Math 118 **Name (Print):** _____
Summer I 2015
Final Exam
7/02/15
Time Limit: 3 hours and 25 minutes. ID number _____

Instructions

- This exam contains 16 pages (including this cover page) and 15 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam(e.g., smartphones, tablets). You are allowed to use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. In the practice part of the exam, a correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- **You must solve problems 1 to 5**. **You must choose only one problem in each of the following pairs: (6,7), (8,9), (10,11), (12,13), (14,15)**. Clearly indicate the problems you want graded by circling the respective number on the "Problem" column on the table to the right. **Do not** write on the "Score" column of the table. The total score for each student is 100.

Problem	Points	Score
1	8	
2	8	
3	8	
4	8	
5	8	
6	12	
7	12	
8	12	
9	12	
10	12	
11	12	
12	12	
13	12	
14	12	
15	12	
Total:	160	

1. (The Mathematics of Social Choice)

   (a) (1 point) Describe the divider-chooser Method.

   (b) (1 point) In the Banzhaf power model, what is the critical count of a player?

   (c) (3 points) Consider the weighted voting system $[q; 8, 4, 1]$. About this system, answer the following:

      a) What are the values of q for which we don't have a gridlock or anarchy?

      b) If $q = 10$, how many players have veto power?

      c) If $q = 11$ what is the critical count of player 2 (the one with weight 4)?

   (d) (3 points) Describe the lone-divider method with 3 players. You may use pictures to support your explanation, but an explanation consisting only of pictures will not be accepted.

2. (Management Science)

(a) (1 point) What are the conditions a graph should satisfy for the existence of an Euler path? And for an Euler circuit?

(b) (1 point) What is the redundancy of a tree?

(c) (3 points) Recall that K_n denotes a complete graph with n vertices.

a) How many egdes are there in K_{12}?

b) How many edges are there in K_{13}?

c) What is the degree of a vertex in K_{35}?

(d) (3 points) Describe Kruskal's Algorithm for finding a MST on a network (i.e., list all of the steps in order).

3. (Growth)

(a) (2 points) In the Logistic Growth model, what is the carrying capacity of an environment?

(b) (3 points) What is the difference between simple and compound interest?

(c) (3 points) Describe a situation in which the linear growth model applies and gives reasonable results. Explain your reasoning.

4. (Geometry)

   (a) (2 points) Can the number of fixed points of an isometry distinguish between a translation and a glide reflection?

   (b) (3 points) What are the symmetry types of the following plane figures:

   a) A square.
   b) A circle.
   c) A hexagon.

   (c) (3 points) How can you distinguish a 180 degrees rotation from a reflection?

5. (Statistics)

(a) (1 point) What is a discrete variable?

(b) (1 point) What is the Placebo Effect?

(c) (3 points) Describe, either by a formula or using words, how do we calculate the following representatives of a data set:

a) The average.

b) The Median.

c) The standard variation.

(d) (3 points) Describe the difference between a permutation, an arrangement, and a combination.

6. (Fairness Criteria) Consider the preference schedules represented by tables A and B below:

Table 1: Table A

Number of voters	6	2	3
1st	A	B	C
2nd	B	C	D
3rd	C	D	B
4th	D	A	A

Table 2: Table B

Number of voters	49	48	3
1st	R	H	F
2nd	H	S	H
3rd	F	O	S
4th	O	F	O
5th	S	R	R

(a) (3 points) Find the winner of the election in Table A under the Borda count Method.

(b) (3 points) Use part (a) to show that the Borda count method violates the Condorcet condition.

(c) (3 points) Find the winner of the election in table B under the plurality method.

(d) (3 points) Eliminate candidate F in the election on table B. Find the winner under the plurality method. Based on your results, does the plurality method satisfy the independence of irrelevant alternatives criterion? Explain.

7. (Power measures) Consider the weighted voting system $[16 : 9, 8, 7]$.

   (a) (6 points) List all the winning coalitions and find the Banzhaf power index of each player.

   (b) (6 points) List all the sequential coalitions, indicate the pivotal players and calculate the Shapley-Shubik power index of each player.

8. (Finding Hamilton Circuits) A sales representative must visit 6 cities in his route: Atlanta, Columbus, Kansas City, Minneapolis, Pierre and Tulsa. Because he arrives and leaves the United States through Atlanta airport, that should be the starting and endpoint of the trip. Naturally, the sales rep is allowed to travel between any two cities. The distances between these cities are listed in Mileage Chart below.

