Computational Physics - Exercise 9

Maurice Donner Lukas Häffner

June 24, 2019

1 Random Numbers – Rolling Dice

We write a simple portable random number generator, using linear congruences:

$$I_{j+1} = aI_j + c \pmod{m} \tag{1}$$

For that, we creat an initiation function, that creates a global Variable.

```
def init(initVal):
    global rand
    rand = initVal
```

This variable will now be rewritten over and over again by a number generated by (1):

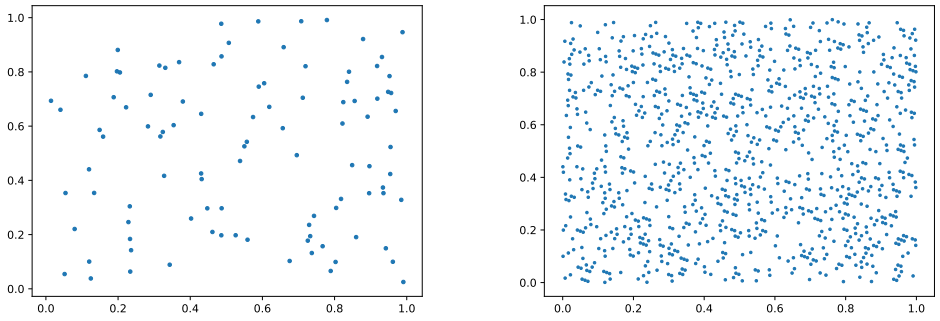
```
def generate_random(a,m,c,initVal):
    global rand
    rand = (a*rand+c)%m
    return rand
```

This function produces homogeneously distributed numbers between 0 and m-1. It can be normalized by $r_i = I_j/(m-1)$, to get homogeneously distributed real numbers between 0 and 1. We try it for example for $a = 106, m = 6075, c = 1283$:

```
Generating random number... a = 106, m = 6075, c = 1283
Random number sequence:
0: 1389
1: 2717
2: 3760
3: 4968
4: 5441
5: 904
6: 5982
7: 3575
8: 3583
9: 4431

Normalizing...
0: 0.22867961804412248
1: 0.4473164306881791
2: 0.6190319394138953
3: 0.8179124135660191
4: 0.8957853144550544
5: 0.14883108330589398
6: 0.9848534738228515
7: 0.5885742509054989
8: 0.5898913401382944
9: 0.7295027988146197
```

A simple way to check 'by eye' that the random number generator really does produce homogeneously distributed numbers, is to create two sequences with different initial values I_0 and J_0. Let $\mathbf{r}$ be a random normalized number sequence generated with $I_0 = 1$ and $\mathbf{s}$ generated with $J_0 = 2$. We plot all pairs (r_i, s_i) of generated numbers in a number plane:



(a) $n = 100$ (b) $n = 1000$

Figure 1: Creating a set of random numbers

Since the eye is quite sensitive to see a good distribution. For $m = 100$ there is no visible pattern. Taking 1000 samples, will result in numbers, that appear random at first, but this time there are small patterns forming, that look like little lines all pointing towards the same direction. To see the deterministic character of this random number sequence, we plot I_{j+1} agains I_j for $I_0 = 1,$ and $n = 1000$:

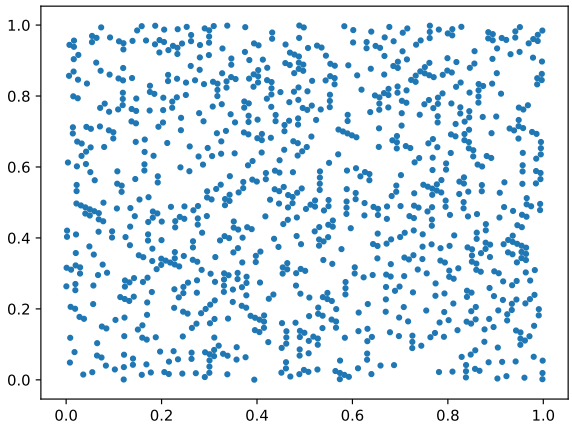


Figure 2: Deterministic Character of random number sequence

Again, overall the generator appears random, yet small patterns can still be seen. This effect can be midigated, by choosing larger values for a, m, and c. For example, for $a = 1060, m = 60751, c = 12835$, the small patterns, formerly visible in the deterministic character of our sequence vanishes:

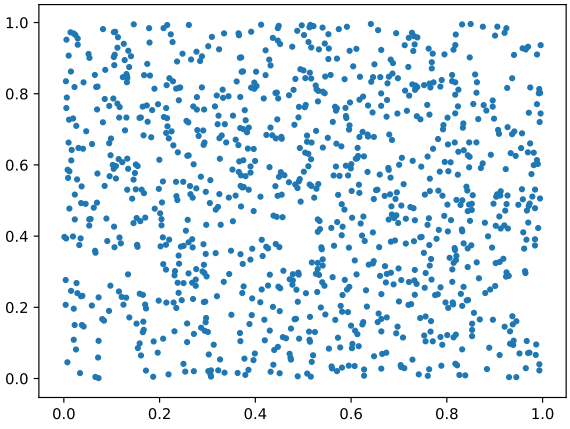


Figure 3: Choosing larger values → The plot becomes ”more random”