Lecture 08:

Multi-Layer Perceptron (MLP)

Markus Hohle

University California, Berkeley

Machine Learning Algorithms

MSSE 277B, 3 Units

Berkeley UNIVERSITY OF CALIFORNIA

Course Map

Lecture 1: Course Overview and Introduction to Machine Learning

classic ML tools & algorithms

Lecture 2: Bayesian Methods in Machine Learning

Lecture 3: Dimensionality Reduction: Principal Component Analysis

Lecture 4: Linear and Non-linear Regression and Classification

Lecture 5: Unsupervised Learning: K-Means, GMM, Trees

Lecture 6: Adaptive Learning and Gradient Descent Optimization Algorithms

ANNs/AI/Deep Learning

Lecture 7: Introduction to Artificial Neural Networks - The Perceptron

Lecture 8: Introduction to Artificial Neural Networks - Building Multiple Dense Layers

Lecture 9: Convolutional Neural Networks (CNNs) - Part I

Lecture 10: CNNs - Part II

Lecture 11: Recurrent Neural Networks (RNNs) and Long Short-Term Memory (LSTMs)

Lecture 12: Combining LSTMs and CNNs

Lecture 13: Running Models on GPUs and Parallel Processing

Lecture 14: Project Presentations

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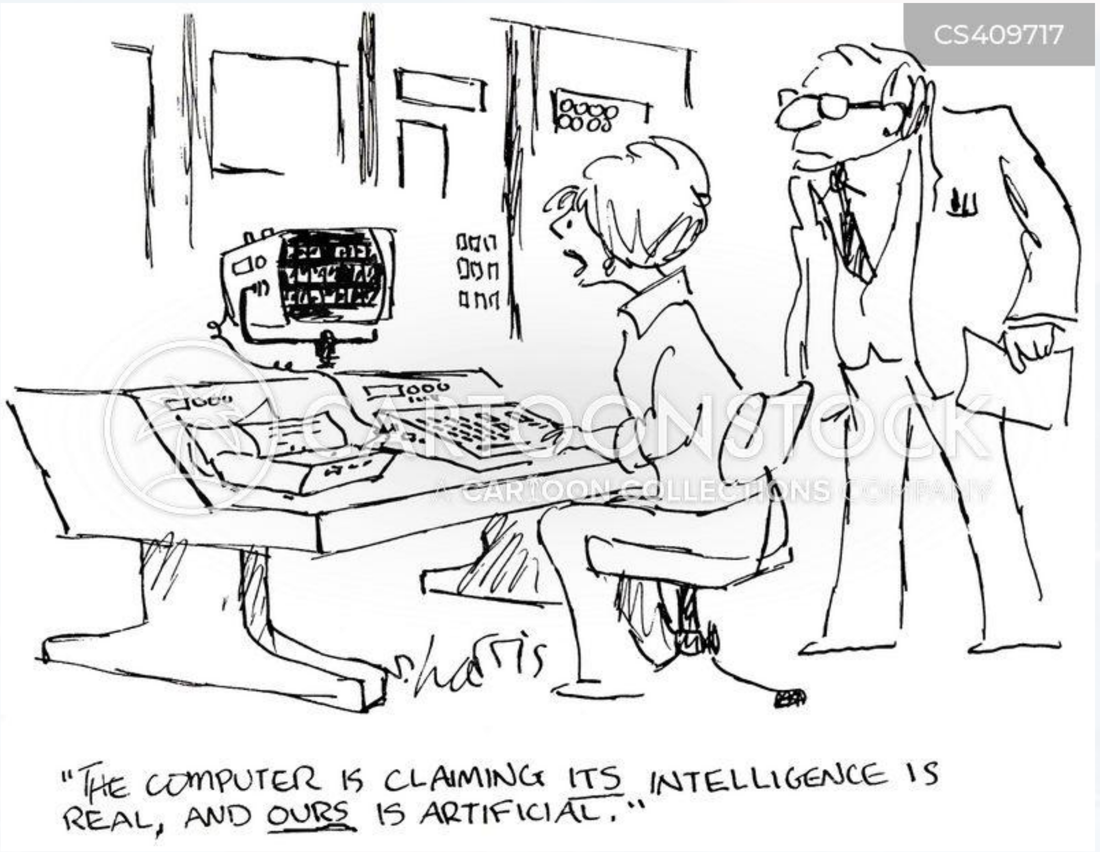
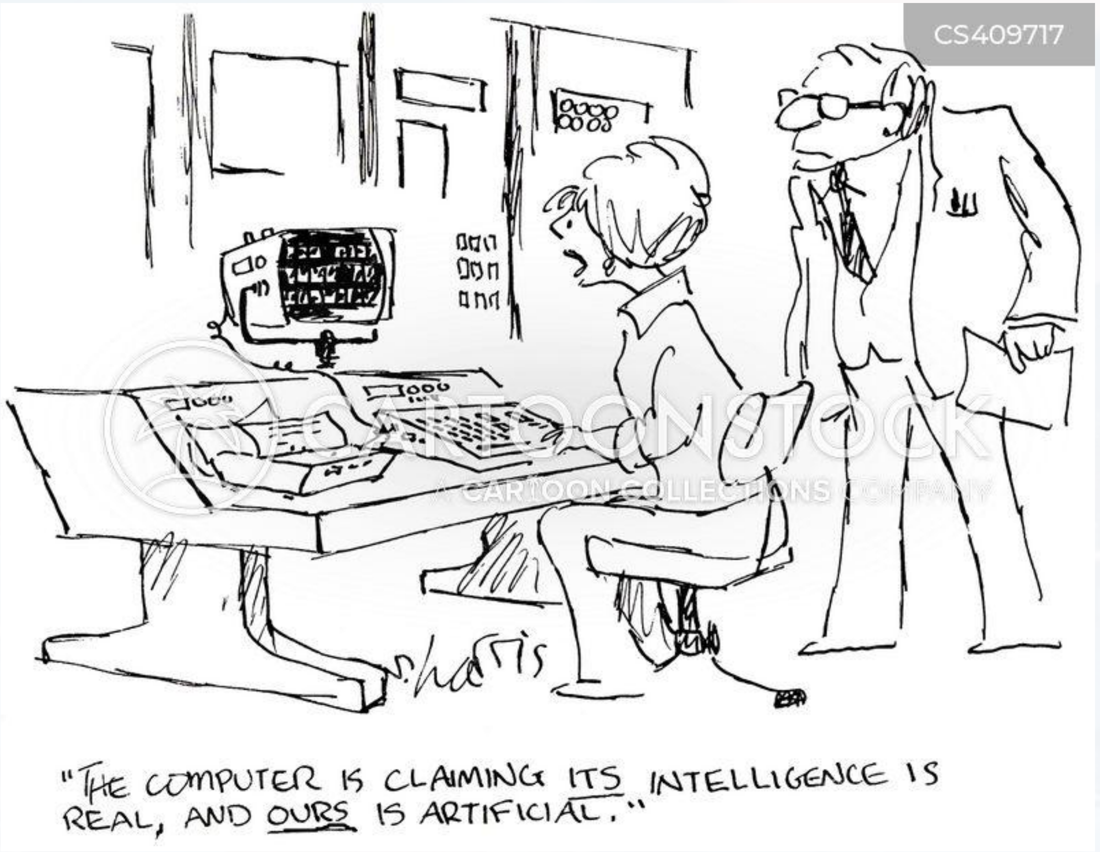


Outline

- **Motivation & Overview**
- **The Minimal Setup**
- **Building a fully functional ANN**



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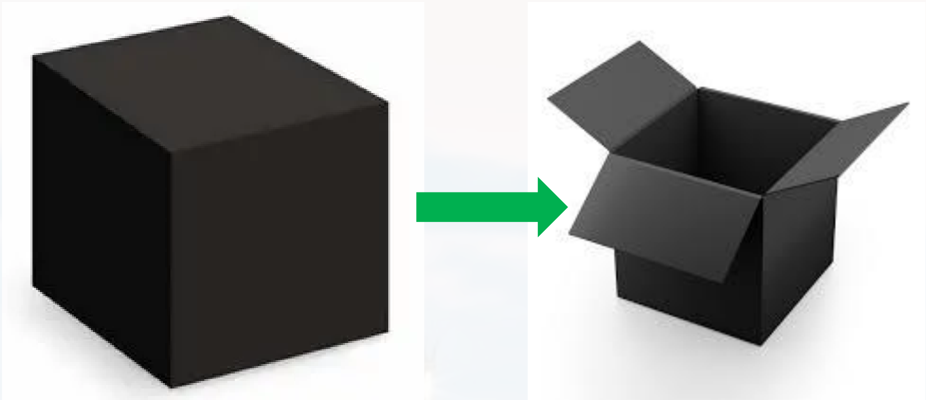
MLP - Intro: Motivation & Overview



ANNs are **not** black boxes!

