Лабораторная работа №6

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Используя явную схему крест и неявную схему, решить начально-краевую задачу для дифференциального уравнения гиперболического типа. Аппроксимацию второго начального условия произвести с первым и со вторым порядком. Осуществить реализацию трех вариантов аппроксимации граничных условий, содержащих производные: двухточечная аппроксимация с первым порядком, трехточечная аппроксимация со вторым порядком, двухточечная аппроксимация со вторым порядком. В различные моменты времени вычислить погрешность численного решения путем сравнения результатов с приведенным в задании аналитическим решением $U(x, t)$. Исследовать зависимость погрешности от сеточных параметров τ, h.

Вариант 6

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2\frac{\partial u}{\partial x} - 2u$$

$$u(0,\ t) = \cos 2t$$

$$u(\tfrac{\pi}{2},\ t) = 0$$

$$u(x,\ 0) = \exp(-x)\cos x$$

$$u_t(x,\ 0) = 0$$

$$U(x,\ t) = \exp(-x)\cos x\cos(2t)$$

In [8]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [9]:
```
eq = {
    'a': 1,
    'b': 2,
    'c': -2,
    'd': 0,
    'l': np.pi / 2,
    'f': lambda: 0,
    'alpha': 1,
    'beta': 0,
    'gamma': 1,
    'delta': 0,
    'psi1': lambda x: np.exp(-x) * np.cos(x),
    'psi2': lambda x: 0,
    'psi1_dir1': lambda x: -np.exp(-x) * np.sin(x) - np.exp(-x) * n
    'psi1_dir2': lambda x: 2 * np.exp(-x) * np.sin(x),
    'phi0': lambda t: np.cos(2 * t),
    'phi1': lambda t: 0,
    'solution': lambda x, t: np.exp(-x) * np.cos(x) * np.cos(2 * t)
}
```

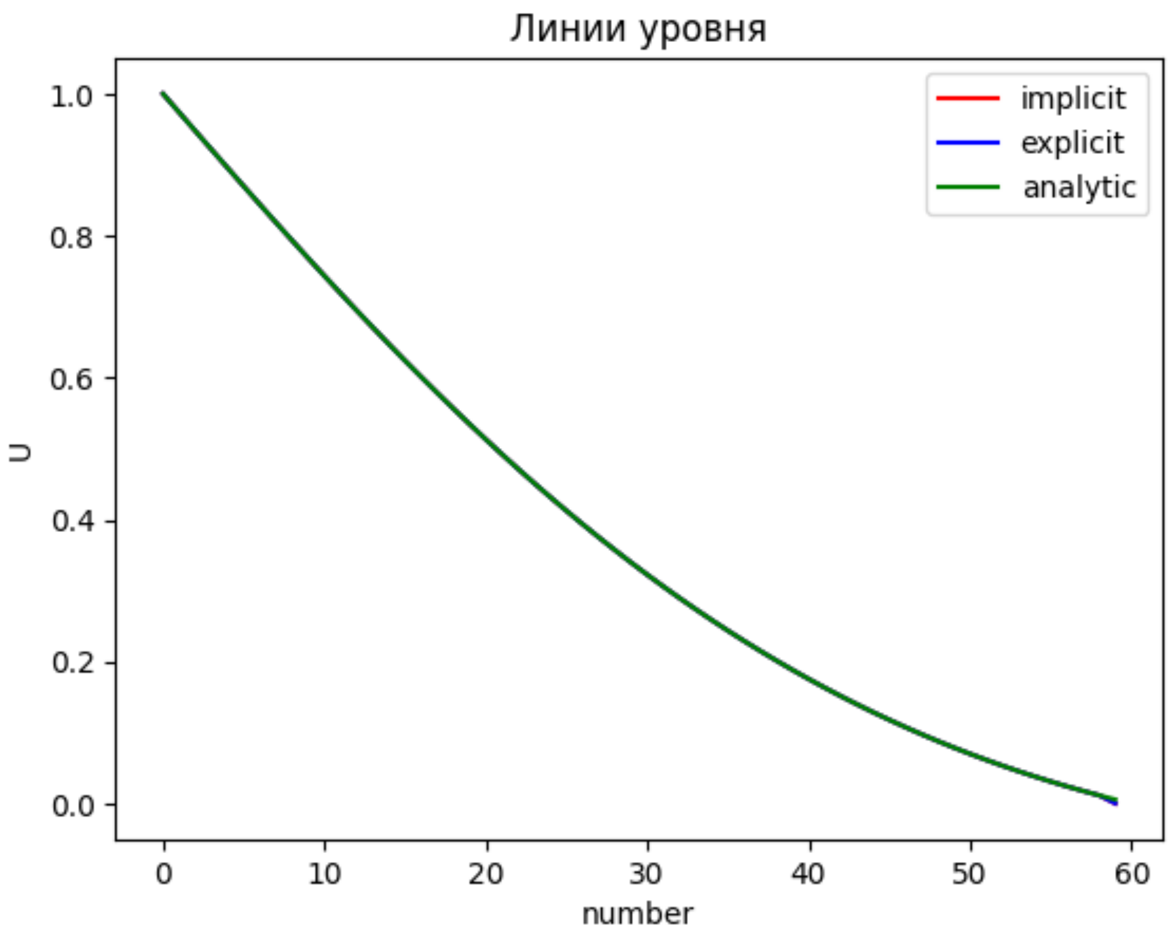
In [10]:
```
def tma(a, b, c, d):
    size = len(a)
    p, q = [], []
    p.append(-c[0] / b[0])
    q.append(d[0] / b[0])

    for i in range(1, size):
        p_tmp = -c[i] / (b[i] + a[i] * p[i - 1])
        q_tmp = (d[i] - a[i] * q[i - 1]) / (b[i] + a[i] * p[i - 1])
        p.append(p_tmp)
        q.append(q_tmp)

    x = [0 for _ in range(size)]
    x[size - 1] = q[size - 1]

    for i in range(size - 2, -1, -1):
        x[i] = p[i] * x[i + 1] + q[i]

    return x

class Eq:
    def __init__(self, args):
        self.a = args['a']
        self.b = args['b']
        self.c = args['c']
        self.d = args['d']
        self.l = args['l']
        self.f = args['f']
        self.alpha = args['alpha']
        self.beta = args['beta']
        self.gamma = args['gamma']
        self.delta = args['delta']
        self.psi1 = args['psi1']
        self.psi2 = args['psi2']
        self.psi1_dir1 = args['psi1_dir1']
        self.psi1_dir2 = args['psi1_dir2']
```

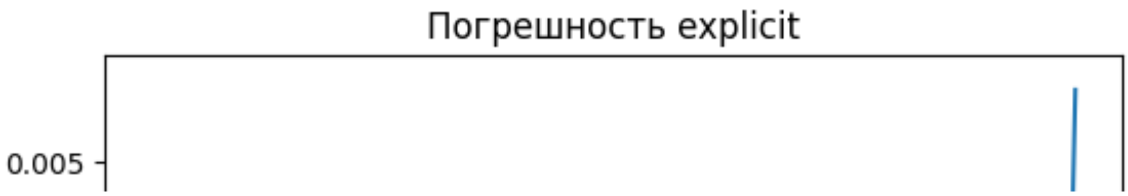
```python
        self.phi0 = args['phi0']
        self.phi1 = args['phi1']
        self.bound_type = args['bound_type']
        self.approximation = args['approximation']
        self.solution = args['solution']


class Hyperbolic:
    def __init__(self, args, N, K, T):
        self.data = Eq(args)
        self.h = self.data.l / N
        self.tau = T / K
        self.sigma = (self.tau ** 2) / (self.h ** 2)

    def solve_analytic(self, N, K, T):
        self.h = self.data.l / N
        self.tau = T / K
        self.sigma = (self.tau ** 2) / (self.h ** 2)
        u = np.zeros((K, N))
        for k in range(K):
            for j in range(N):
                u[k][j] = self.data.solution(j * self.h, k * self.ta
        return u

    def calc(self, N, K):
        u = np.zeros((K, N))

        for j in range(0, N - 1):
            x = j * self.h
            u[0][j] = self.data.psi1(x)

            if self.data.approximation == 'p1':
                u[1][j] = self.data.psi1(x) + self.data.psi2(x) * s
                          (self.tau ** 2 / 2)
            elif self.data.approximation == 'p2':
                u[1][j] = self.data.psi1(x) + self.data.psi2(x) * s
                          (self.data.psi1_dir2(x) + self.data.b * s
                           self.data.c * self.data.psi1(x) + self.d

        return u

    def solve_implicit(self, N, K, T):
        u = self.calc(N, K)

        a = np.zeros(N)
        b = np.zeros(N)
        c = np.zeros(N)
        d = np.zeros(N)

        for k in range(2, K):
            for j in range(1, N):
                a[j] = self.sigma
                b[j] = -(1 + 2 * self.sigma)
                c[j] = self.sigma
                d[j] = -2 * u[k - 1][j] + u[k - 2][j]

            if self.data.bound_type == 'a1p2':
                b[0] = self.data.alpha / self.h / (self.data.beta -
```

