Performance Analysis of Visible Light Communication System With Imperfect CSI

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***Abstract*— A single-input single-output visible light communication (VLC) system corrupted by additive white Gaussian noise is considered in this letter. Closed-form expressions for the symbol error probability (SEP) for the system with perfect knowledge of the channel, and with least square estimate of the channel at the receiver, are derived. The dependence of the SEP on the ratio of the vertical and horizontal distances between the transmitter and the receiver, the semi-angle of the LED transmitter, and on the powers of the transmitted data and pilot symbols provide various insights to the design of the VLC system for reliable communication.**

***Index Terms*— Channel state estimation, least squares, symbol error probability, visible light communication.**

I. Introduction

OVER the past decade visible light communication (VLC), based on the data transmission through light, has emerged as a promising technology to fulfil the unprecedented demand for bandwidth and high-speed internet due to wide spread use of smart and mobile devices. VLC uses light emitting diodes (LEDs) as the source of illumination and data transmission [1], [2] and has found substantial use in indoor wireless communications, hospitals and healthcare systems, and under water communication etc., [3], [4] due to its proficiency in various attractive features such as unlicensed frequency bands, excellent immunity to radio frequency interference, energy-efficiency, low-cost technology and secure data communication [5].

In comparison to the traditional wireless radio frequency communication systems, the indoor channel characteristics of a VLC system [6] are quite distinct as the channel is denser due to the existence of line-of-sight (LOS) and several non-LOS optical multipath reflections from walls and ceiling in an indoor environment. Additionally, the position, shape, and size of the opaque obstacles and the ambiance have profound impact on the VLC channel characteristics. Further, the background shot and thermal noise present in different VLC channels is statistically modeled as additive white Gaussian noise (AWGN) for a channel operating in the frequency range of visible spectrum, i.e., from 380 to 780 nanometers [7]–[10].

Thus, the statistical nature of the channel gain, modeled by a monotonic decreasing distribution [6], [7] along with the additive noise corrupting the channel, poses major challenges to achieve reliable data transmission in a VLC system. A very well-known technique to overcome this issue is to estimate the channel state information (CSI) at the receiver prior to data transmission. Several such studies of channel estimation for VLC systems have been reported in the literature. The authors in [8] have proposed least squares (LS) and minimum mean squared estimation methods for indoor VLC systems using a direct current biased optical orthogonal frequency division multiplexing (DCO-OFDM) scheme in the frequency domain. The authors in [10] have investigated a novel superimposed training channel estimation and LS methods in multiple-input single-output (MISO) DCO-OFDM VLC scenarios. The authors in [11] have considered a LS channel estimation for optical OFDM based multiuser MISO indoor VLC system to optimize the pilot sequence and pilot tones for minimum mean squared error (MSE) of the estimated VLC channel. In [12], the authors have proposed a new channel estimation algorithm referred to as the adaptive statistical Bayesian minimum mean square error channel estimation for indoor DCO-OFDM downlink VLC systems to achieve a superior channel estimation performance in terms of both MSE and bit error rate (BER). Further, the authors in [13] have investigated a channel estimation method based on compressive sensing for multiple-input multiple-output OFDM VLC systems to achieve reduced MSE and improved BER performance with reduced pilot overhead compared to the traditional LS channel estimation method.

However, to the best of our knowledge, there has been no study on the performance of an indoor VLC system with and without the knowledge of the CSI at the receiver, considering the statistical channel model as given in [6]. Therefore, this work considers a single-input single-output (SISO) VLC system with a static receiver, subject to the monotonic decreasing statistical distribution model of the channel gain and corrupted by AWGN noise. Further, the transmitter employs binary phase-shift keying (BPSK) with a direct current (DC) bias for intensity modulation (IM) and the receiver removes the DC biasing prior to symbol detection. The major contributions of this letter can be summarized as follows:

- Maximum-likelihood (ML) receivers for the VLC systems with perfect CSI and with LS estimates of the channel prior to data transmission are proposed.

