Numerical Project

Submitted to: Dr. Amani Abdul Qadir Aziz El Gammal

Team : 9

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Introduction

Solving a system of differential equations can be done by a lot of methods like homogenous with constant coefficients, the method of undetermined coefficients, variation of parameters method, Cauchy Euler, reduction of order, and many more. These methods solve the equations exactly, but most second-order ODEs arising in realistic applications cannot be solved exactly. For these problems, one does a qualitative analysis to get a rough idea of the behavior of the solution, then a numerical method is employed to get an accurate solution. In this way, one can verify the answer obtained from the numerical method by comparing it to the answer obtained from qualitative analysis. In a few fortunate cases, a second-order ODE can be solved exactly, and there are a lot of methods to solve with like Euler's method, Henn's method, Runge Kutta method, predictor-corrector method, Richard's extrapolation, Runge Kutta Nystrom. The accuracy of numerical analysis increases by decreasing the step size and increasing the number of iterations in the iterative method; the accuracy of the most frequently used methods of integrating differential equations is fairly well known.

Numerical Methods and Real-Life Applications

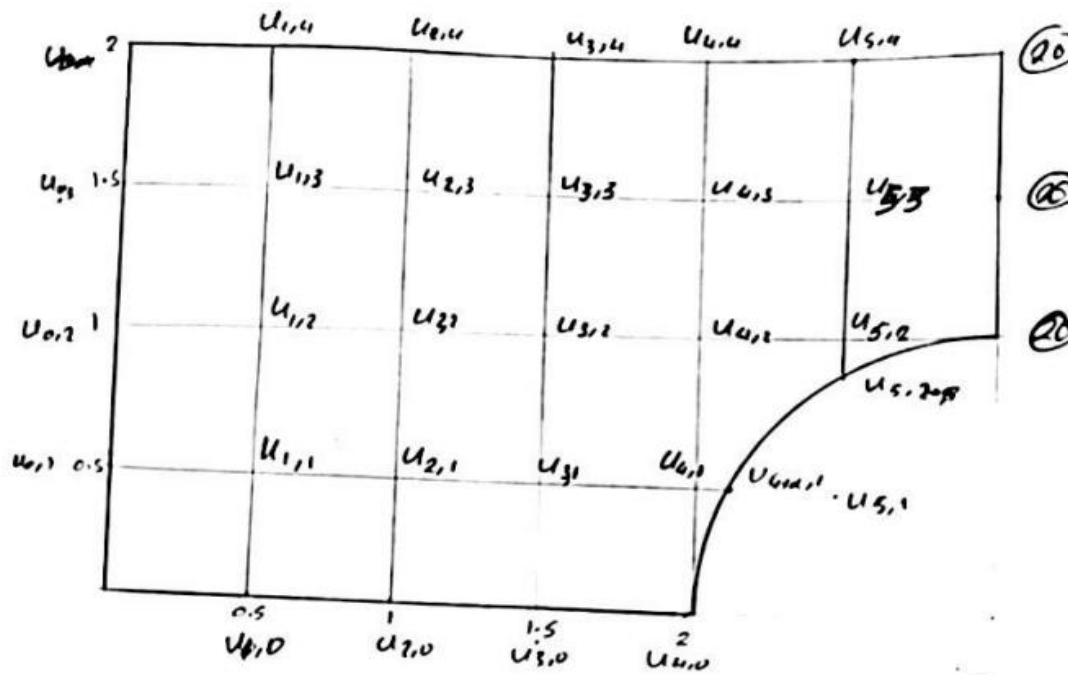
This study will focus on two numerical methods: Runge Kutta and Finite Difference. These methods are used in real-life applications. Runge-Kutta (RK) methods are used widely in many types of research mainly in fluid dynamics and mechanics for better solutions of the fluids. Another real-life application of the Runge-Kutta method is simulation and games. In all present games, we find the motion of objects relatively and vary the position of different objects according to it. If there is only one force acting on the body, or only one acceleration such as gravity, then we can apply simply laws of motion. But, in reality, many constraints change with time or velocity or any other physical quantity. In this type of situation, we need to numerically integrate the equations we have to get the approximation of the moving object.

Problem 1

1) The flow through porous media can be described by the Laplace equation, where h is the head. Use the Finite Difference Method to determine the distribution of heads for the system shown below with the indicated initial and boundary conditions; (Use h=k=0.1)

a) Hand Analysis :

We have got 31 unknown so we needed 31 equations FROM BD AND FD.



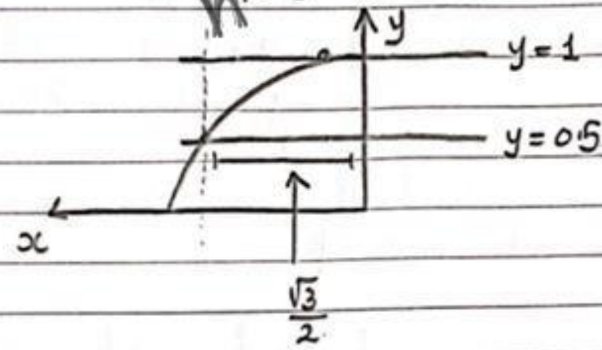
2 8 points req

29 ... with

U5,1

No:------------- Date:-------------

$\alpha = 1 - \sqrt{3}/2 = 0.1339746 \times 2$



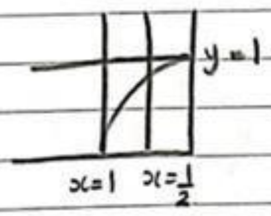
$x^2 + y^2 = 1$

$y = 0.5$

$x^2 + \frac{1}{4} = 1$

$x^2 = \frac{3}{4}$

$x = \frac{\sqrt{3}}{2}$



$x^2 + y^2 = 1$

$x = \frac{1}{2}$

$y = \frac{\sqrt{3}}{2}$

$\beta = \alpha = 2 - \sqrt{3}$

β=α=2-√3

$$U_{xx}P = \frac{2}{h^2}\left[\frac{\overleftarrow{U_{ireg}}}{\alpha(1+\alpha)} - \frac{U_P}{\alpha} + \frac{\overrightarrow{U_{reg}}}{1+\alpha}\right]$$

$U_{xx} + U_{yy} = 0$

$$\rightarrow \overrightarrow{U_{i+1,j}} + \overleftarrow{U_{i-1,j}} + \overset{\uparrow}{U_{i,j+1}} + \overset{\downarrow}{U_{i,j-1}} - 4U_{i,j} = 0$$

at $(0.5, 0) \Rightarrow$ F.D. $U_y = \frac{U_{i,j+1} - U_{ij}}{K}$, $0 = \frac{U_{1,1} - U_{1,0}}{1/2}$

, $\boxed{U_{1,1} = U_{1,0}}$ -- ①

at $(0, 0.5) \Rightarrow$ F.D. $U_x = \frac{U_{i+1,j} - U_{i,j}}{K}$, $1 = \frac{U_{1,1} - U_{0,1}}{1/2}$

$\boxed{U_{0,1} + 1/2 = U_{1,1}}$ -- ②

at $(1,0)$ F.D. $\Rightarrow$ $U_y \Rightarrow 0 = \frac{U_{2,1} - U_{2,0}}{1/2}$, $\boxed{U_{2,1} = U_{2,0}}$ -- ③

at $(1.5, 0)$ F.D. , $\boxed{U_{3,1} = U_{3,0}}$ -- ④

at $(2, 0)$ F.D. , $\boxed{U_{4,1} = U_{4,0}}$ -- ⑤

at $(0,1)$ F.D $U_x = \frac{U_{1,2} - U_{0,2}}{1/2} = 1$, $\boxed{U_{1,2} - U_{0,2} = 1/2}$ -- ⑥

at $(0, 1.5)$ F.D U_x, $\boxed{U_{1,3} - U_{0,3} = 1/2}$ -- ⑦

at $(0.5, 2)$ B.D U_y, $\boxed{U_{1,3} = U_{1,4}}$ -- ⑧

at $(1,2)$ B.D U_y, $\boxed{U_{2,3} = U_{2,4}}$ -- ⑨

at $(1.5, 2)$ B.D. U_y, $\boxed{U_{3,3} = U_{3,4}}$ -- ⑩

at $(2,2)$ B.D. U_y, $\boxed{U_{4,3} = U_{4,4}}$ ⑪

at $(25,2)$ B.D U_y, $\boxed{U_{5,3} = U_{5,4}}$ -- (12)

Using $\overrightarrow{U_{i+1,j}} + \overleftarrow{U_{i-1,j}} + \overset{\uparrow}{U_{i,j+1}} + \overset{\downarrow}{U_{i,j-1}} - 4U_{i,j} = 0$ for regular boundries

