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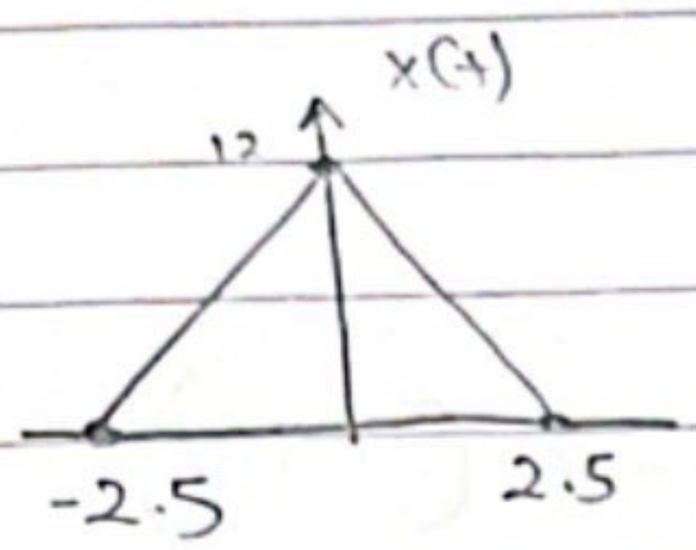
Project 1:



$T_0 = 10$

Show that

$a_k = 3\sin^2(k/4)$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{10} = \boxed{\pi/5}$

$x(t) = \frac{12}{2.5}t + 12$ (from -2.5 to 0)

$\because$ The given Signal is even

$\therefore a_k = \frac{2}{T}\int_{-\frac{T}{2}}^{0} x(t)\cos(k\omega_0 t)\,dt$

$\frac{2}{10}\int_{-2.5}^{0}\underbrace{\left(\frac{12}{2.5}t+12\right)}_{u}\underbrace{\cos(k\pi/5t)\,dt}_{dv}$

$u = \frac{12}{2.5}t + 12$ $\quad dv = \cos(k\pi/5t)$

$du = \frac{12}{2.5}$ $\quad v = \frac{\sin(k\pi/5\,t)}{k\pi/5}$

$\frac{2}{10}\left[\frac{\left(\frac{12}{2.5}t+12\right)\sin(k\pi/5t)}{k\pi/5}\Bigg|_{-2.5}^{0} - \frac{12}{2.5}\int_{-2.5}^{0}\frac{\sin(k\pi/5t)}{k\pi/5}dt\right]$

$a_k = \frac{2}{10}\left[\frac{\cancel{12\sin(0) - (-12+12)\sin(-\frac{k\pi}{2})}}{k\pi/5} + \frac{12}{2.5}\frac{\cos(k\pi/5t)}{(k\pi/5)^2}\Bigg|_{-2.5}^{0}\right]$ $\to 0$

$\frac{24}{25}\left[\frac{1-\cos(\frac{k\pi}{2})}{(k\pi/5)^2}\right]$ $\quad \because \cos(2\theta) = 1 - 2\sin^2(\theta)$

