AMS-511 Foundations of Quantitative Finance

Fall 2020 — Assignment 03

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Question 1

Use FinancialBond[ ] to compute the following:

- A newly issued $10,000 bond with a ten-year term has a 4% coupon rate and pays semi-annually. The current market yield for a ten-year bond of this type is 3.7%. What is the current market price of the bond?
- A $10,000 bond with a five-year term, a 3.7% coupon rate paying semi-annually was issued eight months ago. The current market yield is 3.5%.
  - What is the current price of the bond?
  - What is the accrued interest on the bond?
- Consider a $10,000 semi-annual bond with a twenty-year term and a 4.5% coupon rate. The bond was issued five-years ago and currently trades at a price of $10,500. What is the yield on the bond?

Question 2

Consider two assets with mean returns, standard deviations and correlation matrix:

$$\boldsymbol{\mu} = \begin{pmatrix} 0.08 \\ 0.05 \end{pmatrix}$$

$$\boldsymbol{\sigma} = \begin{pmatrix} 0.10 \\ 0.04 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$$

$$\mathcal{M}_1 = \min_{\boldsymbol{x}} \left\{ \frac{1}{2} \mathbf{x}^T \boldsymbol{\Sigma}\, \mathbf{x} - \lambda\, \boldsymbol{\mu}^T \mathbf{x} \;\middle|\; \mathbf{1}^T \mathbf{x} = 1 \right\}$$

- Compute the covariance matrix $\boldsymbol{\Sigma}$.

Consider the following portfolio optimization problem $\mathcal{M}_1$ (short positions allowed):

$$\mathcal{M}_1 = \min_{\mathbf{x}} \left\{ \frac{1}{2} \mathbf{x}^T \boldsymbol{\Sigma}\, \mathbf{x} - \lambda\, \boldsymbol{\mu}^T \mathbf{x} \;\middle|\; \mathbf{1}^T \mathbf{x} = 1 \right\}$$

- Compute and plot a mean-variance efficient frontier for a portfolio consisting of these two assets.
- If the risk-free rate $r_f = 0.01$, what is the tangent portfolio?

Consider the following portfolio optimization problem $\mathcal{M}_2$ (no short positions allowed):

$$\mathcal{M}_2 = \min_{\mathbf{x}} \left\{ \frac{1}{2} \mathbf{x}^T \boldsymbol{\Sigma}\, \mathbf{x} - \lambda\, \boldsymbol{\mu}^T \mathbf{x} \;\middle|\; \mathbf{1}^T \mathbf{x} = 1 \wedge x_i \geq 0 \right\}$$

- Compute and plot a mean-variance efficient frontier for a portfolio consisting of these two assets.
- Compare the efficient frontiers of $\mathcal{M}_1$ and $\mathcal{M}_2$.

Question 3

Use FinancialData[ ] to download the data required to complete the following:

- Secure the closing price data for Microsoft and Apple to 2019-06-30.
- Plot the closing price of Microsoft and Apple on the same graph.
- Compute the daily returns for Microsoft and Apple and plot them on separate graphs.
- Generate a table which contains for each stock in the Dow Jones Industrial index: the ticker symbol, name, and market capitalization.

Question 4

Recently, yields on the sovereign debt of several European countries have turned negative. In effect, bond holders are paying these governments for holding their capital.

Yields on German Bunds can be found at

https://www.bloomberg.com/markets/rates-bonds/government-bonds/germany.

Recent data for late September 2019 were {term, yield}:

In[•]:=
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mnBundYields = {{2, -0.0076}, {5, -0.007}, {10, -0.0059}, {30, -0.0012}};
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A reasonable estimate for the yield curve can be found using a second-order spline interpolation.

Consider an older Bund with a 10-year term, semi-annual payments, a face amount of €10,000 and a coupon rate of 0.002%. The bond is 3 years into its term. What is the current market price of the bond?