

## Lecture 4:

# Bayesian Signal Detection



Markus Hohle

University California, Berkeley

**Bayesian Data Analysis and  
Machine Learning for Physical  
Sciences**



## Course Map

Module 1	Maximum Entropy and Information, Bayes Theorem
Module 2	Naive Bayes, Bayesian Parameter Estimation, MAP
Module 3	MLE, Lin Regression, Model selection: Comparing Distributions
<b>Module 4</b>	<b>Model Selection: Bayesian Signal Detection</b>
Module 5	Variational Bayes, Expectation Maximization
Module 6	Stochastic Processes
Module 7	Monte Carlo Methods
Module 8	Markov Models, Graphs
Module 9	Machine Learning Overview, Supervised Methods
Module 10	Unsupervised Methods
Module 11	ANN: Perceptron, Backpropagation
Module 12	ANN: Basic Architecture, Regression vs Classification, Backpropagation again
Module 13	Convolution and Image Classification and Segmentation
Module 14	TBD (GNNs)
Module 15	TBD (RNNs and LSTMs)
Module 16	TBD (Transformer and LLMs)



## Outline

**Discrete Signals**

**The Model**

**The Priors**

**Occam Factors**

**A Code Example**





## Outline

**Discrete Signals**

The Model

The Priors

Occam Factors

A Code Example



literature:

A NEW METHOD FOR THE DETECTION OF A PERIODIC SIGNAL OF UNKNOWN SHAPE AND PERIOD

P. C. GREGORY

Department of Physics, University of British Columbia, 6224 Agricultural Road, Vancouver, British Columbia, Canada V6T 1Z1

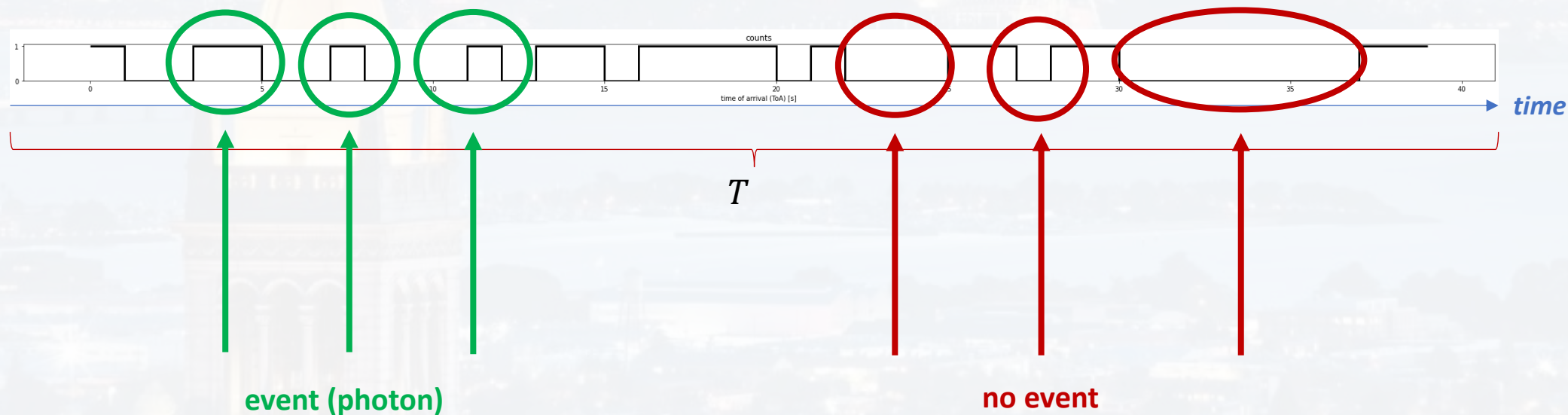
AND

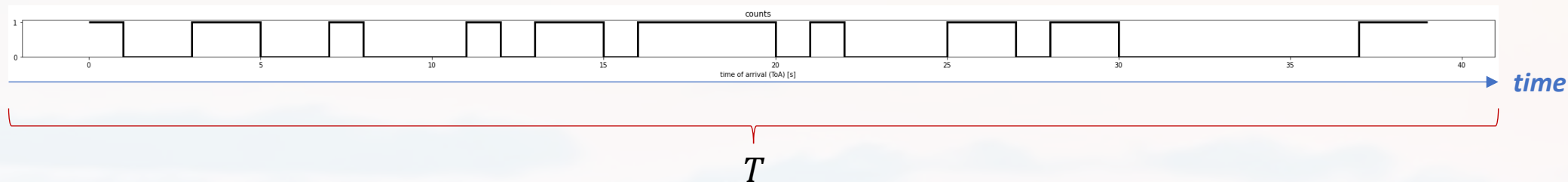
THOMAS J. LOREDO

Department of Astronomy, Space Sciences Building, Cornell University, Ithaca, NY 14853

Received 1992 January 6; accepted 1992 April 20

one application: pulsar timing (X-ray pulsars), Hambaryan et al., 20XX





	0
0	0
1	1
2	2
3	3
4	5
5	7
6	8
7	10
8	11

$T$

if we *had* the frequency  $\omega$

→ calculating the phase  $\varphi_i(t_i) = \omega t_i + \varphi_0$

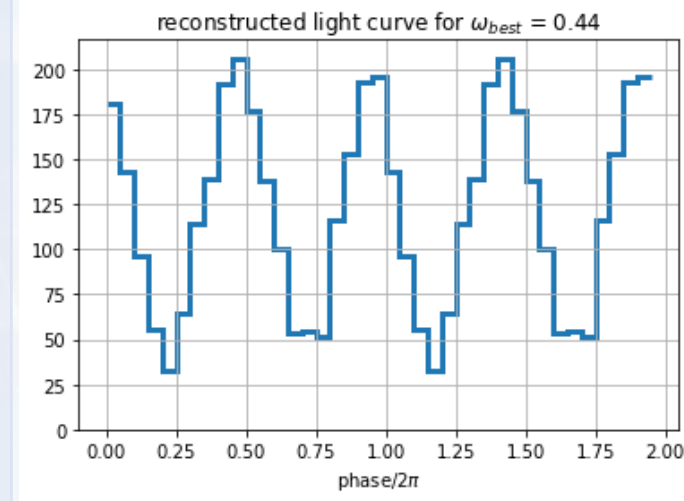
→  $\varphi(\text{norm})_i(t_i) = \varphi_i(t_i)/2\pi$

→ using  $\varphi(\text{norm})_i(t_i) = \varphi(\text{norm})_i(t_i) + 1$

→ histogram of all  $\varphi(\text{norm})_i(t_i)$

→ **phase binned light curve**

actual data:  $t_i$  **Time of Arrival (ToA)**

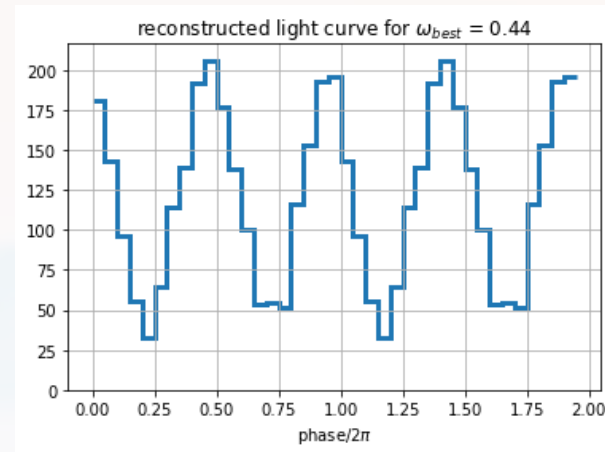






if we *had* the frequency  $\omega$

- calculating the phase  $\varphi_i(t_i) = \omega t_i + \varphi_0$
- $\varphi(\text{norm})_i(t_i) = \varphi_i(t_i)/2\pi$
- using  $\varphi(\text{norm})_i(t_i) = \varphi(\text{norm})_i(t_i) + 1$
- histogram of all  $\varphi(\text{norm})_i(t_i)$
- **phase binned light curve**



