Homework#8

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1 Sample Graph

We will use the following graph for our examples:

- Vertices: $V = \{A, B, C, D\}$
- Edges with weights:
  - $(A, B, 1)$
  - $(A, C, 3)$
  - $(B, C, 3)$
  - $(B, D, 6)$
  - $(C, D, 4)$

2 Prim's Algorithm with Weight Matrix and Unordered Array

2.1 Steps

1. **Initialization:**
   - Start with vertex A.
   - Priority queue (unordered array): $[(A, 0)]$
   - MST: $\emptyset$
2. **Main Loop (runs —V— times):**
   - Extract minimum: $(A, 0)$
   - Add edges $(A, B, 1)$ and $(A, C, 3)$ to the queue.
   - Priority queue: $[(B, 1), (C, 3)]$
   - MST: $\{A\}$
   - Extract minimum: $(B, 1)$

- Add edge $(B, D, 6)$ to the queue.
- Priority queue: $[(C, 3), (D, 6)]$
- MST: $\{A, B\}$
- Extract minimum: $(C, 3)$
- Add edge $(C, D, 4)$ to the queue.
- Priority queue: $[(D, 4)]$
- MST: $\{A, B, C\}$
- Extract minimum: $(D, 4)$
- Priority queue: $\emptyset$
- MST: $\{A, B, C, D\}$

3 Prim's Algorithm with Adjacency List and Min-Heap

3.1 Steps

1. **Initialization:**
   - Start with vertex A.
   - Priority queue (min-heap): $[(0, A)]$
   - MST: $\emptyset$
2. **Main Loop (runs —V— times):**
   - Extract minimum: $(0, A)$
   - Add edges $(A, B, 1)$ and $(A, C, 3)$ to the heap.
   - Priority queue: $[(1, B), (3, C)]$
   - MST: $\{A\}$
   - Extract minimum: $(1, B)$
   - Add edge $(B, D, 6)$ to the heap.
   - Priority queue: $[(3, C), (6, D)]$
   - MST: $\{A, B\}$
   - Extract minimum: $(3, C)$
   - Add edge $(C, D, 4)$ to the heap.
   - Priority queue: $[(4, D), (6, D)]$
   - MST: $\{A, B, C\}$
   - Extract minimum: $(4, D)$
   - Priority queue: $[(6, D)]$
   - MST: $\{A, B, C, D\}$

4 Kruskal's Algorithm with Adjacency List and Fast Sort

4.1 Steps

1. **Initialization:**
   - Sort edges by weight.
   - Edges: $[(A, B, 1), (A, C, 3), (B, C, 3), (C, D, 4), (B, D, 6)]$
   - MST: $\emptyset$
2. **Main Loop (runs —E— times):**
   - Edge $(A, B, 1)$ - add to MST.
   - MST: $\{(A, B, 1)\}$
   - Edge $(A, C, 3)$ - add to MST.
   - MST: $\{(A, B, 1), (A, C, 3)\}$
   - Edge $(B, C, 3)$ - cycle detected, skip.
   - Edge $(C, D, 4)$ - add to MST.
   - MST: $\{(A, B, 1), (A, C, 3), (C, D, 4)\}$
   - Edge $(B, D, 6)$ - cycle detected, skip.