

## Lecture 13:

# Machine Learning Fundamentals



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Numerical Methods for  
Computational Science



## Course Map

<b>Week 1:</b>	Introduction to Scientific Computing and Python Libraries
<b>Week 2:</b>	Linear Algebra Fundamentals
<b>Week 3:</b>	Vector Calculus
<b>Week 4:</b>	Numerical Differentiation and Integration
<b>Week 5:</b>	Solving Nonlinear Equations
<b>Week 6:</b>	Probability Theory Basics
<b>Week 7:</b>	Random Variables and Distributions
<b>Week 8:</b>	Statistics for Data Science
<b>Week 9:</b>	Eigenvalues and Eigenvectors
<b>Week 10:</b>	Simulation and Monte Carlo Method
<b>Week 11:</b>	Data Fitting and Regression
<b>Week 12:</b>	Optimization Techniques
<b>Week 13:</b>	<b>Machine Learning Fundamentals</b>



### Concept

#### Supervised Learning

- Support Vector Machine
- K-nearest

#### Unsupervised Learning

- K – means
- Gaussian Mixture Models



### Concept

#### Supervised Learning

- Support Vector Machine
- K-nearest

#### Unsupervised Learning

- K – means
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### What is data mining?

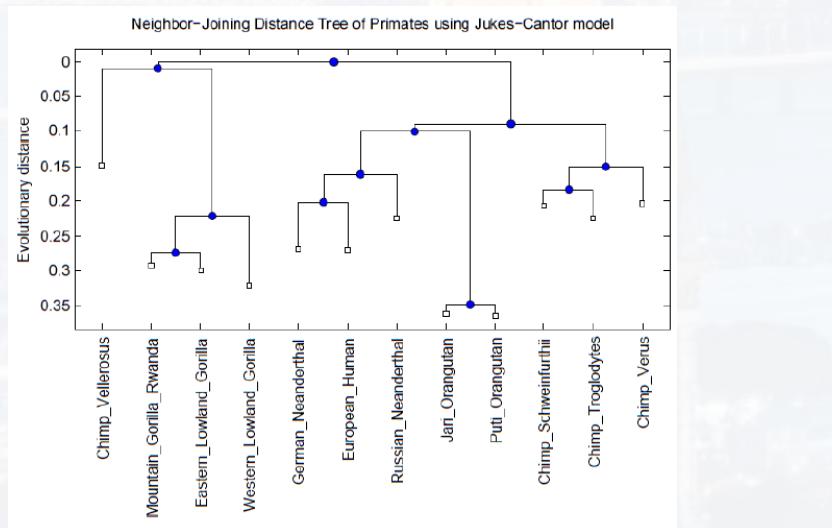
Wiki says:

***"Data mining is the computing process of discovering patterns in large data sets involving methods at the intersection of***

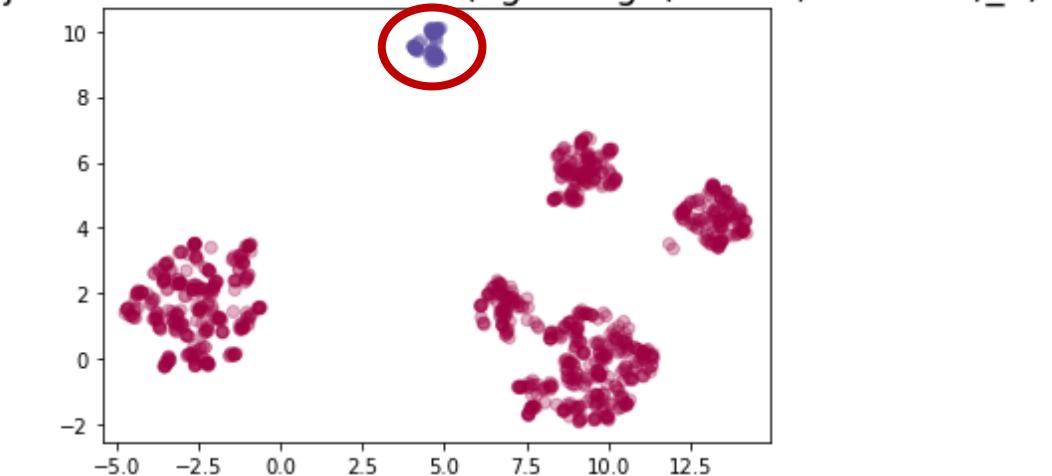
- *artificial intelligence,*
- ***machine learning,***
- *statistics,*
- *and database systems."*

***"computers have the ability to learn without being explicitly programmed"***

cluster and/or categorize data and understand relations/interactions.

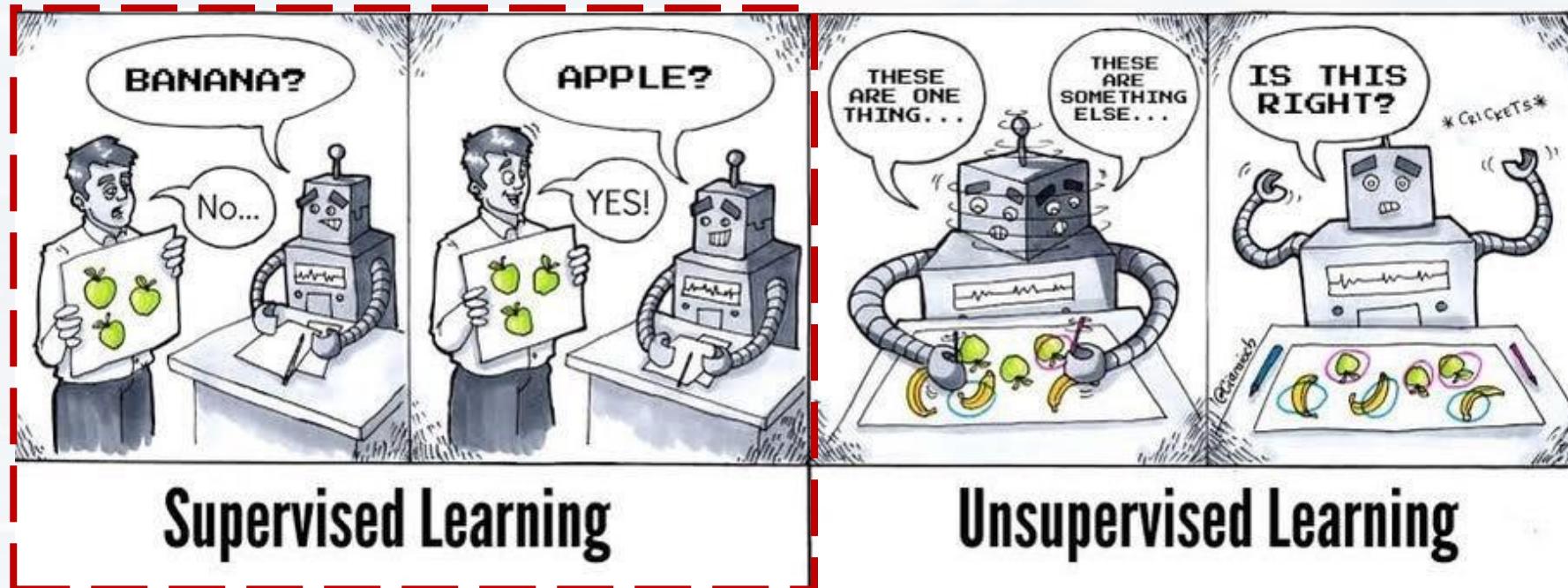


UMAP projection of the AD dataset (age range(0: <75; 1: >=75)\_1)





curve fitting, linear/logistic models:  
training data set and a **test** data set...

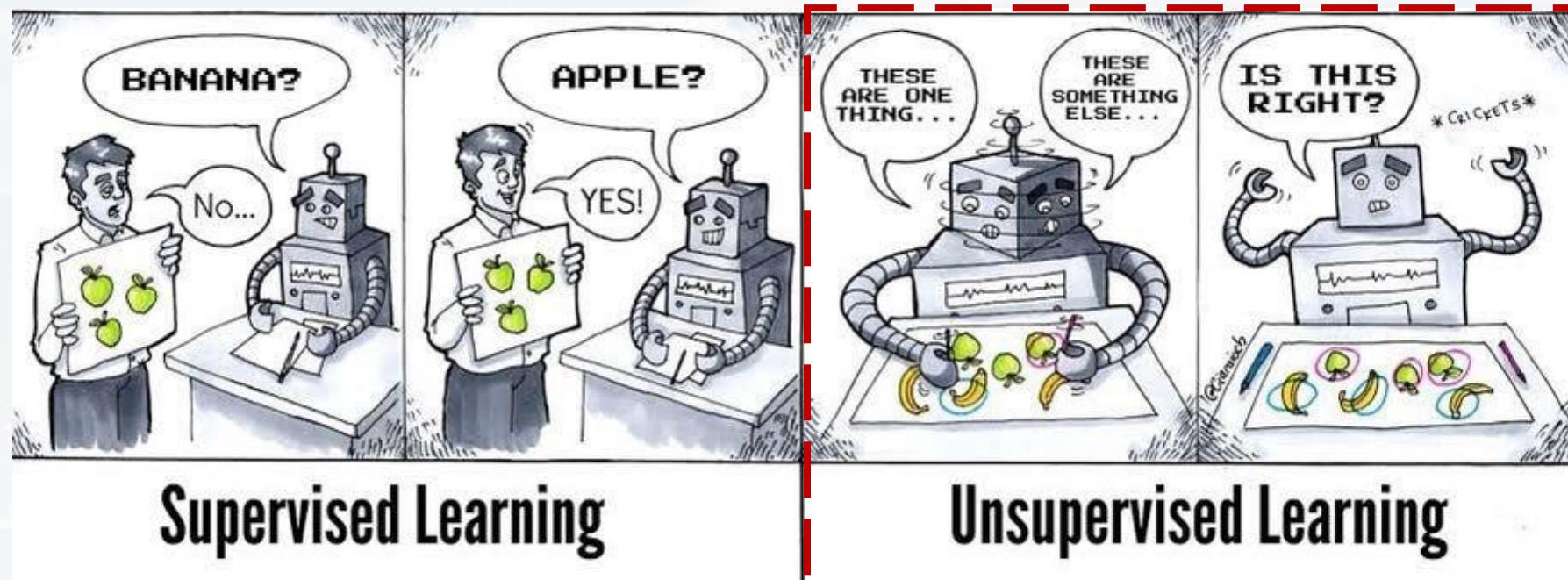


- Support Vector Machine
- K-nearest



curve fitting, linear/logistic models:  
training data set and a **test** data set...

**no training data set required** – we can start right away!

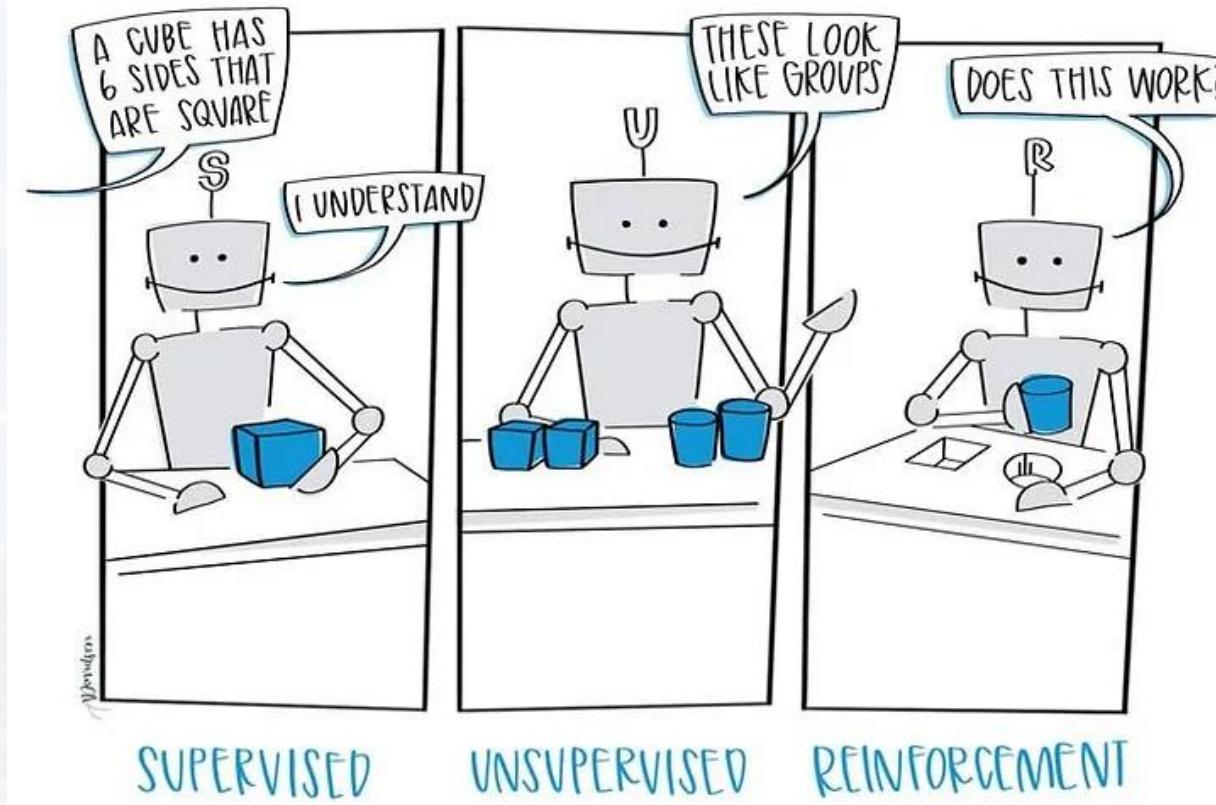


- Support Vector Machine
- K-nearest

- K - means
- GMM



# MACHINE LEARNING





curve fitting, linear/logistic models:

**training** data set and a **test** data set...

1) creating the model:

```
my_model = library.method(argument1 = 'arg1', ... )
```

2) training the model

```
out = my_model.fit(xtrain, ytrain)
```

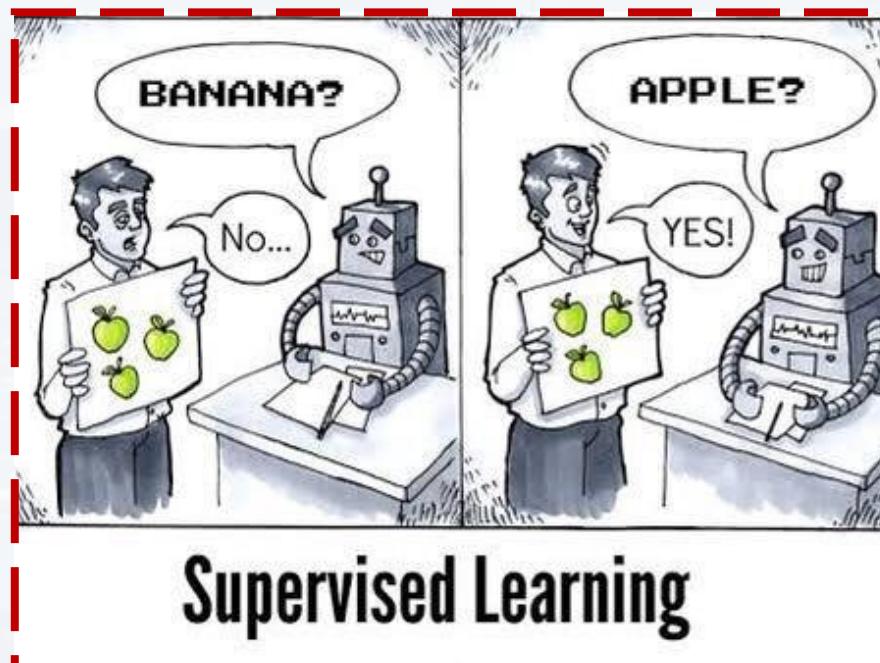
3) evaluation

```
ypred = out.predict(xeval)
```

```
accur = (ypred == yeval).sum()/len(yeval)
```

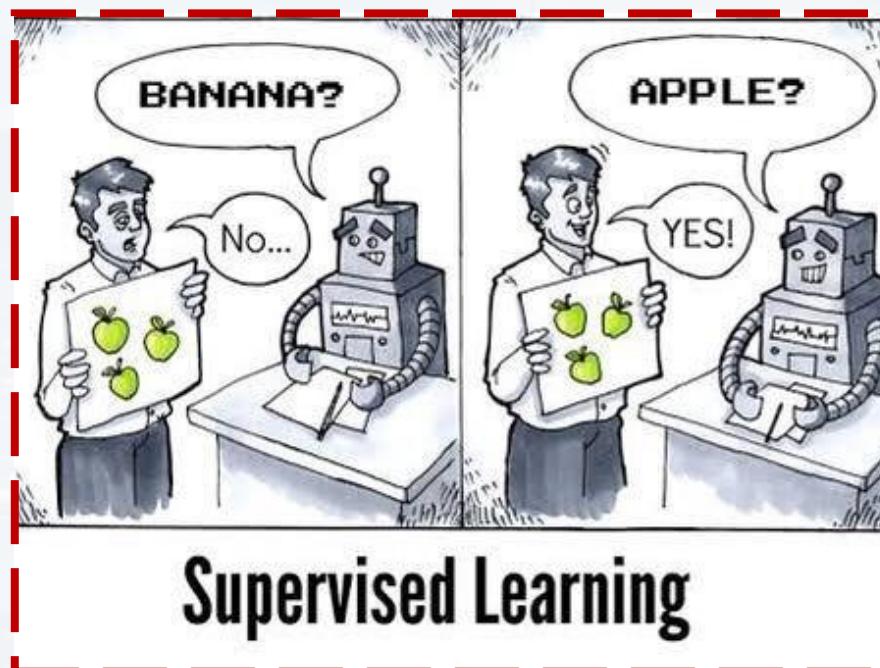
4) prediction (actual application)

```
ypred = out.predict(xnew)
```





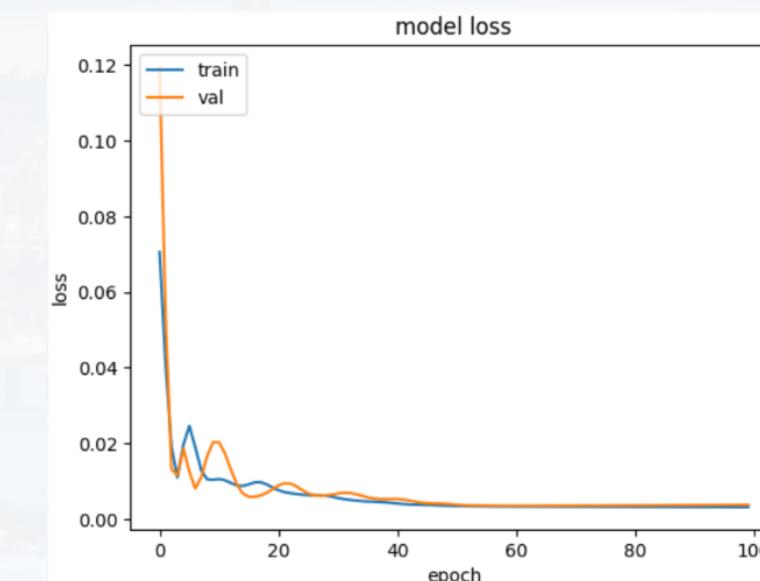
curve fitting, linear/logistic models:  
**training** data set and a **test** data set...



### How to split the data?

- 80% training (60% actual training 20% evaluation)
- 20% test

evaluation while training!

