Discussion 5:

Machine Learning Overview & SVM

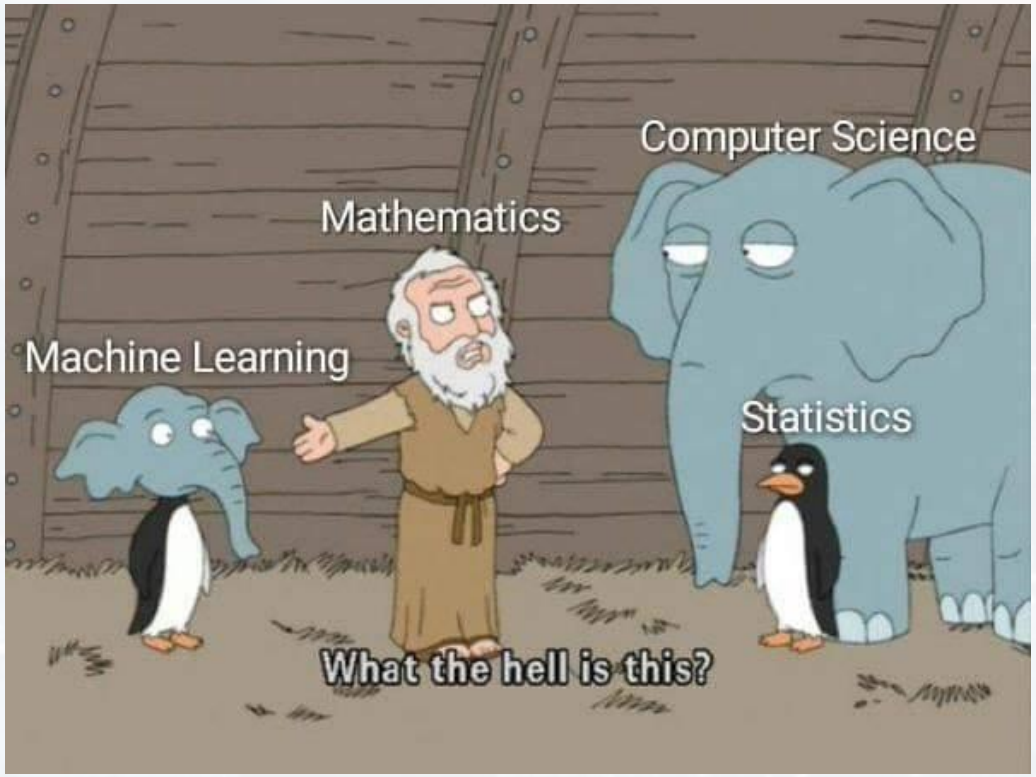
Markus Hohle

University California, Berkeley

Machine Learning Algorithms

MSSE 277B, 3 Units

Machine Learning Overview & SVM

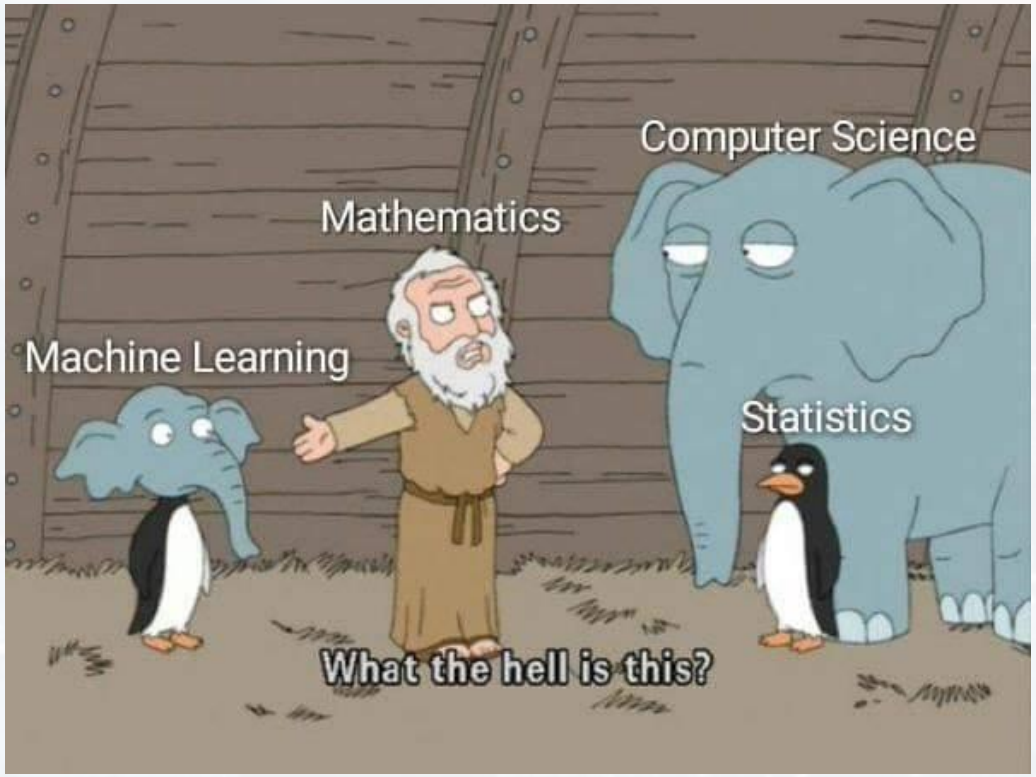


Outline

General Overview

Support Vector Machine
(Supervised Learning)

Machine Learning Overview & SVM



Outline

General Overview

Support Vector Machine
(Supervised Learning)

ML Overview

	supervised	unsupervised	
regression	linear regression ✓ ANNs (soon)		
classification	logistic regression ✓ Naïve Bayes ✓ K-nearest **SVM (today)** ANNs (soon)	GMM ✓ K-means ✓ HMM Trees ✓	Leiden, DBSCAN

Workflow

supervised

1) creating the model:

```
my_model = library.method(argument1 = 'arg1', … )
```

2) training the model

```
out = my_model.fit(xtrain, ytrain)
```

3) evaluation

```
ypred = out.predict(xeval)
accur = (ypred == yeval).sum()/len(yeval)
```

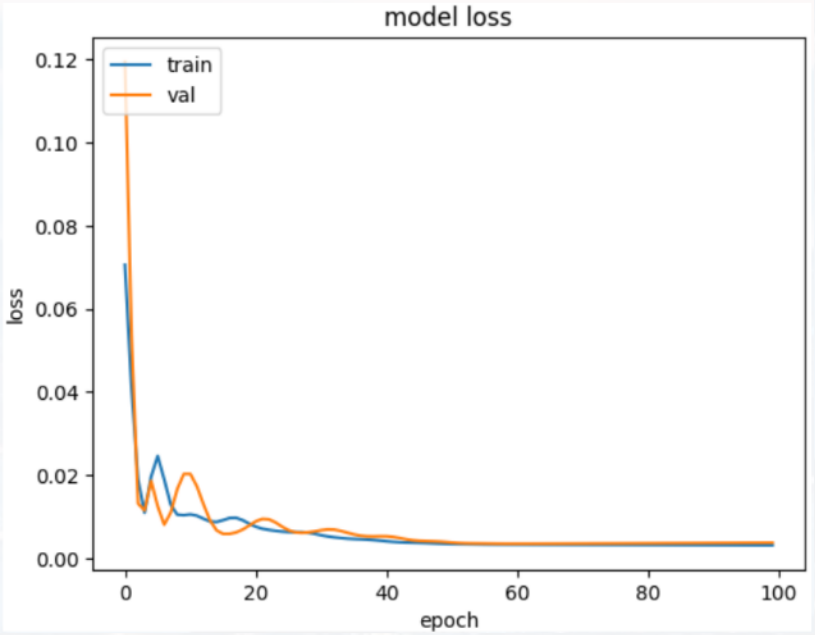
4) prediction (actual application)

```
ypred = out.predict(xnew)
```

How to split the data?

- 80% training **(60% actual training 20% evaluation)**
- 20% test

evaluation while training!



Workflow

supervised

1) creating the model:

```
my_model = library.method(argument1 = 'arg1', … )
```

2) training the model

```
out = my_model.fit(xtrain, ytrain)
```

3) evaluation

```
ypred = out.predict(xeval)
accur = (ypred == yeval).sum()/len(yeval)
```

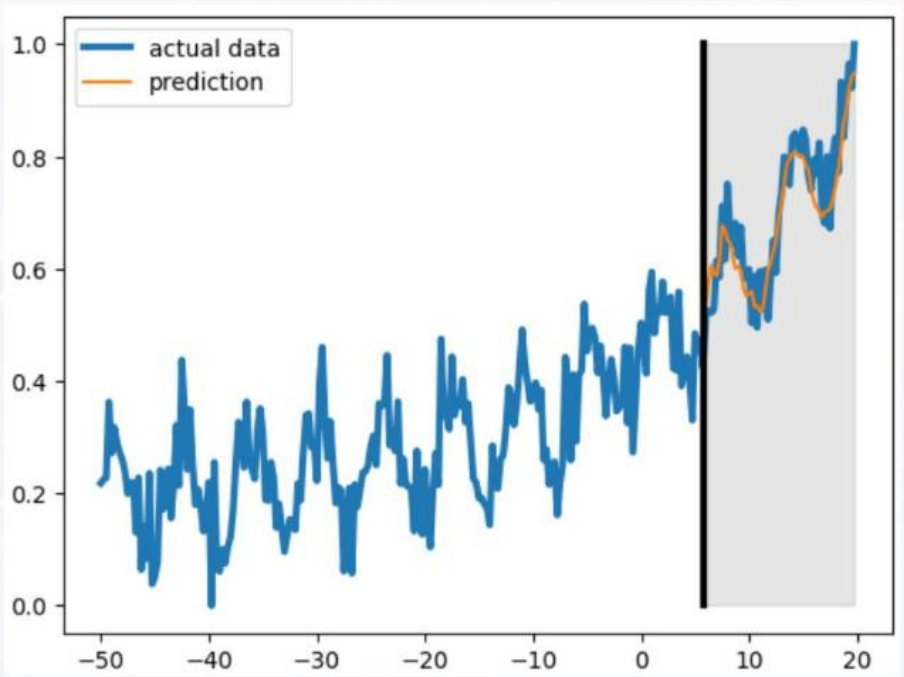
4) prediction (actual application)

```
ypred = out.predict(xnew)
```

How to split the data?

