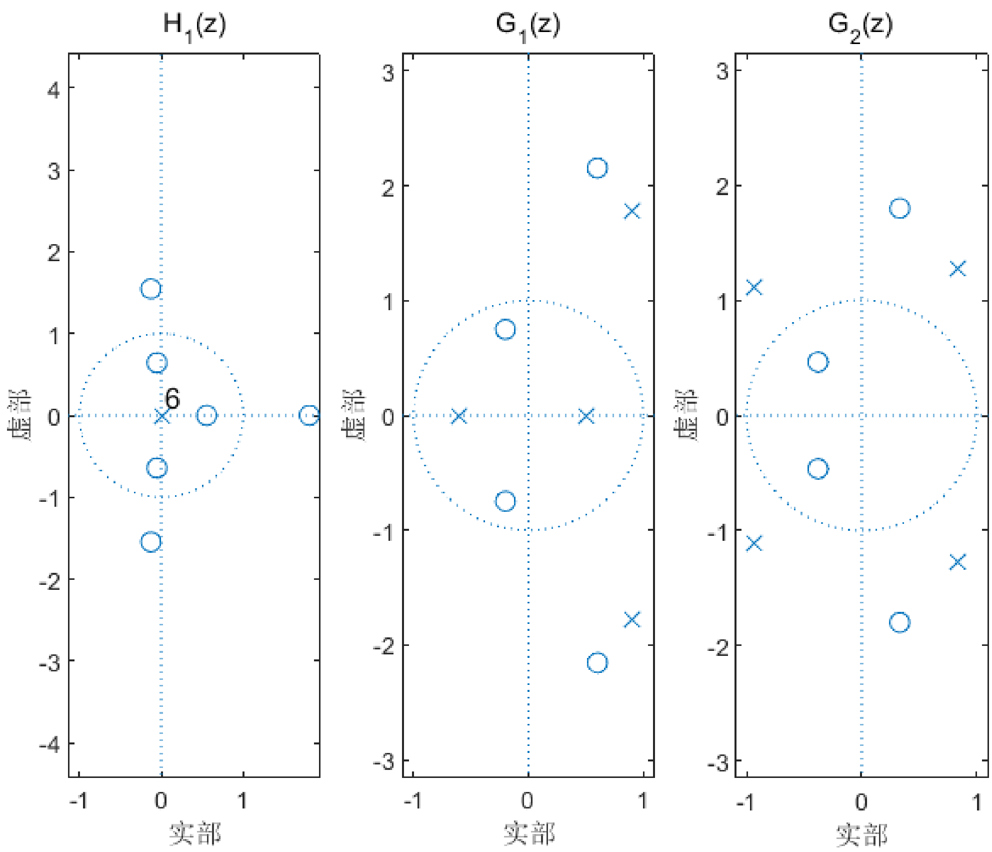
6.2.2



6.2.2 Poles and zeros of z-transform

The poles of $H(z)$ in rational form given in (6.3) are the roots of $D(z)$, and the zeros are the roots of $P(z)$, respectively. If we factorize $H(z)$ to the following form

$$H(z) = \frac{p_0}{d_0} \cdot \frac{\prod_{l=1}^{M}(1-\xi_l z^{-1})}{\prod_{l=1}^{N}(1-\lambda_l z^{-1})} = z^{(N-M)} \frac{p_0}{d_0} \cdot \frac{\prod_{l=1}^{M}(z-\xi_l)}{\prod_{l=1}^{N}(z-\lambda_l)},$$

he system contains N poles at $z = \lambda_l$, for $l = 1,2,..,N$, and M zeros at $z = \xi_l$ for $l = 1,2,..,M$.

max(abs(pole))

Use the Matlab function **roots.m** to find the poles and zeros of the following z-transform

(a) $H_1(z) = -1 + 2z^{-1} - 3z^{-2} + 6z^{-3} - 3z^{-4} + 2z^{-5} - z^{-6}$ $|z|>0$.

