Лабораторная работа №8

Студент	**Лошманов Ю. А.**
Группа	М8О-406Б-19

Используя схемы переменных направлений и дробных шагов, решить двумерную начально-краевую задачу для дифференциального уравнения параболического типа. В различные моменты времени вычислить погрешность численного решения путем сравнения результатов с приведенным в задании аналитическим решением $U(x,t)$. Исследовать зависимость погрешности от сеточных параметров τ, h_x, h_y.

Вариант 6

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + a\frac{\partial^2 u}{\partial y^2}, a > 0$$

$$u(0,\ y,\ t) = \sinh(y)\exp(-3at)$$

$$u_x(\tfrac{\pi}{4},\ y,\ t) = -2\sinh(y)\exp(-3at)$$

$$u_y(x,\ 0,\ t) = \cos(2x)\exp(-3at)$$

$$u(x,\ \ln 2,\ t) = \tfrac{3}{4}\cos(2x)\exp(-3at)$$

$$u(x,\ y,\ 0) = \cos(2x)\sinh(y)$$

$$U(x,\ y,\ t) = \cos(2x)\sinh(y)\exp(-3at)$$

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

args = {
    'a': 1,
    'b': 1,
    'c': 0,
    'd': 0,
    'lx': np.pi,
    'ly': np.pi,
    'f': lambda x, y, t: 0,
    'alpha1': 0,
    'alpha2': 1,
    'beta1': 0,
    'beta2': 1,
    'gamma1': 0,
    'gamma2': 1,
    'delta1': 0,
```

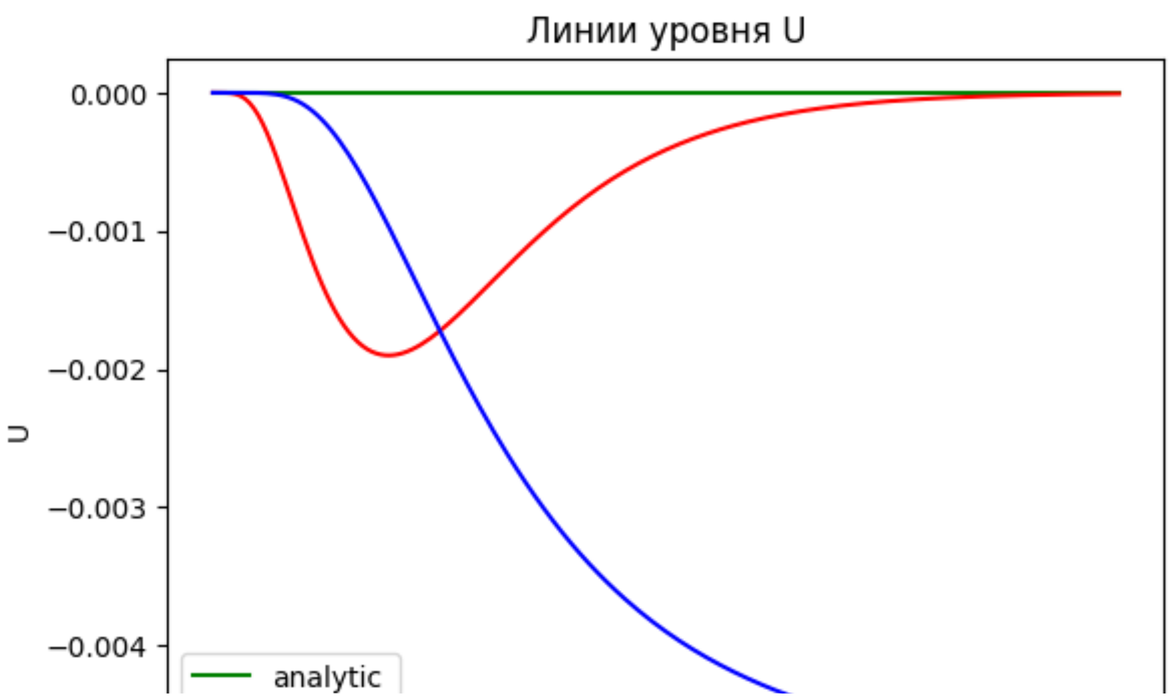
```
    'delta2': 1,
    'phi11': lambda y, t: np.cos(y) * np.exp(-2 * t),
    'phi12': lambda y, t: -1 * np.cos(y) * np.exp(-2 * t),
    'phi21': lambda x, t: np.cos(x) * np.exp(-2 * t),
    'phi22': lambda x, t: -1 * np.cos(x) * np.exp(-2 * t),
    'psi': lambda x, y: np.cos(x) * np.cos(y),
    'solution': lambda x, y, t: np.cos(x) * np.cos(y) * np.exp(-2 *
}

def tma(a, b, c, d):
    size = len(a)
    p, q = [], []
    p.append(-c[0] / b[0])
    q.append(d[0] / b[0])

