2025-06-11--jam99--Complex-inverse-matrix-D-Im-order.nb

```
$Assumptions = {Element[z, Reals], z > 0}
```

Out[28]= $\{z \in \mathbb{R}, z > 0\}$

Cook up a complex matrix containing complex functions that depend on z .

In[29]:=
$$M = \begin{pmatrix} 1/z & 1/(z^{\wedge}2 + I) \\ z^{\wedge}2 + I z & 1 \end{pmatrix}$$

Out[29]= $\left\{\left\{\frac{1}{z}, \frac{1}{i + z^2}\right\}, \left\{i z + z^2, 1\right\}\right\}$

Compute the matrix inverse . It is very complicated.

In[30]:=
```
Inverse[M] // Simplify
```

Out[30]=
$$\left\{\left\{-\frac{z\left(i + z^2\right)}{-i - (1 - i) z^2 + z^3}, \frac{z}{-i - (1 - i) z^2 + z^3}\right\}, \left\{\frac{z^2 (i + z)\left(i + z^2\right)}{-i - (1 - i) z^2 + z^3}, \frac{i + z^2}{i + (1 - i) z^2 - z^3}\right\}\right\}$$

In[34]:=
```
Simplify[ComplexExpand[Im[I Inverse[M]]]]
```

Out[34]=
$$\left\{\left\{-\frac{z\left(1 + z + z^4\right)}{(-1 + z)\left(1 + 2 z + z^2 + z^4\right)}, \frac{z^3}{(-1 + z)\left(1 + 2 z + z^2 + z^4\right)}\right\}, \left\{\frac{z^3\left(1 + z + z^2 + z^4\right)}{(-1 + z)\left(1 + 2 z + z^2 + z^4\right)}, -\frac{1 + z + z^4}{(-1 + z)\left(1 + 2 z + z^2 + z^4\right)}\right\}\right\}$$

Imaginary part first, then derivative .

In[37]:=
```
ans1 = Simplify[D[Simplify[ComplexExpand[Im[I Inverse[M]]]], z] ]
```

Out[37]=
$$\left\{\left\{\frac{1 + 2 z + 2 z^2 + 2 z^3 + 3 z^4 + 6 z^5 - 2 z^7 + z^8}{(-1 + z)^2\left(1 + 2 z + z^2 + z^4\right)^2}, \frac{z^2\left(-3 - 2 z + z^2 + z^4 - 2 z^5\right)}{(-1 + z)^2\left(1 + 2 z + z^2 + z^4\right)^2}\right\}, \left\{\frac{z^2\left(-3 - 6 z - 7 z^2 - 2 z^3 - 2 z^4 - 6 z^5 + 3 z^6 + 4 z^7 - 3 z^8 + 2 z^9\right)}{(-1 + z)^2\left(1 + 2 z + z^2 + z^4\right)^2}, \frac{z\left(2 + 4 z + 2 z^2 + 5 z^3 + 2 z^4 - z^5 + z^7\right)}{(-1 + z)^2\left(1 + 2 z + z^2 + z^4\right)^2}\right\}\right\}$$

Derivative, then imaginary part

In[45]:=
```
ans2 = Simplify[ComplexExpand[ Im[I Simplify[D[Inverse[M], z]]]]]
```

Out[45]=
$$\left\{\left\{\frac{1+2z+2z^2+2z^3+3z^4+6z^5-2z^7+z^8}{(-1+z)^2\left(1+2z+z^2+z^4\right)^2},\ \frac{z^2\left(-3-2z+z^2+z^4-2z^5\right)}{(-1+z)^2\left(1+2z+z^2+z^4\right)^2}\right\},\right.$$
$$\left.\left\{\frac{z^2\left(-3-6z-7z^2-2z^3-2z^4-6z^5+3z^6+4z^7-3z^8+2z^9\right)}{(-1+z)^2\left(1+2z+z^2+z^4\right)^2},\ \frac{z\left(2+4z+2z^2+5z^3+2z^4-z^5+z^7\right)}{(-1+z)^2\left(1+2z+z^2+z^4\right)^2}\right\}\right\}$$

In[47]:=
```
ans1 - ans2 // Simplify
```

Out[47]= {{0, 0}, {0, 0}}

Let us check another relation.

In[55]:=
```
ans3 = - Simplify[Inverse[M] . D[M, z] . Inverse[M]]
```

Out[55]=
$$\left\{\left\{-\frac{1+(1-2i)z^2-2iz^3-(1-i)z^4}{\left(-i-(1-i)z^2+z^3\right)^2},\ -\frac{i-(1-i)z^2+2z^3}{\left(-i-(1-i)z^2+z^3\right)^2}\right\},\right.$$
$$\left.\left\{\frac{z\left(2i+3z+4z^2-6iz^3-(2+2i)z^4-(3-4i)z^5+2z^6\right)}{\left(-i-(1-i)z^2+z^3\right)^2},\ \frac{z\left(-2+3iz+z^3\right)}{\left(-i-(1-i)z^2+z^3\right)^2}\right\}\right\}$$

In[57]:=
```
ans4 = Simplify[D[Inverse[M], z]]
```

Out[57]=
$$\left\{\left\{\frac{-1-(1-2i)z^2+2iz^3+(1-i)z^4}{\left(-i-(1-i)z^2+z^3\right)^2},\ \frac{-i+(1-i)z^2-2z^3}{\left(-i-(1-i)z^2+z^3\right)^2}\right\},\right.$$
$$\left.\left\{\frac{z\left(2i+3z+4z^2-6iz^3-(2+2i)z^4-(3-4i)z^5+2z^6\right)}{\left(-i-(1-i)z^2+z^3\right)^2},\ \frac{z\left(-2+3iz+z^3\right)}{\left(-i-(1-i)z^2+z^3\right)^2}\right\}\right\}$$

In[59]:=
```
ans3 - ans4 // Simplify
```

Out[59]= {{0, 0}, {0, 0}}