# Particle Physics – Exercise Sheet 2 – WS 2020/21

Lecturer: Prof. Dr. Hans-Christian Schultz-Coulon
discussed on Friday 20$^{\text{th}}$, November in the exercise groups

## 2.1 Neutrinos in matter

A single 1 GeV neutrino is fired on an iron ($^{56}_{28}$Fe) target (length l = 2 m, $\rho_{Fe} = 7.9 \cdot 10^3$ kg m$^{-3}$). Assuming an average neutrino-nucleon interaction cross section of $\sigma = 8 \cdot 10^{-39}\ cm^2$, what is the average number of interactions for the neutrino traversing the block and what is the probability of that neutrino to interact in the block?

**Hint:** Assume the average mass of a nucleon to be $1.67 \cdot 10^{-27}$kg.

## 2.2 Phase space integration

Consider the scattering process $a + b \to 1 + 2$. Hereby $E_1, E_2$ and $\vec{p_1}, \vec{p_2}$ denotes the energies and 3-momenta of the final state particles. The magnitude of the momenta of the initial and final state particles in the CMS are $p_i^*$ and $p_f^*$, respectively.

Prove that

$$\sigma = \frac{1}{(2\pi)^2}\frac{1}{4p_i^*\sqrt{s}}\int |M_{fi}|^2\delta(\sqrt{s} - E_1 - E_2)\delta^3(\vec{p_1} + \vec{p_2})\frac{d^3\vec{p_1}}{2E_1}\frac{d^3\vec{p_2}}{2E_2}$$

can be simplified to

$$\sigma = \frac{1}{64\pi^2 s} \cdot \frac{p_f^*}{p_i^*}\int |M_{fi}|^2 d\Omega^*$$

by solving the integration in the center-of-mass frame.

**Hint:** Exploit features of the $\delta$-function among others
$\delta(f(x)) = \left|\frac{df}{dx}\right|^{-1}_{x=x_0}$ with $f(x_0) = 0$;
see Thompson A.3 for derivation.

## 2.3 Fermi's golden rule

The $\eta_c$ meson is a spin-0 bound state of a $c$ and a $\overline{c}$ quark. Consider the following decays:

$$\eta_c \to p + \overline{p} \qquad \text{and} \qquad \eta_c \to \Lambda + \overline{\Lambda}.$$

a) Draw the Feynman diagrams of the both decays.
b) Calculate the magnitude of the momentum $p_f^*$ for the daughter particles in the rest frame of $\eta_c$ for both decays as the function of the mass of the $\eta_c$ and the daughter particles.
c) Use Fermi's golden rule in order to calculate the ratio $r = \frac{\Gamma_{\eta_c \to p+\overline{p}}}{\Gamma_{\eta_c \to \Lambda+\overline{\Lambda}}}$ of the partial widths of both decays. Look up the corresponding branching ratios and compare your result.
d) The total decay width $\Gamma_{\eta_c}$ is 28.6 MeV. Calculate the lifetime of the $\eta_c$.

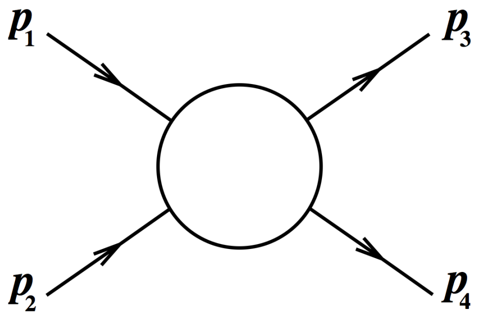
**Hint:** Use PDG listings to look up masses and branching ratios (pdg.lbl.gov).
Use the formular for the partial decay width of a two body decay of a spin-0 particle $A$ with mass $m_A$ derived in the lecture

$$\Gamma = \frac{p_f^*}{32\pi^2 m_A^2} \int |M_{fi}|^2 d\Omega^*$$

where $p_f^*$ is the magnitude of the momenta of any of the both daughter particles in the final state, $M_{fi}$ is the matrix element of the decay.

## 2.4 Mandelstam variables and scattering angle

Two particles (1 and 2) scatter and produce two outgoing particles (3 and 4):




With $p_i$ denoting the 4-momentum vector of particle $i$, the three Mandelstam variables $s,\ t$ and $u$ are defined.

a) Write down the definition of the Mandelstam variables $s,\ t$ and $u$
b) Calculate $s + t + u$.

Prove the following relations for the case that the masses of all particles vanish ($m_i = 0$):

c)
$$\frac{t^2 + u^2}{s^2} = \frac{1 + cos^2\theta^*}{2}$$

d)
$$t = -s\frac{1 - cos\theta^*}{2}$$

e)
$$u = -s\frac{1 + cos\theta^*}{2}$$

The angle $\theta^*$ is the angle between the flight directions of particles 1 and 3 in the center-of-mass frame: $\theta^* = \angle(\vec{p_1^*}, \vec{p_3^*})$.

f) Draw an example of a $s,\ t$ and $u$ channel process.