- 80% training **(60% actual training 20% evaluation)**
- 20% test

prediction after training/evaluation completed



unsupervised

Workflow

1) setting up the model

2) ~~fitting the model:~~ **X** are the **features** ~~(sepal lengths/widths and petal lengths/widths)~~,
Y are the **classes** (setosa, versicolor, virginica)

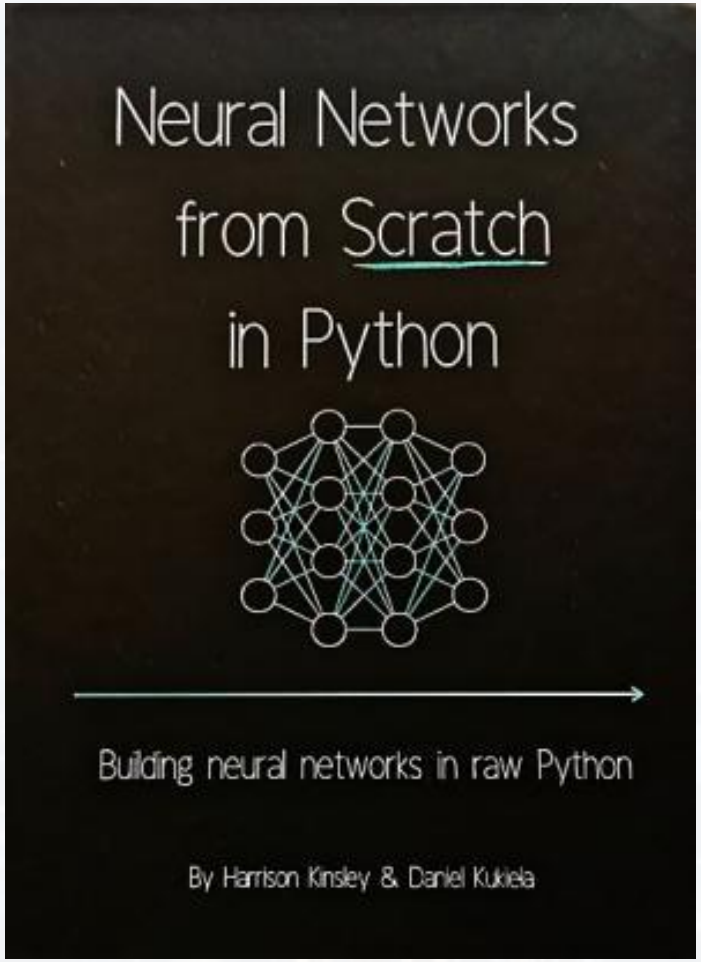
3) evaluating the model

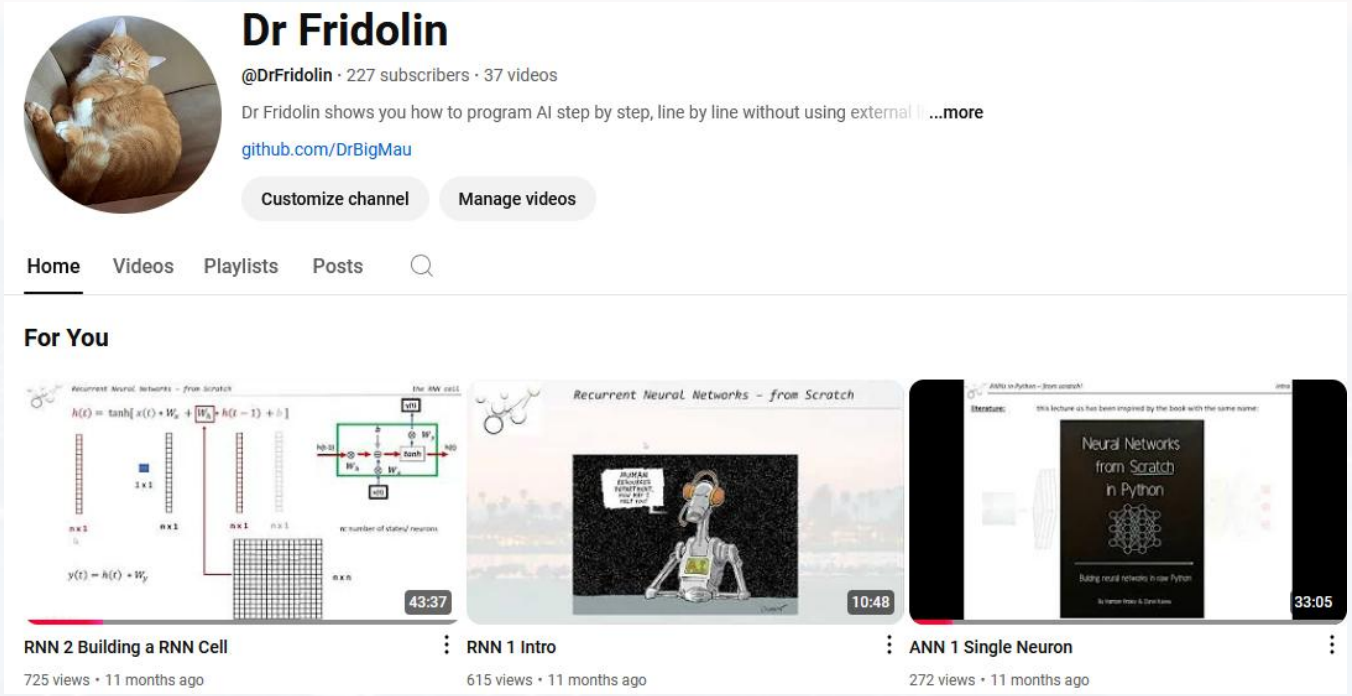
4) applying the model to a new data set

unsupervised learning:
- we don't know the classes
- sometimes we even don't know the **number *k* of classes**

resources for the next lectures to come:

Tutorials and Books





building ANNs, RNNs, LSTMs, CNNs **from scratch**
(only numpy and `matplotlib`)

Tutorials and Books

examples and **application** (CNN/RNN/LSTM and more):

Jason Brownlee
Machine Learning Mastery

all about **transformers, LLM/NLP and way more**

Andrej Karpathy

@AndrejKarpathy · 451K subscribers · 14 videos

FAQ ...more

karpathy.ai and 2 more links

Subscribe

Mısra Turp

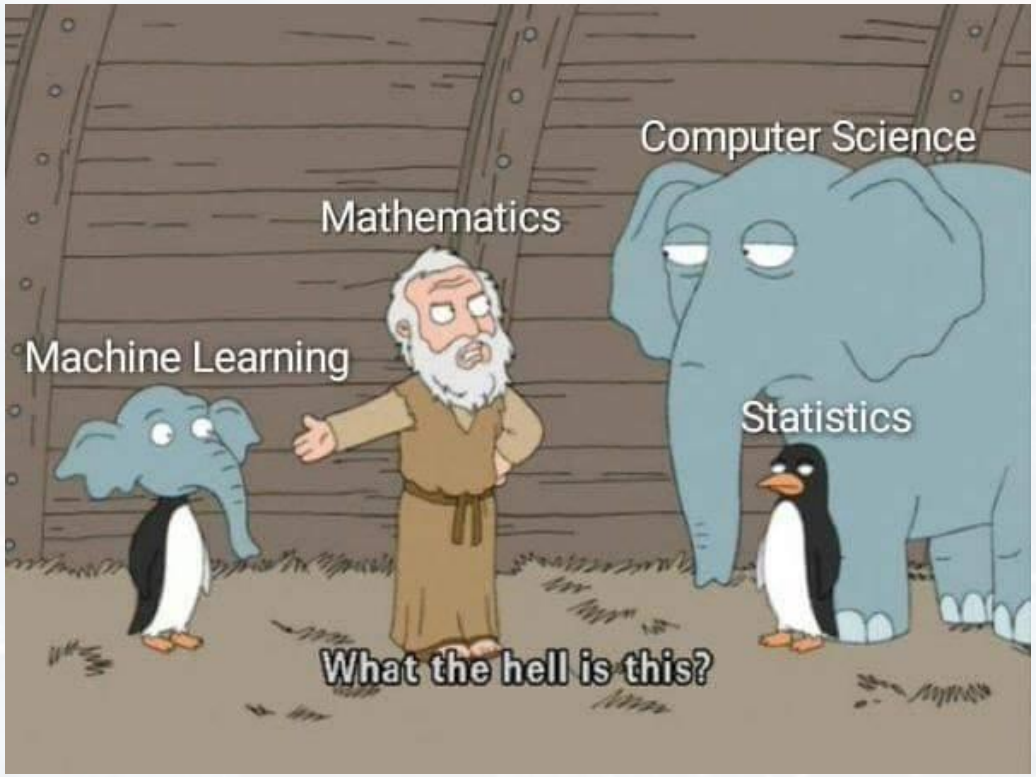
@misraturp · 38.2K subscribers · 163 videos

Here is where we learn! This is a space to take it slow

misraturp.com/roadmap and 3 more links

Subscribe

Machine Learning Overview & SVM



Outline

General Overview

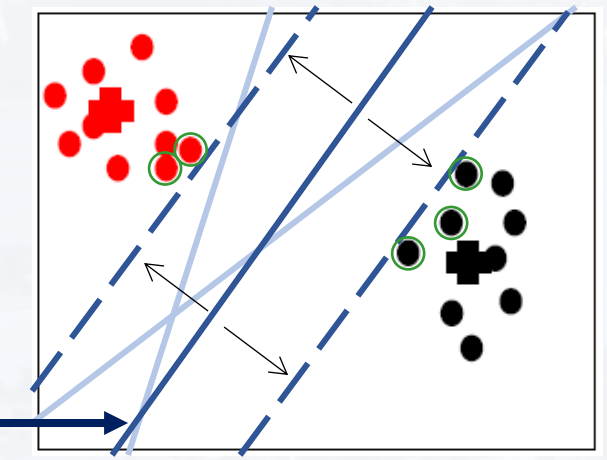
Support Vector Machine
(Supervised Learning)

