Yields
Lorentz-Transformations:

inertial systems move along z-axis relative to each other.

$$t' = (t - \beta z)\gamma \quad x' = x \quad y' = y \quad z' = (z - \beta t)\gamma$$

with $\hbar = c = 1$.

Covariant 4-vectors transform according to: $\tilde{X}' = \Lambda^{-1}\tilde{X}$.

inverse of Lorentz transform with

Examples:

$$\Lambda^{-1} = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

Covariant 4-vectors $\rightarrow$ $X_\mu = (t, -x, -y, -z)$

$\rightarrow$ $p_\mu = (E, -p_x, -p_y, -p_z)$

$X_\mu = g_{\mu\nu} X^\nu$

As $\Lambda^{-1}\Lambda = \mathbb{1}$
any product of a contra- and a covariant 4-vector is Lorentz-invariant:

$$a^\mu b_\mu = \text{invariant under LT.}$$

In particle physics physical predictions are expressed in explicit Lorentz-invariant form.

Imp. Examples:

$$x^\mu x_\mu = (c^2)t^2 - x^2 - y^2 - z^2 = \text{const. in all inertial frames}$$

$$p^\mu p_\mu = E^2 - p_x^2 - p_y^2 - p_z^2 = E^2 - \vec{p}^{\,2} = m^2$$

Energy-momentum relation

relativistic Expressions of energy and momentum $\rightarrow$ with $E = \gamma m$, $\vec{p} = \gamma m \vec{\beta}$ $\rightarrow$ $\boxed{\gamma = \frac{E}{m}, \quad |\vec{\beta}| = \frac{|\vec{p}|}{E}}$

Important!

$\rightarrow$ Also for $p^\mu = \sum p^\mu_i$:

$$p^\mu p_\mu = \left(\sum E_i\right)^2 - \left(\sum \vec{p}_i\right)^2 = m^2$$

Invariant mass of a system; Conserved! imp. for HEP.

<u>Four</u> <u>derivative</u> $\frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$.

$x^\mu \mapsto x^{\mu'}$:

$$\frac{\partial}{\partial z'} = \left(\frac{\partial z}{\partial z'}\right)\frac{\partial}{\partial z} + \left(\frac{\partial t}{\partial z'}\right)\frac{\partial}{\partial t}$$

with $z' = \gamma(z - \beta t)$, $t' = \gamma(t - \beta z)$
$z = \gamma(z' + \beta t')$, $t = \gamma(t' + \beta z')$

$$= \gamma\frac{\partial}{\partial z} + \gamma\beta\frac{\partial}{\partial t}$$

covariant transformation!!

Thus $\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ transforms covariant …
instead of contravariant:

$$\begin{pmatrix} \partial/\partial t' \\ \partial/\partial x' \\ \partial/\partial y' \\ \partial/\partial z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} \partial/\partial t \\ \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}$$

$$\rightarrow \boxed{\partial_\mu = \frac{\partial}{\partial x^\mu} \quad \text{und} \quad \partial^\mu = \frac{\partial}{\partial x_\mu}}$$

Laplacian or D'Alembert Operator: $\partial^\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} = \Box$.

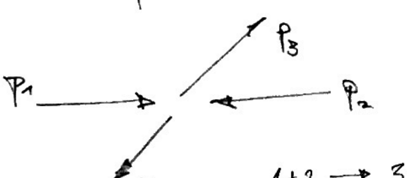
Mandelstam variables:

Cross sections must be Lorentz-invariant !

Life Time & Decay Rates are not !

Particle physics interested in scattering cross sections;
Expressed in terms of Lorentz-invariant products $p^\mu p_\mu$ …

Mandelstam variables:
Convenient choice of Lorentz-invariant expressions.
for 2×2 processes …



$1+2 \rightarrow 3+4$ process

$$s = (p_1+p_2)^2 = (p_3+p_4)^2$$

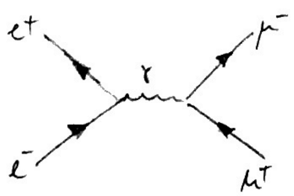
$$t = (p_1-p_3)^2 = (p_2-p_4)^2$$

$$u = (p_1-p_4)^2 = (p_2-p_3)^2$$

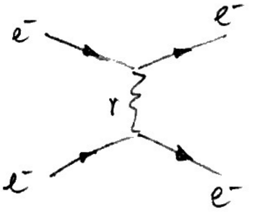
$$s+u+t = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

Used to describe:

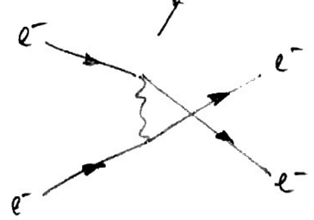
Applies only for identical particles.



$$\sim \frac{\alpha^2}{s} \cdot \left(\frac{t^2+u^2}{s^2}\right)$$



$$\sim \frac{\alpha^2}{s} \left(\frac{s^2+u^2}{t^2}\right)$$



$$\sim \frac{\alpha^2}{s} \cdot \left(\frac{s^2+t^2}{u^2}\right)$$

III. Decay Rates and Cross Sections.

Interaction

Decay

Particle physics concerned with <u>interaction</u> and <u>decays</u>.