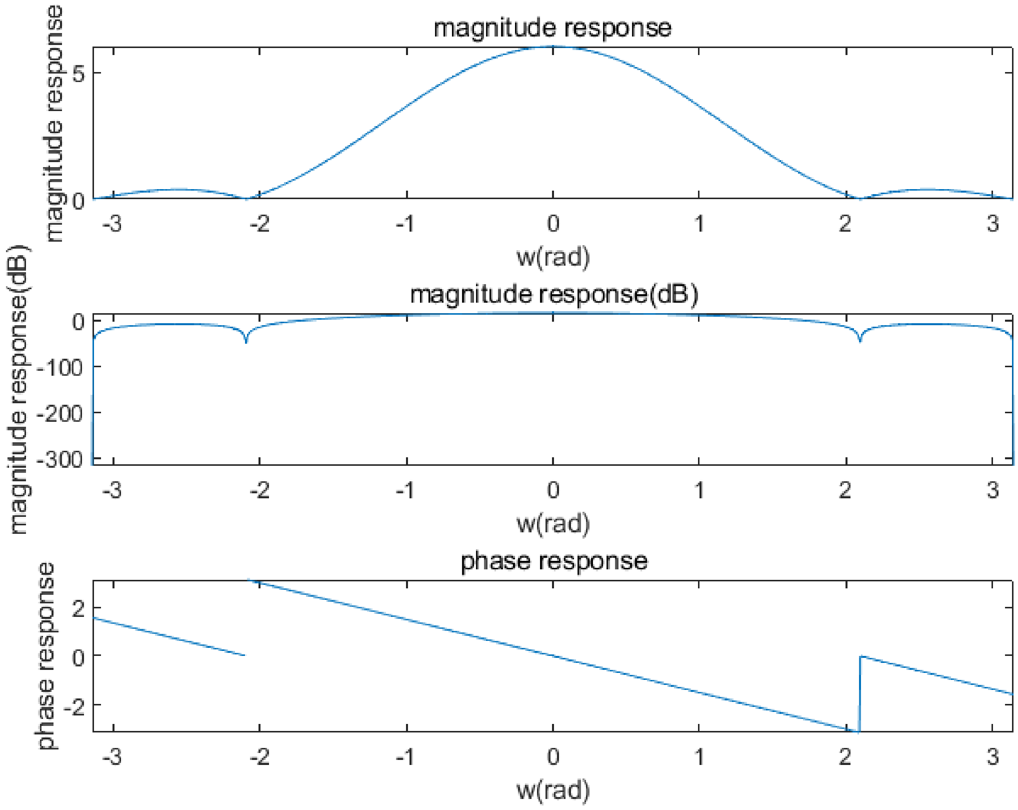
(b) $G_1(z) = \frac{3z^4-2.4z^3+15.36z^2+3.84z+9}{5z^4-8.5z^3+17.6z^2+4.7z-6}$ 左. $|z|<0.5$ 右: $|z|>2$ 双: $0.5\sim2$

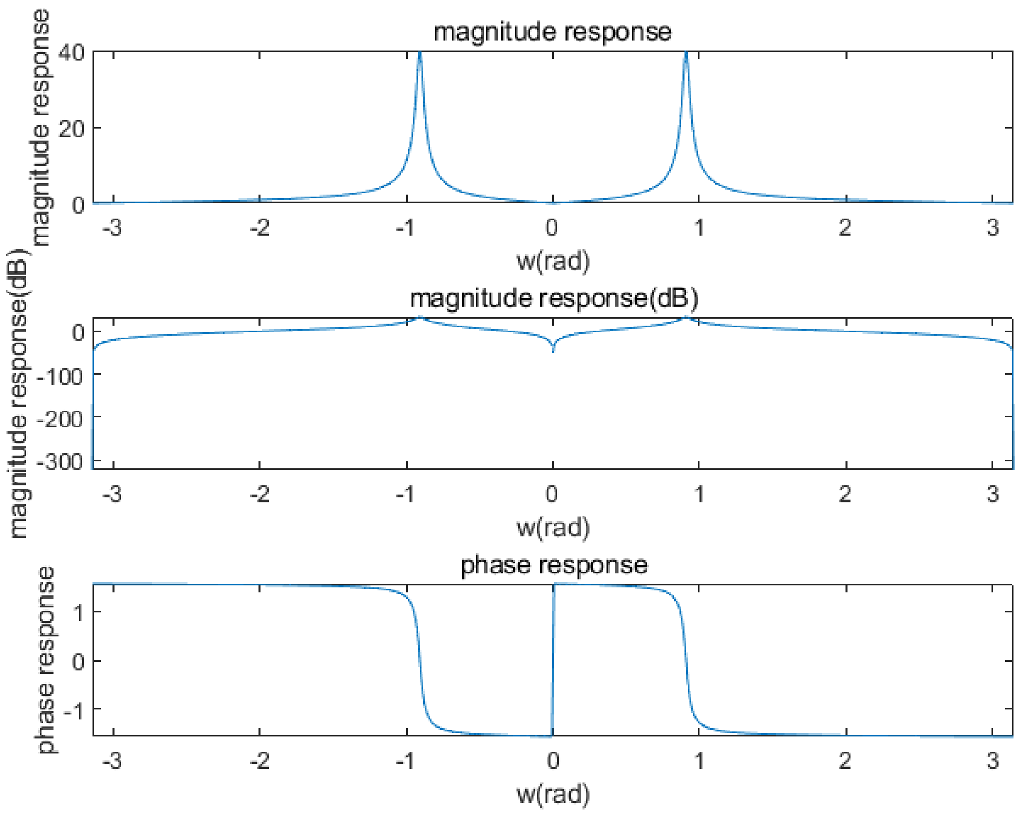
(c) $G_2(z) = \frac{2z^4+0.2z^3+6.4z^2+4.6z+2.4}{5z^4+z^3+6.6z^2+4.2z+24}$ 左 $|z|<1.4421$ 右 $|z|>1.5193$ 双: $1.4421\sim1.5193$.

