An Analysis of Resonance Activity in π^--proton elastic collision and a Brief Review of Quantum Numbers

Shahnaz Abdul Hameed

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1 Summary

In the first part of this report, a brief review is provided on Quantum Numbers-i.e, their origin, history and broader connections to the symmetry of nature. The second part focuses on studying negative pion-proton elastic collision data provided by the Particle Data Group (PDG, `https://pdg.lbl.gov/`), led by the Lawrence Berkeley National Laboratory (LBNL). Energy (in GeV) is calculated from momentum values and plotted against Cross-Section (MB). I've then run algorithms to identify resonances at different prominences, calculated each of their masses and widths, studied and then reported them.

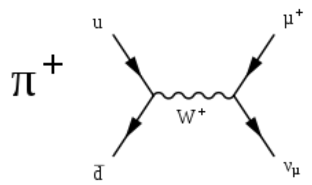

Figure 1: Feynman diagram of the dominant leptonic pion decay

2 Quantum Numbers

2.1 Bohr, 1913

Bohr thought that each electron was in its own unique energy level, which he called a "stationary state," and that each electron would have a unique value of 'n'. He subsequently discovered the first qunatum number, **The Principal Quantum Number** (signified by the letter 'n') in 1913.

Bohr was wrong. It very quickly was discovered that more than one electron could have a given 'n' value. For example, it was eventually discovered that when n = 3, eighteen different electrons could have that value of n. n does not refer to any particular location in space or any particular shape. It is one component (of four) that will uniquely identify each electron in an atom. n will always be a whole number and never less than one.

2.2 Insufficiency of Bohrs 'n', enter Sommerfeld, 1914

In 1914-1915, Arnold Sommerfeld realized that Bohr's 'n' wasn't gonna cut it. In other words, more equations were needed to properly describe how electrons behaved. In fact, Sommerfeld realized that **two more** quantum numbers were needed.

The first of these is the quantum number signified by 'ℓ'. When Sommerfeld started this work, he used n' (n prime), but he shifted it to 'ℓ' after some years (easier to print ℓ)

The rule for selecting the proper values of ℓ is as follows:

- $\ell = 0, 1, 2, \ldots, n - 1$ ℓ will always be a whole number
- ℓ will **never** be as large as the 'n' value it is associated with

Caution: Some sources call this quantum number the "Orbital Quantum Number".

The Magnetic Quantum Number (signified by m_ℓ): Sommerfield discovered this quantum number in 1914-15, but there were other contributors (that I'll get to) towards its origin as well.

The rule for selecting m_ℓ is as follows:

- m_ℓ starts at negative ℓ, runs by whole numbers to 0 and then goes by whole numbers to positive ℓ.
- When $\ell = 2$, the m_ℓ values generated are $2, 1, 0, +1, +2$, for a total of 5 values

2.3 Beyond Sommerfield, The Landé g-factor, 1923

Landé's Frankfurt investigations (December 1920 until April 1921) ended with the discovery of the well-known Landé g-formula and an explanation for the anomalous Zeeman effect.

The Landé g-factor is now defined through m_ℓ, the magnetic quantum number. In 1923, he stated the *Landé interval rule*, a rule dealing with the relation between an electron's spin and orbital angular momentum.

2.4 Pauli's request for a fourth one, 1925

In 1925, Wolfgang Pauli demonstrated the need for a fourth quantum number. He closed the abstract to his paper this way:

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"On the basis of these results one is also led to a general classification of every
electron in the atom by the principal quantum number n and two auxiliary quantum numbers
k1 and k2 to which is added a further quantum number m in the presence of an external
field.  In conjunction with a recent paper by E. C. Stoner this classification leads to
a general quantum theoretical formulation of the completion of electron groups in atoms."
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In late 1925, two young researchers named **George Uhlenbeck** and **Samuel Goudsmit** discovered the property of the electron responsible for the fourth quantum number being needed and named this property **spin**.