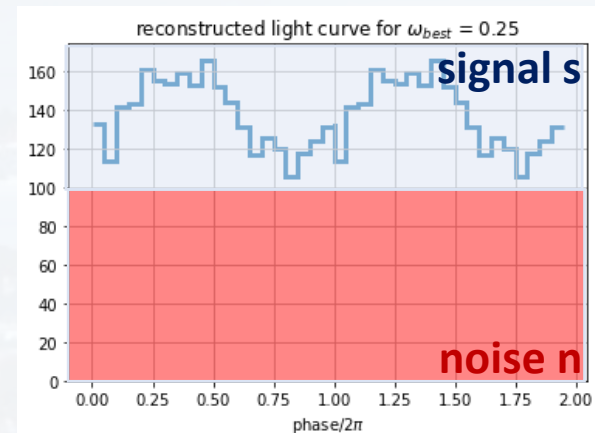
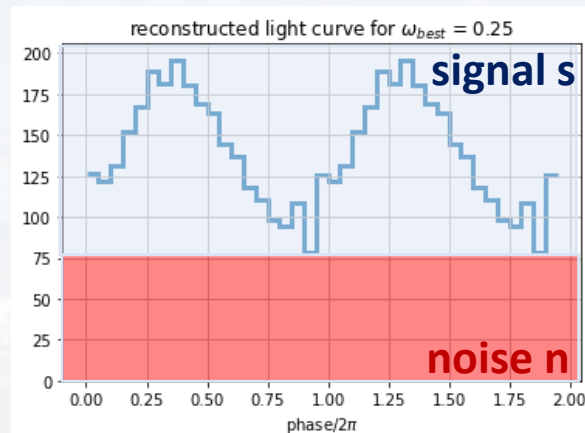
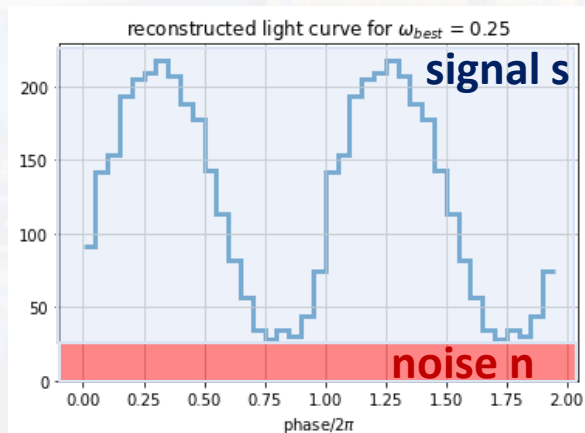
no signal:

- constant rate  $r(t_i) = \text{const}$
- events are evenly distributed over phase bins
- light curve is flat

signal:

- $r(t_i)$  is a function of time
- light curve of any shape

increasing noise level





$$P(M_i|D, I) = \frac{P(D|M_i, I)}{P(D|I)} P(M_i|I)$$

Bayes Theorem

$$\rho_{i,j} = \frac{P(M_i|D, I)}{P(M_j|D, I)} = \frac{P(D|M_i, I)}{P(D|M_j, I)} \frac{P(M_i|I)}{P(M_j|I)}$$

odds ratio

“Bayes Factor”

$D$	: data
$I$	: information
$M_i$	: model i
$M_j$	: model j
$r(t_i)$	: rate
$t_i$	: ToA

We want to compare the **constant rate model**  $M_1$  with  $r(t_i) = \text{const}$  to a set of **periodic models**

for  $N_{mod}$  (disjunct!) models, we can normalize and find:

$$\begin{aligned} \sum_{j=1}^{N_{mod}} P(M_j|D, I) &= 1 = \sum_{j=1}^{N_{mod}} \frac{P(D|M_j, I)}{P(D|M_i, I)} \frac{P(M_j|I)}{P(M_i|I)} P(M_i|D, I) \\ &= \frac{P(M_i|D, I)}{P(D|M_i, I) P(M_i|I)} \sum_{j=1}^{N_{mod}} P(D|M_j, I) P(M_j|I) \end{aligned}$$





$$P(M_i|D, I) = \frac{P(D|M_i, I)}{P(D|I)} P(M_i|I)$$

Bayes Theorem

$$\rho_{i,j} = \frac{P(M_i|D, I)}{P(M_j|D, I)} = \frac{P(D|M_i, I)}{P(D|M_j, I)} \frac{P(M_i|I)}{P(M_j|I)}$$

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for  $N_{mod}$  (disjunct!) models, we can normalize and find:

$$\sum_{j=1}^{N_{mod}} P(M_j|D, I) = 1 = \frac{P(M_i|D, I)}{P(D|M_i, I) P(M_i|I)} \sum_{j=1}^{N_{mod}} P(D|M_j, I) P(M_j|I)$$

$$P(M_i|D, I) = \frac{P(D|M_i, I) P(M_i|I)}{\sum_{j=1}^{N_{mod}} P(D|M_j, I) P(M_j|I)}$$

$$P(M_i|D, I) = \frac{\frac{P(D|M_i, I)}{P(D|M_1, I)} \frac{P(M_i|I)}{P(M_1|I)}}{\sum_{j=1}^{N_{mod}} \frac{P(D|M_j, I)}{P(D|M_1, I)} \frac{P(M_j|I)}{P(M_1|I)}}$$



$$P(M_i|D, I) = \frac{P(D|M_i, I)}{P(D|I)} P(M_i|I)$$

Bayes Theorem

$$\rho_{i,j} = \frac{P(M_i|D, I)}{P(M_j|D, I)} = \frac{P(D|M_i, I)}{P(D|M_j, I)} \frac{P(M_i|I)}{P(M_j|I)}$$

odds ratio

“Bayes Factor”

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We want to compare the **constant rate model**  $M_1$  with  $r(t_i) = \text{const}$  to a set of **periodic models**

for  $N_{mod}$  (disjunct!) models, we can normalize and find:

$$P(M_i|D, I) = \frac{\frac{P(D|M_i, I)}{P(D|M_1, I)} \frac{P(M_i|I)}{P(M_1|I)}}{\sum_{j=1}^{N_{mod}} \frac{P(D|M_j, I)}{P(D|M_1, I)} \frac{P(M_j|I)}{P(M_1|I)}} = \frac{\rho_{i,1}}{\sum_{j=1}^{N_{mod}} \rho_{j,1}} = \frac{\rho_{i,1}}{\sum_{j=2}^{N_{mod}} \rho_{j,1} + 1}$$



$$P(M_i|D, I) = \frac{P(D|M_i, I)}{P(D|I)} P(M_i|I)$$

Bayes Theorem

$$\rho_{i,j} = \frac{P(M_i|D, I)}{P(M_j|D, I)} = \frac{P(D|M_i, I)}{P(D|M_j, I)} \frac{P(M_i|I)}{P(M_j|I)}$$

odds ratio

“Bayes Factor”

$D$	: data
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$r(t_i)$	: rate
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We want to compare the **constant rate model**  $M_1$  with  $r(t_i) = \text{const}$  to a set of **periodic models**

for  $N_{mod}$  (disjunct!) models, we can normalize and find:

$$P(M_i|D, I) = \frac{\rho_{i,1}}{\sum_{j=2}^{N_{mod}} \rho_{j,1} + 1}$$





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$$\rho_{i,j} = \frac{P(M_i|D,I)}{P(M_j|D,I)} = \frac{P(D|M_i,I)}{P(D|M_j,I)} \frac{P(M_i|I)}{P(M_j|I)}$$