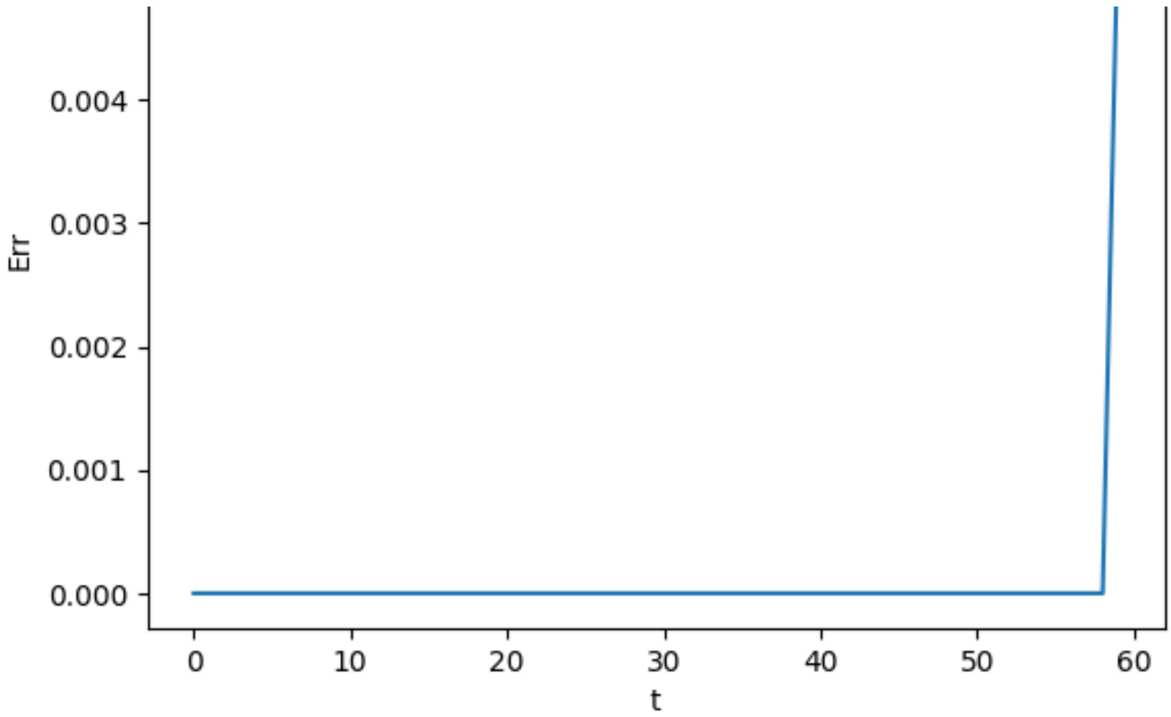
```python
                c[0] = 1
                d[0] = 1 / (self.data.beta - self.data.alpha / self
                a[-1] = -self.data.gamma / self.h / (self.data.delt
                d[-1] = 1 / (self.data.delta + self.data.gamma / se

            elif self.data.bound_type == 'a2p3':
                k1 = 2 * self.h * self.data.beta - 3 * self.data.al
                omega = self.tau ** 2 * self.data.b / (2 * self.h)
                xi = self.data.d * self.tau / 2

                b[0] = 4 * self.data.alpha - self.data.alpha / (sel
                       (1 + xi + 2 * self.sigma - self.data.c * sel
                c[0] = k1 - self.data.alpha * (omega - self.sigma)
                d[0] = 2 * self.h * self.data.phi0(k * self.tau) +
                a[-1] = -self.data.gamma / (omega - self.sigma) * \
                        (1 + xi + 2 * self.sigma - self.data.c * se
                d[-1] = 2 * self.h * self.data.phi1(k * self.tau) -

            elif self.data.bound_type == 'a2p2':
                b[0] = 2 * self.data.a / self.h
                c[0] = -2 * self.data.a / self.h + self.h / self.ta
                       -self.data.d * self.h / (2 * self.tau) + \
                       self.data.beta / self.data.alpha * (2 * self
                d[0] = self.h / self.tau ** 2 * (u[k - 2][0] - 2 *
                       -self.data.d * self.h / (2 * self.tau) * u[k
                       (2 * self.data.a - self.data.b * self.h) / s
                a[-1] = -b[0]
                d[-1] = self.h / self.tau ** 2 * (-u[k - 2][0] + 2
                        self.data.d * self.h / (2 * self.tau) * u[k
                        (2 * self.data.a + self.data.b * self.h) /

            u[k] = tma(a, b, c, d)

        return u

    def _left_bound_a1p2(self, u, k, t):
        coeff = self.data.alpha / self.h
        return (-coeff * u[k - 1][1] + self.data.phi0(t)) / (self.da

    def _right_bound_a1p2(self, u, k, t):
        coeff = self.data.gamma / self.h
        return (coeff * u[k - 1][-2] + self.data.phi1(t)) / (self.da

    def _left_bound_a2p2(self, u, k, t):
        n = self.data.c * self.h - 2 * self.data.a / self.h - self.
            (2 * self.tau) + self.data.beta / self.data.alpha * (2
        return 1 / n * (- 2 * self.data.a / self.h * u[k][1] +
                        self.h / self.tau ** 2 * (u[k - 2][0] - 2 *
                        -self.data.d * self.h / (2 * self.tau) * u[
                        (2 * self.data.a - self.data.b * self.h) /

    def _right_bound_a2p2(self, u, k, t):
        n = -self.data.c * self.h + 2 * self.data.a / self.h + self
            (2 * self.tau) + self.data.delta / self.data.gamma * (2
        return 1 / n * (2 * self.data.a / self.h * u[k][-2] +
                        self.h / self.tau ** 2 * (2 * u[k - 1][-1]
                        self.data.d * self.h / (2 * self.tau) * u[k
```

