Exercise Set 1: Big O Notation

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1 Question 1:

$\mathbf{f(n)} = 3n^2 + 7n + 10$

Big O classiffication: $O(n^2)$

$\mathbf{f(n)} = 5n \log(n) + n^2$

Big O classification: $O(n^2)$

$\mathbf{f(n)} = 2^n + n^2$

Big O classification: $O(2^n)$

2 Question 2:

Growth Rates from slowest to fastest: $n \log(n) \longrightarrow n^2 \longrightarrow 2^n \longrightarrow n!$

3 Question 3:

We define the function:

$$f(n) = n^3 + 100n^2 + 50n + 10$$

To prove $f(n) = O(n^3)$, we need to find constants $c > 0$ and n_0 such that:

$$f(n) \leq cn^3, \quad \forall n \geq n_0.$$

Replace each term with an upper bound proportional to n^3:

$$n^3 + 100n^2 + 50n + 10 \leq n^3 + 100n^3 + 50n^3 + 10n^3.$$

Since for $n \geq 1$, we can deduct that, $n^2 \leq n^3$, $n \leq n^3$, and $1 \leq n^3$

$$= (1 + 100 + 50 + 10)n^3 = 161n^3.$$

Thus, we have:

$$f(n) \leq 161n^3, \quad \forall n \geq 1.$$

By setting $c = 161$ and $n_0 = 1$, we have a c and an n_0 which satisfy the definition of big O notation. Therefore, we can conclude that:

$$f(n) = O(n^3).$$

4   Question 4:

For SElection sort:

runns inner and ourter oop irrespective of the input.(Worst and best input)
Worst-case scenario: n^2
Best-case scenario: n^2

For Insertion sort:

Runs only the outer loop for input that is sorted.(Best Input)
RUns both inner and outer loop for input that is reverse sorted or random.(Worst input)
Worst-case scenario: n^2
Best-case scenario: n