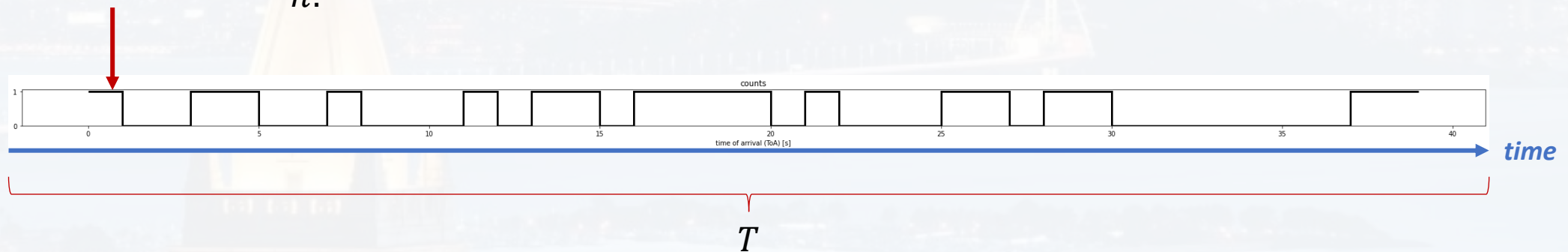
marginalization

$$P(D|M_i) = \int P(D|\{\alpha\}_i, M_i) P(\{\alpha\}_i | M_i) d\Omega_{\{\alpha\}_i}$$

$D$	: data
$I$	: information
$M_i$	: model i
$M_j$	: model j
$r(t)$	: rate
$t$	: ToA
$\{\alpha\}_i$	: set of parameters of model $M_i$
$\Delta t$	: time resolution
$n$	: number of events
$T$	: obs. time span

$$P(n|r(t)) = \frac{(r(t) \cdot \Delta t)^n}{n!} e^{-r(t) \cdot \Delta t}$$

**Poisson distribution**



actual data (ToA)

$N$	intervals with $n = 1$
$Q$	intervals with $n = 0$

$$(N + Q)\Delta t = T$$



$$\rho_{i,j} = \frac{P(M_i|D,I)}{P(M_j|D,I)} = \frac{P(D|M_i,I)}{P(D|M_j,I)} \frac{P(M_i|I)}{P(M_j|I)}$$

$$P(n|r(t)) = \frac{(r(t) \cdot \Delta t)^n}{n!} e^{-r(t) \cdot \Delta t} \quad \text{Poisson distribution}$$

$N$  intervals with  $n = 1$   
 $Q$  intervals with  $n = 0$

$$(N + Q)\Delta t = T$$

$D$	: data
$I$	: information
$M_i$	: model i
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**joint likelihood**

$$P(D| r(t), t) = \prod_{i=1}^N r(t_i) \cdot \Delta t e^{-r(t_i) \cdot \Delta t} \cdot \prod_{i=1}^Q e^{-r(t_i) \cdot \Delta t}$$

$$P(D|M_i) = \int P(D|\{\alpha\}_i M_i) P(\{\alpha\}_i | M_i) d\Omega_{\{\alpha\}_i}$$

$$P(D| r(t), t) = (\Delta t)^N \cdot \prod_{i=1}^N r(t_i) \cdot \exp \left[ - \sum_{i=1}^{Q+N} r(t_i) \cdot \Delta t \right]$$





$$P(n|r(t)) = \frac{(r(t) \cdot \Delta t)^n}{n!} e^{-r(t) \cdot \Delta t}$$

**Poisson** distribution

$N$  intervals with  $n = 1$   
 $Q$  intervals with  $n = 0$

$$(N + Q)\Delta t = T$$

$$P(D| r(t), t) = (\Delta t)^N \cdot \prod_{i=1}^N r(t_i) \cdot \exp \left[ - \underbrace{\sum_{i=1}^{Q+N} r(t_i) \cdot \Delta t}_{= \int_0^T r(t) dt} \right]$$

$$P(D| r(t), t) = (\Delta t)^N \cdot \prod_{i=1}^N r(t_i) \cdot \exp \left( - \int_0^T r(t) dt \right)$$

$D$	: data
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$M_i$	: model i
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$r(t)$	: rate
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$n$	: number of events
$T$	: obs. time span

**any periodic model**



any periodic model

$$P(D | r(t), t) = (\Delta t)^N \cdot \prod_{i=1}^N r(t_i) \cdot \exp\left(-\int_0^T r(t) dt\right)$$

if we had the frequency already:

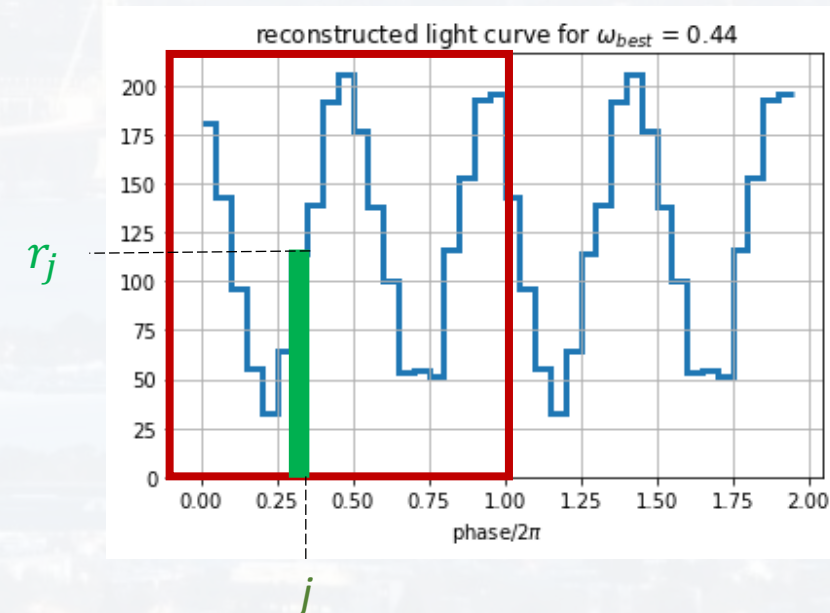
$m$  phase bins:

$r_j$  rate in each phase bin  $j$

$A = \frac{1}{m} \sum_{j=1}^m r_j$  average rate

$f_j = \frac{r_j}{\sum_{j=1}^m r_j} = \frac{r_j}{A m}$  fraction of total rate in each phase bin  $j$

$r(t)$	: rate
$\Delta t$	: time resolution
$n$	: number of events
$T$	: obs. time span
$D$	: data set
$N$	: number of intervals with $n=1$
$r_j$	: rate in each phase bin $j$
$A$	: average rate
$m$	: number of phase bins
$f_j$	: fraction of total rate in $j$



Each light curve of any shape is being fully described by  $f_j$



## any periodic model

$$P(D | r(t), t) = (\Delta t)^N \cdot \prod_{i=1}^N r(t_i) \cdot \exp\left(-\int_0^T r(t) dt\right)$$

$m$  phase bins:

