In[1]:= `Integrate[t^2 Exp[-2 t d / z$r], {t, 0, ∞}]`

Out[1]= $\frac{\text{z\$r}^3}{4\,\text{d}^3}$ if $\text{Re}\left[\frac{\text{d}}{\text{z\$r}}\right] > 0$

In[2]:= `Integrate[ψ^2 Exp[-2 ψ], {ψ, 0, ∞}]`

Out[2]= $\frac{1}{4}$

In[3]:= $\theta\text{\$red} = \frac{1}{\epsilon\text{\$s}} \frac{1}{1 - \frac{\text{I k\$D}^2\,\lambda\text{\$d}^2}{\epsilon\text{\$s}}} \left(1 - \frac{\text{I k\$D}^2\,\lambda\text{\$d}^2}{\epsilon\text{\$s}} \frac{\psi}{\sqrt{\psi^2 + \frac{\text{k\$D}^2\,\text{d}^2}{\epsilon\text{\$s}} + \text{I}\,\frac{\text{d}^2}{\lambda\text{\$d}^2}}}\right)$

Out[3]= $$\frac{1 - \frac{i\,\text{k\$D}^2\,\lambda\text{\$d}^2\,\psi}{\epsilon\text{\$s}\sqrt{\frac{\text{d}^2\,\text{k\$D}^2}{\epsilon\text{\$s}} + \frac{i\,\text{d}^2}{\lambda\text{\$d}^2} + \psi^2}}}{\epsilon\text{\$s}\left(1 - \frac{i\,\text{k\$D}^2\,\lambda\text{\$d}^2}{\epsilon\text{\$s}}\right)}$$

In[39]:= $\xi = \frac{1 - \theta\text{\$red}}{1 + \theta\text{\$red}}$ `// ExpandAll // FullSimplify`

Out[39]= $$\left(i\,(-1 + \epsilon\text{\$s})\,\epsilon\text{\$s}\sqrt{\text{d}^2\left(\frac{\text{k\$D}^2}{\epsilon\text{\$s}} + \frac{i}{\lambda\text{\$d}^2}\right) + \psi^2} + \text{k\$D}^2\,\lambda\text{\$d}^2\left(-\psi + \epsilon\text{\$s}\sqrt{\text{d}^2\left(\frac{\text{k\$D}^2}{\epsilon\text{\$s}} + \frac{i}{\lambda\text{\$d}^2}\right) + \psi^2}\right)\right) \Big/ \left(i\,\epsilon\text{\$s}\,(1 + \epsilon\text{\$s})\sqrt{\text{d}^2\left(\frac{\text{k\$D}^2}{\epsilon\text{\$s}} + \frac{i}{\lambda\text{\$d}^2}\right) + \psi^2} + \text{k\$D}^2\,\lambda\text{\$d}^2\left(\psi + \epsilon\text{\$s}\sqrt{\text{d}^2\left(\frac{\text{k\$D}^2}{\epsilon\text{\$s}} + \frac{i}{\lambda\text{\$d}^2}\right) + \psi^2}\right)\right)$$

`ξ$simpler =`

In[5]:= `Series[Series[ξ, {k$D, 0, 2}], {λ$d, 0, 2}] // Normal // Expand`

Out[5]= $-\frac{1}{1 + \epsilon\text{\$s}} + \frac{\epsilon\text{\$s}}{1 + \epsilon\text{\$s}} - \frac{2\,i\,\text{k\$D}^2\,\lambda\text{\$d}^2}{(1 + \epsilon\text{\$s})^2}$

In[6]:= $$\frac{(1+\epsilon\$r)^2-\epsilon\$i^2}{\left((1+\epsilon\$r)^2-\epsilon\$i^2\right)^2+4\,\epsilon\$i^2\,(1+\epsilon\$r)^2}\ \text{// Simplify}$$

Out[6]= $$\frac{-\epsilon\$i^2+(1+\epsilon\$r)^2}{\left(\epsilon\$i^2+(1+\epsilon\$r)^2\right)^2}$$

In[7]:= $$\left((1+\epsilon\$r)^2-\epsilon\$i^2\right)^2+4\,\epsilon\$i^2\,(1+\epsilon\$r)^2\ \text{// Simplify}$$

Out[7]= $$\left(\epsilon\$i^2+(1+\epsilon\$r)^2\right)^2$$

In[8]:= $$\text{maxpt} = \text{D}\left[\xi\ \text{/. k\$D} \to \frac{\rho\,\text{q\$e}^2}{\epsilon\$0\,\text{k\$b}\,\text{T}},\ \rho\right]\ \text{// Simplify}$$

