A certain process for manufacturing integrated circuits has been in use for a period of time, and it is known that 12% of the circuits it produces are defective. A new process that is supposed to reduce the proportion of defectives is being tested. In a simple random sample of 100 circuits produced by the new process, 12 were defective.

a. One of the engineers suggests that the test proves that the new process is no better than the old process, since the proportion of defectives in the sample is the same. Is this conclusion justified? Explain.

b. Assume that there had been only 11 defective circuits in the sample of 100. Would this have proven that the new process is better? Explain.

c. Which outcome represents stronger evidence that the new process is better: finding 11 defective circuits in the sample, or finding 2 defective circuits in the sample?

Answer:

a. The conclusion is not justified, having the same results regarding defective circuits is not a conclusion that the process is not better, also 100 samples is a small number so the result may vary.
b. Not necessarily, there would be only a difference of 11% to 12% between the new and old method, it is not a very significant difference.
c. Finding 2 defective circuits is even more evidence that the new process is better than the old one, in this case it would be a difference of 2% to 12% which does demonstrate a more significant improvement

A medical researcher wants to determine whether exercising can lower blood pressure. At a health fair, he measures the blood pressure of 100 individuals, and interviews them about their exercise habits. He divides the individuals into two categories: those whose typical level of exercise is low, and those whose level of exercise is high.

a. Is this a controlled experiment or an observational study?

b. The subjects in the low exercise group had considerably higher blood pressure, on the average, than subjects in the high exercise group. The researcher concludes that exercise decreases blood pressure. Is this conclusion well-justified? Explain.

Answer:

a. This is an observational study because the researcher is just observing the blood pressure of the people but not trying to control or manipulate it.
b. The conclusion isn’t well-justified because this information is not enough, there are several factors that must be analyzed first such as the age of the people, their diet, and even with a small group, is hard to get that kind of conclusions

A sample of 100 cars driving on a freeway during a morning commute was drawn, and the number of occupants in each car was recorded. The results were as follows:

| Occupants | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of Cars | 70 | 15 | 10 | 3 | 2 |

a. Find the sample mean number of occupants.
b. Find the sample standard deviation of the number of occupants.
c. Find the sample median number of occupants.
d. Compute the first and third quartiles of the number of occupants.
e. What proportion of cars had more than the mean number of occupants?
f. For what proportion of cars was the number of occupants more than one standard deviation greater than the mean?
g. For what proportion of cars was the number of occupants within one standard deviation of the mean?

Answer:

a. μ = (1/100) * (1 * 70 + 2 * 15 + 3 * 10 + 4 * 3 + 5 * 2) = 2.21
b.
c. Median = (2+3)/2 = 2.5
d.
e. P (X > 2.21) = (1/100) * (15 + 10 + 3 + 2) = 0.3
f. P (X > 2.21 + 1.07) = (1/100) * (10 + 3 + 2) = 0.15
g.

There are 10 employees in a particular division of a company. Their salaries have a mean of $70,000, a median of $55,000, and a standard deviation of $20,000. The largest number on the list is $100,000. By accident, this number is changed to $1,000,000.

a. What is the value of the mean after the change?
b. What is the value of the median after the change?
c. What is the value of the standard deviation after the change?

Answer:

a. $70,000 + ($1,000,000 - $100,000)/10 = $81,000 the new mean value is $81.000
b. The median salary is still $55,000
c.

A section of an exam contains four True-False questions. A completed exam paper is selected at random, and the four answers are recorded.

a. List all 16 outcomes in the sample space.

b. Assuming the outcomes to be equally likely, find the probability that all the answers are the same.

c. Assuming the outcomes to be equally likely, find the probability that exactly one of the four answers is "True."

d. Assuming the outcomes to be equally likely, find the probability that at most one of the four answers is "True."

Answer:

a. TTTT, TTTF, TTFT, TFFT, FTFT, FTTF, FFTT, TTFC, TTCF, TFCF, TFFC, TCFT, CFTF, CFTC, CFCT
b. 2 / 16 = 0.125 This is the number of outcomes in the same answer (TTTT and FFFF) divided by the total number of outcomes (16)
c. 4 / 16=0.25 The number of outcomes with one true answer (TFFF, FFTF, FFFT, FTFF) divided by the total number of outcomes (16)
d. 10 / 16= 0.625 The number of outcomes with at most one true answer (TFFF, FFTF, FFFT, FTFF, TFFF, TTFF, TFTF, TFTT, TTTF, TTTF divide by the total number of outcomes (16)

A computer password consists of eight characters.

a. How many different passwords are possible if each character may be any lowercase letter or digit?

b. How many different passwords are possible if each character may be any lowercase letter or digit, and at least one character must be a digit?

c. A computer system requires that passwords contain at least one digit. If eight characters are generated at random, and each is equally likely to be any of the 26 letters or 10 digits, what is the probability that a valid password will be generated?

Answer:

a. There are 26 lowercase letters and 10 digits so it's 36^8 = 2,821,109,907,456
b. The number of passwords without any digits would be just 26 so the number is 26^8 - 36^8 = 2,814,749,767,104
c. To find the probability its necessary divide the Number of passwords with at least one digit / Total number of possible passwords = (36^8 - 26^8) / 36^8 ≈ 0.9346, so the probability is approximately 93.46%

At a certain college, 30% of the students major in engineering, 20% play club sports, and 10% both major in engineering and play club sports. A student is selected at random.

a. What is the probability that the student is majoring in engineering?

b. What is the probability that the student plays club sports?

c. Given that the student is majoring in engineering, what is the probability that the student plays club sports?

d. Given that the student plays club sports, what is the probability that the student is majoring in engineering?

e. Given that the student is majoring in engineering, what is the probability that the student does not play club sports?

f. Given that the student plays club sports, what is the probability that the student is not majoring in engineering?

