

AMS-511 Foundations of Quantitative Finance

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Question 1.

In the lecture notes we used the example of a firm with the characteristics:

$$\begin{aligned}\mu &= 0.07 \\ \sigma &= 0.20 \\ r &= 0.01 \\ K &= 7\,000\,000 \\ V &= 10\,000\,000\end{aligned}$$

then used the following expression to estimate the default probability.

$$P[V(T) < K] = F_{\text{Normal}}\left[\frac{\log[K/V(0)] - (r - \sigma^2/2)T}{\sqrt{\sigma^2 T}}\right]$$

The example assumed that a default would occur if the value of the firm fell below the face value of the debt at the expiry point; *i.e.*, if $V(T) < K$.

We noted that an alternative approach was to allow defaults to occur at any point $0 \leq t \leq T$ which introduces path dependence into the default process.

Assume now that a default is said to occur if the value of the firm falls below the face value of the debt at any time up to and including the expiry; *i.e.*, if $V(t) < K$ for $0 \leq t \leq T$. Construct a Monte Carlo simulation to estimate the risk neutral default probability under that condition.

Question 2.

Consider a 10-year zero coupon bond with a face value $F = 100\,000$. The risk free rate is 0.01. The credit spread for the bond is 150 basis points. Compute the following at $t = 0$:

- The default probability of the bond.
- The value of the bond if there is no recovery on a default.

- The value of the bond if there is a recovery of 15%.
- The value of the bond if there were no possibility of default.