    for i in range(1, size):
        p_tmp = -c[i] / (b[i] + a[i] * p[i - 1])
        q_tmp = (d[i] - a[i] * q[i - 1]) / (b[i] + a[i] * p[i - 1])
        p.append(p_tmp)
        q.append(q_tmp)

    x = [0 for _ in range(size)]
    x[size - 1] = q[size - 1]

    for i in range(size - 2, -1, -1):
        x[i] = p[i] * x[i + 1] + q[i]

    return x

class Eq:
    def __init__(self, args):
        self.a = args['a']
        self.b = args['b']
        self.c = args['c']
        self.d = args['d']
        self.lx = args['lx']
        self.ly = args['ly']
        self.f = args['f']
        self.alpha1 = args['alpha1']
        self.alpha2 = args['alpha2']
        self.beta1 = args['beta1']
        self.beta2 = args['beta2']
        self.gamma1 = args['gamma1']
        self.gamma2 = args['gamma2']
        self.delta1 = args['delta1']
        self.delta2 = args['delta2']
        self.phi11 = args['phi11']
        self.phi21 = args['phi21']
        self.phi12 = args['phi12']
        self.phi22 = args['phi22']
        self.psi = args['psi']
        self.solution = args['solution']


class Parabolic:
    def __init__(self, args, nx, ny, T, K):
        self.data = Eq(args)
        self.hx = self.data.lx / nx
```

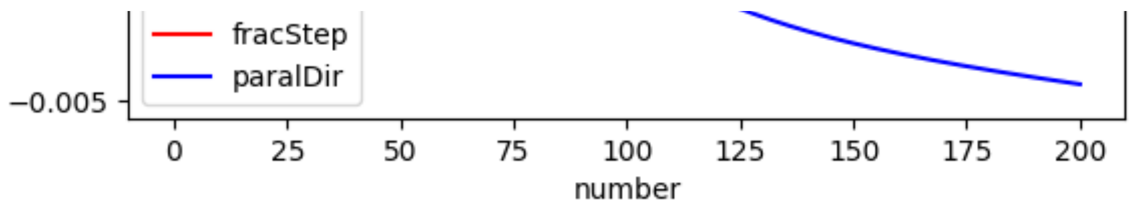
```python
        self.hy = self.data.ly / ny
        self.tau = T / K
        self.x, self.y, self.t = self.prepare(nx, ny, T, K)
        self.uu = self.init_U(self.x, self.y, self.t)

    def get_coeff(self, n):
        aa = np.zeros(len(n))
        bb = np.zeros(len(n))
        cc = np.zeros(len(n))
        dd = np.zeros(len(n))

        return aa, bb, cc, dd

    def compute_coeff(self, x, y, t2, j):
        aa, bb, cc, dd = self.get_coeff(x)
        bb[0] = self.hx * self.data.alpha2 - self.data.alpha1
        bb[-1] = self.hx * self.data.beta2 + self.data.beta1
        cc[0] = self.data.alpha1
        aa[-1] = -self.data.beta1
        dd[0] = self.data.phi11(y[j], t2) * self.hx
        dd[-1] = self.data.phi12(y[j], t2) * self.hx

        return aa, bb, cc, dd

    def prepare(self, nx, ny, T, K):
        self.hx = self.data.lx / nx
        self.hy = self.data.ly / ny
        self.tau = T / K
        x = np.arange(0, self.data.lx + self.hx, self.hx)
        y = np.arange(0, self.data.ly + self.hy, self.hy)
        t = np.arange(0, T + self.tau, self.tau)

        return x, y, t

    def init_U(self, x, y, t):
        u = np.zeros((len(x), len(y), len(t)))
        for i in range(len(x)):
            for j in range(len(y)):
                u[i][j][0] = self.data.psi(x[i], y[j])

        return u

    def analytic(self, nx, ny, T, K):
        x, y, t = self.prepare(nx, ny, T, K)

        uu = np.zeros((len(x), len(y), len(t)))

        for i in range(len(x)):
            for j in range(len(y)):
                for k in range(len(t)):
                    uu[i][j][k] = self.data.solution(x[i], y[j], t[

