MAT 132 **Name (Print):** _____
Summer II 2016
Final Exam
08/18/16
Time Limit: 3 hours and 5 minutes ID number _____

Instructions

- This exam contains 12 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam (e.g., smartphones, smartwatches, tablets). You may *not* use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	20	
Total:	80	

1. Use geometrical interpretation to compute the following integrals.

   (a) (3 points)

$$\int_0^5 (x+2)dx$$

   (b) (3 points)

$$\int_{-\pi}^{\pi} (\sin x)^3 dx$$

   (c) (4 points)

$$\int_0^2 [\sqrt{4-x^2}+1]dx$$

2. Use the Fundamental Theorem of Calculus to solve the following problems. Clearly state which part of the FTC you are using in your solution.

   (a) (5 points) Describe all the continuous functions $f : [0, 1] \longrightarrow \mathbb{R}$ with the following properties:

   1. $f(0) = 0$, $f(1) = \frac{1}{2}$;
   2. $f'(x) < \frac{1}{4}$ for every $x \in (0, 1)$.

   If no such functions exist, explain why.

   (b) (5 points) Let $f(x)$ be a continuous function defined on the interval $[0, +\infty)$, satisfying the following equation:

$$x \ln(x^2 + 1) = \int_1^{x^2} f(t)dt.$$

   Find the value of $f(1)$.

3. Determine if the following improper integrals are convergent or divergent. (Hint: you might not need to evaluate the integrals to answer the problem).

   (a) (5 points)

$$\int_0^\infty xe^{-x^2}dx$$

(b) (5 points)

$$\int_1^\infty \frac{1}{x^2+1}dx$$

4. This question will ask you to compute the volume of an "infinite" solid of revolution.

   (a) (2 points) Sketch the region bounded by the curves $y = \frac{1}{x}$, $y = \frac{1}{x^2}$ and $x = 1$ in the xy-plane.

   (b) (3 points) A solid is obtained by rotating the region described in part (a) around the x-axis. Set up an improper integral that represents the volume of this solid. (You don't need to evaluate the integral in this part.)

(c) (5 points) Show that this solid has finite volume by calculating the improper integral from part (b).

5. In each of the problems below, determine if the series is convergent or not. Clearly state the convergence test used in your solution.

   (a) (3 points)

$$\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$$

   (b) (3 points)

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{4n-1}$$

(c) (4 points)

$$\sum_{n=1}^{\infty} \frac{(-1)^n \arctan(n)}{n^2}$$

6. (10 points) Find the Taylor series of $f(x) = \sin(x)\cos(x)$ at the point $a = 0$. Compute the radius of convergence of this series.

7. Solve the following initial-value problems:

   (a) (10 points)

$$(y^2 + x^2y^2)\frac{dy}{dx} = 1,\ y(0) = 1$$

(b) (10 points)

$$\frac{dy}{dx} + \frac{y}{x^2+1} = e^{-\arctan(x)},\ y(0) = 1$$