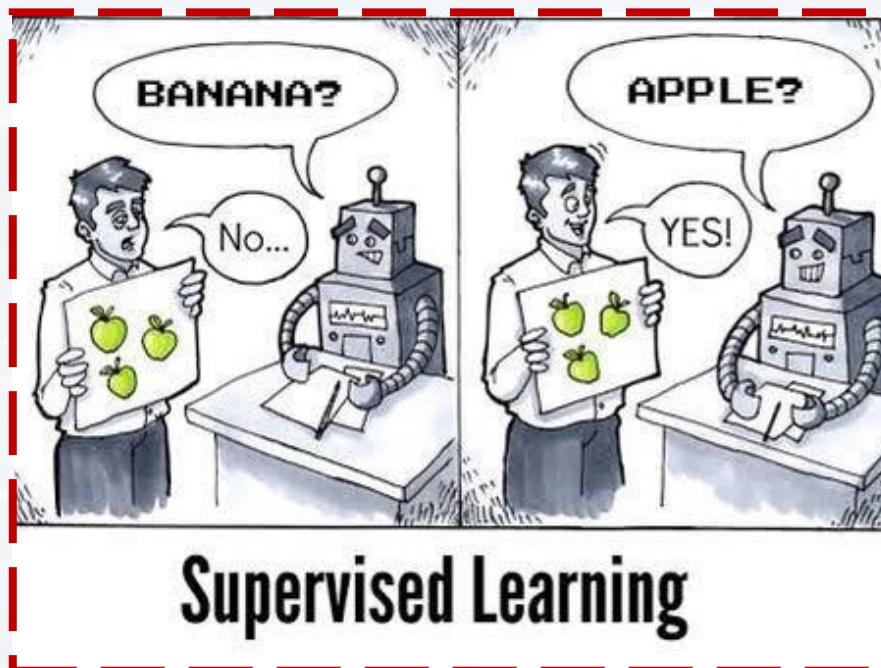




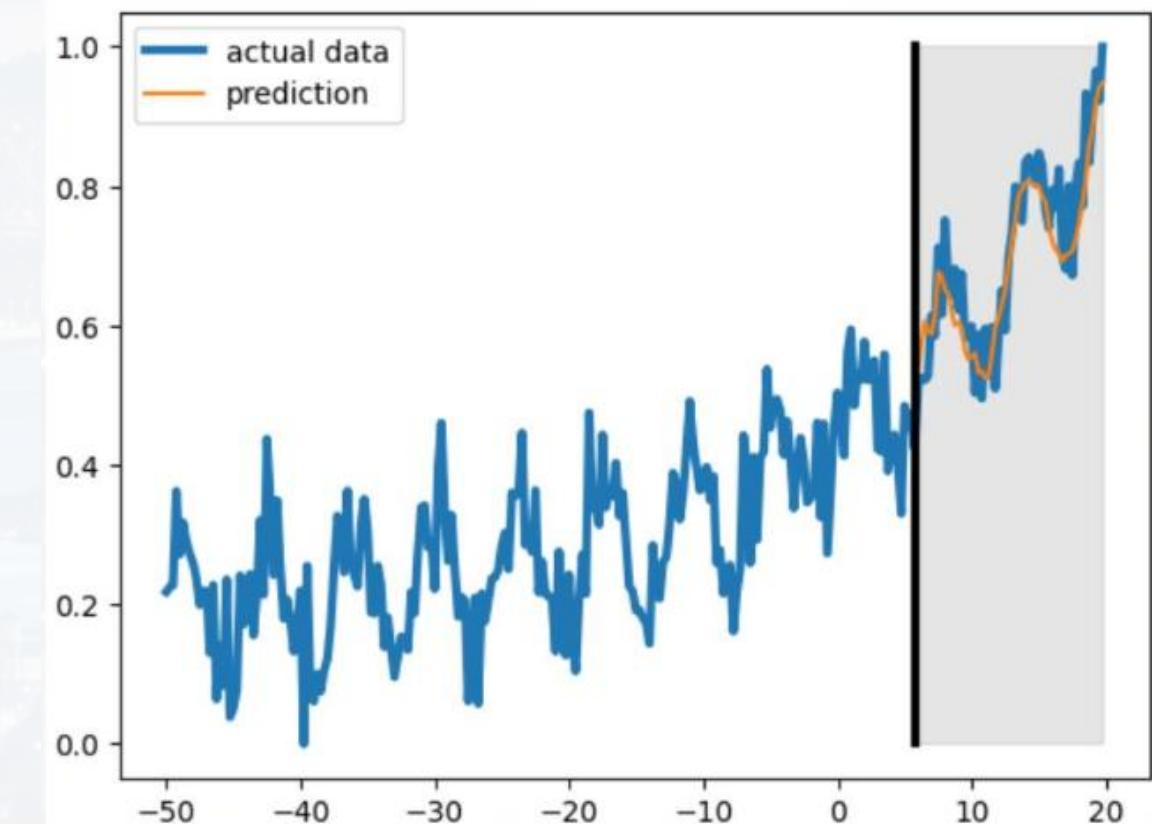
curve fitting, linear/logistic models:  
**training** data set and a **test** data set...

How to split the data?

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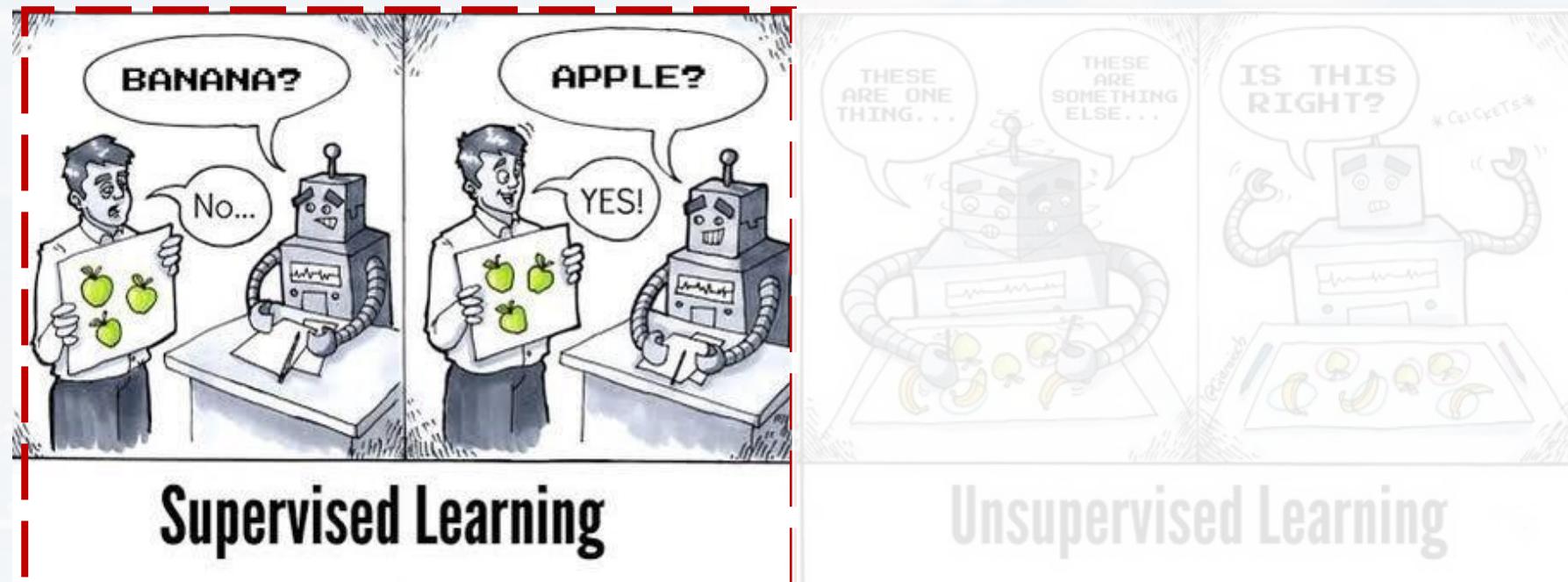
prediction after training/evaluation completed





curve fitting, linear/logistic models:  
training data set and a **test** data set...

**no training** data set required – we can start right away!



- **Support Vector Machine**
- **K-nearest**

- K - means
- GMM



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# Numerical Methods for Computational Science:

Machine Learning Fundamentals



## Concept

### Supervised Learning

- Support Vector Machine
- K-nearest

### Unsupervised Learning

- K – means
- Gaussian Mixture Models



SVM = Support Vector Machine

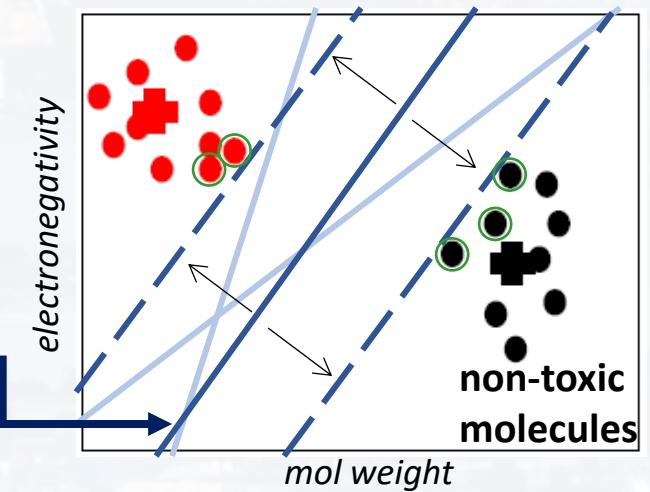
idea:

- 1) finds best **linear** classifier for separating **two** classes by **maximizing margin** using **support vectors**
- 2) assign new data points to these categories
- 3) **supervised** learning
- 4) uses the “kernel trick”

○ support vectors (data points at the edge)

best separator

toxic molecules



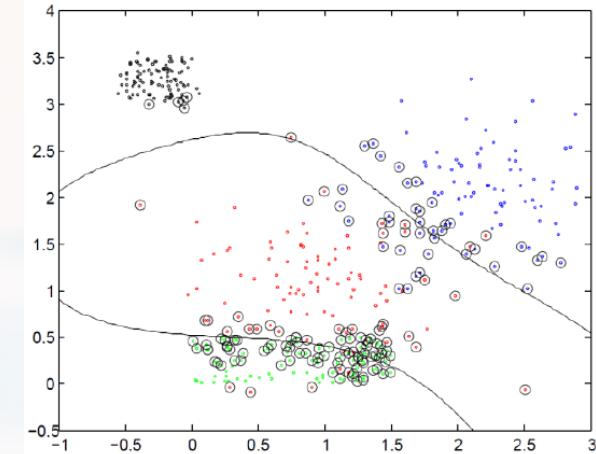


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## Machine Learning Fundamentals:

### Support Vector Machine

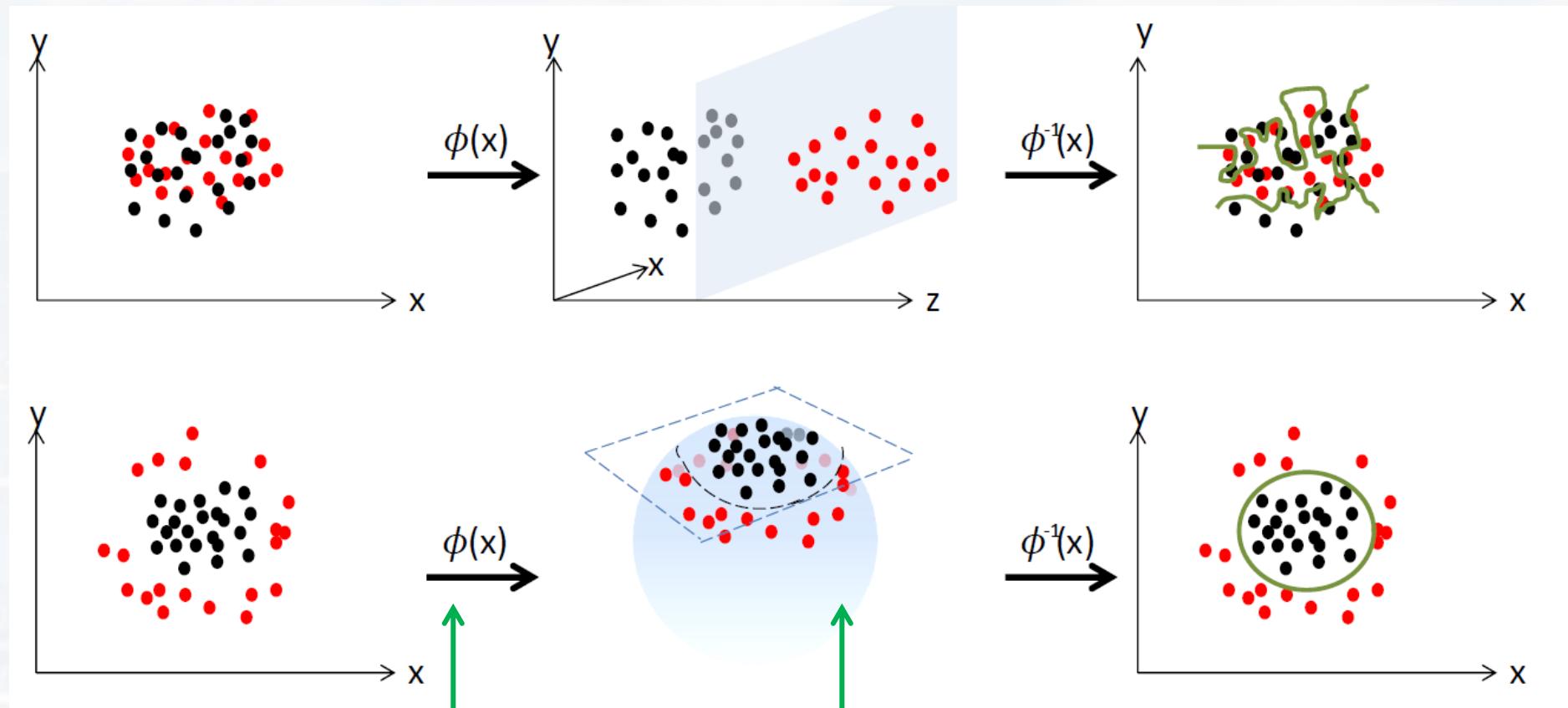
- 1) What if linear separation is not possible?
- 2) Is there a multiclass SVM?



**1) What if linear separation is not possible?**

2) Is there a multiclass SVM?

Theorem: "There exists always a (linear) hyperplane in higher dimensions."



**a)** data space in N-D

**b)** a function  $\phi$  maps the data into a M-D (M>N) **feature space**

**c)** find hyperplane separator in **feature space**

**d)** map back into data space (separator not linear)

**1) What if linear separation is not possible?**

2) Is there a multiclass SVM?

**problems:**

- computationally intensive
- $\phi$  usually unknown

**idea:**

- entire mathematical framework not needed
- for separation: need distances  $d$  in data space and feature space

$$d^2(x, y) = \langle x - y, x - y \rangle$$

$$\begin{aligned} d^2(\phi(x), \phi(y)) &= \langle \phi(x) - \phi(y), \phi(x) - \phi(y) \rangle \\ &= \langle \phi(x), \phi(x) \rangle - 2\langle \phi(x), \phi(y) \rangle + \langle \phi(y), \phi(y) \rangle \end{aligned}$$

kernel  $K(x, y) := \langle \phi(x), \phi(y) \rangle$

$$d^2(\phi(x), \phi(y)) = K(x, x) - 2K(x, y) + K(y, y)$$

**kernel trick:** - we don't know  $K$  either: **we guess it!**



- 1) What if linear separation is not possible?
- 2) Is there a multiclass SVM?

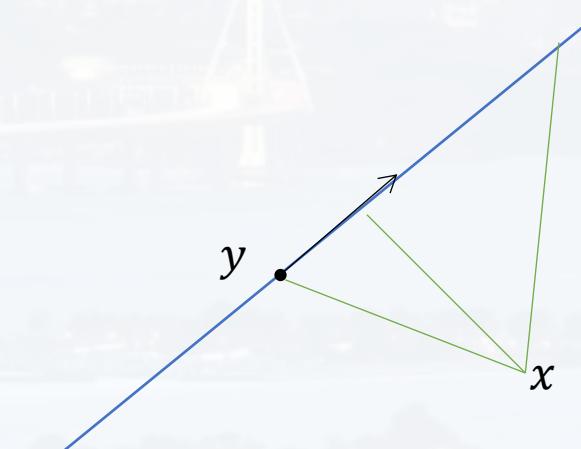
$$d^2(\phi(x), \phi(y)) = K(x, x) - 2K(x, y) + K(y, y)$$

example: linear kernel

$$\phi: x \mapsto x \quad \phi(x) = x$$

$$d^2(x, y) = \langle x, x \rangle - 2\langle x, y \rangle + \langle y, y \rangle = x^2 - 2xy + y^2 = (x - y)^2$$

$$d(x, y) = |x - y|$$



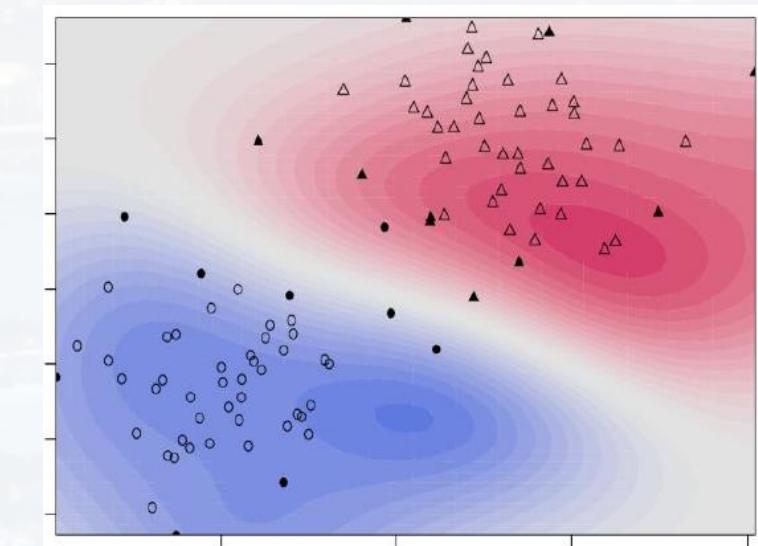
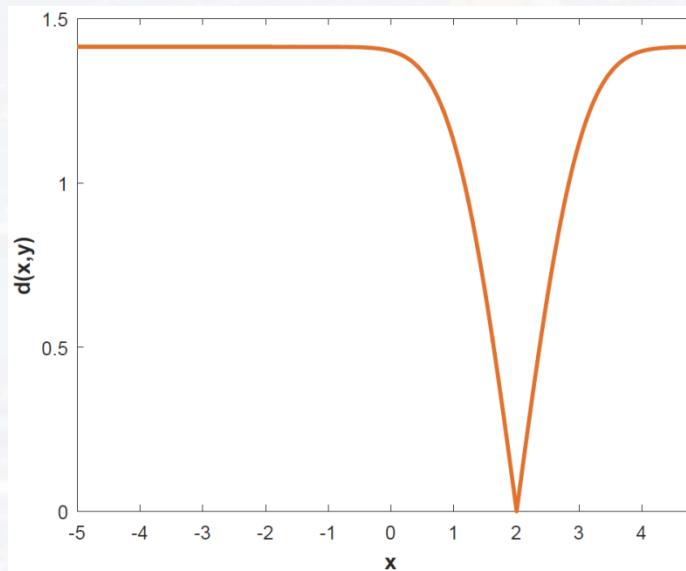


- 1) What if linear separation is not possible?
- 2) Is there a multiclass SVM?

example: : RBF (radial basis function)  $\phi: x \mapsto \zeta$   $\phi(x) = \sim \exp(\sim x)$

$$K(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right)$$

$$d^2(\phi(x), \phi(y)) = 2 \left[ 1 - \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right) \right]$$





- 1) What if linear separation is not possible?
- 2) Is there a multiclass SVM?

kernels available in sklearn:

- linear:

$$K(x, y) = \|x - y\|$$

- Gaussian aka RBF (radial basis function):

$$K(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right)$$

in sklearn we can adjust  $\gamma := \frac{1}{2\sigma^2}$

- polynomial:

$$K(x, y) = \sum_{n=1}^N \|x - y\|^n$$

in sklearn we can adjust  $N$

- sigmoidal:

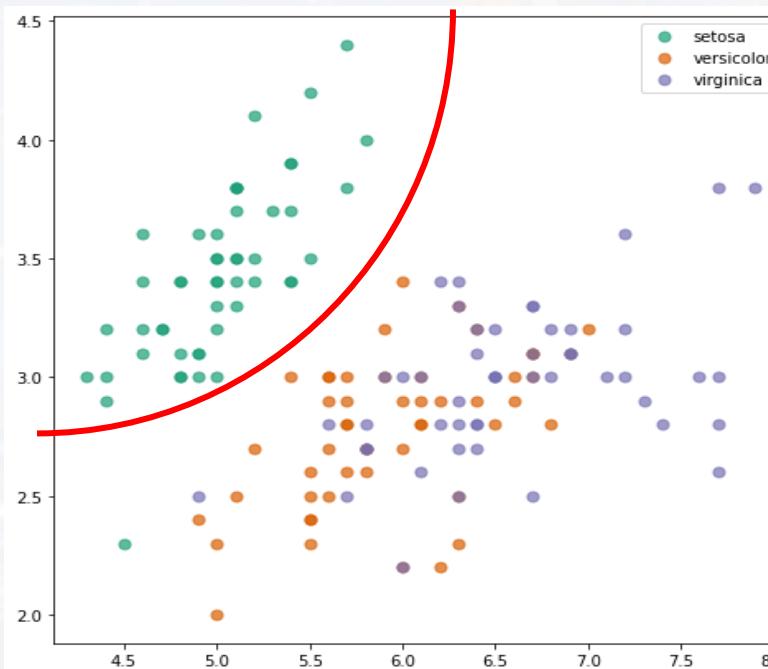
$$K(x, y) = \frac{e^{\|x-y\|}}{1 + e^{\|x-y\|}}$$



- 1) What if linear separation is not possible?
- 2) Is there a multiclass SVM?