        return uu

    def parallel_directions(self):
        for k in range(1, len(self.t)):
            u1 = np.zeros((len(self.x), len(self.y)))
            t2 = self.t[k] - self.tau / 2
```

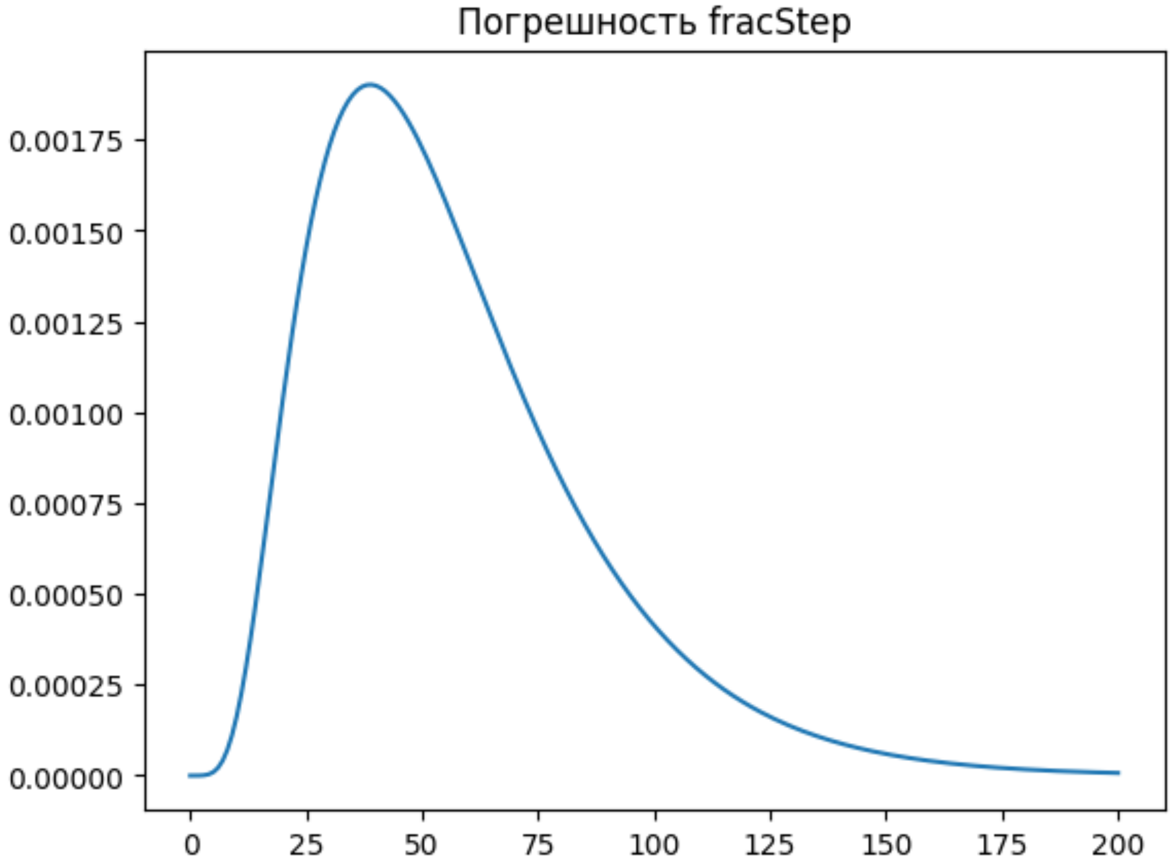
```
        t2 = self.t[k] - self.tau / 2
        for j in range(len(self.y) - 1):
            aa, bb, cc, dd = self.compute_coeff(self.x, self.y,
            for i in range(len(self.x) - 1):
                aa[i] = self.data.a - self.hx * self.data.c / 2
                bb[i] = self.hx ** 2 - 2 * (self.hx ** 2) / sel
                cc[i] = self.data.a + self.hx * self.data.c / 2
                dd[i] = -2 * (self.hx ** 2) * self.uu[i][j][k -
                - self.data.b * (self.hx ** 2) * (self.uu[i][j
                                                  - 2 * self.uu
                - self.data.d * (self.hx ** 2) * (self.uu[i][j
                - (self.hx ** 2) * self.data.f(self.x[i], self.

            xx = tma(aa, bb, cc, dd)
            for i in range(len(self.x)):
                u1[i][j] = xx[i]
                u1[i][0] = (self.data.phi21(self.x[i], t2) - se
                        self.data.gamma2 - self.data.gamma1 / s
                u1[i][-1] = (self.data.phi22(self.x[i], t2) + s
                        self.data.delta2 + self.data.delta1 / s
        for j in range(len(self.y)):
            u1[0][j] = (self.data.phi11(self.y[j], t2) - self.d
                        self.data.alpha2 - self.data.alpha1 / s
            u1[-1][j] = (self.data.phi12(self.y[j], t2) + self.
                        self.data.beta2 + self.data.beta1 / sel
        ####
        u2 = np.zeros((len(self.x), len(self.y)))
        for i in range(len(self.x) - 1):
            aa, bb, cc, dd = self.get_coeff(self.y)
            bb[0] = self.hy * self.data.gamma2 - self.data.gamm
            bb[-1] = self.hy * self.data.delta2 + self.data.del
            cc[0] = self.data.gamma1
            aa[-1] = -self.data.delta1
            dd[0] = self.data.phi21(self.x[i], self.t[k]) * sel
            dd[-1] = self.data.phi22(self.x[i], self.t[k]) * se