SVM = **S**upport **V**ector **M**achine

idea:

1) finds best **linear** classifier for separating **two** classes by **maximizing margin** using **support vectors**
2) assign new data points to these categories
3) **supervised** learning
4) uses the "kernel trick"

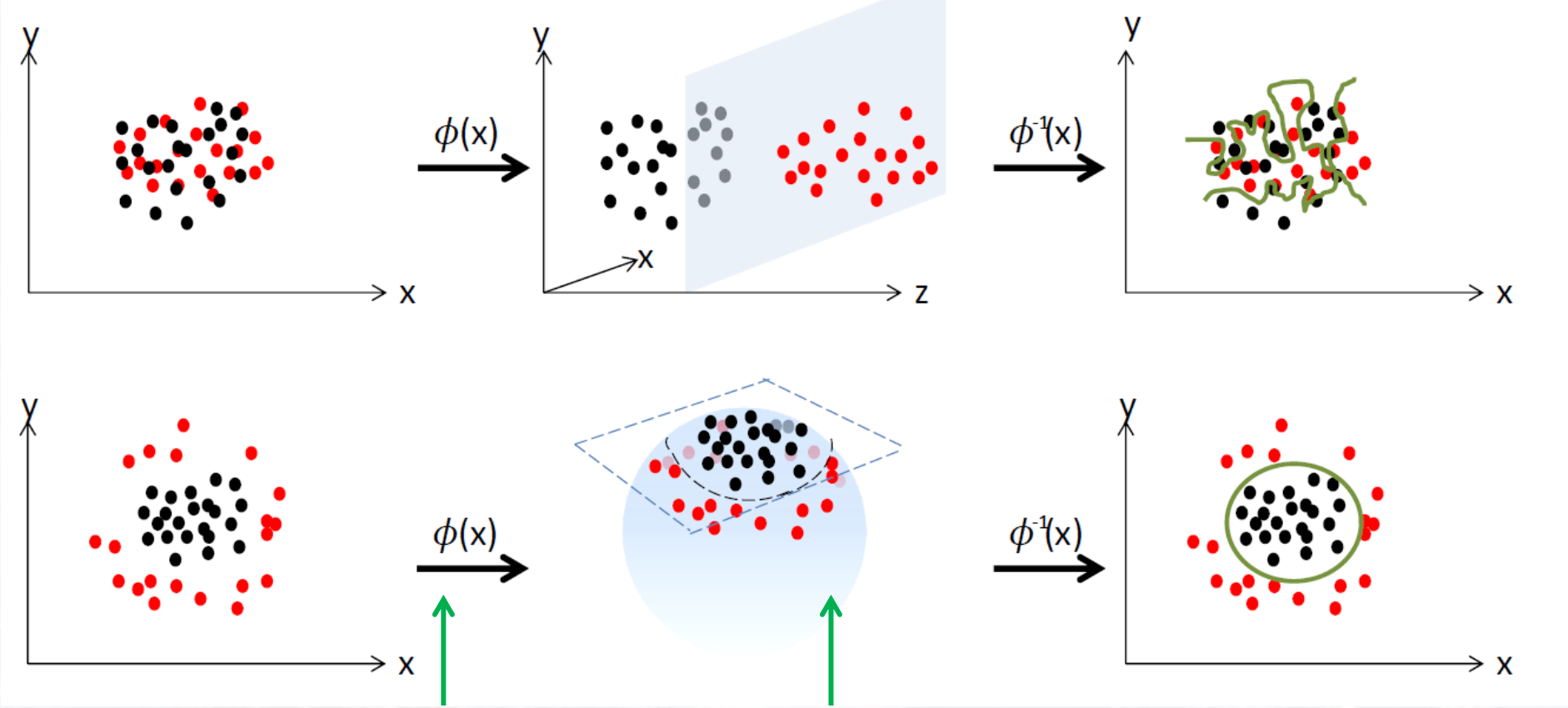
- support vectors (data points at the edge)



1) What if linear separation is not possible?
2) Is there a multiclass SVM?

1) **What if linear separation is not possible?**
2) Is there a multiclass SVM?

idea: mapping data to higher dimensional feature space



a) data space in N-D

b) a function ϕ maps the data into a M-D (M>N) feature space

c) find hyperplane separator in feature space

d) map back into data space (separator not linear)

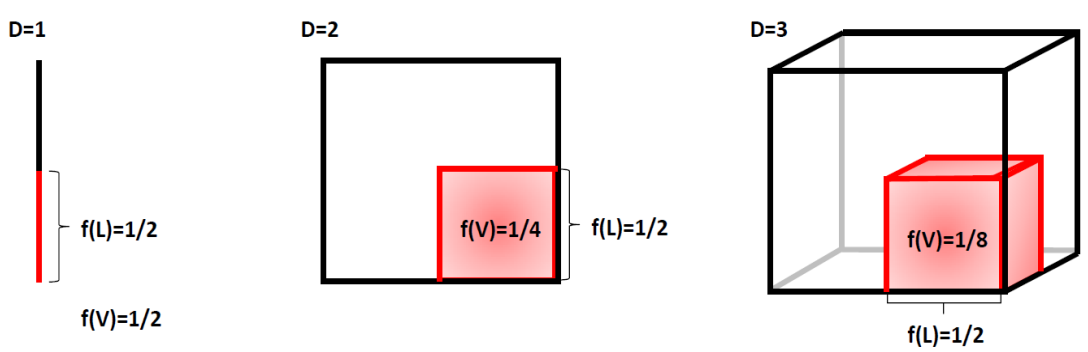
1) **What if linear separation is not possible?**
2) Is there a multiclass SVM?

problems: - computationally intensive
- ϕ usually unknown

…and dimensionality!!

1) **What if linear separation is not possible?**
2) Is there a multiclass SVM?

What is the fraction of volume $f(V)$ covered by a certain fraction of length $f(L) = r/R$ for different dimensions D?



answer: $\boldsymbol{f(V) \sim f(L)^D}$

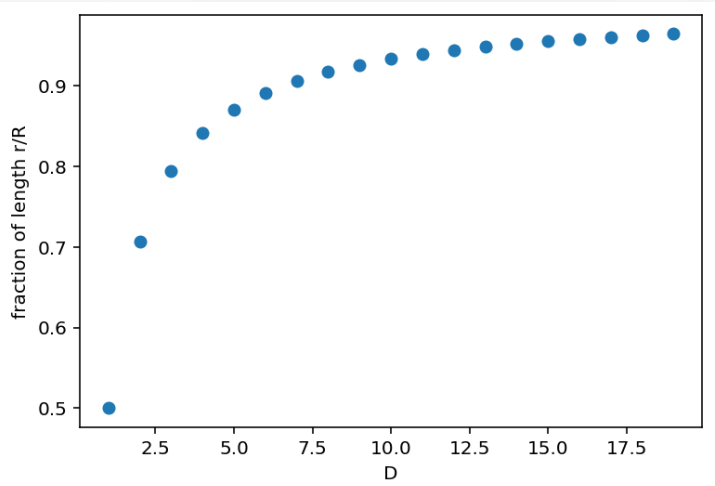
$$f(V) = C(D) f(L)^D$$

$$= C(D) \left(\frac{r}{R}\right)^D$$

$C(D)$: constant that only depends on D

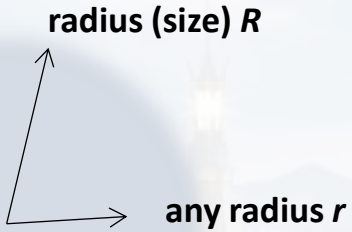
$$\left(\frac{r}{R}\right) \sim \sqrt[D]{f(V)}$$

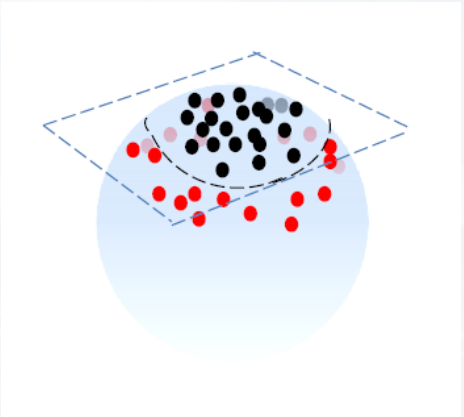
fraction of length that we need to explore in order to find a particular fraction of datapoints **(uniformly distributed in V)**



1) **What if linear separation is not possible?**
2) Is there a multiclass SVM?