	Atlanta	Columbus	Kansas City	Minneapolis	Pierre	Tulsa
Atlanta	*	533	798	1068	1361	772
Columbus	533	*	656	713	1071	820
Kansas City	798	656	*	447	592	248
Minneapolis	1068	713	447	*	394	695
Pierre	1361	1071	592	394	*	760
Tulsa	772	820	248	695	760	*

(a) (6 points) Find the tour (listing the vertices in the order they are travelled) obtained by using the cheapest-link algorithm and calculate its total distance.

(b) (6 points) Find the tour obtained by applying the Nearest-Neighbor Algorithm and calculate its total distance.

9. (Scheduling) A project is divided into 8 tasks: A(8), B(5), C(9), D(12), E(6), F(1), G(2). As usual, the notation X(t) is to indicate that project X takes t units of time to be completed. The tasks satisfy the following list of precendence relations:

   1. A and B must be completed before D;
   2. D must be completed before F;
   3. C must be completed before E;
   4. E must be completed before G;

   (a) (4 points) Draw the project digraph indicating the precedence relations. Display the START and END tasks.

   (b) (4 points) Construct the Decreasing-Time priority list, and the schedule obtained using this method with 3 processors.

   (c) (4 points) Find the critical-times of all vertices, and the critical path of this task list. Construct the Critical-Time priority list and schedule the project using this method with 3 processors.

10. (The logistic model) A population of a variety of fish in a lake grows according to the logistic model, with growth parameter $r = 1.5$. A fish-farmer decides to start a project in this lake and wants to study the viability of such project.

    (a) (3 points) If the initial p-value of the population was 0.4, calculate the p-values for the first 10 generations of the population. Display your result with 3 decimals.

    (b) (3 points) If the initial p-value was 0.8, calculate the p-values for the first 10 generations of the population. Display your results with 3 decimals.

    (c) (3 points) Describe the long term behavior of the population of fish in the lake in both situations.

    (d) (3 points) Interpret the result of part c in terms of cost-efficiency of the fish-culture.

11. (Interest) Consider buying a Certificate of Deposit (CD)with 3.6% APR, compounded quarterly.

(a) (3 points) Convert the rate to a quarterly rate. Round your answer to 4 decimals.

(b) (6 points) Find the future value of the CD if you invest $1500 for a term of 3 quarters. Round your answer to the nearest penny.

(c) (3 points) What would be the necessary rate to obtain the same result as in part B in the same time using simple interest instead of compound interest? Express the rate as an APR and compare with the APR given in the statement of the problem.

12. (Non-Commutativeness of isometries)

(a) (6 points) Construct an example that shows that the result of performing a translation and a rotation depends on the order in which the operations are performed.

(b) (6 points) Construct an example that shows that the result of performing a reflection and a rotation depends on the order in which the operations are performed.

13. (Border Patterns) In each of the border patterns below, identify the symmetries in the picture. If there is a translation symmetry, indicate the vector, with appropriate size, direction and orientation. If there is a reflection symmetry, indicate the line of reflection. If there is a rotation symmetry, indicate the rotocenter. If there is a glide reflection, indicate the translation vector and the line of reflection.

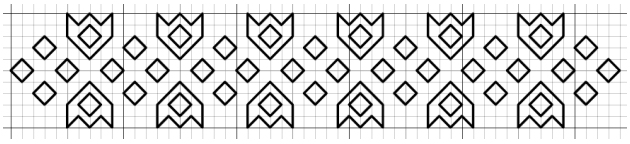

Figure 1:

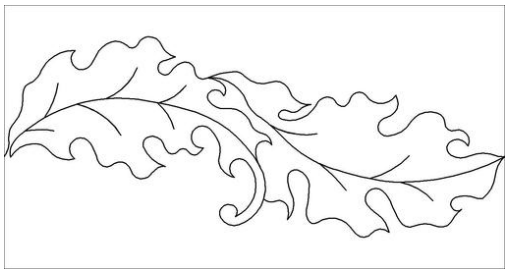

Figure 2:

(a) (6 points) Describe the symmetries of Figure 1.

(b) (6 points) Describe the symmetries of Figure 2.

14. (Distribution of scores in a Math quiz) The following table shows the frequencies of the scores of a group of students that took the Math quiz.

Score	3	4	5	6	7	8	9
Frequency	8	12	16	20	18	14	12

(a) (6 points) Compute the five number summary of this data set.

(b) (6 points) Compute the variance and standard deviation of this data set.

15. (Coin Tossing) An honest coin is tossed three times in a row. Find the probability of each of the following events.

(a) (3 points) The coin comes up heads exactly twice.

(b) (3 points) All three tosses come up the same.

(c) (3 points) Half of the tosses come up heads.

(d) (3 points) The first two tosses come up tails.