            for j in range(len(self.y) - 1):
                aa[j] = self.data.b - self.hy * self.data.d / 2
                bb[j] = self.hy ** 2 - 2 * (self.hy ** 2) / sel
                cc[j] = self.data.b + self.hy * self.data.d / 2
                dd[j] = -2 * (self.hy ** 2) * u1[i][j] / self.t
                - self.data.a * (self.hy ** 2) * (u1[i + 1][j]
                                                  - 2 * u1[i][j
                - self.data.c * (self.hy ** 2) * (u1[i + 1][j]
                - (self.hy ** 2) * self.data.f(self.x[i], self.
            xx = tma(aa, bb, cc, dd)
            for j in range(len(self.y)):
                u2[i][j] = xx[j]
                u2[0][j] = (self.data.phi11(self.y[j], self.t[k
                            self.data.alpha2 - self.data.alpha1
                u2[-1][j] = (self.data.phi12(self.y[j], self.t[
                            self.data.beta2 + self.data.beta1 /
        for i in range(len(self.x)):
            u2[i][0] = (self.data.phi21(self.x[i], self.t[k]) -
                        self.data.gamma2 - self.data.gamma1 / s
            u2[i][-1] = (self.data.phi22(self.x[i], self.t[k])
                        self.data.delta2 + self.data.delta1 / s
        for i in range(len(self.x)):
```

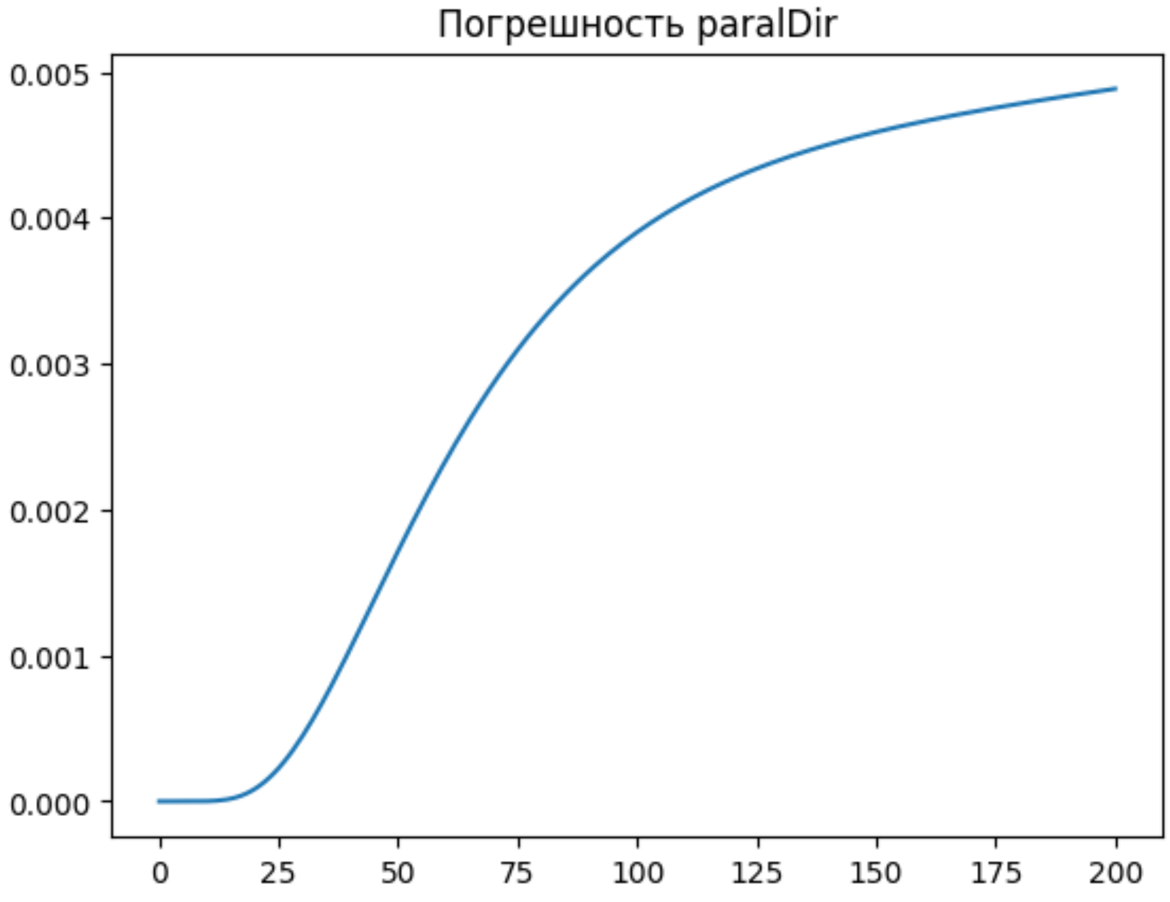
```
                for j in range(len(self.y)):
                    self.uu[i][j][k] = u2[i][j]
        return self.uu

    def fractional_steps(self):
        for k in range(len(self.t)):
            u1 = np.zeros((len(self.x), len(self.y)))
            t2 = self.t[k] - self.tau / 2
            for j in range(len(self.y) - 1):
                aa, bb, cc, dd = self.compute_coeff(self.x, self.y,
                for i in range(len(self.x) - 1):
                    aa[i] = self.data.a
                    bb[i] = -(self.hx ** 2) / self.tau - 2 * self.da
                    cc[i] = self.data.a
                    dd[i] = -(self.hx ** 2) * self.uu[i][j][k - 1] /