- $U_{2,1} + U_{0,1} + U_{1,2} + U_{1,0} - 4U_{1,1} = 0$ -- (13)
- $U_{2,2} + U_{0,2} + U_{1,3} + U_{1,1} - 4U_{1,2} = 0$ -- (14)
- $U_{2,3} + U_{0,3} + U_{1,4} + U_{1,2} - 4U_{1,3} = 0$ -- (15)
- $U_{3,1} + U_{1,1} + U_{2,2} + U_{2,0} - 4U_{2,1} = 0$ -- (16)
- $U_{3,2} + U_{1,2} + U_{2,3} + U_{2,1} - 4U_{2,2} = 0$ -- (17)
- $U_{3,3} + U_{1,3} + U_{2,4} + U_{2,2} - 4U_{2,3} = 0$ -- (18)
- $U_{4,1} + U_{2,1} + U_{3,2} + U_{3,0} - 4U_{3,1} = 0$ (19)
- $U_{4,2} + U_{2,2} + U_{3,3} + U_{3,1} - 4U_{3,2} = 0$ (20)
- $U_{4,3} + U_{2,3} + U_{3,4} + U_{3,2} - 4U_{3,3} = 0$ (21)
- $U_{5,3} + U_{3,3} + U_{4,4} + U_{4,2} - 4U_{4,3} = 0$ (22) $U_{4,1}$
- $U_{5,2} + U_{3,2} + U_{4,3} + U_{4,1} - 4U_{4,2} = 0$ (23) $U_{5,2}$
- $20 + U_{4,3} + U_{5,4} + U_{5,2} - 4U_{5,3} = 0$ (24)

using $U_{xx} = \frac{2}{h^2}\left(\frac{U_{irr}}{\alpha(1+\alpha)} - \frac{U_{i,j}}{\alpha} + \frac{U_{rg}}{1+\alpha}\right) + U_{yy} = \frac{2}{k^2}\left(\frac{U_{irr}}{(\beta+1)\beta} - \frac{U_{ij}}{\beta} + \frac{U_{rg}}{1+\beta}\right.$

•

we $U_{xx} = \frac{2}{h^2}\left(\frac{U_{irr}}{\alpha(1+\alpha)} - \frac{U_{i,j}}{\alpha} + \frac{U_{reg}}{1+\alpha}\right)$ $\qquad \boxed{\beta_2 \alpha = 2-\sqrt{3}}$

$U_{yy} = \frac{2}{k^2}\left(\frac{U_{irr}}{\beta(1+\beta)} - \frac{U_{i,j}}{\beta} + \frac{U_{reg}}{1+\beta}\right).$

@ $U_{4,1} \rightarrow 8\left(\frac{U_{4+\alpha,1}}{\alpha(1+\alpha)} - \frac{U_{4,1}}{\alpha} + \frac{U_{3,1}}{1+\alpha}\right) + \cancel{4}\left(U_{4,2} - 2U_{4,1} + U_{4,0}\right) = 0$

$\Longrightarrow 2\left(\frac{U_{4+\alpha,1}}{\alpha(1+\alpha)} - \frac{U_{4,1}}{\alpha} + \frac{U_{3,1}}{1+\alpha}\right) + 1\left(U_{4,2} - 2U_{4,1} + U_{4,0}\right) = 0.$ (25)

@ $U_{5,2} \rightarrow \frac{20 - 2U_{5,2} + U_{4,2}}{\left(\frac{1}{2}\right)^2} + 8\left(\frac{U_{5,1+\beta k}}{\beta(1+\beta)} - \frac{U_{5,2}}{\beta} + \frac{U_{5,3}}{1+\beta}\right) = 0.$

$\Longrightarrow 20 - 2U_{5,2} + U_{4,2} + 2\left(\frac{U_{5,1+\beta k}}{\beta(1+\beta)} - \frac{U_{5,2}}{\beta} + \frac{U_{5,3}}{1+\beta}\right) = 0$ (26)

@ Boundaries

@ $(2+\alpha h, 0.5)$ $\quad U_x = 0 = \left[\frac{(\sqrt{3}-1)U_{4,1}}{(2-\sqrt{3})(1)} - \frac{(2\sqrt{3}-3)}{3\sqrt{3}-5}U_{4+\alpha h,\alpha p} - \frac{(2-\sqrt{3})U_{5,1}}{(\sqrt{3}-1)(1)}\right]$ (27)

$\alpha_1 = 2-\sqrt{3}$

$\alpha_2 = 1-\alpha_1 = \sqrt{3}-1$

$\alpha_1 + \alpha_2 = 1$

@ $(2.5, 1-\beta k)$, $\quad 0 = \left[\frac{\beta_2 U_{5,2}}{\beta_1(1)} - \frac{(2\sqrt{3}-3)}{3\sqrt{3}-5}U_{5,1+\beta k} - \frac{\beta_1}{\beta_2}U_{5,1}\right].$ (28)

$\beta_2 - \beta_1$, $\beta_1\beta_2$.

$\beta_1 = 2-\sqrt{3}$

$\beta_2 = \sqrt{3}-1$

$\beta_1 + \beta_2 = 1$

@ $U_{0,0} \Rightarrow \frac{U_{01} + U_{10} - \frac{1}{2}}{2}$ (29), $\quad U_{4,0} = \frac{U_{41} + U_{30}}{2}$ (30)

$U_{04} = \frac{U_{03} + U_{14} - \frac{1}{2}}{2}$ (31)

from (28)

$$U_{5,1} = -\frac{\beta_2}{\beta_1}\left[\frac{\beta_2-\beta_1}{\beta_1\beta_2}U_{5,1+\beta k} - \frac{\beta_2}{\beta_1}U_{52}\right].$$

Matlab code FOR FD AND BD

```
%%%%%%%%%%%%%%%%% Question one Using FD , BD for some
points
clc;
clear all;
close all;
syms  U00 U01 U02 U03 U04 U10 U11 U12 U13 U14 U20 U21 U22
U23 U24 U30 U31 U32 U33 U34 U40 U42 U43 U44 U51 UIR52 UIR41
U53 U54 U41339 U0866
alpha=2-sqrt(3);
peta=alpha;
alpha1=alpha;
peta1=peta;
peta2=1-peta1;
alpha2=1-alpha1;
eq1=U01==U11-0.5;%%%%%%%%
eq2=U02==U12-0.5;%%%%%%%%
eq3=U03==U13-0.5;%%%%%%%%%
eq4=U10==U11;%%%%%%%%%%%%%
eq5=0==U21+U01+U10+U12-4*U11;
eq6=0==U02+U22+U13+U11-4*U12;
eq7=0==U23+U03+U14+U12-4*U13;
eq8=U14==U13;%%%%%%%%%
eq9=U20==U21;%%%%%%%%%%
eq10=0==U31+U11+U22+U20-4*U21;
eq11=0==U32+U12+U23+U21-4*U22;
eq12=0==U33+U13+U24+U22-4*U23;
eq13=U24==U23;%%%%%%%%%%%%
eq14=U30==U31;%%%%%%%%%
eq15=U40==(UIR41+U30)/2;%%%%%%%%%%%%%%
eq16=0==UIR41+U21+U32+U30-4*U31;
eq17=0==U42+U22+U33+U31-4*U32;
eq18=0==U43+U23+U34+U32-4*U33;
eq19=U34==U33;%%%%%%%%%%%%%%
eq20=0==2*((U41339/(alpha*(1+alpha)))-
(UIR41/(alpha))+(U31/(1+alpha)))+(U42+U40-2*UIR41);
eq21=0==UIR52+U32+U43+UIR41-4*U42;
eq22=0==U53+U33+U44+U42-4*U43;
eq23=U44==U43;%%%%%%%%%%%%%%%%
eq24=U51==-(peta2/peta1)*(((peta1-
peta2)/(peta1*peta2))*U0866-((peta2/peta1)*UIR52));
eq25=0==2*((U0866/(peta*(1+peta)))-
(UIR52/(peta))+(U53/(1+peta)))+(20-2*UIR52+U42);%%%%%@U5,2
```

```
eq26=0==20+U43+U54+UIR52-4*U53;
eq27=U54==U53;%%%%%%%%%%
eq28=0==((alpha2/alpha1)*UIR41-((alpha1-
alpha2)/(alpha2*alpha1))*U41339-(alpha1/alpha2)*U51);
eq29=0==((peta2/peta1)*UIR52-((peta1-
peta2)/(peta2*peta1))*U0866-(peta1/peta2)*U51);
eq30=U04==(U03+U14-0.5)/2;%%%%%%%%
eq31=U00==(U01+U10-0.5)/2;%%%%%%%%%
[x,y]=equationsToMatrix([eq1,eq2,eq3,eq4,eq5,eq6,eq7,eq8,eq
9,eq10,eq11,eq12,eq13,eq14,eq15,eq16,eq17,eq18,eq19,eq20,eq
21,eq22,eq23,eq24,eq25,eq26,eq27,eq28,eq29,eq30,eq31]);
N=linsolve(x,y);
fprintf('%g\n',abs(N)*2.15)
```

Results:

0.513644

0.513644

0.870938

1.2381

1.2381

5.55922

1.58864

1.58864

1.94594

2.3131

2.3131

2.30635

2.30635

3.01107

3.75527

3.75527

2.31934

2.31934

4.0367

5.94164

5.94164

1.46715

0.710104

4.87476

10.033

10.033

0

19.2825

19.2825

0.614968

4.81443

From these results we can see the distribution of h on our grid which is reasonable as some of them are near to 20 which is the boundaries values.

