MAT 132
Summer II 2016
Midterm
07/28/16
Time Limit: 3 hours and 5 minutes

Name (Print): _____

ID number _____

Instructions

- This exam contains 10 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam (e.g., smartphones, smartwatches, tablets). You may *not* use a calculator.

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	10	
2	10	
3	20	
4	5	
5	15	
Total:	60	

1. Use geometrical interpretation to compute the following integrals.

   (a) (3 points)

$$\int_0^5 (x+2)dx$$

   (b) (3 points)

$$\int_{-\pi}^{\pi} (\sin x)^3 dx$$

   (c) (4 points)

$$\int_0^2 [\sqrt{4-x^2}+1]dx$$

2. (10 points) Describe all the continuous functions $f : [0, 1] \longrightarrow \mathbb{R}$ with the following properties:$f(0) = 0$, $f(1) = 1$ and $f'(x) < \frac{1}{2}$ for every $x \in (0, 1)$. If no such functions exist, explain why.

3. Compute the following indefinite integrals. You can use any technique that was taught in class. Your final answer should not depend on the computation of another integral.

   (a) (5 points)

$$\int x\sqrt{9 - x^2}\, dx$$

(b) (5 points)

$$\int x^2 \sin(x)\,dx$$

(c) (5 points)

$$\int \frac{x^2 + x + 1}{x^3 - x^2 + 2x - 2} dx$$

(d) (5 points)

$$\int \sqrt{x^2 + x + 1}\, dx$$

4. (5 points) If $f(x)$ is a continuous function on $\mathbb{R}$ satisfying the following equation

$$x \sin(\pi x) = \int_0^{x^2} f(t)dt.$$

Find the value of $f(4)$.

5. In each of the problems below, determine if the integral is improper. If it is, determine if it is convergent or divergent. If it is convergent, compute its value.

(a) (5 points)

$$\int_0^\pi \frac{\frac{\cos^2 \theta}{\sin \theta} + \sin \theta}{\csc^2 \theta}\, d\theta$$

(b) (5 points)

$$\int_0^\infty \frac{e^x}{1 + e^{2x}}\, dx$$

(c) (5 points)

$$\int_0^2 \ln x \, dx$$