The rule for selecting ms is as follows:

- after the n, ℓ and m_ℓ to be used have been determined, assign the value $+1/2$ to one electron
- then assign $1/2$ to the next electron, while using the same n, ℓ and m_ℓ values
- For example, when n, ℓ, $m_\ell = 1, 0, 0$; the first m_s value used is $+1/2$. However a second electron can also have n, ℓ, $m_\ell = 1, 0, 0$; so assign $1/2$ to it.

2.4.1 Why call it "spin"?

Spin is a property of electrons that is not related to a sphere spinning. It was first thought to be this way, hence the name spin, but it was soon realized that electrons cannot spin on their axis like the Earth does on its axis. If the electron did this, its surface would be moving at about ten times the speed of light. This is absurd. In any event, the electron's surface would have to move faster than the speed of light and this isn't possible.

2.5 Through the lens of Schrodinger, 1927

In 1927, Schrodinger put forth the idea that the wavelike properties of an electron could be described by mathematical equations called wave functions. These wave functions were used to create orbitals which are probability distribution maps showing where the electron is likely to be found.

Quantum numbers are thus a **way of describing the orbitals and electrons contained in an atom**. Quantum numbers arise in the process of solving the Schrodinger equation by constraints or boundary conditions which must be applied to get the solution to fit the physical situation. The case of a particle confined in a three-dimensional box can be used to show how quantum numbers arise. The solution to this 3-D wave-function is given by:

$$\Psi(x, y, z) = A \sin\left(\frac{n_x \pi x}{L_x}\right) \sin\left(\frac{n_y \pi y}{L_y}\right) \sin\left(\frac{n_z \pi z}{L_z}\right)$$

This yields energy eigenvalues which are determined by three quantum numbers.

$$E = \frac{n_x^2 h_x^2}{8mL_x^2} + \frac{n_y^2 h_y^2}{8mL_y^2} + \frac{n_z^2 h_z^2}{8mL_z^2}$$

So we see how the solution to the Schrodinger equation in three dimensions led to three quantum numbers. In the case of the hydrogen atom, the boundary conditions are much different, but they also lead to three spatial quantum numbers.

2.6 On Relation to Symmetries in nature

Symmetries are basically mathematical transformations that leave the physical laws and properties of a system invariant. What do qunatum numbers have to do with it?

2.6.1 Symmetry Breaking

Symmetry breaking occurs when a physical system transitions from a symmetric state to a lower symmetry state. This can result in the emergence of new quantum numbers and the appearance of new physical phenomena. An example is the *Higgs mechanism* in particle physics, where the breaking of electroweak symmetry gives rise to the masses of particles and introduces new quantum numbers associated with the Higgs field.

2.6.2 Group Theory

Group theory provides a mathematical framework to analyze and classify certain symmetries. Quantum numbers are associated with the irreducible representations of symmetry groups and the properties and behavior of particles can be understood by considering their transformation properties under the symmetry group of the system.

2.6.3 Conservation Laws

Quantum numbers can be associated with the conservation laws observed in particle interactions. As examples, the conservation of electric charge is related to the electric charge quantum number, the conservation of baryon number and lepton number are connected to the baryon number and lepton number quantum numbers, respectively. These conservation laws arise due to the underlying symmetries of nature, such as the conservation of energy, momentum, and angular momentum.

3 Analysing the resonance activity in π^--proton elastic collision

For our analysis, we first extract the pi^--proton elastic cross-section data from the Particle Data Group (PDG).

3.1 Brief theoretical take

INFORMATION ABOUT THE SYSTEM:

- **Reaction**: $\pi^- + P \dashrightarrow P + \pi^-$. It represents the collision of a negative pion (π^-) with a proton (P) resulting in a proton and a negative pion in the final state.
- **Beam Mass**: 0.139570 GeV/c^2. The mass of the incoming particle (pion) in GeV/c^2.
- **Target Mass**: 0.938270 GeV/c^2. The mass of the target particle (proton) in GeV/c^2.
- **Threshold**: 0. The threshold energy for the reaction (likely meaning the reaction is possible at all energies).
- **Final State Multiplicity**: 2. The multiplicity of the final state particles (proton and negative pion).
- **Number of Data Points**: 277. The total number of data points available for this reaction.

