MAT 127
Summer II 2015
Midterm
07/23/15
Time Limit: 2 hours.

Name (Print): _____

ID number _____

Instructions

- This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam(e.g., smartphones, tablets). You may *not* use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. In the practice part of the exam, a correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

1. Determine whether or not the sequences below converge, and if so, calculate their limit.

   (a) (3 points)

$$a_n = 1 + \left(\frac{-2}{e}\right)^n$$

   (b) (3 points)

$$a_n = \frac{e^m + e^{-m}}{e^{2n} - 1}$$

   (c) (4 points)

$$a_n = \frac{(2n-1)!}{(2n+1)!}$$

2. Consider the sequence defined recursively by:

$$a_1 = 2, \quad a_n = \sqrt{12 + a_n}$$

(a) (3 points) Show that this sequence is bounded above by 4.

(b) (3 points) Show that this sequence is increasing. (Hint: consider the quadratic polynomial associated to the recursive formula).

(c) (4 points) Explain why the sequence is convergent, and find its limit.

3. Calculate the values of the following series:

   (a) (5 points)

$$\sum_{n=1}^{\infty} \left(\frac{9}{10} \right)^n$$

   (b) (5 points)

$$\sum_{n=1}^{\infty} \log \left(\frac{n^2 + n}{n^2 + 3n + 2} \right)$$

4. Decide whether the series below are convergent or divergent. Explain your answers. In each case, clearly indicate what convergence test you used.

   (a) (3 points)

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{\pi}{n}\right)}{n^2}$$

   (b) (3 points)

$$\sum_{n=1}^{\infty} \frac{\log n}{n}$$

   (c) (4 points)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$$

5. This question is related to the following power series:

$$\sum_{n=1}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$$

(a) (5 points) Calculate the radius of convergence.

(b) (5 points) Describe what happens in the border line case, i.e., when $|x+2| = R$, the radius of convergence you found on part (a).

6. Express the following functions as power series centered at the points indicated.

(a) (3 points)

$$f(x) = \frac{2}{3-x}, \quad x_0 = 1.$$

(b) (3 points)

$$g(x) = \log(1+x), \quad x_0 = 0$$

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(c) (4 points)

$$h(x) = \arctan(x), \quad x_0 = 0$$

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