AGAIN USING CD:

Hand analysis:

$\cancel{(0,0) \rightarrow U_{0,1} + U_{1,0}}$ $\quad$ $U_{1,0}$

$(0.5,0).\quad 2U_{1,1} + U_{2,0} + U_{0,0}^{*} - 4U_{1,0} = 0 \quad \text{— (1)}$

$U_{2,0}.\quad 2U_{2,1} + U_{3,0} + U_{1,0} - 4U_{2,0} = 0 \quad \text{— (14)}$

$U_{3,0}.\quad 2U_{3,1} + U_{4,0} + U_{2,0} - 4U_{3,0} = 0 \quad \text{— (2)}$

$\cancel{U_{4,0}}$

$U_{1,4}.\quad 2U_{1,3} + U_{2,4} + U_{0,4} - 4U_{1,4} = 0 \quad \text{— (3)}$

$U_{2,4}.\quad 2U_{2,3} + U_{3,4} + U_{1,4} - 4U_{2,4} = 0 \quad \text{— (4)}$

$U_{3,4}.\quad 2U_{3,3} + U_{4,4} + U_{2,4} - 4U_{3,4} = 0 \quad \text{— (5)}$

$U_{4,4}.\quad 2U_{4,3} + U_{5,4} + U_{3,4} - 4U_{4,4} = 0 \quad \text{— (6)}$

$U_{5,4}.\quad 2U_{5,3} + 20 + U_{4,4} - 4U_{5,4} = 0 \quad \text{— (7)}$

$U_{0,3}.\quad U_{0,4} + 2U_{1,3} + U_{0,2} - 4U_{0,3} = 1 \quad \text{(11)}$

$U_{0,2}.\quad U_{0,3} + 2U_{1,2} + U_{0,1} - 4U_{0,2} = 1 \quad \text{(12)}$

$U_{0,1}.\quad U_{0,2} + 2U_{1,1} + U_{0,0} - 4U_{0,1} = 1 \quad \text{(13)}$

@ $U_{4,1} \rightarrow$

$$U_{xx} = 8\left(\frac{U_{4+\alpha,1}}{\alpha(1+\alpha)} - \frac{U_{41}}{\alpha} + \frac{U_{31}}{1+\alpha}\right) + 4\left(\frac{U_{42}}{1} + U_{40} - 2U_{41}\right) = 0$$

$$2\left(\frac{U_{4+\alpha,1}}{\alpha(1+\alpha)} - \frac{U_{41}}{\alpha} + \frac{U_{31}}{1+\alpha}\right) + U_{42} + U_{40} - 2U_{41} = 0$$

$$\boxed{U_{x20} = 8\left(\frac{U_{52}\,\beta_2}{\beta_1(\beta_1+\beta_2)} - \frac{(\beta_1-\beta_2)}{\beta_1\beta_2}U_{5,1+\beta_2} - \frac{\beta_1\,U_{51}}{\beta_2(\beta_1+\beta_2)}\right)} =$$

$$U_{51} = \frac{-\beta_2}{\beta_1}\left(\frac{\beta_1-\beta_2}{\beta_1\beta_2}U_{5,1+\beta_2} - \frac{\beta_2}{\beta_1}U_{52}\right)$$

at $U_{0,0}$

$$U_{1,0} + U_{0,1} + U_{-1,0} + U_{0,-1} - 4U_{0,0} = 0$$

$U_{0,1}$

$U_{1,0} - 1$

$\underline{U_{0,0}}$: $2U_{1,0} + 2U_{0,1} - 4U_{0,0} = 1$ —— 8

$\underline{U_{4,0}}$: $2U_{4,1} + 2U_{3,0} - 4U_{4,0} = 0$ 9

$\underline{U_{0,4}}$: $2U_{0,3} + 2U_{1,4} - 4U_{0,4} = 1$ 10 at (2.5,2) B.D U_y, $\boxed{\cancel{U_{5,3}} = U_{5,4}}$ .. (12)

Using $\overset{\rightarrow}{U_{i+1,y}} + \overset{\leftarrow}{U_{i-1,j}} + \overset{\uparrow}{U_{i,j+1}} + \overset{\downarrow}{U_{i,j-1}} - 4U_{i,j} = 0$ for regular boundries

- $U_{2,1} + U_{0,1} + U_{1,2} + U_{1,0} - 4U_{1,1} = 0$ ..... (13)
- $U_{2,2} + U_{0,2} + U_{1,3} + U_{1,1} - 4U_{1,2} = 0$ ... (14)
- $U_{2,3} + U_{0,3} + U_{1,4} + U_{1,2} - 4U_{1,3} = 0$ ..... (15)
- $U_{3,1} + U_{1,1} + U_{2,2} + U_{2,0} - 4U_{2,1} = 0$ .. (16)
- $U_{3,2} + U_{1,2} + U_{2,3} + U_{2,1} - 4U_{2,2} = 0$ .. (17)
- $U_{3,3} + U_{1,3} + U_{2,4} + U_{2,2} - 4U_{2,3} = 0$ (18)
- $U_{4,1} + U_{2,1} + U_{3,2} + U_{3,0} - 4U_{3,1} = 0$ (19)
- $U_{4,2} + U_{2,2} + U_{3,3} + U_{3,1} - 4U_{3,2} = 0$ (20)
- $U_{4,3} + U_{2,3} + U_{3,4} + U_{3,2} - 4U_{3,3} = 0$ (21)
- $U_{5,3} + U_{3,3} + U_{4,4} + U_{4,2} - 4U_{4,3} = 0$ (22)
- $U_{5,2} + U_{3,2} + U_{4,3} + U_{4,1} - 4U_{4,2} = 0$ (23)
- $20 + U_{4,3} + U_{5,4} + U_{5,2} - 4U_{5,3} = 0$ (24)

eq14, eq4, eq8, 18

$U_{4,1}$

$U_{5,2}$

Using $U_{xx} = \frac{2}{h^2}\left(\frac{U_{irr}}{\alpha(1+\alpha)} - \frac{U_{i,j}}{\alpha} + \frac{U_{reg}}{1+\alpha}\right) + U_{yy} = \frac{2}{k^2}\left(\frac{U_{irr}}{(\beta+1)\beta} - \frac{U_{ij}}{\beta} + \frac{U_{rg}}{1+\beta}\right.$

-

Results: NOTE THESE RESULTS ARE NOT ARRANGED

0.965819

1.06189

1.30904

1.56703

1.67551

2.74945

1.36975

1.48634

1.80363

2.14178

2.28399

1.54051

1.71009

2.27737

2.91245

3.17692

1.3721

1.53615

2.68331

4.05373

4.59877

0.875596

0.437737

2.86597

6.0204

7.11071

0

10.0512

11.8033

0.379092

2.38109

Code for matlab CD

```
%%%%%%%%%%%%%%%%% Question one Using FD , BD for some
points
clc;
clear all;
close all;
syms  U00 U01 U02 U03 U04 U10 U11 U12 U13 U14 U20 U21 U22
U23 U24 U30 U31 U32 U33 U34 U40 U42 U43 U44 U51 UIR52 UIR41
U53 U54 U41339 U0866
alpha=2-sqrt(3);
peta=alpha;
alpha1=alpha;
peta1=peta;
peta2=1-peta1;
alpha2=1-alpha1;
eq31=2*U10+2*U01-4*U00==1;%%%%%%U00%%
eq32=U02+2*U11+U00-4*U01==1;%%%%%%U01
eq33=U03+2*U12+U01-4*U02==1;%%%%%%U02
eq34=U04+2*U13+U02-4*U03==1;%%%%%%U03
eq30=2*U03+2*U14-4*U04==1;%%%%%U04%%
eq1=2*U11+U20+U00-4*U10==0;%%%%%%%  U10
eq9=2*U21+U30+U10+-4*U20==0;%%%%%%%%%%%% U20%%%%%%%
eq2=2*U31+U40+U20-4*U30==0;%%%%%%%% U30
eq15=2*UIR41+2*U30-4*U40==0;%%%%%%%%%%%%%U40
eq5=U01+U21+U12+U10-4*U11==0; %%%%%%% U11
eq6=U02+U22+U13+U11-4*U12==0;%%%%%%%%% U12
eq7=U03+U23+U14+U12-4*U13==0;%%%%%%%% U13
eq3=2*U13+U24+U04-4*U14==0;%%%%%%%%% U14
eq10=U31+U11+U22+U20-4*U21==0;%%%%%%%% U21
eq11=U32+U12+U23+U21-4*U22==0;%%%% U22
eq12=U33+U13+U24+U22-4*U23==0;%%%%%%%%U23
eq13=2*U23+U34+U14+-4*U24==0;%%%%%%%%%% U24%%%%%%%%%%%
eq16=UIR41+U21+U32+U30-4*U31==0;%%%%%%%%%%U31
```

```
eq17=U42+U22+U33+U31-4*U32==0;%%%%%%%%%U32
eq18=U43+U23+U34+U32-4*U33==0;%%%%%%U33
eq19=2*U33+U44+U24+-4*U34==0;%%%%%%%%%%%%%%U34
eq20=2*((U41339/(alpha*(1+alpha)))-
(UIR41/(alpha))+(U31/(1+alpha)))+(U42+U40-
2*UIR41)==0;%%%%%%%U41
eq21=UIR52+U32+U43+UIR41-4*U42==0;%%%U42
eq22=U53+U33+U44+U42-4*U43==0;%%%%%%U43
eq23=2*U43+U54+U34+-4*U44==0;%%%%%%%%%%%%%%%%U44
eq24=-1*(peta2/peta1)*(((peta1-peta2)/(peta1*peta2))*U0866-
((peta2/peta1)*UIR52))==U51;%%%%%%%%%%U51
eq25=2*((U0866/(peta*(1+peta)))-
(UIR52/(peta))+(U53/(1+peta)))+(20-
2*UIR52+U42)==0;%%%%%@U5,2
eq26=20+U43+U54+UIR52-4*U53==0;%%%%%%%%U53
eq27=2*U53+20+U44+-4*U54==0;%%%%%%%%%%U54
eq28=(alpha2/alpha1)*UIR41-((alpha1-
alpha2)/(alpha2*alpha1))*U41339-
(alpha1/alpha2)*U51==0;%%%%%%%%UIRALPHA
eq29=(peta2/peta1)*UIR52-((peta1-
peta2)/(peta2*peta1))*U0866-
(peta1/peta2)*U51==0;%%%%%%%%%UIRPETA
[x,y]=equationsToMatrix([eq1,eq2,eq3,eq5,eq6,eq7,eq9,eq10,e
q11,eq12,eq13,eq15,eq16,eq17,eq18,eq19,eq20,eq21,eq22,eq23,
eq24,eq25,eq26,eq27,eq28,eq29,eq30,eq31,eq32,eq33,eq34]);
N=linsolve(x,y);
fprintf('%g\n',abs(N))
```

