MAT 514 - Summer II 2020
July 30, 2020

Long Quiz 3

The objective of this quiz is to use methods of complex integration to solve a real integral,

$$\int_0^{2\pi} \frac{1}{2+\sin(\phi)}\, d\phi.$$

The problems below will guide you through the solution.

Problem 1 By expressing the sine function as a combination of complex exponentials, rewrite the integrand as a function of $e^{i\phi}$.

Problem 2 Use the substitution $z = e^{i\phi}$ to turn the real-valued integral into a line integral of a rational function in the complex variable z. Your answer should take the form

$$\int_{C[0,1]} \frac{A}{p(z)}\, dz,$$

where A is a constant and $p(z)$ is a quadratic polynomial on z.

Problem 3 Factor the polynomial $p(z)$ found in problem 2. Write the integrand from problem 2 as a sum of partial fractions whose denominators have degree one.

Problem 4 Use Cauchy's Theorem and Cauchy's integral formula to solve the integral

$$\int_0^{2\pi} \frac{1}{2+\sin(\phi)}\, d\phi.$$

via the method developed in problems 1 through 3.