Particle Physics - Exercise Sheet 05

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December 9, 2020

1 Polarized beams

a) Since we are at high energies, we can assume that $E \gg m$.
The chiral nature of QED states, that only two of the four helicity combinations lead to a non-zero 4-vector current. The initial and final state vector currents are

$$\begin{aligned}
e^- : \theta = \varphi = 0 \quad e^+ : \theta = \varphi = \pi \\
j_{(e),\mathrm{RL}} &= 2E\,(0, -1, i, 0) \\
j_{(e),\mathrm{RR}} &= (0,0,0,0) \\
j_{(e),\mathrm{LL}} &= (0,0,0,0) \\
j_{(e),\mathrm{LR}} &= 2E\,(0, -1, -i, 0) \\
\mu^- : \theta, \varphi = 0 \quad \mu^+ : \theta - \pi, \varphi = \pi \\
j_{(\mu),\mathrm{RL}} &= 2E\,(0, -\cos(\theta), i, \sin(\theta)) \\
j_{(\mu),\mathrm{RR}} &= (0,0,0,0) \\
j_{(\mu),\mathrm{LL}} &= (0,0,0,0) \\
j_{(\mu),\mathrm{LR}} &= 2E\,(0, -\cos(\theta), -i, \sin(\theta))
\end{aligned} \tag{1}$$

which lead to the non-zero matrix elements

$$|M_{\mathrm{RL}\to\mathrm{RL}}| = -\frac{e^2}{q^2}\; j_{e,\mathrm{RL}}\; g_{\mu\nu}\; j_{\mu,\mathrm{RL}} = e^4\,[1 + \cos(\theta)] \tag{2}$$

$$|M_{\mathrm{RL}\to\mathrm{LR}}| = -\frac{e^2}{q^2}\; j_{e,\mathrm{RL}}\; g_{\mu\nu}\; j_{\mu,\mathrm{LR}} = e^4\,[1 - \cos(\theta)] \tag{3}$$

$$|M_{\mathrm{LR}\to\mathrm{RL}}| = -\frac{e^2}{q^2}\; j_{e,\mathrm{LR}}\; g_{\mu\nu}\; j_{\mu,\mathrm{RL}} = e^4\,[1 - \cos(\theta)] \tag{4}$$

$$|M_{\mathrm{LR}\to\mathrm{LR}}| = -\frac{e^2}{q^2}\; j_{e,\mathrm{LR}}\; g_{\mu\nu}\; j_{\mu,\mathrm{LR}} = e^4\,[1 + \cos(\theta)] \tag{5}$$

Lastly, we will use the formula for the total cross section to compare production rates

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}\int |M_{fi}|\, d\Omega \tag{6}$$

- If both beams are unpolarized, we have to consider all possible initial to final state configurations

$$\langle |M_{fi}| \rangle = \frac{1}{4} \left(|M_{\mathrm{RL}\to\mathrm{RL}}|^2 + |M_{\mathrm{RL}\to\mathrm{LR}}|^2 + |M_{\mathrm{LR}\to\mathrm{RL}}|^2 + |M_{\mathrm{LR}\to\mathrm{LR}}|^2 \right) \tag{7}$$

  Plugging into (6) yields the known cross section for the annihilation process

$$\sigma = \frac{4\pi\alpha^2}{3s} \tag{8}$$

- If the electron beam has left-handed polarization, one of the two allowed initial state is not possible. Therefore there are only two possible states in total, so we only need to average over two states this time

$$\langle |M_{fi}| \rangle = \frac{1}{2} \left(|M_{\mathrm{LR}\to\mathrm{RL}}|^2 + |M_{\mathrm{LR}\to\mathrm{LR}}|^2 \right) \tag{9}$$

  The total cross section is the same as for an unpolarized beam: $\sigma = 4\pi\alpha^2/3s$

- If the electron beam is left-handed and the positron beam is right handed, there is only one possible initial state. However, there are two possible final states:

$$\langle |M_{fi}| \rangle = \frac{1}{1} \left(|M_{\mathrm{LR}\to\mathrm{RL}}|^2 + |M_{\mathrm{LR}\to\mathrm{LR}}|^2 \right) \tag{10}$$

  Hence the total cross section yields $8\pi\alpha^2/3s$ and is a facter two larger than for the unpolarized beam.