Goal 1: Understanding what "is in the box"

→ important for knowing **which** type of ANN can solve your problem!

```
model = Sequential()

model.add(Embedding(n_words, embedding_vecor_length,\
                    input_length = max_review_length))

model.add(Dropout(DA_rate))
model.add(LSTM(n_neurons))
model.add(Dropout(DA_rate))
model.add(Dense(2, activation = 'softmax'))

opt = optimizers.Adam()
model.compile(loss = 'categorical_crossentropy', optimizer = opt,\
              metrics = ['accuracy'])

model.summary()
```

Goal 2: Understanding Python syntax

→ important for **building the specific** ANN that can solve your problem!

MLP - Intro:

Goal 1: Understanding what "is in the box"

→ important for knowing **which** type of ANN can solve your problem!

Goal 2: Understanding Python syntax

→ important for **building the specific** ANN that can solve your problem!

→ important for **solving error messages** (requires understanding of processes within the ANN)!

I have tried:

stackoverflow

```
training_features = numpy.reshape(
      training_features,
      (training_features.shape[0], 1, training_features.shape[1]))
```

But I get:

```
ValueError: Input 0 is incompatible with layer lstm_1: expected ndim=3, found ndim=4
```

MLP - Intro: Motivation & Overview

Goal 1: Understanding what "is in the box"

→ important for knowing **which** type of ANN can solve your problem!

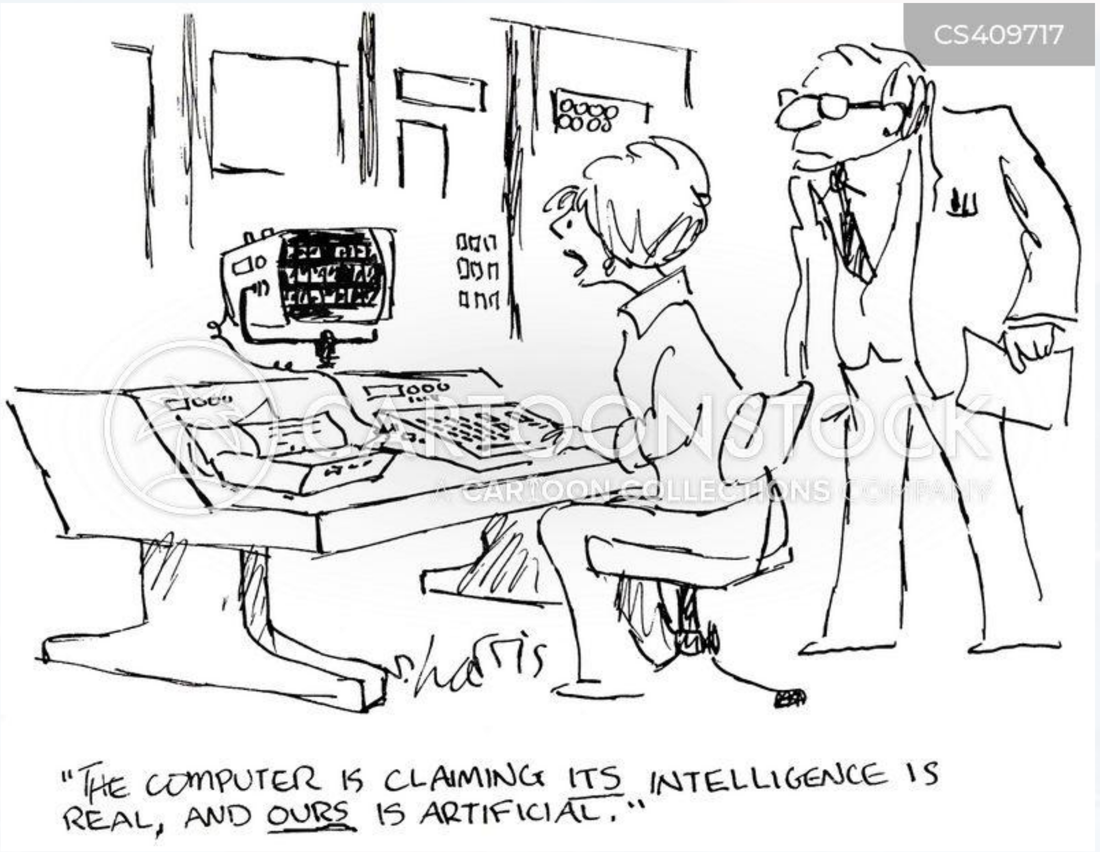
Goal 2: Understanding Python syntax

→ important for **building the specific** ANN that can solve your problem!

→ important for **solving error messages** (requires understanding of processes within the ANN)!

"What I cannot build. I do not understand."

— Richard Feynman



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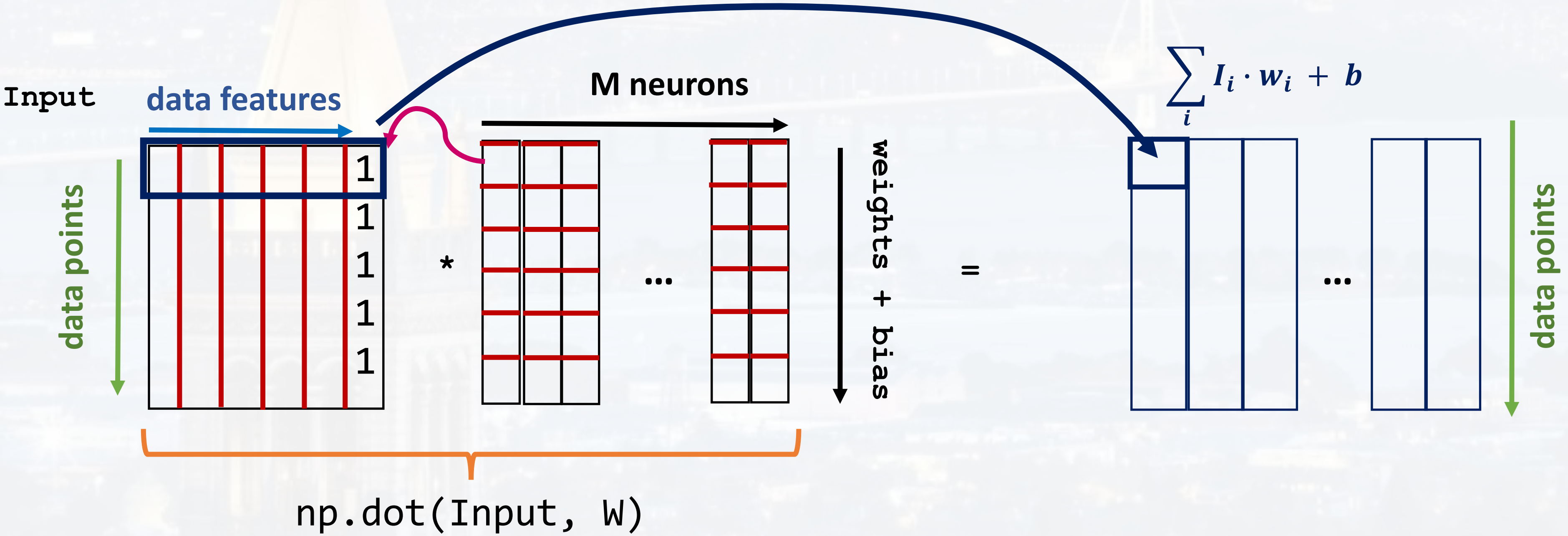
main part of the code:

