Units, cross sections, luminosities

1 Natural units

We have been used to using units in which times are measured in seconds and distances in meters. In such units the speed of light takes the value close to $3 \times 10^8\,\mathrm{ms}^{-1}$.

We could instead have chosen to use unit of time in seconds and distance in light-seconds. In such units the speed of light takes the value $c = 1$ light-second per second, and so c can be left out of equations, provided we are careful to remember the units we are working in.

What is the effect of having units in which $c = 1$? They are going to be useful in relativistic systems, since now the relativistic equations for a particle are

$$
\begin{aligned}
E &= \gamma m \\
p &= \gamma m v \\
E^2 - p^2 &= m^2
\end{aligned}
$$

So for a relativistic system setting $c = 1$ means that energy, mass and momentum all have the same dimensions.

For relativistic systems we have seen that it is useful to choose $c = 1$. Since we are interested in quantum systems, we can go further and look for units for which $\hbar$ is also 1. In such units the angular momentum of a particle $s(s+1)\hbar^2$ will simplify to $s(s+1)$. What quantites does $\hbar$ relate? Remember the time-energy uncertainty relationship $\Delta E\,\Delta t \sim \hbar$. Setting $\hbar = 1$ means that time (and hence distance) must have the same dimensions as E^{-1}.

So in our system **natural units** we have have that

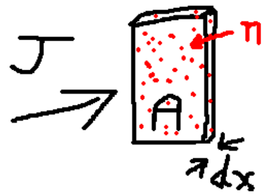
$$\boxed{[\mathrm{Mass}] = [\mathrm{Energy}] = [\mathrm{Momentum}] = [\mathrm{Time}]^{-1} = [\mathrm{Distance}]^{-1}}$$

We are going to use units of energy for all of the quantities above. The nuclear energy levels have typical energies of the order of 10^6 electron-volts, so we shall measure energies and masses in MeV, and lengths and times in MeV^{-1}. At the end of a calculation how can we recover a "real" length from one measured in MeV^{-1}? We can use the conversion factor

$$\boxed{\hbar c \approx 197\ \mathrm{MeV\,fm}}$$

which tells us that one of our MeV^{-1} length units corresponds to $197\ \mathrm{fm}$ where $1\ \mathrm{fm} = 10^{-15}\,\mathrm{m}$.

2 Cross sections



Many experiments take the form of scattering a beam of projectiles into a target.

Provided that the target is sufficiently thin that the flux is approximately constant within that target, the rate of any reaction W_i will be proportional to the flux of incoming projectiles J (number per unit time) the number density of scattering centres n (number per unit volume), and the width of the target

$$W_i = \sigma_i n J \, \delta x$$

where σ_i which has dimensions of area. and is referred to as the *cross section* for process i. We can rewrite the above equation as

$$W_i = \underbrace{(n \, A \, \delta x)}_{N_{\mathrm{target}}} \, J \, \underbrace{\frac{\sigma}{A}}_{P_{\mathrm{scatt}}}$$

where A is the area of the target, which shows that the cross section as the effective area presented to the beam per target for a particular reaction can be expected to occur.

The total rate of loss of beam is given by $W = \Sigma W_i$, and the corresponding total cross section is

$$\sigma = \sum_i \sigma_i .$$

We could chose to quote cross sections in units of e.g. fm^2 or GeV^{-2}, however the most common unit used in nuclear and particle physics is the so-called **barn** where

$$\boxed{1 \, \mathrm{barn} = 10^{-28} \, \mathrm{m}^2}$$

We can convert this to MeV^{-2} units using the usual $\hbar c$ conversion constant as follows

$$\begin{aligned} 1 \, \mathrm{barn} &= 10^{-28} \, \mathrm{m}^2 \\ &= 100 \, \mathrm{fm}^2 / (197 \, \mathrm{MeV} \, \mathrm{fm})^2 \\ &= 0.00257 \, \mathrm{MeV}^{-2} . \end{aligned}$$

3 Luminosity

In a **collider** – a machine which collides opposing beams of particles – the rate of any particular reaction will be proportional to the cross section for that reaction and on various other parameters which depend on the machine set-up. Those parameters will include the number of particles per bunch, their spatial distribution, and the frequency of the collision of those bunches.

We call the constant of proportionality which encompasses all those machine effects the **luminosity**

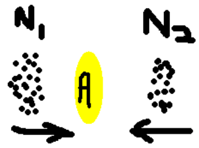
$$\boxed{\mathcal{L} = \frac{W}{\sigma}.}$$

It has dimensions of $[L]^{-2}[T]^{-1}$ and is useful to factor out if you don't care about the details of the machine and just want to know the rates of various processes. The time-integrated luminosity times the cross section gives the expected count of the events of any type

$$N_{\text{events, i}} = \sigma_i \int \mathcal{L}\, dt\ .$$

For a machine colliding opposing bunches containing N_1 and N_2 particles at rate f, you should be able to show that the luminosity is

$$\boxed{\mathcal{L} = \frac{N_1 N_2 f}{A},}$$



where A is the cross-sectional area of each bunch (perpendicular to the beam direction).

We've assumed above that the distributions of particles within each bunch is uniform. If that's not the case (e.g. in most real experiments the beams have approximately Gaussian profiles) then we will have to calculate the effective overlap area A of the bunches with an integral.

Key concepts

- In **natural units**, $\hbar = c = 1$ and

$$[\text{Mass}] = [\text{Energy}] = [\text{Momentum}] = [\text{Time}]^{-1} = [\text{Distance}]^{-1}$$

- A conversion constant worth having to hand is

$$\boxed{\hbar c \approx 197 \text{ MeV fm}}$$

- The **cross section** σ has dimensions of area is defined by

$$\boxed{W = n\sigma J\, \delta x}$$

- Cross sections for sub-atomic physics are often expressed in the unit of **barns**.

$$\boxed{1 \text{ barn} = 10^{-28} \text{ m}^2}$$

- The **luminosity** of a colliding-beam machine is the rate per unit cross-section

$$\boxed{\mathcal{L} = \frac{W}{\sigma} = \frac{N_1 N_2 f}{A}.}$$