

CS 5720 Design and Analysis of Algorithms
Homework #1

Submission requirements:

- Submit your work in PDF format to the appropriate assignment on Canvas.
- **5% extra credit** if your writeup is *typed*.

Assignment:

1. Determine the order of growth $\Theta(\cdot)$ for each of the following functions. Correct answers with no justification may receive no credit.
 - (a) $(n^2 + 1)^{10}$
 - (b) $\sqrt{10n^2 + 7n + 3}$
 - (c) $2n \log((n + 2)^2) + (n + 2)^2 \log(n/2)$
 - (d) $2^{n+1} + 3^{n-1}$
 - (e) $\lfloor \log_2 n \rfloor$
2. Complete Exercise 7 from Exercises 2.2 in the textbook. I.e., prove the following assertions by using the definitions of the notations involved, or disprove them by giving a specific counterexample.
 - (a) If $t(n) \in O(g(n))$, then $g(n) \in \Omega(t(n))$.
 - (b) $\Theta(\alpha g(n)) = \Theta(g(n))$, where $\alpha > 0$.
 - (c) $\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$.
 - (d) For any two nonnegative functions $t(n)$ and $g(n)$ defined on the set of nonnegative integers, either $t(n) \in O(g(n))$ or $t(n) \in \Omega(g(n))$, or both.
3. Determine the order of growth ($\Theta(\cdot)$) for each of the following functions. You must show your work! Correct answers with no justification may receive no credit.
 - (a) $T(n) = \sum_{i=1}^{2n} i$
 - (b) $T(n) = \sum_{i=1}^n \sum_{j=i}^n n$
 - (c) $T(n) = \sum_{i=1}^n n^2$
 - (d) $T(n) = \sum_{i=1}^{n^2} i$
 - (e) $T(n) = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{\ell=1}^n \ell$