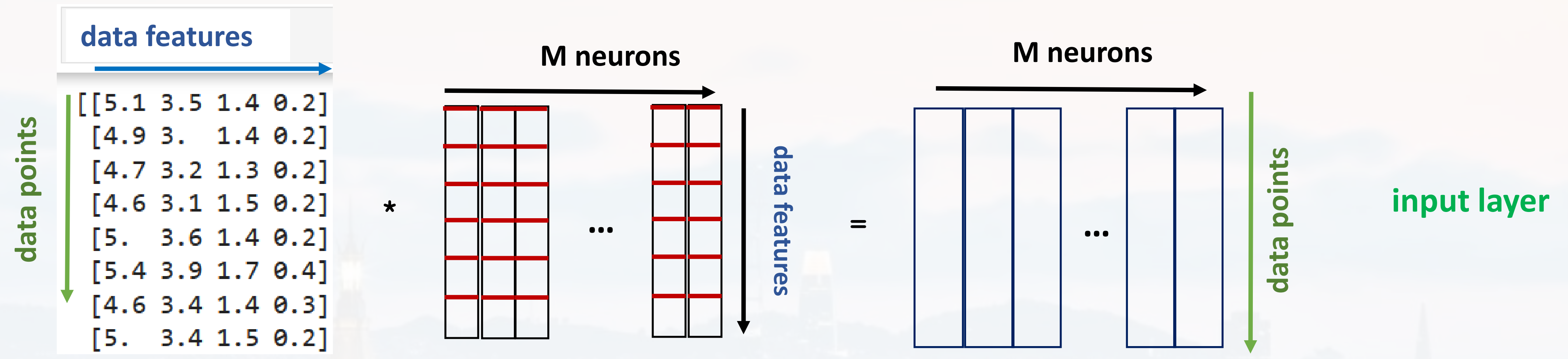
data features

data points

```
[[5.1 3.5 1.4 0.2]
 [4.9 3.  1.4 0.2]
 [4.7 3.2 1.3 0.2]
 [4.6 3.1 1.5 0.2]
 [5.  3.6 1.4 0.2]
 [5.4 3.9 1.7 0.4]
 [4.6 3.4 1.4 0.3]
 [5.  3.4 1.5 0.2]
```

$$\boxed{net = \sum_i \underbrace{I_i \cdot w_i}_{\textit{dot product}} + b}$$




data features
data points
[[5.1 3.5 1.4 0.2]
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[4.6 3.1 1.5 0.2]
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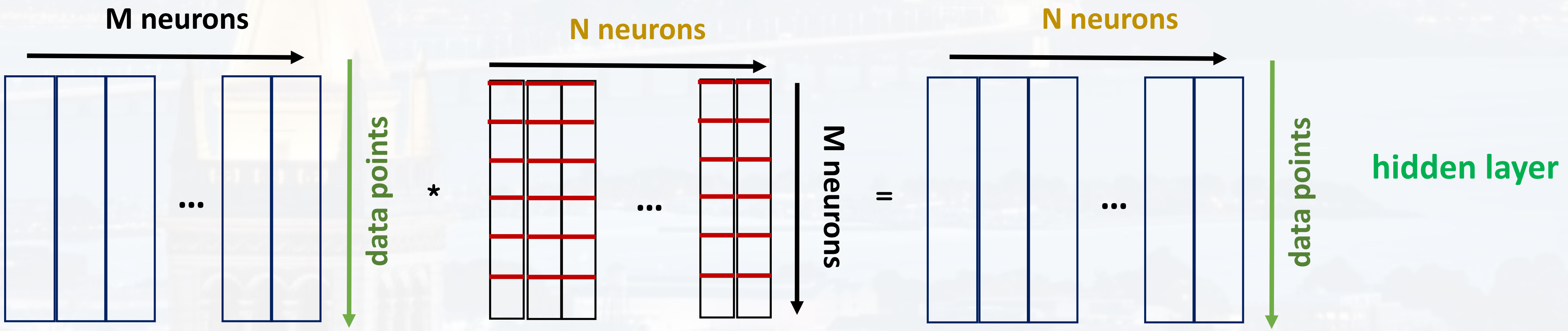
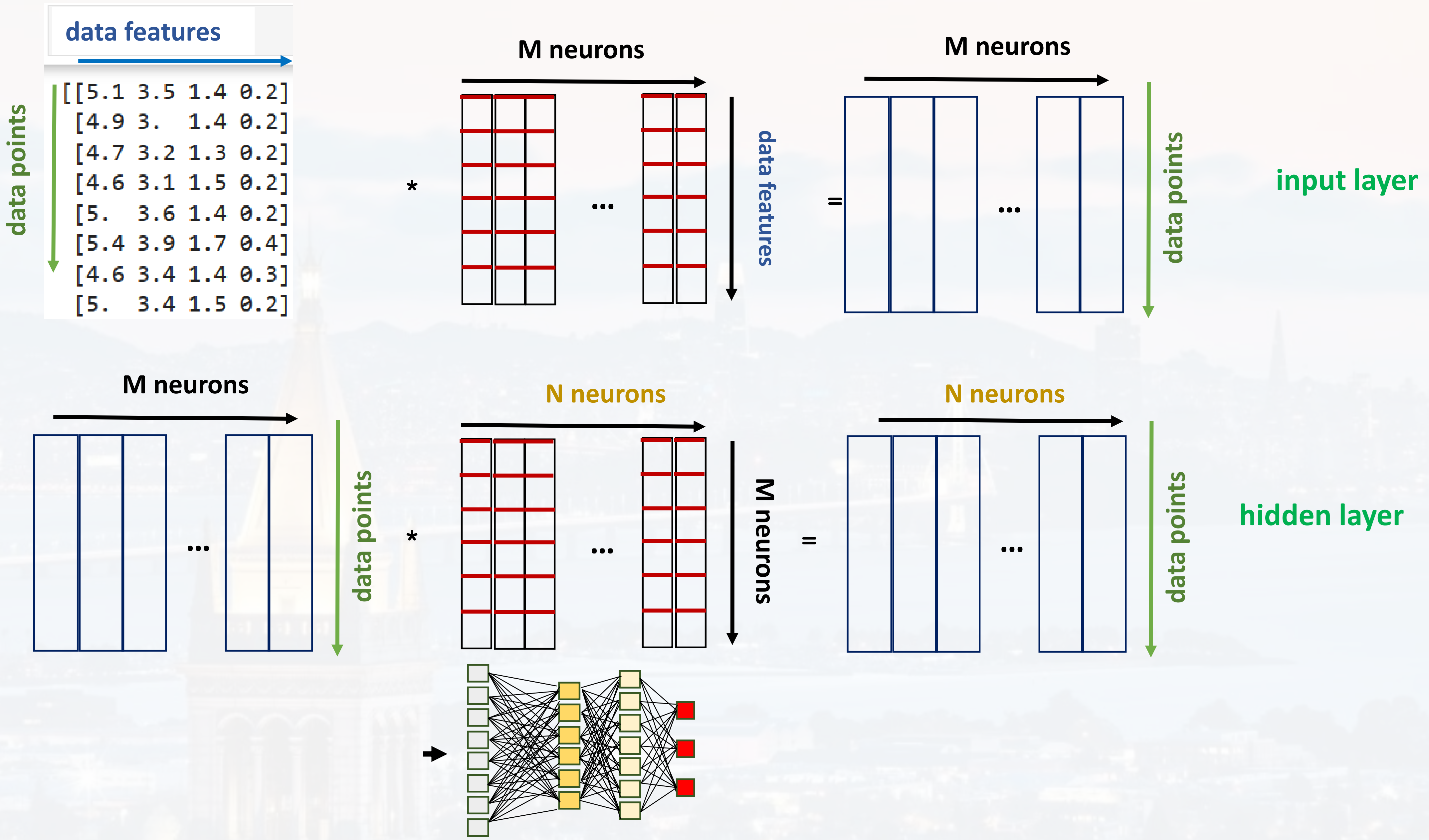
[4.6 3.4 1.4 0.3]
[5. 3.4 1.5 0.2]
*
M neurons
...
data features
=
M neurons
...
data points
input layer