Problem 2

Given $y''=-1001z-1000y, y(0)=1, y'(0)=0, h=0.002, y(5)=?$

put $dy/dx=z$ and differentiate w.r.t. x, we obtain $d2y/dx2=dz/dx$

We have system of equations
$dydx=z=f(x,y,z)$

$dzdx=-1001z-1000y=g(x,y,z)$

Forth order R-K method for second order differential equation

Solution

As we have step of 0.002 -> The needed number of iterations to reach exact solution
Is 2500

The Final Iterations are shown in screenshots below These results :

Command Window

Runge kutta method for 0 iteration
for h=0.000000

K	ValuesK	M	ValuesM
'K1'	0	'M1'	-1000
'K2'	0	'M2'	-1000
'K3'	0	'M3'	-1000
'K4'	0	'M4'	-1000
'Kavg*h'	0	'Mavg*h'	0

ValueOfY	ValueOfYDoubleDash	ValueOfExactY	PercentageOfErrorWithoutDividingByYexact	PercentageOfErrorWithDividingByYexact
1	0	1	0	0

Runge kutta method for 1 iteration
for h=0.002000

K	ValuesK	M	ValuesM
'K1'	0	'M1'	-1000
'K2'	-1	'M2'	1
'K3'	0.001	'M3'	-1000
'K4'	-2	'M4'	1002
'Kavg*h'	-0.0013327	'Mavg*h'	-0.66533

ValueOfY	ValueOfYDoubleDash	ValueOfExactY	PercentageOfErrorWithoutDividingByYexact	PercentageOfErrorWithDividingByYexact
0.99867	-0.66533	0.99887	0.01982	0.019842

Note: we can see here the difference when the step increases the error increases unlike the first one when h=0 the error=0.

'K1'	-0.66533	'M1'	-332.67
'K2'	-0.998	'M2'	0.998
'K3'	-0.66434	'M3'	-332.67
'K4'	-1.3307	'M4'	334.66
'Kavg*h'	-0.0017736	'Mavg*h'	-0.22045

ValueOfY	ValueOfYDoubleDash	ValueOfExactY	PercentageOfErrorWithoutDividingByYexact	PercentageOfErrorWithDividingByYexact
0.99689	-0.88578	0.99699	0.0092888	0.0093169

Runge kutta method for 3 iteration
for h=0.002000

K	ValuesK	M	ValuesM
'K1'	-0.88578	'M1'	-110.23
'K2'	-0.99601	'M2'	0.99601
'K3'	-0.88479	'M3'	-110.23
'K4'	-1.1062	'M4'	112.22
'Kavg*h'	-0.0019179	'Mavg*h'	-0.072156

ValueOfY	ValueOfYDoubleDash	ValueOfExactY	PercentageOfErrorWithoutDividingByYexact	PercentageOfErrorWithDividingByYexact
0.99498	-0.95794	0.99501	0.0034593	0.0034766

Now,

$y_1=y_0+16\left(k_1+2k_2+2k_3+k_4\right)$

$y_1=1+16[0+2(-0.002)+2(0)+(-0.004)]$

$y_1=0.9987$

$z_1=z_0+16\left(m_1+2m_2+2m_3+m_4\right)$

$z_1=0+16[-2+2(0.002)+2(-2)+(2.004)]$

$z_1=-0.6653$

$\therefore y(0.002)=0.9987$

$\therefore y'(0.002)=-0.6653$

Again taking $\left(x_1,y_1,z_1\right)$ in place of $\left(x_0,y_0,z_0\right)$ and repeat the process

$k_1=hf\left(x_1,y_1,z_1\right)=(0.002)\cdot f(0.002,0.9987,-0.6653)=(0.002)\cdot(-0.6653)=-0.0013$

$m_1=hg\left(x_1,y_1,z_1\right)=(0.002)\cdot g(0.002,0.9987,-0.6653)=(0.002)\cdot(-332.668)=-0.6653$

$k_2=hf\left(x_1+h2,y_1+k12,z_1+m12\right)=(0.002)\cdot f(0.003,0.998,-0.998)=(0.002)\cdot(-0.998)=-0.002$

$m_2=hg\left(x_1+h2,y_1+k_12,z_1+m_12\right)=(0.002)\cdot g(0.003,0.998,-0.998)=(0.002)\cdot(0.998)=0.002$

$k_3=hf\left(x_1+h2,y_1+k_22,z_1+m_22\right)=(0.002)\cdot f(0.003,0.9977,-0.6643)=(0.002)\cdot(-0.6643)=-0.0013$

$m_3=hg\left(x_1+h2,y_1+k_22,z_1+m_22\right)=(0.002)\cdot g(0.003,0.9977,-0.6643)=(0.002)\cdot(-332.669)=-0.6653$

$k_4=hf\left(x_1+h,y_1+k_3,z_1+m_3\right)=(0.002)\cdot f(0.004,0.9973,-1.3307)=(0.002)\cdot(-1.3307)=-0.0027$

$m_4=hg\left(x_1+h,y_1+k_3,z_1+m_3\right)=(0.002)\cdot g(0.004,0.9973,-1.3307)=(0.002)\cdot(334.664)=0.6693$

Now,

$y_2=y_1+16\left(k_1+2k_2+2k_3+k_4\right)$

$y_2=0.9987+16[-0.0013+2(-0.002)+2(-0.0013)+(-0.0027)]$

$y_2=0.9969$

$z_2=z_1+16\left(m_1+2m_2+2m_3+m_4\right)$

$z_2=-0.6653+16[-0.6653+2(0.002)+2(-0.6653)+(0.6693)]$

$z_2=-0.8858$

$\therefore y(0.004)=0.9969$

$\therefore y'(0.004)=-0.8858$

Again taking $\left(x_2,y_2,z_2\right)$ in pmace of $\left(x_0,y_0,z_0\right)$ and repeat the process

$k_1=hf\left(x_2,y_2,z_2\right)=(0.002)\cdot f(0.004,0.9969,-0.8858)=(0.002)\cdot(-0.8858)=-0.0018$

$m_1=hg\left(x_2,y_2,z_2\right)=(0.002)\cdot g(0.004,0.9969,-0.8858)=(0.002)\cdot(-110.2253)=-0.2205$

$k_2=hf\left(x_2+h2,y_2+k_12,z_2+m_12\right)=(0.002)\cdot f(0.005,0.996,-0.996)=(0.002)\cdot(-0.996)=-0.002$

$m_2=hg\left(x_2+\frac{h}{2},y_2+\frac{k_1}{2},z_2+\frac{m_1}{2}\right)=(0.002)\cdot g(0.005,0.996,-0.996)=(0.002)\cdot(0.996)=0.002$

$k_3=hf\left(x_2+\frac{h}{2},y_2+\frac{k_2}{2},z_2+\frac{m_2}{2}\right)=(0.002)\cdot f(0.005,0.9959,-0.8848)=(0.002)\cdot(-0.8848)=-0.0018$

$m_3=hg\left(x_2+\frac{h}{2},y_2+\frac{k_2}{2},z_2+\frac{m_2}{2}\right)=(0.002)\cdot g(0.005,0.9959,-0.8848)=(0.002)\cdot(-110.2263)=-0.2205$

$k_4=hf\left(x_2+h,y_2+k_3,z_2+m_3\right)=(0.002)\cdot f(0.006,0.9951,-1.1062)=(0.002)\cdot(-1.1062)=-0.0022$

$m_4=hg\left(x_2+h,y_2+k_3,z_2+m_3\right)=(0.002)\cdot g(0.006,0.9951,-1.1062)=(0.002)\cdot(112.2173)=0.2244$

Now,

$y_3=y_2+\frac{1}{6}\left(k_1+2k_2+2k_3+k_4\right)$

$y_3=0.9969+\frac{1}{6}[-0.0018+2(-0.002)+2(-0.0018)+(-0.0022)]$

$y_3=0.995$

$z_3=z_2+\frac{1}{6}\left(m_1+2m_2+2m_3+m_4\right)$

$z_3=-0.8858+\frac{1}{6}[-0.2205+2(0.002)+2(-0.2205)+(0.2244)]$

$z_3=-0.9579$

$\therefore y(0.006)=0.995$

$\therefore y'(0.006)=-0.9579$

Again taking $\left(x_3,y_3,z_3\right)$ in place of $\left(x_0,y_0,z_0\right)$ and repeat the process

$k_1=hf\left(x_3,y_3,z_3\right)=(0.002)\cdot f(0.006,0.995,-0.9579)=(0.002)\cdot(-0.9579)=-0.0019$

$m_1=hg\left(x_3,y_3,z_3\right)=(0.002)\cdot g(0.006,0.995,-0.9579)=(0.002)\cdot(-36.0791)=-0.0722$

$k_2=hf\left(x_3+\frac{h}{2},y_3+\frac{k_1}{2},z_3+\frac{m_1}{2}\right)=(0.002)\cdot f(0.007,0.994,-0.994)=(0.002)\cdot(-0.994)=-0.002$

$m_2=hg\left(x_3+h2,y_3+k12,z_3+m12\right)=(0.002)\cdot g(0.007,0.994,-0.994)=(0.002)\cdot(0.994)=0.002$

