Capstone Project Proposal: Numerical Simulation of SYK Model

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Proposal

SYK model describes N Majorana fermions with all-to-all interactions. Typically, its Hamiltonian can be written as a $2^{\frac{N}{2}} \times 2^{\frac{N}{2}}$ matrix. Usually, N has to be a even number, there should be $\frac{N}{2}$ types of different Majorana fermions, each with two possible spin states. The Hamiltonian can be obtained through recursion, and we want to implement this process with python. Having the Hamiltonian constructed, one can immediately diagonalize it and see the energy spectrum. Then, we shall have some of the fundamental thermodynamic properties of SYK model, e.g. its internal energy and heat capacitance. Also, SYK model describes a chaotic system, as can be seen in the randomness of the coupling constants. We want to see its chaotic properties, e.g. its out-of-time-order correlations.

Furthermore, with AdS/CFT correspondance, one can relate a conformal theory in dimensions with a gravitational theory in 1+d dimensions, and we will show that SYK model has conformal symmetry which makes it dual to a gravitational theory. Since it's a quantum mechanical theory (1-dimensional field theory), it's supposed to be dual to a 2-dimensional gravitational theory that is known as JT gravity. We will give a very brief introduction on this topic. If there is enough time, we want to study the Schwinger-Dyson equation when N is large (but cannot be too large, at most around 20, otherwise it will beyond the capacity of a laptop) and use it to calculate correlation functions that can reflect features of quantum gravity in 2 dimensions.