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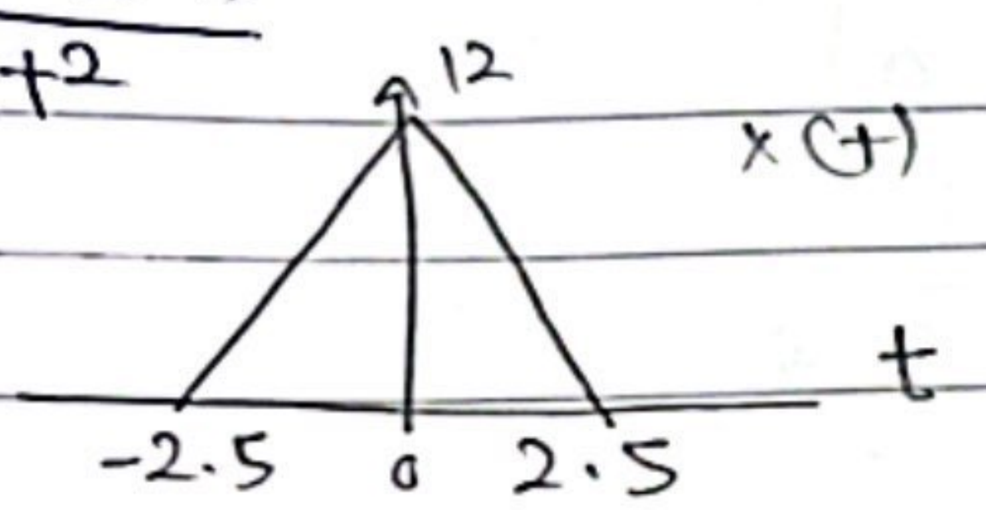
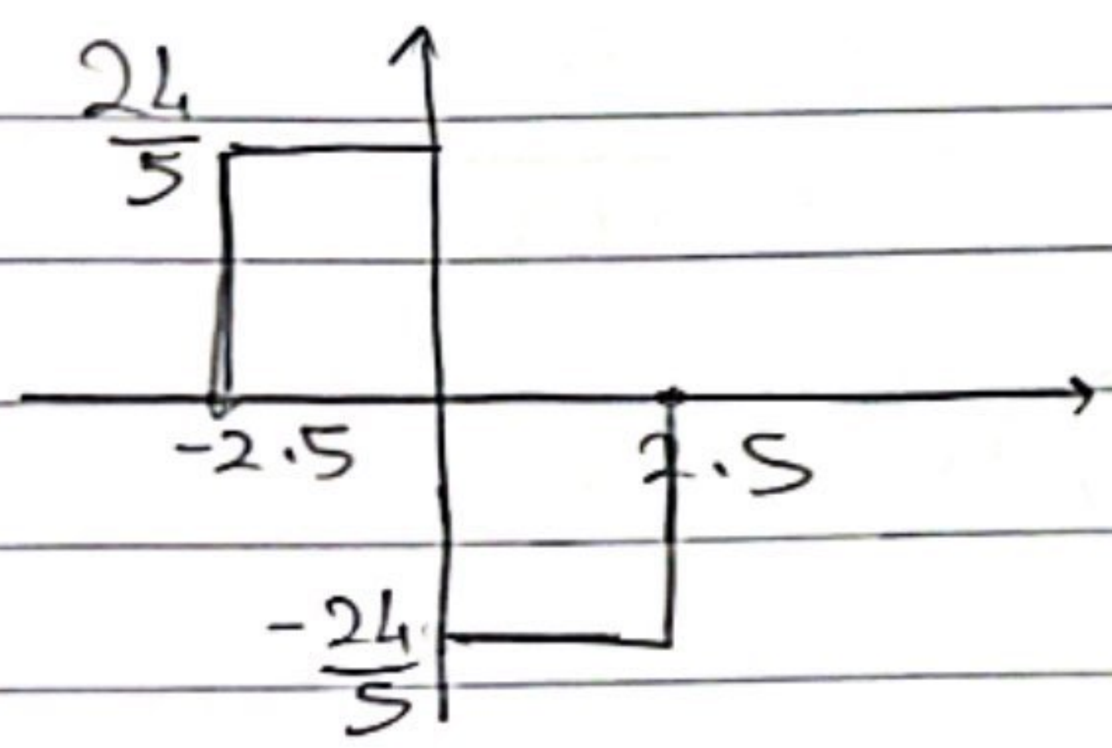
$$a_k = \frac{24}{25} \, \frac{1 - [1 - 2\sin^2(k\pi/4)]}{(k\pi/5)^2}$$

