1 Cross Sections

a) The differential cross-section for $e^+e^- \to \mu^+\mu^-$ scattering can be described as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2\theta) \tag{1}$$

Using $d\Omega = d\varphi d(\cos\theta)$, we get the total cross-section

$$\begin{aligned}
\sigma &= \frac{2\pi\alpha^2}{4s}\int (1 + \cos^2\theta) d(\cos\theta) && (2)\\
&= \frac{2\pi\alpha^2}{4s}\left(\cos\theta\Big|_{-1}^{1} + \frac{1}{3}\cos^3\theta\Big|_{-1}^{1}\right) && (3)\\
&= \frac{2\pi\alpha^2}{4s}\cdot\frac{8}{3} = \frac{4\pi\alpha^2}{3s} && (4)
\end{aligned}$$

The luminosity is $\mathcal{L} = 10^{30}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$. Expressed in natural units, that is

$$\begin{aligned}
\mathcal{L} &= 10^4\ \mathrm{fm}^{-2}\mathrm{s}^{-1}\cdot(\hbar c)^2 && (5)\\
&= 10^4\cdot 197^2\ \mathrm{MeV}^2\mathrm{s}^{-1} && (6)
\end{aligned}$$

With $\alpha = 1/137$, $s = (10^4\ \mathrm{MeV})^2 = 10^8\ \mathrm{MeV}^2$, and (4), we get the production rate

$$\eta = 10^4\cdot 197^2\frac{4\pi}{3\cdot 137^2\cdot 10^8}\ \mathrm{s}^{-1} = 0.87\cdot 10^{-3}\ \mathrm{s}^{-1} = 74.8\ \mathrm{d}^{-1} \tag{7}$$

b) Instead of integrating from -1 to 1 in (3), we integrate from $\cos(150°) = -\sqrt{3}/2$ to $\cos(30°) = \sqrt{3}/2$. That yields

$$\sigma = \frac{2\pi\alpha^2}{4s}\left(\sqrt{3} + \frac{2}{3}\frac{\sqrt{3}^3}{8}\right) = \frac{5\sqrt{3}^3}{12}\frac{2\pi\alpha^2}{4s} = \frac{5\sqrt{3}^3\pi\alpha^2}{24s} \tag{8}$$

Multiplying with the efficiency $\varepsilon = 0.9$ the actual detection rate is

$$\eta' = 0.8\cdot 10^4\cdot 197^2\frac{5\sqrt{3}^3\pi}{24\cdot 137^2\cdot 10^8}\ \mathrm{s}^{-1} = 0.63\cdot 10^3\ \mathrm{s}^{-1} = 54.7\ \mathrm{d}^{-1} \tag{9}$$

The actual detection efficiency is therefore 73%.