

$$\text{In}[*]:= \text{Corr}[n_]=(-1)^{n+1} \frac{1}{z\$r^{n+1}}$$

$$\text{Integrate}\left[\psi^n \text{Exp}\left[-2\psi \frac{d}{z\$r}\right], \{\psi, 0, \infty\}, \text{Assumptions} \rightarrow \{\text{Element}[n, \text{Integers}], n \geq 0, \text{Element}[d, \text{Reals}], d > 0, \text{Element}[z\$r, \text{Reals}], z\$r > 0\}\right]$$

$$\text{Out}[*]=(-1)^{1+n} 2^{-1-n} z\$r^{-1-n} \left(\frac{z\$r}{d}\right)^{1+n} \text{Gamma}[1+n]$$

$$\text{In}[*]:= \text{Corr}[0]$$

$$\text{Out}[*]=-\frac{1}{2d}$$

$$\text{In}[*]:= \text{Corr}[1]$$

$$\text{Out}[*]=\frac{1}{4d^2}$$

$$\text{In}[*]:= \text{Corr}[2]$$

$$\text{Out}[*]=-\frac{1}{4d^3}$$

$$\gamma\$perp = \text{cap}[1]^2 \text{Corr}[0] + 2 \text{cap}[0] \times \text{cap}[1] \times \text{Corr}[1] + \text{cap}[0]^2 \text{Corr}[2] \quad // \quad \text{Expand}$$

$$\text{Out}[*]=-\frac{\text{cap}[0]^2}{4d^3} + \frac{\text{cap}[0] \times \text{cap}[1]}{2d^2} - \frac{\text{cap}[1]^2}{2d}$$