$r_j$  rate in each phase bin  $j$

$A = \frac{1}{m} \sum_{j=1}^m r_j$  average rate

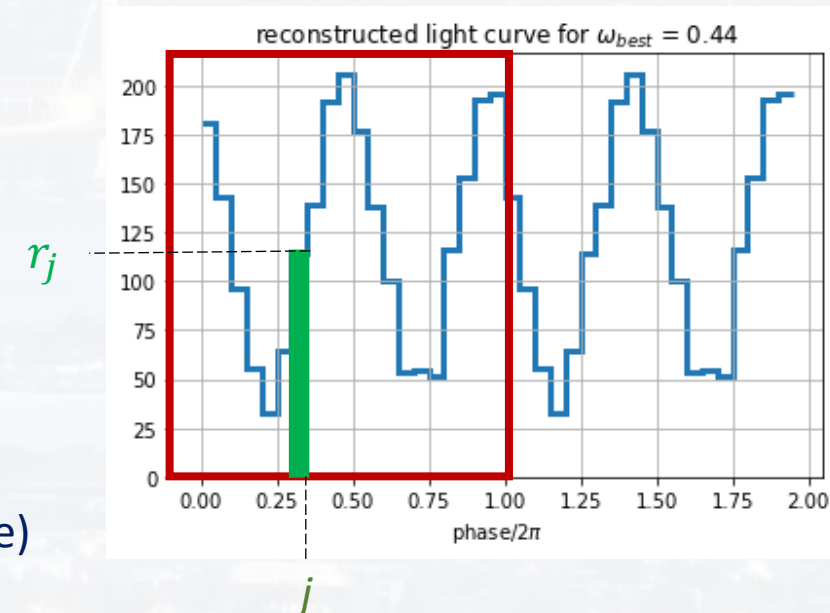
$f_j = \frac{r_j}{\sum_{j=1}^m r_j} = \frac{r_j}{A m}$  fraction of total rate in each phase bin  $j$

constant model:  $r_j = \text{const } \forall j$

$\rightarrow r_j = A$

$P(D | r(t), t) = (\Delta t)^N \cdot A^N \cdot e^{-AT}$  **constant model (poiss noise)**

$r(t)$	: rate
$\Delta t$	: time resolution
$n$	: number of events
$T$	: obs. time span
$D$	: data set
$N$	: number of intervals with $n=1$
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$m$	: number of phase bins
$f_j$	: fraction of total rate in $j$







## any periodic model

$$P(D | r(t), t) = (\Delta t)^N \cdot \prod_{i=1}^N r(t_i) \cdot \exp\left(-\int_0^T r(t) dt\right)$$

$$P(D | r(t), t) = (\Delta t)^N \cdot A^N \cdot e^{-AT} \quad \text{constant model (poiss noise)}$$

## actual signal

- amplitude
- phase
- frequency
- offset

*detection*

-  $t_i$

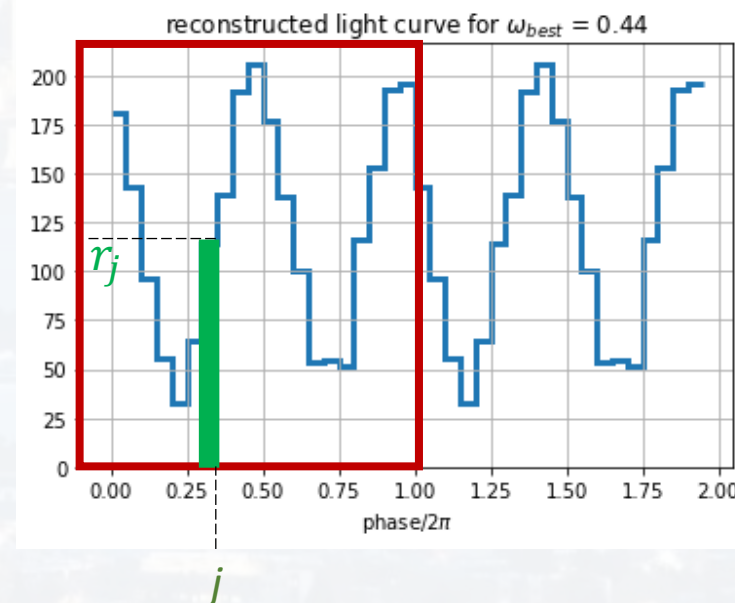
*analysis*

- phase
- frequency
- $f_j(A, m)$

## note:

- we assume a poissonian process
- sampling rate  $\gg r(t)$  throughout the observation
- $r(t)$  does not change within  $\Delta t$
- $T$  is large enough to capture at least  $\approx 10$  periods of the signal
- $m$  is large: we capture finer details of light curve, but increase fluctuations within the bins and vice verse

$r(t)$	: rate
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$r_j$	: rate in each phase bin $j$
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$m$	: number of phase bins
$f_j$	: fraction of total rate in $j$





rewriting the likelihood function in terms of the **light curve parameter**:

$$P(D|A, M_1) = (\Delta t)^N \cdot A^N \cdot e^{-AT} \quad \text{constant model (poiss noise)}$$

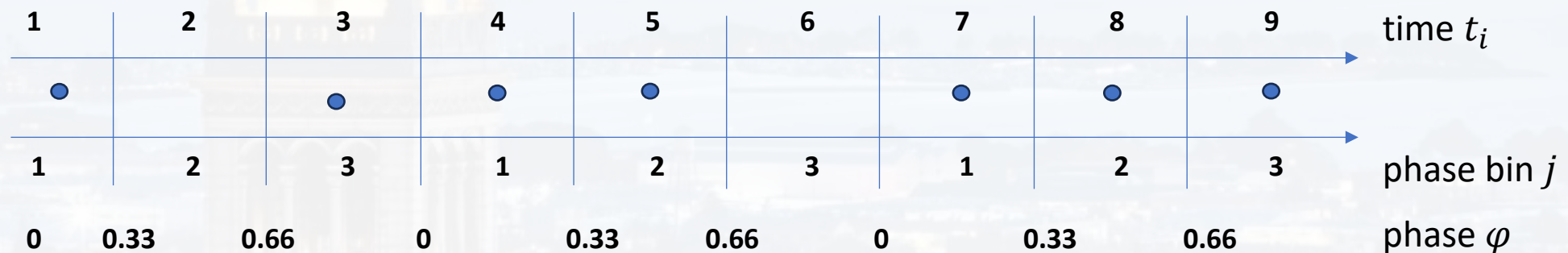
**any periodic model:**

$$P(D| r(t), t) = (\Delta t)^N \cdot \prod_{i=1}^N r(t_i) \cdot \exp\left(-\int_0^T r(t) dt\right)$$

$$P(D|\omega, \varphi, A, f, M_m) = ?$$

$$f_j = \frac{r_j}{\sum_{j=1}^m r_j} = \frac{r_j}{A m}$$

$r(t)$	: rate
$\Delta t$	: time resolution
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$m$	: number of phase bins
$f_j$	: fraction of total rate in $j$
$f$	: full set of $f_j$
$M_1$	: constant model
$M_m$	: periodic model $m$ bins
$\omega$	: frequency
$\varphi$	: phase

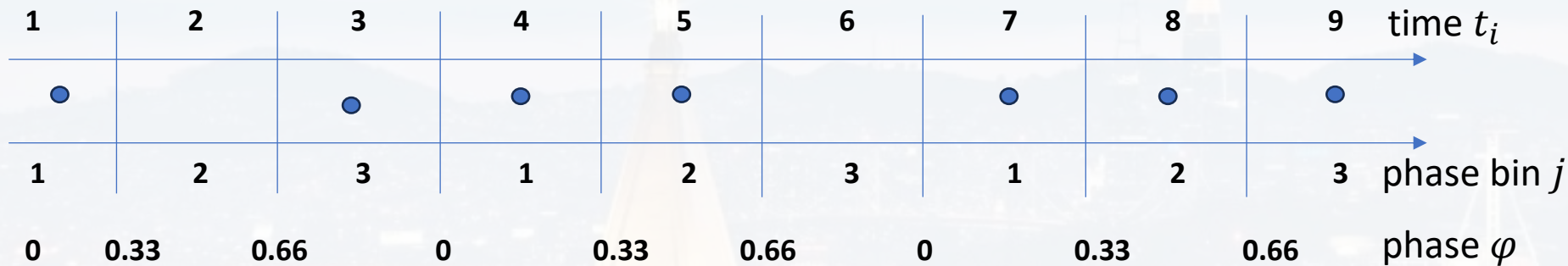




rewriting the likelihood function in terms of the **light curve parameter**:

$$P(D | r(t), t) = (\Delta t)^N \cdot \prod_{i=1}^N r(t_i) \cdot \exp\left(-\int_0^T r(t) dt\right)$$

$$P(D | \omega, \varphi, A, f, M_m) = ?$$



$$f_1 = 3/7$$

$$f_2 = 2/7$$

$$f_3 = 2/7$$

for  $T \gg 1/\omega$  these will be average values

$n_j = n_j(\omega, \varphi)$ : number of events that fall into phase bin  $j$  (here  $n_1 = 3, n_2 = 3$  etc)

$$\prod_{i=1}^N r(t_i) = \prod_{i=1}^N A m f_{j(t_i)} = (A m)^N f_1 f_3 f_1 f_2 f_1 f_2 f_3 = (A m)^N \prod_{j=1}^m f_j^{n_j}$$

