Recap:

$$\langle f\rangle_p = \int_a^b f(x)\,p(x)\,dx \quad \text{with PDF } p(x)$$

normalized $1 = \int_a^b p(x)\,dx$

$$\bar{f}_N = \frac{1}{N}\sum_{i=1}^{N} f(x_i) \quad \text{with } x_i \text{ from PDF } p(x_i)$$

is an approximation for $\langle f\rangle_p$

Special Case: $p(x) = \text{const.} = \frac{1}{b-a}$

$$\langle f\rangle_p = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{b-a}\,I$$

Integral

$$\bar{f}_N = \frac{1}{N}\sum_{f(x_i)} f(x_i) \quad \text{with } x_i \text{ from PDF } p(x_i)$$

is approximation for $\langle f\rangle_p = I/b-a$

Importance Sampling:

$$I = \int_a^b f(x)\,dx = \int_a^b \frac{f(x)}{g(x)}\,g(x)\,dx$$

with normalized PDF $g(x)$: $\int_a^b g(x)\,dx$

$\left\langle \frac{f}{g}\right\rangle_g = I$; is approximated by

$$I_N = \frac{1}{N}\sum_{i=1}^{N} \frac{f(x_i)}{g(x_i)} \quad \text{with } x_i \text{ from PDF } g(x_i)$$