                xx = tma(aa, bb, cc, dd)
                for i in range(len(self.x)):
                    u1[i][j] = xx[i]
                    u1[i][0] = (self.data.phi21(self.x[i], t2) - se
                            self.data.gamma2 - self.data.gamma1 / se
                    u1[i][-1] = (self.data.phi22(self.x[i], t2) + se
                            self.data.delta2 + self.data.delta1 / se
            for j in range(len(self.y)):
                u1[0][j] = (self.data.phi11(self.y[j], t2) - self.da
                        self.data.alpha2 - self.data.alpha1 / self.h
                u1[-1][j] = (self.data.phi12(self.y[j], t2) + self.d
                        self.data.beta2 + self.data.beta1 / self.hx
            #####
            u2 = np.zeros((len(self.x), len(self.y)))
            for i in range(len(self.x) - 1):
                aa, bb, cc, dd = self.get_coeff(self.y)
                bb[0] = self.hy * self.data.gamma2 - self.data.gamma
                bb[-1] = self.hy * self.data.delta2 + self.data.delt
                cc[0] = self.data.gamma1
                aa[-1] = -self.data.delta1
                dd[0] = self.data.phi21(self.x[i], self.t[k]) * self
                dd[-1] = self.data.phi22(self.x[i], self.t[k]) * se

                for j in range(len(self.y) - 1):
                    aa[j] = self.data.b
                    bb[j] = -(self.hy ** 2) / self.tau - 2 * self.da
                    cc[j] = self.data.b
                    dd[j] = -(self.hy ** 2) * u1[i][j] / self.tau -
                xx = tma(aa, bb, cc, dd)
                for j in range(len(self.y)):
                    u2[i][j] = xx[j]
                    u2[0][j] = (self.data.phi11(self.y[j], self.t[k
                            self.data.alpha2 - self.data.alpha1 / se
                    u2[-1][j] = (self.data.phi12(self.y[j], self.t[
                            self.data.beta2 + self.data.beta1 / self
            for i in range(len(self.x)):
                u2[i][0] = (self.data.phi21(self.x[i], self.t[k]) -
                        self.data.gamma2 - self.data.gamma1 / self.h
                u2[i][-1] = (self.data.phi22(self.x[i], self.t[k]) +
                        self.data.delta2 + self.data.delta1 / self.h
            for i in range(len(self.x)):
                for j in range(len(self.y)):
```

```
                for j in range(len(self.y)):
                    self.uu[i][j][k] = u2[i][j]
        return self.uu

def show(dict_, data, args, y_point, time):
  plt.title('Линии уровня U')
  plt.plot(dict_['analytic'][time][y_point], color='g', label='analy
  plt.plot(dict_['fracStep'][time][y_point], color='r', label='frac
  plt.plot(dict_['paralDir'][time][y_point], color='b', label='para
  plt.legend(loc='best')
  plt.ylabel('U')
  plt.xlabel('number')
  plt.show()


  plt.title('Погрешность fracStep')
  plt.plot(abs(dict_['analytic'][time][y_point] - dict_['fracStep']
  plt.show()


  plt.title('Погрешность paralDir')
  plt.plot(abs(dict_['analytic'][time][y_point] - dict_['paralDir']
  plt.show()


data = {'nx': 40, 'ny': 40, 'T': 5, 'K': 200}
nx, ny, T, K = int(data['nx']), int(data['ny']), int(data['T']), in

solver = Parabolic(args, nx, ny, T, K)
solverFrac = Parabolic(args, nx, ny, T, K)
solverParal = Parabolic(args, nx, ny, T, K)
ans = {
    'fracStep': solverFrac.fractional_steps(),
    'paralDir': solverParal.parallel_directions(),
    'analytic': solver.analytic(nx, ny, T, K)
}