→ one vs rest / one vs one

green vs the rest  
→ storing probabilities

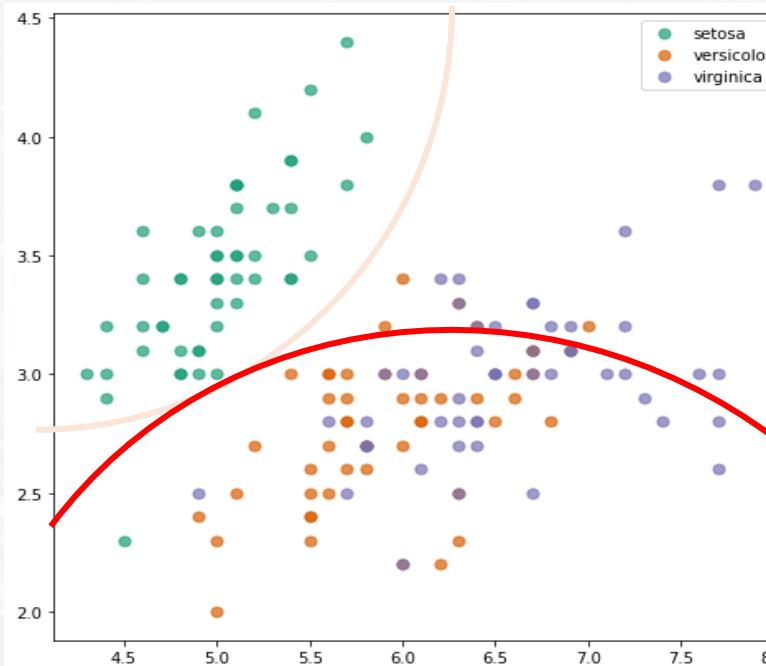




- 1) What if linear separation is not possible?
- 2) Is there a multiclass SVM?

→ one vs rest / one vs one

green vs the rest  
→ storing probabilities



orange vs the rest  
→ storing probabilities

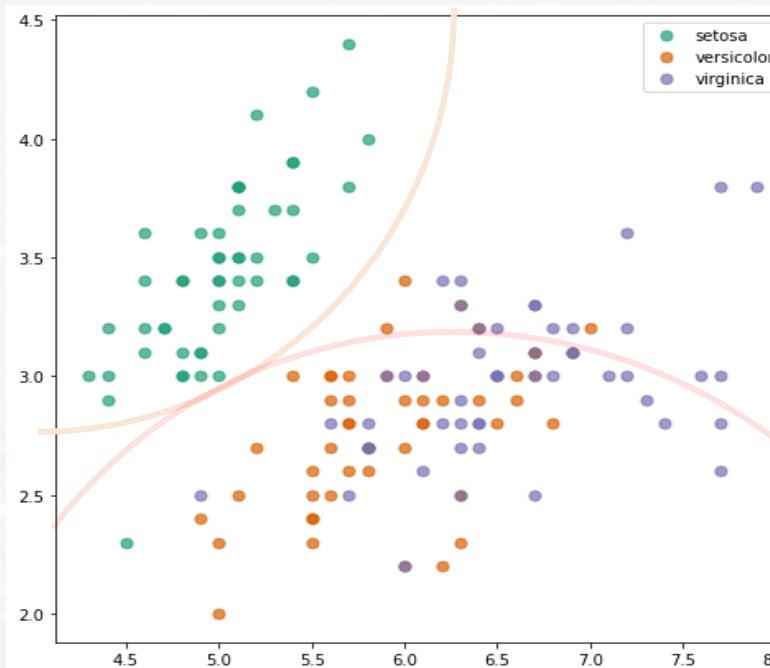




- 1) What if linear separation is not possible?
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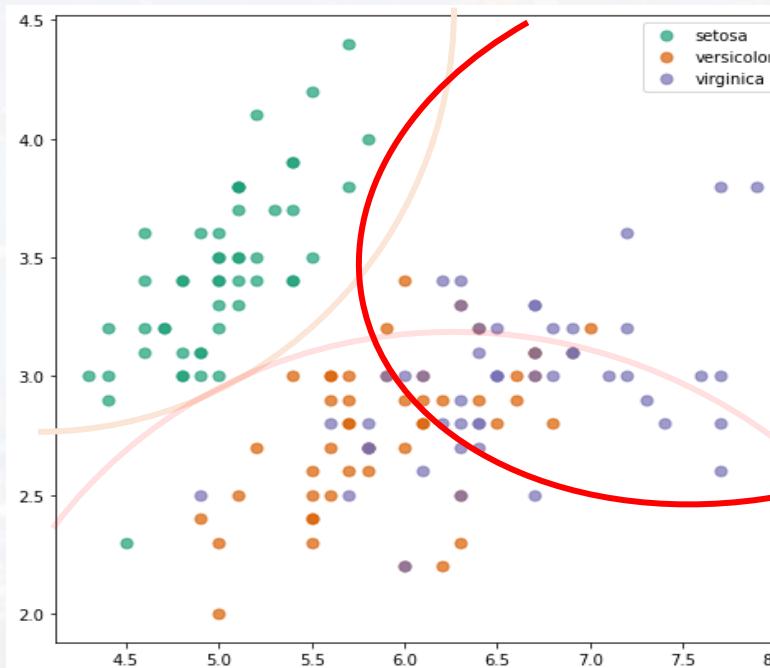




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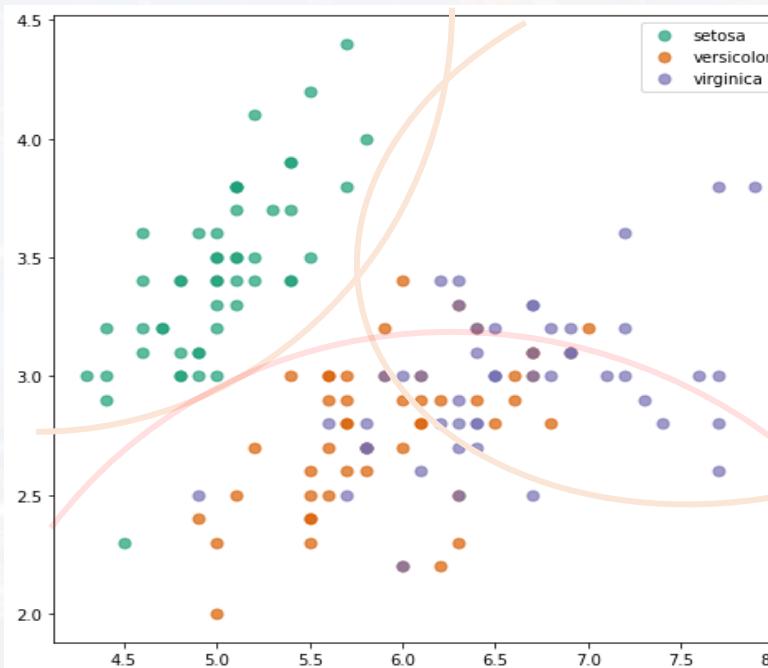




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blue vs the rest  
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orange vs the rest  
→ storing probabilities



three probabilities for each data point  
→ assign class to most probable value



```
import pandas as pd  
import matplotlib.pyplot as plt  
import numpy as np  
import seaborn as sns
```

our standard libraries

```
from sklearn import datasets  
  
iris = datasets.load_iris()  
  
iris.DESCR
```



Iris Versicolor



Iris Setosa



Iris Virginica



```
from sklearn import datasets
```

see [Walk\\_Through\\_SVM.ipynb](#)

loading & exploring the data:

```
iris = datasets.load_iris()  
  
iris.DESCR  
iris.feature_names  
iris.target_names  
  
['sepal length (cm)', four features → 4D  
 'sepal width (cm)',  
 'petal length (cm)',  
 'petal width (cm)']
```

```
array(['setosa', 'versicolor', 'virginica'])
```

Iris plants dataset

-----

\*\*Data Set Characteristics:\*\*

:Number of Instances: 150 (50 in each of three classes)  
:Number of Attributes: 4 numeric, predictive attributes and the class  
:Attribute Information:  
    - sepal length in cm  
    - sepal width in cm  
    - petal length in cm  
    - petal width in cm  
    - class:  
        - Iris-Setosa  
        - Iris-Versicolour  
        - Iris-Virginica

ideal world: three distinct cluster

:Summary Statistics:

	Min	Max	Mean	SD	Class Correlation
sepal length:	4.3	7.9	5.84	0.83	0.7826
sepal width:	2.0	4.4	3.05	0.43	-0.4194
petal length:	1.0	6.9	3.76	1.76	0.9490 (high!)
petal width:	0.1	2.5	1.20	0.76	0.9565 (high!)



```
from sklearn import datasets
```

see [Walk\\_Through\\_SVM.ipynb](#)

## Workflow

- 1) setting up the model
- 2) fitting the model:      **X** are the **features** (sepal lengths/widths and petal lengths/widths),  
                                **Y** are the **classes** (setosa, versicolor, virginica)
- 3) evaluating the model
- 4) applying the model to a new data set



```
from sklearn import svm
```

running analysis with different kernel

1) setting up the model & 2) fitting the model

```
outlinear = svm.SVC(kernel = 'Linear', C = 1, decision_function_shape = 'ovr')
```

```
linear = outlinear.fit(X2D, Y)
```

One Versus rest

```
outrbf = svm.SVC(kernel = 'rbf', gamma = 1, C = 1, \
```

parameter for error  
tolerance when calculating the classifier

```
rbf = outrbf.fit(X2D, Y)
```

$$\gamma := \frac{1}{2\sigma^2}$$

```
outpoly = svm.SVC(kernel = 'poly', degree = 3, C = 1, \
```

```
decision_function_shape = 'ovr')
```

```
poly = outpoly.fit(X2D, Y)
```

refers to  $N$  in  
 $\sum_{n=1}^N \|x - y\|^n$

```
outsig = svm.SVC(kernel = 'sigmoid', C = 1, decision_function_shape = 'ovr')
```

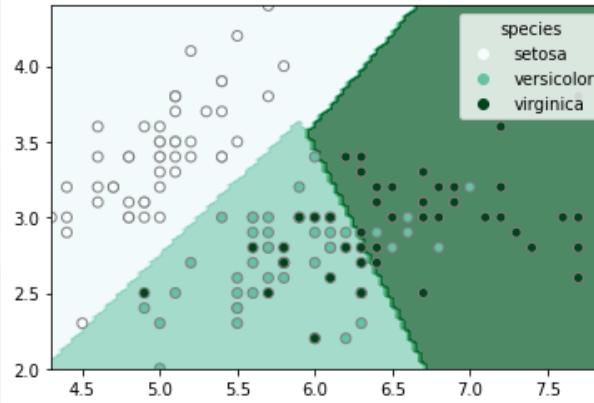
```
sig = outsig.fit(X2D, Y)
```



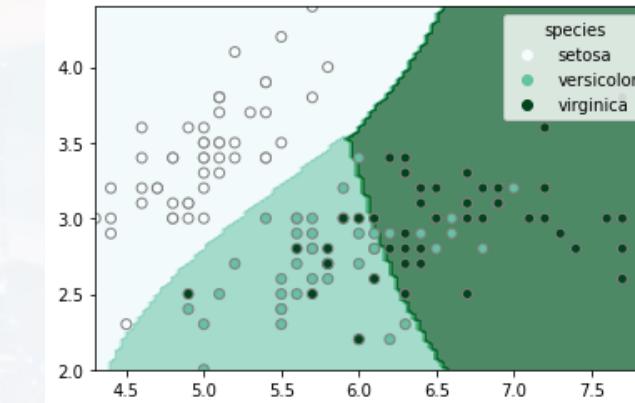
run: SVM.py to visualize the prediction

3) evaluating the model

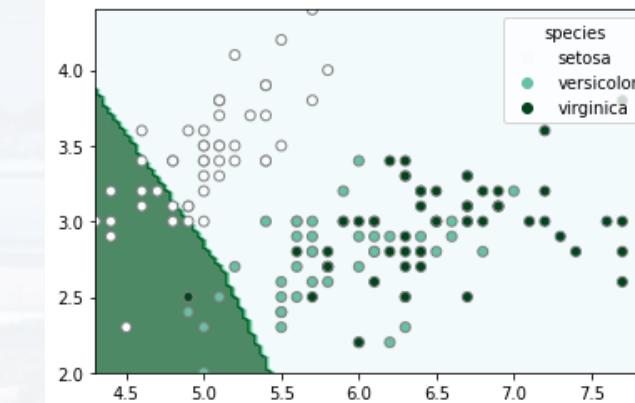
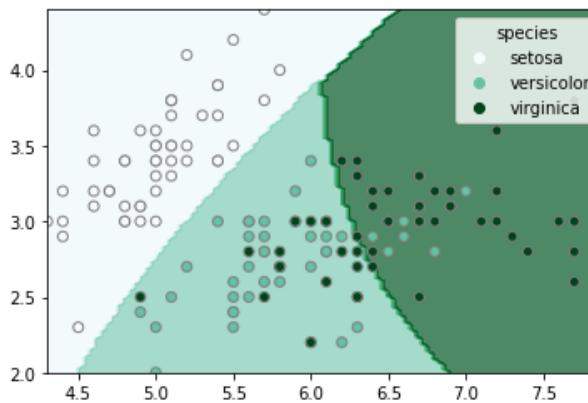
$$K(x, y) = \|x - y\|$$



$$K(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right)$$



$$K(x, y) = \sum_{n=1}^N \|x - y\|^n$$

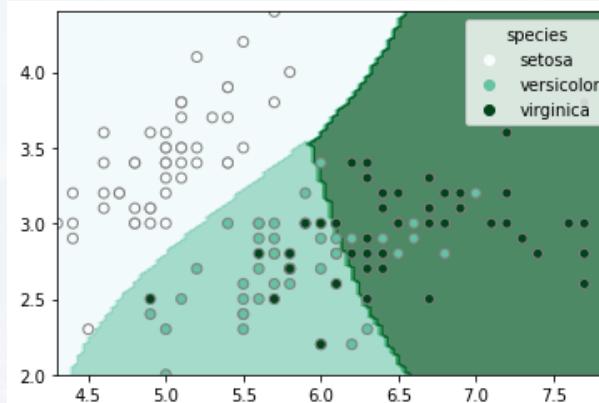




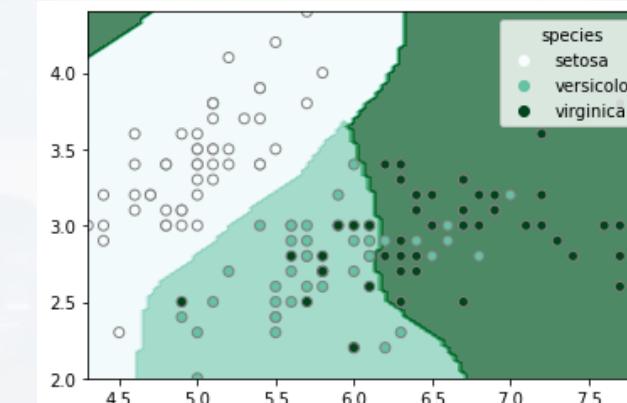
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3) evaluating the model

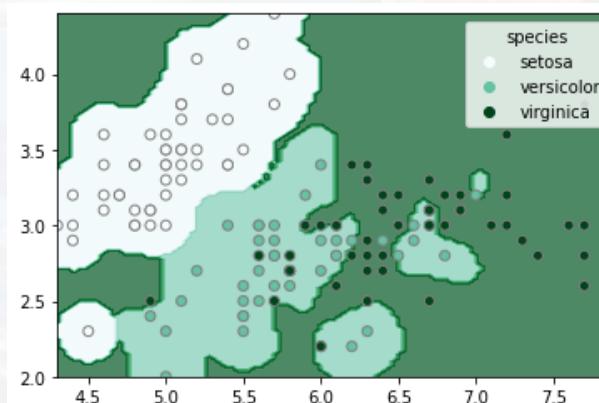
$$K(x, y) = \exp\left(-\frac{1}{2\sigma^2} \|x - y\|^2\right)$$



