3.2

$$C\psi = \psi_c = i\gamma^2\psi^* \qquad \gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}$$

a) $\psi_1 = e^{i(px-Et)} u_1(p)$ $\qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix} \qquad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\psi_2 = e^{i(px-Et)} u_2(p)$ $\qquad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & +i & 0 \\ 0 & +i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$C\psi_1 = \psi_{c1}$

$$= i\gamma^2\psi_1^* = i \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} e^{i(px-Et)} N \begin{pmatrix} 1 \\ 0 \\ p_z/N^2 \\ p_x+ip_y/N^2 \end{pmatrix}$$

$$= iNe^{i(px-Et)} \begin{pmatrix} -ipx \mp p_y/N^2 \\ ip_z/N^2 \\ 0 \\ -i \end{pmatrix} = \begin{pmatrix} p_x+ip_y/N^2 \\ -p_z/N^2 \\ 0 \\ 1 \end{pmatrix} = e^{-i(px-Et)} v_1(p)$$

$$\hat{=} \overline{\psi_1}$$

$C\psi_2 = \psi_{c2}$

$$= i\gamma^2\psi_2^* = i \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} e^{-i(px-Et)} N \begin{pmatrix} 0 \\ 1 \\ p_x+ip_y/N^2 \\ -p_z/N^2 \end{pmatrix}$$

$$= Nie^{-i(px-Et)} \begin{pmatrix} ip_z/N^2 \\ ip_x - p_y/N^2 \\ i \\ 0 \end{pmatrix} = Ne^{-i(px-Et)} \overbrace{\begin{pmatrix} -p_z/N^2 \\ -p_x - ip_y/N^2 \\ -1 \\ 0 \end{pmatrix}}^{-v_2(p)}$$

$$\hat{=} -\overline{\psi_2}$$

The Conjugation operator transforms a particle wave function to its anti-particle wave function. (for u_2, a phase shift of $180°$ occurs)

b) $A^\mu = (\phi, \vec{A})$

$\gamma^\mu(\partial_\mu - ieA_\mu)\psi + im\psi = 0$

$(\Leftrightarrow \gamma^\mu\partial_\mu\psi - \gamma^\mu ieA_\mu\psi + im\psi = 0)$ $\qquad \gamma_\mu = (\gamma^0, \gamma^1, \gamma^2, \gamma^3)$

$\gamma^0 = \begin{pmatrix} \mathbb{1} & \\ & -\mathbb{1} \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}$

$\gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}$

$\gamma^0\gamma^2 = \begin{pmatrix} \mathbb{1} & \\ & \mathbb{1} \end{pmatrix}\begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} = -\gamma^0\gamma^2$

$\gamma^1\gamma^2 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} = \begin{pmatrix} -\sigma_1\sigma_2 & 0 \\ 0 & -\sigma_1\sigma_2 \end{pmatrix} = -i\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} = -\gamma^2\gamma^1$

$\gamma^2\gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} = \begin{pmatrix} -\sigma_2\sigma_2 & 0 \\ 0 & -\sigma_2\sigma_2 \end{pmatrix} = -\begin{pmatrix} \mathbb{1} & \\ & \mathbb{1} \end{pmatrix}$

$\gamma^3\gamma^2 = +i\begin{pmatrix} \sigma_1 & \\ & \sigma_1 \end{pmatrix} = -\gamma^3\gamma^2$

$(\gamma^\mu)^* = \gamma^\mu$ for $\mu = 0, 1, 3$ and $(\gamma^2)^* = -\gamma^2$