Particle Physics – Exercise Sheet 5 – WS 2020/21

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hand in until December 9th (online)
discussed on December 11th in the exercise groups

5.1 Polarized beams

At a particle collider electrons and positrons are collided at the centre-of-mass energy $\sqrt{s} = 5$ GeV. The process $e^+ + e^- \to \mu^+ + \mu^-$ is observed.

a) Compute the relative production rates for the following beam conditions:
   - Both beams are unpolarized.
   - The electron beam has left-handed polarization, the positron beam is unpolarized.
   - The electron has left-handed and the positron beam has right-handed polarization.
   - Both the electron and the positron beam have left-handed polarization.

b) What is the fraction of left- to right-handed outgoing μ^- particles in the four different scenarios in part a) ?

Hint: *Consider only the dominant interaction. Neglect the masses of the leptons.*

5.2 Experimental test of QED

At the Positron-Electron-Tandem-Ring-Accelerator PETRA in Hamburg, muon pair production in electron-positron collisions

$$e^+ + e^- \to \mu^+ + \mu^-$$

at centre-of-mass energies between 2 and 37 GeV was studied.

a) Draw the leading order QED Feynman diagram of this reaction.

b) Suppose there is an energy scale Λ (equivalent to a length scale $\frac{1}{\Lambda}$) at which QED does not describe the data anymore. This would change, for instance, the photon propagator that would be modified as follows :

$$-i\frac{g_{\mu\nu}}{q^2} \to -i\frac{g_{\mu\nu}}{q^2}\left(1 \pm \frac{q^2}{\Lambda^2 - q^2}\right)$$

Write down the modified total cross section and the ratio

$$R_{\mu\mu} = \frac{\sigma(e^+ + e^- \to \mu^+ + \mu^-)_{\text{measured}}}{\sigma(e^+ + e^- \to \mu^+ + \mu^-)_{\text{QED}}}.$$

Hint: *The part of the modified propagator given in parentheses can be treated like a form-factor.*

c) Assume that the experimental cross section is measured to an accuracy of 10% and agrees within this accuracy to the prediction assuming simple photon exchange. What range of values for Λ is still consistent with the experiment?

5.3 Proton form-factor

A compilation of experimental results on the proton electric form-factor $G_E\left(Q^2\right)$ is presented in Figure 1. It shows that early G_E data (PRad, black markers) could be rather well parametrised by a ”dipole function“:

$$G_E(Q^2) = \frac{1}{\left(1 + Q^2/Q_0^2\right)^2}.$$

The $G_E\left(Q^2\right)$ dipole function with $Q_0^2 = 0.71\ \mathrm{GeV}^2$ is denoted as $G_{\text{std.dipole}}$ in the Figure 1. More recent and precise data (from Mainz) show a few percent deviation from this law.