Use the Matlab function **zplane.m** to display the zero-pole plot. *Determine all possible ROCs* each of the above z-transforms, and describe the type of their inverse z-transform (left-sided, right-sided, two sided sequences) associated with each of the ROCs.

INLAB REPORT: For each of the above z-transform, report the poles and zeros, hand in the

6.2.3


magnitude response
magnitude response
w(rad)
magnitude response(dB)
magnitude response(dB)
w(rad)
phase response
phase response
w(rad)


magnitude response
magnitude response
w(rad)
magnitude response(dB)
magnitude response(dB)
w(rad)
phase response
phase response
w(rad)

6.2.4

$$X(z) = \frac{-2.4783}{1-0.7z^{-1}} + \frac{5.4783}{1+1.6z^{-1}}$$

ROC
- $|z| > 1.6$ 右 . $x[n] = -2.4783(0.7)^n u[n] + 5.4783(-1.6)^n u[n]$
- $|z| < 0.7$ 左 $x[n] = 2.4783(0.7)^n u[-n-1] - 5.4783(-1.6)^n u[-n-1]$
- $0.7 < |z| < 1.6$ 双 . $x[n] = -2.4783(0.7)^n u[n] - 5.4783(-1.6)^n u[-n-1]$

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Each above z-transform has several ROCs. Evaluate their respective inverse z-transforms corresponding to each ROC.

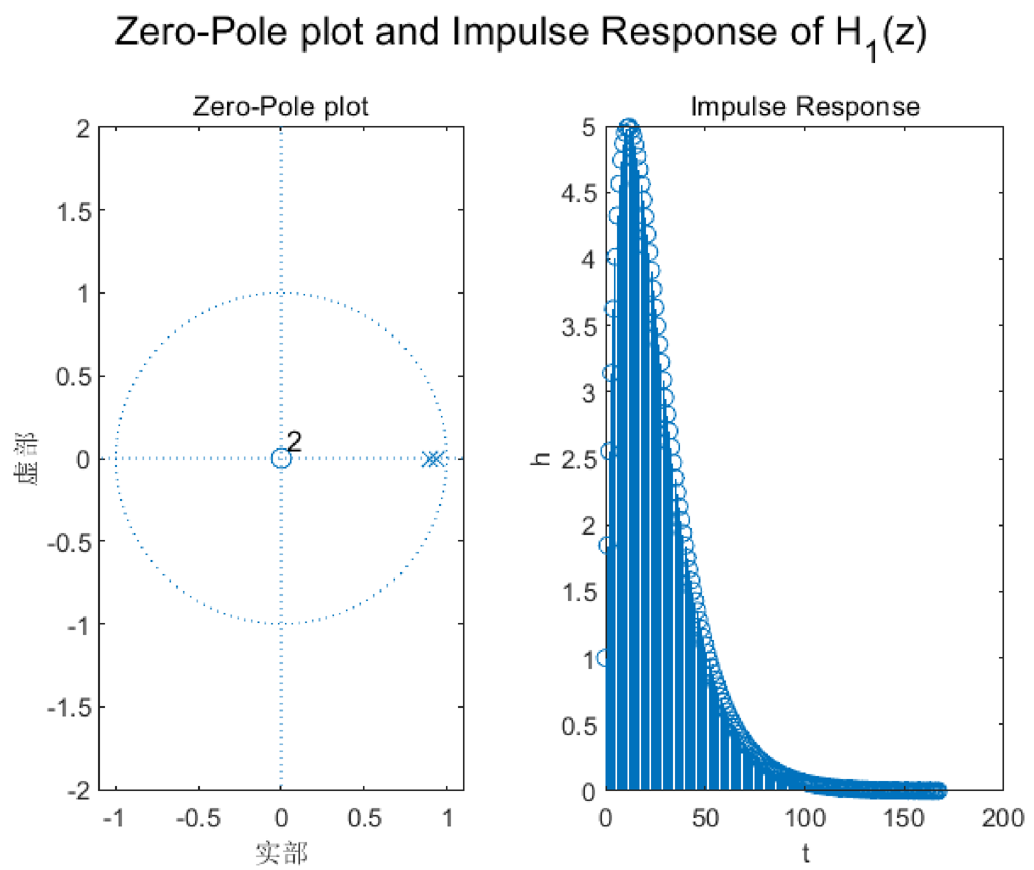
INLAB REPORT: Submit the partial fraction expansion, all possible ROCs, and the corresponding inverse z-transforms. $Y(z) = -6.4 + \frac{2.8889}{1+0.4z^{-1}} + \frac{6.5111}{1-0.5z^{-1}}$

