MAT 203
Summer II 2018
Midterm
07/26/18
Time Limit: 3 hours and 25 minutes

Name (Print): ____________________

ID number ____________________

Instructions

- This exam contains 9 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.
- You may *not* use your books, notes, or any device that is capable of accessing the internet on this exam (e.g., smartphones, smartwatches, tablets). You may not use a calculator.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**.

Problem	Points	Score
1	30	
2	20	
3	10	
4	20	
5	20	
Total:	100	

1. In each of the following problems, determine if the statements are true or false. Explain your reasoning (correct answers without an explanation will be worth only 2 points per statement).
   (a) (5 points) Two vectors in space always determine a unique plane.

   (b) (5 points) Two planes in space always intersect.

   (c) (5 points) The dot product can be used to detect whether two non-zero vectors in space are aligned.

   (d) (5 points) Let $r(t)$ and $s(t)$ be curves in the plane, such that neither has a limit as t converges to 0. Then their cross product $r(t) \times s(t)$ does not have a limit at 0 either.

   (e) (5 points) If all directed limits a scalar-valued, multivariable function at a point exist and coincide, then the function has a limit at the point, in the multivariable sense.

   (f) (5 points) If a scalar-valued, multivariable function is separately continuous with respect to each variable, then it is continuous in the multivariable sense.

2. Consider the lines whose parametric equations are given by

$$L_1\colon x = 2t, y = 4t, z = 6t.$$
$$L_2\colon x = 1 - s, 4 + s, -1 + s.$$

(a) (5 points) Explain why these lines do not intersect.

(b) (5 points) Explain why these lines are not parallel.

(c) (5 points) Find the general equations of two parallel planes, Π_1 and Π_2, containing lines L_1 and L_2, respectively.

(d) (5 points) What is the distance between the lines L_1 and L_2.

3. (10 points) Compute the trajectory of a curve whose velocity vector is given by

$$r'(t) = e^{-t}i + t^2 j + \frac{1}{1+t^2}k,$$

and such that $r(0) = (1, 1, 1)$.

4. Consider the function

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

(a) (5 points) Where are the directed limits of the function along lines $y = kx$, as $x \to 0$?

(b) (5 points) Is this function continuous at the origin?

(c) (5 points) What is the partial derivative of this function relative to x, at points other than the origin?

(d) (5 points) What is the partial derivative of this function relative to y, at points other than the origin?

5. Consider a model of gravitation in which two bodies, A and B, exert gravitational pull on one another. Assume that the mass of A is much bigger than that of B, so that the acceleration felt by A is negligible, and we can regard it as in rest (this is a good approximation for the orbital motion of artificial satelites around the Earth, for instance). We are going to consider our reference frame for 3-space as centered at A, and describe the position of B by a vector-valued function of time, $r(t)$.

   Newton's Law of Universal Gravitation asserts that the gravitational force exerted by A upon B is of the form
   $$F = \frac{K}{||r(t)||^3} r(t),$$
   where K is a (negative) constant depending on the masses of A and B. Meanwhile, Newton's Second Law of Motion says that the acceleration $r''(t)$ felt by B as a result of the gravitational pull from A can be computed by the well-known formula
   $$F = mr''(t),$$
   where m is the mass of B (a constant).

   (a) (10 points) By comparing the two expressions for the force, explain why the cross product of position and velocity vectors, $r(t) \times r'(t)$, is constant.

(b) (10 points) Use part (a) to explain why the position vector $r(t)$ belongs to a fixed plane, for all t.