Transition rate:

Transition Matrix Element

$$\boxed{\Gamma_{fi} = 2\pi \left|T_{fi}\right|^2 \rho(E_f)}$$

Fermi's Golden Rule

[Derivation see ch. 2.3.6. of M. Thomson

rather lengthy with several mathem. tricks and δ-function gymnastics

Density of states accessible with $E = E_f = E_i$

With:

$$T_{fi} = \langle f | \hat{H}' | i \rangle + \sum_{j \neq i} \frac{\langle f|\hat{H}'|j\rangle\langle j|\hat{H}'|i\rangle}{E_i - E_j} + \dots$$

$|i\rangle, |f\rangle$ initial & final states

$\psi = A e^{-ixp}$

This is what we mainly use "Matrix element"

Higher orders ..

$$\rho(E_f) = \left|\frac{dn}{dE}\right|_{E_i}$$

dn: number of states in energy range $[E, E+dE]$ ...

$$= \int \frac{dn}{dE} \delta(E_i - E)\, dE$$

integration over all final states with $E_f = E_i$ ..

$$\rightarrow \boxed{\Gamma_{fi} = 2\pi \int \left|T_{fi}\right|^2 \delta(E_i - E)\, dn}$$

Fermi's Golden Rule [alternative formulation]

Matrix Element

[Here 4-momenta enter]

Energy Conservation

<u>Decay rate:</u>

For a particle 'a' decaying into a final state: $\underbrace{a}_{i} \rightarrow \underbrace{1+2}_{f}$

Γ_{if} represents the <u>decay rate</u> ...

For more than one decay mode: $\Gamma_{tot} = \sum \Gamma_f$

Γ is related to the lifetime of particle a: $N(t) = N_0 e^{-\Gamma t} = N_0 e^{-t/\tau} \rightarrow \boxed{\Gamma = \frac{1}{\tau}}$

Decay rate

Lifetime

Not Lorentz invariant?

Cross section :

Units:

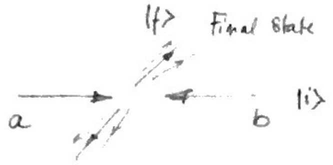
$[r_b] = 1/s$

$[\phi_a] = 1/m^2 s$

$\rightarrow [\sigma] = m^2$

Definition : $r_b = \sigma \, \phi_a$

interaction or transition rate for $|i\rangle \rightarrow |f\rangle$ [per target particle b]



i.e.:

$$\boxed{\sigma = \frac{\text{transition rate } |i\rangle \rightarrow |f\rangle}{\text{incident flux} \cdot \# \text{target particles}}}$$

Cross Section

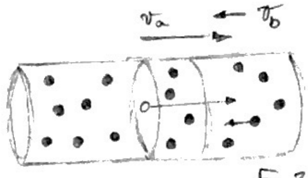


Fig 3.5 M. Thomson

Flux : $\phi_a = n_a \cdot v = n_a (v_a + v_b)$

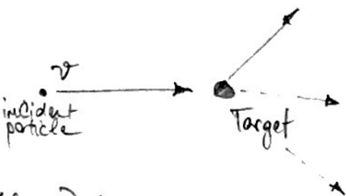
particle density of incoming particles a ..

Also: r_b refers to transition rate per target particle b, but for an incident particle density n_a !

$$r_b = \Gamma_{fi}' \cdot n_a$$

$$\boxed{\sigma = \frac{\Gamma_{fi}}{(v_a + v_b)}}$$

Relation between Cross section and Transition rate



Remember: Energy conservation

$$\boxed{\sigma = \frac{2\pi}{v} \cdot |T_{fi}|^2 \cdot \rho(E_f)}$$

Fermi's Golden Rule for Cross Section ($2 \rightarrow 2$ process)

relative velocity

matrix element

phase space

Reminder: $E_f = E_i$ just in case questions on different notation arise.

Phase space : and wavefunction normalization

$$\rho(E_f) = \left.\frac{dn}{dE}\right|_{E_i} \quad \text{number of accessible states ..}$$

From restricting to $V = a^3$ and requiring periodic boundary conditions:

$\lambda = a/n$

$2\pi/k = a/n$

$\rightarrow p = \frac{2\pi}{a} n$

Periodic BC since we have infinite space (see SSP)

Consider particles in a box with $V = a^3$. Quantization limits number of possible states ..

SLIDES

Boundary conditions imply: $(p_x, p_y, p_z) = (n_x, n_y, n_z) \frac{2\pi}{a}$

Volume occupied by single state in moment space : $d^3\vec{p} = dp_x dp_y dp_z = \left(\frac{2\pi}{a}\right)^3 = \frac{(2\pi)^3}{V}.$