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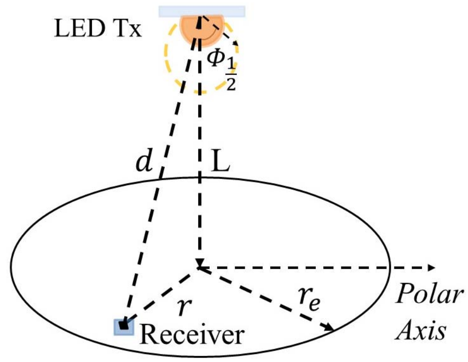

Fig. 1. Model of the proposed indoor VLC system.

- Closed form expressions for symbol error probabilities (SEPs) for the VLC systems with the proposed receivers are derived using the statistics of the multipath VLC channel.
- The asymptotic expression for the SEP is derived for the VLC system with LS channel estimates to study the effect of estimation on the performance of the system.
- The saturation of the SEP of the proposed receiver employing LS estimation at high signal-to-noise (SNR) values and the dependence of the SEPs of the proposed receivers on the ratio of the lengths of the maximum cell radius of the receiver and its vertical distance from the LED are observed and studied.
- The dependence of the performance of the VLC system with and without channel estimation, on the semi-angle of the LED are presented towards the development of a reliable VLC system in practice.

The rest of the letter is organized as follows. The system model of the VLC system under consideration, and the receivers with and without channel estimates are described in Section II. Series form expressions of the SEP for the proposed receivers along with the asymptotic expression of the system with channel estimate are obtained in Section III. The various design parameters for the reliable communication in VLC are discussed by numerical results in Section IV, followed by the concluding remarks of the letter in Section V.

II. System Model

A SISO VLC downlink transmission model is considered as shown in Fig. 1, where the LED is positioned on the ceiling, transmitting BPSK modulated data symbols to the receiver located anywhere within a circular area. Prior to the transmission, the BPSK symbols are converted into unipolar symbols by introducing a positive DC bias level [14] at the transmitter to employ IM and the corresponding reverse process of removing the DC bias is adopted at the receiver to get the bipolar signal constellation points required for threshold detection process. The receiver location is assumed to be static and the receiver employs symbol-by-symbol detection.

Thus, the received signal, r, over the VLC link can be expressed as

$$r = hs + n\,, \tag{1}$$

where s denotes the transmitted symbol, h is the channel fading coefficient of the VLC link, and n denotes the additive noise of VLC system which follows a zero mean Gaussian distribution, i.e., $n \sim \mathcal{N}\left(0, \sigma_n^2\right)$. The user position r as shown in Fig. 1 is random and follows a uniform distribution within a circular plane, with the probability density function (p.d.f) given as $f_r(r) = \frac{2r}{r_e^2}$, where r_e is the maximum cell radius covered by the LED. Consequently, owing to the characteristics of the VLC channel in an indoor scenario, the p.d.f. of the square of the channel gain is given by [6], [7]

$$f_{h^2}(v) = \frac{1}{r_e^2(m+3)}\left(C(m+1)L^{m+1}\right)^{\frac{2}{m+3}} v^{-\frac{1}{m+3}-1}\,, \quad v \in \left[v_{min}, v_{max}\right], \tag{2}$$

where C is a transceiver dependent constant, and L denotes the vertical distance from the LED to the receiving plane. For the condition of the channel power to be unity, i.e., $\mathbf{E}\left[h^2\right] = 1$ ($\mathbf{E}\left[\cdot\right]$ denoting the expectation operator), we have

$$\left(C(m+1)L^{m+1}\right)^2 = \frac{(m+2)\,r_e^2\left(L^2\left(r_e^2+L^2\right)\right)^{m+2}}{\left(r_e^2+L^2\right)^{m+2} - L^{2(m+2)}}, \tag{3}$$

which further leads to the values of v_{min} and v_{max} to be obtained as

$$v_{min} = \frac{(m+2)\;r_e^2\,L^{2(m+2)}}{\left(r_e^2+L^2\right)\left[\left(r_e^2+L^2\right)^{m+2} - L^{2(m+2)}\right]}\,,$$
$$v_{max} = \frac{(m+2)\;r_e^2\;\left(r_e^2+L^2\right)^{m+2}}{L^2\left[\left(r_e^2+L^2\right)^{m+2} - L^{2(m+2)}\right]}\,, \tag{4}$$

where m, denoting the Lambertian radiation pattern of LED, is given as

$$m = -1/\ln\left(\cos\left(\phi_{\frac{1}{2}}\right)\right)\,, \tag{5}$$

with $\phi_{\frac{1}{2}}$ representing the semi-angle of the LED.