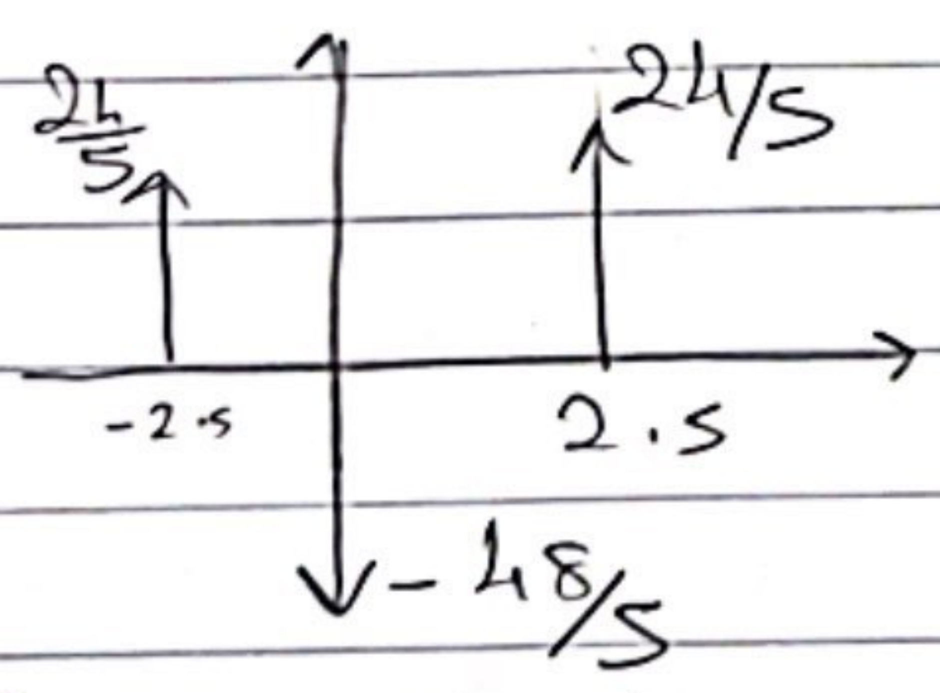
$$a_k = \frac{24}{25} \, \frac{2\sin^2(k\pi/4)}{(k\pi/4)^2 \cdot 16/25}$$

$$a_k = \frac{3\sin^2(k\pi/4)}{(k\pi/4)^2} = 3\,\mathrm{sinc}^2(k/4)$$

b) $\frac{d^2(x(t))}{dt^2}$







$\frac{d^2x(t)}{dt} = y(t)$ $\quad b_k = a_k(j\omega_0 k)^2$

$$b_k = \frac{1}{T}\int_0^T y(t)\, e^{-jk\omega_0 t}\, dt$$

$$b_k = \frac{1}{10}\int_{-5}^{5}\left[\frac{24}{5}\delta(t+2.5) - \frac{48}{5}\delta(t) + \frac{24}{5}\delta(t-2.5)\right] e^{-j\frac{k\pi}{5}t}\, dt$$

$$b_k = \frac{1}{10}\left[\frac{24}{5}e^{jk\pi/2} - \frac{48}{5} + \frac{24}{5}e^{-jk\pi/2}\right]$$

$$b_k = 24/50\left[-2 + e^{jk\pi/2} + e^{-jk\pi/2}\right]$$

$$b_k = \frac{24}{50}\left[-2 + 2\cos(k\pi/2)\right]$$

$$b_k = \frac{48}{50}\left[-1 + \cos(k\pi/2)\right]$$

$$a_k = \frac{b_k}{(jk\pi/5)^2} = \frac{48}{50}\,\frac{\left[1 - \cos(k\pi/2)\right]}{(k\pi/5)^2}$$

$$\cos(2\theta) = 1 - 2\sin^2(\theta)$$

$$a_k = \frac{48}{50}\,\frac{-(1 - 2\sin^2(k\pi/4)) + 1}{(k\pi/5)^2}$$

$$a_k = \frac{48}{50}\,\frac{2\sin^2(k\pi/4)}{(k\pi/4)^2\,\frac{16}{25}}$$

$$a_k = \frac{3\sin^2(k\pi/4)}{(k\pi/4)} \Rightarrow a_k = 3\,\mathrm{sinc}^2(k/4)$$

c) $x(t) = \sum_{-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$$e^{jk\omega_0 t} = \cos(k\omega_0 t) + j\sin(k\omega_0 t)$$

The imag part of $e^{jk\omega_0 t}$ equal to zero as signal is Real

$$x(t) = \sum_{-\infty}^{\infty} a_k \cos(k\omega_0 t)$$

$$x(t) = a_0 + \sum_{-\infty}^{-1} a_k \cos(k\omega_0 t) + \sum_{1}^{\infty} a_k \cos(k\omega_0 t)$$

$\because a_k = 3\sin^2(k/4)$ & $\cos(k\omega_0 t)$ are even functions

$$\therefore x(t) = a_0 + 2\sum_{1}^{\infty} a_k \cos(k\omega_0 t)$$

$$\therefore x(t) = 3 + 2\sum_{k=1}^{\infty} 3\,\mathrm{sinc}^2(k/4)\cos(k\pi/5\, t)$$

$$\therefore x(t) = 3 + 6\sum_{k=1}^{\infty} \mathrm{sinc}^2(k/4)\cos\left(\frac{\pi k}{5}t\right)$$