$k_3=hf\left(x_3+h2,y_3+k22,z_3+m22\right)=(0.002)\cdot f(0.007,0.994,-0.9569)=(0.002)\cdot(-0.9569)=-0.0019$

$m_3=hg\left(x_3+h2,y_3+k22,z_3+m22\right)=(0.002)\cdot g(0.007,0.994,-0.9569)=(0.002)\cdot(-36.0801)=-0.0722$

$k_4=hf\left(x_3+h,y_3+k_3,z_3+m_3\right)=(0.002)\cdot f(0.008,0.9931,-1.0301)=(0.002)\cdot(-1.0301)=-0.0021$

$m_4=hg\left(x_3+h,y_3+k_3,z_3+m_3\right)=(0.002)\cdot g(0.008,0.9931,-1.0301)=(0.002)\cdot(38.0671)=0.0761$

Now,

$y_4=y_3+16\left(k_1+2k_2+2k_3+k_4\right)$

$y_4=0.995+16[-0.0019+2(-0.002)+2(-0.0019)+(-0.0021)]$

$y_4=0.993$

$z_4=z_3+16\left(m_1+2m_2+2m_3+m_4\right)$

$z_4=-0.9579+16[-0.0722+2(0.002)+2(-0.0722)+(0.0761)]$

$z_4=-0.9807$

$\therefore y(0.008)=0.993$

$\therefore y'(0.008)=-0.9807$

Again taking $\left(x_4,y_4,z_4\right)$ in place of $\left(x_0,y_0,z_0\right)$ and repeat the process

$k_1=hf\left(x_4,y_4,z_4\right)=(0.002)\cdot f(0.008,0.993,-0.9807)=(0.002)\cdot(-0.9807)=-0.002$

$m_1=hg\left(x_4,y_4,z_4\right)=(0.002)\cdot g(0.008,0.993,-0.9807)=(0.002)\cdot(-11.365)=-0.0227$

$k_2=hf\left(x_4+h2,y_4+k12,z_4+m12\right)=(0.002)\cdot f(0.009,0.992,-0.992)=(0.002)\cdot(-0.992)=-0.002$

$m_2=hg\left(x_4+h2,y_4+k_12,z_4+m_12\right)=(0.002)\cdot g(0.009,0.992,-0.992)=(0.002)\cdot(0.992)=0.002$

$k_3=hf\left(x_4+h2,y_4+k_22,z_4+m_22\right)=(0.002)\cdot f(0.009,0.992,-0.9797)=(0.002)\cdot(-0.9797)=-0.002$

$m_3=hg\left(x_4+h2,y_4+k_22,z_4+m_22\right)=(0.002)\cdot g(0.009,0.992,-0.9797)=(0.002)\cdot(-11.366)=-0.0227$

$k_4=hf\left(x_4+h,y_4+k_3,z_4+m_3\right)=(0.002)\cdot f(0.01,0.9911,-1.0034)=(0.002)\cdot(-1.0034)=-0.002$

$m_4=hg\left(x_4+h,y_4+k_3,z_4+m_3\right)=(0.002)\cdot g(0.01,0.9911,-1.0034)=(0.002)\cdot(13.3491)=0.0267$

Now,

$y_5=y_4+16\left(k_1+2k_2+2k_3+k_4\right)$

$y_5=0.993+16[-0.002+2(-0.002)+2(-0.002)+(-0.002)]$

$y_5=0.991$

$z_5=z_4+16\left(m_1+2m_2+2m_3+m_4\right)$

$z_5=-0.9807+16[-0.0227+2(0.002)+2(-0.0227)+(0.0267)]$

$z_5=-0.9869$

$\therefore y(0.01)=0.991$

$\therefore y'(0.01)=-0.9869$

Again taking $\left(x_5,y_5,z_5\right)$ in place of $\left(x_0,y_0,z_0\right)$ and repeat the process

$k_1=hf\left(x_5,y_5,z_5\right)=(0.002)\cdot f(0.01,0.991,-0.9869)=(0.002)\cdot(-0.9869)=-0.002$

$m_1=hg\left(x_5,y_5,z_5\right)=(0.002)\cdot g(0.01,0.991,-0.9869)=(0.002)\cdot(-3.1283)=-0.0063$

$k_2=hf\left(x_5+h2,y_5+k_12,z_5+m_12\right)=(0.002)\cdot f(0.011,0.99,-0.99)=(0.002)\cdot(-0.99)=-0.002$

$m_2 = hg\left(x_5 + h2, y_5 + k12, z_5 + m12\right) = (0.002) \cdot g(0.011, 0.99, -0.99) = (0.002) \cdot (0.99) = 0.002$

$k_3 = hf\left(x_5 + h2, y_5 + k22, z_5 + m22\right) = (0.002) \cdot f(0.011, 0.99, -0.9859) = (0.002) \cdot (-0.9859) = -0.002$

$m_3 = hg\left(x_5 + h2, y_5 + k22, z_5 + m22\right) = (0.002) \cdot g(0.011, 0.99, -0.9859) = (0.002) \cdot (-3.1293) = -0.0063$

$k_4 = hf\left(x_5 + h, y_5 + k_3, z_5 + m_3\right) = (0.002) \cdot f(0.012, 0.9891, -0.9932) = (0.002) \cdot (-0.9932) = -0.002$

$m_4 = hg\left(x_5 + h, y_5 + k_3, z_5 + m_3\right) = (0.002) \cdot g(0.012, 0.9891, -0.9932) = (0.002) \cdot (5.1084) = 0.0102$

Now,

$y_6 = y_5 + 16\left(k_1 + 2k_2 + 2k_3 + k_4\right)$

$y_6 = 0.991 + 16[-0.002 + 2(-0.002) + 2(-0.002) + (-0.002)]$

$y_6 = 0.9891$

$z_6 = z_5 + 16\left(m_1 + 2m_2 + 2m_3 + m_4\right)$

$z_6 = -0.9869 + 16[-0.0063 + 2(0.002) + 2(-0.0063) + (0.0102)]$

$z_6 = -0.9877$

$\therefore y(0.012) = 0.9891$

$\therefore y'(0.012) = -0.9877$

Again taking $\left(x_6, y_6, z_6\right)$ in place of $\left(x_0, y_0, z_0\right)$ and repeat the process