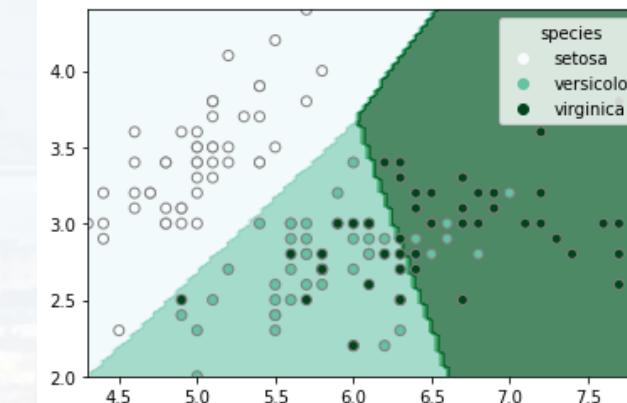
$$\gamma := \frac{1}{2\sigma^2} = 1$$



$$\gamma := \frac{1}{2\sigma^2} = 5$$



$$\gamma := \frac{1}{2\sigma^2} = 50$$



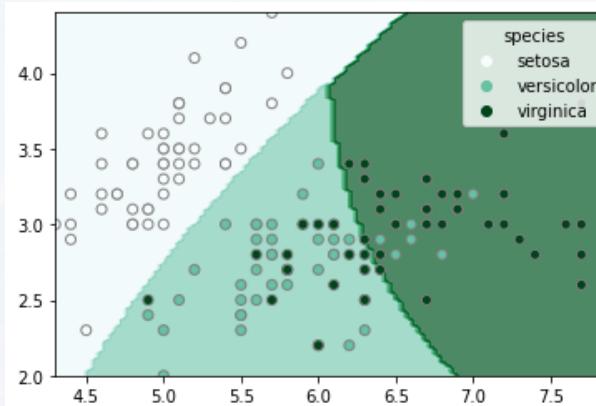
$$\gamma := \frac{1}{2\sigma^2} = 0.1$$



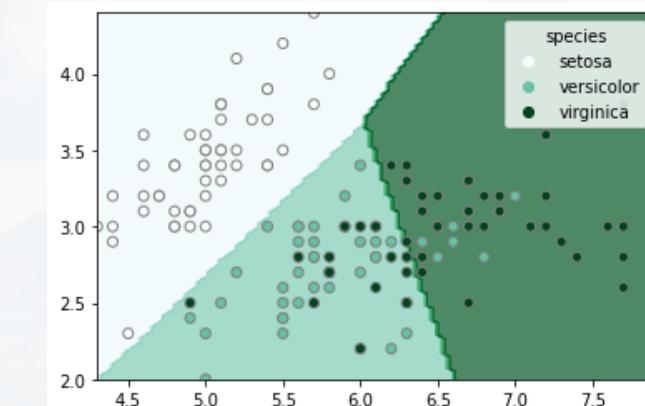
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3) evaluating the model

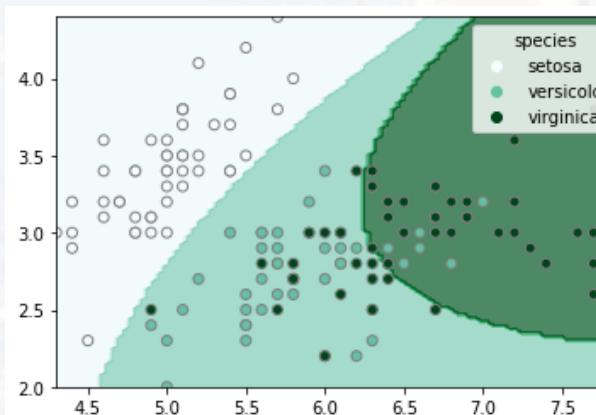
$$K(x, y) = \sum_{n=1}^N \|x - y\|^n$$



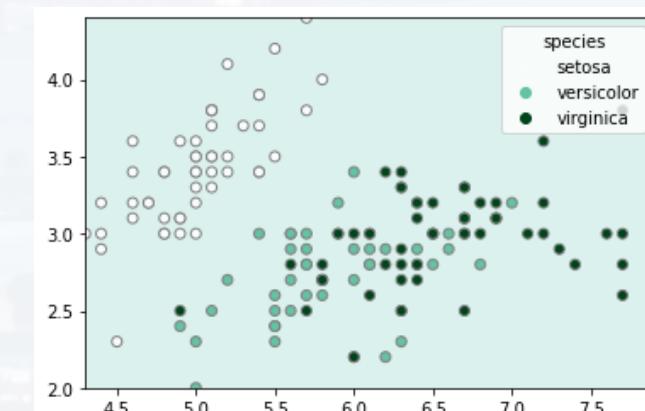
$N = 3$



$N = 1$



$N = 5$



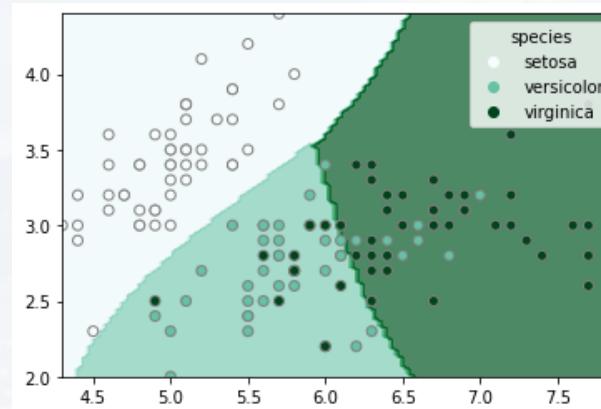
$N = 0$



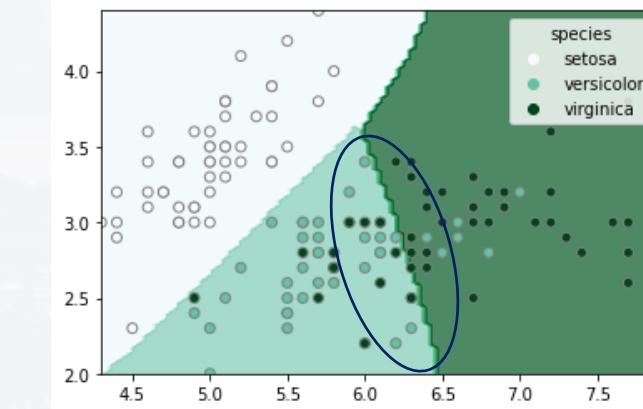
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3) evaluating the model

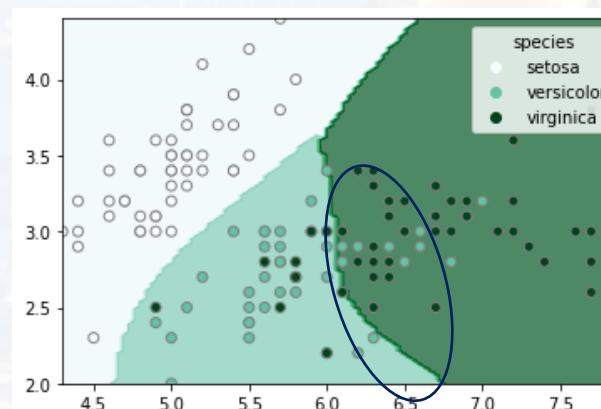
$$K(x, y) = \exp\left(-\frac{1}{2\sigma^2} \|x - y\|^2\right)$$



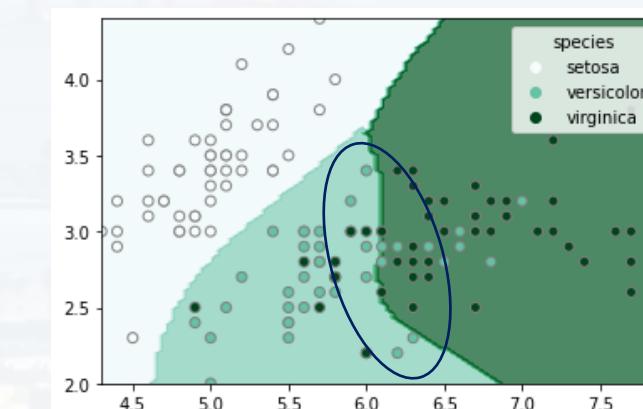
$$C = 1$$



$$C = 0.1$$



$$C = 10$$

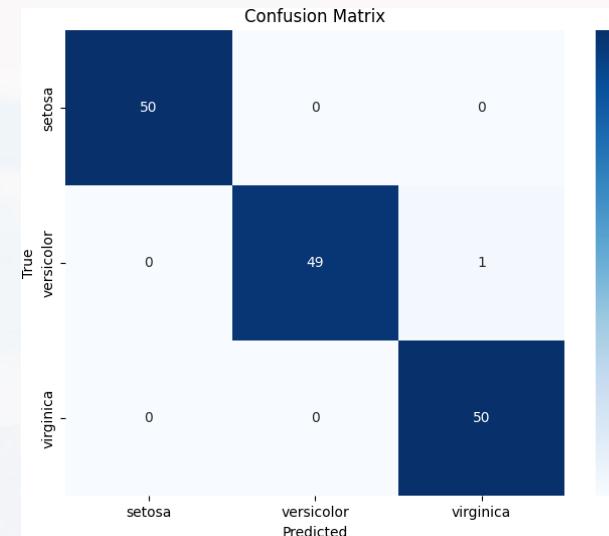


$$C = 20$$

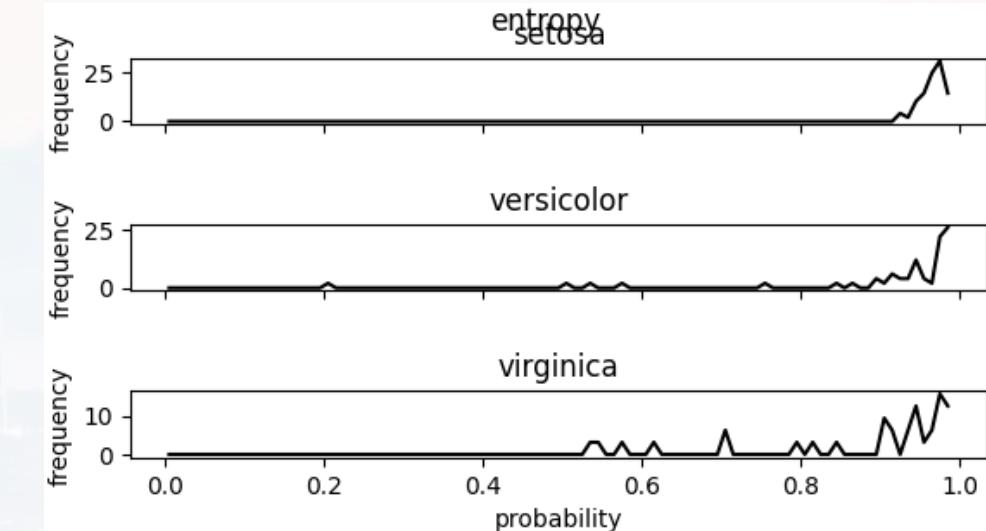


full 4D data set

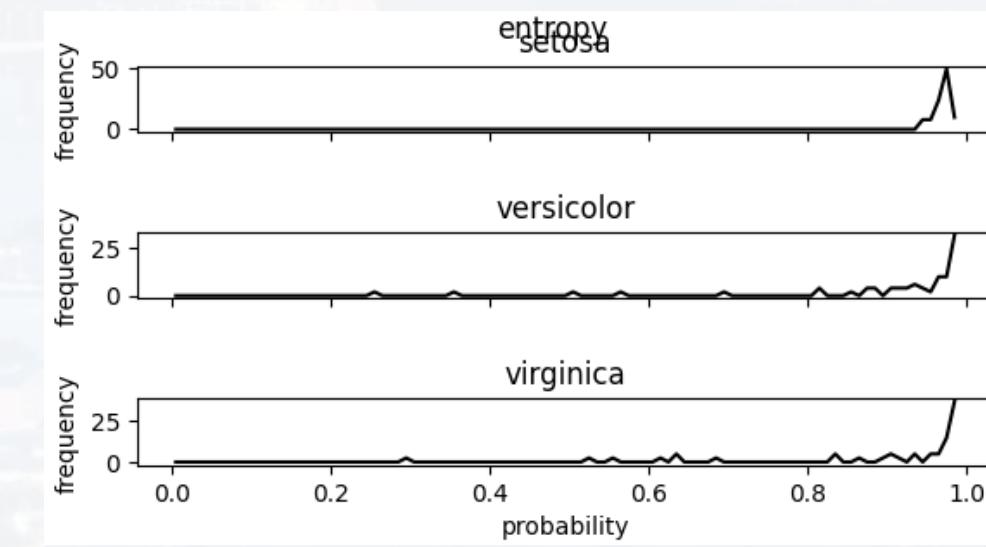
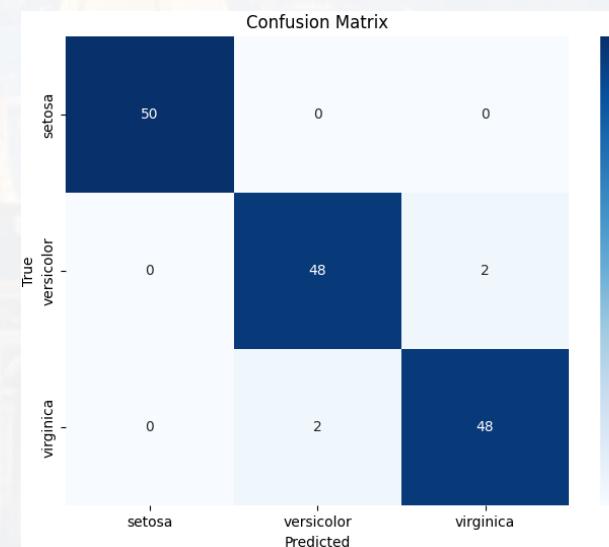
linear  
accuracy: 99.3%



3) evaluating the model



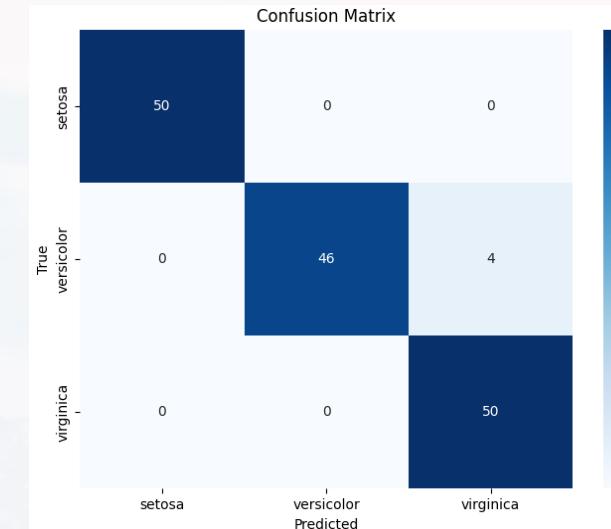
Gaussian ( $\gamma = 1$ )  
accuracy: 97.3%



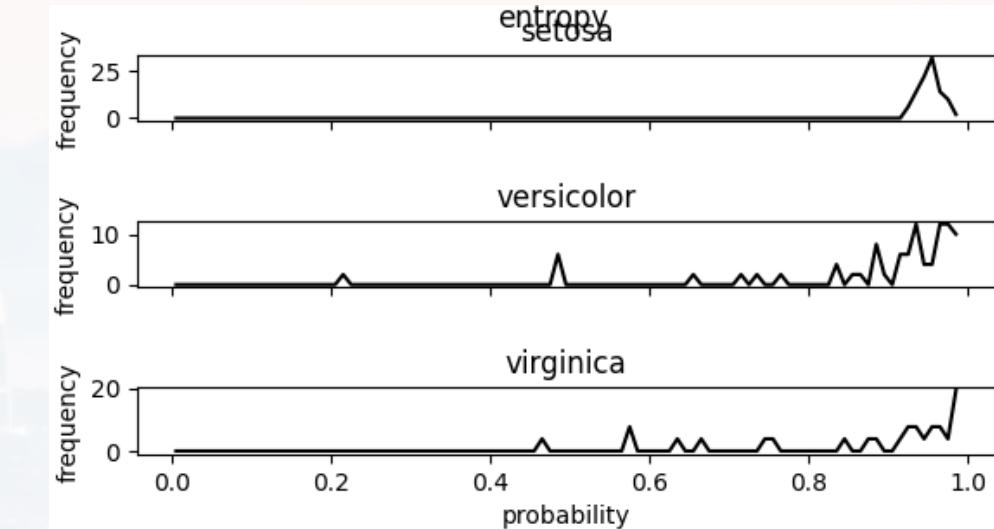


full **4D** data set

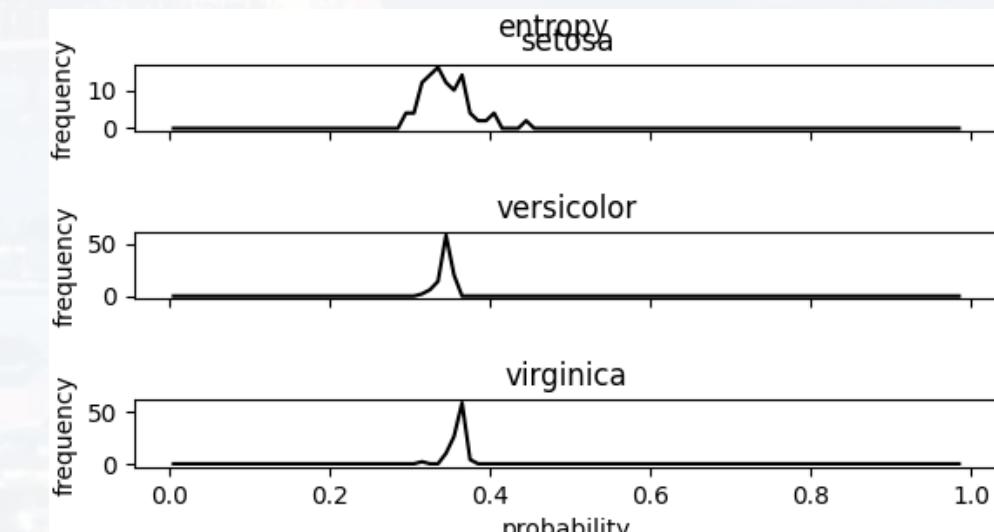
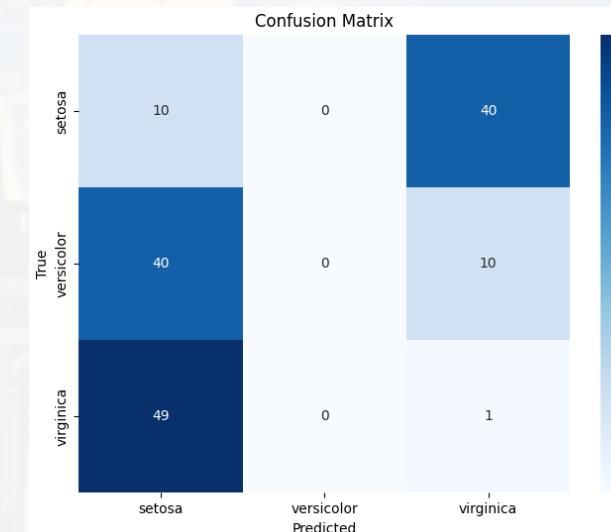
polynomial ( $n = 3$ )  
accuracy: 97.3%