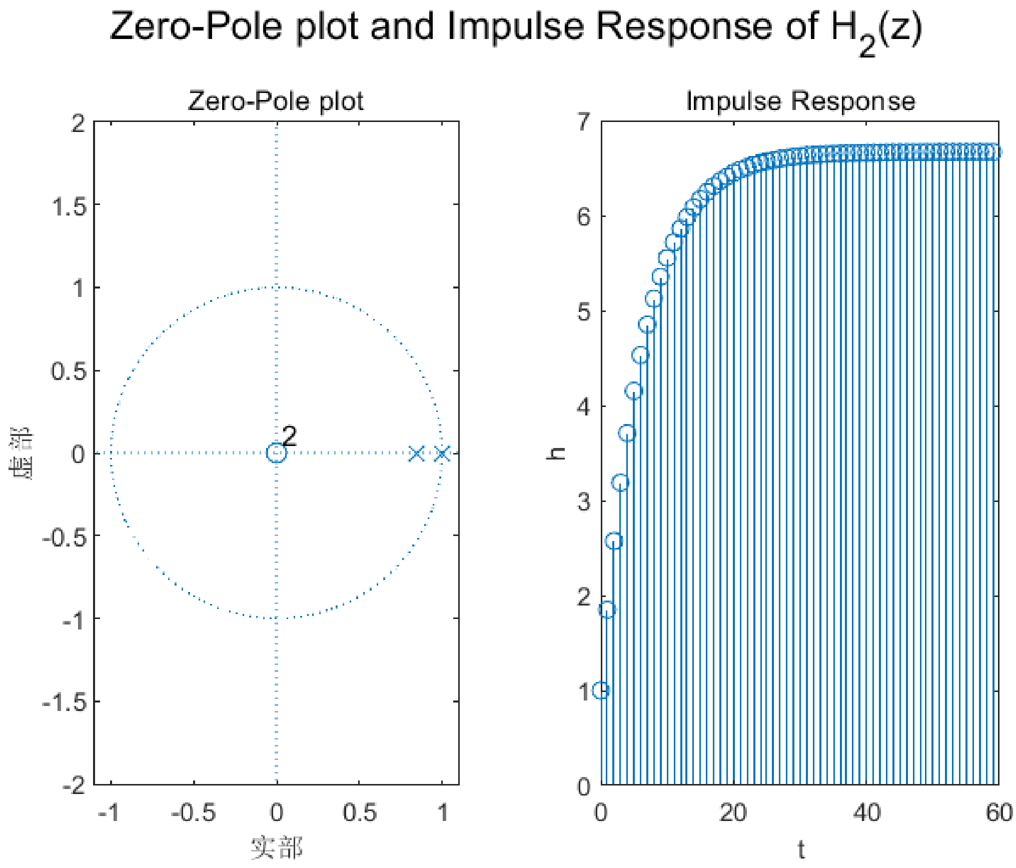
ROC
- $|z| > 0.5$ 右 $y[n] = -6.4\delta[n] + 2.8889(-0.4)^n u[n] + 6.5111(0.5)^n u[n]$
- $|z| < 0.4$ 左 $y[n] = -6.4\delta[n] - 2.8889(-0.4)^n u[-n-1] - 6.5111(0.5)^n u[-n-1]$
- $0.4 < |z| < 0.5$ 双 . $y[n] = -6.4\delta[n] + 2.8889(-0.4)^n u[n] - 6.5111(0.5)^n u[-n-1]$

