

# AMS-511 Foundations of Quantitative Finance

## Fall 2020 — Assignment 04

Robert J. Frey, Research Professor  
Stony Brook University, Applied Mathematics and Statistics

Robert.Frey@StonyBrook.edu  
<http://www.ams.sunysb.edu/~frey>

---

### Question 1

You are given the following annual data on four investments,  $i \in \{1, 2, 3, 4\}$  in a market  $M$ :

- $r_f = 0.02$
- $\mu_M = 0.085$
- $\sigma_M = 0.105$
- $\beta = \{0.9, 1.2, 0.6, 2.1\}$
- $\sigma_\epsilon = \{0.05, 0.07, 0.04, 0.09\}$

Under the assumption that the CAPM applies:

- Compute the mean vector of the asset returns.
  - Compute the correlation and covariance matrices of the asset returns.
  - Compute the mean-variance efficient portfolio such that  $\mathbf{1}^T \mathbf{x} = 1$ . Assume there are no further constraints, specifically, that short positions are permitted
  - Compute the mean and standard deviation of that portfolio.
  - The investor wishes to keep 10% of its assets in cash and place the remainder in the optimal portfolio. Assuming returns are Normally distributed what are the mean and standard deviation of return for this combined cash-risky portfolio?
- 

### Question 2

You have a portfolio with estimated monthly mean return of 0.8% and monthly standard deviation of 3.5%.

- Assuming the portfolio returns follow a Normal distribution, what is the VaR and CVaR at a 99.9% confidence level?

- Assuming the portfolio returns follow a Student  $t$  distribution with 4 degrees of freedom, what is the VaR and CVaR at a 99.9% confidence level?

**Note:** Consider a random variable  $R \approx \text{StudentTDistribution}[\mu, \sigma, \nu]$ . Generally, the parameter  $\mu$  is known as a *location* parameter and  $\sigma$  a *scale* parameter. The mean of the Student  $t$  is  $\mu$ , but its standard deviation is not  $\sigma$ , but

$$\text{StandardDeviation}[R] = \sigma \sqrt{\frac{\nu}{\nu - 2}}$$

Thus, the parameter  $\sigma$  is related to but not identical to the standard deviation. Given the standard deviation as in this problem, you must solve for the Student  $t$  scale parameter  $\sigma$  to specify the distribution correctly.