3) evaluating the model



sigmoid  
accuracy: 7.3%

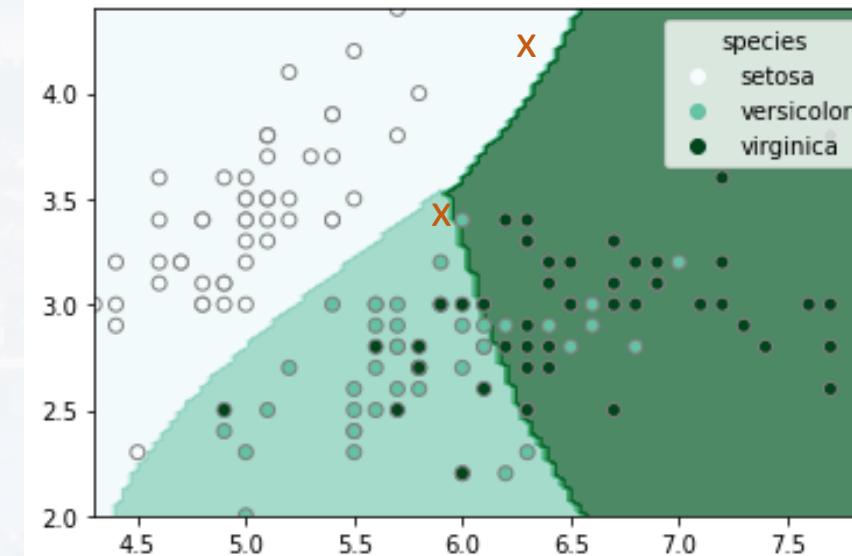




run: SVM.py to visualize the prediction

4) applying the model to a new data set

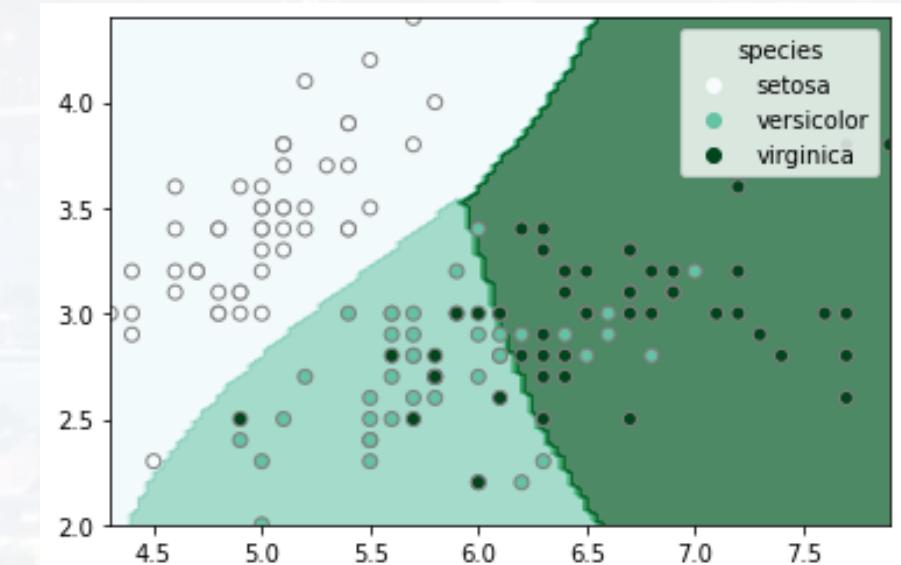
```
ypred = outpoly.predict([[6, 3.5],[6.3, 4.5]])
```





summary:

- simple and fast
- supervised
- *kernel* has to be given
- problems if cluster have unusual shapes  
(elongated, hollow inside, scattered)





### Concept

#### Supervised Learning

- Support Vector Machine
- K-nearest

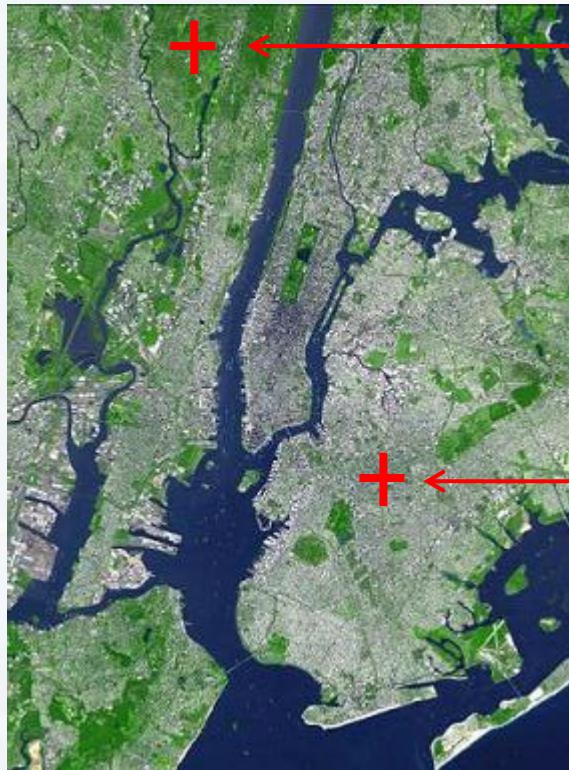
#### Unsupervised Learning

- K – means
- Gaussian Mixture Models



idea:

- 1) determine the **mode** of the categories of the  $k$  nearest neighbors
- 2) assign new data points to this category
- 3) **supervised** learning
- 4) has to be given:  $k$



most  $k$  “neighbors”  
are trees  
→ a new object  
must be a tree too



most  $k$  “neighbors”  
are buildings  
→ a new object  
must be a building too



idea:

- 1) determine the **mode** of the categories of the  $k$  nearest neighbors
- 2) **assign new data points to this category**
- 3) **supervised** learning
- 4) has to be given:  $k$

$$\hat{Y} = \text{mode } \{Y_i\}_{x_i \in N_k(x)}$$

$k$  neighborhood of  $x$   $N_k(x)$





idea:

- 1) determine the **mode** of the categories of the  $k$  nearest neighbors
- 2) **assign new data points to this category**
- 3) **supervised** learning
- 4) has to be given:  $k$





idea:

- 1) determine the **mode** of the categories of the  $k$  nearest neighbors
- 2) assign new data points to this category
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idea:

- 1) determine the **mode** of the categories of the  $k$  nearest neighbors
- 2) assign new data points to this category
- 3) **supervised** learning
- 4) **has to be given:  $k$**





```
import matplotlib.pyplot as plt  
import numpy as np
```

our standard libraries

```
from pyclustering.utils.metric import *  
from sklearn.neighbors import KNeighborsClassifier  
from sklearn import datasets
```

for having different distances available

performing knears classification

calling the “iris” data set



1) + 2) running and fitting the model (again: 2D here only for visualization!)

```
X2D = X[:, 0:2]
```

check out `Walk_Through_Knearest.ipynb`

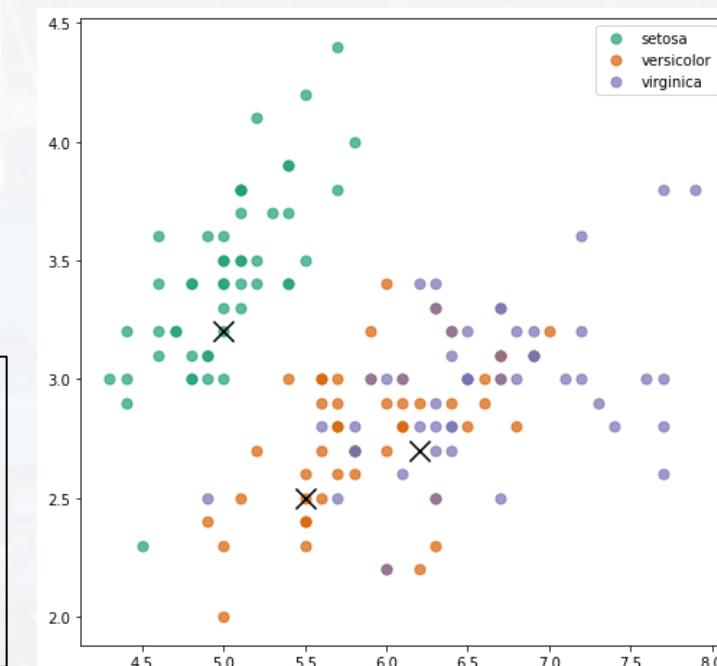
```
k      = 5
out = KNeighborsClassifier(k,
                           metric = distance_metric(type_metric.EUCLIDEAN))
out.fit(X2D, Y)
```

3) + 4) after evaluation: predicting the classes of new data

```
data_new = np.array( [[5, 3.2], [5.5, 2.5], [6.2, 2.7]] )
```

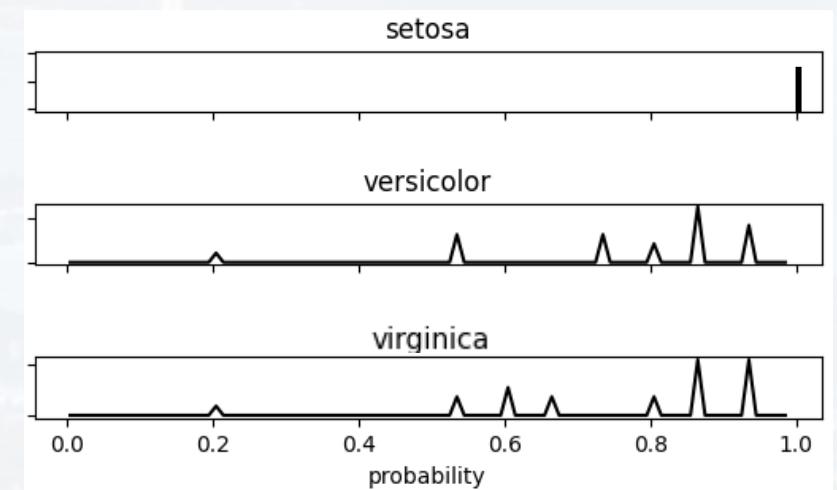
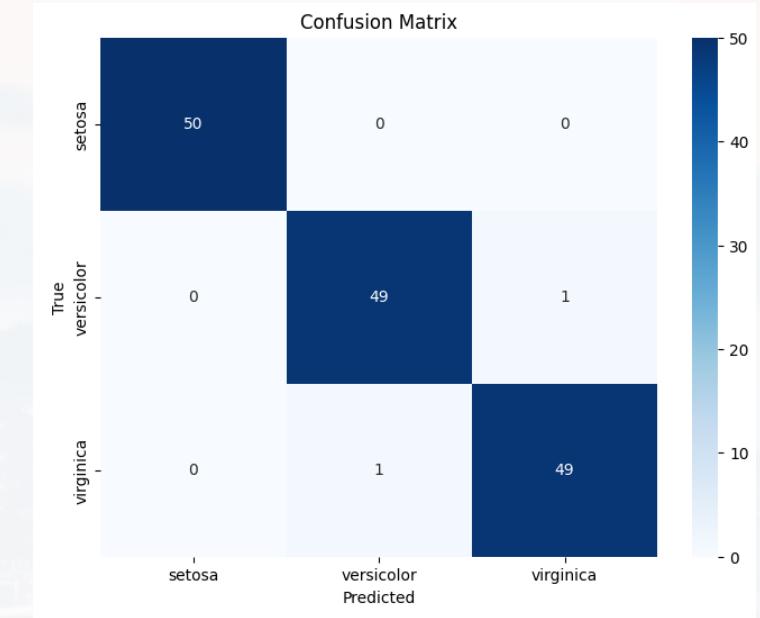
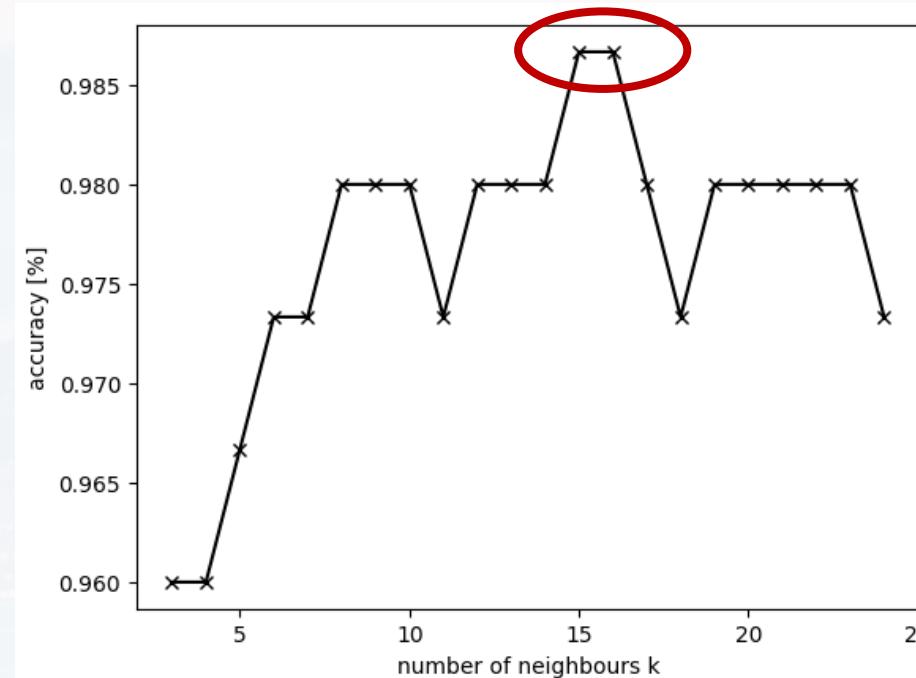
```
Pred      = out.predict(data_new)
Prob      = out.predict_proba(data_new)
print(Prob)
print(Pred)
```

'setosa'	'versicolor'	'virginica'
1.0	0.0	0.0
0.0	1.0	0.0
0.0	0.4	0.6





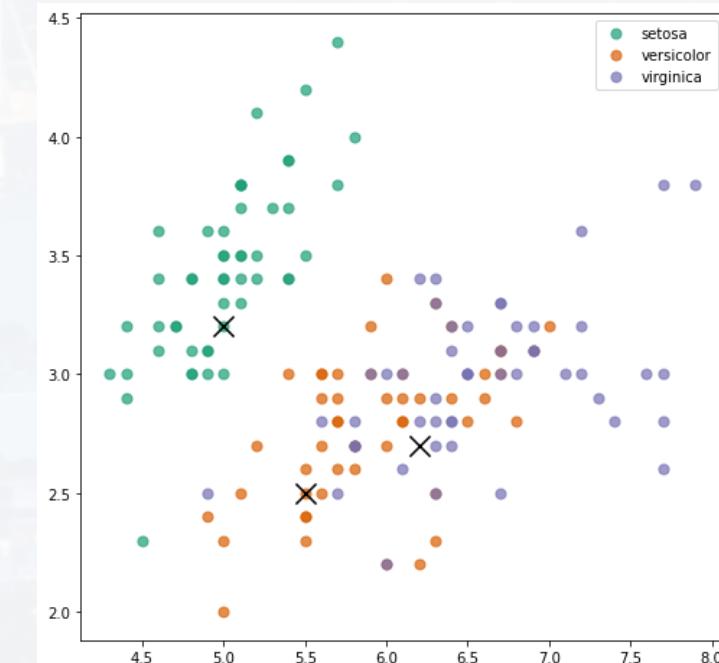
4D dataset and different number of neighbors k





#### summary:

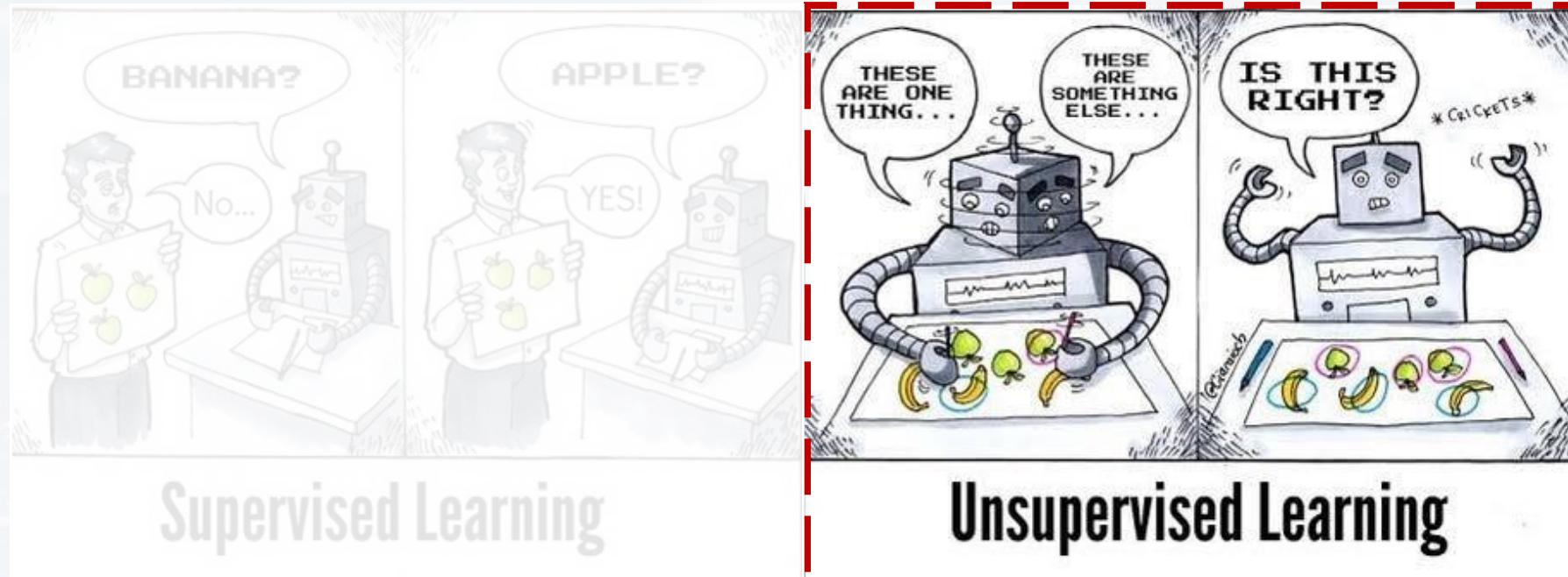
- simple and fast
- supervised
- $k$  has to be given
- problems if cluster are overlapping





curve fitting, linear/logistic models:  
training data set and a **test** data set...

**no training** data set required – we can start right away!



- Support Vector Machine
- K-nearest

- **K - means**
- **GMM**



### Concept

#### Supervised Learning

- Support Vector Machine
- K-nearest

#### Unsupervised Learning

- K – means
- Gaussian Mixture Models



## Workflow

1) setting up the model

2) fitting the model: **X** are the **features** (sepal lengths/widths and petal lengths/widths),  
~~**Y** are the **classes** (*setosa*, *versicolor*, *virginica*)~~

3) evaluating the model

4) applying the model to a new data set

now: **unsupervised** learning

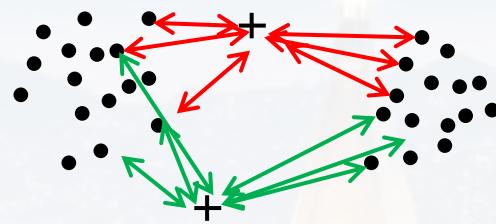
- we don't know the classes
- we don't know the **number k of classes**



idea:



a) assign  $k$  means randomly



b) calculate *distance* from each point to each mean



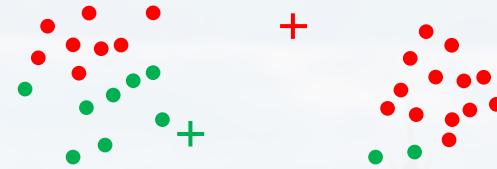
c) assign each point to its closest mean



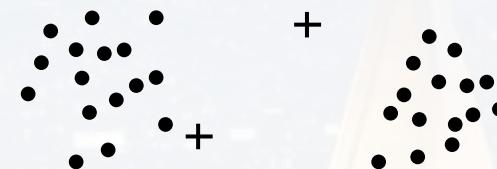
d) update the means accordingly



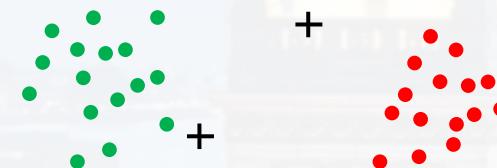
idea:



d) update the means accordingly



e) go back to b)





problem:  $k = \text{number of cluster}$ , is a hyperparameter. How do I know the correct value for  $k$ ?

