MARNO NEL 02/03/2023

Forward & Inverse Kinematics of a turtlebot3 burger

double wheel_radius
double wheel_track

① Keep Track

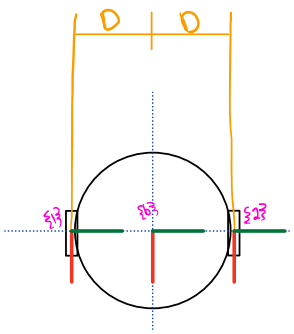
Struct{
double ϕ_l
double ϕ_r
} 3 wheel_position

Struct{
double x
double y
double θ
} 3 robot_configuration

Struct{
double $\dot{\phi}_l$
double $\dot{\phi}_r$
} 3 wheel_velocity

② Forward Kinematics

Given new wheel positions ϕ_l', ϕ_r' UPDATE robot_configuration



$T_{b_1} = (0, -D, 0)$ $T_{b_2} = (0, D, 0)$

$$A_{b1} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_{b2} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- $V_1 = A_{b1} \cdot V_b$

$$\begin{bmatrix} \dot{\theta}_1 \\ V_{x_1} \\ V_{y_1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_b \\ V_{xb} \\ V_{yb} \end{bmatrix} = \begin{bmatrix} \dot{\theta}_b \\ -D \cdot \dot{\theta}_b + V_{xb} \\ V_{yb} \end{bmatrix} \quad \& \quad \begin{bmatrix} r\dot{\phi}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} V_{x_1} \\ V_{y_1} \end{bmatrix}$$

Solve $\Rightarrow$ $$\begin{bmatrix} \dot{\phi}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -D/r & 1/r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_b \\ V_{xb} \\ V_{yb} \end{bmatrix}$$

- $V_2 = A_{b2} \cdot V_b$

$$\begin{bmatrix} \dot{\theta}_2 \\ V_{x_2} \\ V_{y_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_b \\ V_{xb} \\ V_{yb} \end{bmatrix} = \begin{bmatrix} \dot{\theta}_b \\ -D \cdot \dot{\theta}_b + V_{xb} \\ V_{yb} \end{bmatrix} \quad \& \quad \begin{bmatrix} r\dot{\phi}_2 \\ 0 \end{bmatrix} = \begin{bmatrix} V_{x_2} \\ V_{y_2} \end{bmatrix}$$

Solve $\Rightarrow$ $$\begin{bmatrix} \dot{\phi}_2 \\ 0 \end{bmatrix} = \begin{bmatrix} D/r & 1/r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_b \\ V_{xb} \\ V_{yb} \end{bmatrix}$$

- $\dot{\phi} = H \cdot V_b \longrightarrow \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = \frac{1}{r} \cdot \begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_b \\ V_{xb} \\ V_{yb} \end{bmatrix}$

- $u = H \cdot V_b \longrightarrow V_b = H^{\dagger} \cdot u \quad \& \quad H^{\dagger} = H^T \cdot (H \cdot H^T)^{-1}$

$$\begin{bmatrix} \dot{\theta}_b \\ V_{bx} \\ V_{by} \end{bmatrix} = V_b = \frac{r}{2} \cdot \begin{bmatrix} \frac{-1}{D} & \frac{1}{D} \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \& \quad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix}$$

Solve

$$\dot{\theta}_b = \frac{r}{2} \cdot \left[-\frac{\dot{\phi}_1}{D} + \frac{\dot{\phi}_2}{D} \right] \qquad V_{bx} = \frac{r}{2} \cdot [\dot{\phi}_1 + \phi_2] \qquad V_{by} = 0 \quad \text{..... eq (1)}$$

$T_{bb'} = \text{integrateTwist}(\dot{\theta}_b, V_{bx}, V_{by})$ ..... eq (2)

$T_{wb} = (\theta_b, x_b, y_b)$ → Current position ..... eq (3)

$T_{wb'} = T_{wb} \cdot T_{bb'}$ ..... eq (4)

$q = T_{wb'}(\theta_{b'}, x_{b'}, y_{b'})$ → Extract new Configuration ..... eq (5)

③ Inverse Kinematics

① Compute wheel velocities required to make the robot move @ a given Twist.

② If Twist cannot be accomplished without slipping throw an std::logic_error() exception.

Wheel Velocities Twist

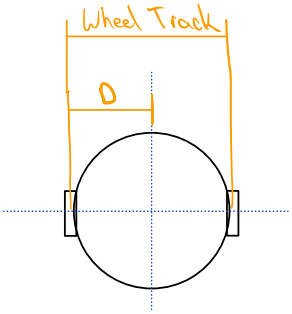
$$\dot{\phi} = H \cdot V_b$$

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}$$

Rotation Velocity ↑ Velocity in x ↑

$$\dot{\phi}_1 = \frac{1}{r}\left[-D \cdot \dot{\theta} + v_x\right] \quad \text{.....} \quad \text{eq (1)}$$

$$\dot{\phi}_2 = \frac{1}{r}\left[D \cdot \dot{\theta} + v_x\right] \quad \text{.....}$$



D = Wheel_track ÷ 2