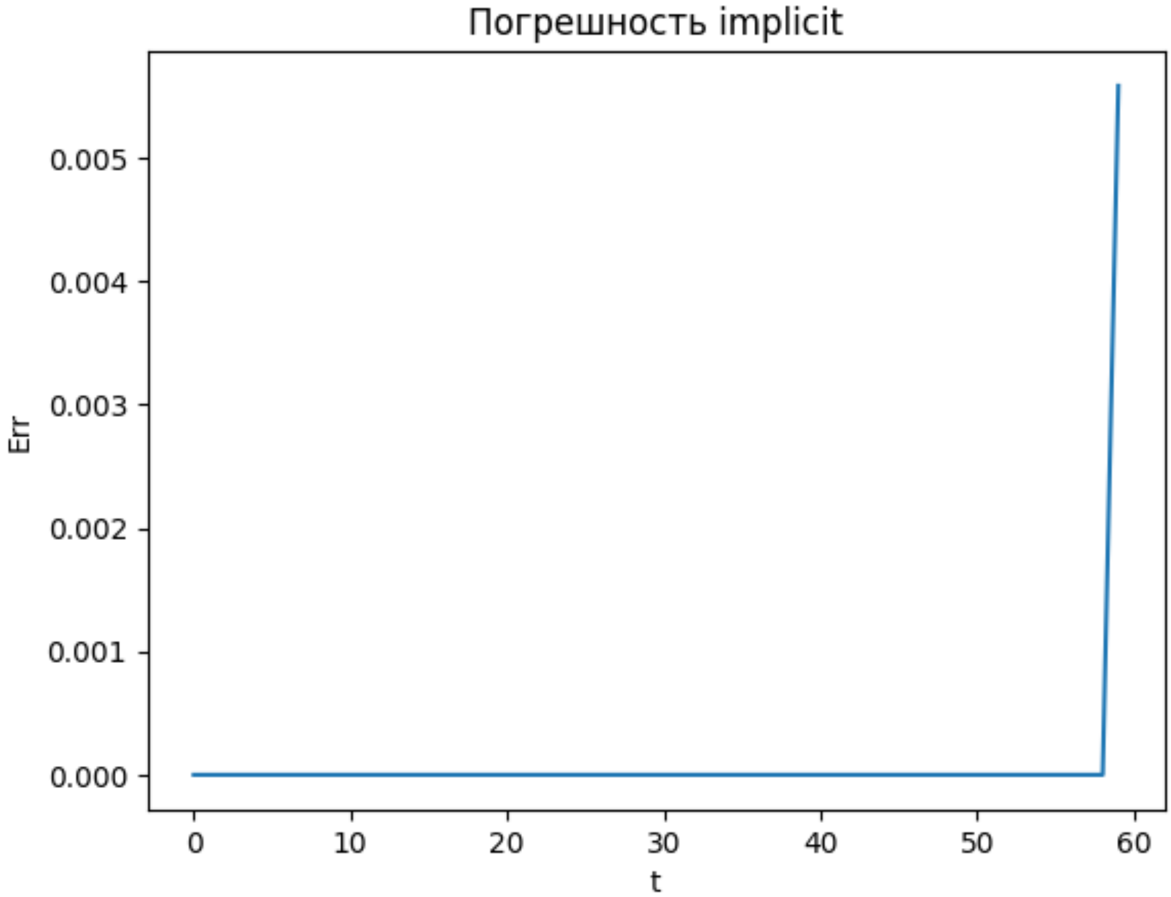
```python
                    (2 * self.data.a + self.data.b * self.h) / 

    def _left_bound_a2p3(self, u, k, t):
        denom = 2 * self.h * self.data.beta - 3 * self.data.alpha
        return self.data.alpha / denom * u[k - 1][2] - 4 * self.data
               2 * self.h / denom * self.data.phi0(t)

    def _right_bound_a2p3(self, u, k, t):
        denom = 2 * self.h * self.data.delta + 3 * self.data.gamma
        return 4 * self.data.gamma / denom * u[k - 1][-2] - self.da
               2 * self.h / denom * self.data.phi1(t)

    def explicit_solver(self, N, K, T):
        global left_bound, right_bound
        u = self.calc(N, K)

        if self.data.bound_type == 'a1p2':
            left_bound = self._left_bound_a1p2
            right_bound = self._right_bound_a1p2

        elif self.data.bound_type == 'a2p2':
            left_bound = self._left_bound_a2p2
            right_bound = self._right_bound_a2p2

        elif self.data.bound_type == 'a2p3':
            left_bound = self._left_bound_a2p3
            right_bound = self._right_bound_a2p3

        for k in range(2, K):
            t = k * self.tau
            for j in range(1, N - 1):
                quadr = self.tau ** 2
                tmp1 = self.sigma + self.data.b * quadr / (2 * self
                tmp2 = self.sigma - self.data.b * quadr / (2 * self
                u[k][j] = u[k - 1][j + 1] * tmp1 + \
                    u[k - 1][j] * (-2 * self.sigma + 2 + self.data.
                    u[k - 1][j - 1] * tmp2 - u[k - 2][j] + quadr * 
            u[k][0] = left_bound(u, k, t)
            u[k][-1] = right_bound(u, k, t)

        return u


def show(dict_, time=0):
    fig = plt.figure()
    plt.title('Линии уровня')
    plt.plot(dict_['implicit'][time], color='r', label='implicit')
    plt.plot(dict_['explicit'][time], color='b', label='explicit')
    plt.plot(dict_['analytic'][time], color='g', label='analytic')
    plt.legend(loc='best')
    plt.ylabel('U')
    plt.xlabel('number')
    plt.show()

    plt.title('Погрешность explicit')
    plt.plot(abs(dict_['explicit'][time] - dict_['analytic'][time])
    plt.ylabel('Err')
    plt.xlabel('t')
```

```
    plt.show()

    plt.title('Погрешность implicit')
    plt.plot(abs(dict_['implicit'][time] - dict_['analytic'][time])
    plt.ylabel('Err')
    plt.xlabel('t')
    plt.show()


data = {'N': 60, 'K': 100, 'T': 1}
N, K, T = int(data['N']), int(data['K']), int(data['T'])

args = eq
args['bound_type'] = 'a1p2'
args['approximation'] = 'p1'
solver = Hyperbolic(args, N, K, T)

ans = {
    'implicit': solver.solve_implicit(N, K, T),
    'explicit': solver.explicit_solver(N, K, T),
    'analytic': solver.solve_analytic(N, K, T)
}

show(ans)
```





