Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2213247

Our model compares the day-over-day values of Bitcoin and gold, and predicts which asset will net maximal profits. We observe the values of the assets in relation to estimated inflation and in relation to each other to produce a more accurate model. We investigate the interdependence of stock value variations with the rate at which these variations themselves change.

We calculate day-over-day percent changes in asset value and synthesize curves to fit the asset value data over time, from which we can calculate second derivatives (concavities). We can compare percent changes with inflation and transaction fees to determine if an asset is viable for an investment.

We establish a network of if-else statements based on the viability of an asset and the various complications of said asset, to conclude which asset should be invested in, which should be held, and which should be sold. Should an event arise in which both assets are trending concave upwards, we use a ratio of gold-to-Bitcoin concavities to determine which asset should be bought.

Simulating our model through a day-to-day asset price data set, we see that our model will increase investments. Our model is adaptable to many industries looking to invest in any asset that they desire.

We observed that relatively recent data is more predictive of near-term future data than relatively older data. This led us to restrict the data used in synthesizing best-fit curves. This contributed to a better model that can more accurately predict the future by avoiding over- and under-fitting the data. We can also conclude that, while our model is good, no model can be perfect, and it is impossible to tell if an asset will increase its value on any given day. All we can do is observe past trends and extrapolate into the future.

This model can be adapted to include more than two assets and a wide range of them as well. This can be applied to whatever the investor desires, and help anyone gain money through investments. A large diversity of investments will only make our model's conclusions more statistically significant as there is more opportunity to maximize profits.

Contents

1 Introduction

1.1 Problem Background

Various assets can be bought, sold, or traded on the open market. Riddled among the seemingly simplistic model of market trading lies a complex system of commissions, price fluctuations, and market activity. How do we navigate this system in order to maximize profits? To solve this question, we need to look at the correlation between past asset prices and whether or not we should invest in the asset, using only past data points. From this, we can predict the future trends of asset value and be able to help maximize profits.

1.2 Restatement of the Problems

1.2.1 Approach

As we approached the problem, the fundamental notion of purchasing assets at low prices and selling at higher prices was at the forefront of our thinking. Initially, we envisioned a model whose core was focused on day-over-day price changes of the assets in question, however such a model often results in buying high and selling low, which directly contradicts our central vision of profit. As such, we moved to synthesizing a best-fit curve for the value versus time data of each asset and calculating its second- and higher-order derivatives, which could subsequently be used for a decision model. Our decision model was simulated using a web of if-else statements regarding the purchase and sale of assets.

1.2.2 Notation

Refer to **Table 1** for our notation frequently used in this solution paper.

Symbol	Meaning
V_C	current value of the dollar relative to 9/11/2016
V_G	current value of gold in dollars
V_B	current value of Bitcoin in dollars
C_G	day-over-day percent change of gold value
C_B	day-over-day percent change of Bitcoin value
Z	decision value; a value of 1 suggests the purchase of a given asset, a value of 0 suggests the holding of a given asset, and a value of -1 suggests the sale of a given asset
Z_G	decision value of gold
Z_B	decision value of Bitcoin

Table 1:
Notation used commonly throughout our paper.

1.3 Basic Assumptions

1. Inflation has a constant continuous compounding rate of 3% per year, which imputes a day-to-day inflation rate of $8.219 * 10^{-3}\%$.
2. The value of cash is exactly equal to the face value of the cash.

1.4 Glossary

- **Percent change**: a calculation of the difference between two consecutive data points, relative to the first.

- **Bitcoin**: a digital currency that can be traded any day of the week with a transaction fee of 2%. Abbr. BTC.
- **Gold**: a material substance that has value for both electronics and aesthetic products. Gold can be traded on weekdays and has a transaction fee of 1%.

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2 Model Development

2.1 Percent Change Model

According to Assumption 1, the value of \$1 USD on any given day from 9/11/2016 to 9/11/2021, P_C, is given exponentially in terms of t:

$$V_C(t) = e^{(8.219*10^{-5})t} \tag{1}$$

Equation 1: Dollar value over time in days since 9/10/2016.

