MAT 132 **Name (Print):** ____________________
Summer II 2017
Midterm
07/27/17
Time Limit: 3 hours and 5 minutes ID number ____________________

Instructions

- This exam contains 13 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam (e.g., smartphones, smartwatches, tablets). You may use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Unsupported answers will not receive full credit**.

Problem	Points	Score
1	20	
2	10	
3	20	
4	20	
5	30	
Total:	100	

1. Use geometrical interpretation and the properties of integration to compute the following integrals. You should not use the fundamental theorem of Calculus.

   (a) (5 points)

   $$\int_0^5 (x+2)dx$$

   (b) (5 points)

   $$\int_0^2 [\sqrt{4-x^2}+1]dx$$

   (c) (10 points)

   $$\int_{-\pi}^{\pi} (\sin x)^3 dx$$

2. (10 points) Describe all the continuously differentiable functions (i.e, differentiable functions whose derivatives are continuous) $f : [0,1] \longrightarrow \mathbb{R}$ with the following properties: $f(0) = 0$, $f(1) = 1$ and $f'(x) < \frac{1}{2}$ for every $x \in (0,1)$. If no such functions exist, explain why.

3. Compute the following indefinite integrals. You can use any technique that was taught in class. Your final answer should not depend on the computation of another integral.

   (a) (5 points)

$$\int x\sqrt{9-x^2}dx$$

(b) (5 points)

$$\int x^2 \sin(x) dx$$

(c) (5 points)

$$\int \frac{x^2 + x + 1}{x^3 - x^2 + 2x - 2} dx$$

(d) (5 points)

$$\int \frac{1}{\sqrt{x^2+4}} dx$$

4. Consider the planar region determined by the following inequalities:

$$\begin{cases} y & \geq 0 \\ x & \geq 1 \\ y & \leq \frac{1}{x} \end{cases}$$

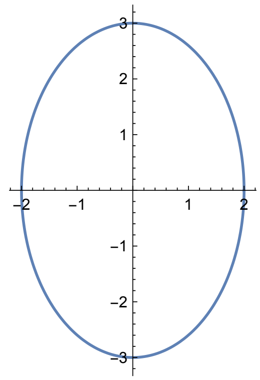
   (a) (10 points) Set up an improper integral that computes the area of this region. Determine if this integral is convergent or divergent. If it converges, compute its value.

(b) (10 points) Consider the solid obtained by rotating this region about the x-axis. Set up an improper integral that computes the volume of this solid. Determine if this integral is convergent or divergent. If it converges, compute its value.

5. The curve given by the parametric equations

$$x(t) = 2\cos(t) \tag{1}$$
$$y(y) = 3\sin(t), \tag{2}$$

for $0 \leq t \leq 2\pi$, is called an ellipse (see the picture below).



The computation of its arc length leads to an integral of a function whose antiderivative cannot be expressed in elementary terms (it is an example of an *elliptic integral*).

(a) (5 points) Set up the integral that computes the arc length of this parametric curve between $0 \leq t \leq \frac{\pi}{2}$ (the length of the ellipse is four times this integral). You should **not** try to evaluate this integral.

(b) (10 points) Compute the fourth derivative of the integrand in the above integral. Estimate an upper bound for the absolute value of this derivative for $x \in [0, 2\pi]$. Your upper bound does not need to be the maximum. *Hint: use the bounds for the trigonometric functions* sin *and* cos.

(c) (5 points) Use the error bound for Simpson's rule to find how many approximation steps are necessary to compute the integral from part (a) with 3 decimal places (i.e., the error should be less than 0.0001).

(d) (10 points) Use Simpson's rule with as many steps as you determined in part (c) to approximate the integral from part (a).