N - D space





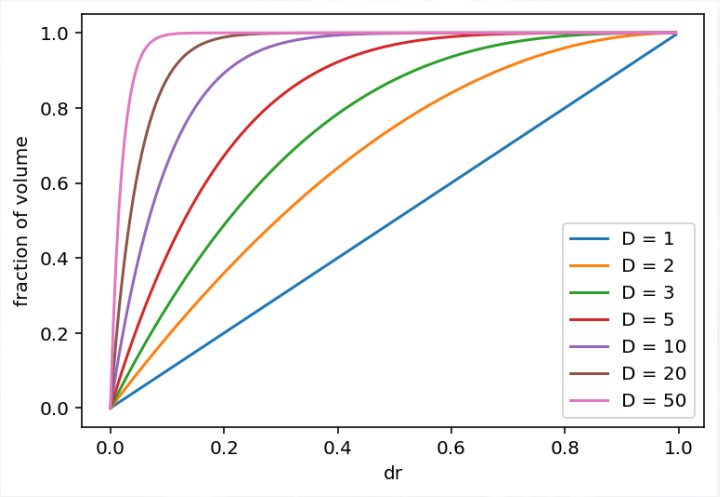
hypersphere:

$C(D)$: constant that only depends on D

$$V_D(r) = C(D)\, r^D$$

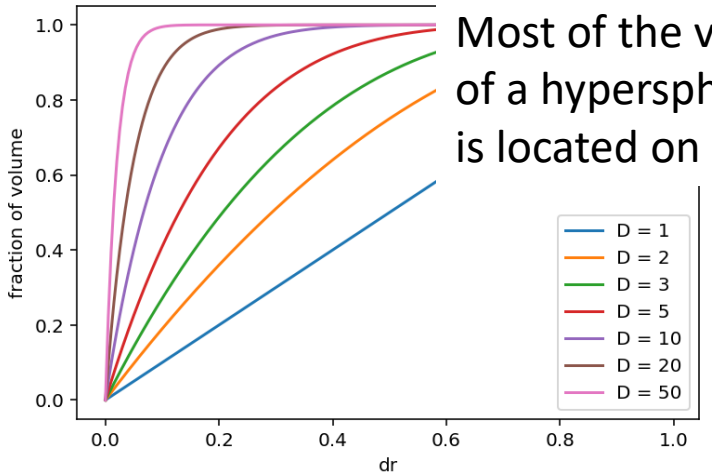
fraction of volume between r and $r - dr$

$$\frac{V_D(r) - V_D(r - dr)}{V_D(r)} = 1 - \frac{(r - dr)^D}{r^D}$$

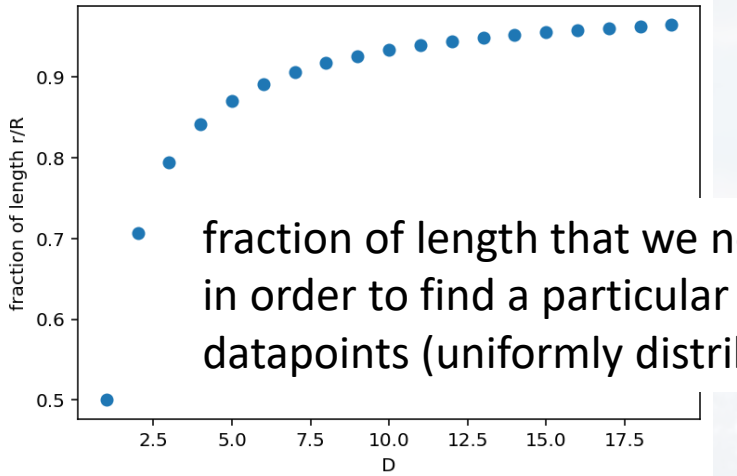


Most of the volume of a hypersphere is located on its surface!

1) **What if linear separation is not possible?**
2) Is there a multiclass SVM?



Most of the volume of a hypersphere is located on its surface!



fraction of length that we need to explore in order to find a particular fraction of datapoints (uniformly distributed in R)

- for **large D (= number of dimensions)**, one has to explore a **larger fraction $\frac{r}{R}$ of the data space** in order to get the **same fraction of data points**
- many algorithms get **less efficient** for large D
- for D→∞, the entire volume is located on the surface of the hyperspace

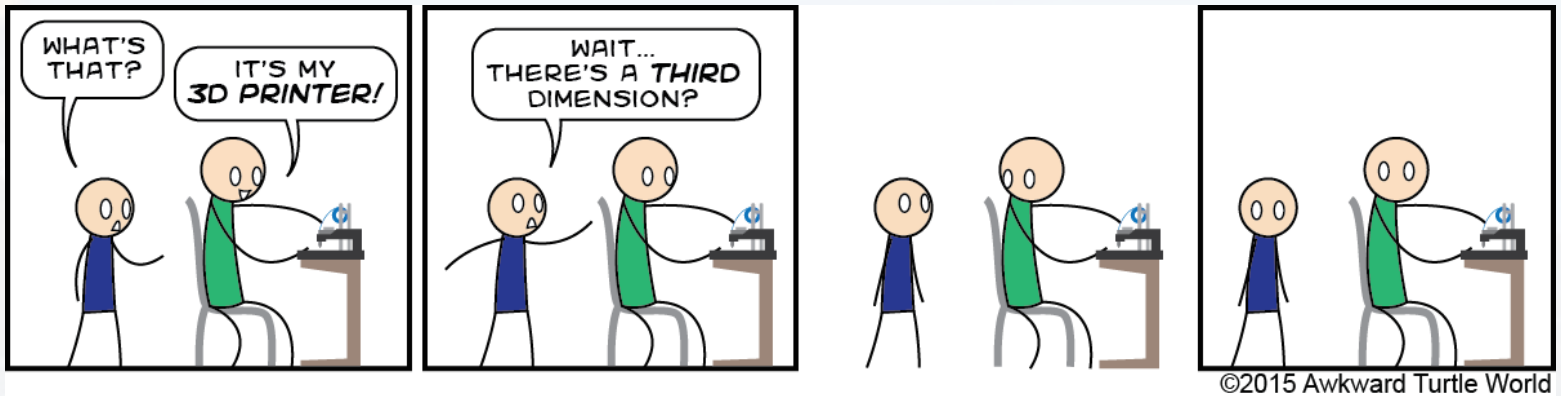
1) **What if linear separation is not possible?**
2) Is there a multiclass SVM?

- for **large D (= number of dimensions)**, one has to explore a **larger fraction $\frac{r}{R}$ of the data space** in order to get the **same fraction of data points**
- many algorithms get **less efficient** for large D
- for D→∞, the entire volume is located on the surface of the hyperspace

Makes many machine learning algorithms less efficient!

"Curse of Dimensionality"

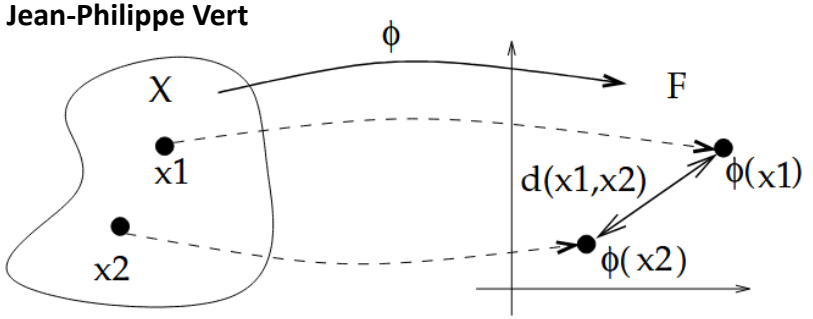
Dimensionality reduction is important!



1) What if linear separation is not possible?

2) Is there a multiclass SVM?

Jean-Philippe Vert



problems: - computationally intensive
- ϕ usually unknown

idea: - entire mathematical framework not needed
- for separation: need distances *d* in data space and feature space

$$d^2(x,y) = \langle x-y, x-y\rangle$$

$$d_\phi^2\big(\phi(x),\phi(y)\big) = \langle \phi(x)-\phi(y), \phi(x)-\phi(y)\rangle$$

$$= \langle \phi(x),\phi(x)\rangle - 2\langle \phi(x),\phi(y)\rangle + \langle \phi(y),\phi(y)\rangle$$

kernel $\mathrm{K}(x,y) := \langle \phi(x),\phi(y)\rangle$

$$d_\phi^2\big(\phi(x),\phi(y)\big) = K(x,x) - 2K(x,y) + K(y,y)$$

kernel trick: - we don't know *K* either: **we guess it!**

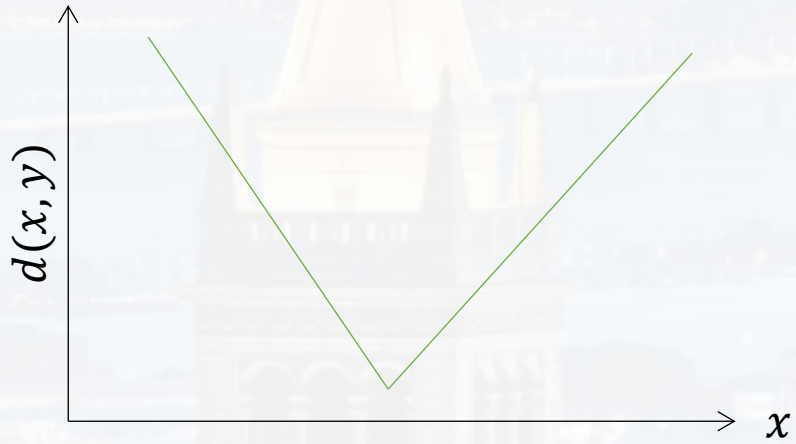
1) **What if linear separation is not possible?**
2) Is there a multiclass SVM?

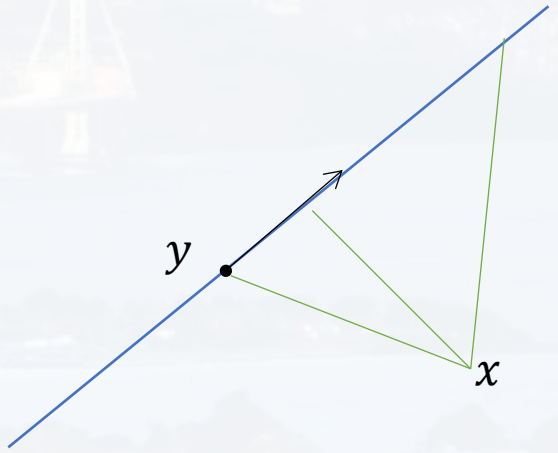
$$d^2\big(\phi(x),\phi(y)\big) = K(x,x) - 2K(x,y) + K(y,y)$$

example: linear kernel ϕ: x ↦ x $\phi(x) = x$

$$d_\phi^2(x,y) = \langle x,x\rangle - 2\langle x,y\rangle + \langle y,y\rangle = x^2 - 2xy + y^2 = (x-y)^2$$

