

Homework 2

Exercise 1 This exercise will get you acquainted with the substitution method. Clearly describe the steps in the solutions (as was done in class).

(a)

$$\int \sqrt{9 - x^2} dx$$

(b)

$$\int \frac{x}{4 + x^2} dx$$

(c)

$$\int \frac{x^2 + 4x}{x^3 + 6x^2 + 2} dx$$

(d)

$$\int (\sec x)^3 \tan x dx$$

Exercise 2 Use integration by parts on the following problems. You may need to use integration by parts more than once on the same problem. Clearly describe the steps in the solutions.

(a)

$$\int_1^2 \frac{\ln x}{x^2} dx$$

(b)

$$\int_0^\pi t \sin(3t) dt$$

(c)

$$\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx$$

(d)

$$\int_0^\pi \sin(2x) \sin(x) dx$$

Exercise 3 Use partial fractions to solve the following problems. Clearly describe the steps in the solutions

(a)

$$\int \frac{x-4}{x^2-5x+6} dx$$

(b)

$$\int \frac{x^2+2x-1}{x^3-x} dx$$

(c)

$$\int \frac{2x^2+5}{(x^2+1)(x^2+4)} dx$$

(d)

$$\int \frac{x^2-x+6}{x^3+3x} dx$$

Exercise 4 In each of the following problems, verify if the integral is improper or not. If it is improper, check if it is convergent or divergent. If it is convergent, compute its value.

(a)

$$\int_0^{2\pi} \frac{\cos(\theta) - \frac{1}{\cos(\theta)}}{\tan^2(\theta)} d\theta$$

(b)

$$\int_0^2 \ln|x-1| dx$$

(c)

$$\int_0^\infty \frac{x}{(x^2+2)^2} dx$$

(d)

$$\int_{-\infty}^\infty \frac{e^x}{e^{2x}+4} dx$$

Exercise 5 Describe all continuous functions $f(x)$, defined on the interval $[0, 1]$, so that the following properties hold:

- $f(0) = 0$
- $f(1) = 1$
- $f'(x) \geq 1$, for all $x \in (0, 1)$.

Explain your answer.