Given the daily prices of gold and Bitcoin from 9/11/2016 to 9/11/2021, a period of five years, we can calculate the percent change in any one given day for gold and Bitcoin respectively to be:

$$C_G(t) = \frac{V_G(t) - V_G(t-1)}{V_G(t-1)} * 100\%$$

$$C_B(t) = \frac{V_B(t) - V_B(t-1)}{V_B(t-1)} * 100\%$$

Where t represents the number of days since the first trading day for each respective asset. Since there is no -1^{st} day value, t must begin at 1.

This model works by comparing the percent change of each asset and deciding which one is trending upwards. We would first make sure that each of the percent changes was positive, and if they were we would use this ratio:

$$R_0 = \frac{C_G}{C_B}$$

If the ratio were greater than one, then it would suggest that one should invest in gold and if it was less than one (but still positive) then it would suggest investing in Bitcoin. If both of the percent changes were less than the percent change of the dollar, given by our **Assumption 1** and by **Equation 1**, then it would suggest taking all of the money out of the assets and putting it back into cash.

This percent change-based model uses the following logic:

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if C_G > 0 and C_B > 0:
    if C_G > C_B:
        BUY GOLD
    else:
        BUY BTC
else:
    SELL ALL
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This model performs well when the prices go up, but falls short when recognizing when to stop buying assets. For example: If the current price of Bitcoin is $750 and the percent change is +0.00085, the model would suggest investing in Bitcoin, without recognizing that the percent change is trending downward from the previous day and that the sale of Bitcoin would be more economical. From this, we were able to determine that we need to find a regression of past data so that we can find a second derivative to map the rate of change of the rate of change of value. This will help us to be able to more accurately report whether a user should or should not invest in a certain asset.

2.2 Second Rate of Change of Price Model

The second derivative of value with respect to time can be applied in order to determine possible times at which the value of gold and Bitcoin will reach a relative minimum or maximum. That is, the second derivative represents the concavity of the original value functions.

Regardless of the specific value of assets at any given point, the second derivative of the curve of best fit of each value graph will indicate whether the value of each asset is expected to rise or fall in the future. Given that the plot of value versus time of each asset is not continuous and thus does not have an intrinsic curve of best fit, it is necessary to artificially synthesize one in order to perform second- and higher-order derivatives.

Our team proposed multiple strategies for synthesizing such a curve. At the most basic level, we can perform a regression fit based on the entire provided data set. **Figure 1** shows this strategy applied using various polynomial degrees. Clearly, while such a regression is a fairly accurate approximation for the behavior of asset value over long periods, it is an ineffective tool for analyzing and predicting short-term value changes and day-to-day changes in the market. And arbitrarily increasing the degree of the polynomial is inefficient and leads to over-fitting, which reduces the predictive power of the model. Furthermore, using the entire provided data set to train the model at all timestamps is only possible in retrospect, so at early timestamps, such as in the years 2016 and 2017, the model breaks down.

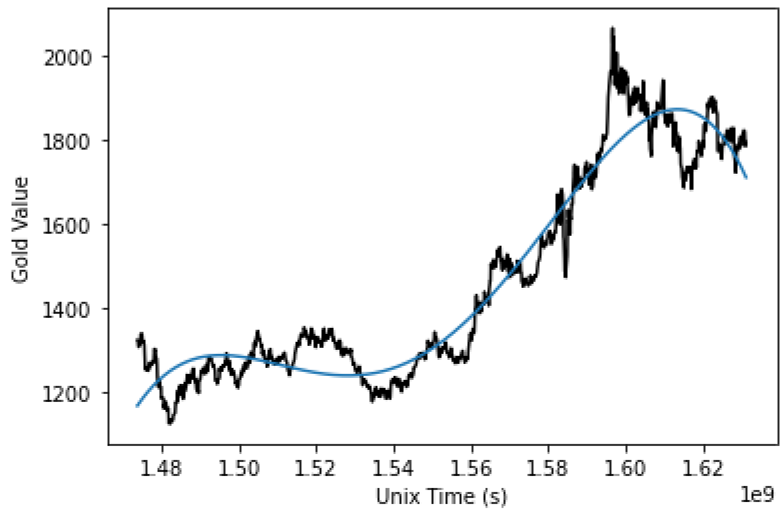

Figure 1:
Polynomial regression curve of degree 4 alongside the plot of actual gold value.