6.2.5 Stability Conditions

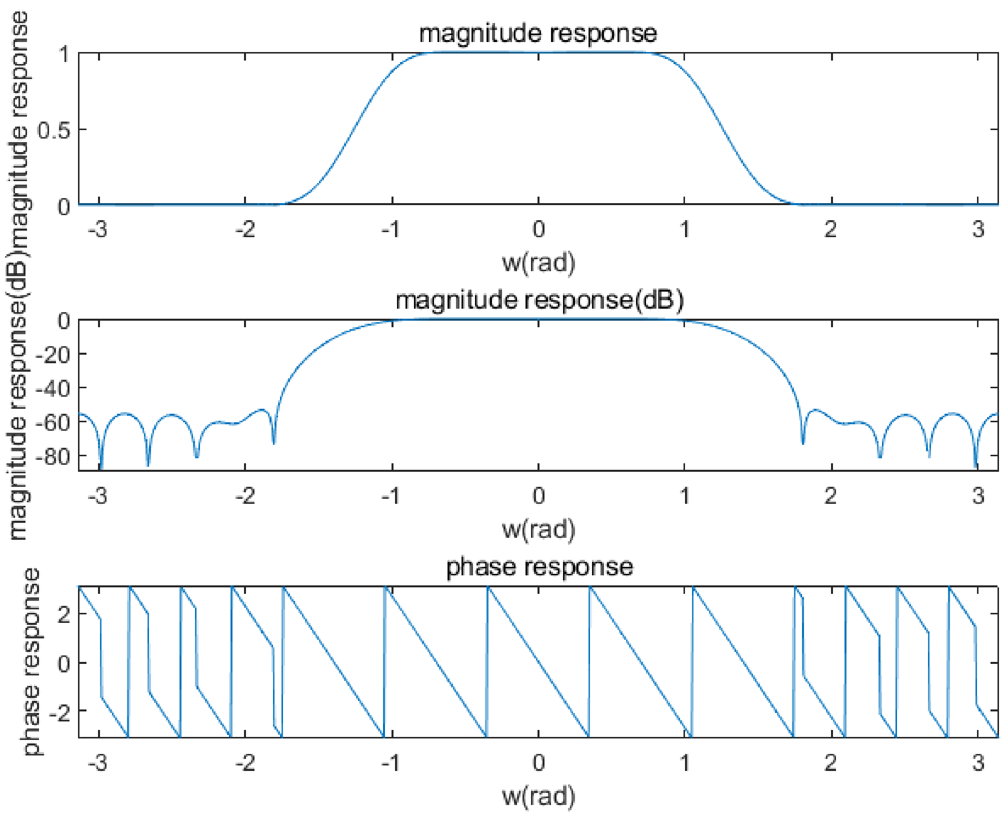
A stable causal LTI system has all poles inside unit circle. This implies that a causal LTI FIR digital filter with bounded impulse response coefficients is always stable, as all its poles are at the origin in the *z*-plane. On the other hand, a causal LTI IIR digital filter may or may not be stable, dependent on its pole locations. In addition, an originally stable IIR filter characterized by infinite precision coefficients and with all poles inside the unit circle may become unstable after implementation due to the unavoidable quantization of all coefficients

