We have that

In[13]:=
$$\theta\mathrm{I} = \frac{\epsilon\$s}{\epsilon\$eff}\left(-\lambda\,\mathrm{Coth}[\theta 2] + \left(\mathrm{Tanh}[\theta 1]\,\mathrm{Tanh}[\theta 2] + \alpha\,\mathrm{Tanh}[\theta 2] - \lambda + 2\lambda\,/\,(\mathrm{Cosh}[\theta 1]\,\mathrm{Cosh}[\theta 2]) - \lambda^2\,\mathrm{Tanh}[\theta 1]\,/\,(\mathrm{Cosh}[\theta 2]\,\mathrm{Sinh}[\theta 2])\right)\Big/\left(\mathrm{Tanh}[\theta 2] + \mathrm{Tanh}[\theta 1]\,(-\lambda + \alpha\,\mathrm{Tanh}[\theta 2])\right)\right);$$

where $\theta 2 = \eta\, h_s$ and $\theta 2 = k\, h_s$. Check the large h_s limit, where $\theta 1$ and $\theta 2$ go to infinity.

In[14]:= `Limit[θI, {θ1 → Infinity, θ2 → Infinity}] // FullSimplify`

Out[14]=
$$\frac{\epsilon\$s - \epsilon\$s\,\lambda}{\epsilon\$eff}$$

This is the expected answer. Now write λ, ϵ\$eff, and ϵ\$s in terms of frequency. All three of these variables diverge as ω goes to zero.

In[15]:=
$$\theta\mathrm{I}\$sub = \theta\mathrm{I}\ /.\ \left\{\lambda \to \frac{\mathrm{I}}{\epsilon\$s}\,\frac{\omega\$0}{\omega}\,\frac{k}{\eta},\ \epsilon\$eff \to \epsilon\$s - \mathrm{I}\,\frac{\omega\$0}{\omega},\ \alpha \to \frac{\epsilon\$s}{\epsilon\$d} - \frac{\mathrm{I}}{\epsilon\$d}\,\frac{\omega\$0}{\omega}\right\}$$

Out[15]=
$$\frac{\epsilon\$s\left(-\frac{i\,k\,\omega\$0\,\mathrm{Coth}[\theta 2]}{\epsilon\$s\,\eta\,\omega} + \frac{-\frac{i\,k\,\omega\$0}{\epsilon\$s\,\eta\,\omega} + \frac{2\,i\,k\,\omega\$0\,\mathrm{Sech}[\theta 1]\,\mathrm{Sech}[\theta 2]}{\epsilon\$s\,\eta\,\omega} + \frac{k^2\,\omega\$0^2\,\mathrm{Csch}[\theta 2]\,\mathrm{Sech}[\theta 2]\,\mathrm{Tanh}[\theta 1]}{\epsilon\$s^2\,\eta^2\,\omega^2} + \left(\frac{\epsilon\$s}{\epsilon\$d} - \frac{i\,\omega\$0}{\epsilon\$d\,\omega}\right)\mathrm{Tanh}[\theta 2] + \mathrm{Tanh}[\theta 1]\,\mathrm{Tanh}[\theta 2]}{\mathrm{Tanh}[\theta 2] + \mathrm{Tanh}[\theta 1]\left(-\frac{i\,k\,\omega\$0}{\epsilon\$s\,\eta\,\omega} + \left(\frac{\epsilon\$s}{\epsilon\$d} - \frac{i\,\omega\$0}{\epsilon\$d\,\omega}\right)\mathrm{Tanh}[\theta 2]\right)}\right)}{\epsilon\$s - \frac{i\,\omega\$0}{\omega}}$$

Take the zero-frequency limit of this equation.

In[16]:=
$$\mathrm{limit} = \mathrm{Limit}\left[\frac{\theta\mathrm{I}\$sub}{\epsilon\$s},\ \omega \to 0\right]\ //\ \mathrm{FullSimplify}$$

Out[16]=
$$\frac{k^2\,\epsilon\$d + k\,\epsilon\$s\,\eta\,\mathrm{Coth}[\theta 2]}{\epsilon\$s^2\,\eta^2 + k\,\epsilon\$d\,\epsilon\$s\,\eta\,\mathrm{Coth}[\theta 2]}$$

When the thickness of the semiconductor goes to zero, the function θ_I/ϵ_s becomes independent of ϵ_s as expected.

In[17]:= `Limit[limit, θ2 → 0]`

Out[17]=
$$\frac{1}{\epsilon\$d}$$

When the thickness of the semiconductor goes to infinity, we get the θ_I/ϵ_s expected for a semi-infinite semiconductor.