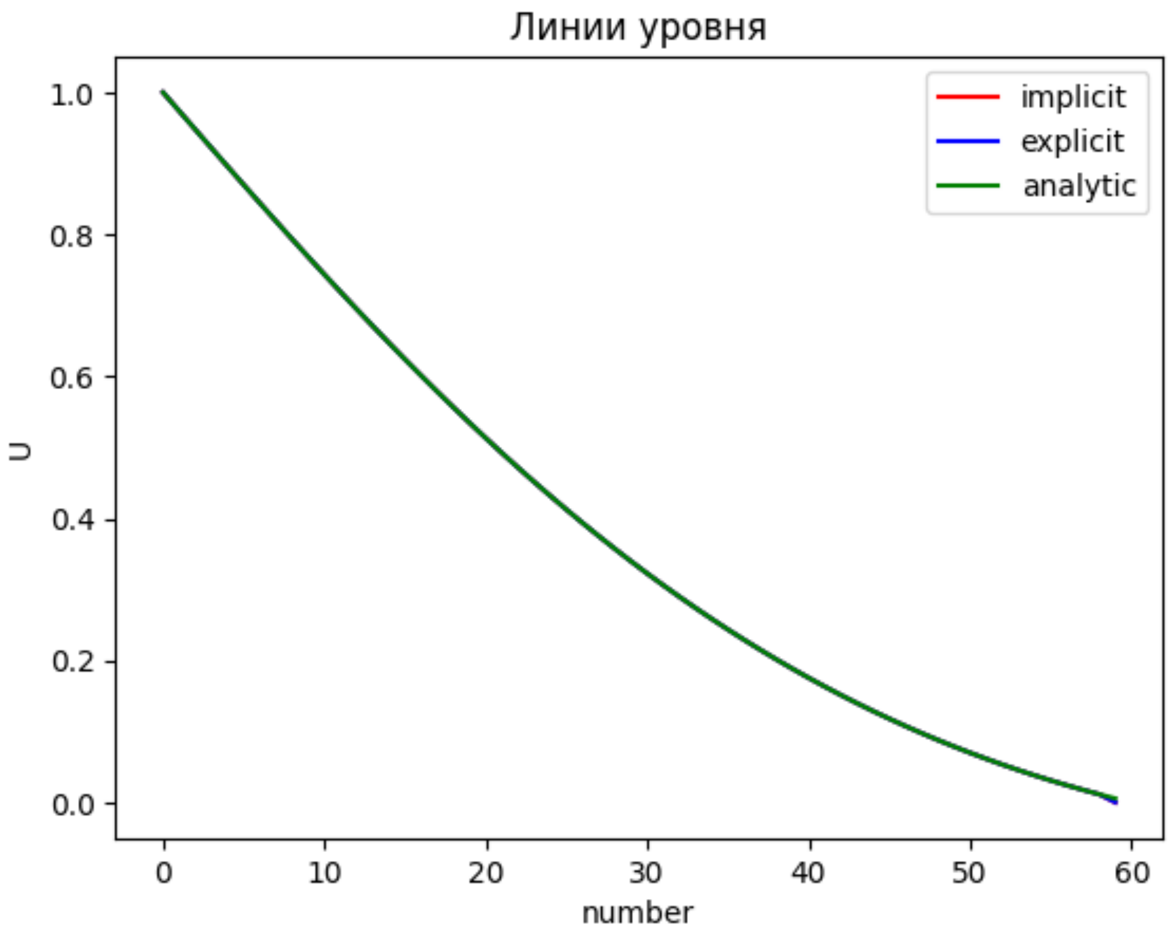


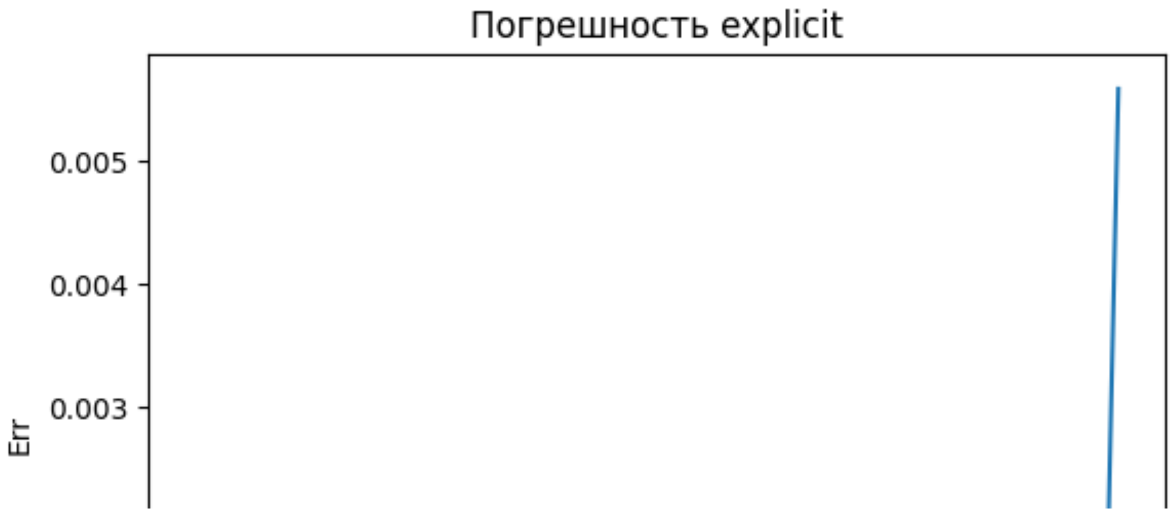


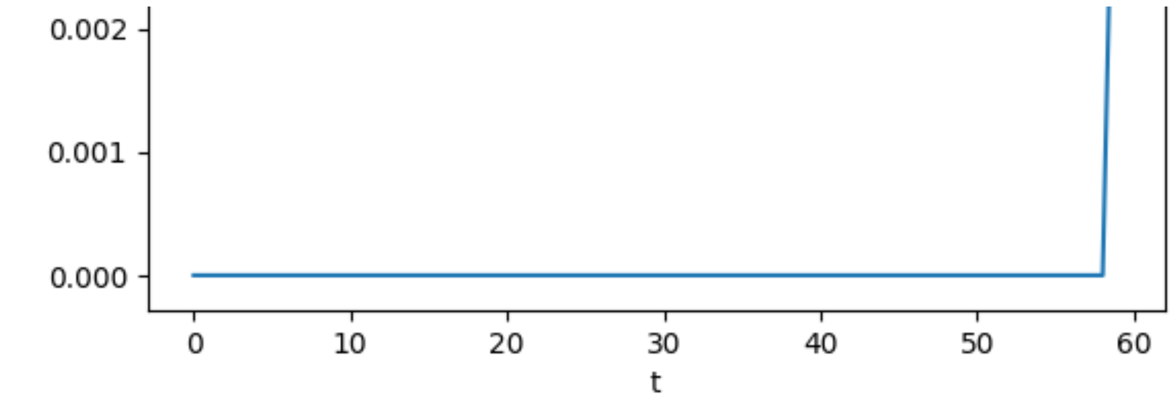
Апроксимация 3-х точечная второго порядка

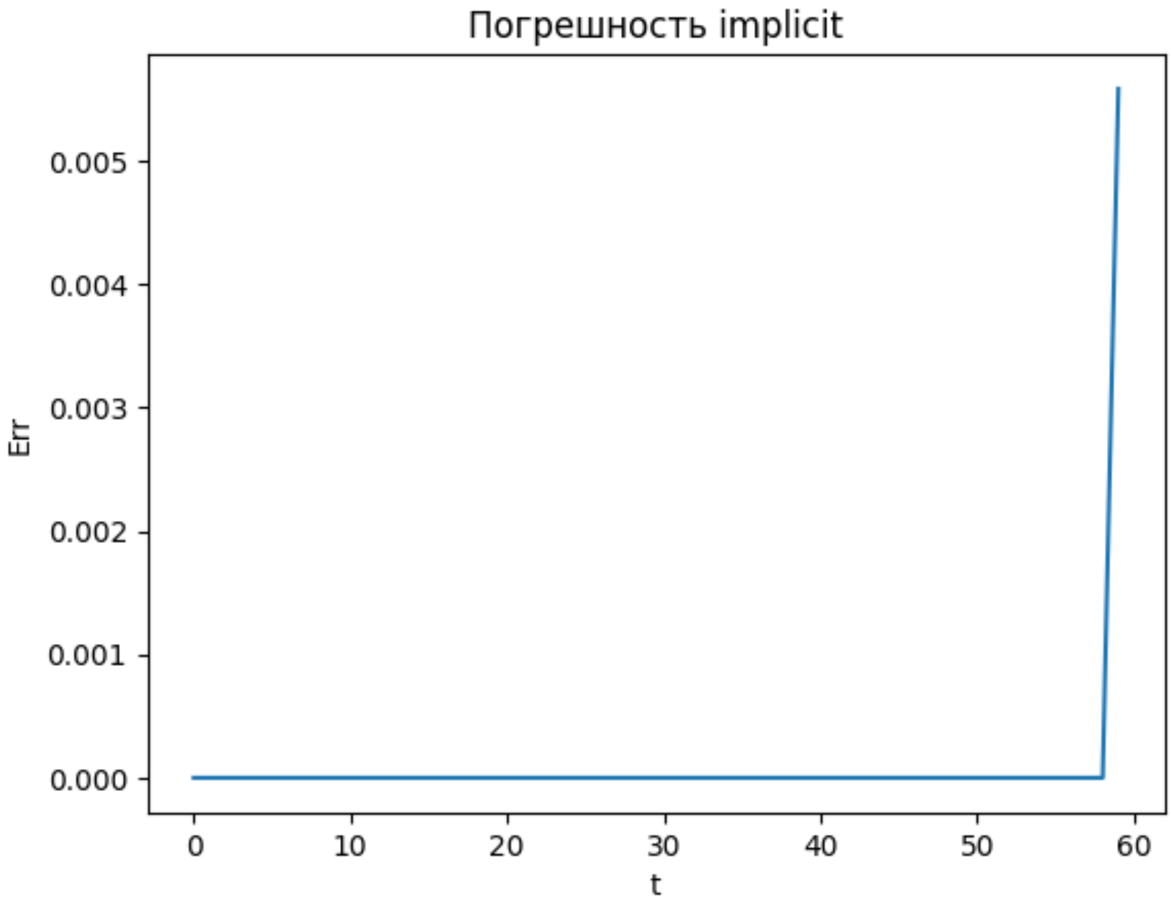
In [11]:

```
data = {'N': 60, 'K': 100, 'T': 1}
N, K, T = int(data['N']), int(data['K']), int(data['T'])

args = eq
args['bound_type'] = 'a2p3'
args['approximation'] = 'p2'
solver = Hyperbolic(args, N, K, T)

ans = {
    'implicit': solver.solve_implicit(N, K, T),
    'explicit': solver.explicit_solver(N, K, T),
    'analytic': solver.solve_analytic(N, K, T)
}

show(ans)
```