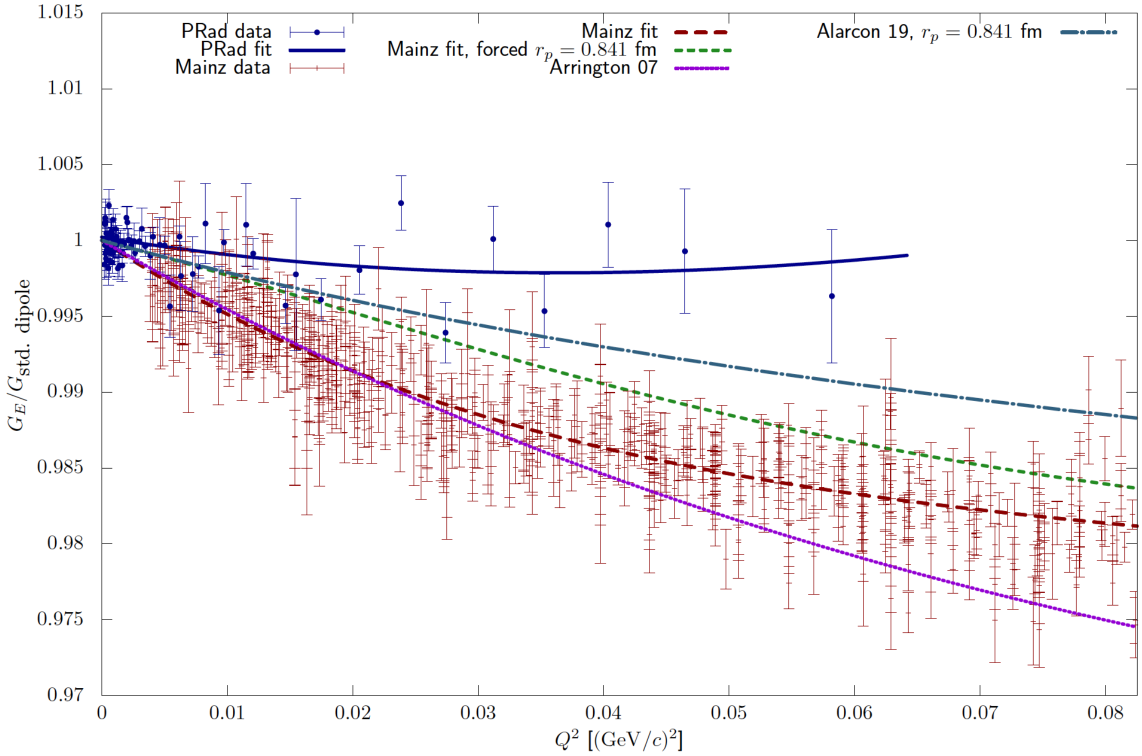

Figure 1: Extracted experimental data and fits for G_E, as the ratio to $G_{\text{std. dipole}}$ to compress the range. Shown are the PRad data and fit, the Mainz data, polynomial fit and experimental uncertainty, a fit to the Mainz data with a radius forced to the muonic spectroscopy value, an fit to pre-Mainz data, the theoretical calculation by Alarcon et al. Plot is taken from the paper https://doi.org/10.1051/epjconf/202023401001, where more details are given.

a) For a spherically symmetric charge distribution $\rho(r) = \rho_0 e^{-r/a}$, where

$$\int \rho(r) d^3\vec{r} = 1\,,$$

show that the form-factor (given in the lecture)

$$F(\vec{q}) = \int \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r}$$

can be expressed as

$$\begin{aligned} F\left(\vec{q}^2\right) &= \frac{4\pi}{q} \int_0^\infty r \sin\left(qr\right) \rho(r)\, dr \\ &\approx 1 - \frac{1}{6} q^2 \langle R^2 \rangle + \ldots\,, \end{aligned}$$

where $\langle R^2 \rangle = \int r^2 \rho(\vec{r}) d^3\vec{r}$ is the mean square charge radius.

Hint: *You will need to use the expansion* $\sin(qr) \approx qr - \frac{1}{3!}(qr)^3 + \ldots$.

Hence show that

$$\langle R^2 \rangle = -6 \left[\frac{dF(\vec{q}^2)}{dq^2} \right]_{q^2=0} .$$

Find the relation between the mean square charge radius of proton $\langle R^2 \rangle$, parameter a of the proton charge distribution and parameter Q_0^2 of the dipole function. Estimate the $\langle R^2 \rangle$ value using the standard dipole function $G_{\text{std.dipole}}$. Compare this value to the proton radius given by the Particle Data Group.

b) Plot the charge distribution of a proton (use the numeric values from part a)) and indicate positions of a and $\sqrt{\langle R^2 \rangle}$ on the x-axis.

5.4 Deep Inelastic Scattering

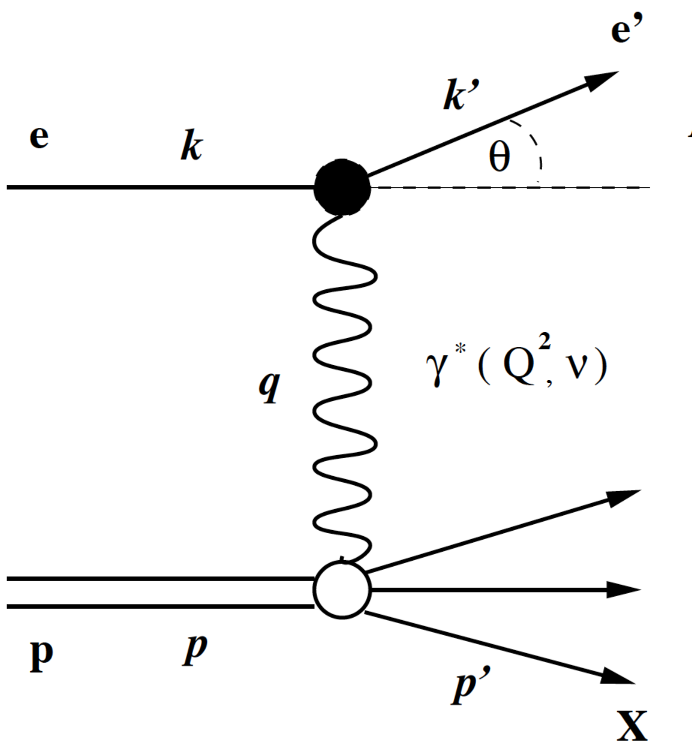

Figure 2: Sketch of the kinematics of Deep Inelastic Scattering. k is the momentum of the incoming electron, k' is the momentum of the outgoing electron, p is the momentum of the incoming proton, X represents the hadronic final state, whose total momentum is p'.

Figure 2 represents the kinematics of ep scattering. In this case we define the following variables:

- $q^2 = (k - k')^2$ is the square of the momentum transfer, where $k(k')$ is the initial (final) electron momentum
- $Q^2 = -q^2$

- $x = \frac{Q^2}{2p \cdot q}$, where p is the momentum of the incoming proton
- $y = \frac{p \cdot q}{p \cdot k}$
- m_p is the mass of the proton
- $W^2 = p'^2 = (p+q)^2$, *i.e.* the square invariant mass of the hadronic final state

Show that the following relations are valid

a) $q^2 < 0$
b) $0 \leq x \leq 1$
c) $0 \leq y \leq 1$
d) $W^2 = m_p^2 + Q^2 \cdot \frac{(1-x)}{x} \geq m_p^2$

Consider the mass of the electron to be negligible.