MAT 132
Summer II 2019
Quiz 1
07/15/19
Time Limit: 50 minutes

Name (Print): ____________________

ID number ____________________

Instructions

- This exam contains 7 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam (e.g., smartphones, smartwatches, tablets). You may not use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	2	
2	7	
3	7	
4	4	
Total:	20	

1. (2 points) Express the area under the graph of the function $f(x) = e^{3x}$, in the region $0 \leq x \leq 2$, as a limit of Riemann sums. Clearly indicate the choice of sampling points, and the width of the subintervals (that is, do not simply write x_i^* and Δx, respectively).

   Solution: We assume a subdivision with n rectangles, and will use right-endpoints as the sampling points. The range of integration has lentgh 2, so the width of each rectangle is

   $$\Delta x = \frac{2}{n}.$$

   The sampling points are

   $$x_i = \frac{2i}{n},$$

   for $1 \leq i \leq n$.

   The area can be expressed as the following Riemann sum

   $$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} e^{\left(\frac{6i}{n}\right)} \frac{2}{n}.$$

2. Compute the indefinite integrals below:

   (a) (3 points)

$$\int \sin(x)\cos(\cos(x))\ dx$$

Solution: Make the substitution

$$\begin{aligned} u &= \cos(x), \\ du &= -\sin(x)dx. \end{aligned}$$

The integral can be computed in terms of the new variable as

$$\begin{aligned} \int \sin(x)\cos(\cos(x))\ dx &= -\int \cos(u)\ du \\ &= -\sin(u) + C \\ &= -\sin(\cos(x)) + C. \end{aligned}$$

(b) (4 points)

$$\int \frac{x-3}{x(x^2+4)}\,dx$$

Solution: The partial fractions decomposition of the integrand is of the form

$$\frac{x-3}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}.$$

Comparing the fractions, we get the following system of equations

$$\begin{cases} A+B &= 0 \\ C &= 1 \\ 4A &= -3, \end{cases}$$

whose solution is $A = -\frac{3}{4}$, $B = 1$, $C = \frac{3}{4}$.
It follows that

$$\begin{aligned}\int \frac{x-3}{x(x^2+4)}\,dx &= -\left(\int \frac{3}{4x}\,dx\right) + \left(\int \frac{\frac{3x}{4}+1}{x^2+4}\,dx\right) \\ &= -\frac{3\ln(|x|)}{4} + \frac{1}{4}\int \frac{3x+4}{x^2+4}\,dx \\ &= -\frac{3\ln(|x|)}{4} + \frac{3}{4}\left(\int \frac{x}{x^2+4}\,dx\right) + \left(\int \frac{1}{x^2+4}\,dx\right) \qquad (1)\end{aligned}$$

To solve the remaining integrals, we use the substitutions $u = x^2+4$, and $x = 2\tan(\theta)$, respectively:

$$\begin{aligned}\int \frac{x}{x^2+4}\,dx &= \int \frac{1}{2u}\,du \\ &= \frac{\ln(|u|)}{2} \\ &= \frac{\ln(x^2+4)}{2};\end{aligned}$$

$$\begin{aligned}\int \frac{1}{x^2+4}\,dx &= \int \frac{2\sec^2(\theta)}{4\tan^2(\theta)+4}\,d\theta \\ &= \int \frac{1}{2}\,d\theta \\ &= \frac{\theta}{2} \\ &= \frac{1}{2}\arctan(\frac{x}{2}).\end{aligned}$$

Combining it all into equation (1), we obtain

$$\int \frac{x-3}{x(x^2+4)}\,dx = -\frac{3\ln(|x|)}{4} + \frac{3\ln(x^2+4)}{8} + \frac{\arctan(\frac{x}{2})}{2} + C$$

3. Compute the definite integrals below:

   (a) (3 points)

$$\int_1^2 \frac{\ln(x)}{x^2}\ dx$$

Solution: By integration by parts, an antiderivative of the integrand is given by

$$\begin{aligned}\int \frac{\ln(x)}{x^2}\ dx &= -\frac{\ln(x)}{x} + \int \frac{1}{x^2}\ dx \\ &= \frac{\ln(x)}{x} - \frac{1}{x}.\end{aligned}$$

Applying the Net Change Theorem we get,

$$\begin{aligned}\int_1^2 \int \frac{\ln(x)}{x^2}\ dx &= -\left(\frac{\ln(x)}{x} - \frac{1}{x}\right)\Big|_1^2 \\ &= -\frac{\ln(2)}{2} + \frac{1}{2}.\end{aligned}$$

(b) (4 points)

$$\int_0^{\frac{\pi}{4}} (\sin(x))^3(\cos(x))^2 \ dx$$

Solution: In order to simplify the integrand, we use the identity

$$\sin^2(x) = 1 - \cos^2(x).$$

An antiderivative can be found by the substitution $u = \cos(x)$,

$$\begin{aligned}
\int (\sin(x))^3(\cos(x))^2 \ dx &= \int \sin(x)(1-\cos^2(x))\cos^2(x) \ dx \\
&= -\int (1-u^2)u^2 \ du \\
&= \int u^4 - u^2 \ du \\
&= \frac{u^5}{5} - \frac{u^3}{3} \\
&= \frac{\cos^5(x)}{5} - \frac{\cos^3(x)}{3}.
\end{aligned}$$

Using the Net Change Theorem, we get

$$\int_0^{\frac{\pi}{4}} (\sin(x))^3(\cos(x))^2 \ dx = \left(\frac{\sqrt{2}}{40} - \frac{\sqrt{2}}{12}\right) - \left(\frac{1}{5} - \frac{1}{3}\right).$$

4. (4 points) Find the derivative of the function $f(x)$, defined by the integral below

$$f(x) = \int_0^{x^2+2x+1} \sqrt{1+3s}\; ds$$

Solution: We use the chain rule and the fundamental theorem of Calculus,

$$f'(x) = \sqrt{1+3(x^2+2x+1)}(2x+2).$$