MAT 203
Summer I 2020
Midterm
06/11/20
Time Limit: 3 hours and 5 minutes

Name (Print): ____________________

ID number ____________________

Instructions

- You may use your textbook and class notes. You may not use calculators.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	30	
2	20	
3	20	
4	10	
5	20	
Total:	100	

1. In each of the following problems, determine if the statements are true or false. Explain your reasoning (correct answers without an explanation will be worth only 2 points per statement).

   (a) (5 points) Two planes in space always intersect along a line.

   (b) (5 points) Non-parallel lines in space always intersect.

   (c) (5 points) The cross product can be used to detect whether two non-zero vectors in space are perpendicular.

   (d) (5 points) Let $r(t)$ and $s(t)$ be curves in the plane, such that neither has a limit as t converges to 0. Then their dot product $r(t) \cdot s(t)$ does not have a limit at 0 either.

   (e) (5 points) If the limit of a scalar-valued, multivariable function exists at a point, then all directed limits at that point exist and coincide.

   (f) (5 points) If all the partial derivatives of a scalar-valued, multivariable function exist at a point, then all directional derivatives at that point exist.

2. Consider the lines whose parametric equations are given by

$$L_1\colon x = 3t, y = 2 - t, z = -1 + t.$$
$$L_2\colon x = 1 + 4s, y = -2 + s, z = -3 - 3s.$$

(a) (5 points) Write symmetric equations for each line

(b) (5 points) Explain why these lines do not intersect.

(c) (5 points) Explain why these lines are not parallel.

(d) (5 points) Find the general equations of two parallel planes, Π_1 and Π_2, containing lines L_1 and L_2, respectively.

3. A line and a plane are said to be parallel if they do not intersect. Consider the line L whose symmetric equations are

$$x - 1 = \frac{y-1}{-2} = z - 1,$$

and the plane Π with equation

$$x + y + z = 1.$$

(a) (3 points) Find a vector perpendicular to the plane.

(b) (3 points) Write parametric equations for the line. Clearly identify a direction vector.

(c) (4 points) Check that the line L and the plane Π are parallel by verifying that the vector perpendicular to the plane you found on part (a) is also perpendicular to the direction vector you found in part (b).

(d) (5 points) The point $P = (1, 1, 1)$ belongs to the line L. Write an equation of a line perpendicular to the plane, passing through P.

(e) (5 points) The line you found on part (d) should intersect the plane at a point Q. Find the distance between P and Q.

4. You are given a curve with velocity vector

$$r'(t) = \sin(t)i + \cos(t)j + tk,$$

and such that $r(0) = (1, 0, 1)$.

(a) (5 points) Compute the trajectory of a curve, as a function of t.

(b) (5 points) Compute the arclength of the curve for $0 \leq t \leq 2\pi$.

5. Recall that the polar coordinate system in the plane is related to Cartesian coordinates by means of the equations

$$x = r\cos(\theta), \tag{1}$$
$$y = r\sin(\theta). \tag{2}$$

(a) (6 points) Differentiate equation (1) relative to y to find a relation between $\frac{\partial r}{\partial y}$ and $\frac{\partial \theta}{\partial y}$.

(b) (6 points) Differentiate equation (2) relative to y to find a second relation between $\frac{\partial r}{\partial y}$ and $\frac{\partial \theta}{\partial y}$.

(c) (8 points) By solving the system of equations obtained in the previous two steps, compute the derivatives $\frac{\partial r}{\partial y}$ and $\frac{\partial \theta}{\partial y}$.