$$d_\phi^2(x,y) = |x-y|$$

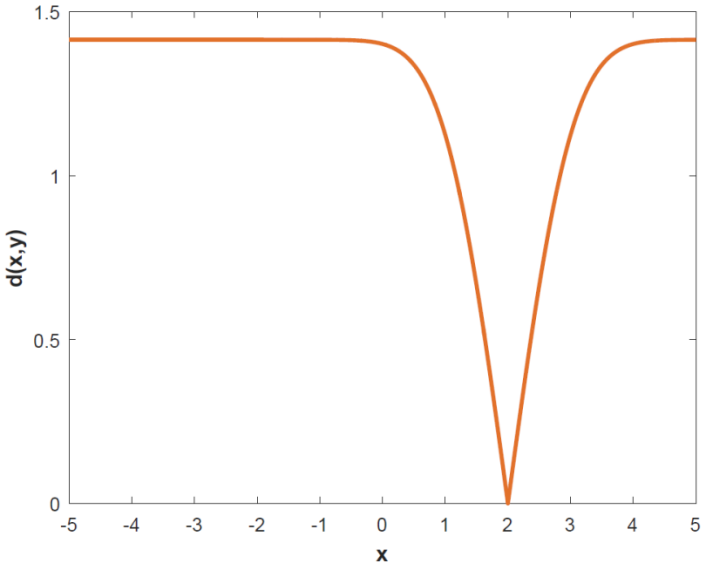


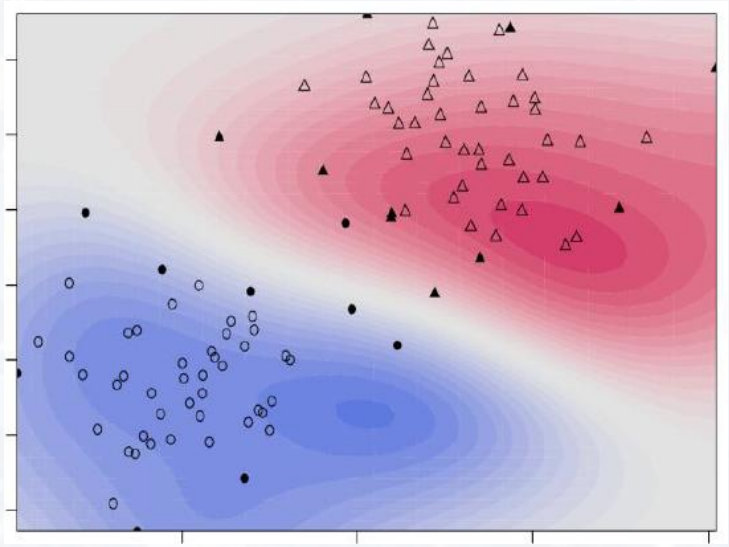


1) **What if linear separation is not possible?**
2) Is there a multiclass SVM?

example: : RBF (**r**adial **b**asis **f**unction) $\phi: x \mapsto \varsigma$ $\phi(x) = \sim\exp(\sim x)$

$$K(x,y) = exp\left(-\frac{\|x-y\|^2}{2\sigma^2}\right) \qquad d_\phi^2(\phi(x),\phi(y)) = 2\left[1 - exp\left(-\frac{\|x-y\|^2}{2\sigma^2}\right)\right]$$





1) **What if linear separation is not possible?**
2) Is there a multiclass SVM?

kernels available in `sklearn`:

- linear: $K(x,y) = \|x - y\|$

- Gaussian aka RBF (**r**adial **b**asis **f**unction): $K(x,y) = exp\left(-\frac{\|x-y\|^2}{2\sigma^2}\right)$

in `sklearn` we can adjust $\boldsymbol{\gamma} := \frac{1}{2\sigma^2}$

- polynomial: $K(x,y) = \sum_{n=1}^{N} \|x - y\|^n$

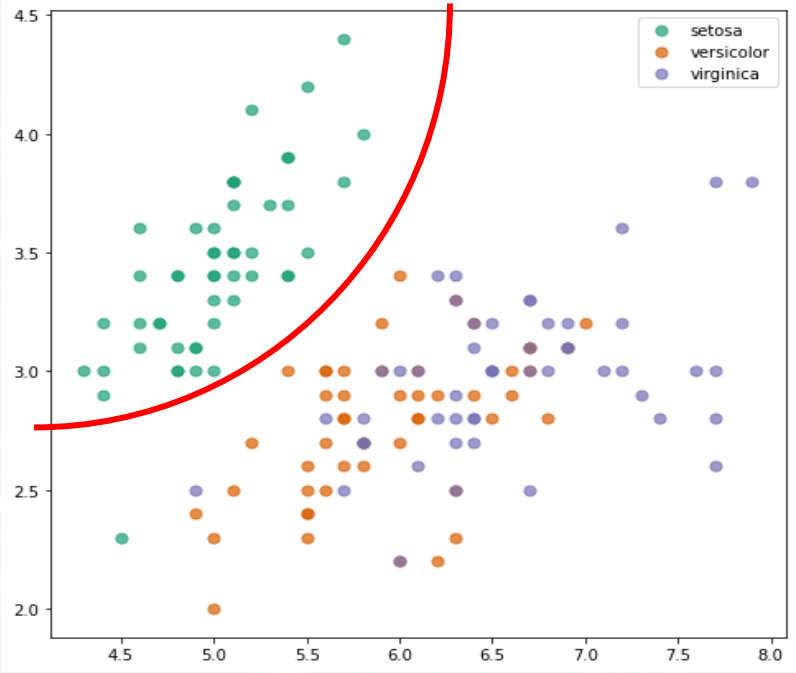
in `sklearn` we can adjust $\boldsymbol{N}$

- sigmoidal: $K(x,y) = \frac{e^{\|x-y\|}}{1 + e^{\|x-y\|}}$

1) What if linear separation is not possible?
2) **Is there a multiclass SVM?**

→ one vs rest / one vs one

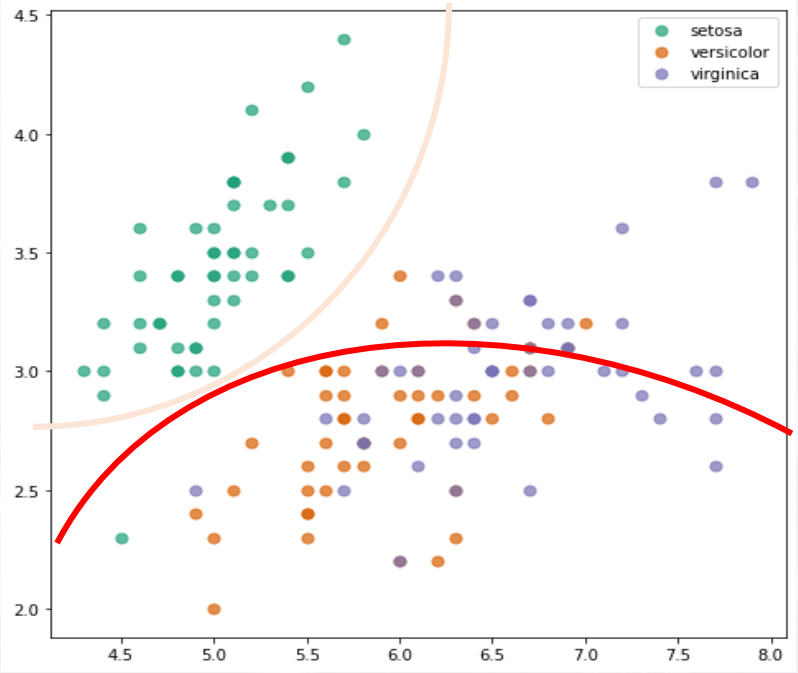
green vs the rest
→ storing probabilities



1) What if linear separation is not possible?
2) **Is there a multiclass SVM?**

→ one vs rest / one vs one

green vs the rest
→ storing probabilities

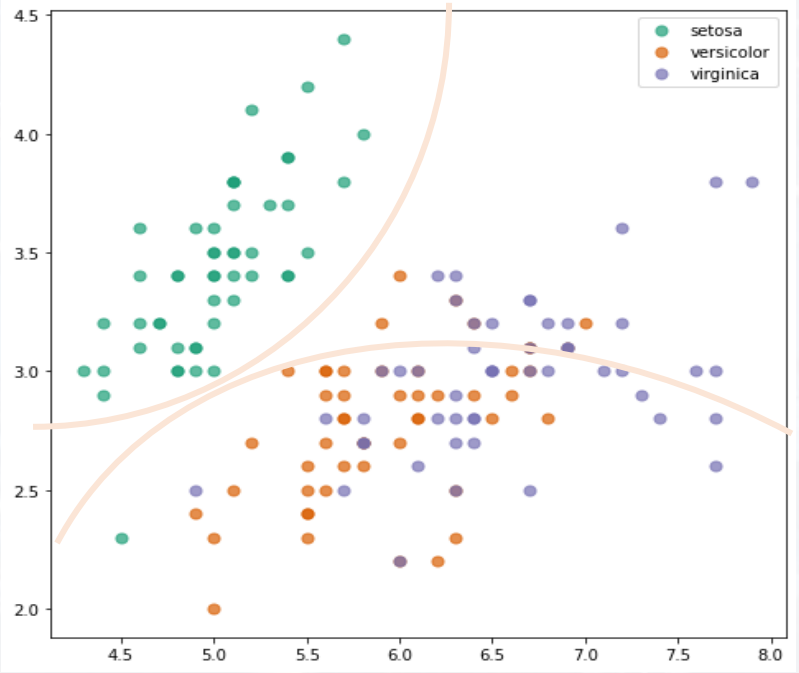


orange vs the rest
→ storing probabilities

1) What if linear separation is not possible?
2) **Is there a multiclass SVM?**

→ one vs rest / one vs one

green vs the rest
→ storing probabilities



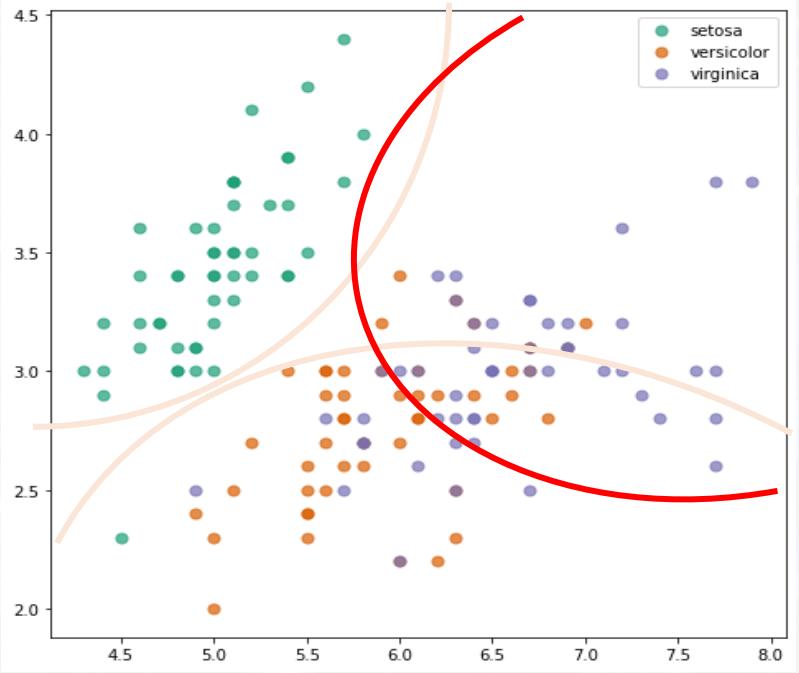
blue vs the rest
→ storing probabilities

