MAT 127 **Name (Print):** _____
Summer II 2015
Final Exam
08/13/15
Time Limit: 3 hours and 25 minutes. ID number _____

Instructions

- This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam(e.g., smartphones, tablets). You may *not* use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. In the practice part of the exam, a correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

1. Determine whether or not the sequences below converge, and if so, calculate their limit.

   (a) (3 points)

$$a_n = 1 + \left(\frac{n^2}{e^n}\right)$$

   (b) (3 points)

$$a_n = \frac{\log(n)}{n}$$

   (c) (4 points)

$$a_n = \frac{\tan(n)}{n}$$

2. Decide whether the series below are convergent or divergent. Explain your answers. In each case, clearly indicate what convergence test you used.

(a) (3 points)

$$\sum_{n=1}^{\infty} \frac{1+4^n}{1+3^n}$$

(b) (3 points)

$$\sum_{n=1}^{\infty} \frac{n^{-}1}{3n^4+1}$$

(c) (4 points)

$$\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^2+1}$$

3. Match the direction fields to the differential equations. Explain your answer.

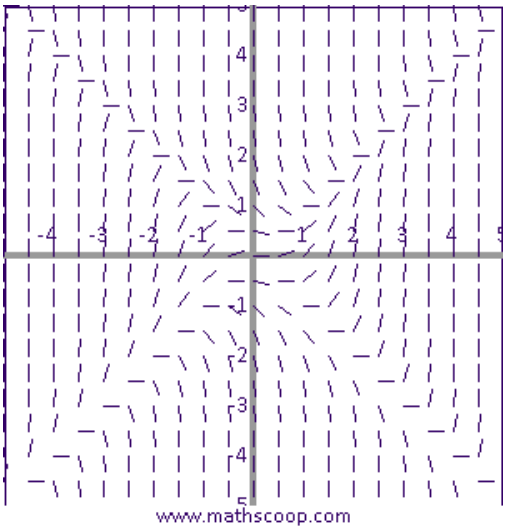

Figure 1:

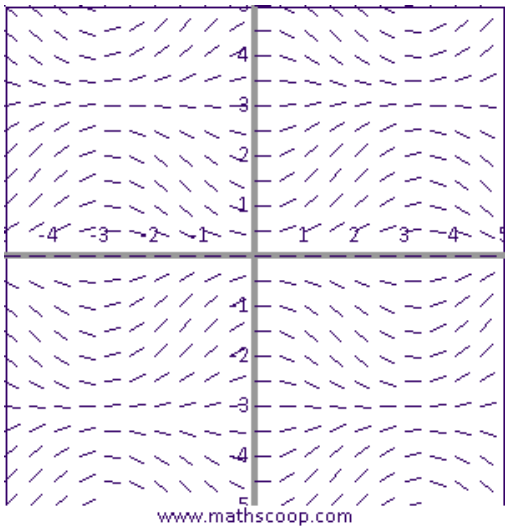

Figure 2:

(a) (5 points)

$$\frac{dy}{dx} = x^2 - y^2$$

(b) (5 points)

$$\frac{dy}{dx} = \sin(x)\sin(y)$$

4. Solve the following equations

   (a) (5 points)

$$\frac{dy}{dx} = \frac{\log(x)}{xy} \qquad y(1) = 2.$$

   (b) (5 points)

$$y(x) = 2 + \int_1^x \frac{1}{ty(t)} dt$$

5. In a murder investigation, the temperature of the corpse was $32.5°C$ at 1:30 PM and $30.3°C$ one hour later. Assume that before death the individual had normal body temperature, $37°C$. The temperature of the surroundings was $20°C$.

   (a) (5 points) Assuming that the cooling process satisfies Newton's Law of Cooling, calculate the body temperature as a function of time.

   (b) (5 points) Use your result from the previous part to estimate the time of death. Use the approximations $\log(\frac{10.3}{12.5}) = -0.084$ and $\log(\frac{17}{12.5}) = 0.134$.

6. Suppose a population $P(t)$ satisfies the following logistic equation:

$$\frac{dP}{dt} = 0.4P - 0.001P^2, \tag{1}$$

where t is measured in years.

(a) (3 points) What is the carrying capacity of this environment?

(b) (3 points) Solve the differential equation, i.e., calculate P(t).

(c) (4 points) When will the population reach 50% of its carrying capacity? Use the approximation $\log(2/7) = -0.544$.