Г1.1

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$H_0: \xi \sim p_0(x) = 1 \{(0,1)\}$

$H_1: \xi \sim p_1(x) = \frac{e}{e-1} e^{-x} \{(0,1)\}$

а) $n=1$ $\quad l = \frac{L_1}{L_0} = \frac{e}{e-1} e^{-x} \geq C \Rightarrow e^{-x} \geq B, \; x \leq A$

$P(x \leq A \mid H_0) = \alpha \Rightarrow \int\limits_0^A dx = A = \alpha$

$G: x \leq \alpha, \; \alpha_1 = \alpha$

$W = P(x \leq A \mid H_1) = \int\limits_0^\alpha \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1}(1 - e^{-\alpha})$

$\alpha_2 = 1 - W$

б) $n=2$ $\quad l = \left(\frac{e}{e-1}\right)^2 e^{-(x_1+x_2)} \geq C \Rightarrow e^{-(x_1+x_2)} \geq B$

$x_1 + x_2 \leq A$

$P(x_1 + x_0 \leq A \mid H_0) = \alpha$

$\iint\limits_{x_1+x_2 \leq A} dx_1 dx_2 = \frac{A^2}{2} = \alpha \Rightarrow A = \sqrt{2\alpha}$

$G: x_1 + x_2 \leq \sqrt{2\alpha}, \; \alpha_1 = \alpha$

$W = P(x_1 + x_2 \leq A \mid H_1) = \iint\limits_{x_1+x_2 \leq A} \left(\frac{e}{e-1}\right)^2 e^{-(x_1+x_2)} dx_1 dx_2 =$

$$= \left(\frac{e}{e-1}\right)^2 \left(1 - e^{-A} - Ae^{-A}\right) = \left(\frac{e}{e-1}\right)^2 \left(1 - e^{-\sqrt{2\alpha}} - Ae^{-\sqrt{2\alpha}}\right)$$

$$\alpha_2 = 1 - W$$

c) $$\rho = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \geq C$$

$$\ln \rho = \sum_{i=1}^{n} \ln \frac{p_1(x_i)}{p_0(x_i)} \geq \ln C$$

$$\frac{\sum_{i=1}^{n} -\text{//}- \; - nM[-\text{//}-]}{\sqrt{n D[-\text{//}-]}} \rightsquigarrow N(0,1)$$

$$G: \; P(\ln \rho \geq \ln \ell \mid H_0) = \alpha$$

$$H_0: \; -\text{//}- = \ln\left(\frac{e}{e-1} e^{-x}\right) = \ln \frac{e}{e-1} - x$$

$$\ln \ell = n \ln \frac{e}{e-1} - \sum_{i=1}^{n} x_i \geq \ln \ell$$

$$G: \; \sum_{i=1}^{n} x_i \leq A \qquad P\left(\sum x_i \leq A \mid H_0\right) = \alpha, \quad M[x] = \frac{1}{2} \quad D[x] = \frac{1}{12}$$

$$P\left(\frac{\sum x_i - nM[x]}{\sqrt{nD[x]}} \leq \frac{A - nM[x]}{\sqrt{nD[x]}} \,\Big|\, H_0\right) = \alpha$$

$$\frac{A - \frac{n}{2}}{\sqrt{\frac{n}{12}}} = u_\alpha \Rightarrow A = \frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$$u_\alpha: \int_{-\infty}^{u_\alpha} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \alpha$$

$$G: \; \sum_{i=1}^{n} x_i \leq \frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}}, \quad \alpha_1 = \alpha$$

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$$W = P\left(\sum x_i \le A \mid H_1\right) = P\left(\frac{\sum x_i - nM[x]}{\sqrt{nD[x]}} \le \frac{A - nM[x]}{\sqrt{nD[x]}} \,\Big|\, H_1\right)$$

$$M[x] = \int_0^1 \frac{xe}{e-1} e^{-x} dx = \frac{e-2}{e-1}$$

$$M[x^2] = \frac{2e-5}{e-1} \qquad D[x] = \frac{e^2-3e+1}{(e-1)^2}$$

$$G: \sum x_i \le \frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$$W = \int_{-\infty}^{B} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \longrightarrow ?$$

$$B = \frac{A - nM[x]}{\sqrt{nD[x]}} = \frac{\frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}} - n\frac{e-2}{e-1}}{n\frac{e^2-3e+1}{(e-1)^2}} \xrightarrow{n\to\infty} \infty$$

$$\Rightarrow W \xrightarrow{n\to\infty} 1$$

T12

$\alpha = 0{,}2$

$H_0: \xi \sim p_0(x) = \frac{1}{4}\{1\} + \frac{1}{4}\{2\} + \frac{1}{6}\{3\} + \frac{1}{3}\{4\}$

$H_1: \xi \sim p_1(x) = \frac{1}{4}\{1\} + \frac{1}{4}\{2\} + \frac{1}{4}\{3\} + \frac{1}{4}\{4\}$

$n = 2 \qquad p = \dfrac{S_1(x_1)S_1(x_2)}{S_0(x_1)S_0(x_2)}$

l:	1	2	3	4
1	1	1	3/2	9/4
2	1	1	9/2	9/4
3	3/2	9/2	9/4	9/8
4	3/4	3/4	9/8	9/16

H_0	1	2	3	4
1	1/16	1/16	1/24	1/12
2	1/16	1/16	1/24	1/12
3	1/24	1/24	1/36	1/18
4	1/12	1/12	1/18	1/9

H_1	1	2	3	4
1	1/16	1/16	1/16	1/16
2	1/16	1/16	1/16	1/16
3	1/16	1/16	1/16	1/16
4	1/16	1/16	1/16	1/16

$l \geq c, \quad c = \frac{3}{2}$

$\alpha_1 = \frac{7}{36}, \quad \alpha_2 = \frac{11}{16}$

$W = \frac{5}{16}$