INFORMATION ABOUT THE DATASET:

- **Point Number**: The sequential number of each data point.
- **PLAB(GEV/C)**: The momentum of the incoming particle (π^-) in GeV/c.
- **PLAB MIN**: The minimum value of the momentum of the incoming particle (π^-) in GeV/c.
- **PLAB MAX**: The maximum value of the momentum of the incoming particle (π^-) in GeV/c.
- **SIG(MB)**: The measured cross-section of the reaction in millibarns.
- **STA ERR+**: The statistical error (+ve) associated with the measured cross-section.
- **STA ERR-**: The statistical error (-ve) associated with the measured cross-section.
- **SY ER+(PCT)**: The systematic error (+ve) expressed as a percentage.
- **SY ER-(PCT)**: The systematic error (-ve) expressed as a percentage.
- **Reference**: The reference or source of the data.
- **Flag**: A flag or additional information related to the data point.

3.2 Studying using Python

Since space restrictions mean the entire code can't be provided in the report, here's a breakdown of what each section of the program intends to do, along with the results.

3.2.1 Energy Calculation

We first need to convert the momentum values (in GeV/c) into energy. We use the relativistic relation between energy and momentum and add both incident and target particle energies. The target particle is at rest and momentum is thus taken to be zero. Value of c is considered to be 1 (in natural units). A function is made for this purpose. This relation is given by:

$$E^2 = (pc)^2 + (mc^2)^2$$

3.2.2 Converting .dat file into pandas DataFrame

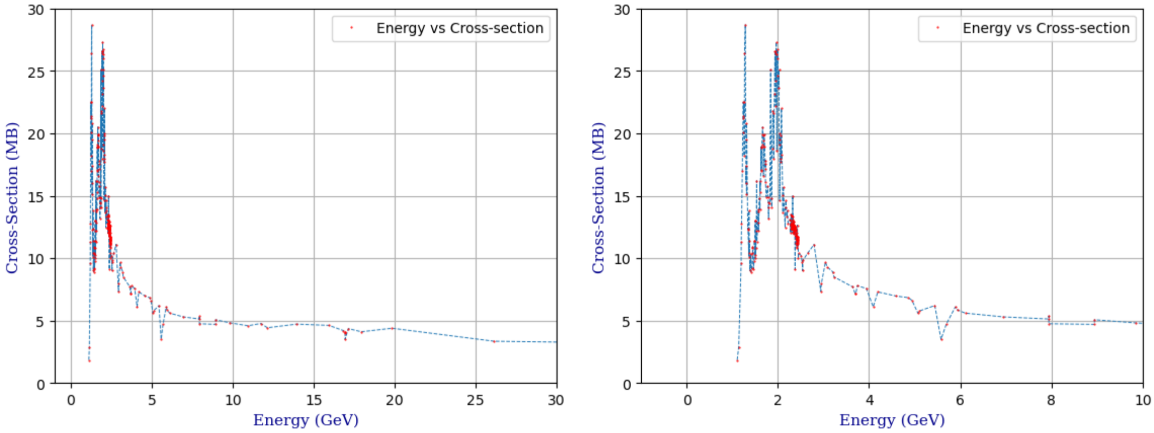
This is done by skipping the first 11 rows of .dat file, manually adding each column, and then converting this <list> type into a pandas Dataframe that can be easily used.

3.2.3 Extracting necessary columns, creating the energy vector

We extract the momentum (in GeV/c), cross-Section (in MB), statistical errors associated with the cross-section (then store it in arrays) and create an energy vector from the previous function.