show(ans, data, args, 20, 20)
```



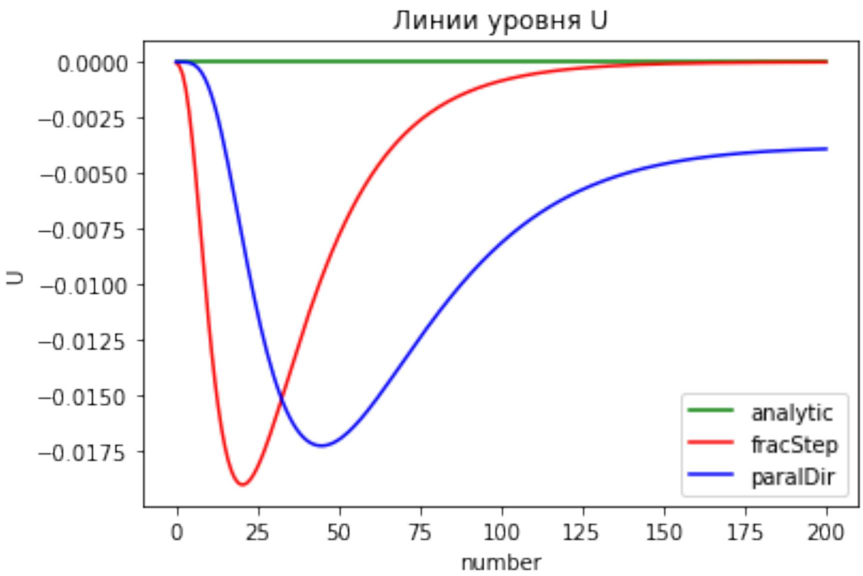

fracStep
paralDir
−0.005
0
25
50
75
100
125
150
175
200
number

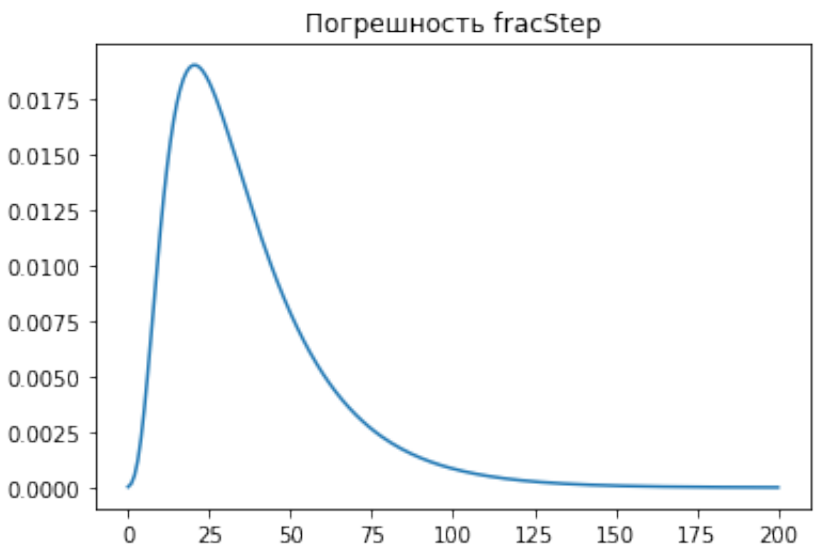

Погрешность fracStep
0.00175
0.00150
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
0
25
50
75
100
125
150
175
200

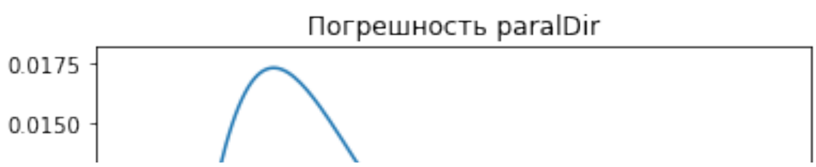

Погрешность paralDir
0.005
0.004
0.003
0.002
0.001
0.000
0
25
50
75
100
125
150
175
200

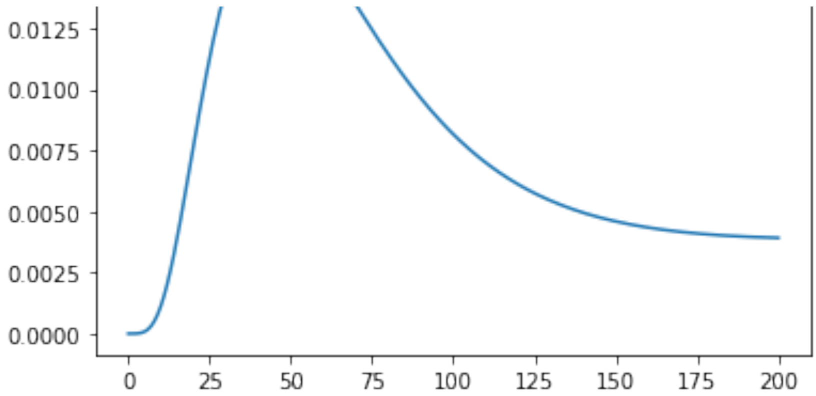
Исследование зависимости погрешности от параметров tau, hx, hy

In [3]:
```
data = {'nx': 80, 'ny': 40, 'T': 5, 'K': 200}
nx, ny, T, K = int(data['nx']), int(data['ny']), int(data['T']), in

solver = Parabolic(args, nx, ny, T, K)
solverFrac = Parabolic(args, nx, ny, T, K)
solverParal = Parabolic(args, nx, ny, T, K)
ans = {
    'fracStep': solverFrac.fractional_steps(),
    'paralDir': solverParal.parallel_directions(),
    'analytic': solver.analytic(nx, ny, T, K)
}