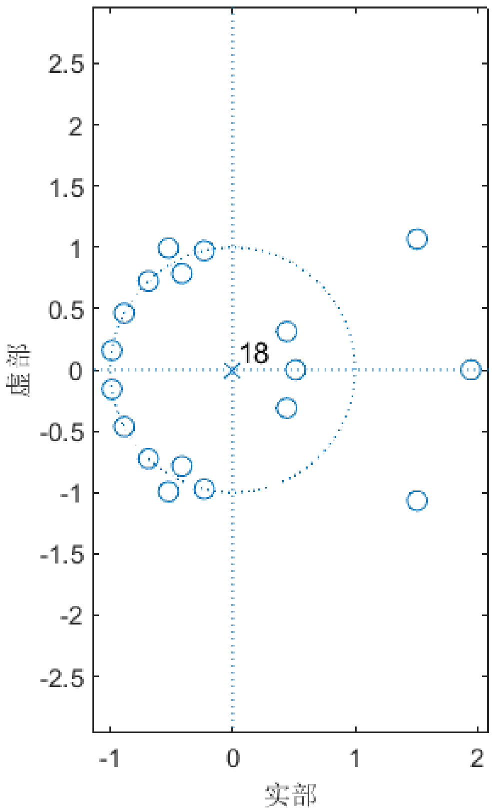
6.2.5



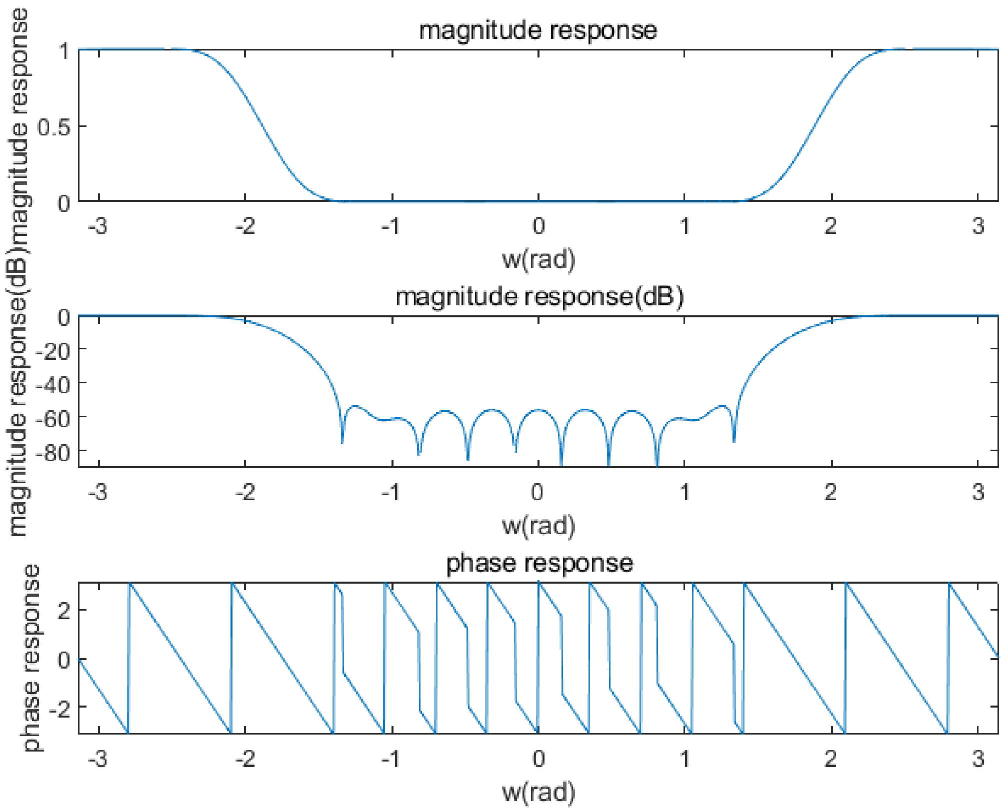

Zero-Pole plot and Impulse Response of $H_2(z)$
Zero-Pole plot
虚部
实部
2
Impulse Response
h
t

6.3.1

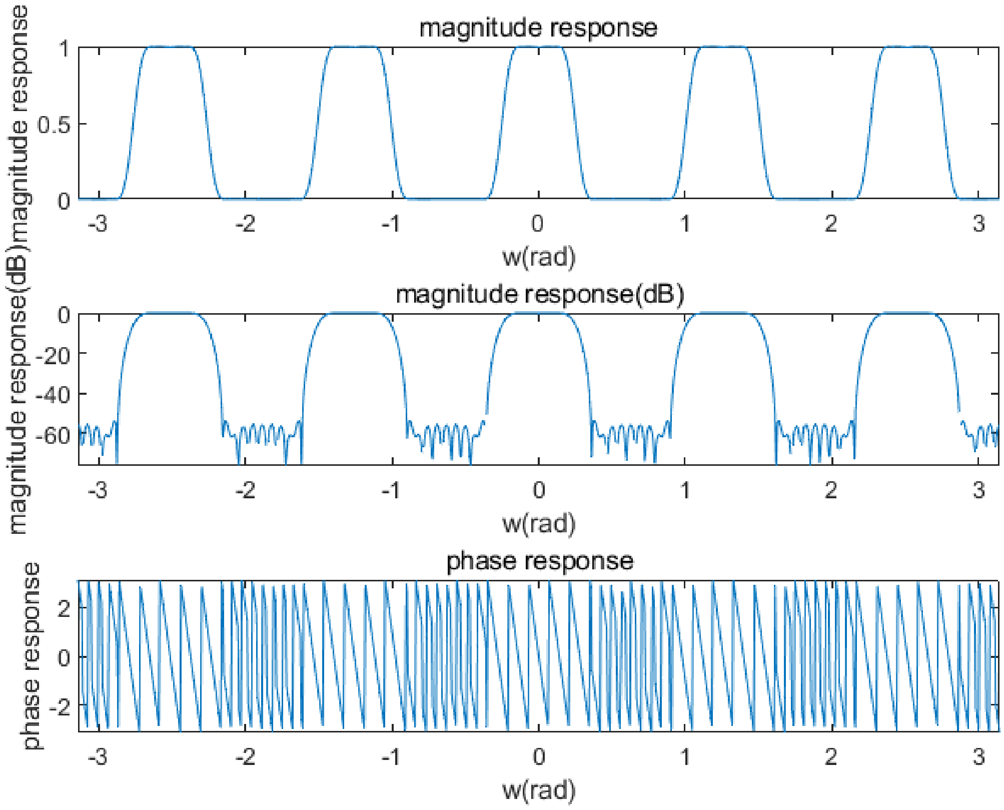

magnitude response
magnitude response
w(rad)
magnitude response(dB)
magnitude response(dB)
w(rad)
phase response
phase response
w(rad)