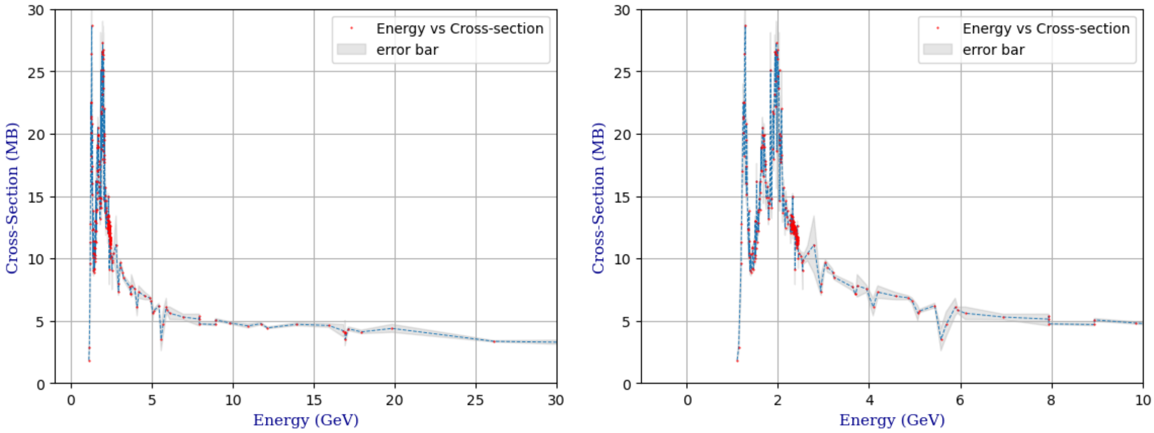
3.2.4 Plotting the results

We first create error bars based on the provided statistical errors. Making use of matplotlib.pyplot's functionalities, functions are created for the plot



(a) (b)

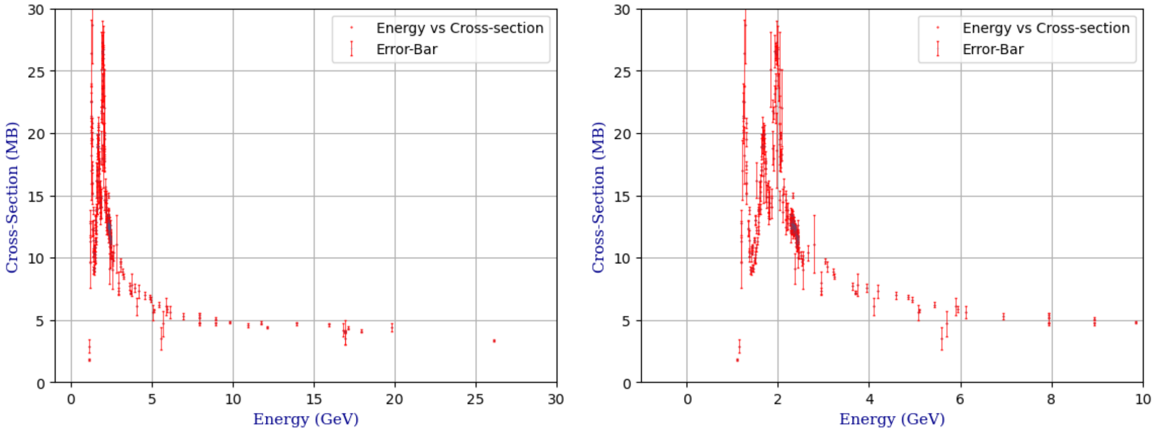
Figure 2: Energy vs Cross-Section at (a) lim 30 GeV (b) lim 10 GeV



(a) (b)

Figure 3: Error-filled (Cross-Section's) Plot at (a) lim 30 GeV (b) lim 10 GeV

Now instead of shading on the statistical error bounds of cross-section, we plot it's error bars. These error bars extend vertically from each data point, indicating the range of possible values due to measurement errors. The length of each error bar is determined by the corresponding positive and negative uncertainties, represented by the sta-err-minus and sta-err-plus variables, respectively. In Figure 3, error-bars were filled between instead.