Instead of using a regression curve fitted to all of the data from the outset, our model uses multiple regression curves to describe a single value plot over time. Specifically, our model synthesizes lines of best fit for each unique day by only using value data up to and including that day. To synthesize day 50's best-fit curve, we only can use the data from days 0 thru 50. In order to maintain a reasonable amount of uniqueness for each best-fit curve, we capped the number of past days considered at 500. So, on day 1000, for example, we only use the data for days 500 thru 1000 to synthesize the best-fit curve. This process is shown graphically in **Figure 2**.

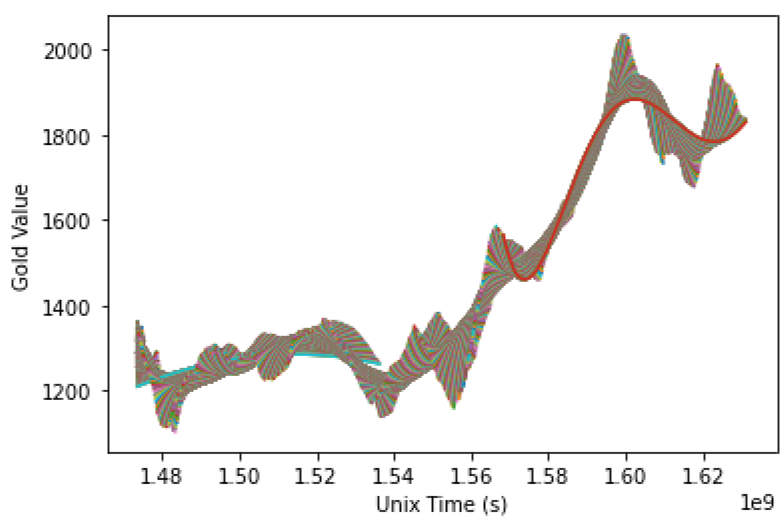

Figure 2:
Unique day-over-day regressions of degree 5.

2.3 Function of Concavity and Percent Change

To find a metric for determining whether or not we should buy a stock we looked at the relationship between concavity and percent change. We want our function to have three outputs, buy, sell, or hold, depending on the parameters of concavity and percent change. We came up with this equation:

$$Z(x,y) = \begin{cases} \frac{x}{|x|} & y = 0, x \neq 0 \\ \frac{y}{|y|} & x = 0, y \neq 0 \\ 0 & x = 0, y = 0 \\ \frac{\frac{x}{|x|}+\frac{y}{|y|}}{2} & x \neq 0, y \neq 0 \end{cases} \tag{2}$$

Equation 2: A function that represents if one should or shouldn't buy an asset, with x representing concavity and y representing percent change. A positive Z value (or decision value) means you should buy an asset, while a negative Z value means you should sell an asset, and a Z value of zero indicates that you should hold and do nothing. Refer to **Figure 3**.

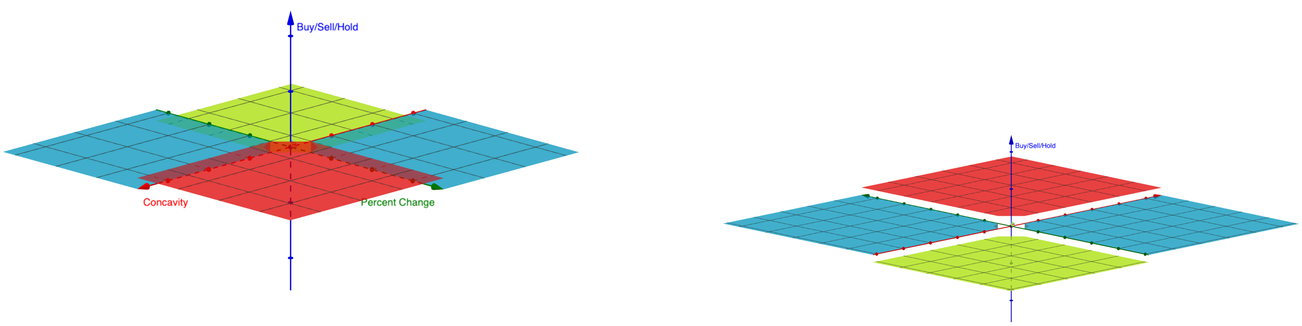

Figure 3:
A three-dimensional representation of **Equation 2**. Green represents a Z value of -1, blue is a value of 0, and red is a value of 1.

