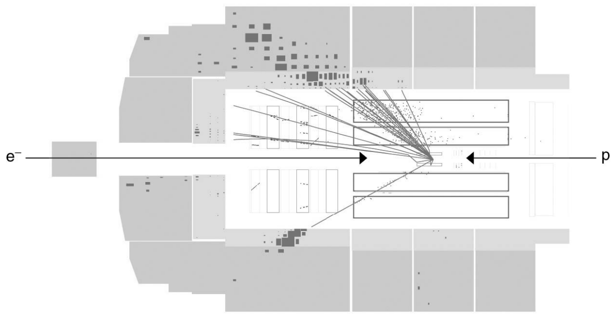
Particle Physics – Exercise Sheet 6 – WS 2020/21

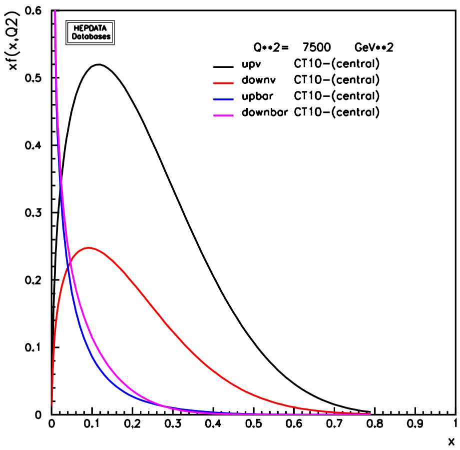
Lecturer: Prof. Dr. Hans-Christian Schultz-Coulon
discussed on Friday 18$^{\text{th}}$, December in the exercise groups

6.1 Deep Inelastic Scattering at HERA

Figure 1(a) shows a deep inelastic scattering event $e^- p \to e^- X$ recorded by the H1 experiment at the HERA collider. The electron beam has an energy of $E_e = 27.5\,\text{GeV}$ and the proton beam has an energy of $E_p = 820\,\text{GeV}$. The energy of the scattered electron is measured to be $E'_e = 72\,\text{GeV}$.



(a)



(b)

Figure 1: (a) A high-energy electron-proton collision in the H1 detector at HERA shown in the x-z plane where the z axis is defined by the direction of the incoming electron. (b) Parton distribution functions for a momentum transfer of $Q^2 = 7500\,\text{GeV}^2$.

a) Show that the Bjorken scaling variable, x, is given by

$$x = \frac{E'_e}{E_p}\left(\frac{1-\cos\theta}{2-\frac{E'_e}{E_e}\left(1+\cos\theta\right)}\right)$$

where θ is the angle between the momentum vector of the incoming electron and the scattered electron.

b) Estimate θ from Figure 1(a) assuming the momentum of the scattered electron lies in the x-z plane. Calculate the values of Q^2, x and y for this event.

c) Draw the lowest order Feynman diagram of this process on parton level. Estimate the relative probabilities of the various quark-level processes for the event using the parton distribution functions $xu_v(x)$, $xd_v(x)$, $x\bar{u}(x)$ and $x\bar{d}(x)$ from Figure 1(b). Neglect contributions from 2$^{\text{nd}}$ and 3$^{\text{rd}}$ generation quarks. Assume that the value of Q^2 calculated in b) is close to $Q^2 = 7500\,\text{GeV}^2$ as indicated in Figure 1(b).
Hint: Remember that the coupling to the virtual photon is proportional to the quark charge Q_q.

6.2 $t\bar{t}$ Production at the Tevatron and the LHC

The Tevatron collider was running until 2011 and collided protons with antiprotons at a centre-of-mass energy $\sqrt{s} = 1.96\,\mathrm{TeV}$. The LHC collides protons with protons, ultimately at $\sqrt{s} = 14\,\mathrm{TeV}$. These two machines are the only ones powerful enough to produce top-antitop quark pairs. To estimate the cross section for $t\bar{t}$ pair production parton distribution functions are needed. They have been measured at high precision at the electron-proton collider HERA at DESY (see problem 6.1). The results can be accessed at `http://hepdata.cedar.ac.uk/pdf/pdf3.html`.

a) Draw the three leading-order parton-level Feynman diagrams for $t\bar{t}$ production.

b) Find the momentum fractions x_1 and x_2 needed by the partons in order to produce a $t\bar{t}$ pair at rest at the Tevatron and at the LHC. What is the center-of-mass energy $\sqrt{\hat{s}}$ of the parton-parton collision?

c) Go to the url given above and plot the parton density functions at the appropriate scale ($Q^2 = (2m_t)^2$). (Use HERAPDF as group and HERAPDF01 as set; note that the plotted parton density function is scaled by x).

d) Use the parton distribution functions and the x values calculated above to determine the most relevant contributions amongst the various Feynman diagrams from part (a) at the Tevatron and the LHC.

e) Is the $t\bar{t}$ pair production cross-section higher at the LHC or at the Tevatron?

