CS 5720 Design and Analysis of Algorithms
Homework #3

Submission requirements:

- Submit your work in PDF format to the appropriate assignment on Canvas.
- **5% extra credit** if your writeup is *typed*.

Assignment:

Gadget Testing. A company wants to determine the *highest* floor of its n-story headquarters from which a gadget can fall without breaking. The company only has one gadget to test with, and if the gadget breaks, it cannot be repaired — that is, by the time the gadget breaks, you better have the answer to your question.

1. *(1 gadget)* Design an algorithm to solve this problem. Provide pseudocode or a precise description of your approach. Determine the **best** and **worst**-case *number of times you need to drop the gadget* to solve the problem. (throughout this problem, best-case means *fewest* and worst-case means *most*.)
2. *(uniform average case)* Suppose that it is equally likely that the gadget will break from any floor. If you perform this experiment many times, what is the *average* number of times you need to drop the gadget?
3. *(skewed average case)* Suppose instead that the probability of breaking when the gadget falls from floor k is proportional to $1/k$.[1] What is the **order of growth** (in terms of $\Theta(\cdot)$ and n) of the average number of gadget drops in this case?
4. *(2 gadgets)* Now, suppose that the company has *two* identical gadgets to experiment with ("identical" means that if gadget 1 could survive a fall from floor k, then so can gadget 2, and vice-versa). If one of the gadgets gets broken, it cannot be repaired, and the experiment will have to be completed with the remaining gadget. Use the 2nd gadget to improve on your algorithm from part (a). That is, design an algorithm that has worst-case time complexity *better* than $\Theta(n)$. What is the **order of growth** of the worst-case number of drops for your redesigned algorithm?
5. *(the tradeoff between best and worst case)* What is the best-case **number of drops** for your 2-gadget algorithm? Discuss the relationship between this answer and the best-case answer you gave in part (1).

[1] Formally: let p_k the probability that the gadget breaks when dropped from floor k. That is, it is more likely to break when it falls from floor 1 than from higher floors. Then $p_k = M/k$, where $M > 0$ is some constant such that $\{p_k\}_{k=1}^{n}$ is a valid probability distribution, or $\sum_{i=1}^{n} p_k = 1$.