M neurons
...
data points
*
N neurons
...
M neurons
=
N neurons
...
data points
hidden layer

ANN - Intro: The Minimal Setup



```
class Layer_Dense():

    def __init__(self, n_inputs, n_neurons):
        self.weights = np.random.randn(n_inputs, n_neurons)
        self.biases  = np.zeros((1, n_neurons))

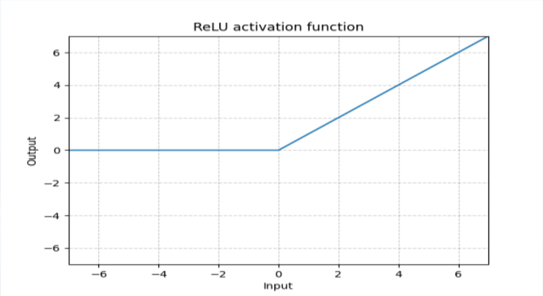

    def forward(self, inputs):
        self.output  = np.dot(inputs, self.weights) + self.biases
        self.inputs  = inputs



class Activation_ReLU():

    def forward(self, inputs):
        self.output  = np.maximum(0, inputs)
        self.inputs  = inputs
```

number of features



ANN - Intro:

The Minimal Setup

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class Layer_Dense():

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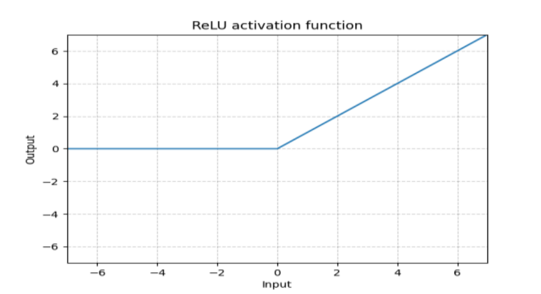
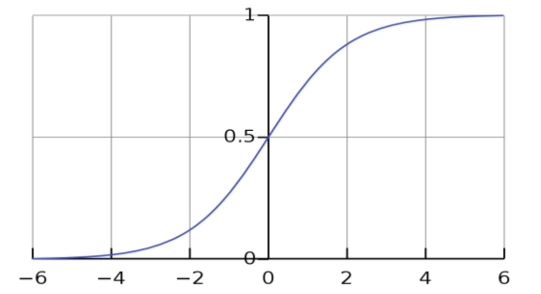


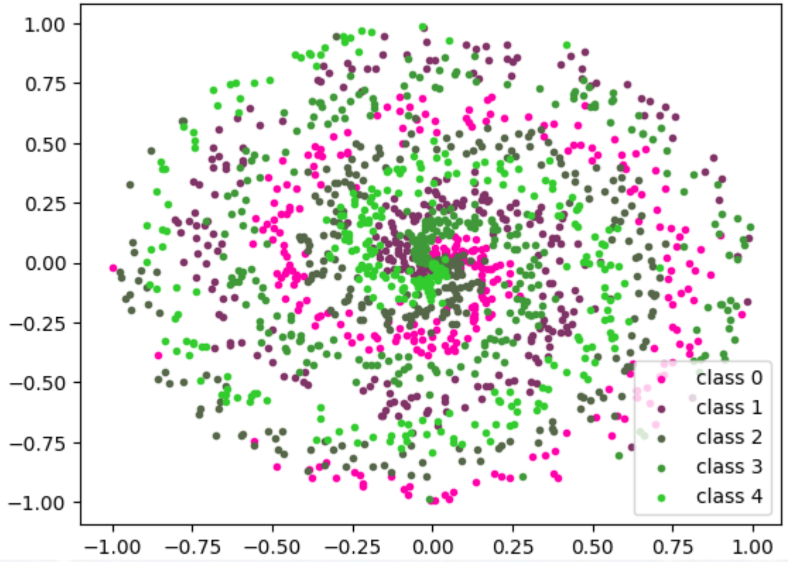



```
class Activation_Sigmoid():

        def forward(self, inputs):
                self.output = np.clip(1/(1 + np.exp(-inputs)), 1e-7, 1-1e-7)
                self.inputs = inputs
```

Goal: fitting the Spiral Dataset

two features, **five** classes



initializing the layers

```
Nneurons1 = 64
Nfeatures = X.shape[1]

dense1    = Layer_Dense(Nfeatures, Nneurons1)

dense_reg = Layer_Dense(Nneurons1, 1)
dense_cla = Layer_Dense(Nneurons1, Nclasses)

ReLU      = Activation_ReLU()
Sigm      = Activation_Sigmoid()
```

for regression: one value for each datapoint

for classification: Nclasses values for each datapoint (probabilities for each class)

ANN - Intro: The Minimal Setup

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ReLU      = Activation_ReLU()
Sigm      = Activation_Sigmoid()
```

building the **forward** part of the ANN

```
dense1.forward(X)
ReLU.forward(dense1.output)
```

regression

```
dense_reg.forward(ReLU.output)
```

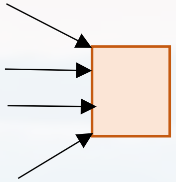
classification

