Homework 3

Exercise 1 Sketch the region enclosed by the following curves. Find the intersection points of the curves. Use integrals to compute the area of the region.

(a) $y = 12 - x^2$ and $y = x^2 - 6$

(b) $y = \cos x$, $y = xe^x$ and $x = 0$.

(c) $y = \sin(x)$, $y = \cos(x)$, $x = 0$ and $x = \frac{\pi}{2}$ (Notice that the region enclosed by these 4 curves has two separate parts.)

Exercise 2 Compute the volumes of the following solids:

(a) The solid obtained by rotating the region bounded by the curves $y = x^3$, $y = \sqrt{x}$ and $x \geq 0$, about the y-axis.

(b) The solid obtained by rotating the region bounded by the curves $x = 2y - y^2$ and $x = 0$, about the $y - axis$.

(c) The solid obtained by rotating the region bounded by the curves $x = 1 + y^2$, $x = 0$, $y = 1$ and $y = 2$, about the $x - axis$. (Hint: Use cylindrical shells).

Exercise 3 Determine whether or not the sequences below converge, and if so, calculate their limit.

(a)
$$a_n = 1 + \left(\frac{-2}{e}\right)^n$$

(b)
$$a_n = \frac{e^m + e^{-m}}{e^{2n} - 1}$$

(c)
$$a_n = \frac{(2n-1)!}{(2n+1)!}$$

Exercise 4 Calculate the values of the following series:

(a)
$$\sum_{n=1}^{\infty} \left(\frac{9}{10}\right)^n$$

(b)

$$\sum_{n=1}^{\infty} \log\left(\frac{n^2+n}{n^2+3n+2}\right)$$

Exercise 5 Decide whether the series below are convergent or divergent. Explain your answers. In each case, clearly indicate what convergence test you used.

(a)

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{\pi}{n}\right)}{n^2}$$

(b)

$$\sum_{n=1}^{\infty} \frac{\log n}{n}$$

(c)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$$