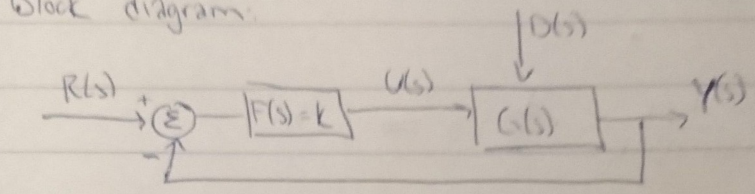
Question 1

$$Y(s) = \frac{k_f}{s(1+Ts)}\left(U(s)+D(s)\right) \qquad k_f, T > 0$$

$$g(t) = \mathcal{L}^{-1}\left\{\frac{k_f}{s(1+sT)}\right\} = k_f\left(1-e^{-t/T}\right)$$

Block diagram:



$$Y(s) = G_c(s)R(s) + G_d(s)D(s)$$

$$U = K(R-Y)$$

$$Y = \frac{k_f}{s(1+sT)}(U+D)$$

$$Y(s) = G(s)\cdot(K(R-Y)+D) = G\,(KR - KY + D) =$$

$$= GKR - GKY + DG$$

$$\Rightarrow GKY + Y = GKR - DG$$

$$\Rightarrow Y(GK+1) = GKR - DG$$

$$\Rightarrow Y = \frac{GKR - GD}{GK+1} = \frac{GKR}{GK+1} - \frac{GD}{GK+1} = \frac{\frac{k_f}{s(1+sT)}K}{\frac{k_f}{s(1+sT)}K+1}R - \frac{\frac{k_f}{s(1+sT)}G}{\frac{k_f}{s(1+sT)}K+1}D =$$

$$= \frac{k_f K}{s(1+sT)+k_f K}R - \frac{k_f G}{s(1+sT)+k_f K}D$$

$$s = \frac{-1}{2T} \pm \sqrt{\frac{1-4T^2k_fK}{2T}}$$

It's stable since the poles has negative real values with $T>0$.

a) $d \equiv 0$, $r = \begin{cases} r_0 & , t \geq 0 \\ 0 & , t < 0 \end{cases}$ $\mathcal{L}\{r\} = \frac{r_0}{s}$

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \left(\frac{k_f K}{s(1+sT) + k_f K} \cdot \frac{r_0}{s} \right) \cdot s =$$

$$= \lim_{s \to 0} \frac{k_f K}{s(1+sT) + K_f K} \cdot r_0 = r_0$$

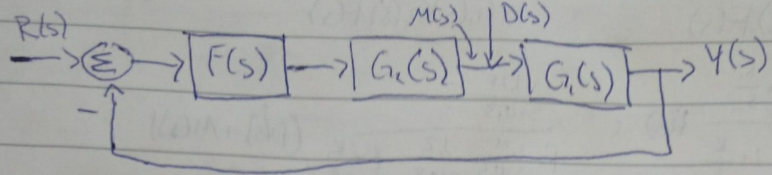
b) $r \equiv 0$, $d = \begin{cases} d_0 & , t \geq 0 \\ 0 & , t < 0 \end{cases}$ $\mathcal{L}\{d\} = \frac{d_0}{s}$

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \left(- \frac{k_f G}{s(1+sT) + k_f K} \cdot \frac{d_0}{s} \right) \cdot s =$$

$$= \lim_{s \to 0} - \frac{k_f \cdot \frac{k_f}{s(1+sT)}}{s(1+sT) + k_f K} \cdot d_0 \cancel{= \lim_{s \to 0}} = - \frac{d_0}{K}$$

Question 2.

$$Y(s) = \frac{G_1(s)G_2(s)F(s)}{1+G_1(s)G_2(s)F(s)}R(s) + \frac{G_1(s)}{1+G_1(s)G_2(s)F(s)}(D(s)+M(s))$$



$$F(s) = K_p + \frac{K_1}{s} \qquad G_2(s) = \frac{3{,}2}{s+0{,}8} \qquad G_1(s) = \frac{1{,}25}{s+0{,}3}$$

$$Y(s) = \frac{\frac{1{,}25}{s+0{,}3}\cdot\frac{3{,}2}{s+0{,}8}\cdot K_p + \frac{K_1}{s}}{1+\frac{1{,}25}{s+0{,}3}\cdot\frac{3{,}2}{s+0{,}8}\cdot K_p+\frac{K_1}{s}}R(s) + \frac{\frac{1{,}25}{s+0{,}3}}{1+\frac{1{,}25}{s+0{,}3}\cdot\frac{3{,}2}{s+0{,}8}\cdot K_p+\frac{K_1}{s}}$$

$$= \frac{1{,}25(s+0{,}8)}{s^2+1{,}1s+0{,}24+4+\frac{K_1}{s}} = \frac{1{,}25(s+0{,}8)}{s^3+1{,}1s^2+4{,}24s+4K_1}$$

Rouths algorithm:

1	$4{,}24$
$1{,}1$	$4K_1$
$\frac{4{,}664-4K_1}{1{,}1}$	

$$c_0 = \frac{1{,}1\cdot 4{,}24 - 4K_1\cdot 1}{1{,}1} \Rightarrow K_1 < 1{,}166$$

$$d_0 = \frac{\frac{4{,}664-4K_1}{1{,}1}\cdot 4K_1 - 1{,}1\cdot 0}{\frac{4{,}664-4K_1}{1{,}1}} = 4K_1$$

For a stable closed loop system K_1 needs to be $0 < K_1 < 1{,}166$.

Stability:
When K is unchanged the system have one pole in the origio the system is marginally stable but with bigger K-values the system becomes stable.

Quickness: With smaller K the system is slow since one pole lies close to 0, with bigger K the system will become faster.

Oscillations: When K is small enought to have poles on the Re-axis there will be no oscillations but when K grows the poles will become complex conjugate pairs and the system will start oscillate.