AMS-511 Foundations of Quantitative Finance

Fall 2020 — Assignment 01

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Question 1

Suppose $1 was invested in 1776 at 3.3% interest compounded annually.

- Approximately how much would that investment be worth today?
- What if the interest rate were 6.6%?

Question 2

(Effective rates) Find the corresponding effective rates for:

- 3% compounded monthly,
- 18% compounded monthly,
- 18% compounded quarterly.

Question 3

Two copy machines are available. Both have useful lives of 5 years. One machine can be either leased or purchased outright, the other must be purchased. Below is a description of the three options with the first year's maintenance included in the initial cost. There are then four additional yearly payments occurring at the beginning of each year, followed by the revenues from any resale. According to a PV analysis the least cost option is B:

	A	B	C
Initial outlay	6000	30 000	35 000
Yearly expense	8000	2000	1600
Resale value	0	10 000	12 000
PV at 10 × %	31 559	30 131	32 621

It is not possible to compute an IRR on these options since the cashflows are all negative, except for the resale value. It is possible, however, to calculate the IRR on an incremental basis. Find the IRR on a change from A to B.

Is this change justified on an IRR basis?

Question 4

Gavin's father, Mr. Jones, has just turned 90 years-old and is applying for a lifetime annuity that will pay $10,000 per year, starting 1 year from now, until he dies. He asks Gavin to analyze it for him. Gavin finds that according to statistical summaries, the probability of that Mr. Jones will die at a particular age is:

age	90	91	92	93	94	95	96	97	98	99	100
prob.	0.07	0.08	0.09	0.1	0.1	0.1	0.1	0.1	0.1	0.07	0.05

What would Gavin's answers be to the following questions:

- What is Mr. Jones's (Gavin's father's) life expectancy?
- What is the PV of an annuity at 8% interest that has a lifetime equal to Mr. Jones's life expectancy? (For an annuity equal to a non-integral number of years use an averaging method.)
- What is the expected PV of the annuity?
- What is the standard deviation of the annuity?

Question 5

The Smith family just took out a variable-rate mortgage on their new home. The mortgage value is $100,000, the term is 30 years, and the initial interest rate is 8%. This rate is guaranteed for 5 years, after which it will be adjusted to the prevailing rates. The mortgage will be adjusted by either modifying the payment amount or the length of the remaining loan.

In the text you are instructed to use yearly payment amounts, but for this assignment assume that the mortgage requires monthly payment,

- What is the monthly payment at the start of the mortgage?
- What is the mortgage balance after 5 years?
- If the prevailing rate is 9% at the readjustment point and the mortgage termination date is kept constant, then what is the new monthly payment?
- Under the same conditions immediately above, if the monthly payment is kept constant, then what it the new term (i.e., years and months remaining) of the mortgage?

Question 6

A firm wishes to purchase a building for $10,000,000. Origination fees and other costs will total $25,000. The firm has $2,000,000 to cover the initial costs and down payment on the building. The mortgage terms available are a 5-year term with monthly payments computed at 7.5% with an amortization of 20 years.

- What is the amount of the mortgage?
- What are the monthly payments and final balloon payment?
- How much interest did the firm pay over the 5 years of the mortgage?

Question 7

Arthur is planning on buying a house and needs to take out a mortgage for $200,000. He has two choices a 30-year with monthly payments of $1,468 and a 15-year with monthly payments of 1,854. Arthur wants to pay the lowest interest rate possible.

- What is the interest rate on each mortgage and which mortgage should he pick?
- He changes his mind and wants to pay the smallest amount of total interest. What are the total interest charges for each mortgage and which mortgage should he now pick?

Question 8

Meg has the opportunity to purchase bond A at origination for $98,930. The bond's term is 10 years and pays semi-annual interest based on a nominal rate of 3.5%. She needs to invest the money for 5 years after which she wants to sell the bonds and put the funds towards the purchase of a house. She chose a 10-year bond to secure a higher interest rate.

- What is bond A's yield?
- Another bond B is offered for sale with the same characteristics except the coupon rate is 3.75%. How would the market price bond B?
- Meg is concerned that yields are at all time lows and is concerned that they will rise in the future. If the long term average yield for 5-year bonds is 4.75%, then how would the prices of bonds A and B be affected when Meg is ready to sell them?

Question 9

Ten years ago Sarah purchased her house and took out a mortgage for $150,000. The term of the mortgage was 30-years with monthly payments at an interest rate of 5.6%. She is selling the house and will use a portion of her proceeds to pay off the mortgage.

- What is the principal balance on the mortgage that must be paid off?
- How much total interest has been paid by Sarah over the past 10 years?

Question 10

Joe purchases a new car every 4 years, paying $25,000 (assume there is no inflation). He can put away money each month into a saving account which pays interest at the rate of 4% compounded monthly and needs to know what his monthly savings payment must be so that he has $25,000 in the account after 4 years. Alternately, he can fund the purchase price of $25,000 with a loan with a term of 4 years with monthly payments based on an interest rate of 6%.

- How much must Joe save each month if he decides to save for the purchase of the car?
- What is the monthly loan payment if Joe decides to borrow the money for the purchase of the car?
- If Joe decides to save each month the same amount he would have had to pay for the loan, how much would he have left over when he purchased the car after 4 years of saving?

Question 11

Consider two investments, both of which require an initial investment of $12,500. The first, A, pays a steady cashflow over the next 4 years. The second, B, pays a gradually increasing cashflow over the same period. These

cash-flows are summarized in the table below:

	0	1	2	3	4
A	−12 500	4000	4000	4000	4000
B	−12 500	0	2500	6000	8500

You need to make an investment decision. Assume annual compounding for your calculations.

- An investment is viable if it has a positive IRR. You decide to pick the viable investment with the largest IRR, but if neither IRR is positive you will hold onto your $12,500.
  - Determine the viability of each investment in terms of its IRR.
  - What is your investment decision on an IRR basis: do not invest, invest in A, or invest in B?
- Alternately, you have a threshold rate of return of 7% that you use to evaluate investment opportunities. An investment is considered viable if its PV is positive. You decide to pick the viable investment with the highest PV, but if neither is viable you will hold onto your $12,500.
  - Determine the viability of each investment in terms of its PV.
  - What is your investment decision on a PV basis: do not invest, invest in A, or invest in B?

Question 12

Perform the following computations in Mathematica:

- Plot $(x^2 \sin x)$ for x from $-\pi$ to 2π.
- Integrate $(x^2 \sin x)$ for x from $-\pi$ to 2π.
- Differentiate $(x^2 \sin x)$.
- Write a function f[x] which, given x, computes $(x^2 \sin x)$.
- Evaluate the function above for x from 0 to 1 in increments of 0.1