**MAT 303** **Name (Print):** _____
**Summer II 2018**
**Final**
**08/16/18**
**Time Limit: 3 hours and 25 minutes** ID number _____

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## Instructions

- This exam contains 13 pages (including this cover page) and 6 problems, one of which (problem 6) is an extra credit problem. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam (e.g., smartphones, smartwatches, tablets). You may not use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**.

| Problem | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |
| 6 | 20 | |
| Total: | 120 | |

1. Find particular solutions of the differential equations below.

   (a) (10 points)

$$y^{(4)} - 5y'' + 4y = e^x - xe^{2x}$$

(b) (10 points)

$$y'' + 9y = 2\sec(3x)$$

2. Consider the initial-value problem

$$(2x-1)y' + 2y = 0,\ y(0) = 1.$$

(a) (10 points) Solve this equation via the power series method

(b) (5 points) Determine the radius of convergence of the power series solution you found.

(c) (5 points) Write the power series solution in terms of elementary functions.

3. (20 points) Find as many independent Frobenius series solutions as possible for the differential equation

$$xy'' + 2y' + xy = 0.$$

4. Find general solutions of the following linear systems by the eigenvalue method.

   (a) (4 points)

$$x' = x - 5y$$
$$y' = x + 3y$$

(b) (6 points)

$$x' = -3x + 5y + 5z$$
$$y' = 3x - y + 3z$$
$$z' = 9x - 8y + 10z$$

(c) (10 points)

$$\begin{aligned} x' &= -x + y + z - 2w \\ y' &= 7x - 4y - 6z + 11w \\ z' &= 5z - y + z + 3w \\ w' &= 6x - 2y - 2z + 6w \end{aligned}$$

5. Find general solutions of the following initial-value problems by the matrix exponential method.

   (a) (6 points)

$$x' = 2x + y$$
$$y' = x + 2y,$$

   with $x(0) = 3, y(0) = -2$.

(b) (7 points)

$$\begin{aligned} x' &= 5x - 6z \\ y' &= 2x - y - 2z \\ z' &= 4x - 2y - 4z, \end{aligned}$$

with $x(0) = 2, y(0) = 1, z(0) = 0$.

(c) (7 points)

$$\begin{aligned} x' &= x + 2y + 3z \\ y' &= y + 2z \\ z' &= z, \end{aligned}$$

with $x(0) = 1, y(0) = 1, z(0) = 0$.

6. (20 points) **Extra credit** Consider the predator-prey system consisting of two species coexisting in the same environment, whose populations at time $t$ are denoted by $x(t)$ and $y(t)$. The two species are assumed to naturally satisfy a logistic equation int he absence of the other. The two species are assumed to cooperate, in such a way that their interaction is described by the system

$$x' = 30x - 3x^2 + xy$$
$$y' = 60y - 3y^2 + 4xy.$$

Find all four equilibrium solutions of the model (1 point for each equilibrium solution). Describe the local behaviour of the linearized system at each such solution (3 points for each equilibrium solution). Determine whether you expect the linearized behavior to be reflected by the nonlinear system (1 point for each equilibrium solution).