MAT 203 **Name (Print):** _____
Summer I 2020
Final Practice
06/29/20
Time Limit: 3 hours and 25 minutes ID number _____

Instructions

- This exam contains 10 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- This is an open-book exam. You may not use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	20	
2	20	
3	20	
4	10	
5	30	
Total:	100	

1. A heated storage room is to be constructed in the shape of a rectangular prism, with total volume 1000ft^3. Because warm air rises, the heat loss per unit of area through the ceiling is five times as great as the heat loss through the floor. The heat loss through each of the four lateral walls is identical, equal to three times as much as the heat loss of the floor.

   (a) (3 points) Sketch the room, labeling the dimensions of its sides. Describe the heat loss function according to your sketch.

   (b) (3 points) Use the volume constraint to reduce the number of variables in the heat loss function to two.

   (c) (4 points) Compute the gradient of the symplified heat loss function found on part (b).

(d) (5 points) Find the critical points of the heat loss function.

(e) (5 points) Use the Second Partials Test to determine which critical point, if any, minimizes heat loss.

2. The table below shows the yield of a chemical reaction (measured in miligrams) in terms of time (measured in minutes):

Time	1	2	3	4	5
Yield	1.2	7.1	9.9	13.1	15.5

   (a) (5 points) Given a line with equation $y = ax + b$, describe a function, in terms of the variables a and b, measuring the sum of the square-distances between the points in the table above and the points with corresponding x-coordinate on the line.

   (b) (5 points) Find the gradient of the function you described in part (a).

(c) (5 points) Find the critical points of the function you described in part (a).

(d) (5 points) Find the equation of the regression line approximating the points in the table by determining which of the critical points you found on part (c), is a minimum.

3. This problem concerns the iterated integral

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{3}{2+y^3}\, dy\, dx.$$

(a) (5 points) Sketch the region of integration in the plane.

(b) (5 points) Switch the order of integration.

(c) (10 points) Evaluate the integral by using whichever order you prefer.

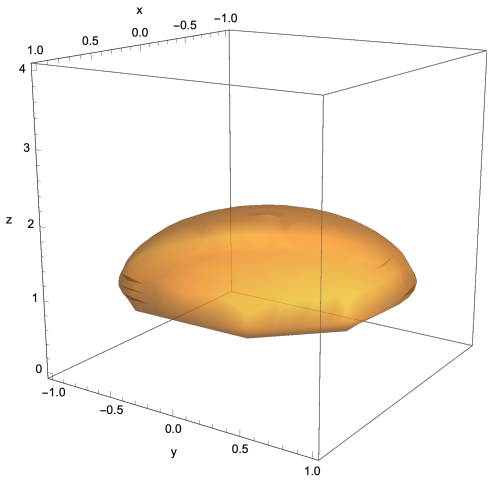
4. (10 points) Use polar coordinates to find the area between outside of the circle of radius 1 and inside the cardioid with equation

$$r = 2 + \cos(\theta)$$

5. This problem concerns the iterated integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{1}^{1+\sqrt{1-x^2-y^2}} x\,dz\,dy\,dx.$$

Below is a plot of the region of integration, for your convenience.



(a) (8 points) Convert it to cylindrical coordinates (do not evaluate).

(b) (8 points) Convert it to spherical coordinates (do not evaluate).

(c) (14 points) Evaluate the integral by using your preferred system of coordinates: Cartesian, cylindrical, or spherical.