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$\Delta t$	: time resolution
$T$	: obs. time span
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$M_m$	: periodic model $m$ bins
$\omega$	: frequency
$\varphi$	: phase

$$f_j = \frac{r_j}{\sum_{j=1}^m r_j} = \frac{r_j}{A m}$$





rewriting the likelihood function in terms of the **light curve parameter**:

$$P(D | r(t), t) = (\Delta t)^N \cdot (A m)^N \left( \prod_{j=1}^m f_j^{n_j} \right) \exp \left( - \int_0^T r(t) dt \right)$$

$$P(D | \omega, \varphi, A, f, M_m) = ?$$

$n_j = n_j(\omega, \varphi)$ : number of events that fall into phase bin  $j$

$$r_j = \text{events/time} = n_j / \tau_j$$

$$\tau_j: \text{total time integrated per phase bin } j \quad \tau_j \approx \frac{T}{m}$$

$$\int_0^T r(t) dt = \sum_{i=1}^{Q+N} r(t_i) \cdot \Delta t = \sum_{i=1}^{Q+N} A m f_{j(t_i)} \Delta t$$

$$= A m \sum_{i=1}^{Q+N} (f_1 + f_3 + f_1 + f_2 + f_1 + f_2 + f_3) \Delta t = A m \sum_{j=1}^m f_j n_j \Delta t = A m \sum_{j=1}^m f_j \tau_j$$

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$M_1$	: constant model
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$\omega$	: frequency
$\varphi$	: phase

$$f_j = \frac{r_j}{\sum_{j=1}^m r_j} = \frac{r_j}{A m}$$



rewriting the likelihood function in terms of the **light curve parameter**:

**any periodic model:**

$$P(D|\omega, \varphi, A, f, M_m) = (\Delta t)^N \cdot (A m)^N \left( \prod_{j=1}^m f_j^{n_j(\omega, \varphi)} \right) \exp \left( -A m \sum_{j=1}^m f_j \tau_j(\omega, \varphi) \right)$$

**constant model (poiss noise)**

$$P(D|A, M_1) = (\Delta t)^N \cdot A^N \cdot e^{-AT}$$

$r(t)$	: rate
$\Delta t$	: time resolution
$T$	: obs. time span
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$N$	: number of intervals with $n=1$
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$f_j$	: fraction of total rate in $j$
$f$	: full set of $f_j$
$M_1$	: constant model
$M_m$	: periodic model $m$ bins
$\omega$	: frequency
$\varphi$	: phase

**note:** - for  $T \gg 1/\omega$  and  $\tau_j \approx \frac{T}{m}$  on can show that

$$P(D|\omega, \varphi, A, f, M_m) \approx (\Delta t)^N \cdot (A m)^N \left( \prod_{j=1}^m f_j^{n_j(\omega, \varphi)} \right) \exp(-A T)$$

see paper page 152/153



## Outline

Discrete Signals

The Model

**The Priors**

Occam Factors

A Code Example





any periodic  
model:

$$P(D|\omega, \varphi, A, f, M_m) = (\Delta t)^N \cdot (A m)^N \left( \prod_{j=1}^m f_j^{n_j(\omega, \varphi)} \right) \exp \left( -A m \sum_{j=1}^m f_j \tau_j(\omega, \varphi) \right)$$

constant model (poiss noise)

$$P(D|A, M_1) = (\Delta t)^N \cdot A^N \cdot e^{-AT}$$

$$P(D|M_i) = \int P(D|\{\alpha\}_i M_i) P(\{\alpha\}_i | M_i) d\Omega_{\{\alpha\}_i}$$

$$P(\omega, \varphi, A, f|M_i) = P(\omega|M_i)P(\varphi|M_i)P(A|M_i)P(f|M_i)$$

max entropy:  $P(\omega|M_i) = \frac{1}{\omega \ln(\omega_{max}/\omega_{min})}$        $\omega_{max} = \frac{2\pi N}{T}$        $\omega_{min} = \frac{2\pi}{T}$

$$P(\varphi|M_i) = \frac{1}{2\pi}$$

$$P(A|M_i) = \frac{1}{A_{max}}$$

$$P(f|M_i) = (m-1)! \delta \left( 1 - \sum_{j=1}^m f_j \right)$$

$r(t)$	: rate
$\Delta t$	: time resolution
$T$	: obs. time span
$D$	: data set
$N$	: number of intervals with $n=1$
$r_j$	: rate in each phase bin $j$
$A$	: average rate
$m$	: number of phase bins
$f_j$	: fraction of total rate in $j$
$f$	: full set of $f_j$
$M_1$	: constant model
$M_m$	: periodic model $m$ bins
$\omega$	: frequency
$\varphi$	: phase

in practice:  $\omega_{min} = 10 \frac{2\pi}{T}$

delta function  $\delta$ ,  
see also page 155



a few notes on pulsar timing:

- rotating dipole  $\rightarrow$  spin down  $\dot{\omega} \approx -const \omega^n$



$$\varphi(t) = \omega (t - t_0) + \frac{1}{2} \dot{\omega} (t - t_0)^2$$

fitting  $\dot{\omega}$  such that phases are consistent: phase coherent timing



## Outline

Discrete Signals

The Model

The Priors

**Occam Factors**

A Code Example





$$P(D|M_i) = \int P(D|\{\alpha\}_i M_i) P(\{\alpha\}_i | M_i) d\Omega_{\{\alpha\}_i}$$

for  $T \gg 1/\omega$  and  $\tau_j \approx \frac{T}{m}$ :

$$P(D|M_m) \approx \frac{1}{A_{max}} \frac{1}{2\pi} \frac{(m-1)!}{\ln\left(\frac{\omega_{max}}{\omega_{min}}\right)} \int \frac{1}{\omega} (\Delta t)^N \cdot (A m)^N \left( \prod_{j=1}^m f_j^{n_j(\omega, \varphi)} \delta\left(1 - \sum_{j=1}^m f_j\right) \right) e^{-AT} d\omega df dA d\varphi$$

we need (estimation of frequency):

$$P(\omega|D, M_m) = P(\omega|M_m) \frac{P(D|\omega, M_m)}{P(D|M_m)} = \frac{1}{\omega \ln(\omega_{max}/\omega_{min})} \frac{P(D|\omega, M_m)}{P(D|M_m)}$$

$$P(D|\omega, M_m) = \frac{\Delta t^N (m-1)!}{2\pi A_{max}} \int (A m)^N \left( \prod_{j=1}^m f_j^{n_j(\omega, \varphi)} \delta\left(1 - \sum_{j=1}^m f_j\right) \right) e^{-AT} df dA d\varphi$$

note: we can also marginalize over  $m$ , if we are **not interested in the shape** of the light curve



