Particle Physics – Exercise Sheet 4 – WS 2020/21

Lecturer: Prof. Dr. Hans-Christian Schultz-Coulon
hand in until December 2nd (online)
discussed on December 4th in the exercise groups

4.1 Cross Sections

The total reaction cross section of $e^+e^- \to \mu^+\mu^-$ scattering is well described by the first-order QED cross section.

a) How many muons do you expect to be produced per day at an e^+e^--collider with luminosity $\mathrm{L} = 10^{30}\ \mathrm{cm}^2\mathrm{s}^{-1}$ at a beam energy of 10 GeV?

b) How many muon pairs can be detected with a typical collider detector that features full azimuthal coverage and a polar angle acceptance of $30^\circ < \theta < 150^\circ$. Assume a detection efficiency of $\epsilon = 90\%$ for a muon with momentum above 3 GeV.

4.2 $e^-\mu^-$ Scattering (part 1)

Consider a beam of muons shot at a thin target. We are interrested in the scattering off electrons.

a) Draw the dominant Feynman diagram for $e^-\mu^- \to e^-\mu^-$ scattering.

b) Using the Feynman rules for QED, calculate the matrix element $(-i\mathcal{M})$ for this process. Use the following notation: p_1 and p_3 for the incoming and outgoing electron 4-vectors, respectively. p_2 and p_4 for the incoming and outgoing μ^- 4-vectors, respectively. μ and ν for the indices at the electron and μ^- vertices, respectively.

c) Analogously to $e^-e^+ \to \mu^-\mu^+$ annihilation as discussed in the lecture, draw the allowed helicity states for $e^-\mu^- \to e^-\mu^-$ scattering. Assume the center-of-mass energy is much greater than the mass of the electron or μ^-.

4.3 $e^-\mu^-$ Scattering (part 2)

Consider $e^-\mu^- \to e^-\mu^-$ scattering as in problem 4.2. Assume that the center-of-mass energy is much greater than the mass of the electron or μ^-. For that limit the 4-vectors for the initial- and final-state electrons are given in the centre-of-mass frame by $p_1 = (E, 0, 0, E)$ and $p_3 = (E, E\sin\theta, 0, E\cos\theta)$.

a) Calculate the electron and μ^- currents for the allowed helicity states in problem 4.2c).

b) Show that $|\mathcal{M}_{RR}|^2$ is given by

$$\frac{4\alpha^2 s^2}{t^2}$$

c) Calculate the equivalent expressions for the other allowed helicity states and find $|\mathcal{M}_{fi}|^2$. *Hint:* Use the ultra-relativistic limit for the spinors given in problem 3.3.

d) Show that $|\mathcal{M}_{fi}|^2$ expressed in terms of the Mandelstam variables is

$$\langle|\mathcal{M}_{fi}|^2\rangle = 2\alpha^2 \left(\frac{s^2 + u^2}{t^2}\right)$$

e) Show that the differential cross section in the centre of mass frame is

$$\frac{d\sigma}{d\Omega} = \frac{2\alpha^2}{s} \cdot \frac{1 + \frac{1}{4}(1 + \cos\theta)^2}{(1 - \cos\theta)^2}$$

4.4 Chirality Conservation in Electromagnetic Interactions

In QED, the fermion current at a vertex is given by

$$j^\mu = \overline{\Psi}\gamma^\mu\Phi,$$

where Ψ is the wave function of the incoming fermion and Φ the wave function of the outgoing fermion. The both wave functions can be split up in their right and left handed chirality components,

$$\Psi = \Psi_R + \Psi_L \text{ and } \Phi = \Phi_R + \Phi_L,$$

thus the fermion current can be written as

$$j^\mu = (\overline{\Psi_R} + \overline{\Psi_L})\gamma^\mu(\Phi_R + \Phi_L).$$

Show that only two of the four possible chirality combinations contribute to the fermion current.