(a) (b)

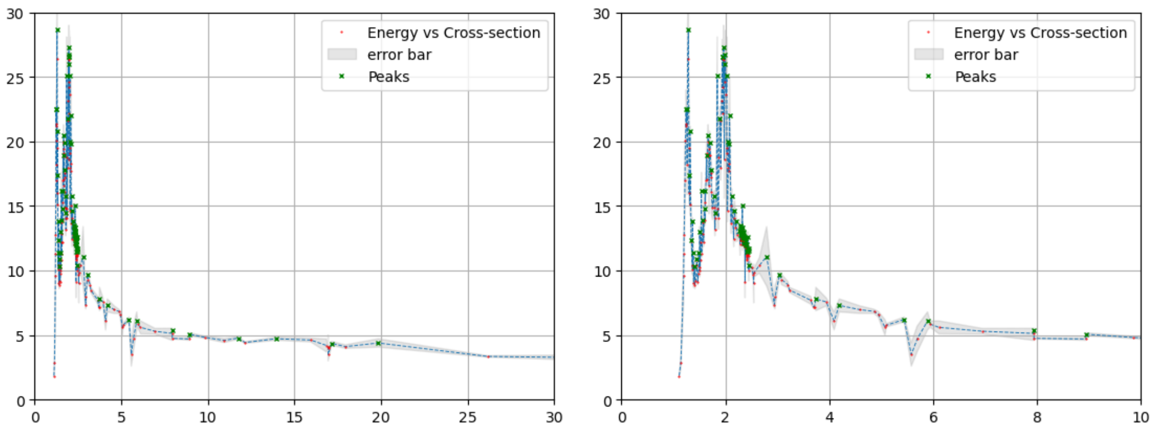
Figure 4: Plot with error-bars at (a) lim 30 GeV (b) lim 10 GeV

3.2.5 Understanding and identifying Resonances

Resonances in the elastic collision between a π^- and a proton refer to specific energy states or excited configurations of particles that exhibit enhanced interaction probabilities. In this collision scenario, they may appear as localized enhancements or peaks in the cross-section measurements, indicating a higher likelihood of interaction at particular energy values.

These deviations from the smooth background behavior can be identified by carefully analyzing the data, looking for systematic patterns, comparing with theoretical models, and considering the resonance width or lifetime. By identifying resonances and extracting their parameters (such as mass and width), valuable insights into the underlying physics and the properties of the interacting particles can be gained, contributing to a deeper understanding of the collision dynamics.

Here we use **scipy.signals** functionalities to parametrise, identify and classify sharp peaks in our data The indices where peaks occur are identified, index values for energy and cross section are extracted, plotted and marked with a green cross. Note here that we do not specify any threshold for prominence, or all peaks are detected naively.

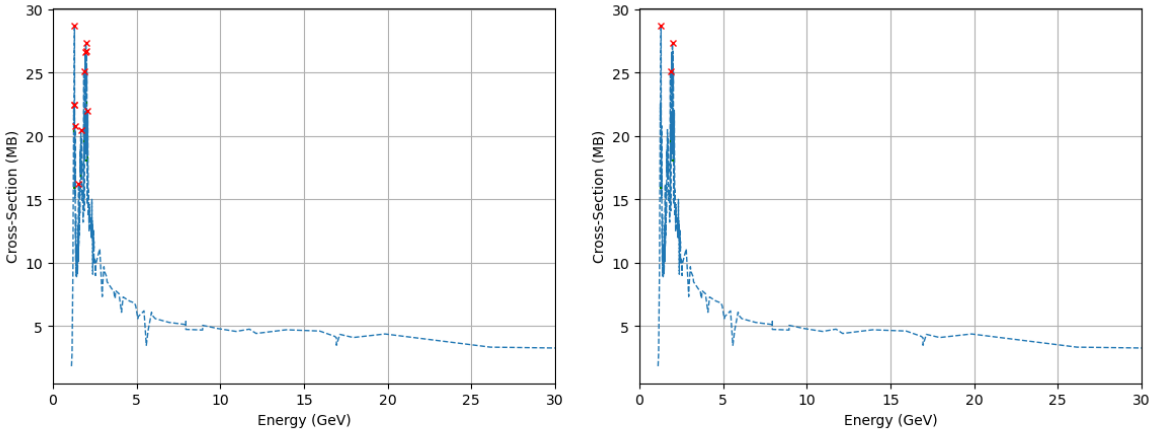


(a) (b)

