AMS-511 Foundations of Quantitative Finance

Fall 2020 — Assignment 07

Robert J. Frey, Research Professor
Stony Brook University, Applied Mathematics and Statistics

Robert.Frey@StonyBrook.edu
http://www.ams.sunysb.edu/~frey

Question 1.

Let $r(t)$ denote the return of an asset in time period t, and assume that time varies over T periods, *i.e.*, $t = 1, 2, \ldots, T-1, T$. Let r_{total} denote the total return over the T periods. Note that the log return of rate r is computed by $\log[1 + r]$. Thus,

$$(1 + r_{\text{total}}) = (1 + r(1))\,(1 + r(2)) \ldots (1 + r(T-1))\,(1 + r(T)) = \prod_{t=1}^{T} (1 + r(t))$$

Show that maximizing the log return of $r(t)$ also maximizes r_{total}.

Question 2.

Consider a stock whose price at time t is $S(t)$ and which has constant volatility $\sigma = 18\%$. The risk free rate is $r_f = 1\%$. Let $S(0) = \$102.25$. Based on public information, the stock is involved in a lawsuit. If it wins the lawsuit its price will rise dramatically, and if it loses will fall dramatically. Your best estimate is that the lawsuit will be resolved in six months.

- Using FinancialDerivative[ ] and vanilla European options, determine the price at $t = 0$ of a suitable long straddle to take advantage of this situation.
- Plot the value of the straddle at three months hence for values of the underlying from \$75 to \$125.
- The lawsuit is settled early at three months and you close out the position. Plot the P&L graph for this strategy for values of the underlying from \$75 to \$125.

Question 3.

In the Notes for Lecture 07 we downloaded the daily price for the S&P 500 for 1960-01-01 to 2020-09-30 and then computed the daily log returns.

- As in the Notes, use EstmatedDistribution[ ] to fit a NormalDistribution[ ] and a StudentTDistribution[ ] to the daily log returns.
- Read the description for DistributionFitTest[ ] in the Documentation Center. Use it to evaluate how well each of the distributions above fit the data.
- Read the description for ProbabilityPlot[ ] in the Documentation Center. Use it to evaluate how well each of the distributions above fit the data.
- Using whichever of the two distributions are the most representative and assuming the daily log return is an i.i.d. random variable, what is the probability a given day's log return is -10% or less?
- Assuming there are 250 trading days in a year, how many years on average will there be between log return losses worse than -10%?
- Using the raw data, what is the probability that a given day's log return is -10% or less?

Question 4.

You are playing game at a casino. Your current wealth is w. Given your bet b (assuming $0 \le b \le w$), you have a 1/4 probability of winning back your original bet plus 3 b and a 3/4 probability of losing the amount bet.

- Use the Kelly criterion to determine your optimal bet size.
- What is your expected wealth after one round of betting?