$k_1 = -0.002, k_2 = -0.002, k_3 = -0.002, k_4 = -0.002, y_7 = 0.9871$

$m_1 = -0.0008, m_2 = 0.002, m_3 = -0.0008, m_4 = 0.0047, z_7 = -0.9866$

$\therefore y(0.014) = 0.9871$

$\therefore y'(0.014) = -0.9866$

---

k_1=-0.002,k_2=-0.002,k_3=-0.002,k_4=-0.002,y_8=0.9851

m_1=0.0011,m_2=0.002,m_3=0.0011,m_4=0.0029,z_8=-0.985

∴$y(0.016)$=0.9851

∴$y'(0.016)$=-0.985

---

k_1=-0.002,k_2=-0.002,k_3=-0.002,k_4=-0.002,y_9=0.9831

m_1=0.0017,m_2=0.002,m_3=0.0017,m_4=0.0023,z_9=-0.9831

∴$y(0.018)$=0.9831

∴$y'(0.018)$=-0.9831

---

k_1=-0.002,k_2=-0.002,k_3=-0.002,k_4=-0.002,y_{10}=0.9812

m_1=0.0019,m_2=0.002,m_3=0.0019,m_4=0.0021,z_{10}=-0.9812

∴$y(0.02)$=0.9812

∴$y'(0.02)$=-0.9812

---

k_1=-0.002,k_2=-0.002,k_3=-0.002,k_4=-0.002,y_{11}=0.9792

m_1=0.0019,m_2=0.002,m_3=0.0019,m_4=0.002,z_{11}=-0.9792

∴$y(0.022)$=0.9792

∴$y'(0.022)$=-0.9792

---

k_1=-0.002,k_2=-0.002,k_3=-0.002,k_4=-0.002,y_{12}=0.9773

m_1=0.0019,m_2=0.002,m_3=0.0019,m_4=0.002,z_{12}=-0.9773

∴$y(0.024)$=0.9773

∴$y'(0.024)$=-0.9773

---

k_1=-0.002,k_2=-0.002,k_3=-0.002,k_4=-0.002,y_{13}=0.9753

m_1=0.002,m_2=0.002,m_3=0.0019,m_4=0.002,z_{13}=-0.9753

∴y(0.026)=0.9753

∴y'(0.026)=-0.9753

---

k_1=-0.002,k_2=-0.0019,k_3=-0.0019,k_4=-0.0019,y_{14}=0.9734

m_1=0.0019,m_2=0.0019,m_3=0.0019,m_4=0.0019,z_{14}=-0.9734

∴y(0.028)=0.9734

∴y'(0.028)=-0.9734

---

k_1=-0.0019,k_2=-0.0019,k_3=-0.0019,k_4=-0.0019,y_{15}=0.9714

m_1=0.0019,m_2=0.0019,m_3=0.0019,m_4=0.0019,z_{15}=-0.9714

∴y(0.03)=0.9714

∴y'(0.03)=-0.9714

---

k_1=-0.0019,k_2=-0.0019,k_3=-0.0019,k_4=-0.0019,y_{16}=0.9695

m_1=0.0019,m_2=0.0019,m_3=0.0019,m_4=0.0019,z_{16}=-0.9695

∴y(0.032)=0.9695

∴y'(0.032)=-0.9695

---

k_1=-0.0019,k_2=-0.0019,k_3=-0.0019,k_4=-0.0019,y_{17}=0.9675

m_1=0.0019,m_2=0.0019,m_3=0.0019,m_4=0.0019,z_{17}=-0.9675

∴y(0.034)=0.9675

∴y'(0.034)=-0.9675

---

k_1=-0.0019,k_2=-0.0019,k_3=-0.0019,k_4=-0.0019,y_{18}=0.9656

m_1=0.0019,m_2=0.0019,m_3=0.0019,m_4=0.0019,z_{18}=-0.9656

$\therefore y(0.036)=0.9656$

$\therefore y'(0.036)=-0.9656$

$k_1=-0.0019, k_2=-0.0019, k_3=-0.0019, k_4=-0.0019, y_{19}=0.9637$

$m_1=0.0019, m_2=0.0019, m_3=0.0019, m_4=0.0019, z_{19}=-0.9637$

$\therefore y(0.038)=0.9637$

$\therefore y'(0.038)=-0.9637$

$k_1=-0.0019, k_2=-0.0019, k_3=-0.0019, k_4=-0.0019, y_{20}=0.9618$

$m_1=0.0019, m_2=0.0019, m_3=0.0019, m_4=0.0019, z_{20}=-0.9618$

$\therefore y(0.04)=0.9618$

$\therefore y'(0.04)=-0.9618$

$k_1=-0.0019, k_2=-0.0019, k_3=-0.0019, k_4=-0.0019, y_{21}=0.9598$

$m_1=0.0019, m_2=0.0019, m_3=0.0019, m_4=0.0019, z_{21}=-0.9598$

$\therefore y(0.042)=0.9598$

$\therefore y'(0.042)=-0.9598$

$k_1=-0.0019, k_2=-0.0019, k_3=-0.0019, k_4=-0.0019, y_{22}=0.9579$

$m_1=0.0019, m_2=0.0019, m_3=0.0019, m_4=0.0019, z_{22}=-0.9579$

$\therefore y(0.044)=0.9579$

$\therefore y'(0.044)=-0.9579$

$k_1=-0.0019, k_2=-0.0019, k_3=-0.0019, k_4=-0.0019, y_{23}=0.956$

$m_1=0.0019, m_2=0.0019, m_3=0.0019, m_4=0.0019, z_{23}=-0.956$

$\therefore y(0.046)=0.956$

$\therefore y'(0.046)=-0.956$

$k_1=-0.0019, k_2=-0.0019, k_3=-0.0019, k_4=-0.0019, y_{24}=0.9541$

$m_1=0.0019, m_2=0.0019, m_3=0.0019, m_4=0.0019, z_{24}=-0.9541$

$\therefore y(0.048)=0.9541$

$\therefore y'(0.048)=-0.9541$

$k_1=-0.0019, k_2=-0.0019, k_3=-0.0019, k_4=-0.0019, y_{25}=0.9522$

$m_1=0.0019, m_2=0.0019, m_3=0.0019, m_4=0.0019, z_{25}=-0.9522$

$\therefore y(0.05)=0.9522$

$\therefore y'(0.05)=-0.9522$

$k_1=-0.0019, k_2=-0.0019, k_3=-0.0019, k_4=-0.0019, y_{26}=0.9503$

$m_1=0.0019, m_2=0.0019, m_3=0.0019, m_4=0.0019, z_{26}=-0.9503$

$\therefore y(0.052)=0.9503$

$\therefore y'(0.052)=-0.9503$

$k_1=-0.0019, k_2=-0.0019, k_3=-0.0019, k_4=-0.0019, y_{27}=0.9484$

$m_1=0.0019, m_2=0.0019, m_3=0.0019, m_4=0.0019, z_{27}=-0.9484$

$\therefore y(0.054)=0.9484$

$\therefore y'(0.054)=-0.9484$

$k_1=-0.0019, k_2=-0.0019, k_3=-0.0019, k_4=-0.0019, y_{28}=0.9465$

$m_1=0.0019, m_2=0.0019, m_3=0.0019, m_4=0.0019, z_{28}=-0.9465$

$\therefore y(0.056)=0.9465$

$\therefore y'(0.056)$=-0.9465

---

k_1=-0.0019,k_2=-0.0019,k_3=-0.0019,k_4=-0.0019,y_{29}=0.9446

m_1=0.0019,m_2=0.0019,m_3=0.0019,m_4=0.0019,z_{29}=-0.9446

$\therefore y(0.058)$=0.9446

$\therefore y'(0.058)$=-0.9446

Some of The last iterations From screenshots :

```
Command Window
 Runge kutta method for 2463 iteration
for h=0.002000
      K          ValuesK        M         ValuesM
   _________   ___________   _________   _________

   'K1'        -0.0072773    'M1'        0.0072773
   'K2'          -0.00727    'M2'          0.00727
   'K3'          -0.00727    'M3'          0.00727
   'K4'        -0.0072627    'M4'        0.0072627
   'Kavg*h'    -1.454e-05    'Mavg*h'    1.454e-05

   ValueOfY     ValueOfYDoubleDash    ValueOfExactY    PercentageOfErrorWithoutDividingByYexact    PercentageOfErrorWithDividingByYexact
   _________    __________________    _____________    ________________________________________    _____________________________________

   0.0072627        -0.0072627          0.0072627                     4.8008e-13                                 6.6103e-11

---------------------------------------------------------------------------------------------------------------------------------------
 Runge kutta method for 2464 iteration
for h=0.002000
      K          ValuesK          M          ValuesM
   _________   ___________   _________   __________

   'K1'         -0.0072627    'M1'         0.0072627
   'K2'         -0.0072555    'M2'         0.0072555
   'K3'         -0.0072555    'M3'         0.0072555
   'K4'         -0.0072482    'M4'         0.0072482
   'Kavg*h'    -1.4511e-05    'Mavg*h'    1.4511e-05

   ValueOfY     ValueOfYDoubleDash    ValueOfExactY    PercentageOfErrorWithoutDividingByYexact    PercentageOfErrorWithDividingByYexact
   _________    __________________    _____________    ________________________________________    _____________________________________

   0.0072482        -0.0072482          0.0072482                     4.7922e-13                                 6.6115e-11
```

```
Command Window
   0.0070199        -0.0070199          0.0070199                     4.6707e-13                                 6.6535e-11

---------------------------------------------------------------------------------------------------------------------------------------
 Runge kutta method for 2481 iteration
for h=0.002000
      K           ValuesK          M          ValuesM
   _________    ___________    _________    __________

   'K1'         -0.0070199    'M1'         0.0070199
   'K2'         -0.0070129    'M2'         0.0070129
   'K3'         -0.0070129    'M3'         0.0070129
   'K4'         -0.0070059    'M4'         0.0070059
   'Kavg*h'    -1.4026e-05    'Mavg*h'    1.4026e-05

   ValueOfY     ValueOfYDoubleDash    ValueOfExactY    PercentageOfErrorWithoutDividingByYexact    PercentageOfErrorWithDividingByYexact
   _________    __________________    _____________    ________________________________________    _____________________________________

   0.0070059        -0.0070059          0.0070059                     4.6638e-13                                 6.6569e-11

---------------------------------------------------------------------------------------------------------------------------------------
```

Command Window

```
 Runge kutta method for 2474 iteration
for h=0.002000
     K         ValuesK       M        ValuesM
  _________   __________   ________   __________

  'K1'        -0.0071189   'M1'        0.0071189
  'K2'        -0.0071118   'M2'        0.0071118
  'K3'        -0.0071118   'M3'        0.0071118
  'K4'        -0.0071047   'M4'        0.0071047
  'Kavg*h'    -1.4224e-05  'Mavg*h'    1.4224e-05

  ValueOfY    ValueOfYDoubleDash   ValueOfExactY   PercentageOfErrorWithoutDividingByYexact   PercentageOfErrorWithDividingByYexact
  _________   __________________   _____________   ________________________________________   _____________________________________

  0.0071047       -0.0071047        0.0071047                    4.7167e-13                                  6.6389e-11

----------------------------------------------------------------------------------------------------------------------------------
 Runge kutta method for 2475 iteration
for h=0.002000
     K         ValuesK       M        ValuesM
  _________   __________   ________   __________

  'K1'        -0.0071047   'M1'        0.0071047
  'K2'        -0.0070976   'M2'        0.0070976
  'K3'        -0.0070976   'M3'        0.0070976
  'K4'        -0.0070905   'M4'        0.0070905
  'Kavg*h'    -1.4195e-05  'Mavg*h'    1.4195e-05

  ValueOfY    ValueOfYDoubleDash   ValueOfExactY   PercentageOfErrorWithoutDividingByYexact   PercentageOfErrorWithDividingByYexact
  _________   __________________   _____________   ________________________________________   _____________________________________

fx 0.0070905      -0.0070905        0.0070905                    4.7089e-13                                  6.6411e-11
```

Command Window

```
  0.0070199       -0.0070199        0.0070199                    4.6707e-13                                  6.6535e-11

----------------------------------------------------------------------------------------------------------------------------------
 Runge kutta method for 2481 iteration
for h=0.002000
     K         ValuesK       M        ValuesM
  _________   __________   ________   __________

  'K1'        -0.0070199   'M1'        0.0070199
  'K2'        -0.0070129   'M2'        0.0070129
  'K3'        -0.0070129   'M3'        0.0070129
  'K4'        -0.0070059   'M4'        0.0070059
  'Kavg*h'    -1.4026e-05  'Mavg*h'    1.4026e-05

  ValueOfY    ValueOfYDoubleDash   ValueOfExactY   PercentageOfErrorWithoutDividingByYexact   PercentageOfErrorWithDividingByYexact
  _________   __________________   _____________   ________________________________________   _____________________________________

  0.0070059       -0.0070059        0.0070059                    4.6638e-13                                  6.6569e-11

----------------------------------------------------------------------------------------------------------------------------------
```


