Particle Physics - Exercise Sheet 02

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1 Neutrinos in matter

The mean free path is described by

$$\lambda = \frac{1}{\sigma n} \tag{1}$$

Where n is the target density of a material. Using $\sigma = 8 \cdot 10^{-39}$ cm^2 and $n = \rho N_A/A$, where $N_A = 6.022 \cdot 10^{23}$ is the Avogadro constant, $\rho = 7.9 \cdot 101^3$ kg/m^3, the density of iron and $A = 56$ its atomic number, we get

$$\lambda = \frac{1}{\sigma n} = \frac{A}{\sigma \rho N_A} = 1.471 \cdot 10^{12} \text{ m} \tag{2}$$

This means, that the neutrino experience one interaction after a distance of λ, which means the probability of interaction in $l = 2m$ of iron is $1.36 \cdot 10^{-12}$.

2 Phase space integration

The interaction cross section σ for a scattering process $a + b \rightarrow 1 + 2$ can be described by

$$\sigma = \frac{1}{(2\pi^2)} \frac{1}{4p_i^* \sqrt{s}} \int |M_{fi}|^2 \, \delta(\sqrt{s} - E_1 - E_2) \delta^3(\vec{p}_1 + \vec{p}_2) \frac{\vec{p}_1}{2E_1} \frac{\vec{p}_2}{2E_2} \tag{3}$$

First, we use

$$\delta(f(x)) = \left| \frac{df}{dx} \right|_{x=x_0}^{-1} \text{ with } f(x_0) = 0 \tag{4}$$

$$\text{and } E_1 = \sqrt{m_1^2 + p_1^2} \text{ and } E_2 = \sqrt{m_2^2 + p_2^2} \tag{5}$$

to rewrite the first delta term and evaluate it for $p_2 = -p_1$:

$$\begin{aligned} \delta\left(\sqrt{s} - E_1 - E_2\right) &= \delta\left(\sqrt{s} - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_2^2}\right) \\ &= \left| -\frac{2p_1}{2\sqrt{m_1^2 + p_1^2}} - \frac{2p_1}{2\sqrt{m_2^2 + p_1^2}} \right|^{-1} \\ &= \left(\frac{p_1}{E_1} + \frac{p_1}{E_2} \right)^{-1} = \left(\frac{p_1 E_2 + p_1 E_1}{E_1 E_2} \right)^{-1} \\ &= \frac{E_1 E_2}{p_1 \sqrt{s}} \end{aligned} \tag{6}$$

Now we transform (3) into spherical coordinates and use (6)

$$\begin{aligned} &\text{with } d^3\vec{p_i} = p_i^2 dp_i d\Omega \\ \Rightarrow \sigma &= \frac{p_f^*}{16\pi^2 p_i^* \sqrt{s}} \frac{E_1 E_2}{4E_1 E_2 \sqrt{s}} \int |M_{fi}|^2 \, d\Omega^* \\ &= \frac{p_f^*}{64\pi^2 p_i^* s} \int |M_{fi}|^2 \, d\Omega^* \end{aligned} \tag{7}$$

3 Fermi's golden rule

b) The energy of the product particles has to match the rest mass of η:

$$\begin{aligned} & m_\eta = E_p + E_{\bar{p}} = 2E_p \\ \Rightarrow & \frac{m_\eta^2}{4} = m_p^2 + p_f^2 \quad \Rightarrow \quad p_f = \left(\frac{m_\eta^2}{4} - m_p^2\right)^{\frac{1}{2}} \end{aligned} \tag{8}$$

c) Fermi's golden rule states that

$$\Gamma_{fi} = \frac{p_f^*}{32\pi^2 m_A * 2} \int |M_{fi}| \, d\Omega^* \tag{9}$$

Assuming a similar matrix element, the ratio of the two decays is

$$\frac{\Gamma_{\eta\to p+\bar{p}}}{\Gamma_{\eta\to\Lambda+\bar{\Lambda}}} = \frac{\sqrt{m_\eta^2 - 4m_p^2}}{\sqrt{m_\eta^2 - 4m_\Lambda^2}} = \frac{33.29 \text{ MeV}}{27.46 \text{ MeV}} = 1.17 \tag{10}$$

The pdg databook suggests a ratio of 1.35, thus the matrix element of the proton decay has to be larger than the matrix element of the Λ decay.
d) With $\tau = 1/\Gamma$ we get $\tau = 2.34 \cdot 10^{-23}$ s

4 Mandelstam variables and scattering angle

a) The Mandelstam varbiales are as follows:

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \tag{11}$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \tag{12}$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \tag{13}$$

b) The square of a particle's four momentum is its mass ($p_i^2 = m_i$), and momentum has to be conserved ($p_1 + p_2 = p_3 + p_4$). Therefore

$$\begin{aligned} s + t + u &= p_1^2 + p_2^2 + 2p_1p_2 + p_1^2 + p_3^2 - 2p_1p_3 + p_1^2 + p_4^2 - 2p_1p_4 \\ &= m_1^2 + m_2^2 + m_3^2 + m_4^2 + 2p_1^2 + 2p_1p_2 - 2p_1p_3 - 2p_1p_4 \\ &= m_1^2 + m_2^2 + m_3^2 + m_4^2 + \underbrace{2p_1(p_1 + p_2 - p_3 - p_4)}_{=0} \\ &= m_1^2 + m_2^2 + m_3^2 + m_4^2 \end{aligned} \tag{14}$$