AMS-511 Foundations of Quantitative Finance

Fall 2020 — Workshop

Examining the CAPM Against Actual Data

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Workshop Description

Overview

The Capital Asset Pricing Model (CAPM) makes some specific predictions about the market for securities. There are two that we will address here. First, that the relationship between the return of an asset i and the market M follows the relationship

$$r_i(t) - r_f = \beta(r_M(t) - r_f) + \epsilon_i(t)$$

And second, the proportion of asset i in the market (*i.e.*, tangent) portfolio is

$$x_i \propto \frac{\beta_i}{\text{Var}[\epsilon_i]}$$

Assignment

The assignment for this Workshop is to perform an analysis which computes the theoretical allocation, the actual allocation, and compares the two. You will be working with daily return data for the Dow Jones Industrial Index, which consists of 30 stocks, beginning 2008-03-31 and ending 2020-10-30, inclusive. The work involved is the following:

1. Load the necessary data. The required data files are on the AMS 511 course page in the syllabus under "Workshop".

1.1. Import[ ] the daily return vector for the returns of the market.

1.2. Import[ ] the daily returns matrix for the thirty members of the DJI. This is a matrix with each row representing a different date and each column a different stock.

1.3. Import[ ] vectors representing the tickers of the DJI members (a vector of 30 strings).

1.4. Import[ ] the calendar of dates with each element a date {yyyy, mm, dd} vector.

1.5. All of the above are aligned properly.

2. Perform a CAPM linear regression (using FitLinearModel[ ] to estimate the betas and error variances of each member of the DJI. For simplicity assume an annual risk free rate of 1%.

2.1. Use the *daily* rate estimated as (1 + 0.01)^(1/250) − 1.

2.2. Use the option IncludeConstantBasis→False in LinearModelFit[ ] to fit a linear regression without an intercept term.

3. Use these estimates of the betas and error variances to compute the proportional allocations of each as predicted by the CAPM and normalize them so they sum to one.

3.1. Check the documentation for LinearModelFit[ ] for the properties of a FittedModel object.

3.2. The value of β_i and ${\sigma_{\epsilon_i}}^2$. are both available from the FittedModel.

4. Use FinancialData[*ticker*, "MarketCap"] to download the market capitalization of the members of the DJI and use them to compute the actual proportion of each in the DJI. Again, normalize their sum to one.

5. Plot the CAPM allocations estimated in (**3.**) against the market's observed in (**4.**).

6. Perform whatever additional comparisons you feel would be useful to characterize what you found.

7. What discrepancies do you note? How might you explain them?

Important

It's important to remember that clarity and organization counts when submitting your work. As a professional if you can't communicate what you've done effectively, then you often might as well not have done it. Please organize and annotate the notebook you submit for this Workshop.