Applied to a single asset, at one single day, this model determines whether an asset should be bought, sold, or held at any single instant in time.

- To buy an asset, the concavity must be positive with a positive percent change.
- To sell an asset, the concavity must be negative with a negative percent change.
- To hold an asset, either the concavity must be less than zero with a positive percent change or the concavity must be positive with a negative percent change.

However, this model fails to establish a relationship between an asset trading decision and time. Given that the markets for both gold and bitcoin fluctuate over time, our model must compare the concavity and percent change for both assets given time. Inflation is also not accounted for in this model, meaning a tilted axis must take into account the constant 3% inflation rate according to **Assumption 1**. Represented with inflation, the function of concavity and percent change now becomes:

$$Z(x,y)=\begin{cases}\frac{x}{|x|} & y=8.219*10^{-5}, x\neq 0\\ \frac{y-8.219*10^{-5}}{|y-8.219*10^{-5}|} & x=0, y\neq 8.219*10^{-5}\\ 0 & x=0, y=8.219*10^{-5}\\ \frac{\frac{x}{|x|}+\frac{y-8.219*10^{-5}}{|y-8.219*10^{-5}|}}{2} & x\neq 0, y\neq 8.219*10^{-5}\end{cases} \tag{3}$$

Equation 3: Equation 2 modified to account for constant inflation rate according to **Assumption 1**. This will make sure that our model will not suggest buying or holding an asset that is growing at a slower rate than inflation.

The function of concavity and percent change also does not compare which asset should be invested in, should there be an event when the function says to buy both. In order to determine the correct decision for both assets at any given time, we can represent the ratio of the concavity as

$$R_1=\frac{\frac{d^2V_G(t)}{dt^2}}{\frac{d^2V_B(t)}{dt^2}} \tag{4}$$

Equation 4: Ratio of Second Derivatives

This will give us a definitive choice of which asset investing in should the model predict to buy both. If the R-value is greater than one you should invest in gold and if it is less than one you should invest it in Bitcoin. At exactly one it does not matter which one you invest in, so we arbitrarily chose Bitcoin. Here is a flow chart mapping all of the possible Z values and how they interact:

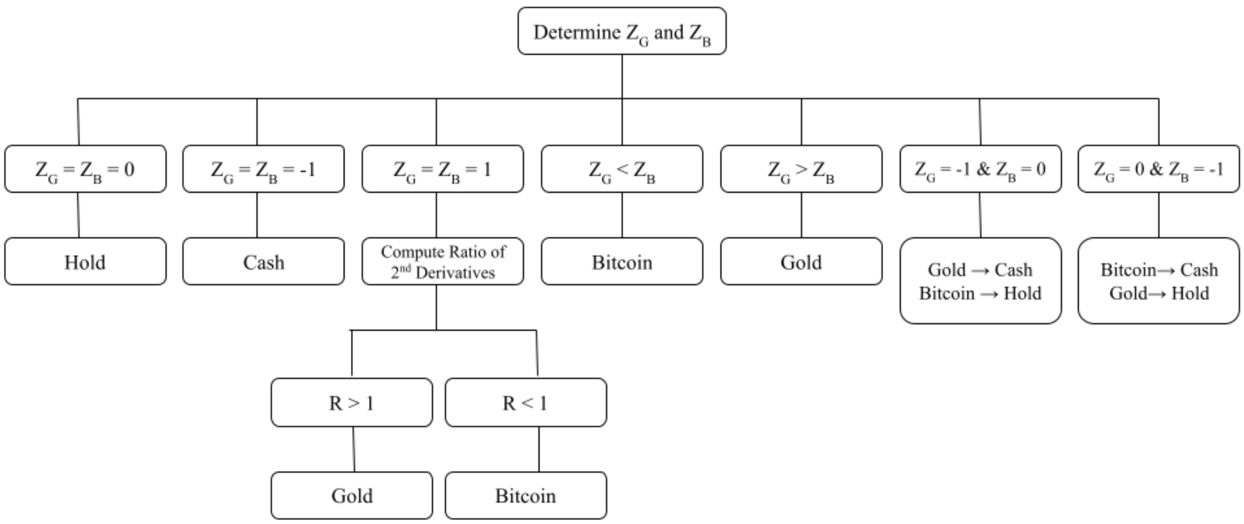

Figure 4:
Decision diagram for both assets at one instant in time

2.4 Implementation of Transaction Fee

Every time an asset is bought or sold, a transaction cost is applied: 1% for gold and 2% for bitcoin. To account for this loss in the form of transactions, the price of buying assets was adjusted to account for the additional 2% charge, and the price of selling assets was adjusted to account for the additional 1% charge.