6.3 Quark-Parton Model

In this problem we discuss structure functions in deep-inelastic scattering in the quark-parton model. Figure 2 shows measurements of the structure functions in electron-proton (F_2^{ep}) and electron-deuteron (F_2^{eD}) scattering made at the Stanford Linear Accelerator Center (SLAC). Consider only u, $\bar{u}$, d, and $\bar{d}$ quarks for this problem.

a) Write down the structure functions F_2^{ep} and F_2^{en} in the quark-parton model in terms of the parton distributions $q_i^{\mathrm{p}}(x)$ of the proton.

b) Determine the quark-parton model prediction for

$$R = \frac{\int_0^1 F_2^{\mathrm{eD}}(x)\,\mathrm{d}x}{\int_0^1 F_2^{\mathrm{ep}}(x)\,\mathrm{d}x}.$$

Experimentally one finds $R \approx 0.84$. What does this imply for the ratio $f_{\mathrm{d}}/f_{\mathrm{u}}$ where $f_{\mathrm{q}} := \int_0^1 x(q(x) + \bar{q}(x))\,\mathrm{d}x$?

c) Assume $f_{\mathrm{u}} = 0.36$. What fraction of the proton's momentum is carried by quarks?

d) Show that

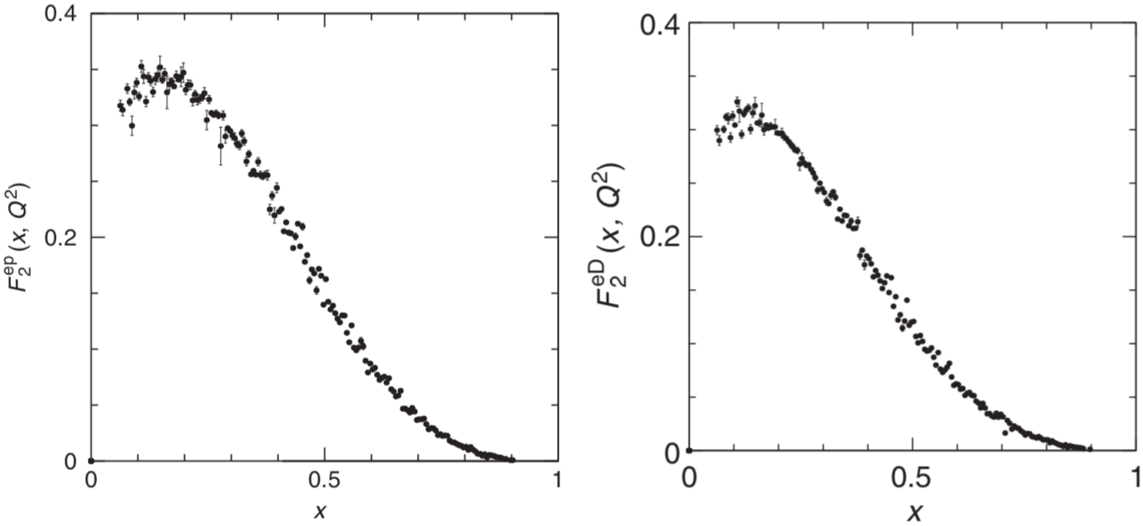
$$\int\limits_0^1 \frac{[F_2^{\mathrm{ep}}(x) - F_2^{\mathrm{en}}(x)]}{x}\,\mathrm{d}x \approx \frac{1}{3} + \frac{2}{3}\int\limits_0^1 \left[\bar{u}(x) - \bar{d}(x)\right]\,\mathrm{d}x.$$

Interpret the measured value of 0.24 ± 0.03.

e) In the limit $x \to 1$ valence quarks are expected to dominate. Write the ratio

$$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)}$$

in this limit in terms of the valence quark distributions $u_{\mathrm{v}}(x)$ and $d_{\mathrm{v}}(x)$. Experimentally one finds $F_2^{\mathrm{en}}(x)/F_2^{\mathrm{ep}}(x) \to 0.25$ for $x \to 1$. What does this imply for the ratio $d_{\mathrm{v}}(x)/u_{\mathrm{v}}(x)$?



(a) (b)

Figure 2: $F_2^{\mathrm{ep}}(x)$ (a) and $F_2^{\mathrm{eD}}(x)$ (b) measured for $2 < Q^2/\mathrm{GeV}^2 < 30$. Data from Whitlow et al., Phys. Lett. B282 (1992) 475–482.