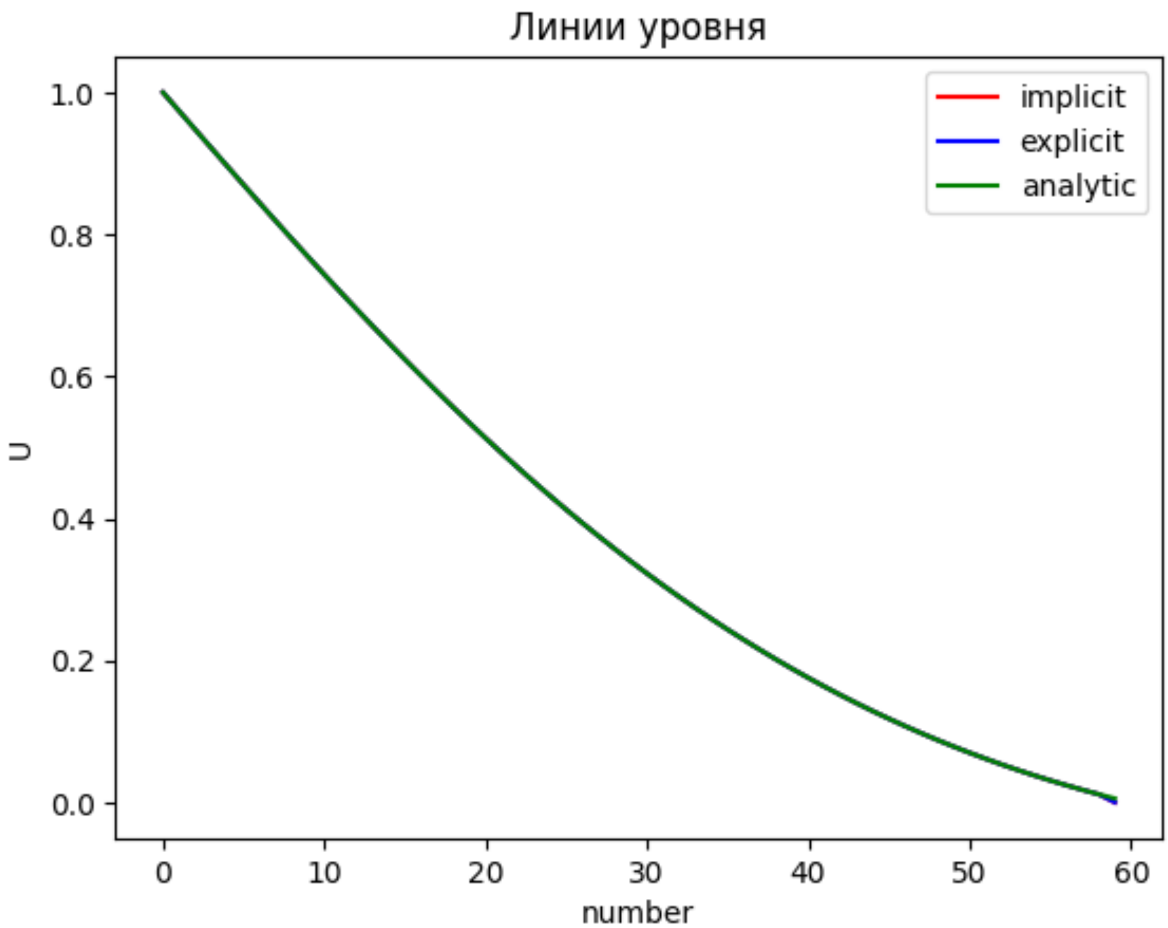


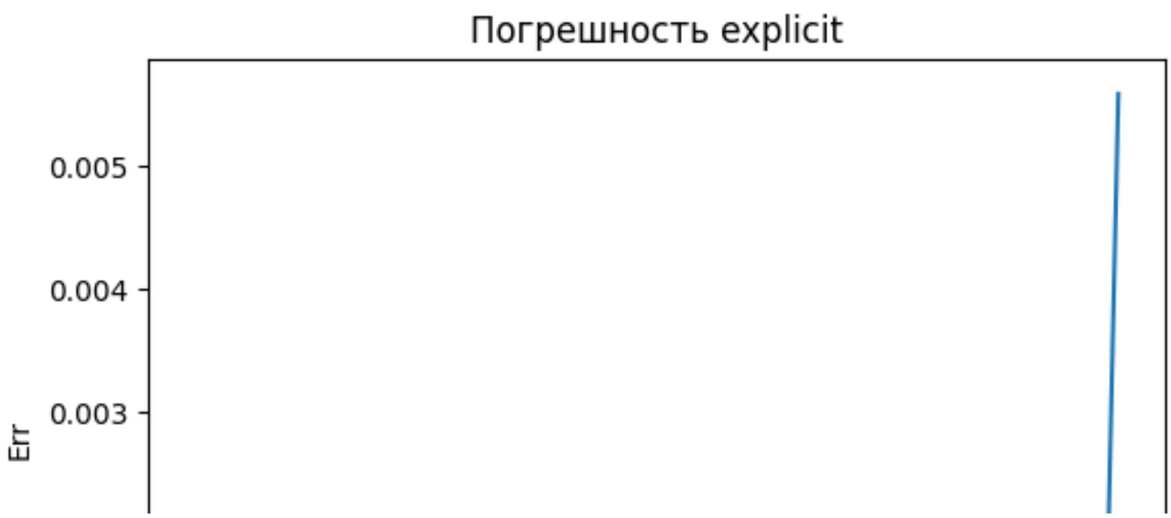


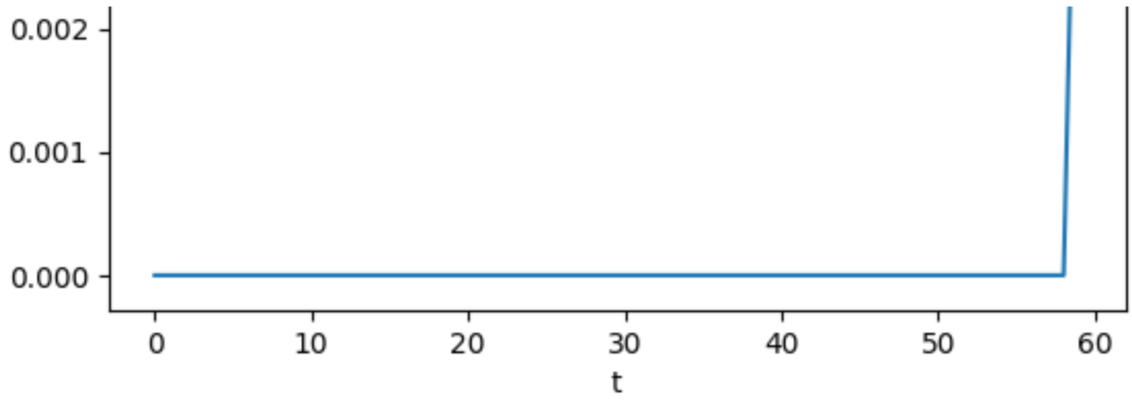
Апроксимация 2-х точечная второго порядка

