MAT 203 **Name (Print):** ____________________
Summer I 2020
Midterm Practice
06/11/20
Time Limit: 3 hours and 25 minutes ID number ____________________

Instructions

- This exam contains 10 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- This is an open-book exam. You may not use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	30	
2	20	
3	10	
4	20	
5	20	
Total:	100	

1. In each of the following problems, determine if the statements are true or false. Explain your reasoning (correct answers without an explanation will be worth only 2 points per statement).

   (a) (5 points) Two vectors in space always determine a unique plane.

   (b) (5 points) Two planes in space always intersect.

   (c) (5 points) The dot product can be used to detect whether two non-zero vectors in space are aligned.

   (d) (5 points) Let $r(t)$ and $s(t)$ be curves in the plane, such that neither has a limit as t converges to 0. Then their cross product $r(t) \times s(t)$ does not have a limit at 0 either.

   (e) (5 points) If all directed limits a scalar-valued, multivariable function at a point exist and coincide, then the function has a limit at the point, in the multivariable sense.

   (f) (5 points) If a scalar-valued, multivariable function is separately continuous with respect to each variable, then it is continuous in the multivariable sense.

2. Consider the lines whose parametric equations are given by

$$L_1\colon x = 2t, y = 4t, z = 6t.$$
$$L_2\colon x = 1 - s, 4 + s, -1 + s.$$

(a) (5 points) Write symmetric equations for each line

(b) (5 points) Explain why these lines do not intersect.

(c) (5 points) Explain why these lines are not parallel.

(d) (5 points) Find the general equations of two parallel planes, Π_1 and Π_2, containing lines L_1 and L_2, respectively.

3. (10 points) Compute the trajectory of a curve whose velocity vector is given by

$$r'(t) = e^{-t} i + t^2 j + \frac{1}{1+t^2} k,$$

and such that $r(0) = (1, 1, 1)$.

4. Consider the function

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

(a) (5 points) Where are the directed limits of the function along lines $y = kx$, as $x \to 0$?

(b) (5 points) Is this function continuous at the origin?

(c) (5 points) What is the partial derivative of this function relative to x, at points other than the origin?

(d) (5 points) What is the partial derivative of this function relative to y, at points other than the origin?

5. Recall that the polar coordinate system in the plane is related to Cartesian coordinates by means of the equations

$$x = r\cos(\theta), \tag{1}$$
$$y = r\sin(\theta). \tag{2}$$

(a) (6 points) Differentiate equation (1) relative to x to find a relation between $\frac{\partial r}{\partial x}$ and $\frac{\partial \theta}{\partial x}$.

(b) (6 points) Differentiate equation (2) relative to x to find a second relation between $\frac{\partial r}{\partial x}$ and $\frac{\partial \theta}{\partial x}$.

(c) (8 points) By solving the system of equations obtained in the previous two steps, compute the derivatives $\frac{\partial r}{\partial x}$ and $\frac{\partial \theta}{\partial x}$.