Introduction to Computational Physics SS2019

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Exercise 6 from May 29, 2019
Return by noon of June 7, 2019

1 Mathematica exercises

- Repeat and vary the `Mathematica` examples in Section 3.2.6 of the 2008 lecture manuscript (link in our webpage). Solve the Volterra-Lotka system as explained there.
- Use the help function of `Mathematica` to get information about the functions used.
- Solve the 2-body problem with `Mathematica`. Do this by solving the differential equations for the vector components in 2 dimensions and plot the resulting objects (the orbits).

2 Population dynamics (homework)

In this exercise we study the following equation for population dynamics:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN(1 - N/K) - \frac{BN^2}{A^2 + N^2} \tag{2.1}$$

where all parameters r, K, A and B are positive.

1. (10 pt) Dimensional analysis: Determine the dimension of the parameters and rewrite the equation in dimensionless form. Note that there are different possibilities. Please formulate a dimensionless time τ that is *not* defined on the basis of r. Use $n = N/A$ as the dimensionless version of N.
2. (10 pt) Determine the stationary points n^* for $K/A = 7.5$. Note that for $n^* \neq 0$ these values are solutions of a cubic equation; it depends on n and the remaining free parameter. The cubic equation should be derived by yourself analytically; its zero points you can obtain numerically / graphically by using e.g. `Mathematica`. When do one or three real solutions exist as a function of the remaining free parameter? (Hint: we do not ask for some analytical formula here! It is enough to vary the free parameter and check using `Mathematica` which three solutions for the stationary points you get; as stationary points only real solutions are valid. Only one digit after the comma is enough, in other words you vary the free parameter by about 0.05.). Which of the stationary points is stable and unstable?