A. LS Based Channel Estimation

A sequence of L_p pilot symbols is transmitted from the transmitter of the VLC system to estimate the channel coefficient h, prior to data transmission. Denoting the $L_p \times 1$ pilot symbol and the additive noise vectors in the channel by $\mathbf{s}_p$ and $\mathbf{n}_p$, respectively, the received symbol vector $\mathbf{r}_p$ is obtained as

$$\mathbf{r}_p = h\,\mathbf{s}_p + \mathbf{n}_p. \tag{6}$$

From the statistics of the additive noise, the elements of $\mathbf{n}_p$ are i.i.d. zero mean Gaussian distributed random variables, implying that $\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}_{L_p}, \sigma_n^2\mathbf{I}_{L_p}\right)$, where $\mathbf{0}_{L_p}$ and $\mathbf{I}_{L_p}$ denote the $L_p \times 1$ vector of zeros and $L_p \times L_p$ identity matrix, respectively. Thus, the LS estimate of the channel coefficient is given by

$$\hat{h}_{ls} = h + \epsilon_{ls}\,, \tag{7}$$

where the estimation error $\epsilon_{ls} = \mathbf{s}_p^T\,\mathbf{n}_p/\|\mathbf{s}_p\|^2$ is a Gaussian random variable with $\epsilon_{ls} \sim \mathcal{N}\left(0, \Gamma_p^{-1}\right)$, with $(\cdot)^T$ denoting the transpose operator and Γ_p denoting the pilot symbol SNR, given as $\Gamma_p = \|\mathbf{s}_p\|^2/\sigma_n^2$.

B. Receiver Structures

Owing to the statistics of the noise and employing coherent ML rule at the receiver [15] the optimal decision rule for the VLC system with perfect CSI knowledge at the receiver is obtained as

$$\hat{s} = \arg\max_{s} \left(2rhs - s^2\right), \tag{8}$$

and the proposed suboptimal receiver structure for the VLC system estimating the channel prior to data transmission is obtained as

$$\hat{s} = \arg\max_{s} \left(2r\hat{h}_{ls}s - s^2\right). \tag{9}$$

III. Error Analysis

In this section, we derive the expressions for the SEPs for equiprobable BPSK modulated symbols, i.e., $s \in \{-A, +A\}$ transmitted over the VLC channel under consideration for the cases of the receiver with and without the perfect knowledge of the CSI.

A. Error Analysis With Perfect CSI

Utilizing the receiver structure in (8) for the case of BPSK transmission, and from the statistics of the additive noise in the VLC system, the expression of the conditional SEP conditioned on the VLC channel gain can be expressed as

$$P_{e\left|h\right.} = Q\left(\sqrt{\frac{A^2h^2}{\sigma_n^2}}\right), \tag{10}$$

where $Q(x)$ denotes the Gaussian-Q function. Thus, unconditioning on the channel gain by using (2) along with the condition that $\mathbf{E}\left[h^2\right] = 1$, the expression for SEP can be written as

$$\begin{aligned} P_e &= \int_{v_{min}}^{v_{max}} P_{e\left|h\right.} f_{|h|^2}(v)\,\mathrm{d}v \\ &= \left[\frac{\left[(m+2)\,r_e^2\left(L^2\left(r_e^2+L^2\right)\right)^{m+2}\right]^{\frac{1}{m+3}}}{r_e^2\,(m+3)\left[\left(r_e^2+L^2\right)^{m+2} - L^{2(m+2)}\right]^{\frac{1}{m+3}}}\right] \\ &\quad \times \int_{v_{min}}^{v_{max}} \frac{Q\left(\sqrt{\Gamma_{av}v}\right)}{v^{\frac{m+4}{m+3}}}\mathrm{d}v, \end{aligned} \tag{11}$$

where the average SNR per symbol is given as $\Gamma_{av} = A^2/\sigma_n^2$. Simplifying the expression in (11) by using the Craig's formula for the Gaussian-Q function [16] and carrying out algebraic manipulations by employing integration by parts technique followed by Leibniz integral rule, the closed-form expression for P_e is obtained as

