MAT 303 **Name (Print):** ______________________
Summer II 2018
Midterm
07/26/18
Time Limit: 3 hours and 25 minutes ID number ______________________

Instructions

- This exam contains 11 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam (e.g., smartphones, smartwatches, tablets). You may not use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. Find the general solution to each of the following first-order differential equations. Note: solutions can be written implicitely, or depend on the computation of certain integrals which can't be expressed in terms of elementary functions. Primes denote derivatives relative to the independent variable t.

   (a) (5 points)
$$(t^2+1)x' + 2tx = t$$

   (b) (5 points)
$$2txx' = t^2 + x^2$$

(c) (10 points)

$$3t^2 + x^2 + (4tx + 6x^2)x' = 0$$

2. Match the differential equations and slope fields below. Explain your reasoning in the space provided below each equation.

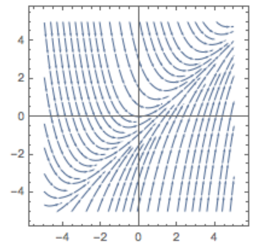

Figure 1: Slope field 1

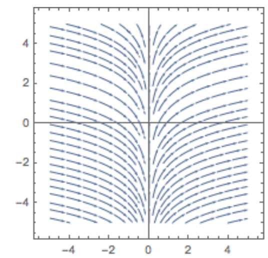

Figure 2: Slope field 2

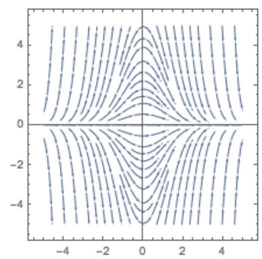

Figure 3: Slope field 3

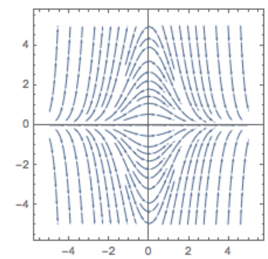

Figure 4: Slope field 4

(a) (5 points)

$$y' = -xy$$

(b) (5 points)

$$y' = \frac{1}{x}$$

(c) (5 points)

$$y' = x - y$$

(d) (5 points)

$$y' = x^2 - y$$

3. A certain logistic population naturally satisfies the equation

$$\frac{dP}{dt} = 4P - P^2.$$

Answer the following questions about this populational model:

(a) (2 points) What are the values of the equilibrium solutions? Classify each of them into stable, semistable or unstable.

(b) (3 points) What is the value of initial population which yields maximal initial populational growth (i.e., maximizes $P'(0)$)?

(c) (5 points) If the population is to be harvarsted at a constant rate $h > 0$, what is the new differential equation modelling this phenomenon?

(d) (4 points) Regarding the equation you found in part (c), how does the number of equilibrium solutions vary in terms of h?

(e) (4 points) How does the stability classification vary in terms of h?

(f) (2 points) Sketch the bifurcation diagram for the equation from part (c).

4. Find general solutions to the following higher-order linear differential equations:

   (a) (5 points)

$$x^{(3)} + x - 10 = 0$$

   (b) (5 points)

$$x^{(3)} + 4x'' + 5x' + 2x = 0$$

(c) (5 points)

$$x^{(4)} - 4x = 0$$

(d) (5 points)

$$x^{(4)} + 2x'' + x = 0$$

5. Consider the initial-value problem

$$\begin{aligned} y' &= x + y, \\ y(0) &= 1. \end{aligned}$$

(a) (10 points) Estimate the value of $y(1)$ using Euler's method with 3 steps.

(b) (10 points) Estimate the value of $y(1)$ using the 3rd order Picard approximation.