$$P(D|\omega, M_m) = \frac{\Delta t^N (m-1)!}{2\pi A_{max}} \int (A m)^N \left( \prod_{j=1}^m f_j^{n_j(\omega, \varphi)} \delta \left( 1 - \sum_{j=1}^m f_j \right) \right) e^{-AT} df dA d\varphi$$

$$P(D|\omega, M_m) = \frac{\Delta t^N (m-1)!}{2\pi A_{max}} \underbrace{\int (A m)^N e^{-AT} dA}_{\Gamma(a, b) = \int_a^b y^{a-1} e^{-y} dy} \int \prod_{j=1}^m f_j^{n_j(\omega, \varphi)} \delta \left( 1 - \sum_{j=1}^m f_j \right) df d\varphi$$

$$\Gamma(a, b) = \int_a^b y^{a-1} e^{-y} dy$$

$$\int (A m)^N e^{-AT} dA = \Gamma(N+1, A_{max} T) \frac{1}{T^{N+1}}$$

$$\int \prod_{j=1}^m f_j^{n_j(\omega, \varphi)} \delta \left( 1 - \sum_{j=1}^m f_j \right) df d\varphi \quad \text{generalized } \beta \text{ integral with constrain } \sum_{j=1}^m f_j = 1$$

$$\int \prod_{j=1}^m f_j^{n_j(\omega, \varphi)} \delta \left( 1 - \sum_{j=1}^m f_j \right) df d\varphi = \frac{m^N}{(N+m-1)!} \int_0^{2\pi} \prod_{j=1}^m n_j(\omega, \varphi)! d\varphi$$



$$P(D|\omega, M_m) = \frac{\Delta t^N (m-1)!}{2\pi A_{max}} \Gamma(N+1, A_{max} T) \frac{1}{T^{N+1}} \frac{m^N}{(N+m-1)!} \int_0^{2\pi} \prod_{j=1}^m n_j(\omega, \varphi)! d\varphi$$

$$P(D|\omega, M_m) = \frac{\Delta t^N (m-1)!}{2\pi A_{max}} \Gamma(N+1, A_{max} T) \frac{1}{T^{N+1}} \frac{m^N}{(N+m-1)!} N! \int_0^{2\pi} \frac{1}{N!} \prod_{j=1}^m n_j(\omega, \varphi)! d\varphi$$

$$\text{multiplicity: } \Omega_m(\omega, \varphi) = \frac{N!}{\prod_{j=1}^m n_j(\omega, \varphi)!}$$

$$P(D|\omega, M_m) = \frac{\Delta t^N (m-1)!}{2\pi A_{max}} \Gamma(N+1, A_{max} T) \frac{1}{T^{N+1}} \frac{m^N}{(N+m-1)!} N! \int_0^{2\pi} \frac{1}{\Omega_m(\omega, \varphi)} d\varphi$$

**“Occam Factors”** penalize complex models  
(large values for number of bins  $m$ )





$$P(D|\omega, M_m) = \frac{\Delta t^N (m-1)!}{2\pi A_{max}} \Gamma(N+1, A_{max} T) \frac{1}{T^{N+1}} \frac{m^N}{(N+m-1)!} \int_0^{2\pi} \prod_{j=1}^m n_j(\omega, \varphi)! d\varphi$$

we still need  $P(D|M_m)$ , but it is the same steps as for  $P(D|\omega, M_m)$  just with a  $\int \frac{d\omega}{\omega}$  term

finally:

$$\text{using: } \Omega_m(\omega, \varphi) = \frac{N!}{\prod_{j=1}^m n_j(\omega, \varphi)!} \quad P(\omega|D, M_m) = \frac{1}{\omega} \frac{1}{\int \frac{d\omega}{\omega} \frac{d\varphi}{\Omega_m(\omega, \varphi)}} \int \frac{d\varphi}{\Omega_m(\omega, \varphi)}$$

- note:**
- the multiplicity  $\Omega_m$  of the binned ToA contains all the information about  $\omega$  (and  $\varphi$ )
  - neither  $A_{max}$  nor  $\ln\left(\frac{\omega_{max}}{\omega_{min}}\right)$  appear in the above equation  $\rightarrow$  rel. insensitive to these priors
  - more complex models (large  $m$ ) are penalized (more detail: paper, Sect. 5.2)



## Outline

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**A Code Example**



python package BayesianSignalDetect.py

```
from BayesianSignalDetect import *
```

a *decorator* that measures runtime of a function

*function* creates a test dataset for illustration

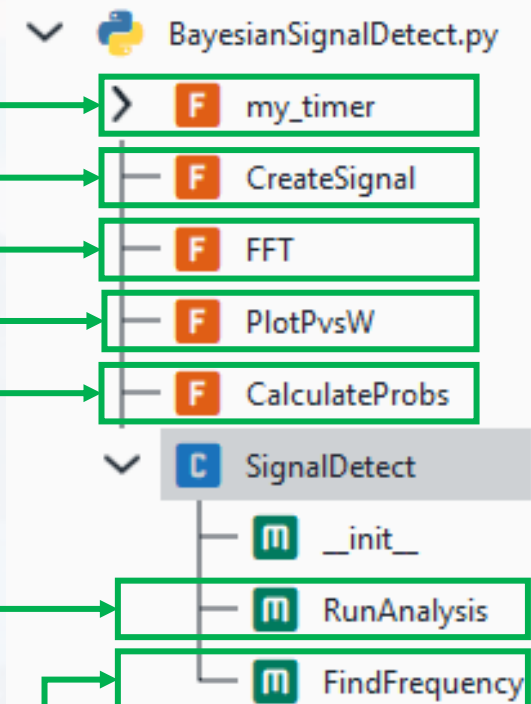
FFT for comparison

plot routine for *periodogram*

calculates  $P(D|\omega, \varphi, A, m)$

calls *CalculateProbs* and calculates  $P(D|M_i)$

calls main part of the code – runs *RunAnalysis* on multiple cpus in parallel





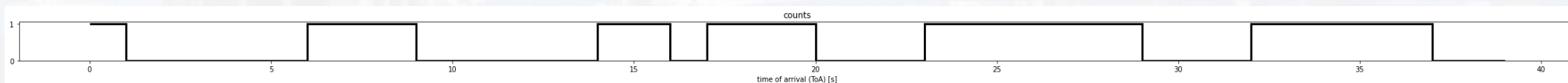
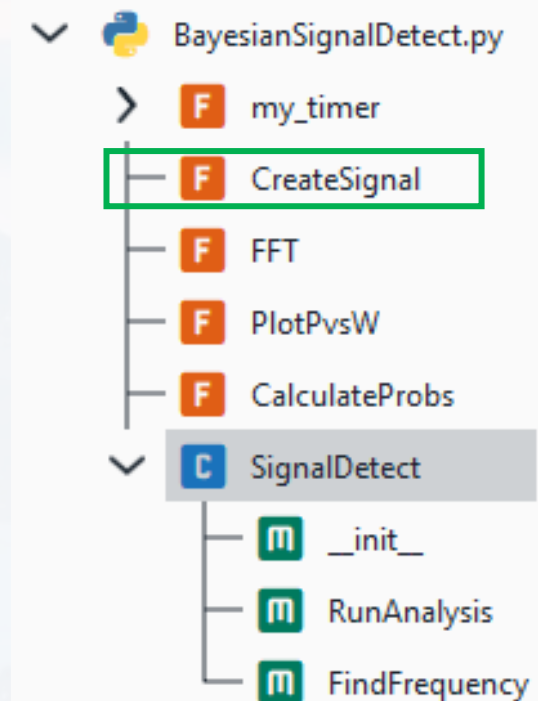
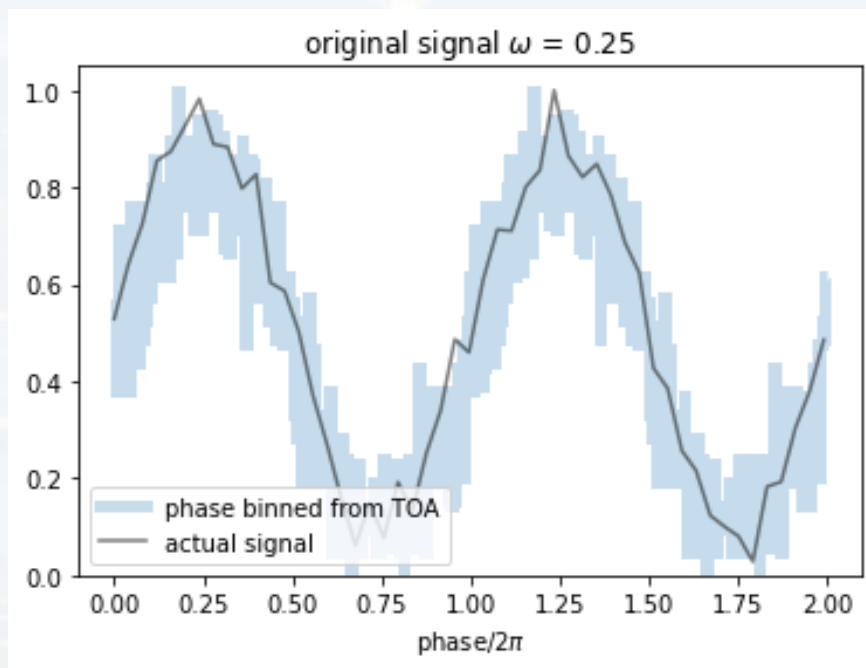


