Particle Physics - Exercise Sheet 01

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1 Neutral pion decay

a) The momentum of a particle can be described with its momentum 4-vector ($\hbar = c = 1$):

$$\bar{p} = (E, p_x, p_y, p_z) \tag{1}$$
$$\text{with } p_i = \beta_i \gamma m c \tag{2}$$

Assuming we are in the CMS frame of the pion, the two photons emitted along the y-axis will have the following 4-vectors

$$\bar{p}_{\gamma_1} = \begin{pmatrix} E \\ 0 \\ E \\ 0 \end{pmatrix} \quad \bar{p}_{\gamma_2} = \begin{pmatrix} E \\ 0 \\ -E \\ 0 \end{pmatrix} \tag{3}$$

Next we transform into the lab frame. For that we build our transformation matrix to boost only in x-direction:

$$\text{General Case:} \quad \Lambda^\mu_\nu(\bar{\beta}) = \begin{pmatrix} \gamma & -\gamma\beta_j \\ -\gamma\beta_i & \delta_{ij} + \left(\gamma - 1\frac{\beta_i\beta_j}{\beta^2}\right) \end{pmatrix} \tag{4}$$

$$\Rightarrow \text{Boost in x-direction} \quad \Lambda^\mu_\nu(\bar{\beta}) = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

This yields for the momenta of the photons in the lab frame:

$$\bar{p}'_{\gamma_1} = E \begin{pmatrix} \gamma \\ -\beta\gamma \\ +1 \\ 0 \end{pmatrix} \quad \bar{p}'_{\gamma_1} = E \begin{pmatrix} \gamma \\ -\beta\gamma \\ -1 \\ 0 \end{pmatrix} \tag{6}$$

Looking only at the three space components ($\vec{p}$), and Dividing by E, one can get the angle by simple vector analysis:

$$\alpha = \arccos\left(\frac{\vec{p}_{\gamma_1} \cdot \vec{p}_{\gamma_2}}{|\vec{p}_{\gamma_1}| \cdot |\vec{p}_{\gamma_2}|}\right) = \arccos\left(\frac{(\beta\gamma)^2 - 1}{(\beta\gamma)^2 + 1}\right) \tag{7}$$

b) To find the upper energy limit at which the angle between photons is exactly 5°, one has to invert (7):

$$(\beta\gamma)^2 = -\left(\frac{\cos(\alpha) + 1}{\cos(\alpha) - 1}\right) \tag{8}$$

For $\alpha = 5°$ this yields a $\beta\gamma$ of

$$(\beta\gamma)^2 = 524.6 \Rightarrow \beta\gamma = 22.9$$

2 β decay of triton

a) The heisenberg uncertainty principle states, that

$$\Delta p \Delta x \geq \frac{\hbar}{2} \tag{9}$$

If we assume the electron to be confined inside of the nucleon diameter, $\Delta x = 4.3$ fm, then

$$\Delta p \geq 4.9 \cdot 10^{-20}\ \mathrm{kg\,m\,s^{-1}} \equiv 91.8\ \mathrm{MeV/c} \tag{10}$$

That is way larger than the rest mass of an electron, so that we can assume the kinetic Energy to be $E_{\text{kin}} = 91.8$ MeV/c

b) We want to compare the upper estimated energy to the maximum energy of an electron emitted in the β decay of triton ${}^3\mathrm{H} \rightarrow^3 \mathrm{He} + e^- + \bar{\nu}_\mathrm{e}$. The energy can be calculated from the mass difference of ^{3}H and ^{3}He considering an electron rest mass of 511 keV/c^2, and assuming that during the decay, only negligible energy is transferred to the electron neutrino:

$$\begin{aligned} E_{max} &= m({}^3\mathrm{H}) - m({}^3\mathrm{He}) - m_\mathrm{e} \\ &= 2808.9211306\ \mathrm{MeV/c^2} - 2808.391554\ \mathrm{MeV/c^2} - 0.511\ \mathrm{MeV/c^2} \\ &= 0.018\ \mathrm{MeV/c^2} \end{aligned} \tag{11}$$

3 Conservation laws - See Figure 1

4 Resonance decay width

a) From the principle of uncertainty we get

$$\Delta t \geq \frac{\hbar}{\Gamma} \quad \Rightarrow \tau = \frac{\hbar}{\Gamma} \tag{12}$$

$$\Rightarrow \tau = \frac{\hbar}{120\ \mathrm{MeV}} = \frac{\hbar}{1.922 \cdot 10^{-11}\ \mathrm{J}} = 5.46 \cdot 10^{-24}\ \mathrm{s}. \tag{13}$$

