Quiz 2

Problem 1 Find the norm and polar angle of the complex number

$$z = (-3 + \sqrt{3}i)^3$$

Solution: The easier way to solve this problem is to find the norm and polar angle of $(-3+\sqrt{3}i)$, and operate with these according to the rules of complex multiplication in polar form. The norm is

$$|(-3 + \sqrt{3}i)| = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3},$$

while the polar angle θ satisfies

$$\cos(\theta) = \frac{-3}{2\sqrt{3}}$$
$$\sin(\theta) = \frac{\sqrt{3}}{2\sqrt{3}},$$

thus $\theta = \frac{5\pi}{6}$.

In polar form, complex multiplication works by multiplying the norms and adding the polar angles, therefore z has norm

$$|z| = (2\sqrt{3})^3 = 24\sqrt{3},$$

and polar angle

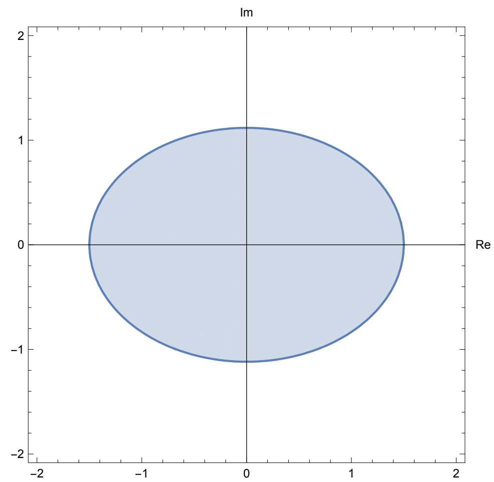
$$\phi = 3\theta = \frac{5\pi}{2}.$$

It is conventional to limit polar angles to $[0, 2\pi)$, so we can replace $\frac{5\pi}{2}$ by $\frac{\pi}{2}$, its coterminal angle in the desired range.

Problem 2 Sketch the following set on the complex plane. Determine the following topological properties: is it open? is it closed? is it bounded? is it compact? it is connected?

$$\{z \in \mathbb{C} | |z+1| + |z-1| < 3\}$$

Solution: Below is a plot of the region in the complex plane.



Below we discuss its topological properties:

- This is an open subset. Indeed, given a point z within it, the disk centered at z whose radius is half the distance between z and the boundary elipse is entirely contained within this set.
- It is not closed, as it does not include its boundary points (the elipse).
- It is bounded, as it is contained in a disk of radius 2 centered at the origin.
- It is not compact, since it is not closed.
- It is connected, and in fact convex (any pair of points may be joined by a line segment entirely contained within the set).