MAT 514 - Summer II 2020
July 16, 2020

Solutions to Quiz 3

Problem 1 Consider the function $f : \{z = x + iy \in \mathbb{C} | x, y \in \mathbb{R}, y \neq 0\} \longrightarrow \mathbb{C}$, defined by

$$f(x + iy) = \frac{ix + 1}{y}.$$

Determine if this function has a limit at 0.

Solution: This function does not have a complex number[1] as its limit at 0. Indeed, as $z \to 0$, the numerator is bounded, whereas the denominator is unbounded.

Problem 2 Determine all poles (infinite discontinuities) of the function

$$f(z) = \frac{z^2 + z - 2}{2z^2 + z - 3}.$$

Solution: As we learned in class, ratios of continuous functions are continuous, except perhaps at zeros of the denominator. In this case, the denominator has factorization

$$2z^2 + z - 3 = (2z + 3)(z - 1),$$

hence its zeros are located at $z = -\frac{3}{2}$ and $z = 1$. One of these points is an apparent discontinuity, since the expression of f can be simplified to

$$f(z) = \frac{(z + 2)(z - 1)}{(2z + 3)(z - 1)} = \frac{z + 2}{2z + 3}.$$

It follows that the only pole of the function is located at $z = -\frac{3}{2}$.

[1] Infinite limits will be dealt with in the following chapter.