$$\begin{aligned} P_e &= \frac{(1+R)}{R}G\left(\frac{\Gamma_{av}}{1+R}\right) - \frac{1}{R}G\left((1+R)^{m+2}\,\Gamma_{av}\right) \\ &\quad -\Gamma_{av}^{\frac{1}{m+3}}\left[H\left(\frac{m+1}{2(m+3)}, \frac{\Gamma_{av}}{1+R}\right)\right. \\ &\quad \left. -H\left(\frac{m+1}{2(m+3)}, (1+R)^{(m+2)}\,\Gamma_{av}\right)\right], \end{aligned} \tag{12}$$

where

$$R \triangleq \frac{r_e^2}{L^2}, \tag{13}$$

and the functions $G(\cdot)$ and $H(\cdot,\cdot)$ are defined as

$$\begin{aligned} G(x) &\triangleq Q\left(\sqrt{\frac{R(m+2)\,x}{(1+R)^{m+2}-1}}\right) \\ H(y,z) &\triangleq \frac{1}{\sqrt{\pi}}\left(\frac{(m+2)(1+R)^{m+2}}{2^{m+4}R^{m+2}\left[(1+R)^{m+2}-1\right]}\right)^{\frac{1}{m+3}} \\ &\quad \times\Gamma\left(y, \frac{R(m+2)\,z}{2\left[(1+R)^{m+2}-1\right]}\right), \end{aligned} \tag{14}$$

where $\Gamma(\cdot,\cdot)$ denotes the upper incomplete Gamma function.

B. Error Analysis Using LS Estimates

The data transmission is carried out in the VLC system following the channel estimation by the LS method. Utilizing the receiver structure in (9) for the case of BPSK transmission, the expression of the SEP can be written as

$$\begin{aligned} P_{e,ls} &= \Pr\left(r\hat{h}_{ls} < 0\right) \\ &= \Pr\left(hA+n<0\,,\,h+\epsilon_{ls}>0\right) \\ &\quad + \Pr\left(hA+n>0\,,\,h+\epsilon_{ls}<0\right). \end{aligned} \tag{15}$$

It is to be noted that the additive noise n and the estimation error ϵ_{ls} in (15) are independent of each other due to the assumption of independence of the noise over time samples. Thus, the conditional SEP of the VLC system using LS estimation, conditioned on the channel gain is obtained from the statistics of n, ϵ_{ls} and (15), as

$$\begin{aligned} P_{e,ls\left|h\right.} &= Q\left(\sqrt{h^2\Gamma_{av}}\right) + Q\left(\sqrt{h^2\Gamma_p}\right) \\ &\quad -2Q\left(\sqrt{h^2\Gamma_{av}}\right)Q\left(\sqrt{h^2\Gamma_p}\right). \end{aligned} \tag{16}$$

Further, simplifying the expression in (16) by utilizing the Craig's formula for the Gaussian-Q function [16] and carrying out algebraic manipulations by employing integration by parts technique followed by the Leibniz integral rule, the closed-form expression for $P_{e,ls}$ is given as in (17), as shown at the bottom of the next page, where the functions $G(\cdot)$ and $H(\cdot,\cdot)$ are given in (14).

It is to be observed that unlike the conditional expression of SEP in (10) which tends to zero for high values of SNR Γ_{av}, the conditional expression of SEP for the VLC system utilizing LS channel estimation in (16) can be approximated at high SNR as $P_{e,ls\left|h,\Gamma_{av}\gg 1\right.} \approx Q\left(\sqrt{h^2\Gamma_p}\right)$ and thus, the asymptotic expression of the SEP at high SNR can be expressed as

$$\begin{aligned} P_{e,ls\left|\Gamma_{av}\gg 1\right.} &\approx \frac{(1+R)}{R}G\left(\frac{\Gamma_p}{1+R}\right) - \frac{1}{R}G\left((1+R)^{m+2}\,\Gamma_p\right) \\ &\quad -\Gamma_p^{\frac{1}{m+3}}\left[H\left(\frac{m+1}{2(m+3)}, \frac{\Gamma_p}{1+R}\right)\right. \\ &\quad \left. -H\left(\frac{m+1}{2(m+3)}, (1+R)^{(m+2)}\,\Gamma_p\right)\right]. \end{aligned} \tag{18}$$