Command Window

```
 Runge kutta method for 2474 iteration
for h=0.002000
     K         ValuesK       M        ValuesM
  _________   __________   ________   __________

  'K1'        -0.0071189   'M1'        0.0071189
  'K2'        -0.0071118   'M2'        0.0071118
  'K3'        -0.0071118   'M3'        0.0071118
  'K4'        -0.0071047   'M4'        0.0071047
  'Kavg*h'    -1.4224e-05  'Mavg*h'    1.4224e-05

  ValueOfY    ValueOfYDoubleDash   ValueOfExactY   PercentageOfErrorWithoutDividingByYexact   PercentageOfErrorWithDividingByYexact
  _________   __________________   _____________   ________________________________________   _____________________________________

  0.0071047       -0.0071047        0.0071047                    4.7167e-13                                  6.6389e-11

----------------------------------------------------------------------------------------------------------------------------------
 Runge kutta method for 2475 iteration
for h=0.002000
     K         ValuesK       M        ValuesM
  _________   __________   ________   __________

  'K1'        -0.0071047   'M1'        0.0071047
  'K2'        -0.0070976   'M2'        0.0070976
  'K3'        -0.0070976   'M3'        0.0070976
  'K4'        -0.0070905   'M4'        0.0070905
  'Kavg*h'    -1.4195e-05  'Mavg*h'    1.4195e-05

  ValueOfY    ValueOfYDoubleDash   ValueOfExactY   PercentageOfErrorWithoutDividingByYexact   PercentageOfErrorWithDividingByYexact
  _________   __________________   _____________   ________________________________________   _____________________________________

fx 0.0070905      -0.0070905        0.0070905                    4.7089e-13                                  6.6411e-11
```

Editor - C:\Users\20112\Desktop\rungekutta_method_second_Output.m

Command Window

```
    'K4'       -0.0067582    'M4'       0.0067582
    'Kavg*h'   -1.353e-05    'Mavg*h'   1.353e-05

    ValueOfY     ValueOfYDoubleDash    ValueOfExactY    PercentageOfErrorWithoutDividingByYexact    PercentageOfErrorWithDividingByYexact
    _________    __________________    _____________    ________________________________________    _____________________________________

    0.0067582        -0.0067582          0.0067582                     4.5328e-13                                6.7072e-11

 ------------------------------------------------------------------------------------------------------------------------------------
  Runge kutta method for 2500 iteration
 for h=0.002000
        K          ValuesK        M          ValuesM
    ________    ___________    ________    __________

    'K1'        -0.0067582     'M1'         0.0067582
    'K2'        -0.0067514     'M2'         0.0067514
    'K3'        -0.0067514     'M3'         0.0067514
    'K4'        -0.0067447     'M4'         0.0067447
    'Kavg*h'    -1.3503e-05    'Mavg*h'    1.3503e-05

    ValueOfY     ValueOfYDoubleDash    ValueOfExactY    PercentageOfErrorWithoutDividingByYexact    PercentageOfErrorWithDividingByYexact
    _________    __________________    _____________    ________________________________________    _____________________________________

    0.0067447        -0.0067447          0.0067447                     4.525e-13                                 6.709e-11

 ------------------------------------------------------------------------------------------------------------------------------------
  after solving the equation to get Yexact=
 (1000*exp(-x))/999 - exp(-1000*x)/999
```

THE REASON FOR THE INCRIDIBLE ERROR:

STIFFNESS AND MULTISTEP METHODS

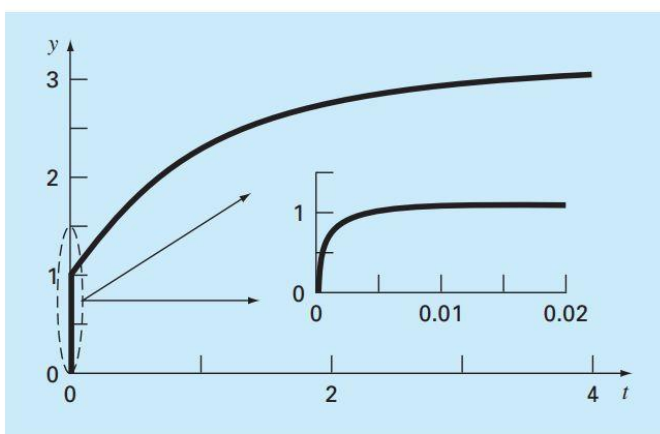


FIGURE 26.1

Plot of a stiff solution of a single ODE. Although the solution appears to start at 1, there is actually a fast transient from $y = 0$ to 1 that occurs in less than 0.005 time unit. This transient is perceptible only when the response is viewed on the finer timescale in the inset.

Stiffness is a special problem that can arise in the solution of ordinary differential equations. A stiff system involves rapidly changing components and slowly changing ones. In many cases, the rapidly varying components are ephemeral transients that die away quickly, after which the solution becomes dominated by the slowly varying components. Although the transient phenomena exist for only a short part of the integration interval, they can dictate the time step for the entire solution.

In mathematics, a stiff equation is a differential equation for which certain numerical methods for solving the equation are numerically unstable unless the step size is taken to be extremely small. It has proven difficult to formulate a precise definition of stiffness, but the main idea is that the equation includes some terms that can lead to rapid variation in the solution.

When integrating a differential equation numerically, one would expect the requisite step size to be relatively small in a region where the solution curve displays many variations and to be relatively large where the solution curve straightens out to approach a line with a slope of nearly zero. For some problems, this is not the case. For a numerical method to give a reliable solution to the differential system sometimes the step size is required to be at an unacceptably small level in a region where the solution curve is very smooth. The phenomenon is known as *stiffness*. In some cases, there may be two different problems with the same solution, yet one is not stiff and the other is. The phenomenon cannot, therefore, be a property of the exact solution, since this is the same for both problems, and must be a property of the differential system itself. Such systems are thus known as *stiff systems*.

As in Fig. 26.1, the solution is initially dominated by the fast exponential term
(e21000t). After a short period (t , 0.005), this transient dies out and the solution becomes
dictated by the slow exponential (e2t).44
If y(0) 5 0, the analytical solution can be developed as
y 5 3 2 0.998e21000t 2 2.002e2t (26.2)
As in Fig. 26.1, the solution is initially dominated by the fast exponential term
(e21000t). After a short period (t , 0.005), this transient dies out and the solution becomes
dictated by the slow exponential (e2t).

Conclusions and comparing results

a) The error increases significantly with the increase in step sizes , for example when we used h=0.2 the results were as follows:

ommand Window

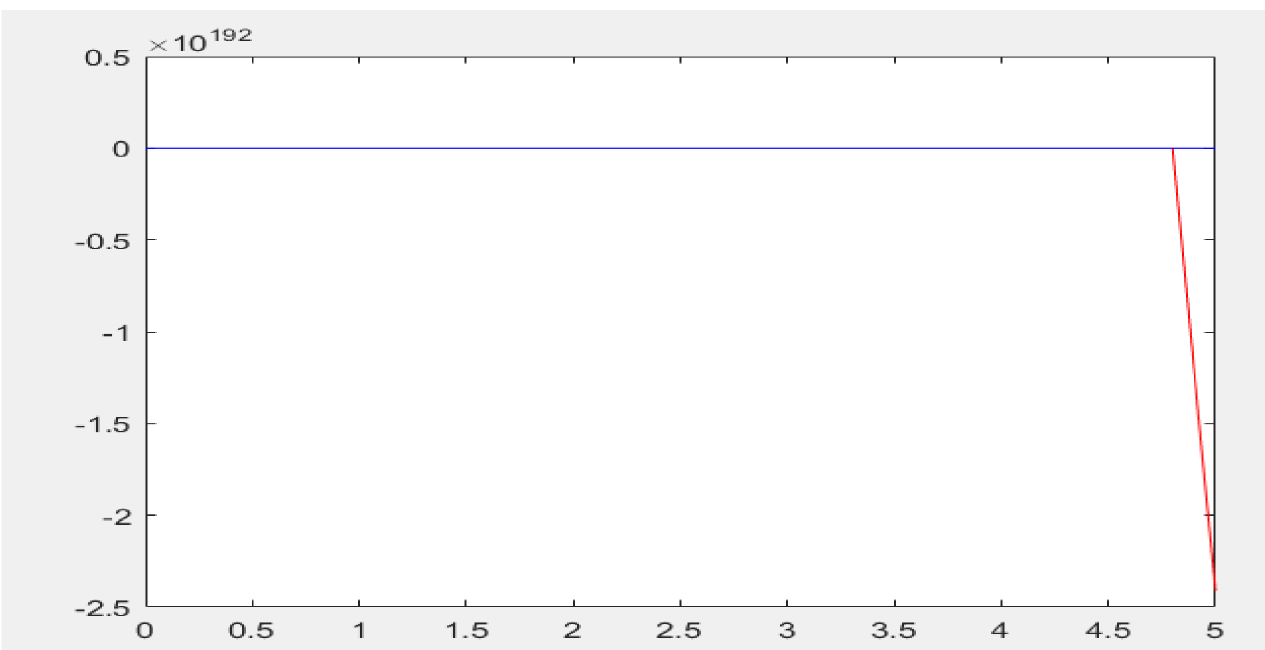
'K1'	5.6437e+179	'M1'	-5.6437e+182
'K2'	-5.5872e+181	'M2'	5.5872e+184
'K3'	5.5878e+183	'M3'	-5.5878e+186
'K4'	-1.1176e+186	'M4'	1.1176e+189
'Kavg*h'	-3.6883e+184	'Mavg*h'	3.6883e+187