18
虚部
实部



magnitude response
magnitude response
w(rad)
magnitude response(dB)
magnitude response(dB)
w(rad)
phase response
phase response
w(rad)



magnitude characteristic of the filter, and (2) if a type III filter may be designed to be a lowpass filter, and justify your answer. $h[n] = -h[N-1-n]$

关于 $w=0, 2\pi$ 奇对称. $w=\pi$ 奇对称

在 $z=1$ 有零点，不能设计低通带阻

The impulse response of the $h_1[n]$ and $h_2[n]$ is obtained by

$$h_1[n] = \begin{cases} h[n], & \text{for even } n \\ -h[n], & \text{for odd } n \end{cases}$$

$$h_2[n] = \begin{cases} h[n/5], & \text{for } n = 5k \text{ for interger } k \\ 0, & \text{otherwise} \end{cases}$$

INLAB REPORT: Express the z-transform of $h_1[n]$ and $h_2[n]$ in terms of $H(z)$, and express the frequency response of $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ in terms of $H(e^{j\omega})$. Verify the frequency response expressions by plotting the frequency response of $h_1[n]$ and $h_2[n]$. Describe the magnitude characteristic of the filter $H_1(z)$ and $H_2(z)$ with respect to that of $H(z)$.

$$H(-z) = \sum_{-\infty}^{\infty} h[n](-z)^{-n}$$
$$= \sum_{-\infty}^{\infty} h[n](z)^{-n}(-1)^{-n}$$
$$= \sum_{-\infty}^{\infty} h[n_{even}](z)^{-n_{even}} - \sum_{n=-\infty}^{\infty} h[n_{odd}](z)^{-n_{odd}}$$
$$= H_1(z) = H(-e^{jw}) = H(e^{j(w+\pi)})$$

$$H_2(z) = \sum_{r=-\infty}^{\infty} h_2[r \cdot 5] e^{-jwr5}$$
$$h_2[rk] = h[r].$$
$$H_2(e^{jw}) = \sum_{-\infty}^{\infty} h_2[r] e^{-j(5w)r} = H(e^{j5w})$$
$$H_2(e^{jw}) = H(e^{j5w})$$

2022 Aug. by Yu Yajun @ SUSTech

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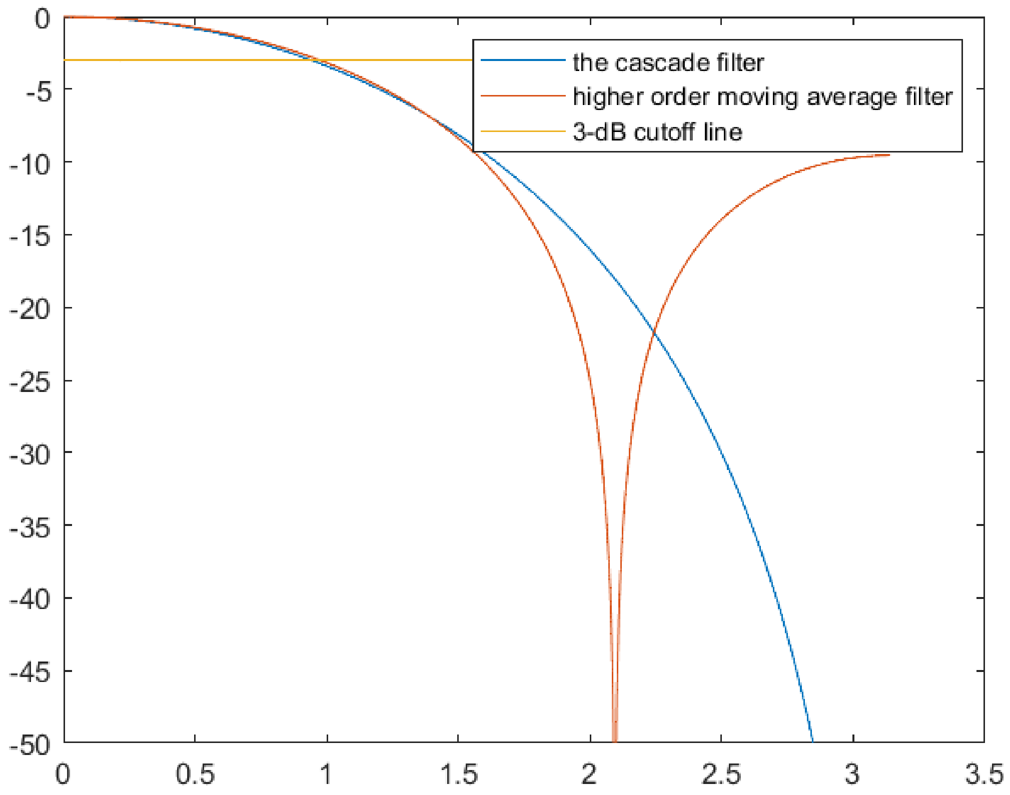
1. right shift π.
2. high pass.