In[18]:= `Limit[limit, θ2 → Infinity]`

Out[18]=
$$\frac{k}{\epsilon\$s\,\eta}$$

Pick out the leading terms by hand

In[22]:=
$$\text{limit2} = \text{Limit}\left[\frac{1}{\epsilon\$s}\,\frac{\epsilon\$s\left(-\frac{i\,k\,\omega\$0\,\text{Coth}[\theta 2]}{\epsilon\$s\,\eta\,\omega}+\frac{\frac{k^2\,\omega\$0^2\,\text{Csch}[\theta 2]\,\text{Sech}[\theta 2]}{\epsilon\$s^2\,\eta^2\,\omega^2}}{\left(-\frac{i\,k\,\omega\$0}{\epsilon\$s\,\eta\,\omega}+\left(\frac{\epsilon\$s}{\epsilon\$d}-\frac{i\,\omega\$0}{\epsilon\$d\,\omega}\right)\text{Tanh}[\theta 2]\right)}\right)}{\epsilon\$s-\frac{i\,\omega\$0}{\omega}},\ \omega\to 0\right]\ \text{// Simplify}$$

Out[22]=
$$\frac{k^2\,\epsilon\$d + k\,\epsilon\$s\,\eta\,\text{Coth}[\theta 2]}{\epsilon\$s^2\,\eta^2 + k\,\epsilon\$d\,\epsilon\$s\,\eta\,\text{Coth}[\theta 2]}$$

In[23]:= **limit === limit2**

Out[23]= True

In[34]:=
$$\text{limit3} = \frac{1}{\epsilon\$s}\,\frac{k}{\eta}\,\text{Coth}[\theta 2] - \left(\frac{1}{\epsilon\$s}\,\frac{k}{\eta}\right)^2\text{Csch}[\theta 2]\,\text{Sech}[\theta 2]\,\frac{1}{\frac{1}{\epsilon\$s}\,\frac{k}{\eta}+\frac{1}{\epsilon\$d}\,\text{Tanh}[\theta 2]}\ \text{// FullSimplify}$$

Out[34]=
$$\frac{k^2\,\epsilon\$d + k\,\epsilon\$s\,\eta\,\text{Coth}[\theta 2]}{\epsilon\$s^2\,\eta^2 + k\,\epsilon\$d\,\epsilon\$s\,\eta\,\text{Coth}[\theta 2]}$$

In[35]:= **limit === limit3**

Out[35]= True

In[37]:=
$$\text{limit4} = \frac{1}{\epsilon\$s}\,\frac{k}{\eta}\left(\text{Coth}[\theta 2] - \frac{\epsilon\$d\,k}{\text{Cosh}[\theta 2]\,\text{Sinh}[\theta 2]}\,\frac{1}{\text{Tanh}[\theta 2]}\,\frac{1}{\epsilon\$s\,\eta + \epsilon\$d\,k\,\text{Coth}[\theta 2]}\right)\ \text{// FullSimplify}$$

Out[37]=
$$\frac{k^2\,\epsilon\$d + k\,\epsilon\$s\,\eta\,\text{Coth}[\theta 2]}{\epsilon\$s^2\,\eta^2 + k\,\epsilon\$d\,\epsilon\$s\,\eta\,\text{Coth}[\theta 2]}$$

In[38]:= **limit === limit4**

Out[38]= True

In[43]:=
$$\text{limit5} = \frac{1}{\epsilon\$s}\,\frac{k}{\eta}\left(\text{Coth}[\theta 2] - \frac{1}{\text{Sinh}[\theta 2]^2}\,\frac{\epsilon\$d\,k}{\epsilon\$s\,\eta + \epsilon\$d\,k\,\text{Coth}[\theta 2]}\right)\ \text{// FullSimplify}$$

Out[43]=
$$\frac{k^2\,\epsilon\$d + k\,\epsilon\$s\,\eta\,\text{Coth}[\theta 2]}{\epsilon\$s^2\,\eta^2 + k\,\epsilon\$d\,\epsilon\$s\,\eta\,\text{Coth}[\theta 2]}$$

In[42]:= **limit === limit5**

Out[42]= True

$$\text{Coth}[\theta 2]\,\text{Sinh}[\theta 2]^2\,(\epsilon\$s\,\eta + \epsilon\$d\,k\,\text{Coth}[\theta 2]) - \epsilon\$d\,k$$

Out[50]=
$$\text{Sinh}[\theta 2]\,(\epsilon\$s\,\eta\,\text{Cosh}[\theta 2] + k\,\epsilon\$d\,\text{Sinh}[\theta 2])$$

In[61]:=
$$\text{Cosh}[\theta 2]^2 - \left(\text{Sinh}[\theta 2]^2 + 1\right)\ \text{// Simplify}$$

Out[61]= 0