orange vs the rest
→ storing probabilities

1) What if linear separation is not possible?
2) **Is there a multiclass SVM?**

→ one vs rest / one vs one

green vs the rest
→ storing probabilities



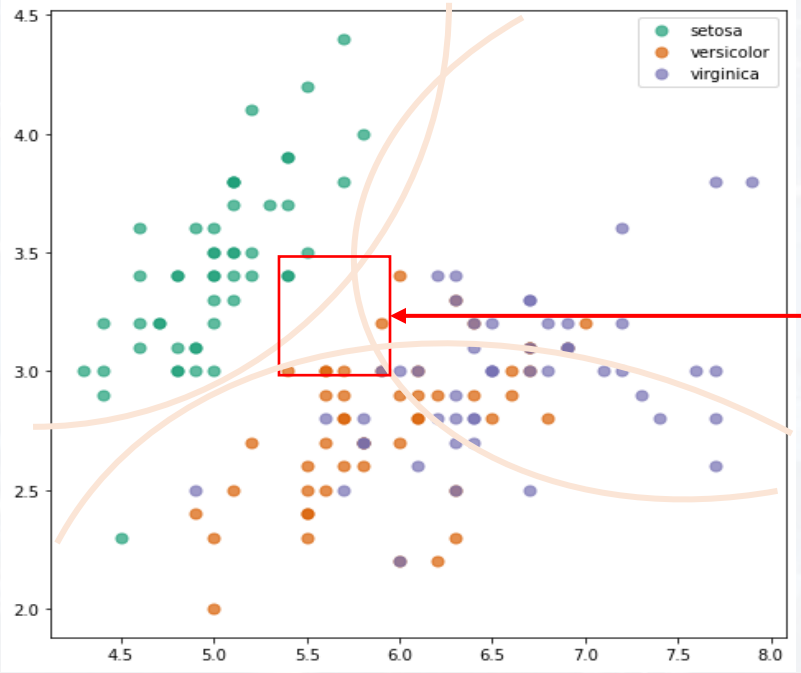
blue vs the rest
→ storing probabilities

orange vs the rest
→ storing probabilities

1) What if linear separation is not possible?
2) **Is there a multiclass SVM?**

→ one vs rest / one vs one

green vs the rest
→ storing probabilities



three probabilities for each data point
→ assign class to most probable value

k class classification with two-class discriminant functions is ambiguous!

```
from sklearn import svm
```

see **Walk_Through_SVM.ipynb**

1) setting up the model & 2) fitting the model

running analysis with different kernel

```
outlinear = svm.SVC(kernel = 'linear', C = 1, decision_function_shape = 'ovr')
linear    = outlinear.fit(X2D, Y)


outrbf    = svm.SVC(kernel = 'rbf', gamma = 1, C = 1, \
                    decision_function_shape = 'ovr')
rbf       = outrbf.fit(X2D, Y)


outpoly   = svm.SVC(kernel = 'poly', degree = 3, C = 1,\
                    decision_function_shape = 'ovr')
poly      = outpoly.fit(X2D, Y)


outsig    = svm.SVC(kernel = 'sigmoid', C = 1, decision_function_shape = 'ovr')
sig       = outsig.fit(X2D, Y)
```

one **v**ersus **r**est

L2 regularization parameter for error tolerance when calculating the classifier

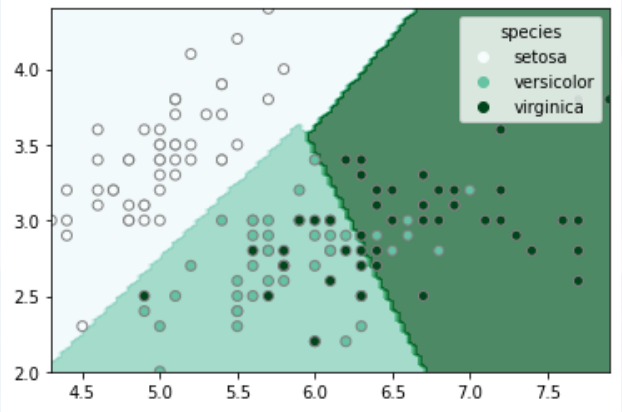
$\gamma := \frac{1}{2\sigma^2}$

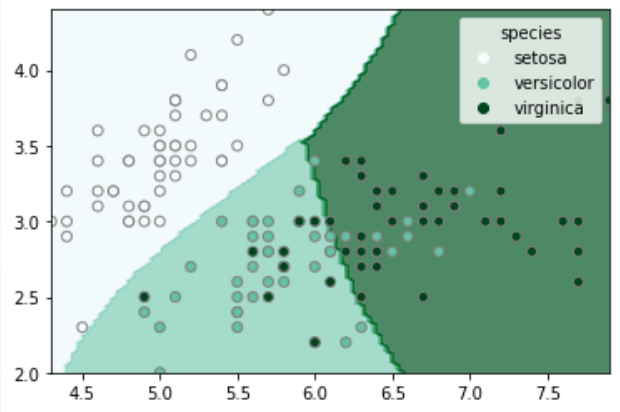
refers to *N* in $\sum_{n=1}^{N} \|x - y\|^n$

```
from sklearn import svm
```

see **Walk_Through_SVM.ipynb**

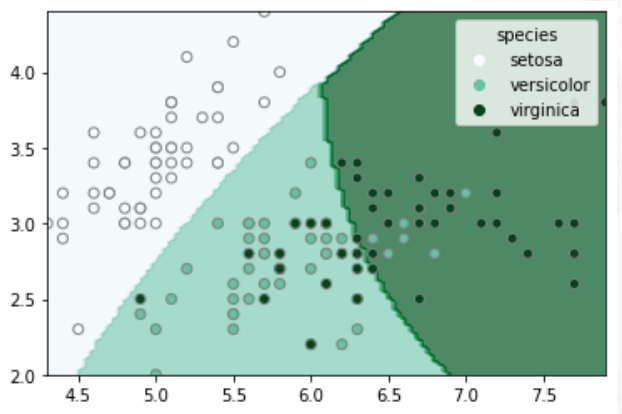
$K(x, y) = \|x - y\|$

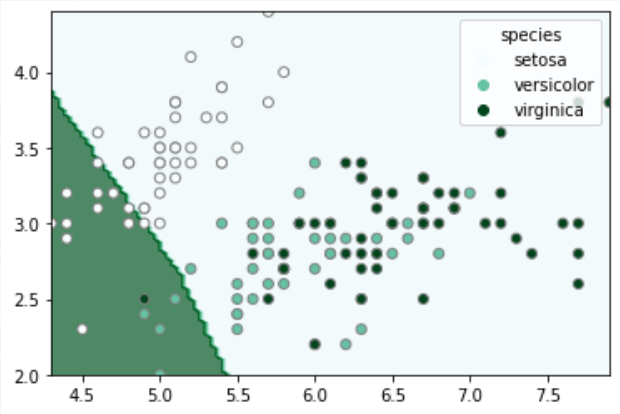




$K(x, y) = exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right)$

$K(x, y) = \sum_{n=1}^{N} \|x - y\|^n$



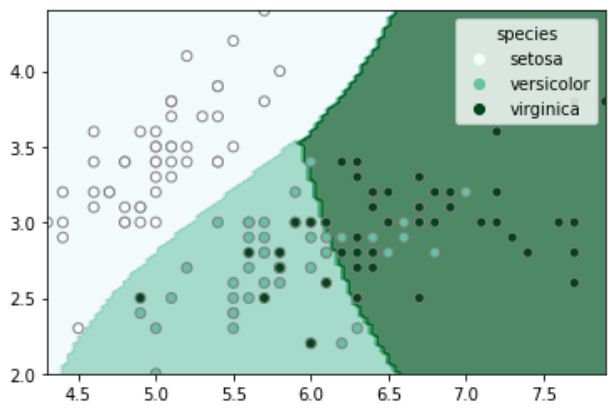


$K(x, y) = \frac{e^{\|x-y\|}}{1 + e^{\|x-y\|}}$

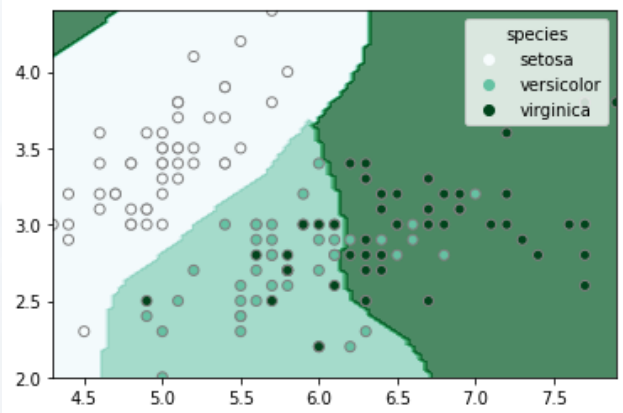
```
from sklearn import svm
```

see **Walk_Through_SVM.ipynb**

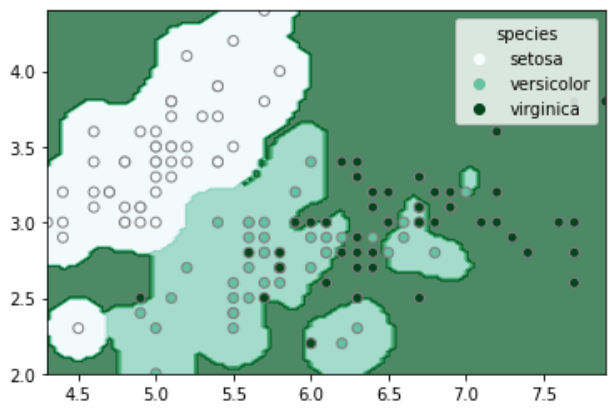
$$K(x, y) = exp\left(-\frac{1}{2\sigma^2}\|x - y\|^2\right)$$