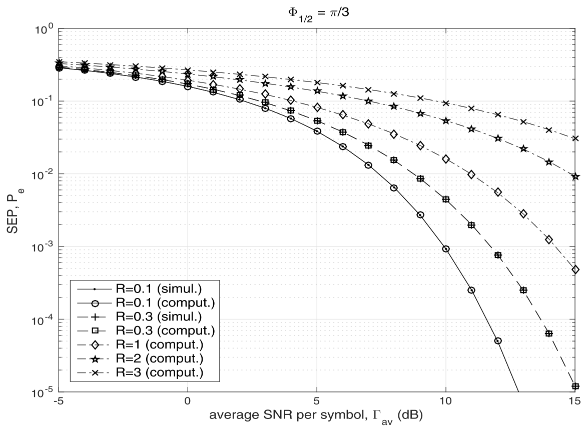

Fig. 2. SEP versus the average SNR per symbol with perfect CSI at the receiver for $R = 0.1, 0.3, 1, 2, 3$ and $\Phi_{1/2} = \pi/3$.

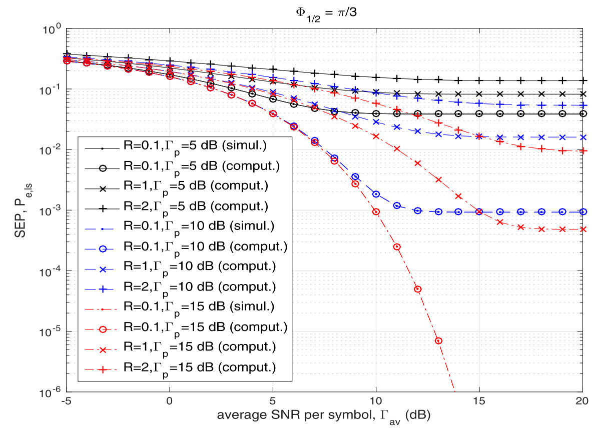

Fig. 3. SEP versus the average SNR per symbol with imperfect CSI at the receiver for $R = 0.1, 1, 2$, $\Gamma_p = 5, 10, 15$ dB, and $\Phi_{1/2} = \pi/3$.

Thus, the expression of SEP for the VLC system utilizing LS estimates saturates to the value as obtained from (18) at high values of the average SNR per symbol Γ_{av}.

IV. Numerical Results

This section provides the numerical studies of the derived performance metrics of the VLC system with varying system parameters. Fig. 2 shows the plots of SEP versus the average SNR per symbol, Γ_{av} for the VLC system with perfect CSI at the receiver. It is observed that the computation plots of SEP as obtained in (12) coincide with the simulation plots of SEP obtained by using Monte Carlo simulations of the receiver in (8), thus implying the correctness of the analysis. Furthermore, it is observed that the performance of the VLC system degrades with the increase in the ratio R of the squares of the lengths r_e and L as defined in (13).

The plots of the variation of SEP with Γ_{av} for the VLC system employing LS estimation at the receiver are presented in Fig. 3. The computation plots obtained from (17) and the simulation plots obtained by using the Monte Carlo technique on the receiver proposed in (9) coincide with each other, thus implying the correctness of the analysis. Apart from having a similar trend of the SEP with the variation of R and the superior performance of the VLC system with channel estimation, it is interesting to observe that the SEP plots tend to saturate at higher values of Γ_{av}. Further, this saturation tends to be prominent for higher values of R and lower values of the pilot symbol SNR, Γ_p.

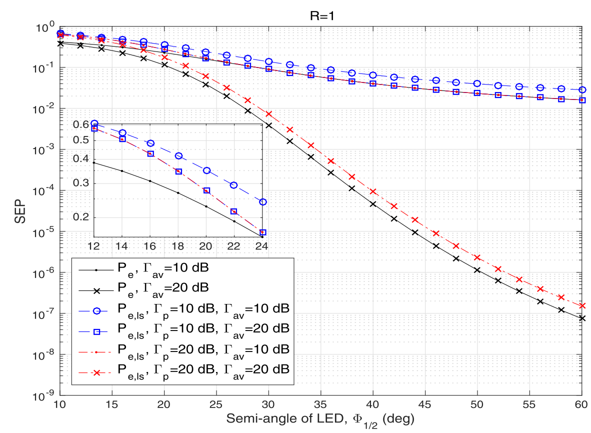

Fig. 4. SEP versus the semi-angle of the LED transmitter for the VLC system with and without CSI at the receiver for $R = 1$, $\Gamma_{av} = 10, 20$ dB, $\Gamma_p = 10, 20$ dB.