→ silhouette  $\Psi$

- distance  $d_1$  of a data point  $x_0$  to *its assigned cluster  $S_i$*   
vs distance  $d_2$  to *closest cluster (here  $S_j$ )*

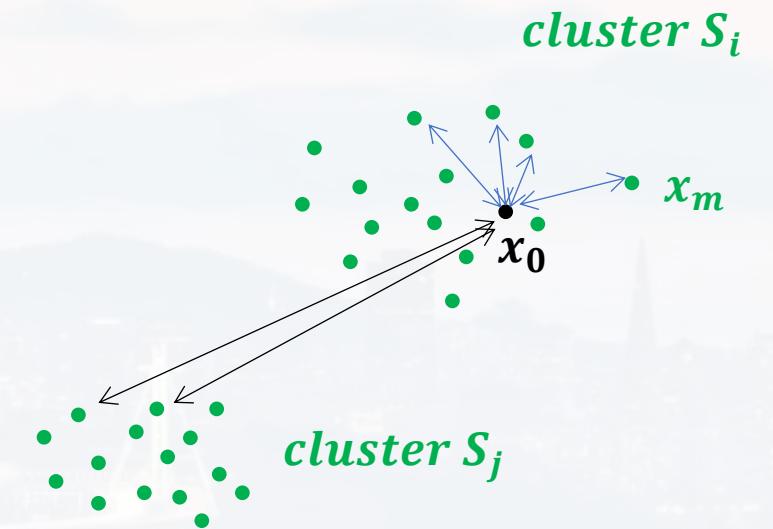
$$\Psi(x_0) = \begin{cases} 0 & \text{if } d_1 = 0 \\ \frac{d_2 - d_1}{\max[d_1; d_2]} & \text{otherwise} \end{cases}$$

- average over all points →  $\Psi_{tot}$

if

$$\begin{aligned} \Psi_{tot} &= 0.75 \dots 1.00 \\ \Psi_{tot} &= 0.50 \dots 0.75 \\ \Psi_{tot} &= 0.25 \dots 0.50 \\ \Psi_{tot} &< 0.25 \end{aligned}$$

- well clustered
- medium clustered
- poorly clustered
- data has no structure



problem:  $k$  is a hyperparameter. How do I know the correct value for  $k$ ?

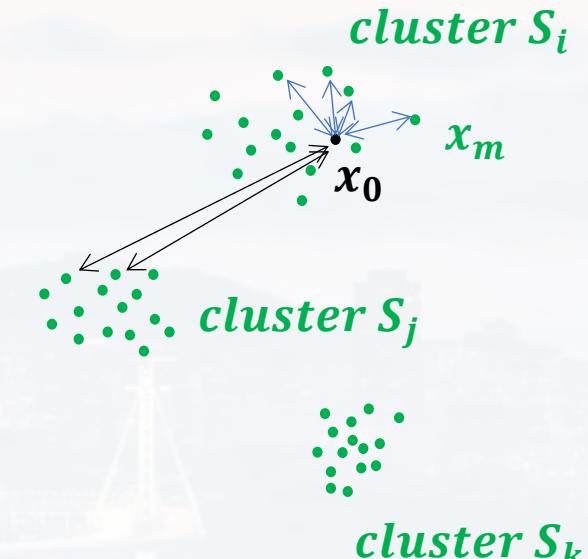
→ silhouette  $\Psi$

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vs distance  $d_2$  to *closest cluster (here  $S_j$ )*

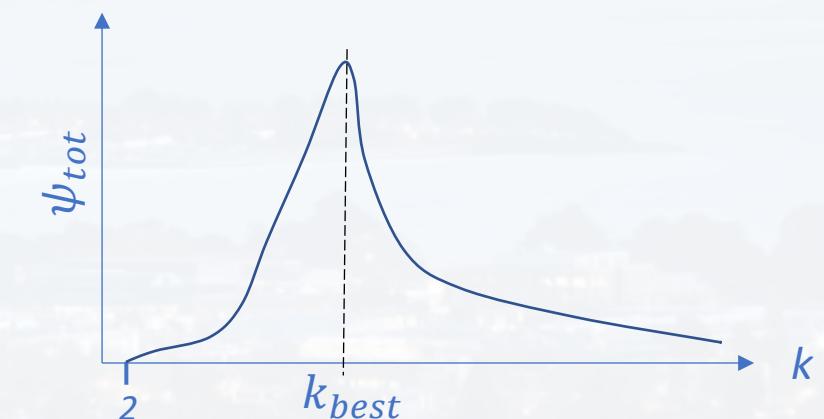
$$\Psi(x_0) = \begin{cases} 0 & \text{if } d_1 = 0 \\ \frac{d_2 - d_1}{\max[d_1; d_2]} & \text{otherwise} \end{cases}$$

- average over all points →  $\psi_{tot}$

$\psi_{tot} = 0.75 \dots 1.00$	→ well clustered
$\psi_{tot} = 0.50 \dots 0.75$	→ medium clustered
$\psi_{tot} = 0.25 \dots 0.50$	→ poorly clustered
$\psi_{tot} < 0.25$	→ data has no structure



ideal world →





```
import pandas as pd  
import matplotlib.pyplot as plt  
import numpy as np  
import seaborn as sns
```

our standard libraries

```
from pyclustering.utils.metric import *  
from nltk.cluster.kmeans import KMeansClusterer  
from sklearn.metrics import silhouette_samples, silhouette_score
```

for having different distances available

performing k-means

calculating silhouette coefficient for different k

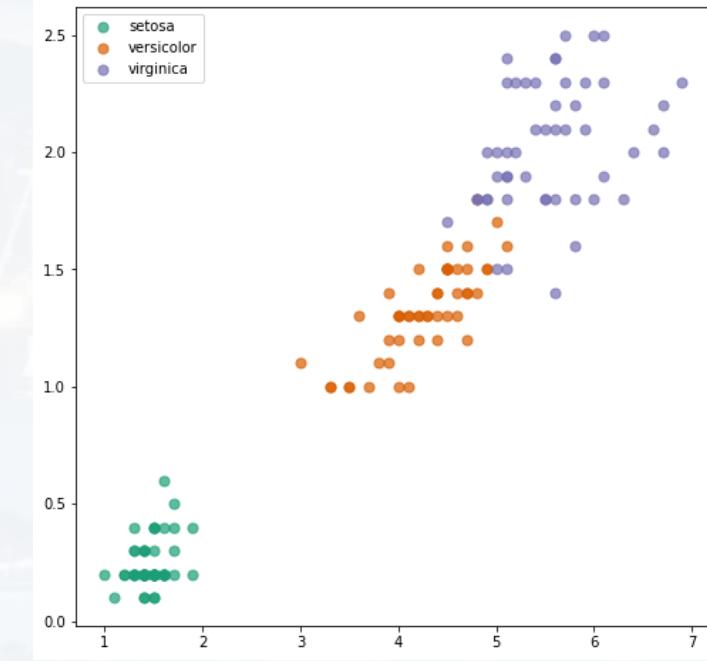
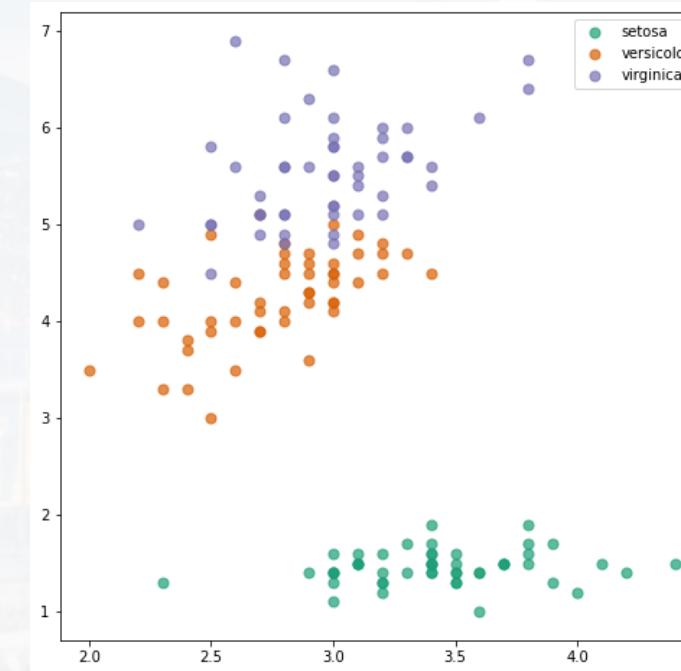
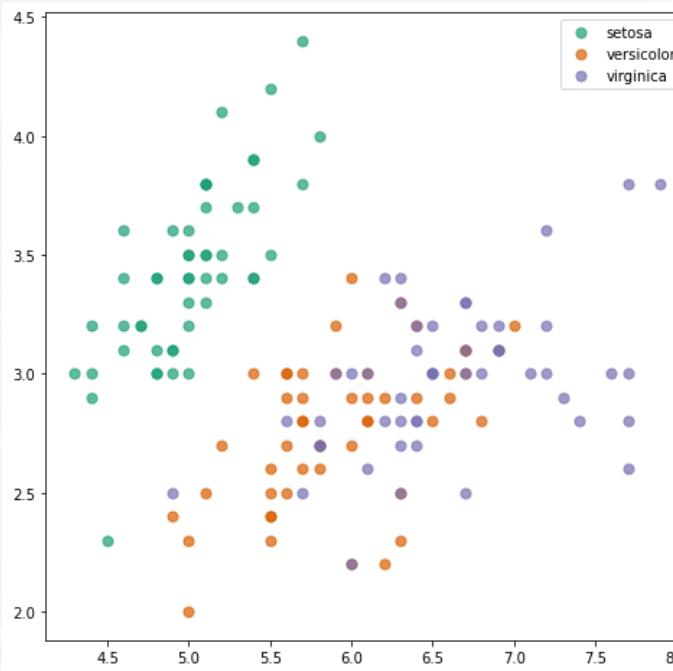


check out the Jupyter Notebook [Walk\\_Through\\_Kmeans.ipynb](#)

```
['sepal length (cm)',  
 'sepal width (cm)',  
 'petal length (cm)',  
 'petal width (cm)']
```

4D dataset → plotting two components

- plotting the data
- running k-means
- evaluating the result





```
nClust      = 3
rep         = 25
dist        = distance_metric(type_metric.EUCLIDEAN)
```

- plotting the data
- running k-means
- evaluating the result

we need to “guess” the number of cluster

```
my_model    = KMeansClusterer(nClust, distance = dist,\n                             repeats  = rep,\n                             avoid_empty_clusters = True)
```

the initial means are assigned randomly.  
→ repeat the procedure 25 times  
→ avoiding local minimum,

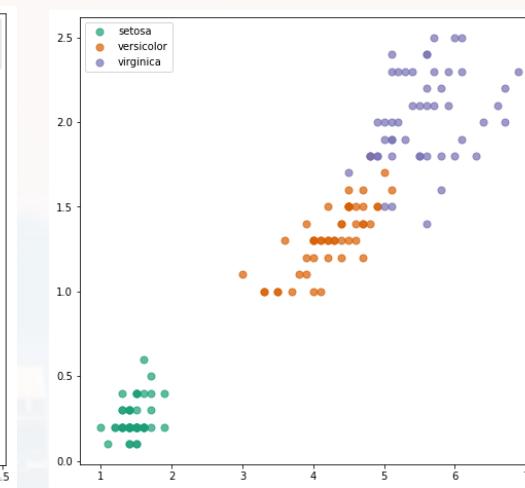
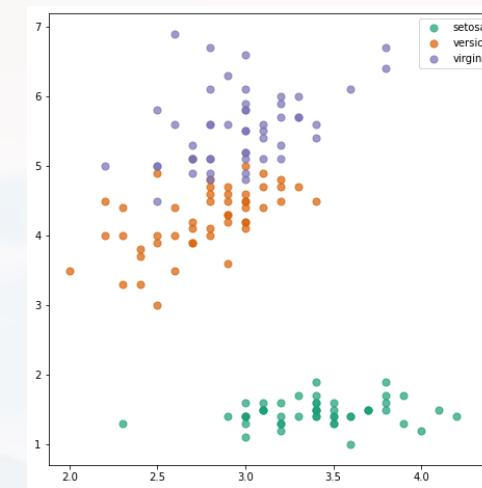
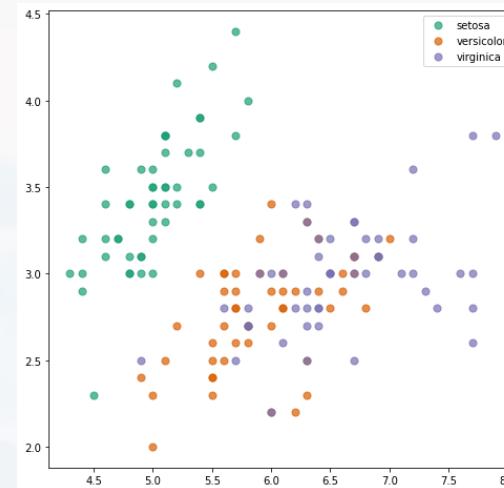
```
PredLabels = my_model.cluster(X2D,\n                           assign_clusters = True)
```

```
Center      = my_model.means()
```

the features are measured in cm, i. e. the correct distance to pick here is Euclidean

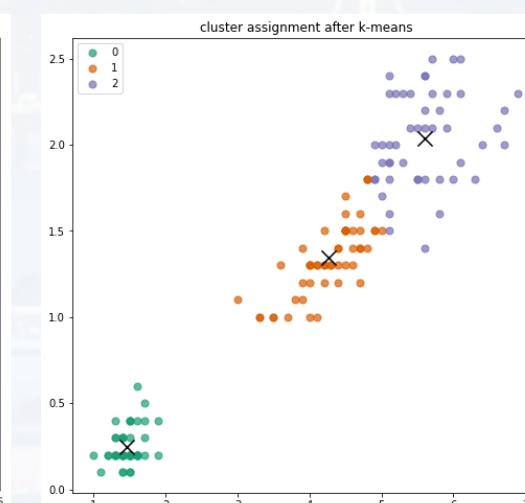
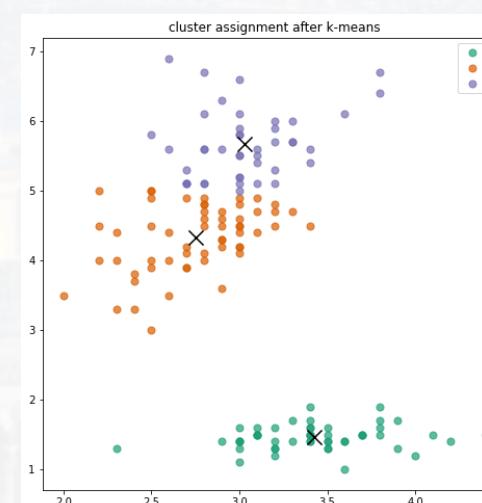
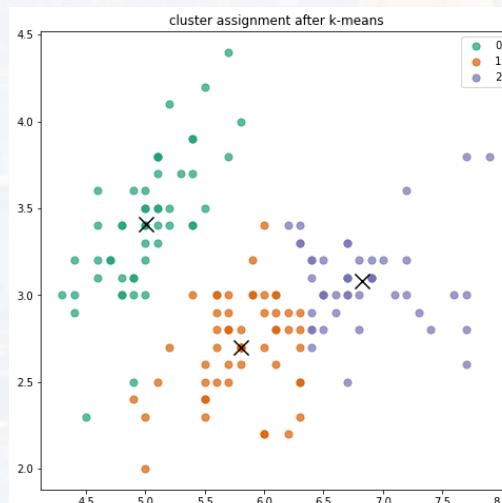


true classes



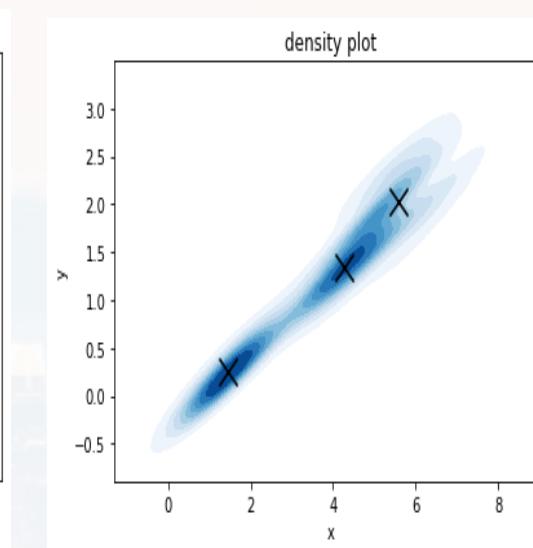
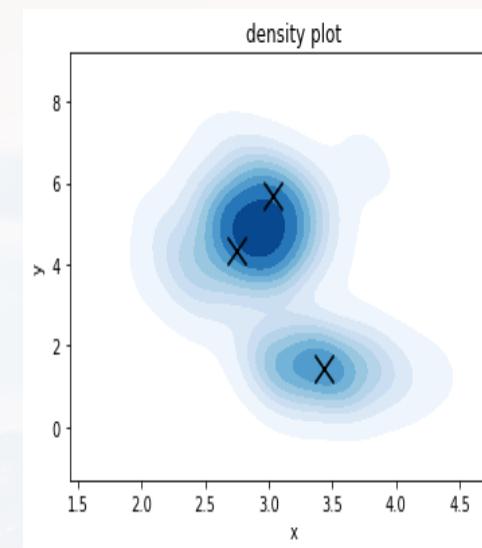
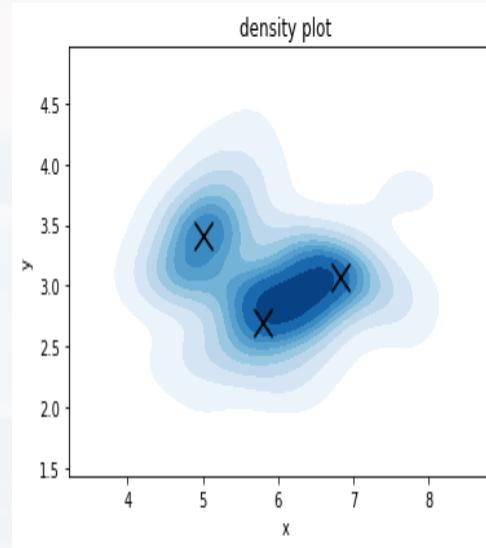
- plotting the data
- running k-means
- evaluating the result

assigned classes



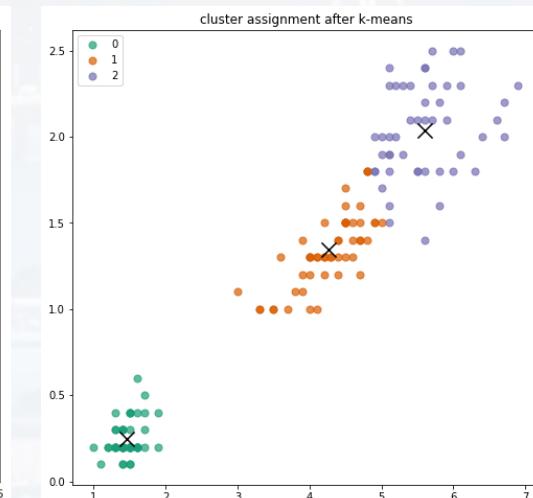
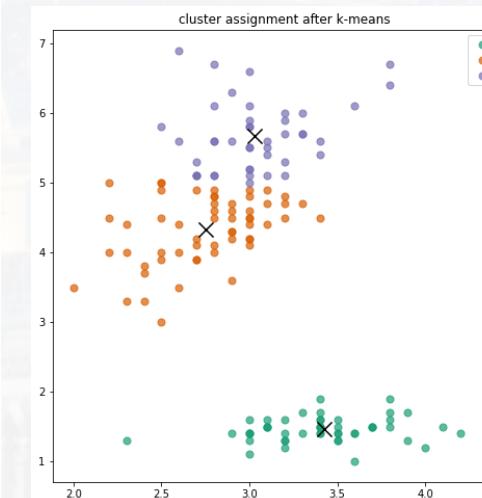
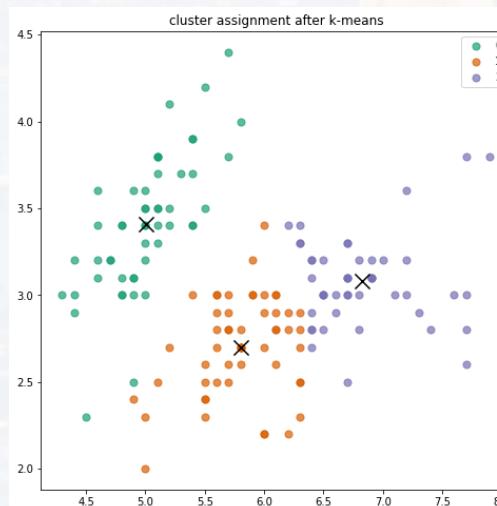