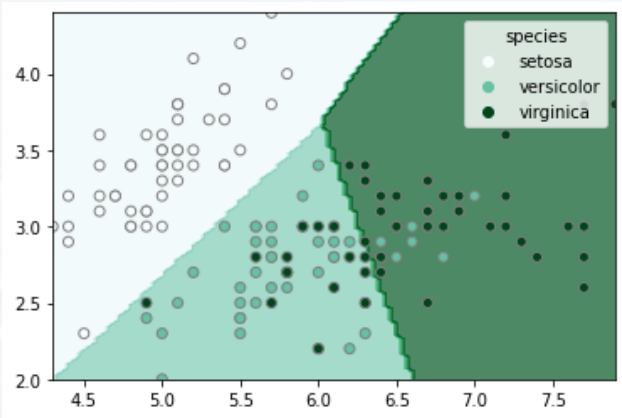
$$\gamma := \frac{1}{2\sigma^2} = 1$$



$$\gamma := \frac{1}{2\sigma^2} = 5$$



$$\gamma := \frac{1}{2\sigma^2} = 50$$

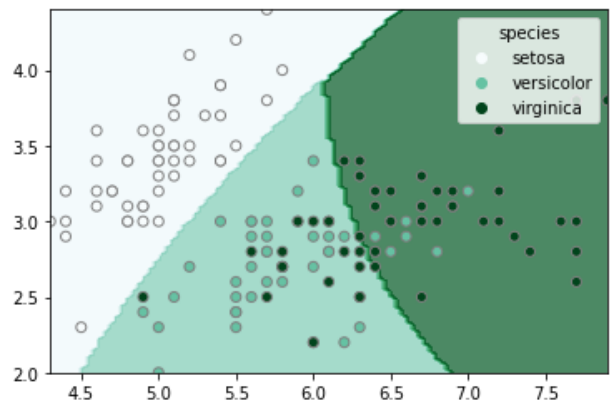


$$\gamma := \frac{1}{2\sigma^2} = 0.1$$

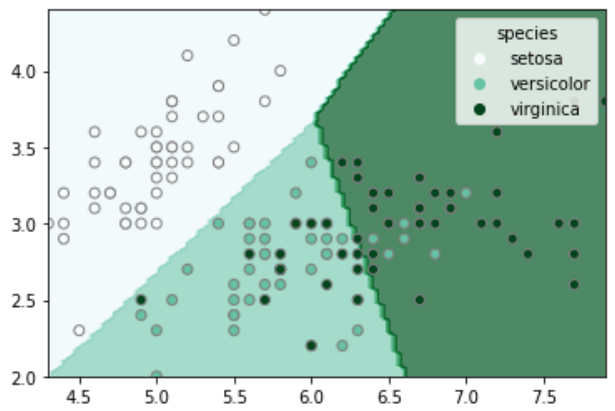
```
from sklearn import svm
```

see **Walk_Through_SVM.ipynb**

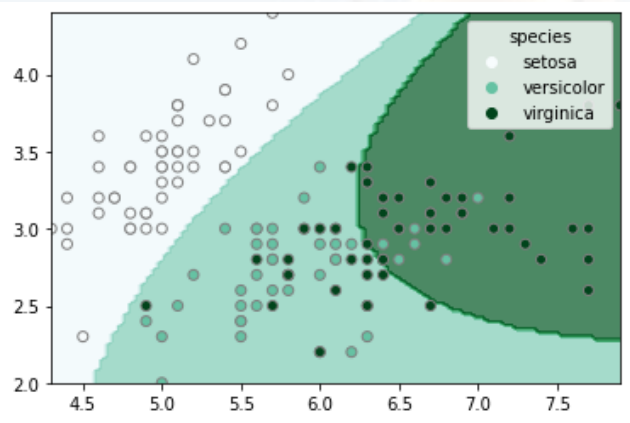
$$K(x, y) = \sum_{n=1}^{N} \|x - y\|^n$$