6.4 The Drell-Yan Process

Figure 3 shows the Feynman-Diagram for the process $p\overline{p} \to \mu^+\mu^- X$, also known as Drell-Yan process. The differential hadronic cross-section for this process can be expressed as:

$$\frac{d\sigma}{dm^2_{\mu\mu}}(pp \to \mu^+\mu^- X) = \underbrace{\frac{1}{3}}_{①} \cdot \sum_q \hat{\sigma}(q\overline{q} \to \mu^+\mu^-) \underbrace{\int_0^1 dx_1 \int_0^1 dx_2 f_q(x_1) f_{\overline{q}}(x_2)}_{②} \cdot \underbrace{\delta(m^2_{\mu\mu} - x_1 x_2 s)}_{③}. \tag{1}$$

Here $m_{\mu\mu}$ is the invariant mass of the muon pair in the final state, s is the centre-of-mass energy of the colliding protons, x_1 and x_2 are the fractions of momenta carried by the quark and the anti-quark with respect to the momentum of the protons, $f(x_1)$ and $f(x_2)$ denote the parton density functions for the quark and the anti-quark in the proton. Finally, $\hat{\sigma}(q\overline{q} \to \mu^+\mu^-)$ denotes the cross-section for the underlying process $q\overline{q} \to \mu^+\mu^-$. This can be written in analogy to the cross section for the process $e^+e^- \to q\overline{q}$ as

$$\hat{\sigma}(q\overline{q} \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3m^2_{\mu\mu}} Q^2_q. \tag{2}$$

using the electric charge Q_q of the involved quarks in units of the electron charge. Note that equation 2 assumes the scattering of a quark-antiquark pair with allowed color-anticolor combination.

a) Explain why the differential cross section is expressed as a function of the invariant muon mass $m_{\mu\mu}$, contrary to the process $e^+e^- \to q\overline{q}$ which is usually given as a function of the centre-of-mass energy.

b) Explain the origin of the terms ①, ② and ③ in the hadronic cross section of equation 1.

c) Now, look at the measurement by the CMS-collaboration shown in figure 3. While equation 1 is valid over the entire invariant mass range equation 2 needs to be modified at invariant masses larger than about 60 GeV. Discuss qualitatively which effect(s) need to be taken into account to describe the data correctly.

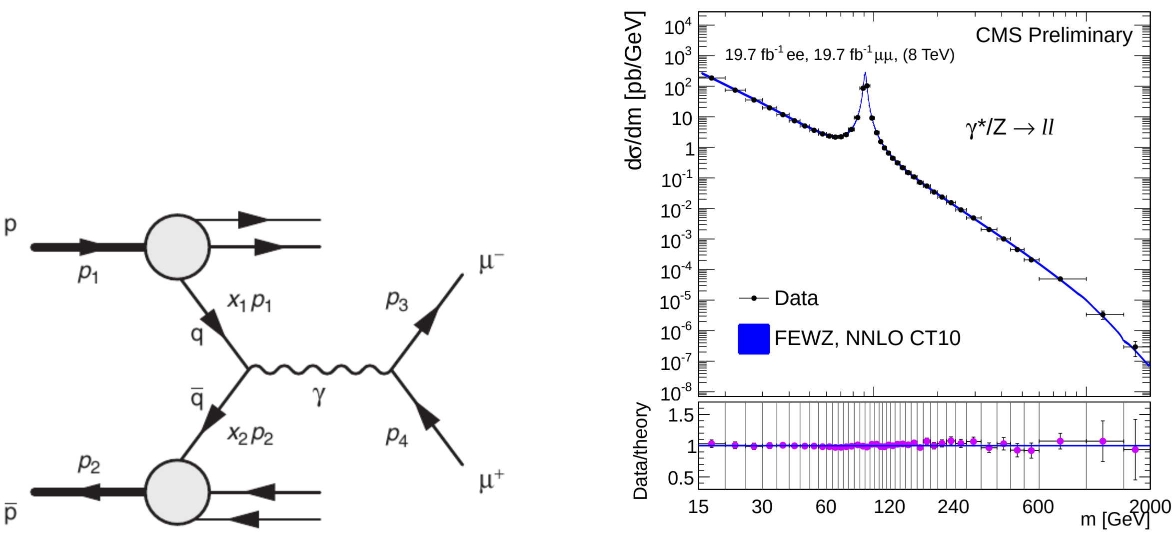


Figure 3: Left: Feynman-Graph for the Drell-Yan process $p\overline{p} \to \mu^+\mu^- X$. Right: Measurement of the differential cross-section for the Drell-Yan process by the CMS collaboration (taken from `http://dx.doi.org/10.1140/epjc/s10052-015-3364-2`[Eur. Phys. J. C 75 (2015) 147)