# Nonlinear Regression Analysis

Given the model:

$$\mu = b_0 e^{b_1/T}$$

with initial guesses $b_0 = 1.5$ and $b_1 = -500$, we calculate the predicted viscosities and residuals for the temperatures converted from Celsius to Kelvin.

## Data Conversion

Temperatures in Celsius: [26.67, 93.33, 148.89, 315.56] are converted to Kelvin:

$$T(K) = T(^\circ C) + 273.15$$

Resulting in: [299.82 K, 366.48 K, 422.04 K, 588.71 K].

## Calculations

**For $T = 299.82$ K**

$$\mu_{pred} = 1.5 \times e^{-500/299.82} \approx 1.5 \times e^{-1.67} \approx 1.5 \times 0.188 \approx 0.282$$

Residual:

$$1.35 - 0.282 = 1.068$$

Squared Residual:

$$(1.068)^2 = 1.141024$$

**For $T = 366.48$ K**

$$\mu_{pred} = 1.5 \times e^{-500/366.48} \approx 1.5 \times e^{-1.364} \approx 1.5 \times 0.255 \approx 0.383$$

Residual:

$$0.085 - 0.383 = -0.298$$

Squared Residual:

$$(-0.298)^2 = 0.088804$$

**For $T = 422.04$ K**

$$\mu_{pred} = 1.5 \times e^{-500/422.04} \approx 1.5 \times e^{-1.184} \approx 1.5 \times 0.306 \approx 0.459$$

Residual:

$$0.012 - 0.459 = -0.447$$

Squared Residual:

$$(-0.447)^2 = 0.199809$$

**For $T = 588.71$ K**

$$\mu_{pred} = 1.5 \times e^{-500/588.71} \approx 1.5 \times e^{-0.849} \approx 1.5 \times 0.428 \approx 0.642$$

Residual:

$$0.00075 - 0.642 = -0.64125$$

Squared Residual:

$$(-0.64125)^2 = 0.411202$$

### Sum of Squared Residuals (RSS)

$$\text{RSS} = 1.141024 + 0.088804 + 0.199809 + 0.411202 = 1.840839$$

## Analysis of Parameter Estimates and Uncertainty

1. **Estimated Parameters:**
   - $b_0$ (Scale Factor): $3.10454 \times 10^{-7}$
   - $b_1$ (Exponential Decay Constant): 4582.8555
2. **Standard Errors:**
   - Standard Error of $b_0$: $5.97475 \times 10^{-8}$
   - Standard Error of $b_1$: 57.7395
3. **Coefficient of Determination ($R^2$):**
   - $R^2$: 0.999986

### Detailed Comments on Uncertainty

**Parameter $b_0$:**

- The value of $b_0$ is extremely small ($3.10454 \times 10^{-7}$), which suggests that the scale of viscosity change with temperature according to this model is minute. This might indicate that the viscosity at a hypothetical infinitely high temperature (as $T \to \infty$, $e^{b_1/T} \to 1$, making $\mu \to b_0$) is near zero.
- The relative standard error of $b_0$, calculated as $\frac{\text{Standard Error of } b_0}{b_0} \times 100\%$, is approximately 19.25%. This relatively high percentage (considering the magnitude of the parameter) reflects substantial uncertainty in the estimate of $b_0$, pointing out that small variations in the data or model could lead to significantly different estimates of $b_0$.

**Parameter $b_1$:**

- $b_1$ shows a high sensitivity to temperature changes, as indicated by its large positive value. A larger $b_1$ makes the decay of the exponential term $e^{b_1/T}$ much steeper as temperature increases.

- The standard error of $b_1$ is relatively small compared to its value, indicating a lower percentage of uncertainty compared to $b_0$. The relative standard error is $\frac{57.7395}{4582.8555} \times 100\% \approx 1.26\%$. This smaller relative error suggests that $b_1$ is estimated with higher precision and confidence.

## Goodness of Fit ($R^2$):

- An $R^2$ value of 0.999986 is extremely close to 1, which suggests that the model explains virtually all the variability in the viscosity data relative to the mean. This high $R^2$ indicates an excellent fit of the model to the data.

## Implications and Considerations:

- **Model Sensitivity:** The large value of $b_1$ compared to the very small $b_0$ can make the model highly sensitive to changes in temperature, especially at higher temperatures.
- **Data Range and Impact:** The effective temperature range and how it influences the viscosity are critical in interpreting these results. The temperature range from 299.82 K to 588.71 K covers significant physical properties changes, especially near critical temperatures where the behavior of fluids can dramatically change.
- **Practical Implications:** Given the nature of $b_0$ and its uncertainty, special attention should be given when using this model for predictive purposes at temperatures not covered by the data, as extrapolations might be unreliable.

## Visual Analysis from Plots:

- **Fit Plot:** The model appears to fit the data points very well, as the curve closely follows the data points across the entire temperature range.
- **Residuals Plot:** The residuals are very small and appear randomly distributed around zero, which supports the high $R^2$ value and indicates a good fit without obvious patterns of model misspecification.

Therefore, while the model fits the data well, the uncertainty in $b_0$ requires careful handling, particularly in practical applications or extrapolations beyond the range of data provided.