Out[8]= $$\Bigg(2\,\text{q\$e}^4\,\rho\,\Bigg(2\,\mathbb{i}\,\text{k\$b}^4\,\text{T}^4\,\epsilon\$0^4\,\epsilon\$s^2\,\lambda\$d^2\,\psi^2\left(-\psi+\sqrt{d^2\left(\frac{\mathbb{i}}{\lambda\$d^2}+\frac{\text{q\$e}^4\,\rho^2}{\text{k\$b}^2\,\text{T}^2\,\epsilon\$0^2\,\epsilon\$s}\right)+\psi^2}\right)-$$
$$d^2\left(\text{k\$b}^2\,\text{T}^2\,\epsilon\$0^2\,\epsilon\$s-\mathbb{i}\,\text{q\$e}^4\,\lambda\$d^2\,\rho^2\right)\Bigg(-\mathbb{i}\,\text{q\$e}^4\,\lambda\$d^2\,\rho^2\,\psi+$$
$$2\,\text{k\$b}^2\,\text{T}^2\,\epsilon\$0^2\,\epsilon\$s\left(-\psi+\sqrt{d^2\left(\frac{\mathbb{i}}{\lambda\$d^2}+\frac{\text{q\$e}^4\,\rho^2}{\text{k\$b}^2\,\text{T}^2\,\epsilon\$0^2\,\epsilon\$s}\right)+\psi^2}\right)\Bigg)\Bigg)\Bigg)\Bigg/$$
$$\Bigg(\sqrt{d^2\left(\frac{\mathbb{i}}{\lambda\$d^2}+\frac{\text{q\$e}^4\,\rho^2}{\text{k\$b}^2\,\text{T}^2\,\epsilon\$0^2\,\epsilon\$s}\right)+\psi^2}$$
$$\Bigg(\mathbb{i}\,\text{k\$b}^3\,\text{T}^3\,\epsilon\$0^3\,\epsilon\$s\,(1+\epsilon\$s)\,\sqrt{d^2\left(\frac{\mathbb{i}}{\lambda\$d^2}+\frac{\text{q\$e}^4\,\rho^2}{\text{k\$b}^2\,\text{T}^2\,\epsilon\$0^2\,\epsilon\$s}\right)+\psi^2}+\text{k\$b}$$
$$\text{q\$e}^4\,\text{T}\,\epsilon\$0\,\lambda\$d^2\,\rho^2\left(\psi+\epsilon\$s\,\sqrt{d^2\left(\frac{\mathbb{i}}{\lambda\$d^2}+\frac{\text{q\$e}^4\,\rho^2}{\text{k\$b}^2\,\text{T}^2\,\epsilon\$0^2\,\epsilon\$s}\right)+\psi^2}\right)\Bigg)^2\Bigg)$$

In[10]:=
```
ζ$new = ζ /. {k$D → 1/λ$D}
```

Out[10]=
$$\frac{1-\frac{1-\frac{\mathbb{i}\,\lambda\$d^2\,\psi}{\epsilon\$s\,\lambda\$D^2\,\sqrt{d^2\left(\frac{\mathbb{i}}{\lambda\$d^2}+\frac{1}{\epsilon\$s\,\lambda\$D^2}\right)+\psi^2}}}{\epsilon\$s-\frac{\mathbb{i}\,\lambda\$d^2}{\lambda\$D^2}}}{1+\frac{1-\frac{\mathbb{i}\,\lambda\$d^2\,\psi}{\epsilon\$s\,\lambda\$D^2\,\sqrt{d^2\left(\frac{\mathbb{i}}{\lambda\$d^2}+\frac{1}{\epsilon\$s\,\lambda\$D^2}\right)+\psi^2}}}{\epsilon\$s-\frac{\mathbb{i}\,\lambda\$d^2}{\lambda\$D^2}}}$$

In[28]:=
```
ans = Series[ζ$new, {λ$D, 0, 2}] // Normal // PowerExpand // Expand
```

Out[28]=
$$1-\frac{2\,\mathbb{i}\,\lambda\$D^2}{\lambda\$d^2}-\frac{2\,\lambda\$D\,\psi}{d\,\sqrt{\epsilon\$s}}+\frac{2\,\lambda\$D^2\,\psi^2}{d^2\,\epsilon\$s}$$

In[29]:=
```
Collect[ans, {λ$D, λ$D^2}]
```

Out[29]=
$$1-\frac{2\,\lambda\$D\,\psi}{d\,\sqrt{\epsilon\$s}}+\lambda\$D^2\left(-\frac{2\,\mathbb{i}}{\lambda\$d^2}+\frac{2\,\psi^2}{d^2\,\epsilon\$s}\right)$$

In[16]:=
```
Integrate[ψ^3 Exp[-2 ψ], {ψ, 0, ∞}]
```

Out[16]=
$$\frac{3}{8}$$

In[18]:=
```
Series[Sqrt[1 - I a], {a, 0, 1}]
```

Out[18]=
$$1-\frac{\mathbb{i}\,a}{2}+O[a]^2$$