

AMS-511 Foundations of Quantitative Finance

Fall 2020 — Assignment 09

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Question 1.

The hyperbolic absolute risk aversion (HARA) class of utility functions has the form

$$U[x] = \frac{1-\gamma}{\gamma} \left(\frac{ax}{1-\gamma} + b \right)^\gamma, \quad b > 0$$

Show that the parameters of the HARA utility function can be chosen to obtain the following special cases:

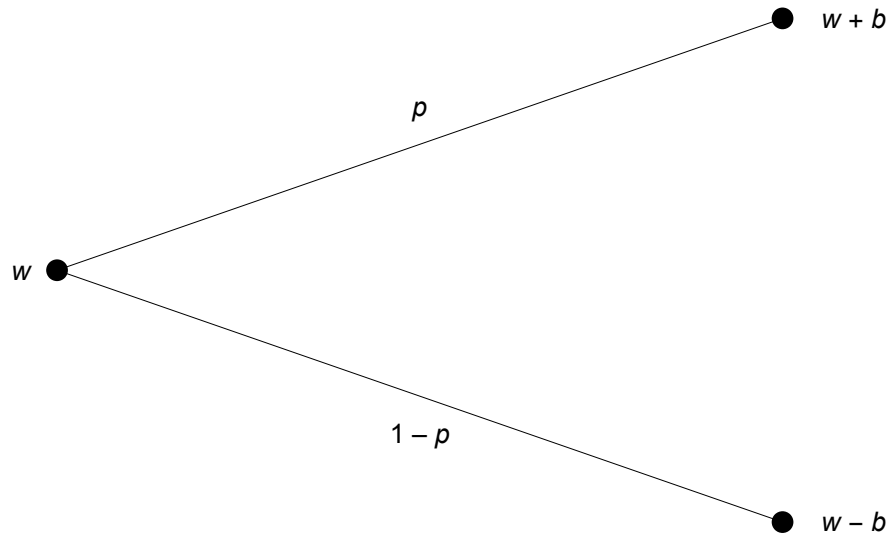
- Risk neutral: $U[w] = w$
 - Exponential: $U[w] = -e^{-aw}$ [Hint: Consider the limit $\gamma \rightarrow -\infty$]
 - Power: $U[w] = a w^\gamma$
 - Log: $U[w] = \ln[w]$ [Hint: Consider $U[w] = (1-\gamma)^{1-\gamma} ((x^\gamma - 1)/\gamma)$ and the limit $\gamma \rightarrow 0$]
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Question 2.

You are given a random variate X which has a Student t distribution with PDF with $0 < \nu$ degrees of freedom. Let $\mathbb{E}[X^n]$ represent the n^{th} moment of X . Show mathematically which moments of X exist and which are indeterminate.

Question 3.

Consider the betting game below that we used to illustrate Kelly betting where we sought to maximize the growth rate.



This led to solving for the bet b^* which maximized the expected log wealth; *i.e.*,

$$b^* = \operatorname{argmax}_b [p \log[w + b] + (1 - p) \log[w - b]]$$

Now consider this game in the more general setting of utility theory where our goal will be to maximize the expected utility $\mathbb{E}[U[X]]$; *i.e.*,

$$b^* = \operatorname{argmax}_b [p U[w + b] + (1 - p) U[w - b]]$$

Let $U[x] = \sqrt{x}$.

- Solve for the optimal bet for a player with square root utility for the game above.
- Assume that the probability of winning $p = 0.6$. Compare the Kelly bet with the square root utility bet.
- For this game is the Kelly bet more aggressive, less aggressive, or sometimes more and sometimes less aggressive than the square root bet? Show how you came up with your answer.