show(ans, data, args, 20, 20)
```

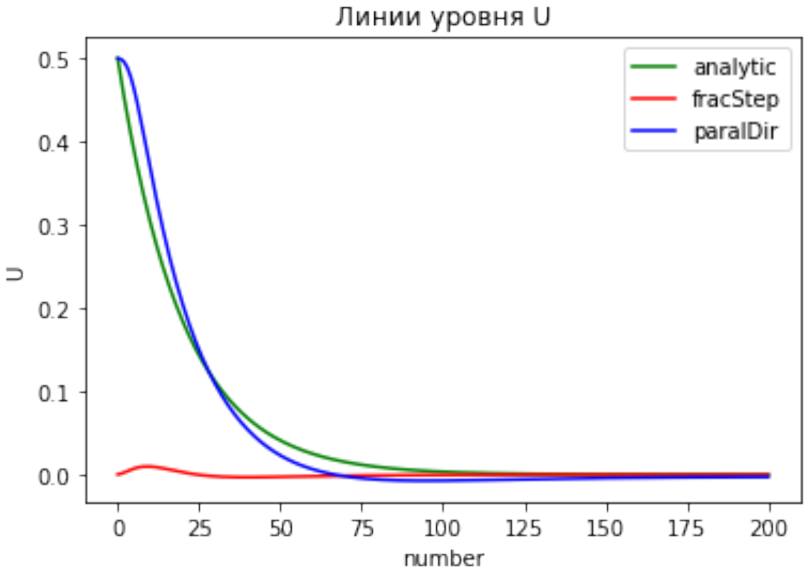


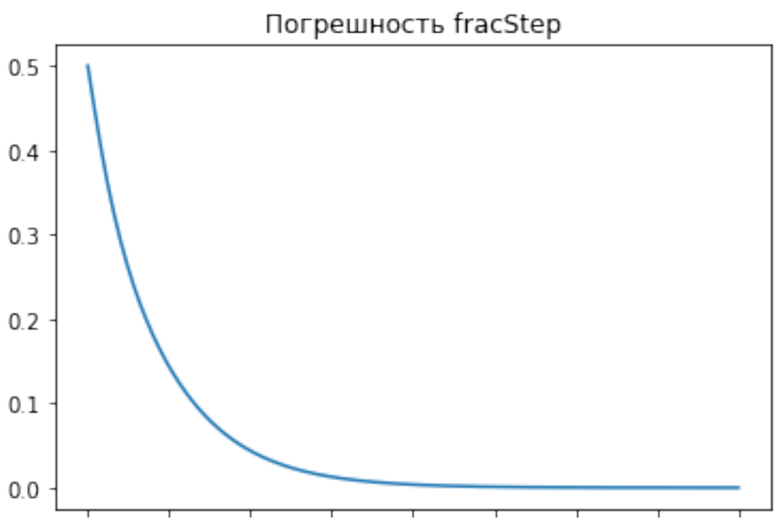


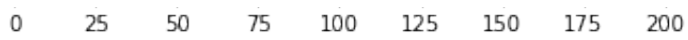


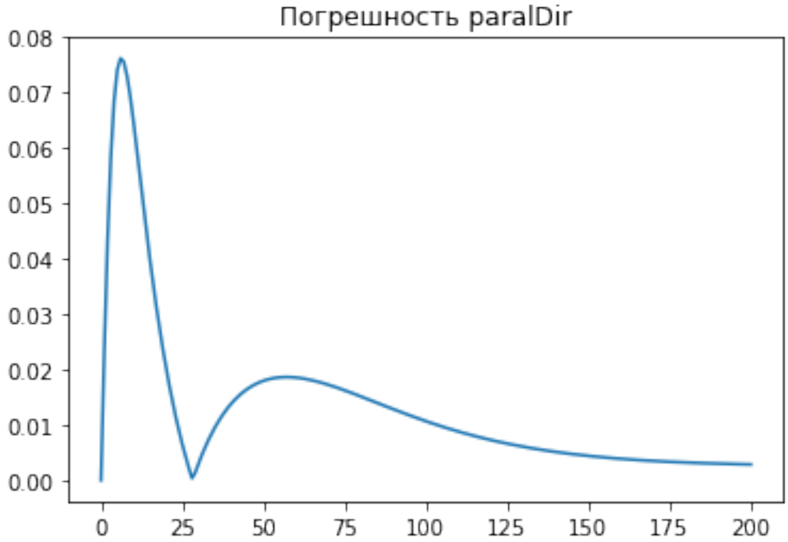



```
In [4]: data = {'nx': 80, 'ny': 80, 'T': 5, 'K': 200}
        nx, ny, T, K = int(data['nx']), int(data['ny']), int(data['T']), in

        solver = Parabolic(args, nx, ny, T, K)
        solverFrac = Parabolic(args, nx, ny, T, K)
        solverParal = Parabolic(args, nx, ny, T, K)
        ans = {
            'fracStep': solverFrac.fractional_steps(),
            'paralDir': solverParal.parallel_directions(),
            'analytic': solver.analytic(nx, ny, T, K)
        }