```
dense_cla.forward(ReLU.output)
```

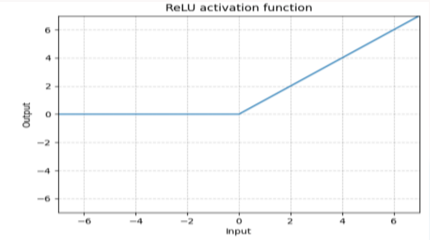
see ANNI.ipynb for details

ANN - Intro: The Minimal Setup

for training: building the backpropagation part



$$\sum_i I_i w_i + b = net$$



$$y = f(net)$$

...

loss

$L(w_i, b, I_i)$

e. g.:
- MSE
- accuracy
- etc

$\boxed{dy}$

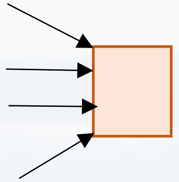
$\boxed{dL}$

$$\Delta w_i = -\alpha \frac{\partial L}{\partial w_i}$$

$$\Delta w_i = -\alpha \frac{dL}{dy} \frac{dy}{dnet} \frac{\partial net}{\partial w_i}$$

$$\Delta w_i = -\alpha \boxed{\frac{dL}{dy}} \boxed{\frac{dy}{dnet}} I_i$$

for training: building the backpropagation part



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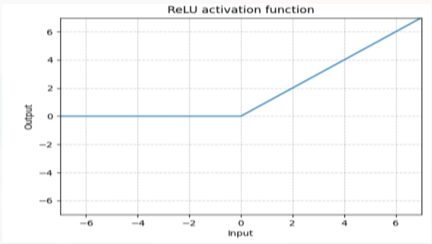

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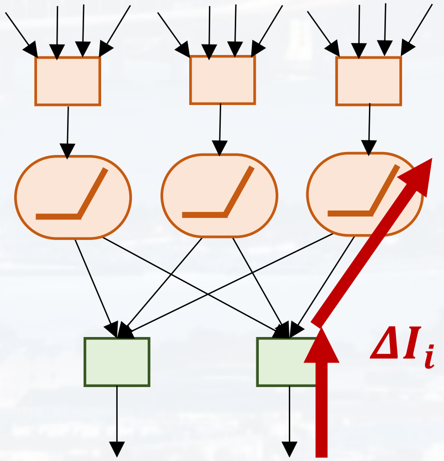
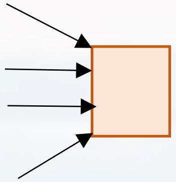
$$\Delta w_i = -\alpha \boxed{\frac{dL}{dy}} \boxed{\frac{dy}{dnet}} I_i$$

$$\Delta I_i = -\alpha \boxed{\frac{dL}{dy}} \boxed{\frac{dy}{dnet}} w_i \quad \text{from} \quad \frac{\partial L}{\partial I_i}$$

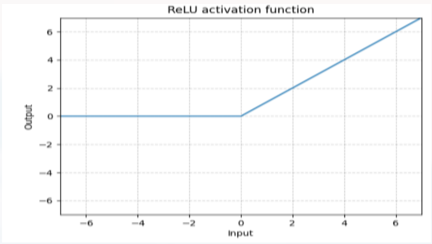
$$\Delta b = -\alpha \boxed{\frac{dL}{dy}} \boxed{\frac{dy}{dnet}} 1 \quad \text{from} \quad \frac{\partial L}{\partial b}$$



for training: building the backpropagation part



$$\sum_i I_i w_i + b = net$$



$$y = f(net)$$

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e. g.:
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$\boxed{dy}$ $\boxed{dL}$

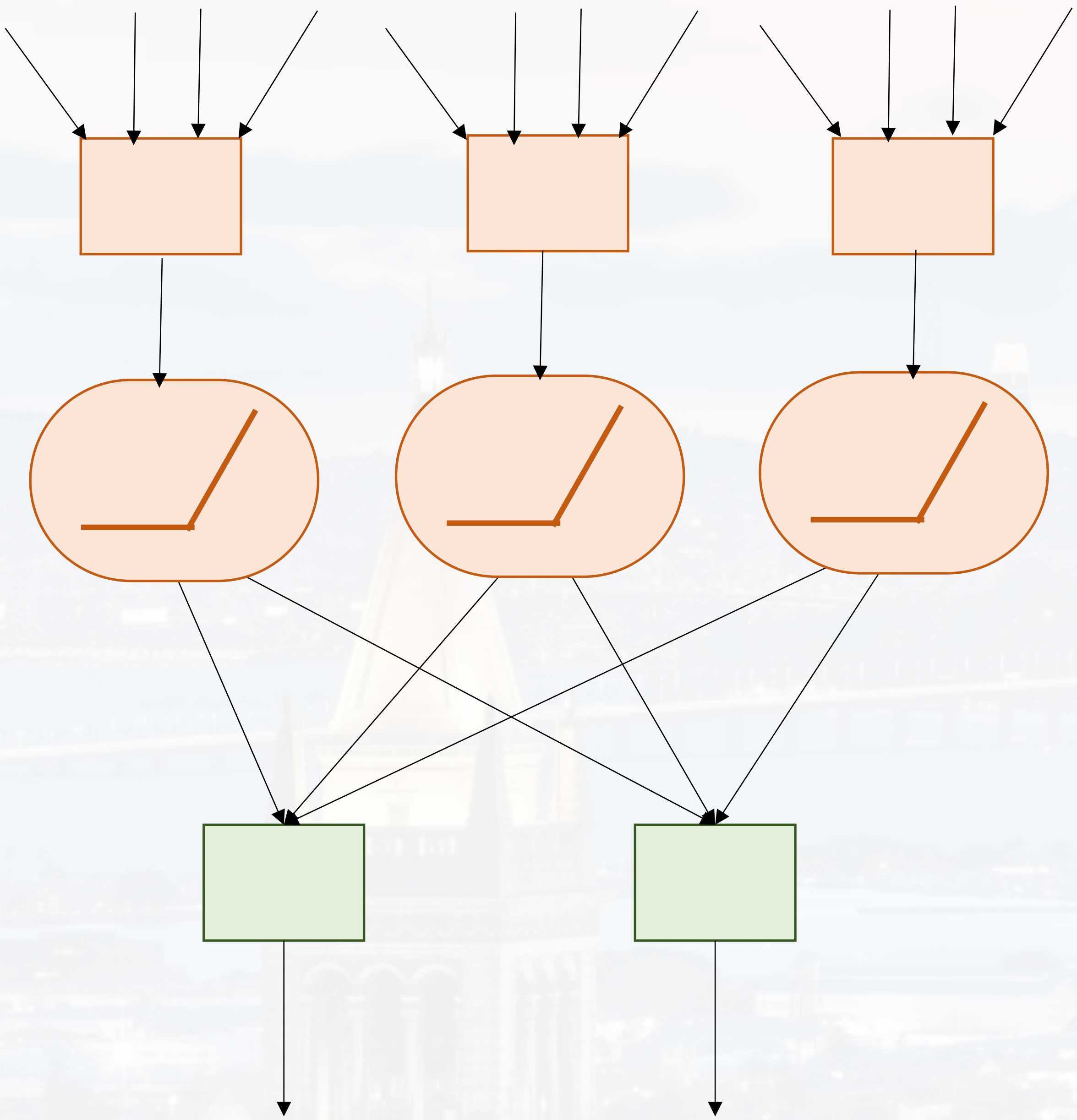
$$\Delta w_i = -\alpha \boxed{\frac{dL}{dy}} \boxed{\frac{dy}{dnet}} I_i$$

$$\Delta I_i = -\alpha \boxed{\frac{dL}{dy}} \boxed{\frac{dy}{dnet}} w_i \quad \text{from} \quad \frac{\partial L}{\partial I_i}$$

$$\Delta b = -\alpha \boxed{\frac{dL}{dy}} \boxed{\frac{dy}{dnet}} 1 \quad \text{from} \quad \frac{\partial L}{\partial b}$$

hence, we need to include the following structure for backpropagation:

self.dweights	=	inputs	*	dvalues
self.dinputs	=	weights	*	dvalues
self.dbiases	=	1	*	dvalues
		inner derivative		**product of outer derivatives**



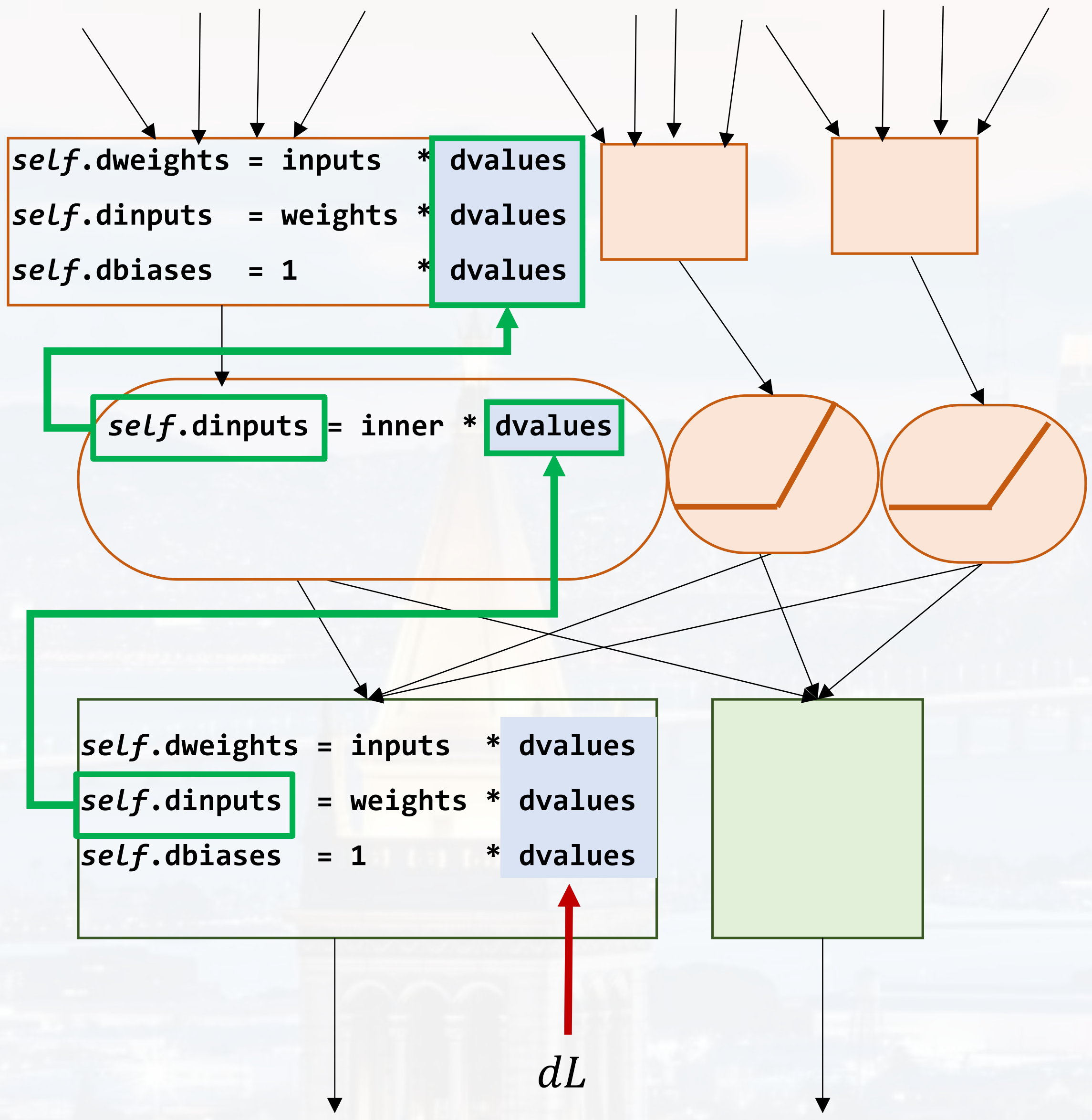
hence, we need to include the following structure for backpropagation:

