1/3{)}{ENTER}

Using a TI-82:

(*b*) {2^{nd}}{10^x}{(−)}2{ENTER} (*d*) {2^{nd}}{10^x}1.5{ENTER}

9.5. Determine the logarithm of: (*a*) 463.8, (*b*) 0.00396, (*c*) $(5.893)(10^{-5})$.

(*a*) We first express 463.8 in scientific notation: $463.8 = (4.638)(10^2)$. The exponent of 10 = characteristic = 2. To find the mantissa, we turn to the table on page 465. Locate the number 46 in the first column and the number 6656 along the same line under the column labeled 3. Continue along the same line to proportional parts column labeled 8 and find the digit 7. Adding, $6656 + 7 = 6663$. Thus, the mantissa of $\log 4.638 = 0.6663$. Therefore, $\log 463.8 = 0.6663 + 2$.

Another way of writing this logarithm is $\log 463.8 = 2.6663$.

(*b*) Similarly, $0.003967 = 3.967(10^{-3})$. The characteristic of the logarithm of this number is -3 and the mantissa is 0.5985. Therefore, $\log 0.003967 = 0.5985 - 3$.

There are three additional ways in which this negative logarithm can be written: $\bar{3}.5985$, $7.5985 - 10$, and -2.4015.

(c) Since the number is already in scientific notation, the characteristic $= -5$. The mantissa from the table is 0.7703. Therefore, $\log(5.893)(10^{-5}) = 0.7703 - 5$. The three other ways of writing this logarithm are $\bar{5}.7703$, $5.7703 - 10$, and -4.2297.

9.6. Determine the antilogarithm of:

(a) 4.5378 (b) 0.6484 − 3 (c) −4.3842 (d) 8.0170 − 10

(a) We write $4.5378 = 0.5378 + 4$ and note in the table that $\log 3.45 = 0.5378$. Hence the required antilogarithm is $3.45(10^4)$ or 34,500.

(b) Similarly, $0.6484 - 3$ is seen to be the logarithm of $4.45(10^{-3})$ and so the desired antilogarithm is 0.00445.

(c) We emphasize again that the mantissa of a logarithm is always positive, and so a logarithm must be expressed with a positive mantissa if its antilogarithm is to be found. Adding and subtracting 5 to -4.3842 will give a positive mantissa. Thus, $5 + (-4.3842) - 5 = 0.6158 - 5$. A glance at the table reveals that $\log 4.129 = 0.6158$, and so the desired antilogarithm is $4.129(10^{-5})$ or 0.00004129.

(d) From the table, $\log 1.04 = 0.0170$. The characteristic is $8 - 10 = -2$. The desired antilogarithm is $1.04(10^{-2}) = 0.0104$.

The antilogarithms of the numbers 4.5378 and −4.3842 can be found with a calculator as indicated below.

Using an HP-38G:

(a) {SHIFT}{10^x}4.5378{ENTER} (b) {SHIFT}{10^x}{−x}4.3842{ENTER}

Due to internal approximations the results obtained with the calculator may differ slightly of those obtained by table. In this case for (a) the calculator shows 34498.4831363. For (b) the result is 4.1257330378E − 5. In calculators the letter E is used to represent powers of 10. This expression is equivalent to $4.1257330378 \times 10^{-5}$.

Using a TI-82:

(a) {2^{nd}}{10^x}4.5378{ENTER} (b) {2^{nd}}{10^x}{(−)}4.3842{ENTER}

In this case, internal errors and approximations also affect the results. These results are for (a) 34498.48314; for (b) 4.128573304E − 5. This latter value is equivalent to $4.128573304 \times 10^{-5}$.

9.7. Use logarithms to compute 243.2×12.54.

$$\begin{aligned} \log 243.2 &= \log 2.432(10^2) = 0.3860 + 2 \\ \log 12.54 &= \log 1.254(10^1) = \underline{0.0983 + 1} \\ \text{sum of logarithms} &= 0.4843 + 3 = \text{logarithm of product} \end{aligned}$$

The desired product = antilogarithm of $0.4843 + 3 = 3.050(10^3) = 3050$.

To calculate the logarithm of this product using a calculator follow the steps indicated below. Remember that due to internal errors and approximations the results may differ from those obtained when using a table of logarithms.

Using an HP-38G:

{SHIFT}{LOG}243.2{*}12.54{)}{ENTER}

The result is 3.4842611071.

STO▶{A . . . Z}{A}{ENTER} ← This sequence stores the result into the variable A.

Check: $(49.48)^2 \stackrel{?}{=} 60^2 + (-32)(18)$

$\stackrel{?}{=} 2448$

$2448.2 \stackrel{?}{=} 2448$

10.2. Determine the nature of the solutions in the following quadratic equations:

(*a*) $p^2 - 9p = 0$ (*c*) $6x^2 - x - 2 = 0$ (*e*) $3s^2 + 5s + 2 = 0$

(*b*) $t^2 - 2t + 1 = 0$ (*d*) $2r^2 - 5r = -4$

(*a*) $a = 1 \quad b = -9 \quad c = 0$

$b^2 - 4ac = 9^2 - (4)(1)(0)$ evaluate the discriminant

$= 81$

Since $81 > 0$, the solutions are real and unequal.

(*b*) $a = 1 \quad b = -2 \quad c = 1$

$b^2 - 4ac = (-2)^2 - 4(1)(1) = 0$ evaluate the discriminant

Since the discriminant is zero, the solutions are real and equal.

(*c*) $a = 6 \quad b = -1 \quad c = -2$

$b^2 - 4ac = (-1)^2 - 4(6)(-2)$ evaluate the discriminant

$= 49$

Since the discriminant is > 0, the solutions are real and unequal.

(*d*) $2r^2 - 5r + 4 = 0$ put the equation in standard form

$a = 2 \quad b = -5 \quad c = 4$

$b^2 - 4ac = (-5)^2 - 4(2)(4)$ evaluate the discriminant

$= -7$

Since $-7 < 0$, the solutions are imaginary.

(*e*) $3s^2 + 5s + 2 = 0$ put the equation in standard form

$a = 3 \quad b = -5 \quad c = 2$

$b^2 - 4ac = (5)^2 - 4(3)(2) = 1$ evaluate the discriminant

Since the discriminant is > 0, the solutions are real and unequal.

10.3. Solve by factoring and check.

(*a*) $x^2 - 4x + 4 = 0$ (*c*) $y^2 + 7y + 12 = 0$ (*e*) $3r^2 = 10r + 8$

(*b*) $4t^2 - 25 = 0$ (*d*) $3b + 5b^2 = 2$

(*a*) $x^2 - 4x + 4 = 0$ left member is the square of the binomial $x - 2$

$(x - 2)(x - 2) = 0$

$x - 2 = 0$ a factor must be zero

$x = 2$ transpose -2

The two roots are equal.

Check: $2^2 - 4(2) + 4 \stackrel{?}{=} 0$
$4 - 8 + 4 \stackrel{?}{=} 0$
$0 = 0$

(*b*) $4t^2 - 25 = 0$ left member is the difference of two squares
$(2t + 5)(2t - 5) = 0$ factor the given equation
$2t + 5 = 0$ a factor must be zero
$t = -5/2$ solve for t
$2t - 5 = 0$ solve for the second value of t
$t = 5/2$

Check: $4(-5/2)^2 - 25 \stackrel{?}{=} 0$ $\quad 4(5/2)^2 - 25 \stackrel{?}{=} 0$
$4(25/4) - 25 \stackrel{?}{=} 0$ $\quad 4(25/4) - 25 \stackrel{?}{=} 0$
$0 = 0$ $\quad 0 = 0$

(*c*) $y^2 + 7y + 12 = 0$
$(y + 4)(y + 3) = 0$ factor the left member
$y + 4 = 0$ a factor must be zero
$y = -4$
$y + 3 = 0$ solve for the two values of y
$y = -3$

Check: $(-4)^2 + 7(-4) + 12 \stackrel{?}{=} 0$ $\quad (-3)^2 + 7(-3) + 12 \stackrel{?}{=} 0$
$16 - 28 + 12 \stackrel{?}{=} 0$ $\quad 9 - 21 + 12 \stackrel{?}{=} 0$
$0 = 0$ $\quad 0 = 0$

(*d*) $3b + 5b^2 = 2$
$5b^2 + 3b - 2 = 0$ put equation in standard form
$(5b - 2)(b + 1) = 0$ factor the left member
$5b - 2 = 0$ a factor must be zero
$b = 2/5$ solve for the two values of b
$b + 1 = 0$
$b = -1$

Check: $5(2/5)^2 + (3)(2/5) \stackrel{?}{=} 2$ $\quad 5(-1)^2 + 3(-1) \stackrel{?}{=} 2$
$20/25 + 6/5 \stackrel{?}{=} 2$ $\quad 5 - 3 \stackrel{?}{=} 2$
$20/25 + 30/25 \stackrel{?}{=} 2$ $\quad 2 = 2$
$50/25 \stackrel{?}{=} 2$
$2 = 2$

(*e*) $3r^2 = 10r + 8$
$3r^2 - 10r - 8 = 0$ put equation in standard form
$(3r + 2)(r - 4) = 0$ factor
$3r + 2 = 0$ a factor must be zero
$r = -2/3$
$r - 4 = 0$ solve for the two values of r
$r = 4$

Check: $3(-2/3)^2 \stackrel{?}{=} 10(-2/3) - 8$ $\qquad$ $3(4^2) \stackrel{?}{=} 10(4) + 8$

$+12/9 \stackrel{?}{=} -20/3 + 8$ $\qquad$ $48 = 48$

$4/3 = 4/3$

10.4. Use the quadratic formula to solve the following equations:

(*a*) $x^2 + x - 6 = 0$ $\qquad$ (*c*) $2x^2 - x + 1 = 0$ $\qquad$ (*e*) $3q^2 - 12 = -1$

(*b*) $3r^2 + 5r = 2$ $\qquad$ (*d*) $-3t^2 - 2t + 5 = 0$

To use the formula, each equation must be in the standard form, $ax^2 + bx + c = 0$. Substitute the values of a, b, and c in the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(*a*) $a = 1 \quad b = 1 \quad c = -6$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-6)}}{2(1)} \qquad \text{substitute in the formula}$$

$$x_1 = \frac{-1 + \sqrt{25}}{2} = 2$$

$$x_2 = \frac{-1 - \sqrt{25}}{2} = -3$$

Check: $2^2 + 2 - 6 \stackrel{?}{=} 0$ $\qquad$ $(-3)^2 + (-3) - 6 \stackrel{?}{=} 0$

$4 + 2 - 6 \stackrel{?}{=} 0$ $\qquad$ $9 - 3 - 6 \stackrel{?}{=} 0$

$0 = 0$ $\qquad$ $0 = 0$

(*b*) $3r^2 + 5r - 2 = 0$ $\qquad$ put equation in standard form

$a = 3 \quad b = 5 \quad c = -2$

$$r = \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2 \cdot 3} \qquad \text{substitute in the formula}$$

$$= \frac{-5 \pm \sqrt{49}}{6}$$

$$r_1 = \frac{-5 + 7}{6} = 1/3$$

$$r_2 = \frac{-5 - 7}{6} = -2$$

Check: $3(1/3)^2 + 5(1/3) \stackrel{?}{=} 2$ $\qquad$ $3(-2)^2 + 5(-2) \stackrel{?}{=} 2$

$3/9 + 5/3 \stackrel{?}{=} 2$ $\qquad$ $12 - 10 \stackrel{?}{=} 2$

$3/9 + 15/9 \stackrel{?}{=} 2$ $\qquad$ $2 = 2$

$2 = 2$

(*c*) $2x^2 - x + 1 = 0$

$a = 2 \quad b = -1 \quad c = 1$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(1)}}{(2)(2)} \qquad \text{substitute in the formula}$$

$$= \frac{1 \pm \sqrt{1-8}}{4}$$

$$= \frac{1 \pm \sqrt{-7}}{4}$$

The solutions are *not* real numbers since $\sqrt{-7}$ is not a real number.

(*d*) $-3t^2 - 2t + 5 = 0$

$a = -3 \quad b = -2 \quad c = 5$

$$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-3)(5)}}{2(-3)} \qquad \text{substitute in the formula}$$

$$= \frac{2 \pm \sqrt{4+60}}{-6}$$

$$= \frac{2 \pm \sqrt{64}}{-6}$$

$$t_1 = \frac{2+8}{-6} = -5/3$$

$$t_2 = \frac{2-8}{-6} = 1$$

Check: $-3(-5/3)^2 - 2(-5/3) + 5 \stackrel{?}{=} 0 \qquad -3(1)^2 - 2(1) + 5 \stackrel{?}{=} 0$

$-75/9 + 10/3 + 5 \stackrel{?}{=} 0 \qquad -3 - 2 + 5 \stackrel{?}{=} 0$

$0 = 0 \qquad 0 = 0$

(*e*) $3q^2 - 12q + 1 = 0$ put equation in standard form

$a = 3 \quad b = -12 \quad c = 1$

$$q = \frac{-(-12) \pm \sqrt{(-12^2) - 4(3)(1)}}{2(3)} \qquad \text{substitute in the formula}$$

$$q = \frac{12 \pm \sqrt{144-12}}{6}$$

$$q_1 = \frac{12 + \sqrt{132}}{6} \qquad \text{look up } \sqrt{132} \text{ in the table on page 464}$$

$$= \frac{12 + 11.48}{6} = \frac{23.48}{6}$$

$$= 3.91$$

$$q_2 = \frac{12 - \sqrt{132}}{6}$$

$$= \frac{12 - 11.48}{6} = \frac{0.52}{6}$$

$$= 0.0867$$

Check: $3(3.91)^2 - 12(3.91) + 1 \stackrel{?}{=} 0 \qquad 3(0.0867)^2 - 12(0.0867) + 1 \stackrel{?}{=} 0$

$3(15.28) - 46.92 + 1 \stackrel{?}{=} 0 \qquad 0.0225 - 1.040 + 1 \approx 0$

$45.84 - 46.92 + 1 \stackrel{?}{=} 0 \qquad -0.0175 \approx 0$

$-0.08 \approx 0$

10.5. If a 40-cm tube closed at one end is thrust, open and downward, 10 cm into mercury, the pressure of air inside the tube depresses the mercury level by h cm according to the equation $h^2 + 106h - 760 = 0$. Solve for h in the equation.

In this equation $a = 1$, $b = 106$, $c = -760$. Application of the quadratic formula gives:

$$h = \frac{-106 \pm \sqrt{(+106)^2 - 4(1)(-760)}}{2} = \frac{-106 \pm \sqrt{11{,}236 + 3040}}{2}$$

$$= \frac{-106 \pm \sqrt{14{,}276}}{2} \approx \frac{-106 \pm 119.5}{2}$$

$$h \approx \frac{13.5}{2}\ \text{cm} \approx 6.75\ \text{cm} \quad \text{and} \quad h \approx \frac{-225.5}{2}\ \text{cm} \approx -112.75\ \text{cm}$$

Check: $(6.75)^2 + 106(6.75) - 760 \stackrel{?}{=} 0$

$$45.6 + 714 - 760 \stackrel{?}{=} 0$$

$$759.6 - 760 \approx 0$$

In many physical science problems one of the solutions is frequently discarded because it has no physical meaning. In the above problem, $-225.5/2$ or -112.75 cm could not be a solution since the entire tube is only 40 cm long.

10.6. Solve and check the equation: $\dfrac{50}{d^2} = \dfrac{150}{(2-d)^2}$.

$\dfrac{1}{d^2} = \dfrac{3}{(2-d)^2}$	divide both sides by 50
$(2-d)^2 = 3d^2$	product of means equals product of extremes
$4 - 4d + d^2 = 3d^2$	remove parentheses
$4 - 4d - 2d^2 = 0$	transpose $3d^2$ to left side
$2 - 2d - d^2 = 0$	divide both members by 2
$-d^2 - 2d + 2 = 0$	put in standard form
$d = \dfrac{-(-2) \pm \sqrt{(-2)^2 - (4)(-1)(2)}}{2(-1)}$	use quadratic formula

$$= \frac{2 \pm \sqrt{12}}{-2} = \frac{2 \pm 3.464}{-2}$$

$$d = +0.732 \quad \text{and} \quad d = -2.732.$$

Check: $\dfrac{50}{(0.732)} \stackrel{?}{=} \dfrac{150}{(2-0.732)^2}$ and $\dfrac{50}{(-2.732)^2} \stackrel{?}{=} \dfrac{150}{(4.732)^2}$

$$\frac{50}{0.537} \stackrel{?}{=} \frac{150}{1.61} \qquad \frac{50}{7.46} \stackrel{?}{=} \frac{150}{22.4}$$

$$93.2 = 93.2 \qquad 6.70 = 6.70$$

10.7. A problem on the reflection of light leads to the equation:

$$10 = \frac{(n-1)^2}{(n+1)^2}\,100$$

where n is the index of refraction relative to air. Determine n.

$$\begin{aligned}
10(n+1)^2 &= (n-1)^2(100) && \text{simplify the equation}\\
10(n^2+2n+1) &= (n^2-2n+1)100 && \text{expand the two binomials}\\
10n^2+20n+10 &= 100n^2-200n+100 && \text{simplify}\\
90n^2-220n+90 &= 0\\
9n^2-22n+9 &= 0
\end{aligned}$$

$$\begin{aligned}
n &= \frac{22 \pm \sqrt{(22)^2-4(9)(9)}}{(2)(9)} && \text{substitute in the quadratic formula}\\
&= \frac{22 \pm \sqrt{484-324}}{18}\\
&= \frac{22 \pm \sqrt{160}}{18} = \frac{22 \pm 12.6}{18}
\end{aligned}$$

$n = \dfrac{34.6}{18}$ and $\dfrac{9.4}{18}$, or $n = 1.92$ and $n = 0.52$.

Mathematically both solutions are correct; but the solution 0.52 has no physical meaning, as the index of refraction cannot be less than 1.

Check: $10 \stackrel{?}{=} \dfrac{(1.92-1)^2}{(1.92+1)^2}(100) = \dfrac{(0.92)^2}{(2.92)^2}(100)$

$10 \stackrel{?}{=} \dfrac{84.6}{8.53}$

$10 \approx 9.92$

10.8. The distance d in feet covered by a freely falling object is given by the equation $d = 16t^2$, where t is the time of fall in seconds. How long will it take for an object to fall 9 feet?

$$\begin{aligned}
9 &= 16t^2 && \text{substitute 9 for } s\\
\frac{9}{16} &= t^2 && \text{divide both sides by 16}\\
\pm\sqrt{9/16} &= t && \text{take the square root of both sides}\\
\pm 3/4 &= t && \text{since 9 and 16 are perfect squares, take the square root of each and divide}
\end{aligned}$$

The time of fall is 3/4 second. The negative root has no physical meaning.

10.9. In the equation $a = \dfrac{v^2}{R}$, if $a = 4$ and $R = 500$, calculate v.

$$\begin{aligned}
4 &= \frac{v^2}{500} && \text{substitute 4 for } a \text{ and 500 for } R\\
2000 &= v^2 && \text{multiply both sides by 500}\\
\pm\sqrt{2000} &= v && \text{take the square root of both sides}\\
44.7 &\approx v
\end{aligned}$$

Check: $4 \stackrel{?}{=} \dfrac{(44.7)^2}{500} = \dfrac{1998}{500}$

$4 \approx 3.996$

10.10. Solve for R: $\dfrac{100}{250} = \dfrac{(6400)^2}{R^2}$.

$100R^2 = (250)(6400)^2$	cross-multiply
$R^2 = \dfrac{(250)(6400)^2}{100}$	divide both sides by 100 and multiply
$R^2 = (2.5)(6.4 \times 10^3)^2$	use exponential notation
$R = \pm\sqrt{(2.5)(6.4 \times 10^3)^2}$	take the square root of both sides
$R = \pm\sqrt{2.5}\sqrt{(6.4 \times 10^3)^2}$	take the square root of each factor
$R = \pm(1.58)(6.4 \times 10^3)$	
$R = \pm 1.01 \times 10^4$	

Check: $\dfrac{100}{250} \stackrel{?}{=} \dfrac{(6400)^2}{(1.01 \times 10^4)^2}$

$0.40 \approx 0.402$

10.11. Solve for t: $(t + 0.80)^2 = 2.5t + t^2$.

$t^2 + 1.6t + 0.64 = 2.5t + t^2$	expand the binomial
$t^2 - t^2 + 1.6t - 2.5t + 0.64 = 0$	transpose
$-0.9t + 0.64 = 0$	simplify
$t = \dfrac{0.64}{0.9}$	solve for t
$t = 0.71$	

Check: $(0.71 + 0.80)^2 \stackrel{?}{=} 2.5(0.71) + (0.71)^2$

$2.28 \stackrel{?}{=} 1.775 + 0.504$

$2.28 \approx 2.279$

10.12. In the equation $v^2 = v_0^2 + 2as$, $v = 50$; $a = -10$; $s = 150$. Compare the value of v_0.

$50^2 = v_0^2 + 2(-10)(150)$	substitute the given values
$2500 = v_0^2 - 3000$	simplify
$v_0^2 = 5500$	solve for v
$v_0 = \pm\sqrt{5500}$	take the square root of each side
≈ 74.16	
≈ 74.2	

Check: $(50)^2 \stackrel{?}{=} (74.2)^2 + 2(-10)150$

$2500 \stackrel{?}{=} 5506 - 3000$

$2500 \approx 2506.$

10.13. Solve: $\dfrac{0.20}{(0.50-d)^2}=\dfrac{0.5}{d^2}$.

$0.20d^2 = 0.5(0.25 - d + d^2)$	cross-multiply and expand the binomial
$0.20d^2 = 0.125 - 0.5d + 0.5d^2$	simplify
$(0.20d^2 - 0.5d^2) + 0.5d - 0.125 = 0$	put equation in standard form
$-0.3d^2 + 0.5d - 0.125 = 0$	
$-30d^2 + 50d - 12.5 = 0$	multiply by 100
$a = -30 \quad b = +50 \quad c = -12.5$	
$d = \dfrac{-(+50) \pm \sqrt{(+50)^2 - 4(-30)(-12.5)}}{2(-30)}$	use quadratic formula
$d_1 = 0.306$ and $d_2 = 1.360$	simplify

Check: $\dfrac{0.20}{(0.50-0.306)^2} \stackrel{?}{=} \dfrac{0.5}{(0.306)^2} \quad 5.31 \approx 5.34 \qquad \dfrac{0.20}{(0.50-1.360)^2} \stackrel{?}{=} \dfrac{0.5}{(1.360)^2} \quad 0.270 \approx 0.270$

10.14. Solve the following equations for the indicated symbol:

(*a*) $S = (1/2)at^2$ for t (*c*) $hf = W + (1/2)mv^2$, for v (*e*) $A = \pi(R_1^2 - R_2^2)$, for R_2

(*b*) $2\pi fL = \dfrac{1}{2\pi fC}$, for f (*d*) $m = m_0/\sqrt{1 - v^2/c^2}$, for v

(*a*) $S = (1/2)at^2$

$2S = at^2$	multiply both sides by 2
$2S/a = t^2$	divide both sides by a
$\pm\sqrt{2S/a} = t$	extract the square root of each side
$t = \pm\sqrt{2S/a}$	

(*b*) $2\pi fL = \dfrac{1}{2\pi fC}$

$4\pi^2 f^2 LC = 1$	multiply both sides by $2\pi fC$
$f^2 = \dfrac{1}{4\pi^2 LC}$	divide both sides by$4\pi^2 LC$
$f = \pm\sqrt{\dfrac{1}{4\pi^2 LC}}$	take the square root of each side
$f = \pm\dfrac{1}{2\pi}\dfrac{1}{\sqrt{LC}}$	take $\dfrac{1}{2\pi}$ outside the radical sign since $\sqrt{\dfrac{1}{4\pi^2}} = \dfrac{1}{2\pi}$

(*c*) $hf = W + (1/2)mv^2$

$hf - W = (1/2)mv^2$	transpose W
$\dfrac{2(hf - W)}{m} = v^2$	multiply both sides by 2 and divide by m
$\pm\sqrt{\dfrac{2(hf - W)}{m}} = v$	extract the square root of each side
$v = \pm\sqrt{\dfrac{2(hf - W)}{m}}$	

(*d*) $m = m_0 \Big/ \sqrt{1 - \dfrac{v^2}{c^2}}$

$m\sqrt{1 - \dfrac{v^2}{c^2}} = m_0$	multiply both sides by $\sqrt{1 - \dfrac{v^2}{c^2}}$
$\sqrt{1 - \dfrac{v^2}{c^2}} = \dfrac{m_0}{m}$	divide both sides by m
$1 - \dfrac{v^2}{c^2} = \dfrac{m_0^2}{m^2}$	square both sides
$\dfrac{-v^2}{c^2} = \dfrac{m_0^2}{m^2} - 1$	transpose 1
$\dfrac{v^2}{c^2} = 1 - \dfrac{m_0^2}{m^2}$	multiply both sides by -1
$v^2 = c^2\left(1 - \dfrac{m_0^2}{m^2}\right)$	multiply both sides by c^2
$v = \pm\sqrt{c^2\left(1 - \dfrac{m_0^2}{m^2}\right)} = \pm c\sqrt{1 - \dfrac{m_0^2}{m^2}}$	extract the square root of each side

(*e*) $A = \pi(R_1^2 - R_2^2)$

$\dfrac{A}{\pi} = R_1^2 - R_2^2$	divide both sides by π
$\dfrac{A}{\pi} - R_1^2 = -R_2^2$	transpose R_1^2
$R_1^2 - \dfrac{A}{\pi} = R_2^2$	multiply both sides by -1
$\pm\sqrt{R_1^2 - \dfrac{A}{\pi}} = R_2$	extract the square root of each side
$R_2 = \pm\sqrt{R_1^2 - \dfrac{A}{\pi}}$	

Note: Since the above five formulas deal with intrinsically positive physical quantities, the negative solutions have no physical meaning in any case.

10.15. Using the table of square roots and interpolation, determine the square roots of the following to three decimal places: (*a*) 36.4, (*b*) 118.2, (*c*) 80.35, (*d*) 7.9, (*e*) 12.08.

(*a*)

$$\begin{aligned}
\sqrt{37} &\approx 6.083 \\
\sqrt{36} &= \underline{6.000} \\
\text{difference} &= 0.083 \\
(0.4)(0.083) &= 0.0332 \\
\sqrt{36.4} &\approx 6.000 + 0.0332 \\
&\approx 6.033
\end{aligned}$$

Check: $6.033^2 \stackrel{?}{=} 36.4$

$36.4 \approx 36.4$

(*b*)

$$\begin{aligned}
\sqrt{119} &= 10.909 \\
\sqrt{118} &= \underline{10.863} \\
\text{difference} &= 0.046 \\
(0.2)(0.046) &= 0.0092 \\
\sqrt{118.2} &\approx 10.863 + 0.0092 \\
&\approx 10.872
\end{aligned}$$

Check: $(10.872)^2 \stackrel{?}{=} 118$

$118.2 \approx 118$