ValueOfY	ValueOfYDoubleDash	ValueOfExactY	PercentageOfErrorWithoutDividingByYexact	PercentageOfErrorWithDividingByYexact
-3.6883e+184	3.6883e+187	0.008238	3.6883e+186	4.4772e+188

Runge kutta method for 25 iteration

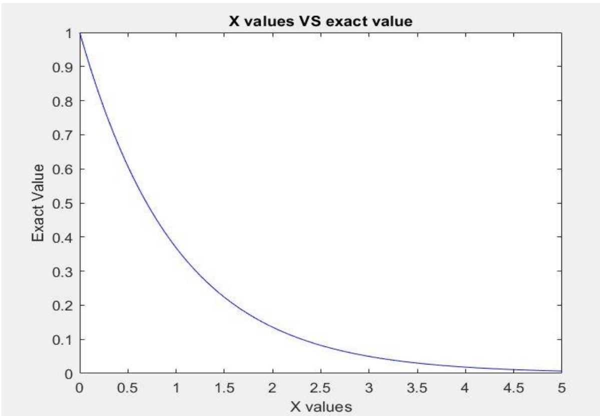
K	ValuesK	M	ValuesM
'K1'	3.6883e+187	'M1'	-3.6883e+190
'K2'	-3.6514e+189	'M2'	3.6514e+192
'K3'	3.6518e+191	'M3'	-3.6518e+194
'K4'	-7.3036e+193	'M4'	7.3036e+196
'Kavg*h'	-2.4104e+192	'Mavg*h'	2.4104e+195

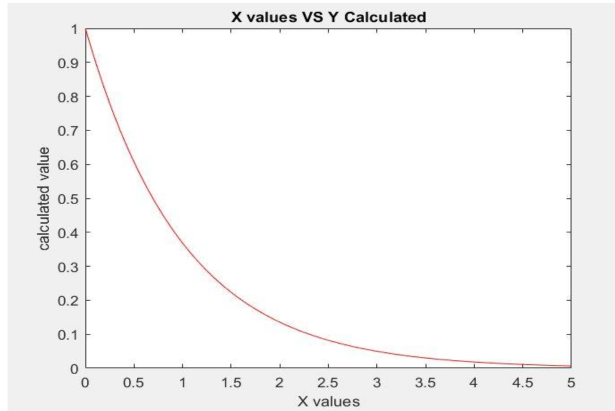
ValueOfY	ValueOfYDoubleDash	ValueOfExactY	PercentageOfErrorWithoutDividingByYexact	PercentageOfErrorWithDividingByYexact
-2.4104e+192	2.4104e+195	0.0067447	2.4104e+194	3.5738e+196

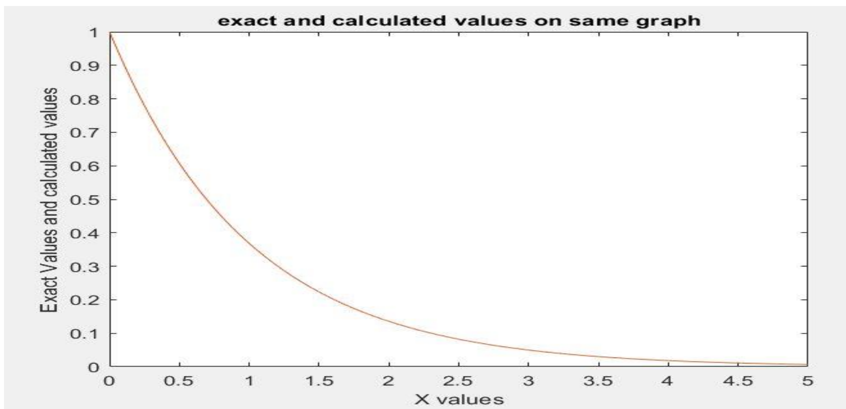


It seems that the error was x10^192

b) But when we estimated the results using smaller step sizes , the Yexact solution and the predicted Ones became nearly the same as shown in the following graphs :







Runge kutta method for 2500 iteration
for h=0.002000

K	ValuesK	M	ValuesM
'K1'	-0.0067582	'M1'	0.0067582
'K2'	-0.0067514	'M2'	0.0067514
'K3'	-0.0067514	'M3'	0.0067514
'K4'	-0.0067447	'M4'	0.0067447
'Kavg*h'	-1.3503e-05	'Mavg*h'	1.3503e-05

ValueOfY	ValueOfYDoubleDash	ValueOfExactY	PercentageOfErrorWithoutDividingByYexact	PercentageOfErrorWithDividingByYexact
0.0067447	-0.0067447	0.0067447	4.525e-13	6.709e-11

----

after solving the equation to get Yexact=
(1000*exp(-x))/999 - exp(-1000*x)/999

Matlab Code

```
%% a) Solve this differential equation Numerically using Runge-Kutta's method
for x = 0 to 5 with a step size of 0.002.
%% Note that the initial conditions are y(0)=1 and y'(0)=0.
%%c) Compare and comment.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%if we are dealing with each value of x as
if it is the X0 with new h greater than the old one with
0.002%%%%%%%%%%%%%%%%%%%%%%%%%
function rungekutta_method
% calculates the numerical values of second order ODE using RK
% you have to enter equations of z and y double dash at the end of this file
% you have to enter intial conditions at the beginings of file
clc % clears the command window
clc % clears the command window
clear % clears the workspace
x0=0; % xn value
y0=1; % yn value
z0=0; % y'n value
counter=1;
n=1;
for i=0:0.002:5
    H(n)=i;
end
for x1=0:0.002:5
    yexact(counter)=(1/999)*(1000*exp((-x1))-exp((-1000*x1)));
    h=x1-x0;
    H(counter)=h;
    k1=ydash(z0);% F( Zn )
    m1=zdash(y0,z0); % G( Yn , Zn )
    k2=ydash(z0+(m1*(h/2)));% F(Zn +(m1×h/2))
    m2=zdash(y0+(k1*(h/2)),z0+(m1*h/2));% G(Yn +(k1×h/2),Zn +(m1×h/2))
    k3=ydash(z0+(m2*(h/2))); % G(Yn +(k2×h/2), Zn +(m2×h/2))
    m3=zdash(y0+(k2*h/2),z0+(m2*(h/2)));% G(Yn +(k2×h/2), Zn +(m2×h/2))
    k4=ydash(z0+m3*h); % F(Yn +(k3×h), Zn +(m3×h))
    m4=zdash(y0+k3*h,z0+m3*h);% G(Yn +(k3×h), Zn +(m3×h))
    h_kavg=(h/6)*(k1+2*k2+2*k3+k4);
    h_mavg=(h/6)*(m1+2*m2+2*m3+m4);
    y1(counter)=y0+h_kavg;
    z1=z0+h_mavg;
    z0=z1;
    x0=x1;
    y0=y1(counter);
    fprintf(' Runge kutta method for %d iteration \n',counter-1);
    fprintf('for h=%f \n',h)
    K={'K1';'K2';'K3';'K4';'Kavg*h'};
    ValuesK=[k1;k2;k3;k4;h_kavg];
    M={'M1';'M2';'M3';'M4';'Mavg*h'};
    ValuesM=[m1;m2;m3;m4;h_mavg];
    ValueOfY=y1(counter);
    ValueOfYDoubleDash=z1;
    ValueOfExactY=yexact(counter);
    t=table(K,ValuesK,M,ValuesM);
```

```
    disp(t);

    erroryWith=(abs((y1(counter)-yexact(counter))/yexact(counter)))*100;
    erroryWithout=(abs((y1(counter)-yexact(counter))))*100;
    error(counter)=erroryWith;
    % disp('THE ERROR PERCENTAGE=');
    % fprintf('%10.2e %% \n',errory);
    counter=counter+1;
    PercentageOfErrorWithoutDividingByYexact=erroryWithout;
    PercentageOfErrorWithDividingByYexact=erroryWith;
    c=table(ValueOfY,ValueOfYDoubleDash,ValueOfExactY,PercentageOfErrorWithou
tDividingByYexact,PercentageOfErrorWithDividingByYexact);
    disp(c);
    disp('-------------------------------------------------------------------
-----------------------------------------------------------------------------
----')
end
n=1;
for i=0:0.002:5
    x1(n)=i;
    n=n+1;
end
figure
plot(x1,yexact,x1,y1)
title('exact and calculated values on same graph');
xlabel('X values');
ylabel('Exact Values and calculated values');
figure
plot(x1,yexact,'b')
title('X values VS exact value');
xlabel('X values');
ylabel('Exact Value');
figure
plot(x1,y1,'r')
title('X values VS Y Calculated');
xlabel('X values');
ylabel('calculated value');
figure
plot(error,H)
title('step size VS error');
xlabel('Error');
ylabel('Step Size');

syms y(x)
Dy = diff(y);
ode = diff(y,x,2) == -1001*Dy-1000*y;
cond1 = y(0) ==1;
cond2 = Dy(0) == 0;
Yexact=dsolve(ode,cond1,cond2);
fprintf(' after solving the equation to get Yexact=\n');
disp(Yexact);
function f=ydash(z)
% enter the equation of y' here
f=z;
function g=zdash(y,z)
% enter the equation of z' here
g=-1001*z-1000*y
```