```
self.dweights  = inputs  * dvalues
self.dinputs   = weights * dvalues
self.dbiases   = 1       * dvalues
```

inner derivative

product of outer derivatives

ANN - Intro:



hence, we need to include the following structure for backpropagation:

self.dweights	=	inputs	*	dvalues
self.dinputs	=	weights	*	dvalues
self.dbiases	=	1	*	dvalues
		inner derivative		**product of outer derivatives**

$$\Delta I_i = -\alpha \boxed{\frac{dL}{dy}} \boxed{\frac{dy}{dnet}} w_i$$

for training: building the backpropagation part

```
class Layer_Dense:

    def __init__(self, n_inputs, n_neurons):
    self.weights = np.random.rand(n_inputs, n_neurons)
        self.biases  = np.zeros((1, n_neurons))

    def forward(self, inputs):

        self.output  = np.dot(inputs, self.weights) + self.biases
        self.inputs  = inputs

    def backward(self, dvalues):

        self.dweights  = np.dot(self.inputs.T, dvalues)
        self.dinputs   = np.dot(dvalues, self.weights.T)
        self.dbiases   = np.sum(dvalues, axis = 0, keepdims = True)
```

outer derivative

see ANNII.ipynb for details

ANN - Intro: The Minimal Setup

```
alpha = 0.001
```

forward

```
dense1.forward(X)
ReLU.forward(dense1.output)
dense_reg.forward(ReLU.output)
```

evaluation

```
Ypred = dense_reg.output
dE    = Ypred - Target
MSE   = np.sum(abs(dE))/(Nsample*Nclasses)
print('MSE = ' + str(MSE))
```

backpropagation

```
dense_reg.backward(dE)
ReLU.backward(dense_reg.dinputs)
dense1.backward(ReLU.dinputs)
```

optimization

```
dense_reg.weights -= alpha * dense_reg.dweights
dense_reg.biases  -= alpha * dense_reg.dbiases
dense1.weights    -= alpha * dense1.dweights
dense1.biases     -= alpha * dense1.dbiases
```

ANN - Intro:


```
dense1    = Layer_Dense(Nfeatures, Nneurons1)
dense_reg = Layer_Dense(Nneurons1, 1)
```

```
print(Ypred)

[[2.00173615]
 [2.00173615]
 [2.00173615]
 ...
 [2.00173615]
 [2.00173615]
 [2.00173615]]
```

```
print(Target)

[[0]
 [0]
 [0]
 ...
 [4]
 [4]
 [4]]
```

for regression: one value for each datapoint

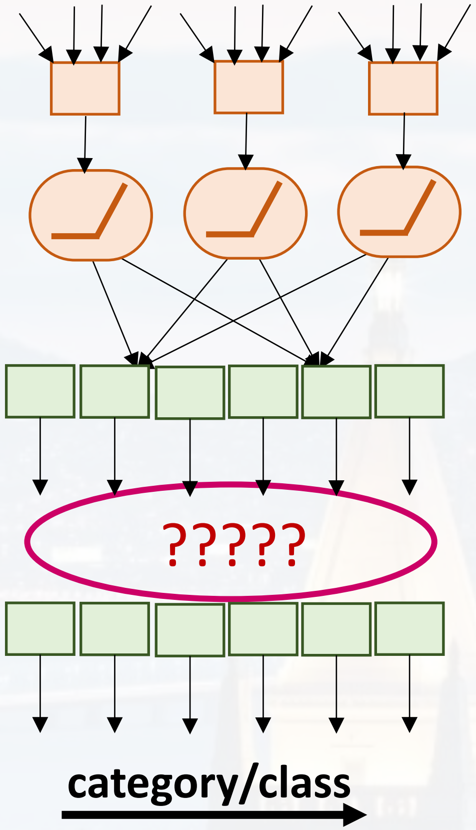
```
dense_cla = Layer_Dense(Nneurons1, Nclasses)
```

for classification: Nclasses values for each datapoint (**probabilities** for each class)

ANN - Intro: The Minimal Setup