Figure 5: Sharp peaks in data at (a) lim 30 GeV (b) lim 10 GeV

3.2.6 Interpolation of indices and filtering peaks

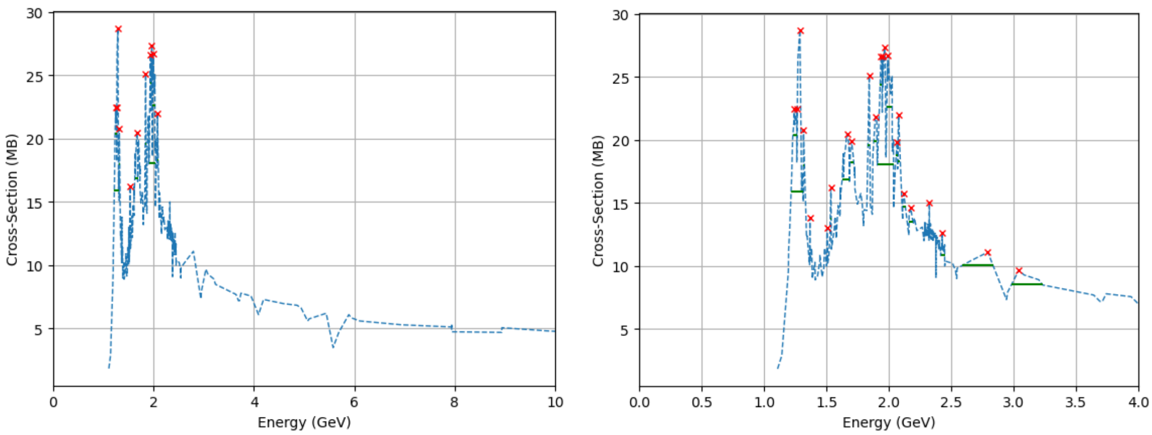
Now there's a need for conversion of peak indices to corresponding energy values. We do this by interpolation. Next, peak widths are identified, inflection points detected and identified peaks plotted with their widths. Additionally, the function is made flexible for the specification of **height** and **threshold**. These modifications enable more precise peak identification and visualization, facilitating a deeper analysis of resonances in the energy versus cross-section data. These figures will help identify what the threshold parameters do. The widths of each resonance is also marked.



(a) (b)

Figure 6: Peaks filtered at energy band (GeV) [0,30] and parameters (a) , height:1, prominence:4 (b) height:1, prominence:10

Closing in on the energy values:



(a) (b)

Figure 7: Peaks filtered (a) energy band [0,10], height:1, prominence:4 (b) energy band [0,4] height:1, prominence:12

In a nutshell, lower threshold valued identify many more peaks. These parameters must be set optimal to the reaction setting being studied. We now take a look at the mass and widths of identified peaks and study them.

3.2.7 Mass and Width of Resonances

In particle physics, a resonance is the peak located around a certain energy found in differential cross sections of scattering experiments. These peaks are associated with subatomic particles, which include a variety of bosons, quarks, and hadrons (such as nucleons, delta baryons, or upsilon mesons) and their excitations.

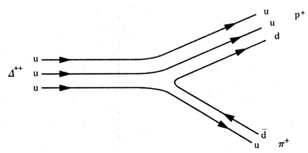


Figure 8: A quark model diagram of the decay Delta++ to p + pi+

In common usage, "resonance" only describes particles with very short lifetimes, mostly high-energy hadrons existing for 10^{-23} seconds or less. It is also used to describe particles in intermediate steps of a decay, so-called virtual particles.

The width of the resonance (Γ) is related to the mean lifetime (τ) of the particle (or its excited state) by the relation

$$\Gamma = \frac{\hbar}{\tau}$$

where $\hbar$ is the reduced Planck constant given by $\hbar = \frac{h}{2\pi}$, and h is the Planck constant.