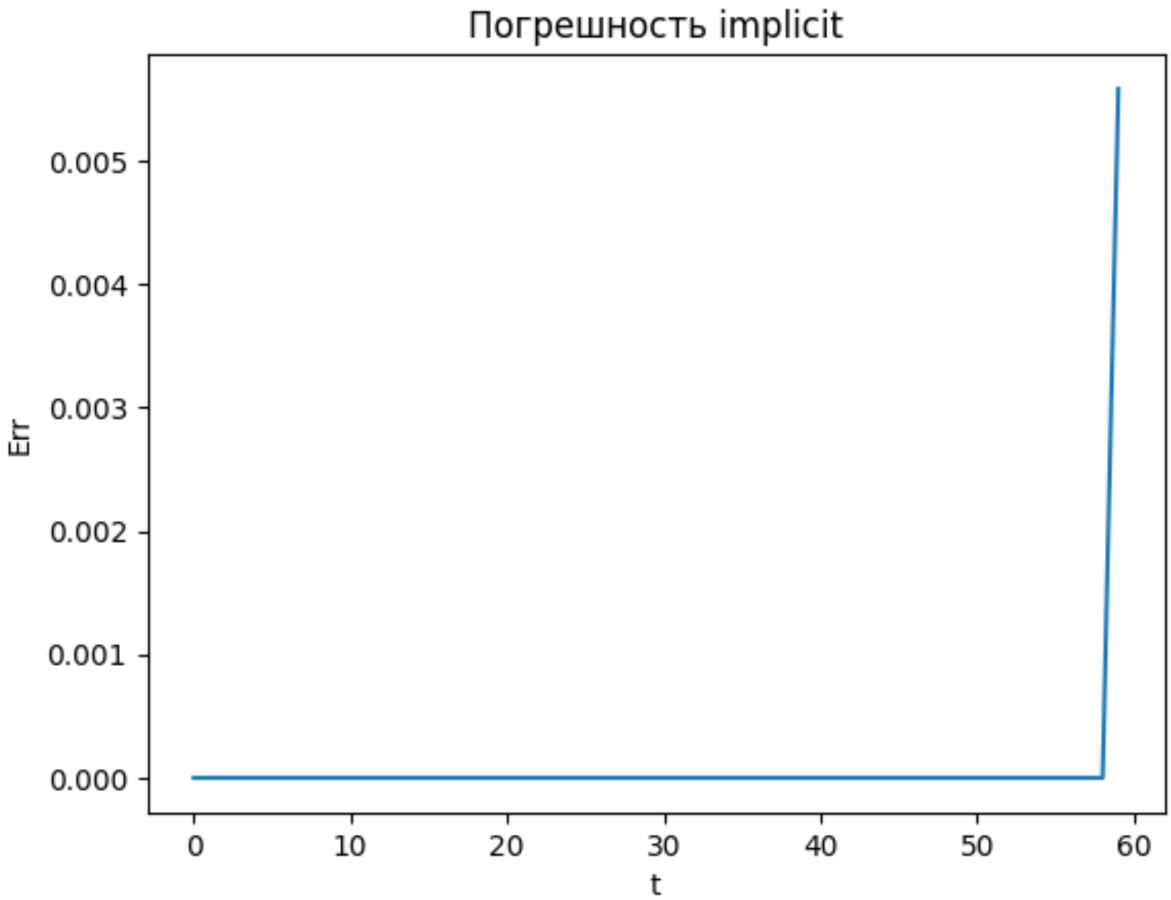
In [12]:

```
data = {'N': 60, 'K': 100, 'T': 1}
N, K, T = int(data['N']), int(data['K']), int(data['T'])

args = eq
args['bound_type'] = 'a1p2'
args['approximation'] = 'p2'
solver = Hyperbolic(args, N, K, T)

ans = {
    'implicit': solver.solve_implicit(N, K, T),
    'explicit': solver.explicit_solver(N, K, T),
    'analytic': solver.solve_analytic(N, K, T)
}

show(ans)
```