density



- plotting the data
- running k-means
- evaluating the result

assigned classes

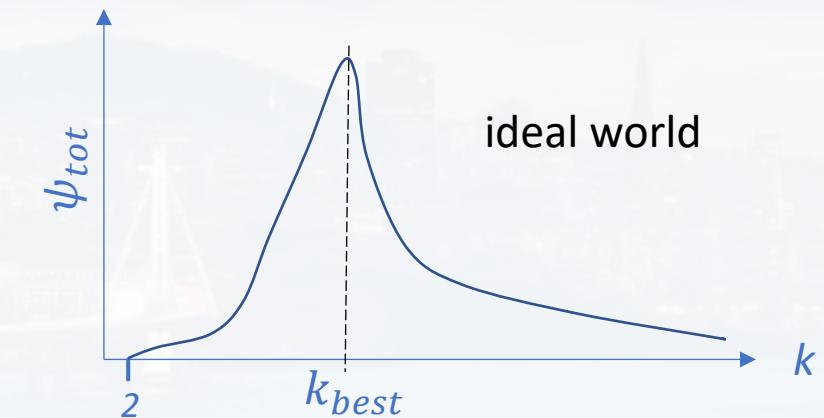
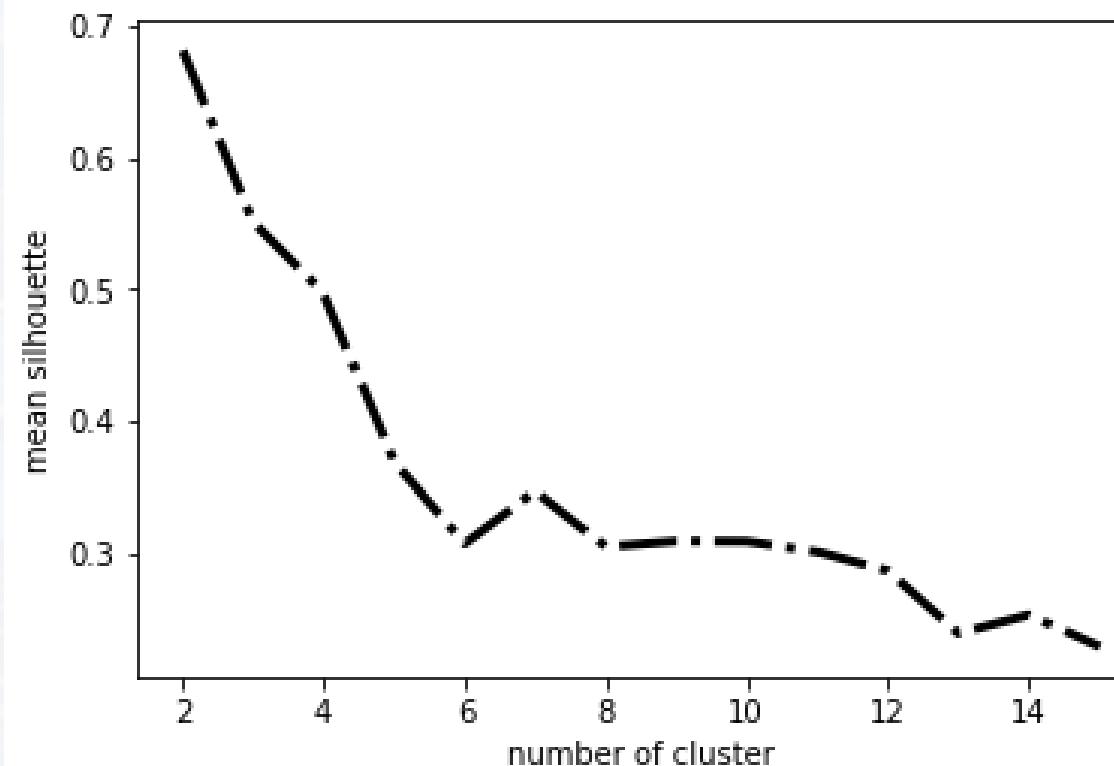




we run k-means now for the **full 4D** dataset  
+ evaluate clustering with silhouette

- plotting the data
- running k-means
- evaluating the result**

`silhouette_score(X, Labels)`

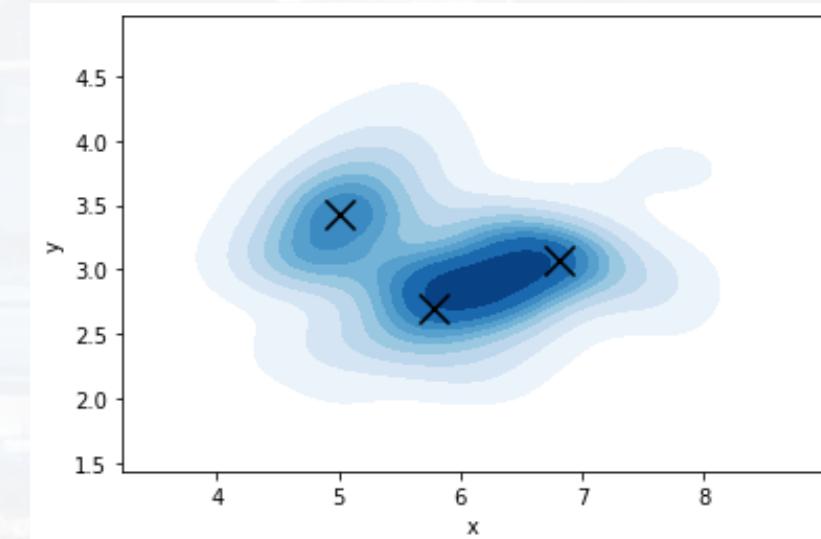
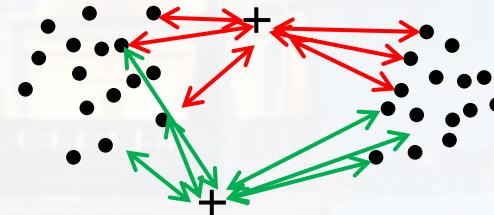


**accuracy for 4D  $k = 3$ : 90%**



### summary:

- simple and fast
- unsupervised
- $k$  has to be given → silhouette for determining best  $k$
- problems if cluster have unusual shapes  
(elongated, hollow inside, scattered)





### Concept

#### Supervised Learning

- Support Vector Machine
- K-nearest

#### Unsupervised Learning

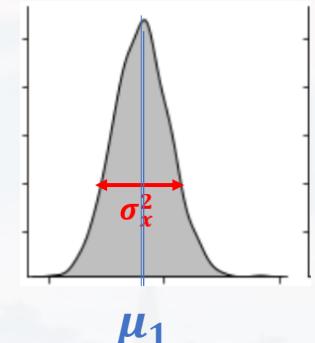
- K – means
- Gaussian Mixture Models



## Gaussian Mixture Models

one feature

$$N_1(x_1) = \frac{1}{\sqrt{2\pi \sigma_{x1}^2}} \exp\left[-\frac{1}{2}\left(\frac{x_1 - \mu_1}{\sigma_{x1}}\right)^2\right]$$

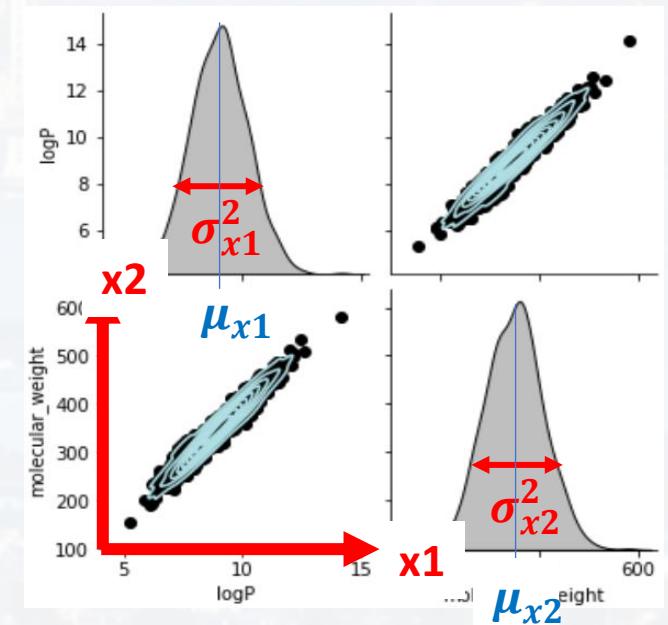


two features

$$\Sigma = \begin{pmatrix} \sigma_{x1}^2 & cov(x_1, x_2) \\ cov(x_2, x_1) & \sigma_{x2}^2 \end{pmatrix} \quad \text{covariance matrix}$$

$$\begin{pmatrix} x_1 - \mu_{x1} \\ x_2 - \mu_{x2} \end{pmatrix}^T \Sigma^{-1} \begin{pmatrix} x_1 - \mu_{x1} \\ x_2 - \mu_{x2} \end{pmatrix}$$

$$N_2(x_1, x_2) = \frac{1}{2\pi \det(\Sigma)^{1/2}} \exp\left\{-\frac{1}{2} \left[ \begin{pmatrix} x_1 - \mu_{x1} \\ x_2 - \mu_{x2} \end{pmatrix}^T \Sigma^{-1} \begin{pmatrix} x_1 - \mu_{x1} \\ x_2 - \mu_{x2} \end{pmatrix} \right] \right\}$$



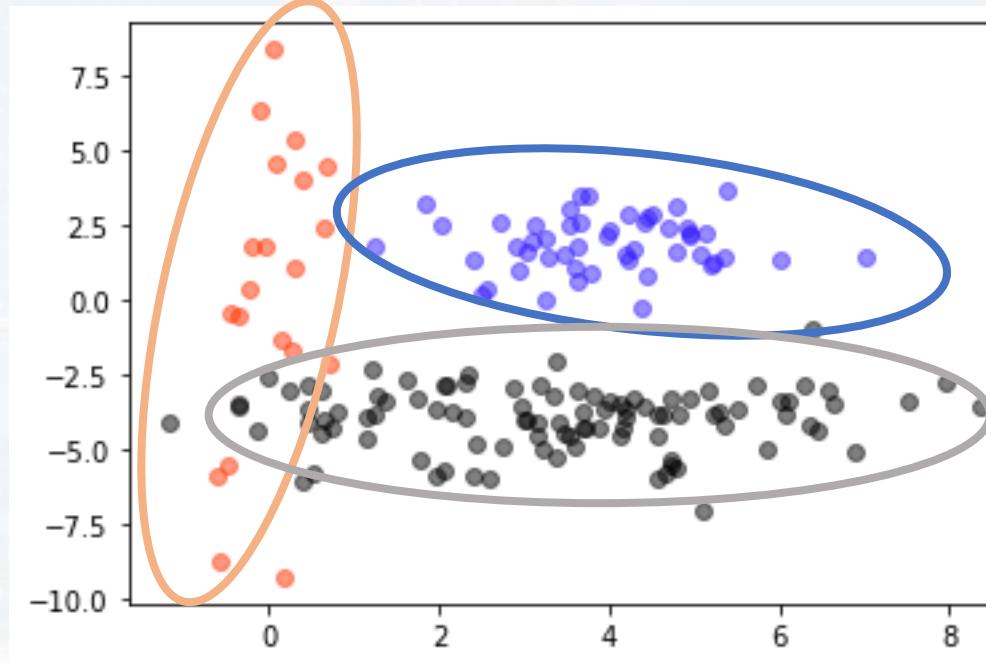


## Gaussian Mixture Models

n features

$$N_k(x_1, x_2 \dots x_n) = \frac{1}{(2\pi)^{n/2} \det(\Sigma)^{1/2}} \exp \left[ -\frac{1}{2}(x - \mu)^T \Sigma^{-1} (x - \mu) \right]$$

vectors  $x$  and  $\mu$

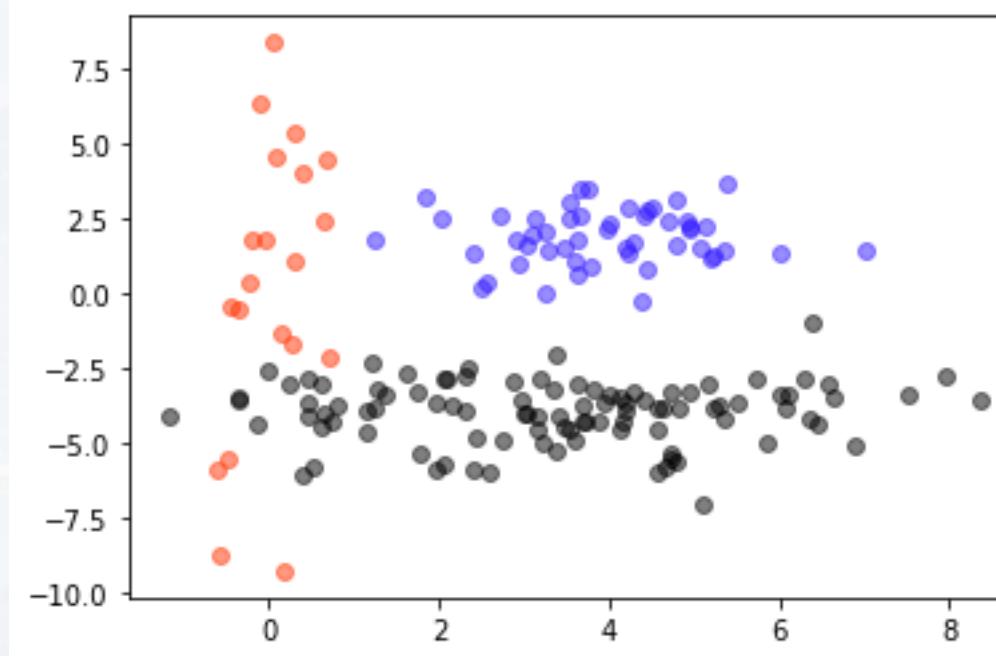


two features,  $k=3$  components

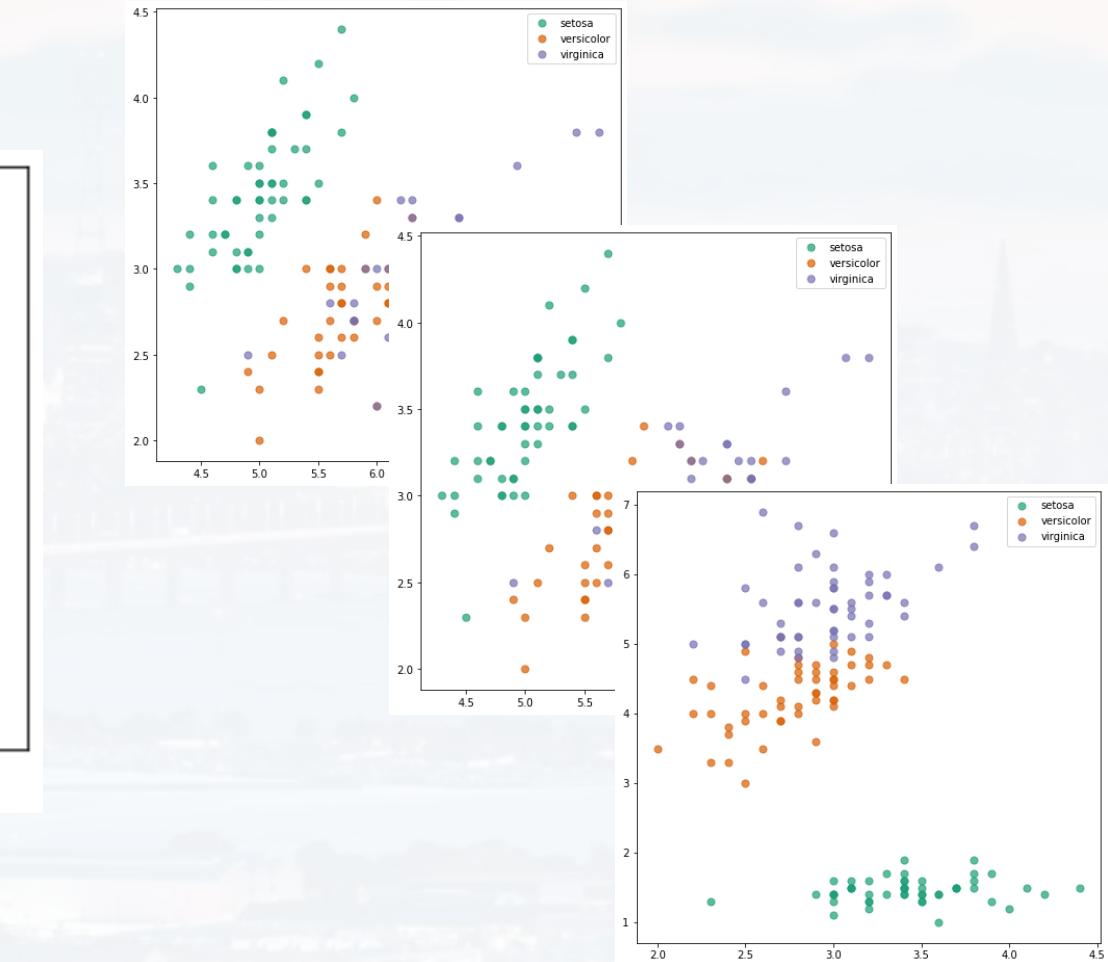


### Gaussian Mixture Models

two features,  $k=3$  components



four features,  $k=3$  components





## Gaussian Mixture Models

idea: fitting the data to a GMM → analytical functions to **calculate** probabilities for labels

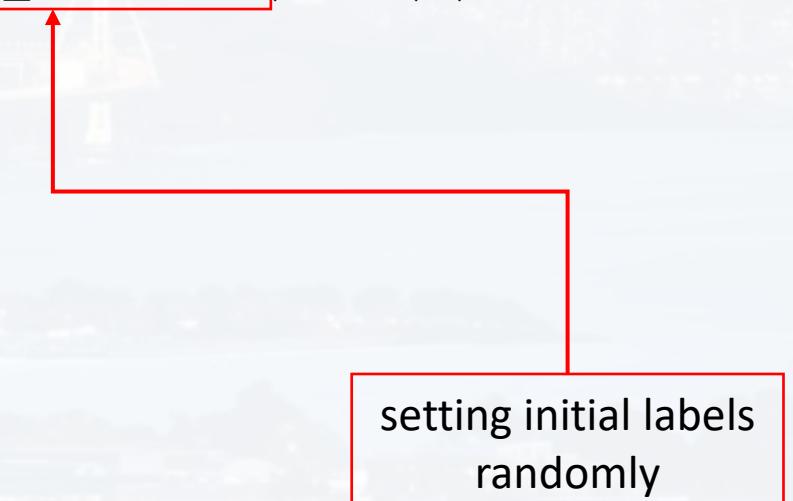
different algorithms:

