AMS-511 Foundations of Quantitative Finance

Fall 2020 — Assignment 06

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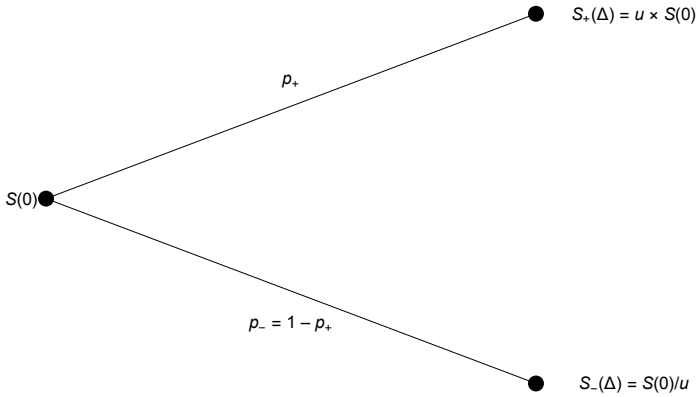
Question 1.

We are given a single-step geometric binomial pricing lattice as the model for the price dynamics of a stock with current price $S(0)$:



Consider a case in which $S(0) = 95$, $\Delta = 0.25$, $u = 1.1$, and the risk free rate of return $r = 1.5\%$, then...

- Show that the market for $S(t)$ is arbitrage free.
- What is the risk neutral measure?
- What is the price at $t = 0$ for an at-the-money put?

Question 2.

For an underlying whose price dynamic follows the following SDE

$$d\,S(t) = \mu\, S(t)\, d\,t + \sigma\, S(t)\, d\,W(t)$$

Let

$$X(t) = S(t)^{-1}$$

Use Itô's lemma to determine the SDE which describes the price dynamics of $X(t)$, assuming $S(0) > 0$.

Question 3.

Consider a European *capped-call option*. With $S(t)$ the price of the underlying and $F(t)$ the price of the option, the capped-call gives the holder the right to exercise the option at expiry with value of a put with strike K whose value, however, is capped at a maximum pay-out C where $K < C$. Thus, the value at expiry T can be expressed by

$$F(T) = \min[\, \max[\, S(T) - K,\ 0\,],\ C\,]$$

Assume the risk free rate is r with continuous compounding. Express the value of the option at time $t < T$ in terms of vanilla European options with appropriately chosen parameters and, if necessary, any cash position.

Question 4.

Consider a security priced $S(t)$ with $\sigma = 0.22$ where the risk free rate $r = 0.01$.

- Use FinancialDerivative[ ] to plot the values of a European call option for a strike $K = 102$ and expiry $T = 0.5$ at $t = 0.25$ for $60 \le S(t) \le 140$.
- With the same parameters above, use FinancialDerivative[ ] to plot the values of an at-the-money European call option at $t = 0.25$ for values of $0.1 \le \sigma \le 0.3$.