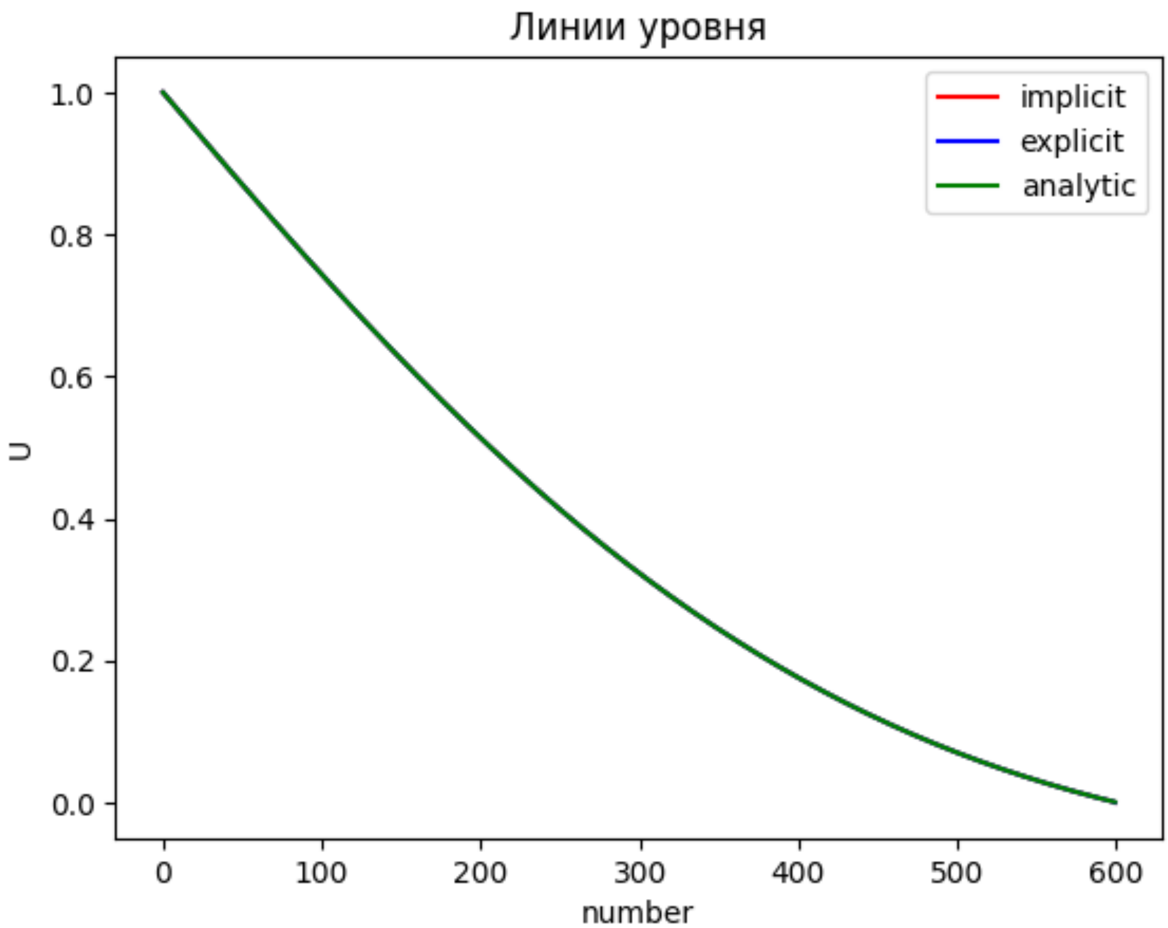


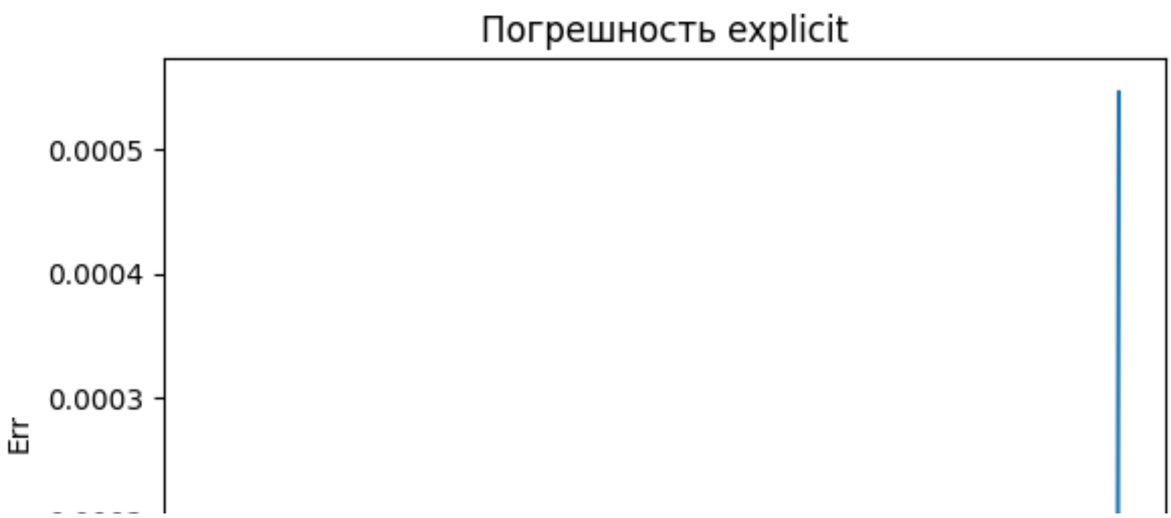


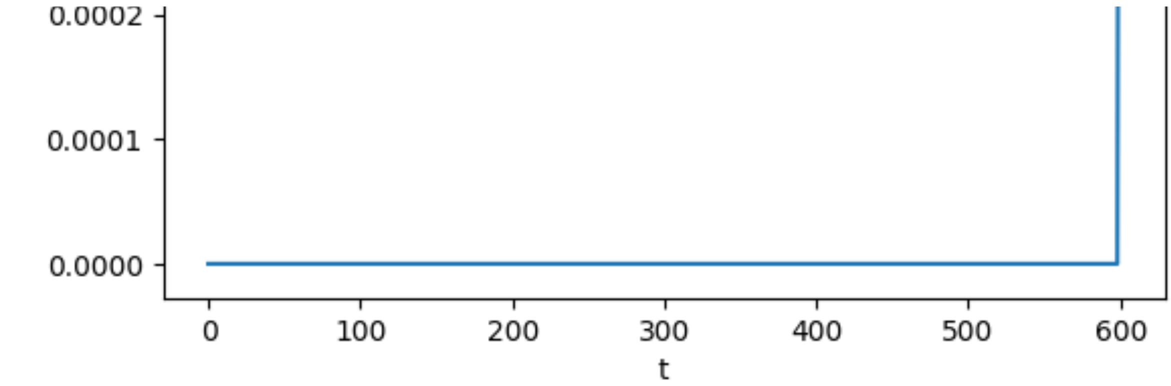
Исследование зависимости погрешности от параметров tau и h

In [13]:

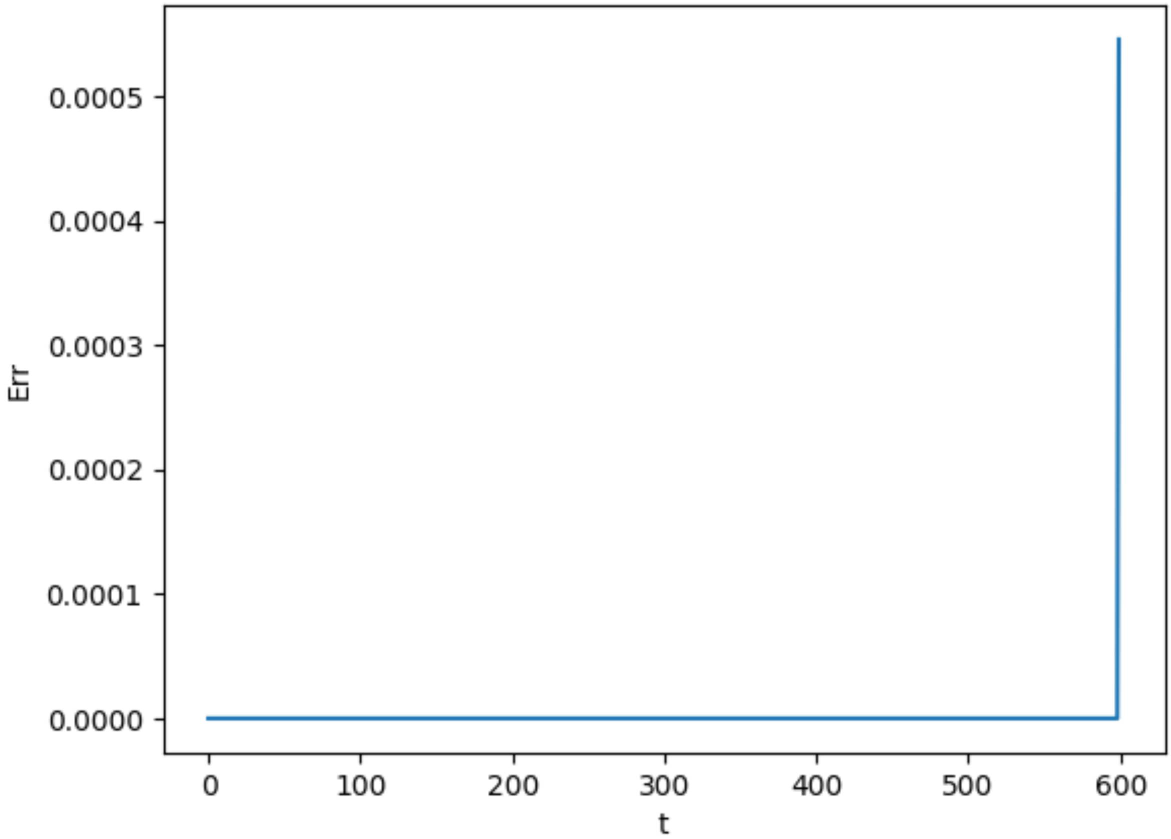
```
data = {'N': 600, 'K': 100, 'T': 1}
N, K, T = int(data['N']), int(data['K']), int(data['T'])

args = eq
args['bound_type'] = 'a1p2'
args['approximation'] = 'p1'
solver = Hyperbolic(args, N, K, T)

ans = {
    'implicit': solver.solve_implicit(N, K, T),
    'explicit': solver.explicit_solver(N, K, T),
    'analytic': solver.solve_analytic(N, K, T)
}

show(ans)
```