1. ~~band~~ period of $H_2 = \frac{1}{5} H$.

6.3.2



INLAB REPORT: Submit your derivation of (6.5) and the number of stages in the cascade design. Show the steps to determine the order of the filter in the higher order moving average filter design. Plot the gain responses of both designed filters on the same figure. Indicate the actual 3-dB cutoff frequency. Give your comments if any.

The simplest highpass FIR filter is obtained by replacing z with $-z$ in (6.4), resulting in a transfer function given by

$$H_1(z) = \frac{1 - z^{-1}}{2} \tag{6.7}$$

(6.4) $H(z) = \left(\frac{1+z^{-1}}{2}\right)^3 = e^{-j3\omega/2} \cos^3(\omega/2)$.

(6.6) $H(e^{j\omega}) = \frac{1}{3} \cdot \frac{\mathrm{sinc}(\frac{3\omega}{2})}{\mathrm{sinc}(\omega/2)} e^{-j\frac{2\omega}{2}}$.

The corresponding frequency response is given by

$$H_1(e^{j\omega}) = je^{-j\omega/2}\sin(\omega/2)$$

3-dB cutoff freq

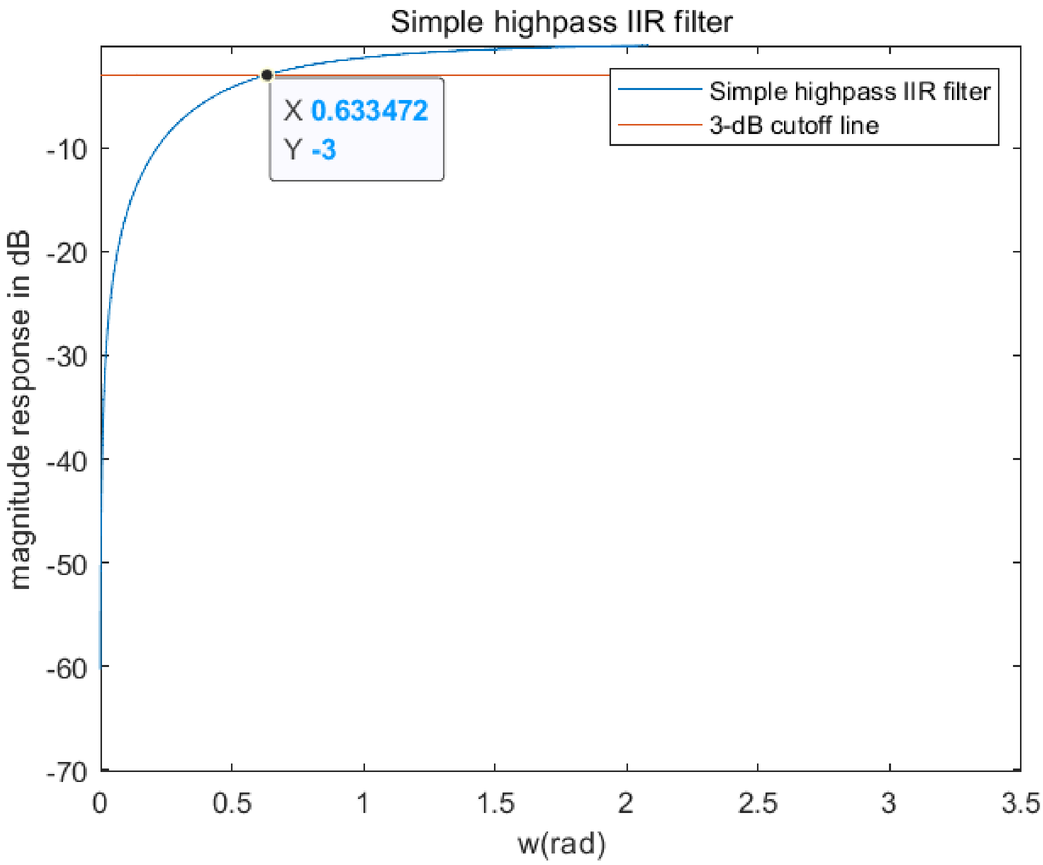
0.9432 cas ~ 0.9740 avg.

Improved highpass frequency response can be obtained by cascading several sections of the simple highpass filter of (6.7), or replacing z with $-z$ in (6.6).

6.4 IIR Filters

The first ...

6.4



Simple highpass IIR filter
Simple highpass IIR filter
3-dB cutoff line
X 0.633472
Y -3
magnitude response in dB
-10
-20
-30
-40
-50
-60
-70
0
0.5
1
1.5
2
2.5
3
3.5
w(rad)