In[195]:=
```
Clear[θ, ϵ$s, λ$d, λ$D, ψ];
```

$$\theta := \frac{1}{\epsilon\$s - I\,\frac{\lambda\$d^2}{\lambda\$D^2}}\left(\epsilon\$s - I\,\frac{\lambda\$d^2}{\lambda\$D^2}\,\frac{\psi}{\sqrt{\psi^2 + \frac{1}{\epsilon\$s} + I\,\frac{\lambda\$D^2}{\lambda\$d^2}}}\right)\ /.\ \lambda\$D \to 1/k\$D$$

In[197]:=

$$rp := \frac{\epsilon\$s - \theta}{\epsilon\$s + \theta}$$

In[198]:=
```
rp$approx = Series[rp, {k$D, 0, 1}] // Normal // PowerExpand // FullSimplify
```

Out[198]=

$$\frac{-1 + \epsilon\$s}{1 + \epsilon\$s}$$

In[203]:=
```
rp$approx = Series[rp, {k$D, 0, 2}] // Normal // PowerExpand
```

Out[203]=

$$\frac{-1 + \epsilon\$s}{1 + \epsilon\$s} - \frac{2\,i\,k\$D^2\,\lambda\$d^2}{(1 + \epsilon\$s)^2}$$

These terms factor out front.

In[205]:=

$$rp\$approx\$x = rp\$approx\ /.\ \lambda\$d \to \frac{1}{k\$D}\sqrt{\epsilon\$s\$real\,x}\ \ //\ PowerExpand$$

Out[205]=

$$\frac{-1 + \epsilon\$s}{1 + \epsilon\$s} - \frac{2\,i\,x\,\epsilon\$s\$real}{(1 + \epsilon\$s)^2}$$

Look at the two coefficient

In[207]:=

$$Im\left[\frac{-1 + \epsilon\$s}{1 + \epsilon\$s}\ /.\ \epsilon\$s \to 20 - 0.02\,I\right]\ //\ N$$

Out[207]= -0.0000907029

In[208]:=

$$Im\left[-\frac{2\,i\,\epsilon\$s\$real}{(1 + \epsilon\$s)^2}\ /.\ \{\epsilon\$s \to 20 - 0.02\,I,\ \epsilon\$s\$real \to 20\}\right]\ //\ N$$

Out[208]= -0.0907027

In[145]:=
```
Clear[ans, q, h];
ans[q_] := FullSimplify[
      Integrate[k^q Exp[- 2 k h], {k, 0, ∞},
       Assumptions → {Element[h, Reals], h > 0, Element[q, Integers], q ≥ 0}],
  Assumptions → {Element[q, Integers], q ≥ 0}]
```

In[147]:=
```
Table[{q, ans[q]}, {q, 0, 2}] // Simplify
```

Out[147]=

$$\left\{\left\{0, \frac{1}{2\,h}\right\}, \left\{1, \frac{1}{4\,h^2}\right\}, \left\{2, \frac{1}{4\,h^3}\right\}\right\}$$

Try another approximation

In[223]:= $\texttt{Series}\left[\theta\ /.\ \lambda\$d \to \frac{1}{k\$D}\sqrt{\epsilon\$s\$real\ x}\ ,\ \{x, 1, 1\}\right]$

Out[223]=
$$\frac{\epsilon\$s - \frac{i\,\epsilon\$s\$real\,\psi}{\sqrt{\frac{i\,\epsilon\$s+\epsilon\$s\$real+\epsilon\$s\,\epsilon\$s\$real\,\psi^2}{\epsilon\$s\,\epsilon\$s\$real}}}}{\epsilon\$s - i\,\epsilon\$s\$real} + \left(\frac{\epsilon\$s\$real\,\psi\left(3\,\epsilon\$s - 2\,i\,\epsilon\$s\$real - 2\,i\,\epsilon\$s\,\epsilon\$s\$real\,\psi^2\right)}{2\,(\epsilon\$s - i\,\epsilon\$s\$real)\left(i\,\epsilon\$s + \epsilon\$s\$real + \epsilon\$s\,\epsilon\$s\$real\,\psi^2\right)\sqrt{\frac{i\,\epsilon\$s+\epsilon\$s\$real+\epsilon\$s\,\epsilon\$s\$real\,\psi^2}{\epsilon\$s\,\epsilon\$s\$real}}} + \frac{i\,\epsilon\$s\$real\left(\epsilon\$s - \frac{i\,\epsilon\$s\$real\,\psi}{\sqrt{\frac{i\,\epsilon\$s+\epsilon\$s\$real+\epsilon\$s\,\epsilon\$s\$real\,\psi^2}{\epsilon\$s\,\epsilon\$s\$real}}}\right)}{(\epsilon\$s - i\,\epsilon\$s\$real)^2}\right)(x-1) + O[x-1]^2$$

The ψ - based integral doesn’t look doable. Stop here.