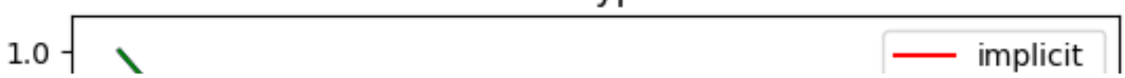


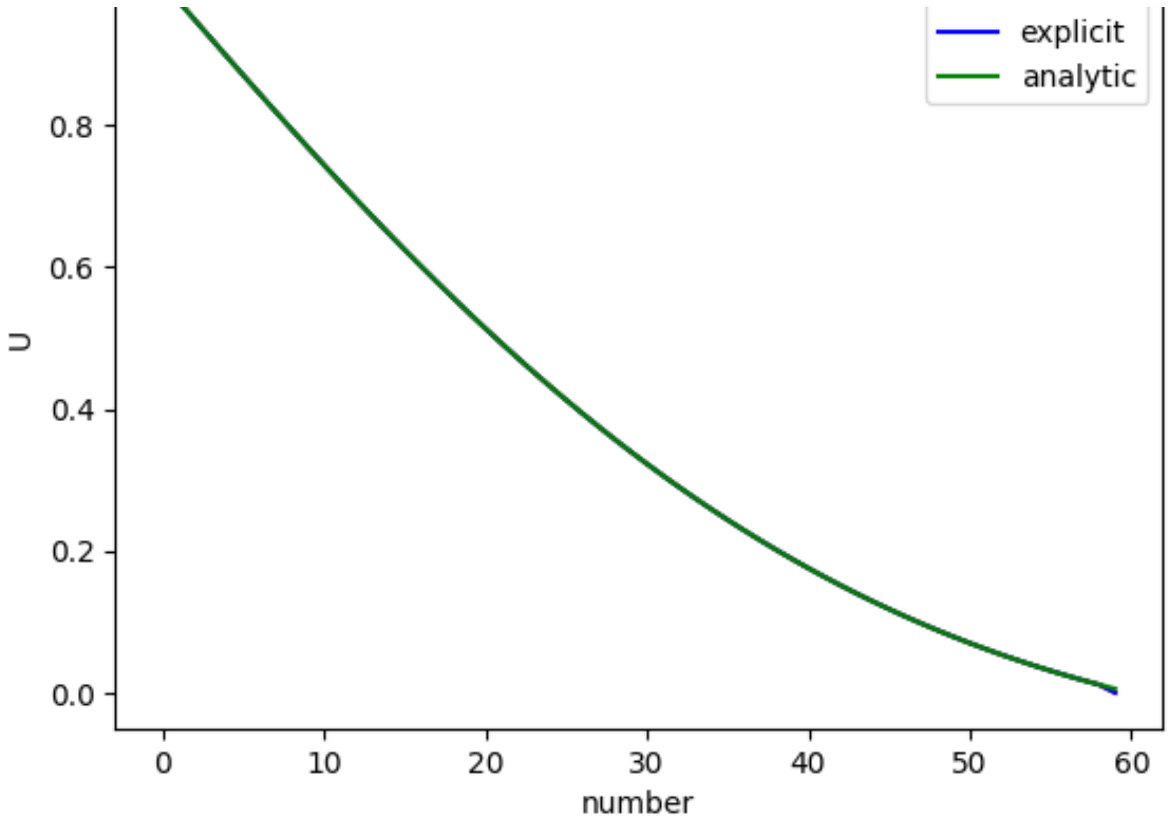
Погрешность implicit



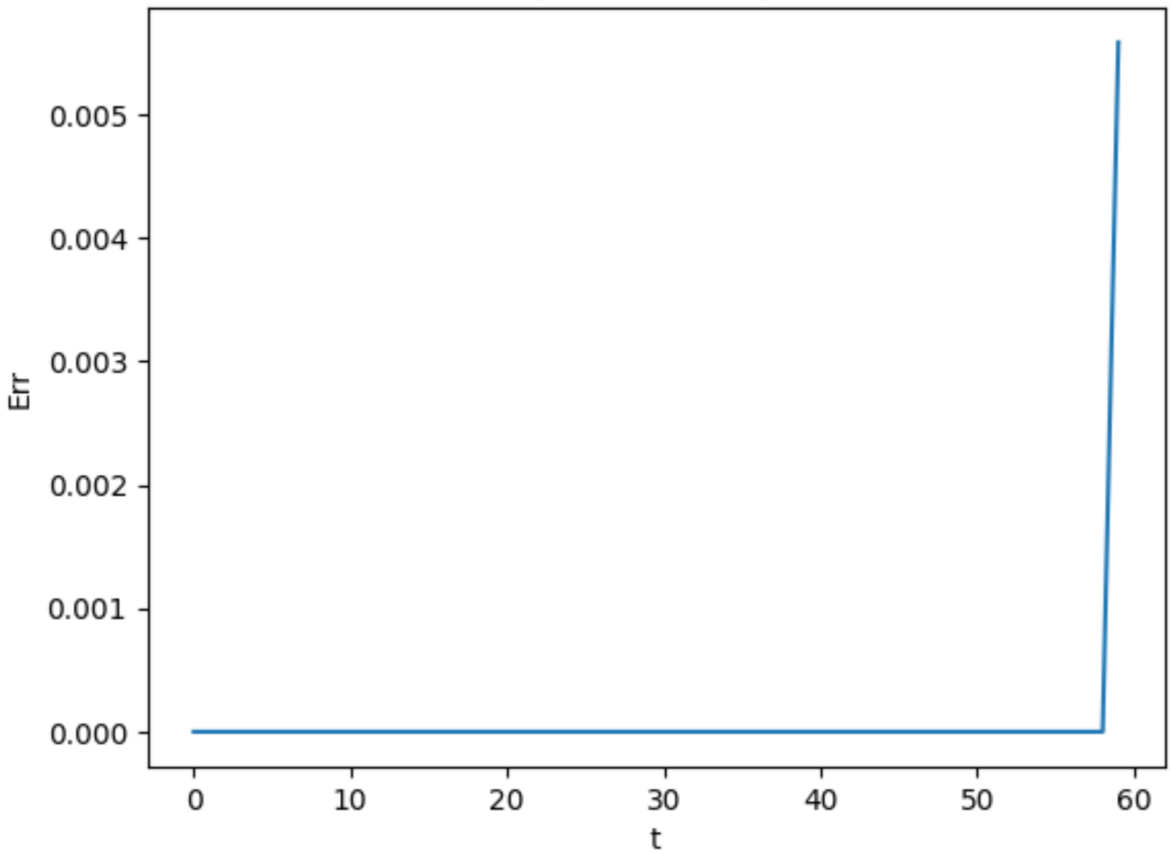
In [14]:
```
data = {'N': 60, 'K': 10000, 'T': 1}
N, K, T = int(data['N']), int(data['K']), int(data['T'])

args = eq
args['bound_type'] = 'a1p2'
args['approximation'] = 'p1'
solver = Hyperbolic(args, N, K, T)

ans = {
    'implicit': solver.solve_implicit(N, K, T),
    'explicit': solver.explicit_solver(N, K, T),
    'analytic': solver.solve_analytic(N, K, T)
}

show(ans)
```

Линии уровня

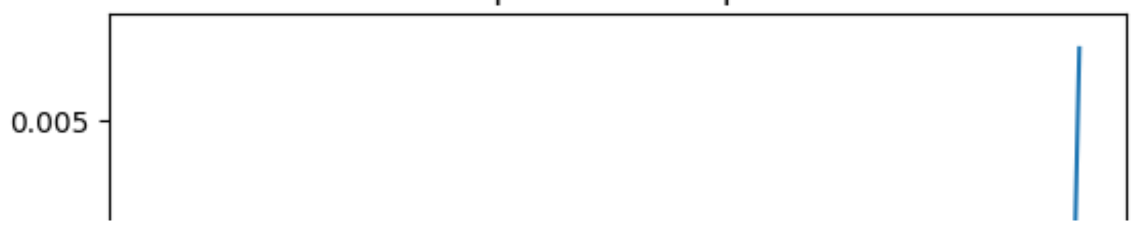


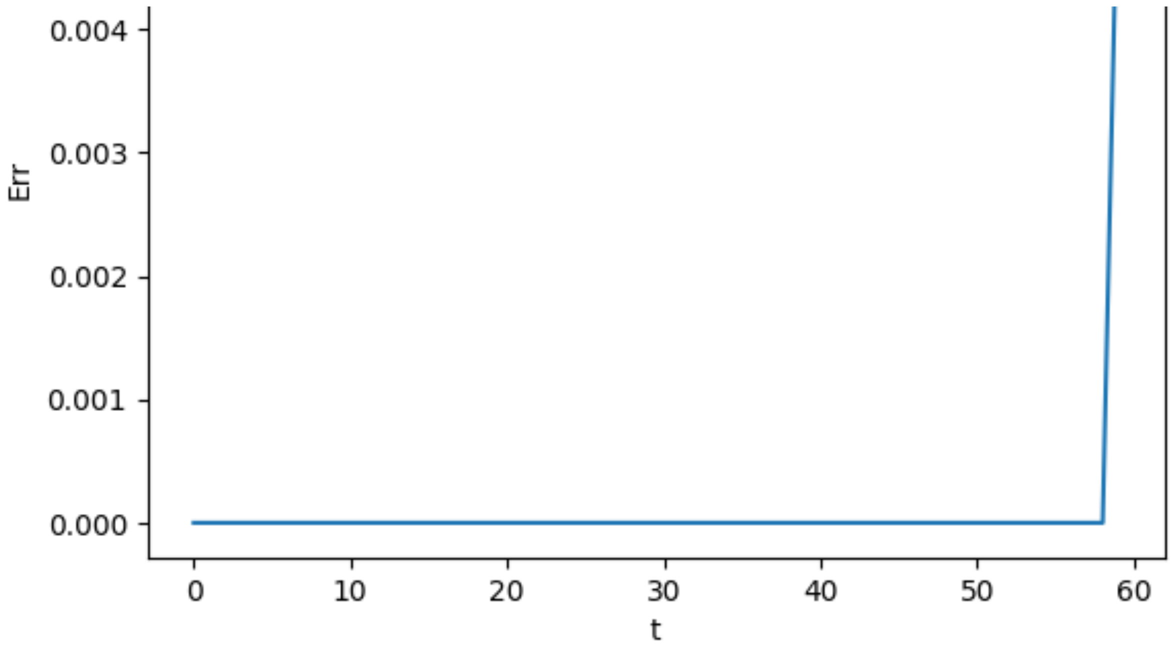


Погрешность explicit



Погрешность implicit





Выводы:

В процессе выполнения ЛР, что шаг h имеет больший вес при подсчёте погрешности, уменьшив его в сто раз, я уменьшил погрешность в 10 раз, а вот τ заметного эффекта не оказало на моё решение

```
In [14]:
```