python package BayesianSignalDetect.py

```
from BayesianSignalDetect import *
```

```
T = CreateSignal(5000, 0.25, 0.1)
```

$N + Q$



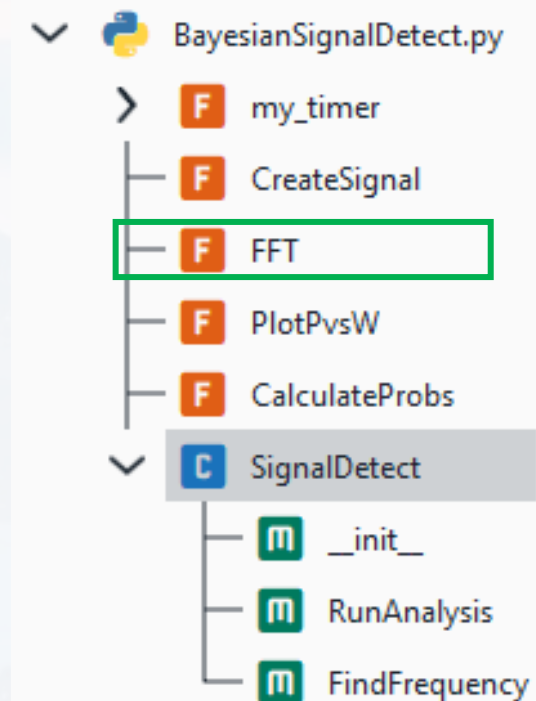
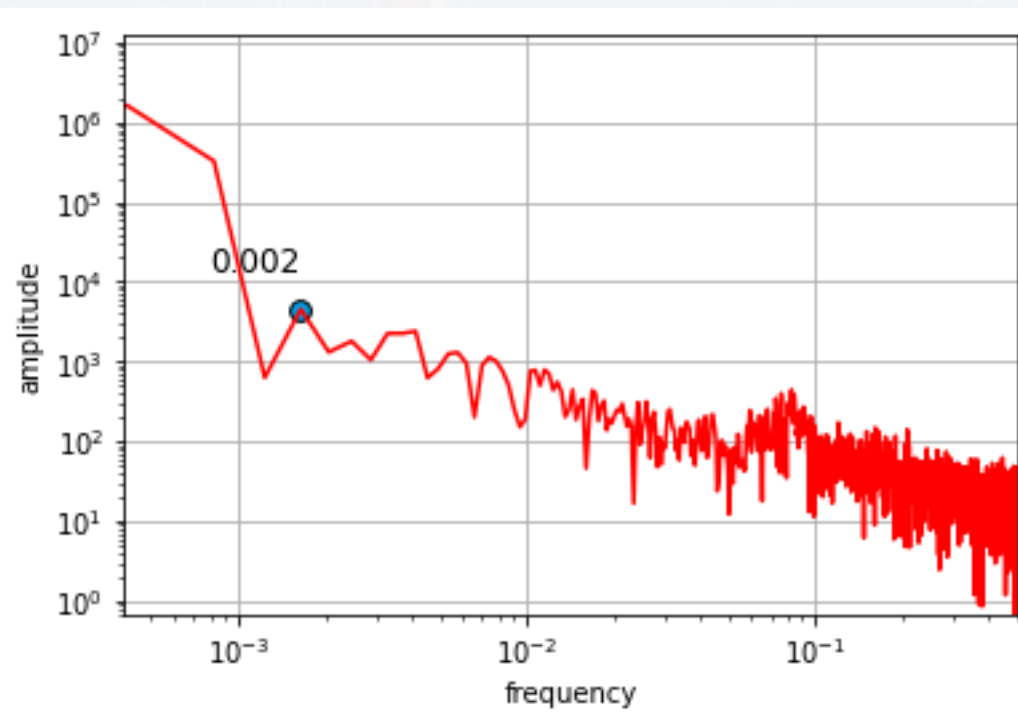


python package BayesianSignalDetect.py

```
from BayesianSignalDetect import *
```

```
T = CreateSignal(5000, 0.25, 0.1)
```

```
FFT(T)
```