Fig. 4 shows the plots of the SEP with and without channel estimation for the variation of the semi-angle of the LED. It is observed that the performance of the VLC system is better for higher values of the semi-angle of the LED, and this superior performance becomes quite significant for higher values of Γ_p and Γ_{av}. Further, for lower values of $\Phi_{1/2}$, the performance of the system is almost the same for various values of the SNRs. This gives us a practical implementation of utilizing less power for transmission of data and pilot symbols for the LED transmitter functioning at a lower semi-angle.

$$
\begin{aligned}
P_{e,ls}=&\frac{(1+R)}{R}\left[G\left(\frac{\Gamma_{av}}{1+R}\right)+G\left(\frac{\Gamma_p}{1+R}\right)\right]-\frac{1}{R}\left[G\left((1+R)^{m+2}\Gamma_{av}\right)+G\left((1+R)^{m+2}\Gamma_p\right)\right]\\
&-\Gamma_{av}^{\frac{1}{m+3}}\left[H\left(\frac{m+1}{2(m+3)},\frac{\Gamma_{av}}{1+R}\right)-H\left(\frac{m+1}{2(m+3)},(1+R)^{m+2}\Gamma_{av}\right)\right]\\
&-\Gamma_{p}^{\frac{1}{m+3}}\left[H\left(\frac{m+1}{2(m+3)},\frac{\Gamma_{p}}{1+R}\right)-H\left(\frac{m+1}{2(m+3)},(1+R)^{m+2}\Gamma_{p}\right)\right]\\
&-\frac{(\Gamma_p+\Gamma_{av})^{\frac{1}{m+3}}}{\sqrt{2}(m+3)}\left[H\left(-\frac{1}{(m+3)},\frac{\Gamma_p+\Gamma_{av}}{1+R}\right)-H\left(-\frac{1}{(m+3)},(1+R)^{(m+2)}(\Gamma_p+\Gamma_{av})\right)\right]
\end{aligned}
\tag{17}
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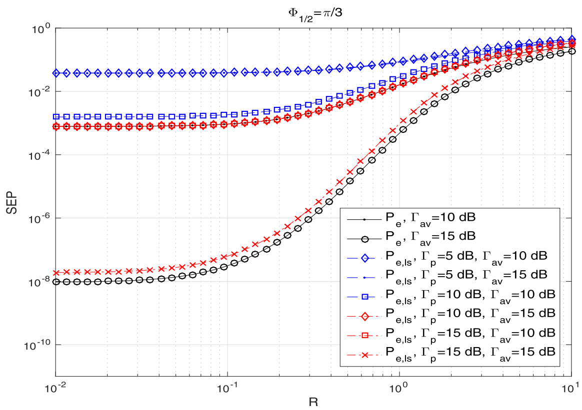

Fig. 5. SEP versus R for VLC system with and without CSI at the receiver for $\Phi_{1/2} = \pi/3$, $\Gamma_{av} = 10, 15$ dB, $\Gamma_p = 10, 15$ dB.

The plots of the variation of SEP with R for the VLC system with and without channel estimates are presented in Fig. 5. It is interesting to observe that the performance of the VLC system without utilizing channel estimates is superior to the system with LS estimates for lower values of R. Further, the performance of both the VLC systems are similar for higher values of R, without much dependence on Γ_{av} and Γ_p.

V. Conclusion

In this letter, a SISO VLC communication system is considered with the receiver with and without the knowledge of the CSI. The closed form expressions for the SEP for the VLC system with perfect CSI and for the system estimating the channel gain using the LS estimation technique are obtained. It is observed that the performance of both the systems are dependent on the ratio of the cell radius of the system and the vertical distance between the LED and the receiver's plane. Further, the SEP performance saturates at lower values of the pilot symbol power for the VLC system utilizing LS estimates. The reduced dependence of the VLC systems on channel estimation at lower values of the semi-angle of the LED transmitter and at higher values of the ratio of the distances, and other design insights to develop a reliable communication system are illustrated by studying the performance both the VLC systems with various system parameters numerically.

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