When calculating the value of Bitcoin and gold when buying an asset, we implemented the transaction fees of 2% and 1%, respectively. That is, if we had 10 USD and wanted to buy gold at a price of 5 USD per troy ounce, then we would apply the formula:

$$N_G = N_C * (\frac{1 \text{ Troy Ounce}}{5 \text{ USD}}) * 0.99 \tag{5}$$

Equation 5: This shows the methodology of how we do transactions with the transaction fees, by using a conversion factor. N_G represents the amount of gold in troy ounces and N_C is the amount of cash

2.5 Final Asset Value Prediction Model

All together, the various equations can be applied every day from 9/11/21 to 9/10/21 to determine the maximum profit that can be made with investments in gold and bitcoin. **Equation 3** is used to determine one of the three outcomes given asset concavity and percent change for a given day. Corresponding Z values for gold and bitcoin on any given day determine whether each asset should be bought, sold, or held, according to **Figure 4**. If **Equation 3** suggests both assets should be bought, **Equation 4** determines which asset will be more profitable at the given time.

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3 Model Application

We tested the model against the assets provided and recorded a total final value on 9/10/2021 of around $19000 USD for a total profit of around $18000 USD. We iterated over the entirety of the provided database, only using past points to attain this profit. This proves that our model can predict the future prices of an asset.

This model can be easily applied, and you only need to know the past prices of an asset. It can be applied to various numbers of assets over any industry. For example, on day 2 (9/13/2016), the first day gold can be traded, applying our model results in holding:

$$V_G(2) = \$1323.65$$
$$V_B(2) = \$610.92$$

$$C_G(2) = \frac{V_G(2) - V_G(2-1)}{V_G(2-1)} * 100\% = -0.071\%$$
$$C_B(2) = \frac{V_B(2) - V_B(2-1)}{V_B(2-1)} * 100\% = 0.205\%$$

$$\frac{d^2V_G(2)}{dt^2} = V_G^2(2) = 4.067 * 10^{-15}$$
$$\frac{d^2V_B(2)}{dt^2} = V_B^2(2) = -3.774 * 10^{-13}$$

$$Z_G(4.067 * 10^{-15}, -0.071) = \frac{\frac{4.067*10^{-15}}{|4.067*10^{-15}|} + \frac{-0.071-8.219*10^{-5}}{|-0.071-8.219*10^{-5}|}}{2} = 0$$
$$Z_B(-3.774 * 10^{-13}, 0.205) = \frac{\frac{-3.774*10^{-13}}{|-3.774*10^{-13}|} + \frac{0.205-8.219*10^{-5}}{|0.205-8.219*10^{-5}|}}{2} = 0$$

According to **Figure 4**, both Z values suggest holding on the second day.

4 Model Effectiveness

To find a measure of how well our model is working we need to come up with a theoretical yield for our model. We achieve this by finding the lowest price for each asset and comparing it with the highest price in the future, in a traditional buy low and sell high model. We determined that Bitcoin will grow more than gold. Then we were able investing in Bitcoin on September 23rd, 2016 when the price per Bitcoin was $594.08 USD, with a transaction fee of 2%. We then sold at the highest point between then and 9/10/2021, which was on April 16th, 2021 when the value was $63554.44 USD. This will give us a theoretical yield of $104840.01 USD. Now we can measure how well our model is operating and against a perfect standard.

Our final value on 9/10/2021 was consistently around $19,000 USD, which means that our percent yield was 18.12%. While our model doesn't produce the same output as the maximum theoretical yield, money is still gained. We can see this represented in **Figure 5**.

