MAT 511   Name (Print): _____
Summer I 2019
Final exam
07/02/19
Time Limit: 3 hours and 25 minutes   ID number   _____

Instructions

- This exam contains 8 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam (e.g., smartphones, smartwatches, tablets). You may not use a calculator.

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	4	
2	4	
3	4	
4	4	
5	4	
Total:	20	

1. Construct relations on the set $A = \{1, 2, 3\}$ satisfying the following properties:

   (a) (1 point) It is not reflexive, not symmetric, and transitive.

   (b) (1 point) It is reflexive, not symmetric, and not transitive.

(c) (1 point) It is anti-symmetric, not reflexive, and not irreflexive.

(d) (1 point) It is not symmetric and not anti-symmetric.

2. (a) (2 points) Give an example of a surjective function which does not admit a left-inverse.

(b) (2 points) Prove that if an injective function has a right-inverse, then it is a bijection.

3. (a) (2 points) Using the definitions of finite and infinite sets given in class, prove that the union of two finite sets is a finite set.

(b) (2 points) Prove, by induction, that

$$A_0, A_1, \cdots, A_{n+1}$$

are finite sets, then their union is also a finite set.

4. Let $\mathbb{N}^*$ denote the set of non-zero natural numbers. Define a relation $D \subset \mathbb{N}^* \times \mathbb{N}^*$ as follows:

$$D = \{(a,b) \in \mathbb{N}^* \times \mathbb{N}^* | \ \exists \ k \in \mathbb{N}^*, b = ka\},$$

that is, aDb if and only if a divides b.

(a) (1 point) Prove that this relation is reflexive.

(b) (1 point) Prove that this relation is anti-symmetric.

(c) (1 point) Prove that this relation is transitive.

(d) (1 point) Does this ordering have the comparability property? Justify your answer.

5. In each of the following problems, a statement about sets is given. Determine whether the statement is true or false. If true, prove it, if false, give a counter example.

   (a) (1 point) If A and B are infinite sets, then $A \cap B$ is an infinite set.

   (b) (1 point) If A is infinite and B is finite, then $A \cap B$ is finite.

   (c) (1 point) If A is infinite and B is finite, then $A - B$ is infinite.

   (d) (1 point) If A is infinite, and B is finite, $A \cup B$ is infinite.