Let's take a look at our identified peaks widths and masses with very high thresholds.

```
---------------------------------
Energy band:( 0 , 30 )
Prominence: 10
Height: 1
Peaks detected at indices: [13 76 89]
Energies (in GeV) corresponding to detected peaks: [1.28784122 1.84326711 1.96484204]
Mass of detected resonances (in GeV/c^2): [1.28784122 1.84326711 1.96484204]
Mass of detected resonances (in MeV/c^2): [1287.84121578 1843.26711259 1964.84203668]
Width of detected peaks (in GeV): [1.27123564 1.14538195 1.31598149]

---------------------------------
Energy band:( 1 , 1.5 )
Prominence: 4
Height: 1
Peaks detected at indices: [  8  10  13  18 ]
Energies (in GeV) corresponding to detected peaks: [1.24284132 1.26284105 1.28784122 1.31683756]
Mass of detected resonances (in GeV/c^2): [1.24284132 1.26284105 1.28784122 1.31683756]
Mass of detected resonances (in MeV/c^2): [1242.84132005 1262.84104892 1287.84121578 1316.83755659]
Width of detected peaks (in GeV): [1.2107591  1.2107591  1.27123564 1.13729109]
```

We observe the resonance at around Energy 1240 GeV showing up at both very difference parameter settings, and might possibly be the **Delta** (Δ) baryon.

The Mass (in GeV/c^2) and mean-lifetime (s) we identified and error wrt Delta Baryon:

```
Mass (in GeV/c^2) 1.24284132
Mean lifetime: 5.4726577318312125e-24 s
Error in lifetime: 1.5734226816878737e-25
Error in Mass (GeV/c^2): 0.010000000000000009
```

Table 1: Properties of the Delta Baryon

Particle	Charge	Mass (GeV/c^2)	Spin	Width (GeV)	Mean Lifetime (s)
Δ^{++}	+2	1.232	3/2	0.117	$(5.63 \pm 0.14) \times 10^{-24}$
Δ^{+}	+1	1.232	3/2	0.117	$(5.63 \pm 0.14) \times 10^{-24}$
Δ^{0}	0	1.232	3/2	0.117	$(5.63 \pm 0.14) \times 10^{-24}$
Δ^{-}	-1	1.232	3/2	0.117	$(5.63 \pm 0.14) \times 10^{-24}$

We find both the error bars are in predefined statistical bounds. For more resonances the thresholds can be decreased further.

```
--------------------------------
Energy band:( 0 , 30 )
Prominence: 4
Height: 1
Peaks detected at indices: [  8  10  13  18  42  58  76  84  89  94 105]
Energies (in GeV) corresponding to detected peaks: [1.24284132 1.26284105 1.28784122 1.31683756
1.53677128 1.66577361 1.84326711 1.93783212 1.96484204 1.99283672 2.07783929]
Mass of detected resonances (in GeV/c^2): [1.24284132 1.26284105 1.28784122 1.31683756
1.53677128 1.66577361 1.84326711 1.93783212 1.96484204 1.99283672 2.07783929]
Mass of detected resonances (in MeV/c^2): [1242.84132005 1262.84104892 1287.84121578 1316.83755659
1536.77128229 1665.77361167 1843.26711259 1937.83212303 1964.84203668 1992.83672098
 2077.83928925]
Width of detected peaks (in GeV): [1.2107591  1.2107591  1.27123564 1.13729109 1.14845545
1.24671754 1.14538195 1.19607791 1.31598149 1.21962453 1.16621001]
```

By changing around parameters depending on our specific requirements, more major resonances can be identified.

4 SUMMARY

In this report, beyond the brief overview of Quantum Numbers and Analysis of Resonance Activity in π^-- proton collision, a general framework is demonstrated on how to perform said analysis for different kinds of datasets: by making dataframes, extracting required values, plotting and finding spikes/peaks, extrapolating based on initial x-values, parametrizing thresholds, filtering resonances and comparing. All the code developed for this report is uploaded on my github: `https://github.com/MarlaJahari/ResonanceAnalysis`. I heartily thank Prof. Maxim Mai for his guidance in the theoretical part.