- Having both the electron and the positron beam polarized into the same direction, yields an initial state that is forbidden

$$|M_{\mathrm{LL}}| = \underbrace{|M_{\mathrm{LL}\to\mathrm{LL}}|}_{=0} + \underbrace{|M_{\mathrm{LL}\to\mathrm{LR}}|}_{=0} + \underbrace{|M_{\mathrm{LL}\to\mathrm{RL}}|}_{=0} + \underbrace{|M_{\mathrm{LL}\to\mathrm{RR}}|}_{=0} = 0 \tag{11}$$

  Therefore the cross section of the matrix element $|M_{fi}|$ is also equal to zero. No production is observed.

b) To calculate the fraction of right- to left-handed μ^-, we look exemplary at the integral over the single matrix elements

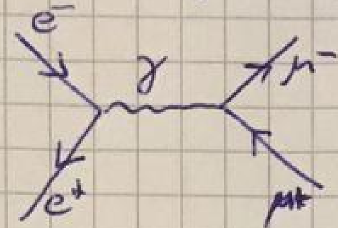
$$\int |M_{\mathrm{LR}\to\mathrm{LR}}|^2 = \int_{-1}^{1} e^4 \left(1 + \cos(\theta)^2\right) d(\cos\theta) = \frac{8}{3}$$
$$\int |M_{\mathrm{LR}\to\mathrm{RL}}|^2 = \int_{-1}^{1} e^4 \left(1 - \cos(\theta)^2\right) d(\cos\theta) = \frac{8}{3} \tag{12}$$
$$\tag{13}$$

The fraction of $\mu^-_\downarrow$ and $\mu^-_\uparrow$ is therefore equal to 1.

5.2 Experimental test of QED

$e^+ + e^- \to \mu^+ + \mu^-$

a)



b)

$$-i\frac{g_{\mu\nu}}{q^2} \to -i\frac{g_{\mu\nu}}{q^2}\left(1 \pm \frac{q^2}{\Lambda^2 - q^2}\right)$$

$$\sigma = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i}\int |M_{fi}|^2_{new}\, d\Omega$$

$$|M_{fi}|^2_{new} = |M_{fi}|^2_{old}\cdot\left(1 \pm \frac{q^2}{\Lambda^2 - q^2}\right)^2$$

$$\Rightarrow \sigma_{new} = \frac{4\pi\alpha^2}{3s}\left(1 \pm \frac{q^2}{\Lambda^2 - q^2}\right)^2$$

$$R_{\mu\mu} = \frac{\sigma_{new}}{\sigma_{QED}} = \left(1 \pm \frac{q^2}{\Lambda^2 - q^2}\right)^2$$

c) $$0{,}1 > \left(1 \pm \frac{q^2}{\Lambda^2 - q^2}\right)^2 - 1$$

$$\sqrt{1{,}1} > 1 \pm \frac{q^2}{\Lambda^2 - q^2}$$ (only look at positiv)

$$\sqrt{1{,}1} - 1 > \frac{q^2}{\Lambda^2 - q^2}$$ (assume $\Lambda^2 > q^2$)

$$(\sqrt{1{,}1} - 1)(\Lambda^2 - q^2) > q^2$$

$$(\sqrt{1{,}1} - 1)\Lambda^2 > q^2(\sqrt{1{,}1})$$

$$\Lambda^2 > q^2\frac{\sqrt{1{,}1}}{\sqrt{1{,}1} - 1}$$

$$\Lambda > \sqrt{s}\left(\frac{\sqrt{1{,}1}}{\sqrt{1{,}1} - 1}\right)^{\frac{1}{2}}$$

$$\Lambda > \sqrt{s}\cdot 171{,}5$$

5.4

a) $q^2 < 0$

In Proton restframe ($p = (m_p, 0, 0, 0)$)

$k = (E, 0, 0, E)$

$k' = (E', 0, \sin\theta E', \cos\theta E')$

$$q^2 = (k-k')^2$$

$$= \underbrace{k^2 + k'^2}_{\substack{=0 \\ \text{because we} \\ \text{assume } m_e = 0}} - 2kk' = -2\left(EE' - (EE'\cos\theta)\right)$$

$$= -2EE'\underbrace{(1-\cos\theta)}_{>0,\text{ for } \cancel{\theta \neq 2k\pi + \frac{\pi}{2}\ k\in\mathbb{Z}}\ \theta \neq 2k\pi\ k\in\mathbb{Z}}$$

$\Rightarrow q^2 < 0$

b) $x = \frac{Q^2}{2pq}$

$$W^2 = (p+q)^2 = p^2 + q^2 + 2pq$$

$$\Rightarrow W^2 + Q^2 - m_p = 2pq$$

$$x = \frac{Q^2}{W^2 + Q^2 - m_p}$$

$\Rightarrow 0 \le x \le 1$, if $0 \le Q^2 \le W^2 - m_p + Q^2$

$$\Rightarrow \underbrace{-Q^2 \le 0}_{\text{true (see a))}} \underbrace{\le W^2 - m_p}$$

because the proton is the lightest baryon the resulting state X has to have the same or a higher invariant mass than m_p, thus b) and d) $W^2 \ge m_p$ are true

c) $y = \frac{p \cdot q}{p \cdot k}$

In Proton restframe: (like in a))

$$y = \frac{m_p (E - E')}{m_p \cdot E} = 1 - \frac{E'}{E}$$

$\Rightarrow$ Because $\frac{E'}{E} \ge 0$ and $E' \le E$

$0 \le y \le 1$