        show(ans, data, args, 20, 20)
```

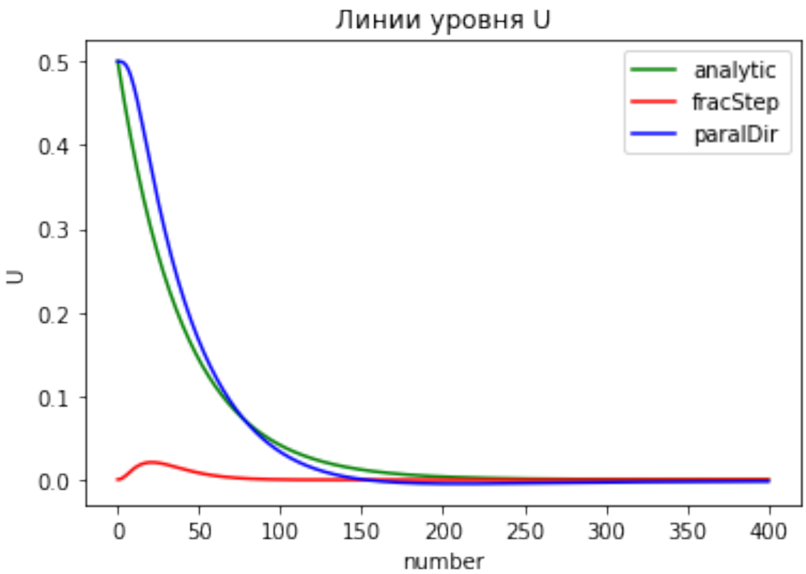


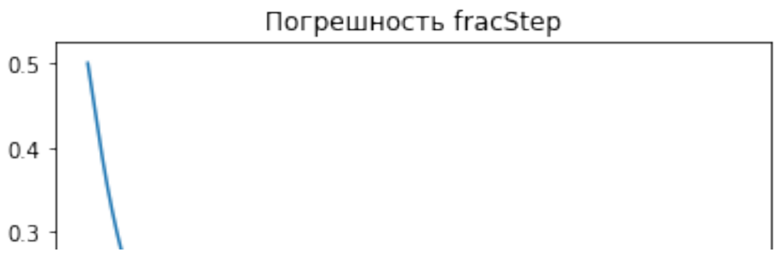


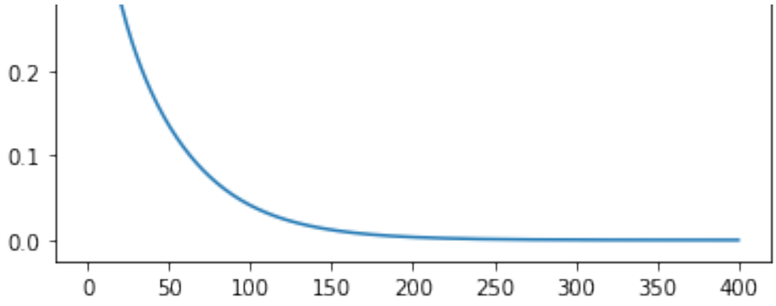


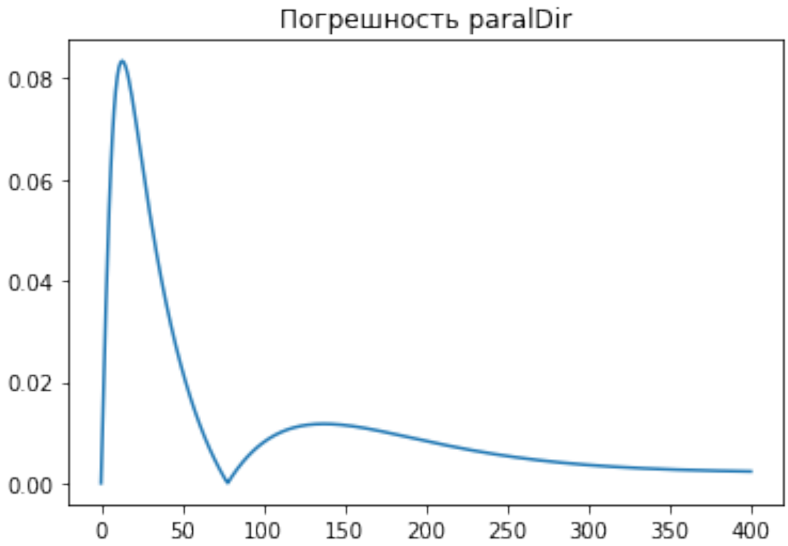


In [5]:
```
data = {'nx': 80, 'ny': 80, 'T': 5, 'K': 400}
nx, ny, T, K = int(data['nx']), int(data['ny']), int(data['T']), in


solver = Parabolic(args, nx, ny, T, K)
solverFrac = Parabolic(args, nx, ny, T, K)
solverParal = Parabolic(args, nx, ny, T, K)
ans = {
    'fracStep': solverFrac.fractional_steps(),
    'paralDir': solverParal.parallel_directions(),
    'analytic': solver.analytic(nx, ny, T, K)
}

show(ans, data, args, 20, 20)
```









Выводы:

По показателям можно понять, что при увелечении h_x и h_y погрешность сначала падает, но после какого-то значения начинает расти, при этом мелкость τ не оказывает сильного влияния на погрешность

In [ ]: