$\boxed{\text{Т4}}$ $\quad \xi \sim \frac{\theta}{2}\{(-1,1)\setminus\{0\}\} + \frac{1-\theta}{2}\{0\} + \frac{1-\theta}{2}\{2\}$

$\theta \in (0,N)$ $\quad \alpha_1 = M[\xi] = \int\limits_{-\infty}^{+\infty} x f(x,\theta)\,dx = \underbrace{\int\limits_{-1}^{1} \frac{\theta}{2} x\,dx}_{=0} +$

$+ \frac{1-\theta}{2}\cdot 0 + \frac{1-\theta}{2}\cdot 2 = 1-\theta$

$\alpha_2 = M[\xi^2] = \frac{\theta}{2}\int\limits_{-1}^{1} x\,dx + \frac{1-\theta}{2}\cdot 2^2 = \frac{\theta}{3} + 2(1-\theta) =$

$= 2 - \frac{5}{3}\theta$

$\mu_2 = \alpha_2 - \alpha_1^2 = D[\xi] = 2 - \frac{5}{3}\theta - \theta^2 + 2\theta - 1 = 1 + \frac{\theta}{3} - \theta^2$

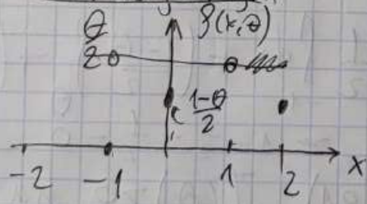
а) ОММ 1: $\alpha_1(\theta) = \overline{\alpha_1} = \overline{x} \Rightarrow 1-\theta = \overline{x} \Rightarrow \tilde{\theta} = 1-\overline{x}$

несмещ.: $M[\tilde{\theta}_1] = M[1-\overline{x}] = 1 - M[\xi] = 1 - 1 + \theta = \theta$ $\Rightarrow$ несмещ

Сост.: $D[\tilde{\theta}_1] = D[1-\overline{x}] = \frac{1}{n} D[\xi] \xrightarrow[n\to\infty]{} 0$

$\Rightarrow$ сост, но дост. усл

ОМП:



$L = \left(\frac{\theta}{2}\right)^{n-m_1-m_2} \cdot \left(\frac{1-\theta}{2}\right)^{m_1+m_2}$

$\Downarrow$

$\ln L = (n-m_1-m_2)\ln\frac{\theta}{2} + (m_1+m_2)\ln\left(\frac{1-\theta}{2}\right)$

$(\ln L)' = \frac{n-m_1-m_2}{\theta} + \frac{m_1+m_2}{\theta-1} = \frac{n\theta - n + m_1 + m_2}{\theta(\theta-1)}$

$\begin{cases} (\ln L)' = 0 \Rightarrow n\theta - n + m_1 + m_2 = 0 \Longrightarrow \theta = 1 - \lambda_1 - \lambda_2 \\ (\ln L)'' = \frac{m_1+m_2-n}{\theta^2} - \frac{m_1+m_2}{(\theta-1)^2} = \{\theta = 1-\lambda_1-\lambda_2\} = \frac{(\lambda_1+\lambda_2)(\lambda_1+\lambda_2-1)}{n\theta^4(\theta-1)^2} < 0 \end{cases}$

$\Rightarrow \theta = 1 - \eta_1 - \eta_2$ — это максимум

$\Rightarrow \tilde{\theta}_2 = 1 - \eta_1 - \eta_2$

в) Несмещенность

потому

$M[\tilde{\theta}_2] = 1 - M[\eta_1] - M[\eta_2] = 1 - \frac{1-\theta}{2} - \frac{1-\theta}{2} = \theta$

$D[\tilde{\theta}_2] = D[\eta_1 + \eta_2] = \frac{(1-\theta)\theta}{n} \xrightarrow[n\to\infty]{} 0$

$\Rightarrow$ сост. по дост. усл.

с) Проверка на эффективность / регулярность

1) $\rho(x,\theta) \in C^\infty(0,1)$

2) $\frac{\partial}{\partial\theta}\int \rho(x,\theta)\,dx = \int \frac{\partial}{\partial\theta}\rho(x,\theta)\,dx$ на $(0,1)$

$\int\limits_{-\infty}^{+\infty} \frac{\partial\rho(x,\theta)}{\partial\theta} = \frac{1}{2}\int\limits_{-1}^{1} dx + \left(\frac{1-\theta}{2}\right)'_\theta + \left(\frac{1-\theta}{2}\right)'_\theta =$

$= 1 - \frac{1}{2} - \frac{1}{2} = 0$

3) $I(\theta) \in C(0,1)$. $I(\theta) > 0$ на $(0,1)$.

$\frac{\partial \ln \rho(x,\theta)}{\partial\theta} = \frac{\theta}{\partial\theta}\left(\ln\frac{\theta}{2}\right) = \frac{1}{\theta} \cdot \frac{\partial \ln P_{0,2}}{\partial\theta} - \frac{\partial}{\partial\theta}\left(\ln\left(\frac{1-\theta}{2}\right)\right) =$

$= \theta \Rightarrow I(\theta) = \int\limits_{-1}^{1}\frac{1}{2\theta}\,dx = \frac{1}{(\theta-1)^2}\cdot\left(\frac{1-\theta}{2}\right)\cdot 2 =$

$$= \frac{1}{\theta} + \frac{1}{1-\theta} = \frac{1}{\theta(1-\theta)} \overset{>0 \text{ на } (0,1)}{\in} C(0,1)$$

$\Rightarrow$ модель – регулярна

компакте

$\tilde{\theta}_1 = 1 - \overline{X}$ – несмещ $D[\tilde{\theta}_1]$ – огр на $\forall$ подотрезке из $[0,1]$

$\Longrightarrow \tilde{\theta}_1$ – регулярна

$\tilde{\theta}_2 = 1 - \nu_1 - \nu_2$ – несмещ $D[\tilde{\theta}_2]$ – огр на $\forall$ компакте из $(0,1)$

$\Rightarrow \tilde{\theta}_2$ – регулярна

Неравенство Крамера Рао:

$$D[\tilde{\theta}_2] \geq \frac{1}{n I(\theta)} = \frac{\theta(1-\theta)}{n}$$

$$\frac{\theta(1-\theta)}{n} \geq \frac{\theta(1-\theta)}{n} \Rightarrow \text{по дост. усл. эфф-сти}$$

$\tilde{\theta}_2$ – эффективна

$\tilde{\theta}_2$ – эффективна, $\tilde{\theta}_1 \neq \tilde{\theta}_2 \Rightarrow \tilde{\theta}_1$ – НЕэффективна