Computational Physics - Exercise 10

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Exercise 2: 1. Importance Sampling

Integration with equal distributet random numbers

We have to compute the following integral:

$$I = \frac{1}{\pi} \int_{-\infty}^{\infty} \exp(-y_1^2 - y_2^2) dy_1 dy_2 \tag{1}$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \exp(-y_1^2) dy_1 \int_{-\infty}^{\infty} \exp(-y_2^2) dy_2 \tag{2}$$

$$= \frac{1}{\pi} (\int_{-\infty}^{\infty} \exp(-y^2) dy)^2 \tag{3}$$

So the problem is solved after the determination of $\int_{-\infty}^{\infty} \exp(-y^2) dy$. With equally distributed random numbers the integral of a function is

$$I = \frac{b-a}{N} \sum_{i=0}^{N} f(x_i) \tag{4}$$

with a, b finite bounds of integration and x_i equally distributed random numbers inside the bounds of a,b. For creating the x_i's we use our random number generator from exercise sheet 9. They'll be computed in equally distributed in bounds of [0,1] and then adapted to the integration bounds [a,b] with:

$$x_i = a + (b-a) * x_{i,[0,1]} \tag{5}$$

We compute the integral $\int_{-\infty}^{\infty} \exp(-y^2) dy$ using equation 4 with n=100000 number of iterations and [a,b]=[-5,5]:

$$I = 1.72558741019$$

Integration with weighted random numbers

Second we compute the same integral using importance sampling. Here the integral is:

$$I = \frac{1}{N} \sum_{i=0}^{N} \frac{f(x_i)}{g(x_i)} \tag{6}$$

with $g(x_i)$ normalized probability density function, near to $f(x_i)$. We use

$$g(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-y^2/2) dy \tag{7}$$

We compute the integral $\int_{-\infty}^{\infty} \exp(-y^2) dy$ using equation 6 with n=10000 number of iterations and [a,b]=[-5,5]:

$$I = 1.78053316266$$

The theoretical solution is $\int_{-5}^{5} \exp(-y^2) dy = 1.77245....$

Solutions

Now we calculate the results for equation 3 with both solutions of the integral and round them:

$$\begin{aligned} \text{Method equally distributed RN: } \mathrm{I} &= 0.9478 \\ \text{Method weighted RN: } \mathrm{I} &= 1.0091 \\ \text{Theoretical solution: } \mathrm{I} &= 1.0000 \end{aligned}$$

Using importance sampling we get the better solution.