MAT 511   **Name (Print):** _____
Summer I 2019
Midterm
06/11/19
Time Limit: 1 hour and 55 minutes   ID number   _____

Instructions

- This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam (e.g., smartphones, smartwatches, tablets). You may not use a calculator.

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	4	
2	4	
3	4	
4	4	
5	4	
Total:	20	

1. (4 points) In the following riddle, identify the universe of discourse, the propositions and conditional statement involving them, and solve the riddle by syllogisms and contrapositives.

<div align="center">

"Animals, that do not kick, are always unexcitable.

Donkeys have no horns.

A buffalo can always toss one over a gate.

No animals that kick are easy to swallow.

No hornless animal can toss one over a gate.

All animals are excitable, except buffaloes.."

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2. (4 points) Use a truth-table to show that the following propositional forms

$$(p \Rightarrow q) \Rightarrow r,$$

and

$$(p \wedge \neg q) \vee r$$

are equivalent.

3. The goal of this problem is to show that $\sqrt{2}$ is an irrational number. We will use the language of predicate logic, and our domain of discourse is that of the Real numbers (with the usual operations of algebra). You may assume the following facts.

    1. (Definition of even number) An integer number n is called even if there exists an integer k for which $n = 2k$.

    2. (Definition of rational number) A real number x is called *rational* if it can be written as a quotient of two integers, a and b, such that $b \neq 0$, and a, b have no common factors.

  (a) (2 points) Show that if m is an integer number such that m^2 is even, then m is even.

  (b) (2 points) Show, by contradiction, that $\sqrt{2}$ is an irrational number.

4. (4 points) Prove that for any triple of sets A, B and C,

$$(A - B) - C = (A - C) - (B - C),$$

by using properties of set membership or reduction to propositional logic.

5. (4 points) Recall that a (non-zero) polynomial d divides another polynomial D, if there exists a third polynomial q for which the relation

$$D(x) = d(x)q(x)$$

holds for all real numbers x. In this exercise, use Mathematical Induction to show that $d(x) = x - 1$ divides the polynomials $D_n(x) = x^n - 1$, for all $n \in \mathbb{N}$.