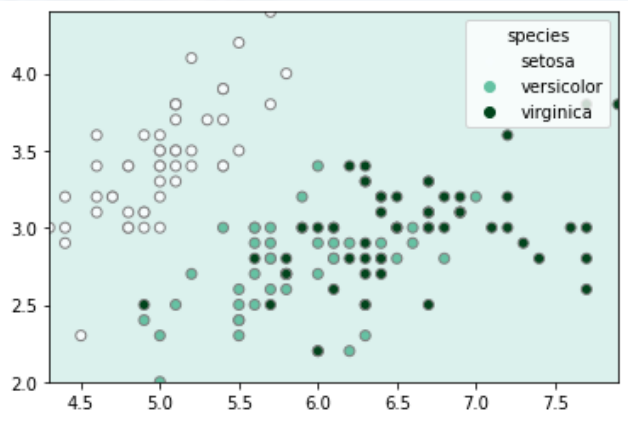
N = 3



N = 1



N = 5



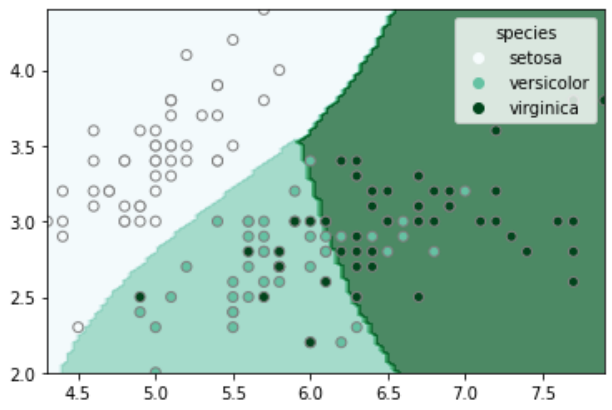
N = 0

```
from sklearn import svm
```

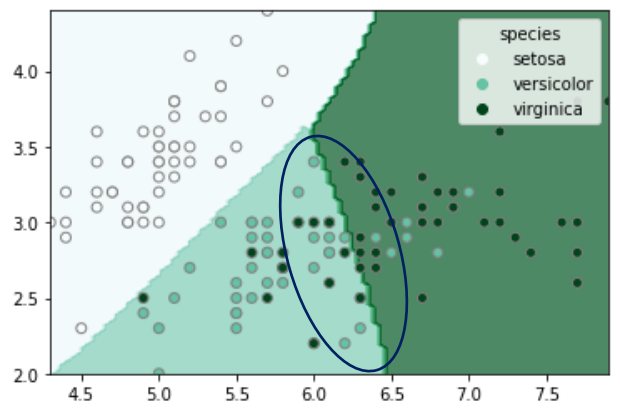
see **Walk_Through_SVM.ipynb**

$$K(x, y) = exp\left(-\frac{1}{2\sigma^2}\|x - y\|^2\right)$$

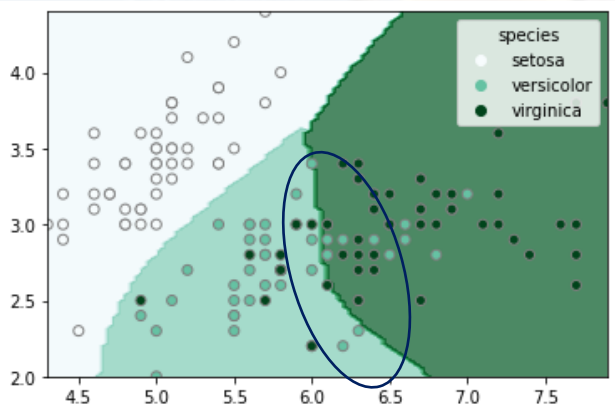
C is a L2 regularization parameter



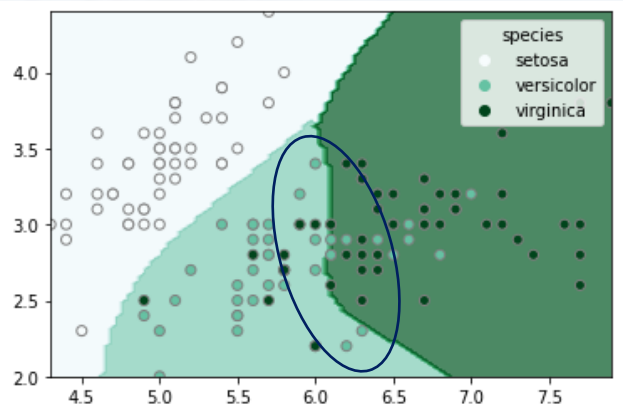
$C = 1$



$C = 0.1$



$C = 10$



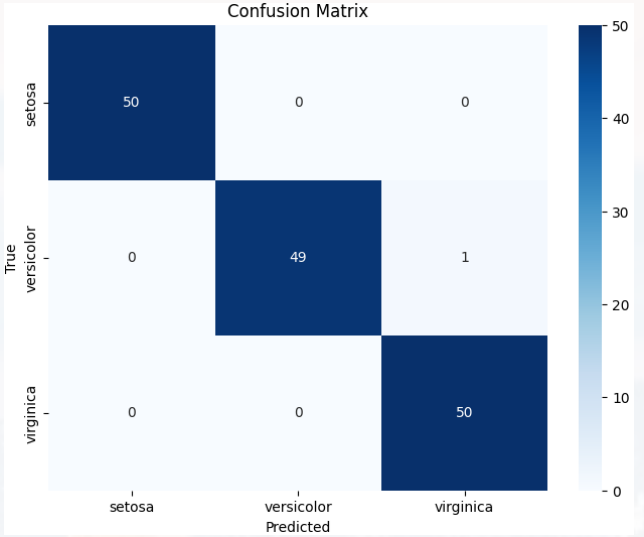
$C = 20$

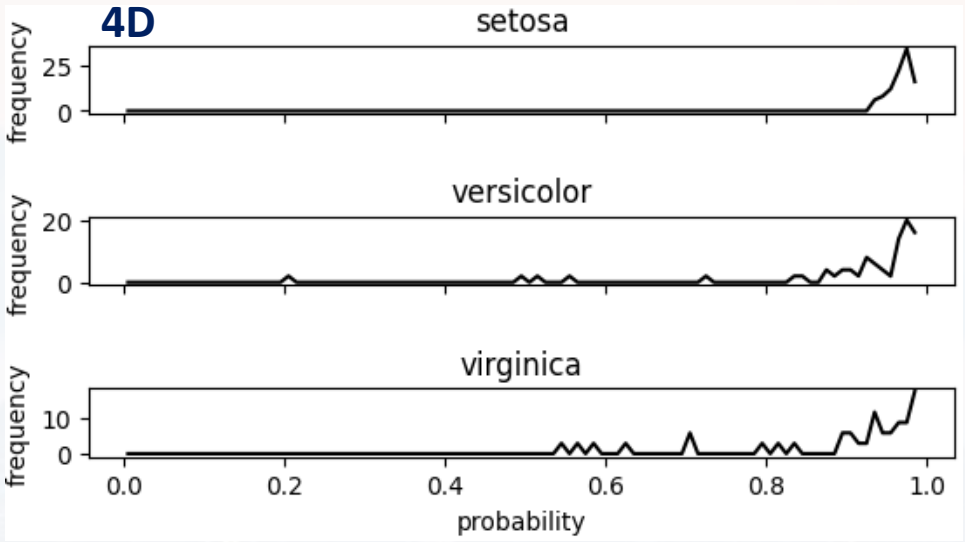
full **4D** data set

3) evaluating the model

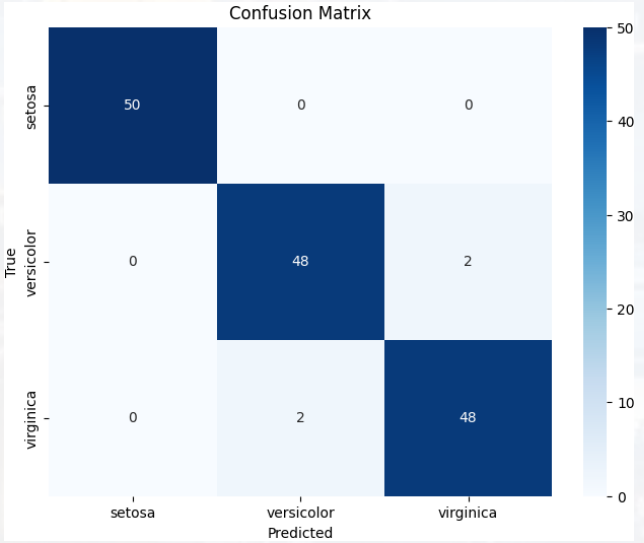
see **`Walk_Through_SVM.ipynb`**

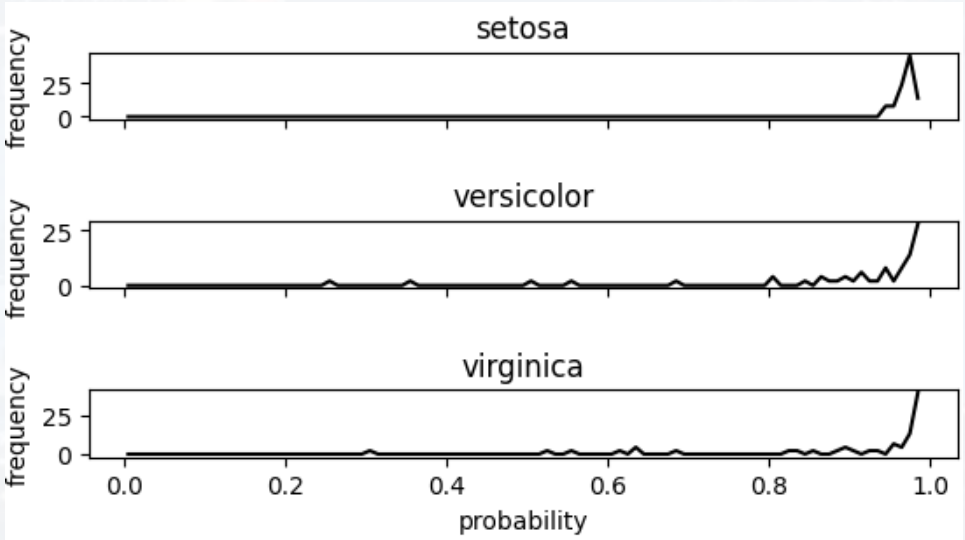
linear
accuracy: 99.3%





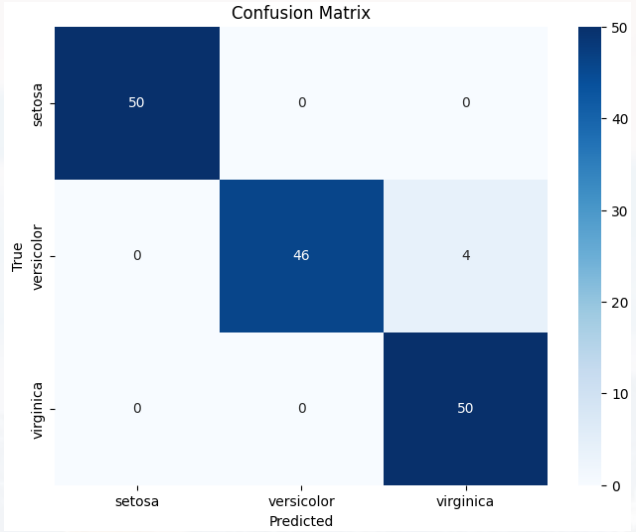
Gaussian ($\gamma = 1$)
accuracy: 97.3%





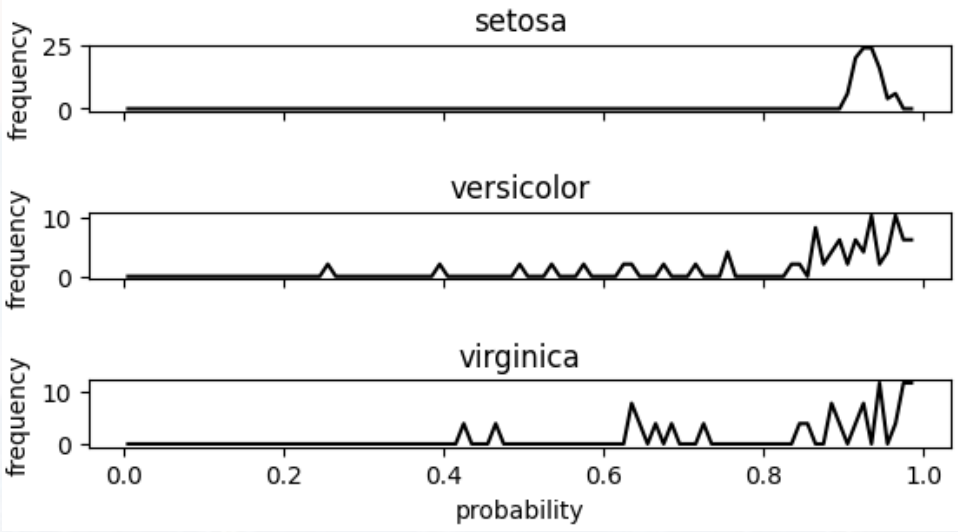
full **4D** data set

polynomial (n = 3)
accuracy: 97.3%



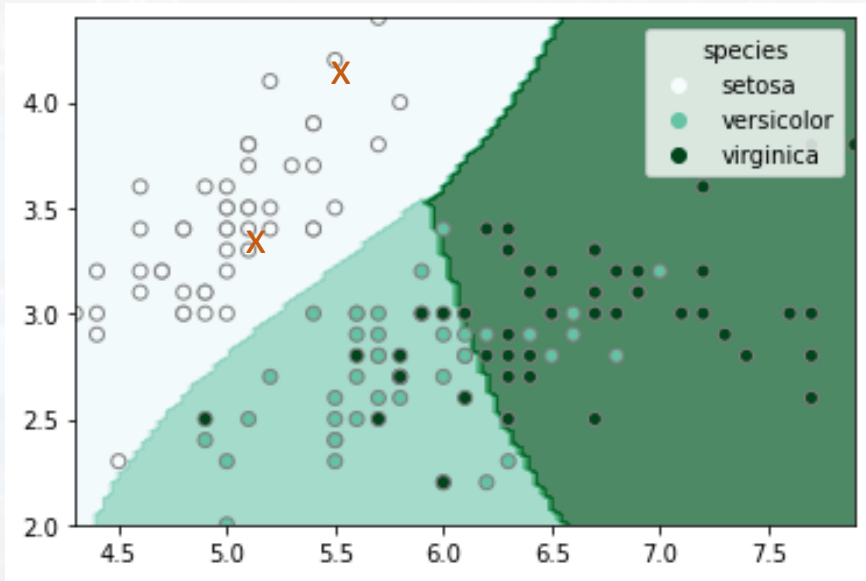
3) evaluating the model

see **`Walk_Through_SVM.ipynb`**



4) applying the model to a new data set

```
ypred = outpoly.predict([[6, 3.5],[6.3, 4.5]])
```



Thank you very much for your attention!