MAT 203
Summer II 2018
Midterm
08/16/18
Time Limit: 3 hours and 25 minutes

Name (Print): ______________________

ID number ______________________

Instructions

- This exam contains 9 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam (e.g., smartphones, smartwatches, tablets). You may not use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. Classify the statements below as true or false.

   (a) (2 points) The gradient of a smooth function of two variables is tangent to the level curves.

   (b) (2 points) If the Laplacian of a smooth function of two variables is positive at an isolated critical point, then the critical point is a local minimum.

   (c) (2 points) The area element in polar coordinates (r, θ) is given by

   $$dA = r\ dr d\theta.$$

   (d) (2 points) The integrals below are equivalent (regardless of the function f)

   $$\int_0^1 \int_0^{\sqrt{1-y^2}} f(x,y) dx dy, \quad \int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y) dy dx$$

   (e) (2 points) The integral below is well-defined,

   $$\int_0^1 \int_0^x (x+y) dx dy.$$

   (f) (2 points) The volume element in spherical coordinates (r, ϕ, θ) is

   $$dV = r \sin(\phi) dr d\phi d\theta$$

   (g) (2 points) A vector field F in the plane satisfying the closedness criterion on a region R is the gradient of a function defined in R.

   (h) (2 points) The line integral of a gradient vector field depends only on initial and final position of the curve.

   (i) (2 points) The divergence of a curl is always zero.

   (j) (2 points) The curl of a gradient vector field is always non-zero.

2. (20 points) Find and classify all the critical points of the function

$$f(x, y) = (x^2 + 3y^2)e^{1-x^2-y^2}$$

3. Compute the following multivariable integrals.

   (a) (10 points)

$$\int_1^2 \int_0^{\ln(x)} (x-1)\sqrt{1+e^{2y}}\ dydx.$$

(b) (10 points)

$$\iiint_R \sqrt{x^2+y^2+z^2}\,e^{-x^2-y^2-z^2}\,dV,$$

where R is the solid region bounded by the surfaces $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$.

4. Compute the following line and surface integrals

   (a) (5 points)

   $$\int_C (x^2 + y^2 + z^2)ds,$$

   where C is the parametrized curve $C\colon r(t) = (\cos(t), \sin(t), t)$, $0 \le t \le 2\pi$.

   (b) (5 points)

   $$\int_C (y^2\mathbf{i} + x^2\mathbf{j}) \cdot dr,$$

   where C is the plane curve bounding the region lying between the graphs of $y = x$ and $y = \sqrt{x}$, $x \ge 0$, oriented counterclockwise.

(c) (10 points)

$$\iint_S F \cdot N dS,$$

where F is the vector field

$$F(x, y, z) = \frac{x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}\mathbf{i} + \frac{y}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}\mathbf{j} + \frac{z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}\mathbf{k},$$

S is the surface of the sphere of radius 1 centered at the origin, and N denotes the outward pointing normal vector field along the sphere.

5. Consider the vector field

$$F(x,y,z) = \frac{-x}{(x^2+y^2+z^2)^{\frac{3}{2}}}\mathbf{i} + \frac{-y}{(x^2+y^2+z^2)^{\frac{3}{2}}}\mathbf{j} + \frac{-z}{(x^2+y^2+z^2)^{\frac{3}{2}}}\mathbf{k},$$

defined for all points (x, y, z) other than the origin.

(a) (10 points) Compute its line integral along the curve which bounds the surface

$$S\colon z = \sqrt{9 - x^2 - y^2}.$$

The orientation on the curve is so that when viewed from above, the curve is oriented counterclockwise.

(b) (10 points) Compute its flux integral (relative to outward pointing normal vectors) on the sphere of radius 1 centered at $(0, 0, 2)$.