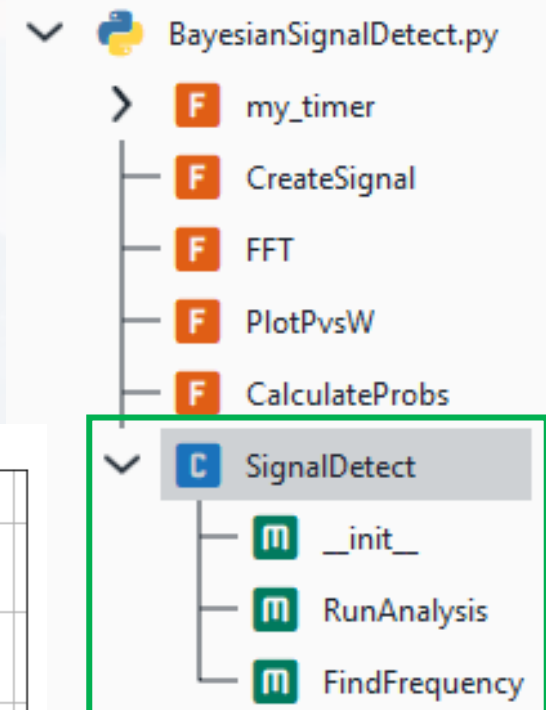
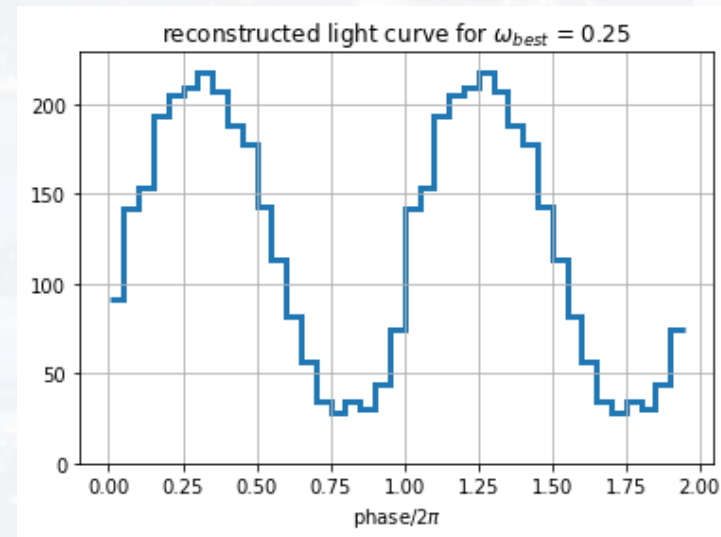
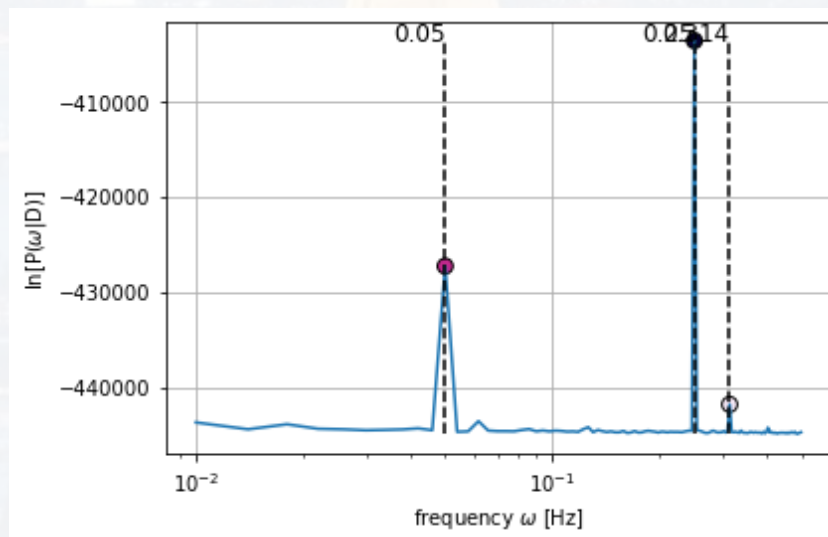


python package BayesianSignalDetect.py

```
from BayesianSignalDetect import *
```

```
T = CreateSignal(5000, 0.25, 0.1)
```

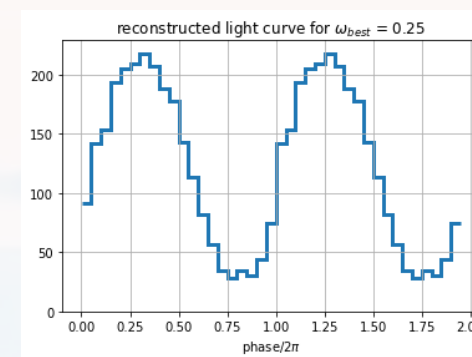
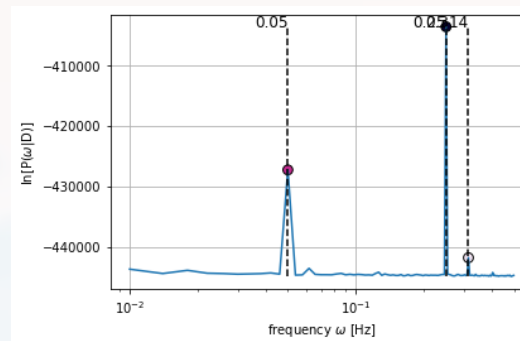
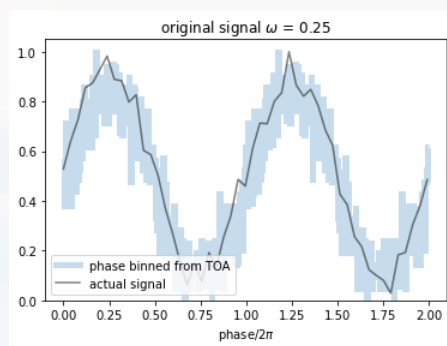
```
S = SignalDetect(T, w_end = 0.5, w_start = 0.01)  
[Omega, P] = S.FindFrequency()
```



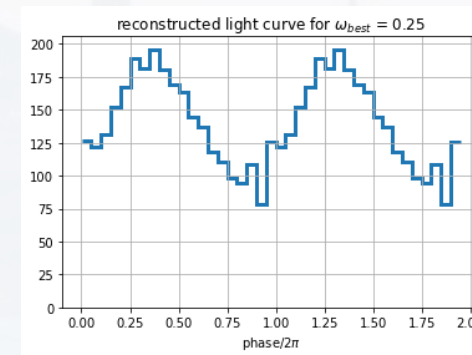
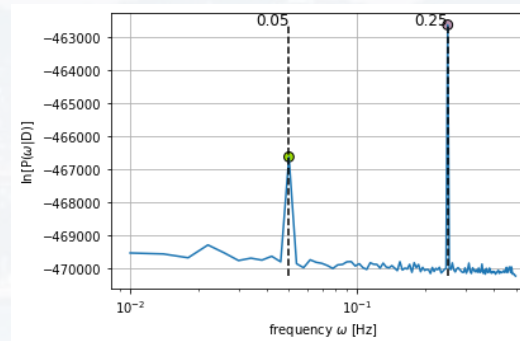
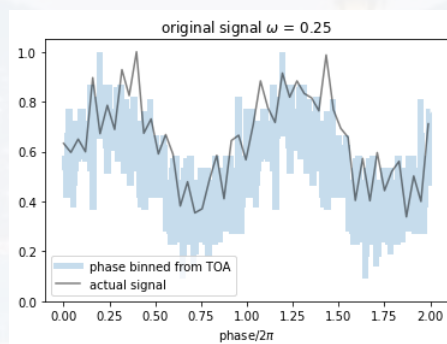




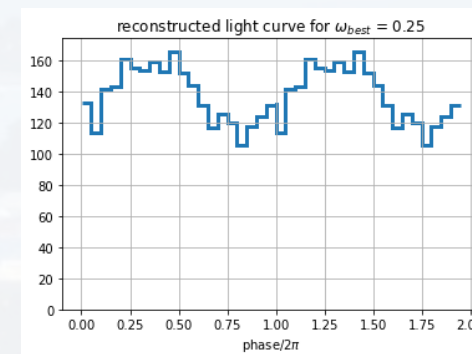
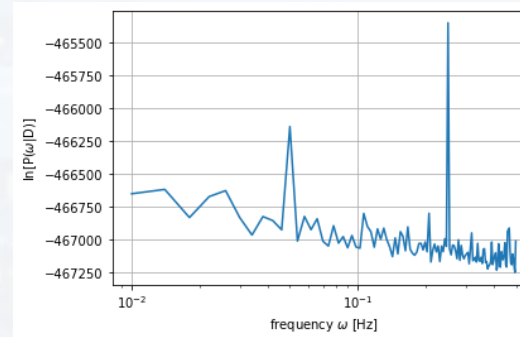
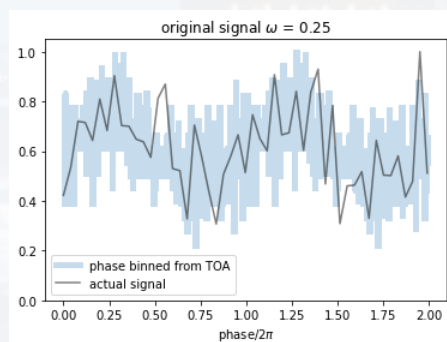
`T = CreateSignal(5000, 0.25, 0.1)`



`T = CreateSignal(5000, 0.25, 0.5)`



`T = CreateSignal(5000, 0.25, 1)`



Thank you very much for your attention!