Answer:

a. The probability that students are majority in engineering is 30%
b. The probability that students play club sports is 20%
c. The probability is 10%
d. This probability is the probability of majoring in engineering minus the probability of majoring in engineering and playing club sports. So it’s 0.3 - 0.1 = 0.2 or 20%
e. The probability is 10% because 30 (students major in engineering) - 10% (students in engineering and club sports) so it’s 10%
f. The probability is 10% since there are a 10% of students in club sports and majoring engineering is also 10%

A car dealer sold 750 automobiles last year. The following table categorizes the cars sold by size and color and presents the number of cars in each category. A car is to be chosen at random from the 750 for which the owner will win a lifetime of free oil changes.

| | Color | | | |
|---|---|---|---|---|
| **Size** | **White** | **Black** | **Red** | **Grey** |
| **Small** | 102 | 71 | 33 | 134 |
| **Midsize** | 86 | 63 | 36 | 105 |
| **Large** | 26 | 32 | 22 | 40 |

a. If the car is small, what is the probability that it is black?

b. If the car is white, what is the probability that it is midsize?

c. If the car is large, what is the probability that it is red?

d. If the car is red, what is the probability that it is large?

e. If the car is not small, what is the probability that it is not grey?

Answer:

a. There are 71 black cars divided by 242 (total number of small cars) = 29.34%
b. There are 87 midsize cars divided by 312 (total number of white cars) = 27.56%
c. There are 22 red cars divided by 118 (total number of large cars) = 18.64%
d. There are 22 large red cars divided by 91 (total number of red cars) = 24.18%
e. There are 118 midsize cars + 118 large cars – 105 gray midsize cars – 40 grey large cars and this is divided by 508 (total number of cars that aren't small) so 191/508 = 37.61%

A lot of 1000 components contains 300 that are defective. Two components are drawn at random and tested. Let $A$ be the event that the first component drawn is defective, and let $B$ be the event that the second component drawn is defective.

a. Find $P(A)$.
b. Find $P(B|A)$.
c. Find $P(A \cap B)$.
d. Find $P(A^c \cap B)$.
e. Find $P(B)$.
f. Find $P(A|B)$.
g. Are $A$ and $B$ independent? Is it reasonable to treat $A$ and $B$ as though they were independent? Explain.

Answer:

a. P(A), is equal to the proportion of defective components in the lot, which is 300/1000 = 0.3
b. P(B), is equal to the proportion of defective components in the lot, which is 300/1000 = 0.3.
c.
d.
e. P(B), is equal to the proportion of defective components in the lot, which is 300/1000 = 0.3.
f. P(A|B), is equal to the probability that both components are defective divided by the probability that the second component is defective. This is equal to 0.3 / 0.3 = 1.
g.

After manufacture, computer disks are tested for errors. Let $X$ be the number of errors detected on a randomly chosen disk. The following table presents values of the cumulative distribution function $F(x)$ of $X$.

| $x$ | $F(x)$ |
|---|---|
| 0 | 0.41 |
| 1 | 0.72 |
| 2 | 0.83 |
| 3 | 0.95 |
| 4 | 1.00 |

a. What is the probability that two or fewer errors are detected?
b. What is the probability that more than three errors are detected?
c. What is the probability that exactly one error is detected?
d. What is the probability that no errors are detected?
e. What is the most probable number of errors to be detected?

Answer:

a. Probability of detecting zero error (0.41) probability of detecting one error (0.72) probability of detecting two errors (0.83) so 0.41 + (0.72 - 0.41) + (0.83 - 0.72) = 0.41 + 0.31 + 0.11 = 0.83
b. The probability that more than three errors are detected is 1- the probability that here or fewer errors are detected, so 1-0.95 = 0.05
c. Probability of detecting one error (0.72) and probability of detecting two errors (0.83) so 0.72 - 0.41 = 0.31
d. From the table the probability that no errors are detected is 0.41
e.

Three components are randomly sampled, one at a time, from a large lot. As each component is selected, it is tested. If it passes the test, a success (S) occurs; if it fails the test, a failure (F) occurs. Assume that 80% of the components in the lot will succeed in passing the test. Let $X$ represent the number of successes among the three sampled components.

a. What are the possible values for $X$?
b. Find $P(X = 3)$.
c. The event that the first component fails and the next two succeed is denoted by FSS. Find $P(\text{FSS})$.
d. Find $P(\text{SFS})$ and $P(\text{SSF})$.
e. Use the results of parts (c) and (d) to find $P(X = 2)$.
f. Find $P(X = 1)$.
g. Find $P(X = 0)$.
h. Find $\mu_X$.

a. The possible values for X are 0, 1, 2, or 3
b. The probability that all three components selected will pass the test is $(0.8)^3 = 0.512$
c. The probability that the first component fails is $1 - 0.8 = 0.2$, and the probability that the next two components succeed is $(0.8)^2 = 0.64$. So $P(FSS) = 0.2 * 0.64 = 0.128$.
d.
e.
f.
g.
h.