AMS-511 Foundations of Quantitative Finance

Fall 2020 — Assignment 05

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Question 01

Assuming continuous compounding, an asset has a carrying charge per unit of time proportional to the spot price. This condition might occur, for example, if the charge represented insurance for the safe storage of the asset. Let the charge be $q\, S[t]$. Show that the theoretical price of the forward contract with delivery date T is

$$F = S\, e^{(r+q)\,T}$$

Question 02

The risk free interest rates in the UK $r_{\text{UK}} = 0.04$ and US $r_{\text{US}} = 0.06$, compounded continuously. The spot price of the UK pound is \$1.60 and the forward price for the UK pound deliverable in 6-months is \$2.00.

- Does an arbitrage opportunity exist. Show clearly why one is or is not available.
- If there is such an opportunity, describe the trade and show what the risk-free profit would be.

Question 03

The stock of the ABC Corp. has a current price $S[0] = \$120.00$ and an annual volatility of $\sigma = 20\%$. The risk-free return is $r_f = 2\%$. What is the value of an call option expiring in $T = 0.5$ years with a strike price of $K = \$115$? Use a geometric binomial model with $n = 20$ time steps.

Question 04

For the stock above, plot the current price of a put option expiring in 6-months for prices from \$90.00 to \$150.00. On the same graph, plot the final pay-off function for the option at expiry. Use a geometric binomial model with $n = 20$ time steps.

Question 05

Given the Itô process with initial condition $X(0) = 0$.

$$d\!X(t) = \mu\, d\!t + \sigma\, d\!W(t)$$

and the transformation

$$Y(t) = a + b\, X(t)$$

Find the Itô process governing $Y(t)$ and its initial condition $Y(0)$.

Question 06

Given the Itô process with initial condition $X(0) = 4$.

$$d\!X(t) = \mu\, X(t)\, d\!t + \sigma \mathrm{X}(t)\, d\!W(t)$$

and the transformation

$$Y(t) = \sqrt{X(t)}$$

Find the Itô process governing $Y(t)$ and its initial condition $Y(0)$.