- Bayesian
- Expectation Maximization
- ...

```
my_model = GaussianMixture(n_components = k, random_state = 0).fit(X)
```

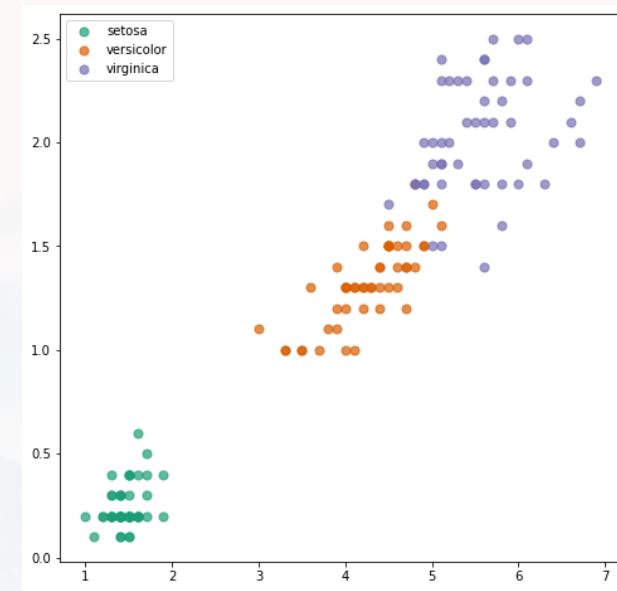
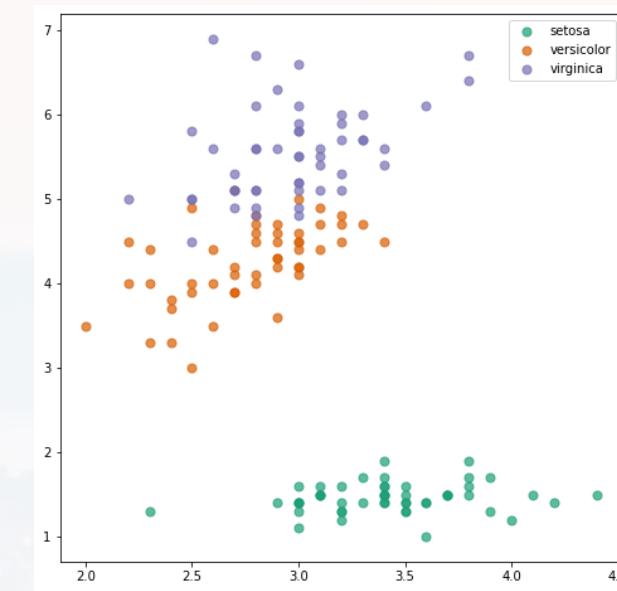
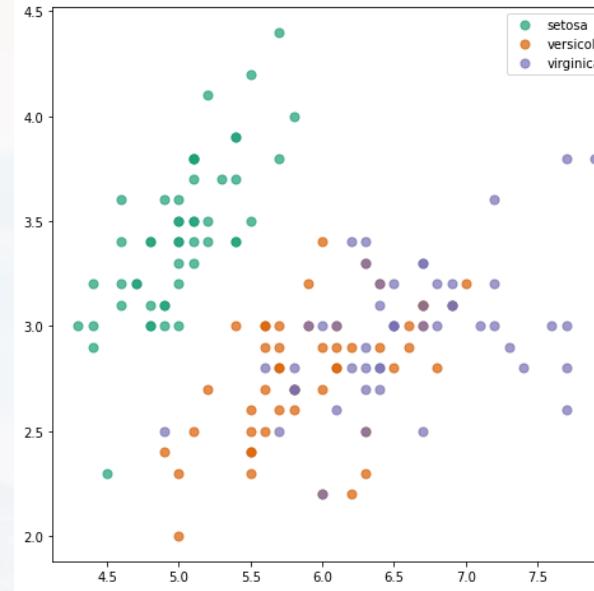
```
Center = my_model.means_
PredLabels = my_model.predict(X)
```

see [Walk\\_Through\\_GMM.ipynb](#)

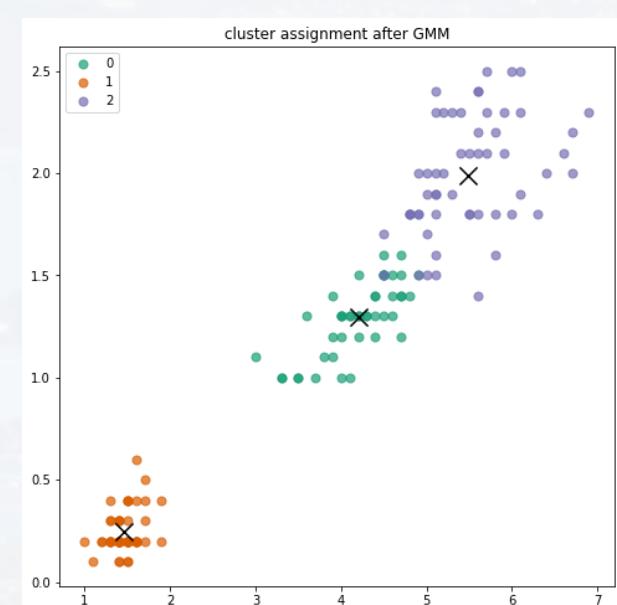
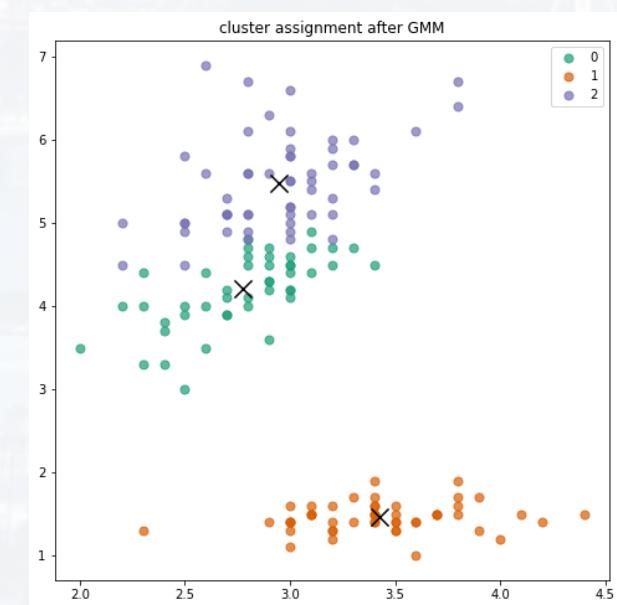
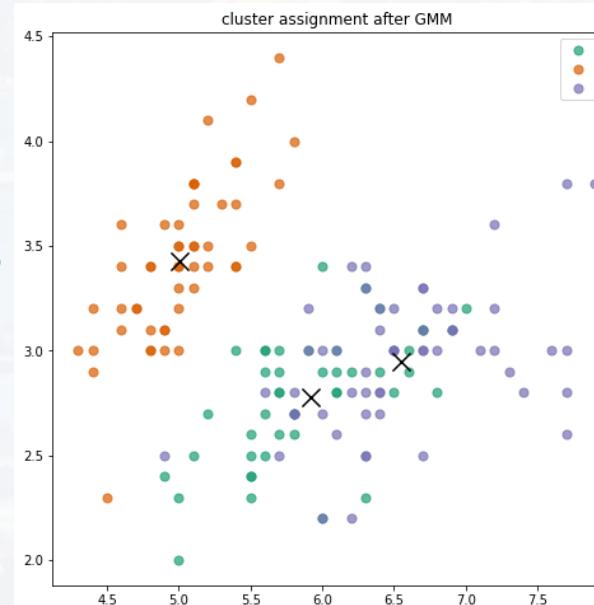




true classes

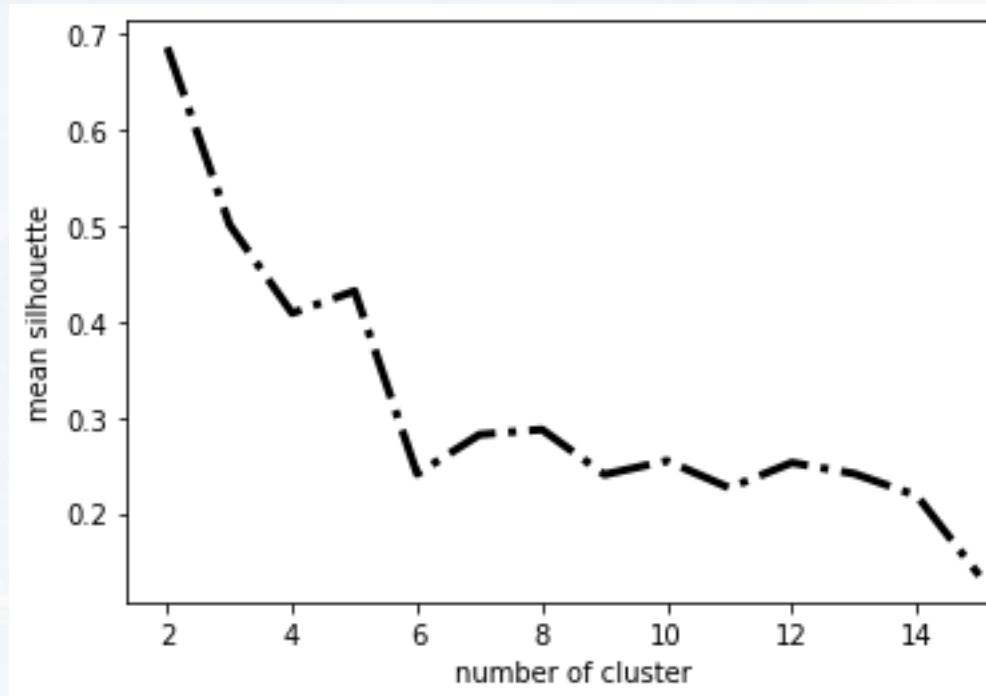


assigned classes





we run k-means now for the **full 4D** dataset  
+ evaluate clustering with silhouette

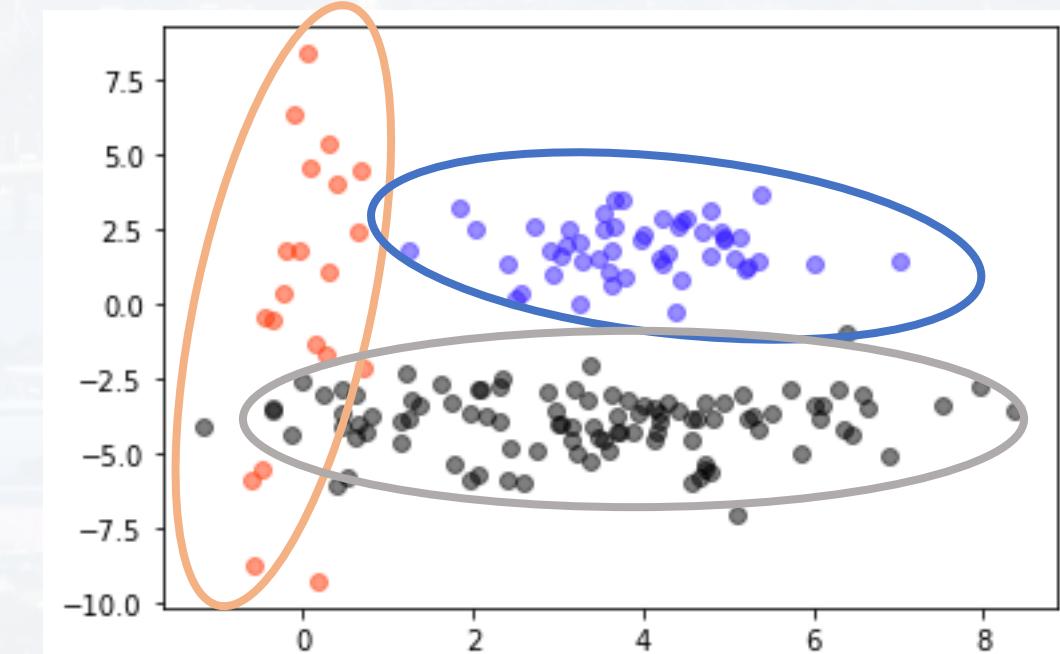


accuracy for 4D k = 3: 93%



### summary:

- simple and fast
- unsupervised
- $k$  has to be given → silhouette for determining best  $k$
- problems if cluster have unusual shapes  
(elongated, hollow inside, scattered)
- Gaussian: max entropy



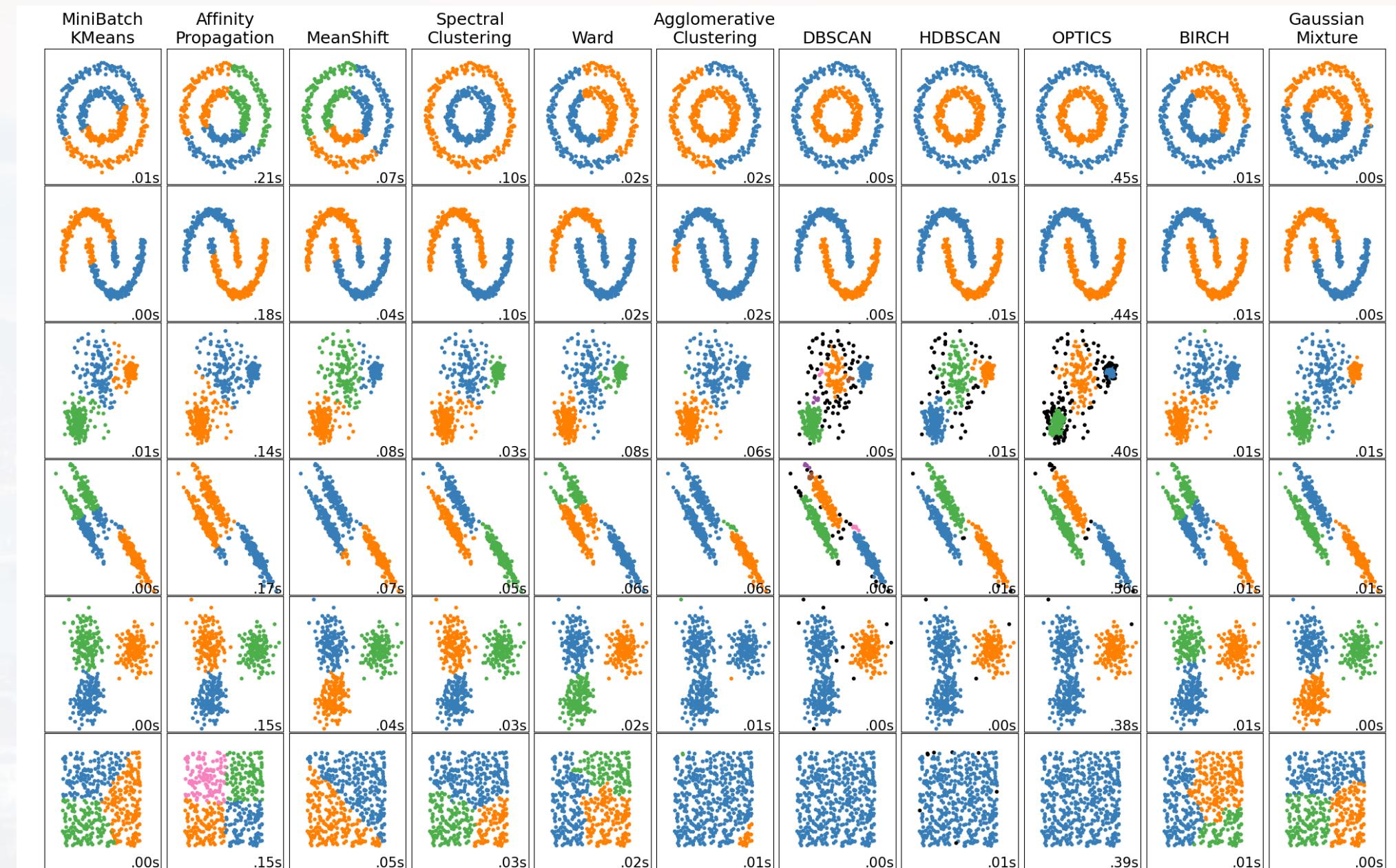


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# Numerical Methods for Computational Science:

Machine Learning Fundamentals

there is a lot more...





there is a lot more...

## Chem 277B “Machine Learning Algorithms”



### LLM & Transformer:

that was attention → now: encoder

<https://jalammar.github.io/illustrated-transformer/>

*h*

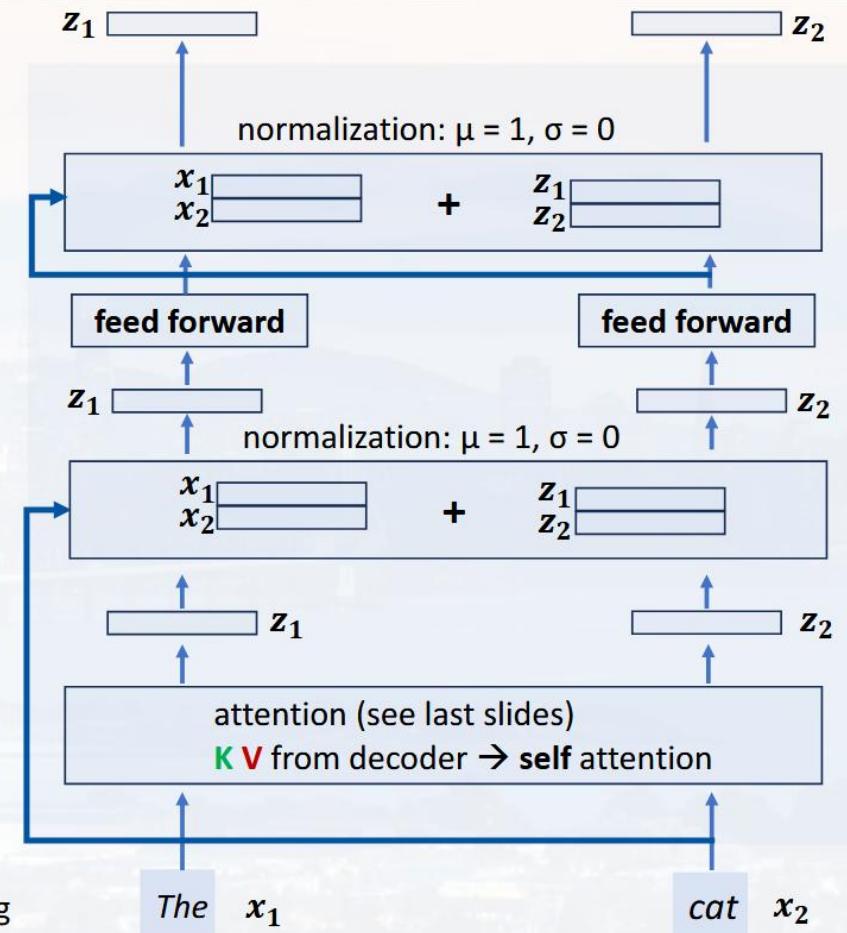
ENCODER #1

depending  
the output  
may have  
lengths t  
input fea

positional encoding

+ word embedding

### Transformer Architecture





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# Numerical Methods for Computational Science:

Machine Learning Fundamentals

Thank you very much for your attention!

