## Ex 6.1 Deep inelastic scattering

$e^- p \to e^- X$

$E_e = 27{,}5\ \mathrm{GeV}$ ; $E_p = 820\ \mathrm{GeV}$

$E_e' = 72\ \mathrm{GeV}$

a) show that:

$$x = \frac{E_e'}{E_p}\left(\frac{1-\cos\theta}{2-\frac{E'}{E_e}(1+\cos\theta)}\right)$$

$$x = \frac{Q^2}{2p_2\cdot q} = \frac{-q^2}{2p\cdot q} \qquad \text{with } q = (p_1 - p_3)$$

$p_1 = (E_e, 0, 0, E_e)$ , $p_3(E_e', E_e'\sin\theta, 0, E_e'\cos\theta)$

$p_2 = (E_p, 0, 0, -E_p)$ $\quad p_1 - p_3 = (E_e - E_e', -E_e'\sin\theta, E_e - E_e'\cos\theta)$

$$x = \frac{p_1 p_3}{p_2(p_1-p_3)} = \frac{E_e E_e'(1-\cos\theta)}{E_p(E_e-E_e') + E_p(E_e - E_e'\cos\theta)}$$

$$= \frac{E_e E_e'(1-\cos\theta)}{E_p(E_e-E_e')(1+\cos\theta)}$$

$$= \frac{E_e'(1-\cos\theta)}{E_p\left(1-\frac{E_e'}{E_e}\right)(1+\cos\theta)} = \frac{E_e'}{E_p}\left(\frac{1-\cos\theta}{\phantom{x}}\right)$$

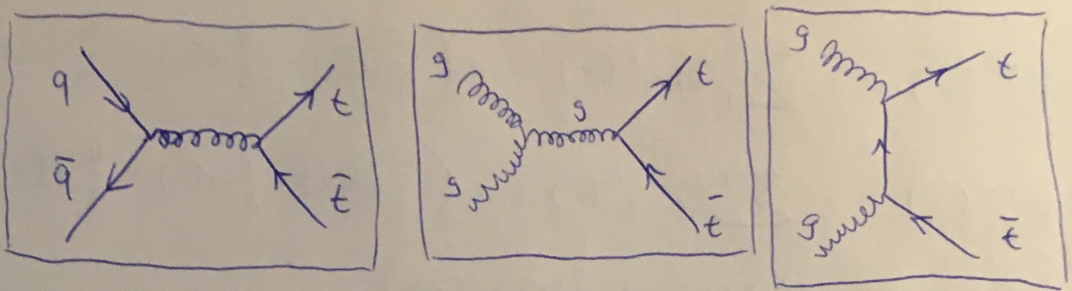
$$= \frac{E_e E_e'(1-\cos\theta)}{E_p(E_e - E_e' + E_e - E_e'\cos\theta)}$$

$$= \frac{E_e E_e'(1-\cos\theta)}{E_p(2E_e - E_e'(1+\cos\theta))}$$

$$= \frac{E_e'}{E_p}\left(\frac{1-\cos\theta}{2-\frac{E_e'}{E_e}(1+\cos\theta)}\right)$$

6.2

a)




b) $Q^2 = -q^2 \qquad x = \frac{Q^2}{2 p_2 q}$

$q^2 = (350\ \text{GeV})^2 = 122500\ \text{GeV}^2$ two times $m_t$

**Tevatron**

$p_2 \approx E_2 = \frac{1.96}{2}$ TeV

$\Rightarrow x = 0.35$

**LHC**

$p_2 \approx E_2 = 7$ TeV

$\Rightarrow x = 0.05$

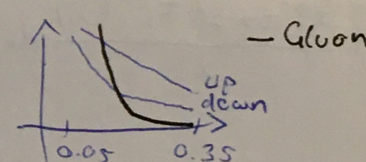
Center of mass energy of parton - parton collision

$\sqrt{\hat{s}} = 350$ GeV

(Minimum Energy required if $f_u = 0.18$: 1.94 TeV) → close to TeVatr

c)

| x | Gluon | up | down | all |
|---|---|---|---|---|
| 0.05 | 1.973 | 0.644 | 0.436 | ≈3 |
| 0.35 | 0.027 | 0.236 | 0.035 | 0.358 |




d) Tevatron: Gluon 3x higher contribution

LHC : up : down : gluon ≈ 10 : 4 : 1

e) $\frac{d^2\sigma}{dx\,dQ} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right] \sum Q_i^2 q_i^p(x)$ $\quad (*)\ y = \frac{p_2 q}{p_2 p_1} \approx 1$

$$= \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2^{pp}(x,Q^2)}{x} + y^2 F_1^{pp}(x,Q^2)\right]$$

$$\overset{(*)}{\approx} \frac{4\pi\alpha^2}{Q^4} F_1^{pp}(x,Q^2)$$ → Cross section at Tevatron is ≈8x Higher

6.3

a) $F_2^{ep}(x) = x \sum_i Q_i^2 q_i^p(x) \approx x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)\right)$

$F_2^{en}(x) = x \sum_i Q_i^2 q_i^n(x) \approx x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}\bar{u}(x)\right)$

b) $F_2^{eD}(x) \approx x\frac{5}{9}\left(d(x) + u(x) + \bar{d}(x) + \bar{u}(x)\right)$

$\int F_2^{eD}(x) = \frac{1}{4}(f_d + f_u) \qquad F_2^{ep}(x) = \frac{4}{9}f_u + \frac{1}{9}f_d$

$$R = \frac{9f_d + 9f_u}{16 f_u + 4f_d} = \frac{f_d\left(9 + 9\frac{f_u}{f_d}\right)}{f_d\left(16\frac{f_u}{f_d} + 4\right)} = \frac{9 + 9\frac{f_u}{f_d}}{4 + 16\underbrace{\frac{f_u}{f_d}}_{=a}} \Rightarrow \frac{f_d}{f_u} = 0.79$$

$$\cancel{\Rightarrow (4+16a)R = (9+9a) \Rightarrow 4R - 9 = 16aR + 9a}$$

$$\cancel{\Rightarrow a = \frac{4R-9}{16R+9} = \quad R = 0.84}$$

c) $f_u = 0.36 \Rightarrow f_d = 0.28 \Rightarrow$ 64% of the momentum is carried by quarks. The rest by gluons

d) $\int_0^1 \frac{F_2^{ep}(x) - F_2^{en}(x)}{x}\,dx = \cancel{\frac{1}{3}f_u - \frac{1}{3}f_d}$

e) $\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{4d_v(x) + u_v(x)}{4u_v(x) + d_v(x)} \rightarrow \frac{1}{4}$ for $x \rightarrow 1$

$\Rightarrow \frac{d_v(x)}{u_v(x)} \rightarrow 0$ as $x \rightarrow 1$

Proton: if $d_v(x) = \frac{1}{2}u_v(x)$, $\frac{F_2^{en}(x)}{F_2^{ep}(x)} \rightarrow \frac{2}{3}$ ↯