Derivadas Logarítmicas

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$f(x) = \ln(\ln^2 x) \Rightarrow \ln(\ln x)^2 \Rightarrow 2(\ln|\ln x|)$

$\Rightarrow f' = 2\left(\frac{1}{\ln x}\right) \cdot \cos = 2\cot x$

5) $f(x) = \sqrt[5]{\ln(x)}$

$f' = \frac{1}{5}(\ln x) \Rightarrow f' = \frac{1}{5x}$

9) $g(x) = \ln(x e^{-2x})$

11) $f(t) = (\ln t^3)\ln t$

$F' = 2(\ln t)\left(\frac{1}{t}\right)\ln t + (\ln t)^2(\cos t)$

$= \ln t\left(\frac{2}{t}\ln t + \ln t \cos t\right)$

$f'(t) = \ln t\left(\frac{2\ln t}{t} + \ln t \cos t\right)$

e) $f(x) = \log_{\sqrt{x}} x$ $\quad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

$$f'(x) = \frac{1}{\sqrt{x}\ln\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$$

$$f' = \frac{1}{2} \cdot \frac{1}{\sqrt{x}\ln\sqrt{x}} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2x\ln\sqrt{x}}$$

f) $f(x) = \log_{10}(\cos x)$

$$f' = \frac{1}{(\cos x)\ln 10}(-\sin x) = \frac{-\sin x}{(\cos x)\ln 10}$$

g) $f(x) = \frac{x}{1-\ln(x-1)}$

$$f' = \frac{1(1-\ln(x-1)) - x\left(0 - \frac{1}{x-1}\right)}{[1-\ln(x-1)]^2} =$$

$$f' = \frac{(x-1)[1-\ln(x-1)] + x}{(x-1)[1-\ln(x-1)]^2}$$

$$f' = \frac{(-1)(1-\ln(x-1)) + x}{(x-1)(1-\ln(x-1))^2} = \frac{1-x+(x-1)\ln(x-1)}{(x-1)(1-\ln(x-1))^2}$$

28) $f(x) = \sqrt{2 + \ln x}$

$f'(x) = \frac{1}{2(2+\ln x)^{\frac{1}{2}}} \cdot \left(0 + \frac{1}{x}\right)$

$f'(x) = \frac{1}{2x\sqrt{2+\ln(x)}}$

$[x \mid x > 0, \; 2 + \ln(x) > 0]$

$\ln(x) > -2$

$e^x > e^{-2}$

$[x \mid x > 0, \; x > e^{-2}] \Rightarrow (e^{-2}, \infty)$

29) $f(x) = \ln(x^2 - 2x)$

$f' = \frac{1}{x^2 - 2x}(2x - 2) \Rightarrow \frac{2x-2}{x^2-2x} \Rightarrow \frac{2(x-1)}{x(x-2)}$

$[x \mid x > 0, \; x - 2 \neq 0]$

$[x \mid x > 0, \; x \neq +2] \Rightarrow (-\infty, -2) \cup$

$(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$

$(-\infty, 0) \cup (0, 2) \cup (2, \infty) = \mathbb{R} \; (0, 2)$

$\mathbb{R} - (0, 2)$

$(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

33) $f(x) = \ln(x + \ln x)$

$$f' = \frac{1}{x + \ln x} \cdot \left(1 + \frac{1}{x}\right)$$

$$f'(1) = \frac{1}{1 + \ln 1} \cdot \left(1 + \frac{1}{1}\right) = 2$$

34) $y = x^2 \ln x \to (1, 0)$

$$y' = 2x(\ln x) + x^2 \cdot \frac{1}{x} = 2x(\ln x) + x$$

$$f'(1) = 2x(\ln x) + x^2 \cdot \frac{1}{x}$$

$$f' = 2x \ln x + x = 1$$

$$y - 0 = 1(x - 1)$$

$$y = x - 1$$

42) $f(x) = \sqrt{x}\, e^{x^2 - x} (x+1)^{2/3}$

$$\ln(y) = \ln\left(\sqrt{x}\, e^{x^2 - x} (x+1)^{2/3}\right)$$

$$\frac{1}{y} y' = \frac{1}{2} \ln(x) + x^2 - x + \frac{2}{3} \ln(x+1)$$

$$\frac{1}{y} y' = \frac{1}{2x} + (2x - 1) + \frac{2}{3} \frac{1}{x+1}$$

→

42) $\frac{1}{y} y' = \frac{1}{2x} + \frac{2x-1}{} + \frac{2}{3(x+1)}$

$y' = y(u)$

$y' = \left(\sqrt{x}\, e^{x^2-x} (x+1)^{2/3}\right)\left(\frac{1}{2x} + \frac{2x-1}{} + \frac{2}{3(x+1)}\right)$

38) Sea $f(x) = \log_b (3x^2 - 3)$

$f' = \frac{c\,1}{b \ln(3x^2-2)} \cdot (6x)$

$3 = \frac{(6x)^2 \cdot 1}{b \ln 3x^2-2} \Rightarrow b \ln(3x^2-2) = \frac{(6x)^2}{3}$

$3 = \frac{6}{\ln b} \Rightarrow$

$3' \ln b = \frac{2}{e}$

$b = e^2$

$b = e^2$