```
dense_cla = Layer_Dense(Nneurons1, Nclasses)
```

for classification: Nclasses values for each datapoint (**probabilities** for each class)



$$p_i = \frac{\exp(\varepsilon_i)}{\sum_i \exp(\varepsilon_i)}$$

Boltzmann distribution

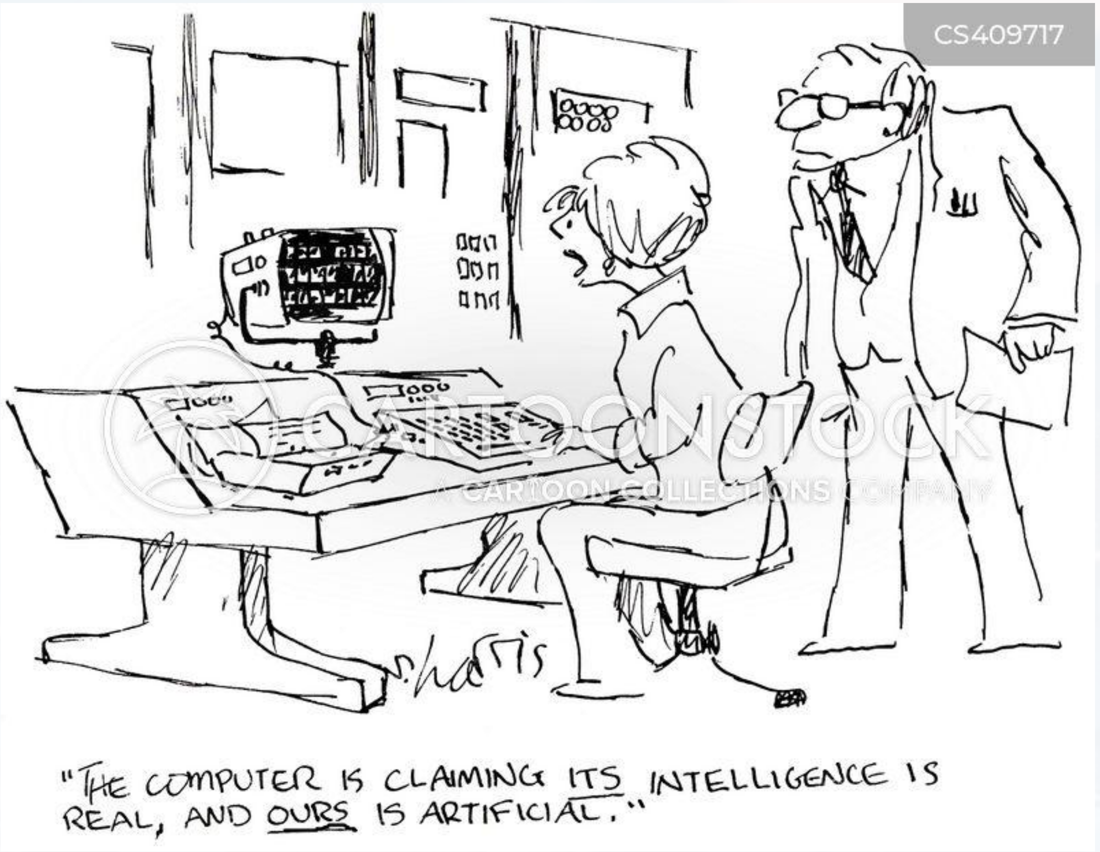
aka softmax

(from max entropy)

prob_pred = $\begin{pmatrix} \mathbf{0.8} & 0.1 & 0.1 \\ \mathbf{0.9} & 0.05 & 0.05 \\ 0.1 & 0.15 & \mathbf{0.75} \\ 0.2 & \mathbf{0.7} & 0.1 \\ \cdots & \cdots & \end{pmatrix}$

samples

ε_i: output from the last hidden layer



Outline

- Motivation & Overview

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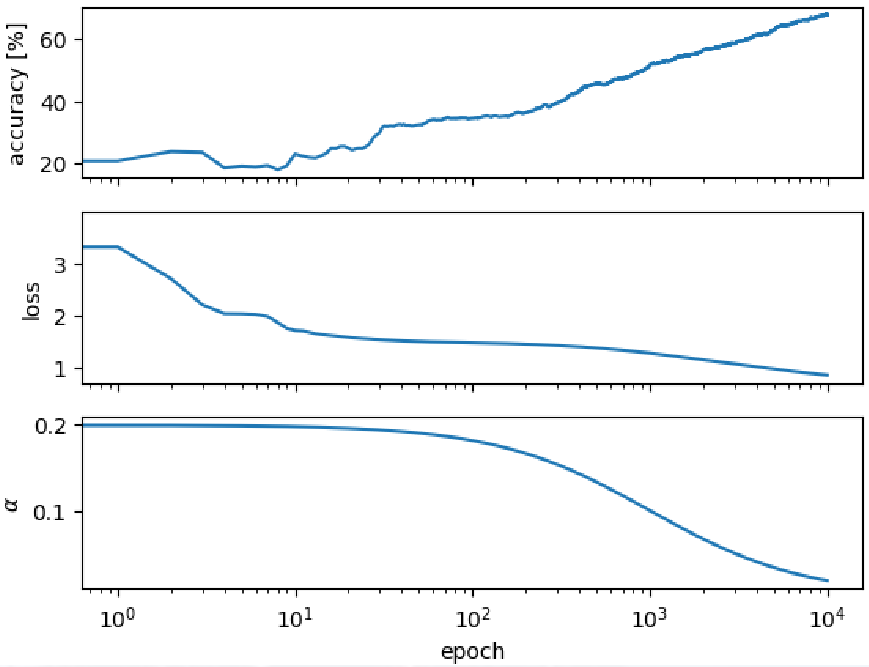
- Building a fully functional ANN

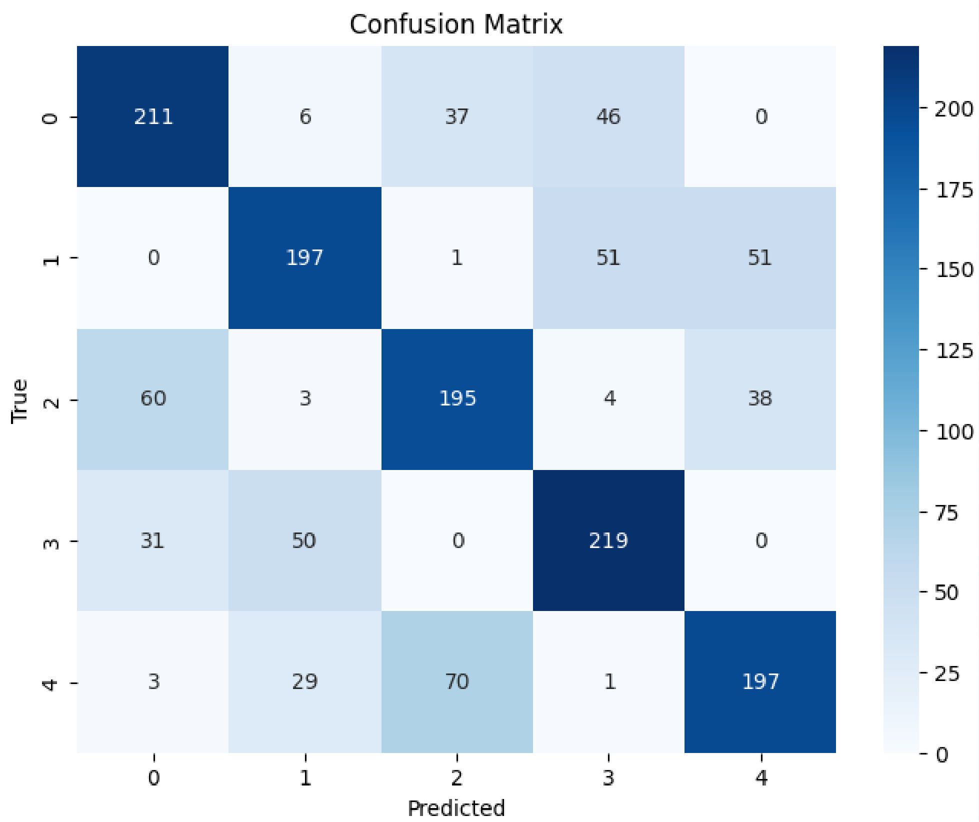
Finally, we have all the ingredients for a fully functional ANN!

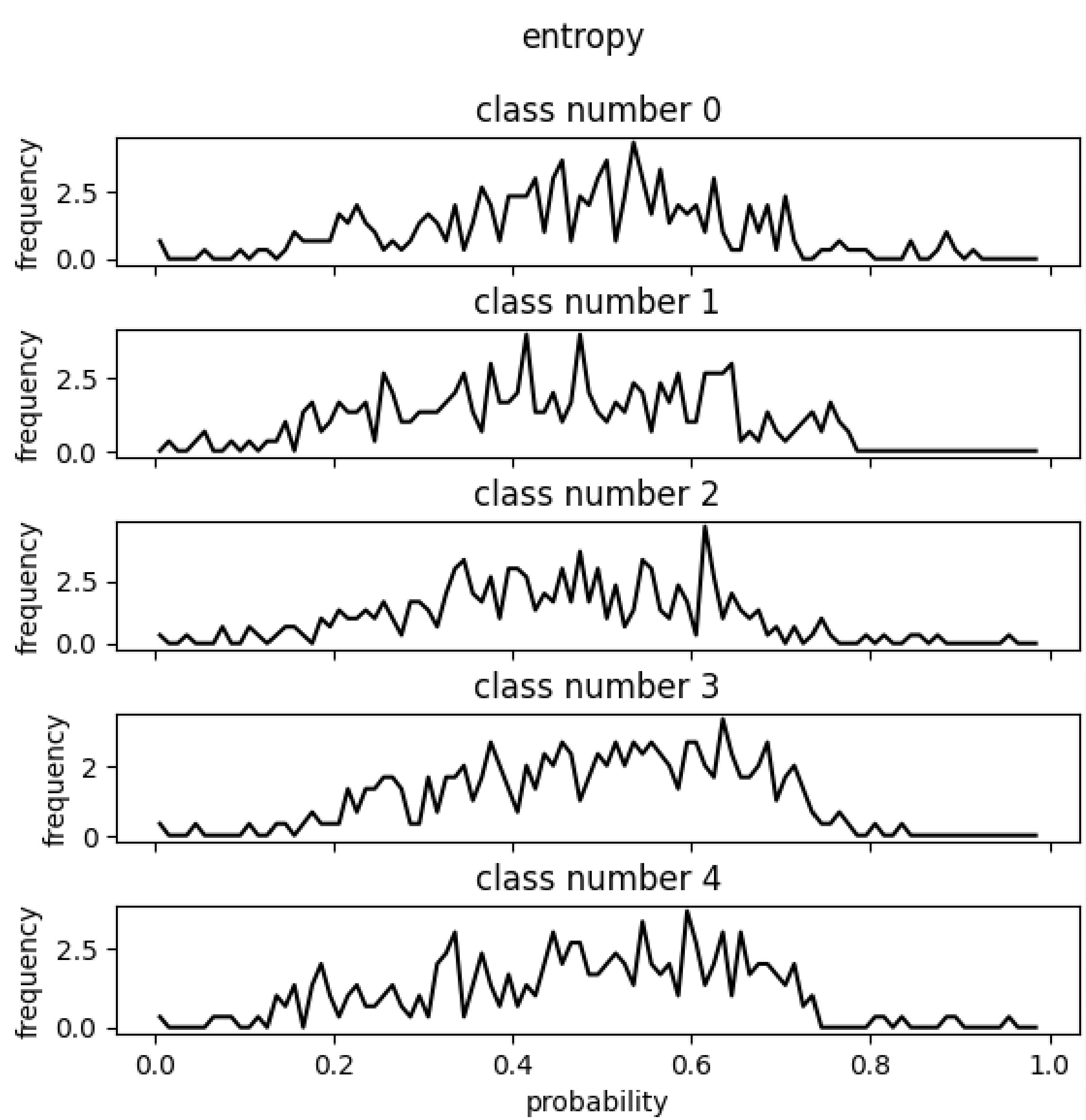
see `ANNIII.ipynb` for details