To calculate the distance travelled however, one has to consider the relativistic proper time of the Δ^{++}:

$$\begin{aligned} t' &= (t - \beta \underbrace{z}_{=0})\gamma \quad \text{and} \quad \gamma = E/m \\ \Rightarrow \gamma &= \frac{200\ \mathrm{GeV}}{1.232\ \mathrm{GeV}} = 162.3 \\ \Rightarrow t' &= 4.45 \cdot 10^{-22}\ \mathrm{s} \end{aligned} \tag{14}$$

With $m = 1232$ GeV/c^2. The travel distance is therefore limited to $x \geq 133.5$ fm.

b) From (12) we get

$$\tau \geq \frac{\hbar}{6.5\ \mathrm{GeV}} = 1.01 \cdot 10^{-28}\ \mathrm{s} \tag{15}$$

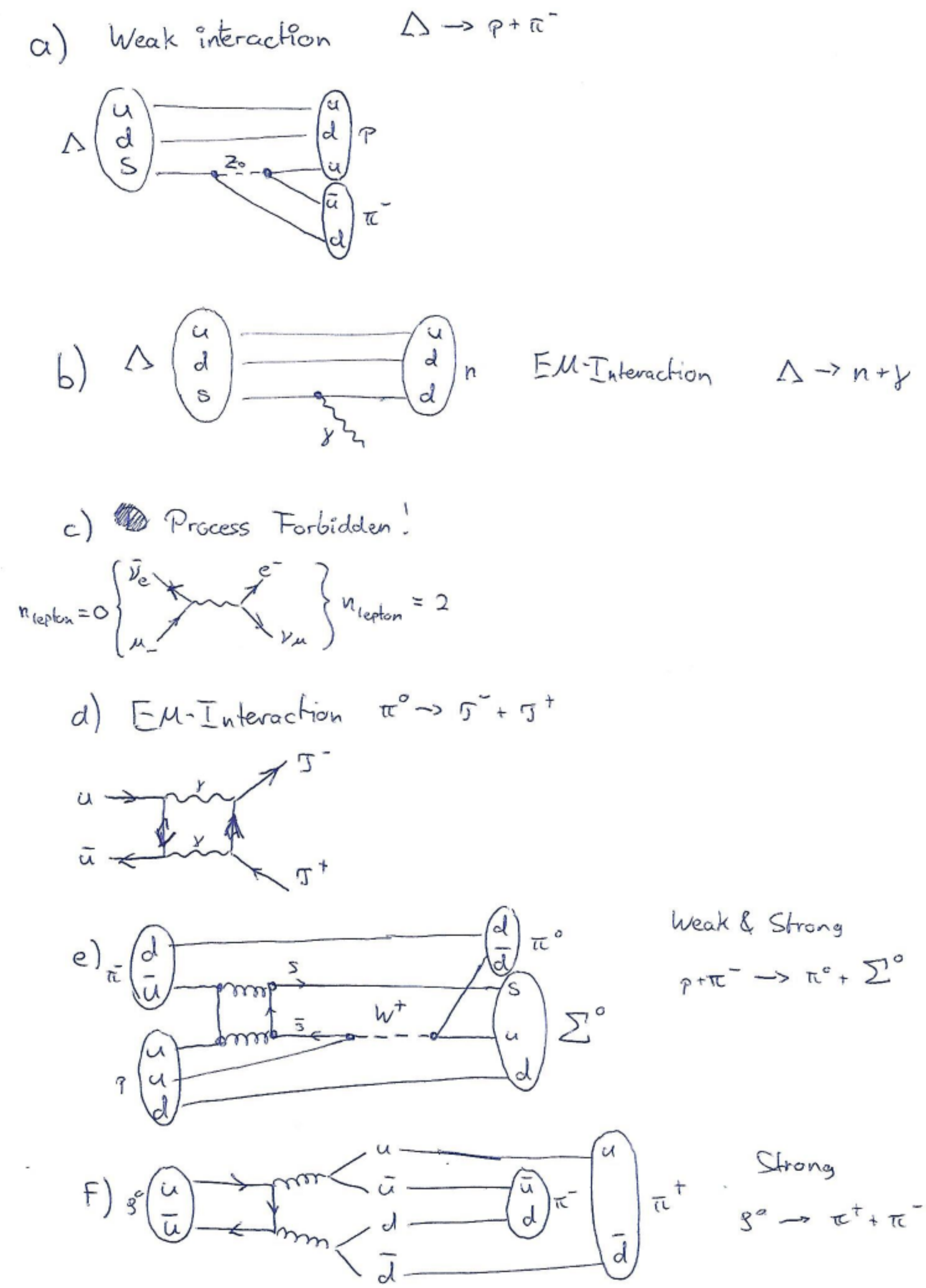

Figure 1: Feynman Diagrams for the given processes