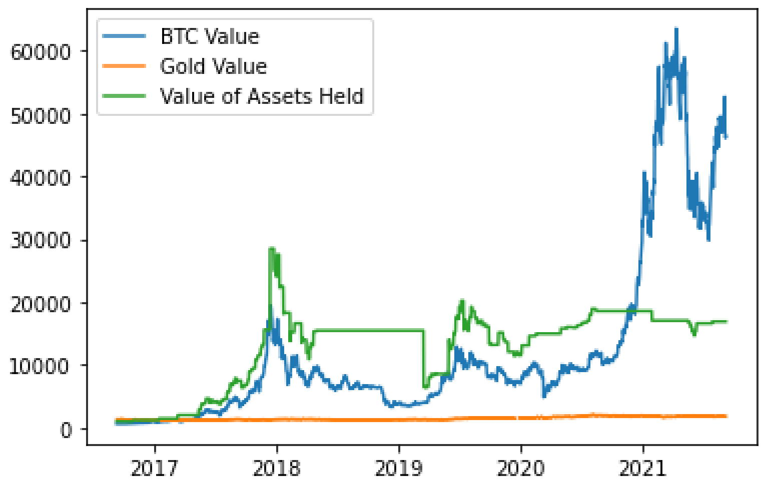

Figure 5:
Net profit over time, including daily values for Bitcoin and gold

4.1 Topics Our Model Does Not Address:

- National holidays when gold cannot be traded on the open market.
- Unexpected events in the market, such as a new source of gold being discovered or bitcoin mining taking a leap forward, which would cause massive drops in the value of either asset.
- Leap days are not included in our model, but it could be applied to such days if one has the data for it.
- Our model does not address how much of a certain asset one should buy. As it stands now we make it assuming one would invest all of their money in the asset, regardless of how much there is or the transaction fees. This is a very risky way to trade as you are not diversifying your assets and could lead to bankruptcy.

5 Memorandum

To whom it may concern,

The seemingly unpredictable, often risky markets for gold and bitcoin are daunting, but our model allows you to navigate the daily fluctuations while pocketing money. Simply put, our model will maximize your profits with regular investments in gold and Bitcoin. Using only past market activity for both assets, our model predicts when you should buy, sell, or hold each asset at any given time with a high degree of confidence. Specifically, using a series of regressions that any given number of days, our model determines precise trends in market activity for both gold and Bitcoin. The period of days in which a regression is performed can be changed based on desired precision of daily changes in asset value. The degree of regression accuracy can also be altered to fit various market predictions of the curve, all based on desired precision. Although our model uses the most well-rounded system of regression degree and regression iterations, our model can fit any needs through slight corresponding customizations.

Given $1,000 USD on 9/10/21, daily trends for both gold and bitcoin are evaluated in terms of rate of change using functions that closely model daily fluctuations in value. For any given day, the model's three outputs - buy, sell, or hold - are determined with two inputs: value concavity and percent change. Concavity is determined by calculating the rate of change of the trend in value throughout the day, and percent change is determined by how much each asset will change day-to-day. For general positive percent changes in value with a positive trend, the model predicts that you should buy an asset. On the other hand, general negative percent changes in value with a negative trend encourages selling an asset. Holding an asset results from either:

- Positive percent change and a negative overall trend
- Negative percent change and a positive overall trend

Using this model will help to maximize profits in the long run and not have to worry about your investments. Furthermore, there are nine different scenarios when comparing the decision value of Bitcoin versus gold, meaning the correct decision is made at any given time. Simply put, our model determines when to invest, what to invest in, and when to sell.

As proof of our model's success, we simulated a five year market trading scenario involving gold and Bitcoin. Using only current and past trends in the data, our model projected a profit of $10829.18 upon initially starting with $1000 USD in cash. Inflation of the USD is also accounted for, in which we assume that inflation compounds at a constant rate of 3%. Thus, more precise and accurate predictions of real-time market activity can be determined in relation to the overall change in price of cash.

Perhaps you might wonder why our model is more profitable than simply investing in either gold or Bitcoin without required market activity. As mentioned before, the market is unpredictable, and so is the security of your money placed in assets. Why blindly trust the risk associated with market trends when a model can predict future prices every day, given asset values, with a high degree of confidence. With little input and implementation, you can sit back, relax, and watch the value of your assets increase.

6 References

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