CS 5720 Design and Analysis of Algorithms

Project #0

Submission requirements:

- A .zip file containing your source code. You may use any language you would like.
- A PDF (submitted separately to the Canvas assignment) containing each item below that is listed as a **Deliverable.** For each item contained in your PDF, clearly mark which deliverable it is associated with. Plots should be clearly labeled and have descriptive captions.

Visualizing Big-O and friends. In class, the concept of Big-O was introduced visually: the instructor sketched a function like $f(n) = \frac{1}{2}n(n-1)$, showed it next to a function like $g(n) = n^2$, and then argued that the two were the same order of growth (that is, $f(n) \in \Theta(g(n))$) by imagining what would happen if you "zoomed out" on the plot. In particular, if you plot two functions and then "zoom way out," the resulting picture can tell you whether they're the same order of growth. If neither of the functions squish all the way down to 0 after zooming way out, then they're probably the same order of growth. Unfortunately, in class the instructor hand-drew all the plots. In this project, you'll explore this concept in code to convince yourself that it's true and meaningful.

Assignment:

1. Consider functions similar to what I discussed above: let $f(n) = \frac{1}{2}n(n-1) + 10$, and let $g(n) = n^2$. There are 2 approaches to show that these have the same order of growth. First, see what happens if you plot these two functions on the same axis with n ranging from 1 to 10. Second, see what happens if you plot them on the same axis with n ranging from 1 to 10^6.

   Deliverable 1: Two plots. Each plot should have **both** $f(n)$ and $g(n)$; the first plot should have n ranging from 1 to 10 and the 2nd plot should have n ranging from 1 to 10^6. Does the first plot indicate that f and g are the same order of growth? Does the 2nd?

2. Now, let's explore the 2nd approach; this time, you'll perform an *empirical limit test* on the functions. This is very similar to how you perform an *analytical limit test*[1] in class to determine order of growth, where you investigate the behavior of

$$\lim_{n \to \infty} \frac{f(n)}{g(n)}.$$

   Of course, in code you can't just plug in ∞ to see what happens; however, you can plug in big numbers and generate a plot to watch the behavior of $\frac{f(n)}{g(n)}$ when n gets large.

   Deliverable 2: Two plots. Each plot should have a single trace of $\frac{f(n)}{g(n)}$; the first plot should have n ranging from 1 to 10 and the 2nd plot should have n ranging from 1 to 10^6. Does the first plot indicate that f and g are the same order of growth? Does the 2nd?

[1] see Levitin, Section 2.2

3. Now, let's investigate two functions that have *different* orders of growth. In particular, this time let $f(n) = \sqrt{n^5 + 3n + 1}$, and let $g(n) = 5n^2$. Repeat the experiments of Deliverables 1 and 2 with these functions, and then based on your experiments report the relationship between the orders of growth of f and g.

   Deliverable 3: Four plots (2 plots like Deliverable 1, 2 plots like Deliverable 2) comparing the orders of growth of the new f and g functions. Explain the relationship between the orders of growth of these functions based on your plots. If you feel that 10^6 is not high enough to show the relationship you'd expect, feel free to use a larger number.

4. This time, use functions that grow much more slowly. Let $f(n) = \log n$, and let $g(n) = \sqrt{n}$. Repeat the experiments of Deliverables 1 and 2 with these functions, and then based on your experiments report the relationship between the orders of growth of f and g.

   Deliverable 4: Four plots (2 plots like Deliverable 1, 2 plots like Deliverable 2) comparing the orders of growth of the new f and g functions. Explain the relationship between the orders of growth of these functions based on your plots. If you feel that 10^6 is not high enough to show the relationship you'd expect, feel free to use a larger number. If you get a divide-by-zero error with $n = 1$, feel free to start at $n = 2$.

5. Something that can feel confusing is the fact that all logarithms have the same order of growth. To show yourself this, let $f(n) = \log_2 n$, and let $g(n) = \log_{10} n$. Repeat the experiments of Deliverables 1 and 2 with these functions, and then based on your experiments report the relationship between the orders of growth of f and g.

   Deliverable 5: Four plots (2 plots like Deliverable 1, 2 plots like Deliverable 2) comparing the orders of growth of the new f and g functions. Explain the relationship between the orders of growth of these functions based on your plots. If you feel that 10^6 is not high enough to show the relationship you'd expect, feel free to use a larger number. If you get a divide-by-zero error with $n = 1$, feel free to start at $n = 2$.