```
epoch: 18200, accuracy: 0.767, loss: 0.662, actual learning rate: 0.010416666666
epoch: 18300, accuracy: 0.767, loss: 0.662, actual learning rate: 0.010362694300
epoch: 18400, accuracy: 0.768, loss: 0.661, actual learning rate: 0.010309278350
epoch: 18500, accuracy: 0.768, loss: 0.661, actual learning rate: 0.010256410256
epoch: 18600, accuracy: 0.769, loss: 0.660, actual learning rate: 0.010204081632
epoch: 18700, accuracy: 0.769, loss: 0.660, actual learning rate: 0.010152284263
epoch: 18800, accuracy: 0.770, loss: 0.660, actual learning rate: 0.010101010101
epoch: 18900, accuracy: 0.770, loss: 0.659, actual learning rate: 0.010050251256
epoch: 19000, accuracy: 0.769, loss: 0.659, actual learning rate: 0.010000000000
epoch: 19100, accuracy: 0.769, loss: 0.658, actual learning rate: 0.009950248756
epoch: 19200, accuracy: 0.769, loss: 0.658, actual learning rate: 0.009900990099
epoch: 19300, accuracy: 0.769, loss: 0.658, actual learning rate: 0.009852216748
epoch: 19400, accuracy: 0.769, loss: 0.657, actual learning rate: 0.009803921568
epoch: 19500, accuracy: 0.769, loss: 0.657, actual learning rate: 0.009756097560
epoch: 19600, accuracy: 0.769, loss: 0.656, actual learning rate: 0.009708737864
epoch: 19700, accuracy: 0.768, loss: 0.656, actual learning rate: 0.009661835748
epoch: 19800, accuracy: 0.769, loss: 0.656, actual learning rate: 0.009615384615
epoch: 19900, accuracy: 0.770, loss: 0.655, actual learning rate: 0.009569377990
```



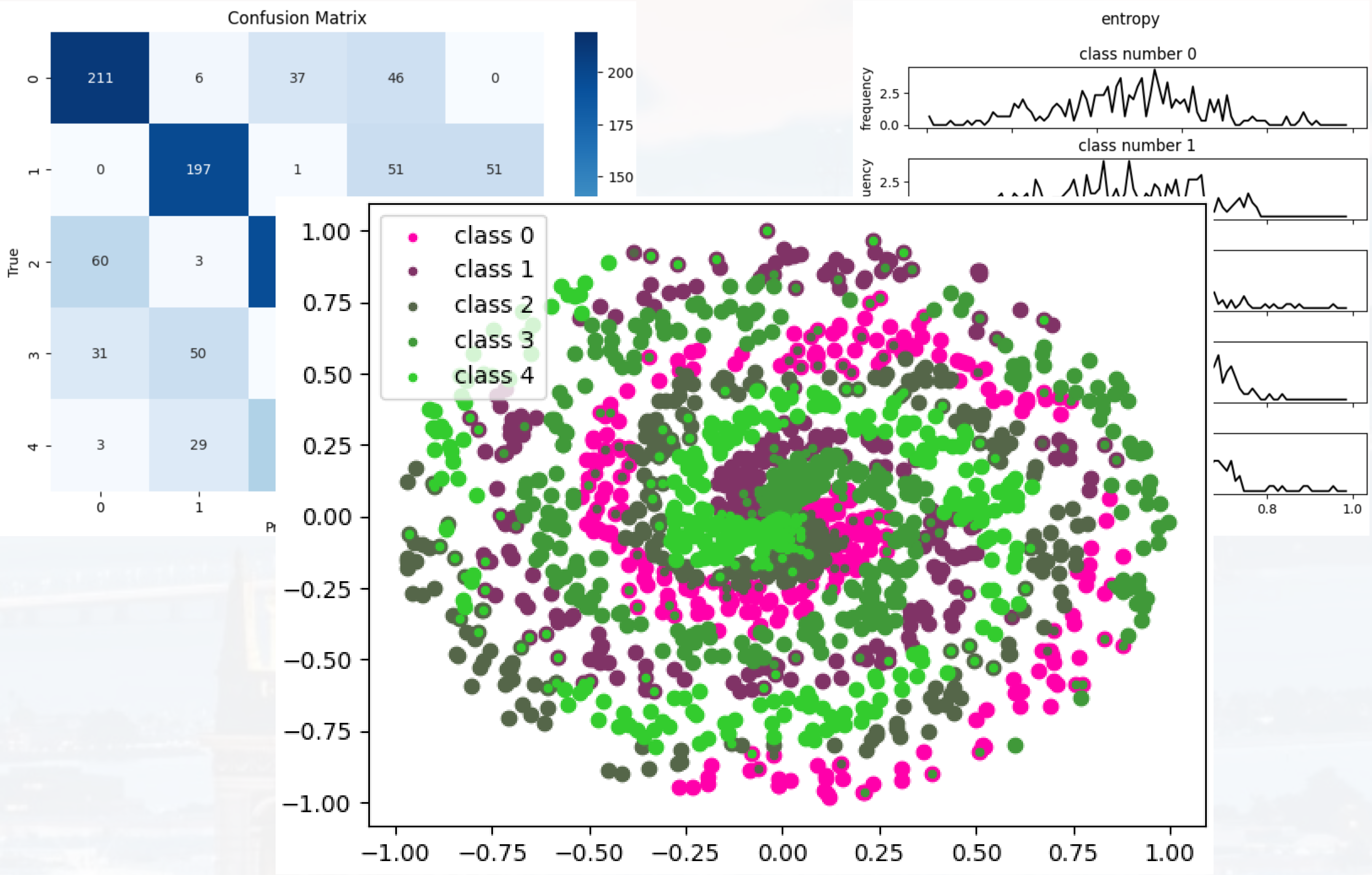
Finally, we have all the ingredients for a fully functional ANN! see `ANNIII.ipynb` for details





ANN - Intro: Fully Functional ANN

Finally, we have all the ingredients for a fully functional ANN! see `ANNIII.ipynb` for details



Thank you very much for your attention!