MAT 203 - Summer I 2020
June 2, 2020

Homework 1 solutions

Exercise 1 Find the distance between the points $(-2, 1, -5)$ and $(4, -1, -1)$.

Solution: The distance formula yields

$$d = \sqrt{(-2-4)^2 + (1-(-1))^2 + (-5-(-1))^2} = \sqrt{56}$$

Exercise 2 Initial and terminal points are given, (6,2,0), (3,-3,8), respectively.
(a) Sketch the directed line segment.

(b) Find the component form of the vector.

(c) Write the vector using standard unit vector notation.

(d) Sketch the vector with its initial point at the origin.

Solution:
(a) See class notes for similar problem.

(b) $v = (3, -3, 8) - (6, 2, 0) = (-3, -5, 8)$.

(c) $v = -3i - 5j + 8k$.

(d) See class notes for similar problem.

Exercise 3 Use vectors to determine whether the points (5,-4,7), (8,-5,5) and (11,6,3) are collinear.

Solution: Taking the point $(5, -4, 7)$ as our base-point, we may define vectors

$$u = (8, -5, -5) - (5, -4, 7) = (3, -1, -12)$$
$$v = (11, 6, 3) - (5, -4, 7) = (6, 10, -4)$$

A direct comparison between corresponding coordinates shows that these vectors are not scalar multiples of each other, hence the three given points are not collinear.

Exercise 4 You are given points P = (2,-1,3), Q = (0,5,1), R = (5,5,0). Let u be the vector from P to Q, v be the vector from P to R. Find
(a) the component forms of u and v

(b) $u \cdot v$

(c) $v \cdot v$.

Solution:
(a) The vectors u and v are

$$\begin{aligned} u &= (0,5,1) - (2,-1,3) = (-2,6,-2) \\ v &= (5,5,0) - (2,-1,3) = (3,6,-3) \end{aligned}$$

(b)
$$u \cdot v = (-2,6,-2) \cdot (3,6,-3) = (-2) \cdot 3 + 6 \cdot 6 + (-2) \cdot (-3) = 36.$$

(c)
$$v \cdot v = (3,6,-3) \cdot (3,6,-3) = 3^2 + 6^2 + (-3)^2 = 54$$

Exercise 5 Find the angle between the vectors u=(1,0,-3) and v=(2,-2,1)
(a) in radians;

(b) in degrees.

Solution: The angle between two vectors may be computed by means of the formula

$$u \cdot v = \|u\| \|v\| \cos(\theta).$$

Using the given values of u and v, we find

$$\begin{aligned} \cos(\theta) &= \frac{1 \cdot 2 + 0 \cdot (-2) + (-3) \cdot 1}{\sqrt{1^2 + 0^2 + (-3)^2}\sqrt{2^2 + (-2)^2 + 1^2}} \\ &= -\frac{1}{3\sqrt{10}} \end{aligned}$$

We find the values of the angle by using the arc-cosine function:
(a) 1.676 radians

(b) 96.05°

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Exercise 6 You are given vectors u = (1,-1,1), v=(2,0,2). Find
(a) the projection of u onto v;

(b) the vector component of u orthogonal to v.

Solution:

(a) Recall that the norm of the projection vector is given by

$$\begin{aligned}\|\text{proj}(u,v)\| &= \frac{u\cdot v}{\|v\|}\\ &= \frac{1\cdot 2+(-1)\cdot 0+1\cdot 2}{\sqrt{2^2+0^2+2^2}}\\ &= \frac{4}{2\sqrt{2}}\\ &= \sqrt{2}\end{aligned}$$

It follows that the projection vector is

$$\begin{aligned}\text{proj}(u,v) &= \frac{\sqrt{2}}{\|v\|}v\\ &= \frac{\sqrt{2}}{2\sqrt{2}}(2,0,2)\\ &= (1,0,1)\end{aligned}$$

(b) The complement vector w is defined by the equation

$$\text{proj}(u,v)+w=u.$$

Using the given value of u and the value we found for the projection on part (a), we conclude

$$w=(1,-1,1)-(1,0,1)=(0,-1,0).$$

Exercise 7 You are given vectors u = (0,2,1), v=(1,-3,4). Find

(a) $u\times v$;

(b) $v\times u$;

(c) $v\times v$.

Solution:

(a) We will use the determinants in this part:

$$\begin{aligned}u\times v &= \begin{vmatrix}2 & 1\\ 1 & 4\end{vmatrix}i+\begin{vmatrix}0 & 1\\ 1 & 4\end{vmatrix}j+\begin{vmatrix}0 & 2\\ 1 & -3\end{vmatrix}k\\ &= (2\cdot 4-1\cdot(-3))i-(0\cdot 4-1\cdot 1)j+(0\cdot(-3)-1\cdot 2)k\\ &= 11i+j-2k\end{aligned}$$

(b) In this case we will use multiplication rules for i, j and k

$$\begin{aligned} v \times u &= (i - 3j + 4k) \times (2j + k) \\ &= 2i \times j - 6j \times j + 8k \times j + i \times k - 3j \times k + 4k \times k \\ &= 2k - 8i - j - 3i \\ &= -11i - j + 2k \end{aligned}$$

(c) As we saw in class, the cross product between a vector and itself is 0.

Exercise 8 You are given points (-1,4,3), (8,10,5). Find sets of

(a) parametric equations, and

(b) symmetric equations.

for the line that passes through the points (write the direction numbers as integers).

Solution:

(a) A parametric equation can be found by describing a base-point and a direction vector. We will choose $(-1, 4, 3)$ as our base point. The difference

$$u = (8, 10, 5) - (-1, 4, 3) = (9, 6, 2)$$

serves a direction vector. The parametric equation with this data is

$$\begin{aligned} (x, y, z) &= (-1, 4, 3) + \lambda(9, 6, 2) \\ &= (-1 + 9\lambda, 4 + 6\lambda, 3 + 2\lambda). \end{aligned}$$

(b) Emphasizing the parameter λ in the equations found on part (a), we find

$$\begin{aligned} \lambda &= \frac{x-1}{9}, \\ \lambda &= \frac{y-4}{6}, \\ \lambda &= \frac{z-3}{2}. \end{aligned}$$

This yields the symmetric equations:

$$\frac{x-1}{9} = \frac{y-4}{6} = \frac{z-3}{2}$$

Exercise 9 Find a set of parametric equations for the line that passes though the point $(1, 2, 3)$ and is parallel to the line given by $x = y = z$.

Solution: A direction vector for the line $x = y = z$ can be found by choosing two points on the line and finding their difference, e.g.

$$v = (1, 1, 1) - (0, 0, 0) = (1, 1, 1).$$

We can thus express the parametric equations for the desired line as

$$(x, y, z) = (1, 2, 3) + \lambda(1, 1, 1).$$

Exercise 10 Find the equation of the plane that passes through the point $(-2, 3, 1)$ and is perpendicular to $n = 3i - j + k$.

Solution: The general form of the equation of a plane perpendicular to $n = 3i - j + k$ takes the form

$$3x - y + z = D.$$

To find D, we substitute the coordinates of the point $(-2, 3, 1)$:

$$3 \cdot (-2) - 3 + 1 = D,$$

hence the equation for the plane is

$$3x - y + z = -8.$$