# Contraction Hierarchies - Detailed Assignment Solutions

#### Automated Computation

February 23, 2025

### 1 Edge Difference Computation

Below is the table of computed edge differences for each node, including the number of edges removed and the shortcuts added:

Node	Neighbors	Edges Removed	Shortcuts Added	Edge Difference
A	[B]	1		-1
В	[A, C, G]	3	[(A, C), (A, G), (C, G)]	0
$\mathbf{C}$	[B, D]	2	[(B, D)]	-1
D	[C, E, I]	3	[(C, E), (C, I), (E, I)]	0
$\mathbf{E}$	[D, J]	2	[(D, J)]	-1
$\mathbf{F}$	[G]	1		-1
G	[F, B, H, L]	4	[(F, B), (F, H), (F, L), (B, H), (B, L), (H, L)]	2
H	[G]	1		-1
$\mathbf{L}$	[G, M, K]	3	[(G, M), (G, K), (M, K)]	0
I	[D, J, N]	3	[(D, J), (D, N), (J, N)]	0
J	[I, E, O]	3	[(I, E), (I, O), (E, O)]	0
O	[J, N]	2	[(J, N)]	-1
N	[O, I, M]	3	[(O, I), (O, M), (I, M)]	0
${ m M}$	[N, L]	2	[(N, L)]	-1
K	[L, P]	2	[(L, P)]	-1
Р	[K, Q]	2	[(K, Q)]	-1
Q	[P, R, V]	3	[(P, R), (P, V), (R, V)]	0
R	[Q, S]	2	[(Q, S)]	-1
V	[Q, U, W]	3	[(Q, U), (Q, W), (U, W)]	0
U	[V]	1		-1
W	[V, X]	2	[(V, X)]	-1
X	[W, Y]	2	[(W, Y)]	-1
Y	[X, T]	2	[(X, T)]	-1
${ m T}$	[Y, S]	2	[(Y, S)]	-1
S	[T, R]	2	[(T, R)]	-1

### 2 Offline Ordering Computation (Edge Difference Heuristic)

The offline node ordering based on edge difference is determined as follows:

Node	Neighbors	Edges Removed	Shortcuts Added	Edge Difference
A	[B]	1		-1
Y	[X, T]	2	[(X, T)]	-1
X	[W, Y]	2	[(W, Y)]	-1
W	[V, X]	2	[(V, X)]	-1
U	[V]	1		-1
$\mathbf{R}$	[Q, S]	2	[(Q, S)]	-1
P	[K, Q]	2	[(K, Q)]	-1
K	[L, P]	2	[(L, P)]	-1
M	[N, L]	2	[(N, L)]	-1
${ m T}$	[Y, S]	2	[(Y, S)]	-1
O	[J, N]	2	[(J, N)]	-1
$\mathbf{S}$	[T, R]	2	[(T, R)]	-1
Η	[G]	1		-1
$\mathbf{F}$	[G]	1		-1
$\mathbf{E}$	[D, J]	2	[(D, J)]	-1
$\mathbf{C}$	[B, D]	2	[(B, D)]	-1
I	[D, J, N]	3	[(D, J), (D, N), (J, N)]	0
L	[G, M, K]	3	[(G, M), (G, K), (M, K)]	0
Q	[P, R, V]	3	[(P, R), (P, V), (R, V)]	0
V	[Q, U, W]	3	[(Q, U), (Q, W), (U, W)]	0
D	[C, E, I]	3	[(C, E), (C, I), (E, I)]	0
В	[A, C, G]	3	[(A, C), (A, G), (C, G)]	0
J	[I, E, O]	3	[(I, E), (I, O), (E, O)]	0
N	[O, I, M]	3	[(O, I), (O, M), (I, M)]	0
G	[F,B,H,L]	4	[(F, B), (F, H), (F, L), (B, H), (B, L), (H, L)]	2

#### Shortcuts Computed Using Offline Ordering

Shortcuts Computed Using Offline Ordering

Each node is contracted in the following order, and the corresponding shortcuts are added:

Node 1	Node 2
X	
W	${ m T}$
V	${ m T}$
Q	$\mathbf{S}$
K	Q Q N
L	Q
L	N
$\mathbf{S}$	V
J	N
D	J
В	D
D	N
G G	Q
G	N
$_{ m V}^{ m Q}$	N
V	G
V	N
J	В
В	N
G	J

Total shortcuts created using offline ordering:  ${f 20}$ 

## 3 Online Ordering Computation (Edge Difference Heuristic)

For the online ordering, edge differences are recomputed dynamically after each node contraction. Below is the computed ordering:

Node
A
В
В С
$\mathbf{E}$
D
$\mathbf{F}$
Η
G
I
J
O
N
L
E D F H G I J O N L M K P Q R U
K
Р
Q
R
U
V
W
X
${ m Y} \ { m T}$
T.
S

### Shortcuts Computed Using Online Ordering

Shortcuts Computed Using Online Ordering Shortcuts were dynamically added as follows:

Node 1	Node 2
С	G
D	G
D	J
I	G
G	J
L	I
L	J
J	N
N	L
O	L
${ m M}$	K
R	V
$\mathbf{S}$	V
W	S
X	S
Y	S

Total shortcuts created using online ordering: 16

# 4 Online Ordering Computation (Shortcut-Minimization Heuristic)

Here, instead of using edge difference, we minimize the number of shortcuts added at each step. The resulting ordering is:

Node
A
$\mathbf{F}$
H
U
В
F H U B C
$\mathbf{E}$
G
O
I
J
D
N
M
L
K
Р
Q
Ř
V
W
E G O I J D N M L K P Q R V W X
Y
$\bar{\mathrm{T}}$
$\bar{\mathrm{S}}$

### Shortcuts Computed Using Shortcut-Minimization Ordering

Shortcuts Computed Using Shortcut-Minimization Ordering The following shortcuts were added dynamically:

Node 1	Node 2
С	G
D	G
D	J
L	D
J	N
D	N
L	N
R	V
$\mathbf{S}$	V
W	$\mathbf{S}$
X	$\mathbf{S}$
Y	S

Total shortcuts created using shortcut-minimization ordering:  ${\bf 12}$