In[3]:=
```
Integrate[Exp[-y] Exp[2 π I y Y],
  {y, 0, ∞}, Assumptions → Element[Y, Reals]] // Simplify
```
Out[3]=
$$\frac{i}{i + 2\pi Y}$$

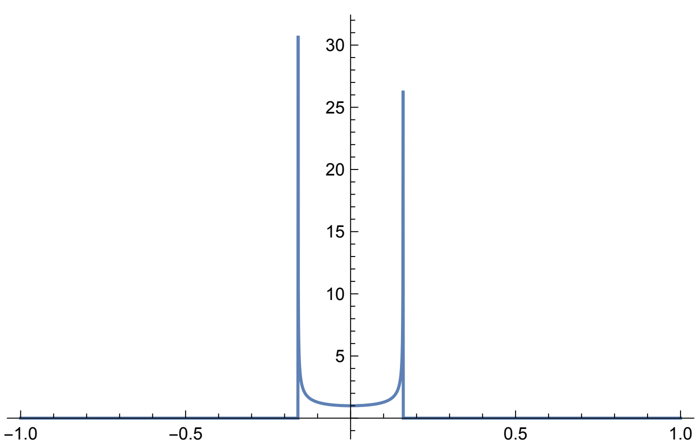
In[4]:=
```
Integrate[ y Exp[-y] Exp[2 π I y Y],
  {y, 0, ∞}, Assumptions → Element[Y, Reals]] // Simplify
```
Out[4]=
$$\frac{1}{(1 - 2 i \pi Y)^2}$$

In[5]:=
```
Integrate[ y^2 Exp[-y] Exp[2 π I y Y],
  {y, 0, ∞}, Assumptions → Element[Y, Reals]] // Simplify
```
Out[5]=
$$\frac{2}{(1 - 2 i \pi Y)^3}$$

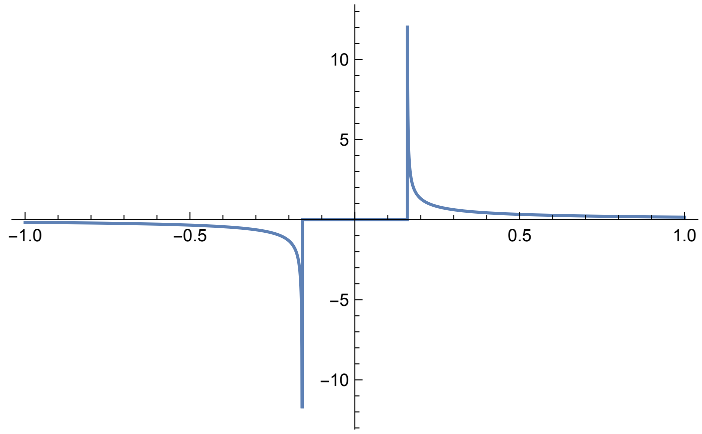
In[6]:=
```
Integrate[BesselJ[0, x y] Exp[2 π I y Y], {y, 0, ∞},
  Assumptions → {Element[Y, Reals], Element[x, Reals], x > 0}] // Simplify
```
Out[6]=
$$\frac{\sqrt{\pi}\left(\left(\begin{cases} \frac{x}{\sqrt{\pi}\,\sqrt{x^2-4\pi^2 Y^2}} & 4\pi^2 Y^2 < x^2 \\ 0 & \text{True} \end{cases}\right) + i\left(\begin{cases} 0 & x^2 \geq 4\pi^2 Y^2 \\ \frac{x}{\sqrt{-\pi x^2+4\pi^3 Y^2}} & \text{True} \end{cases}\right)\text{Sign}[Y]\right)}{x}$$

In[12]:=
```
func[Y_] = Integrate[BesselJ[0, y] Exp[2 π I y Y],
  {y, 0, ∞}, Assumptions → {Element[Y, Reals]}] // Simplify
```
Out[12]=
$$\sqrt{\pi}\left(\left(\begin{cases} \frac{1}{\sqrt{\pi-4\pi^3 Y^2}} & 4\pi^2 Y^2 < 1 \\ 0 & \text{True} \end{cases}\right) + i\left(\begin{cases} 0 & 4\pi^2 Y^2 \leq 1 \\ \frac{1}{\sqrt{\pi}\,\sqrt{-1+4\pi^2 Y^2}} & \text{True} \end{cases}\right)\text{Sign}[Y]\right)$$

In[20]:=
```
Plot[Re[N[func[Y]]], {Y, -1, 1}]
```
Out[20]=



In[22]:=
```
Plot[Im[N[func[Y]]], {Y, -1, 1}, PlotRange → All]
```

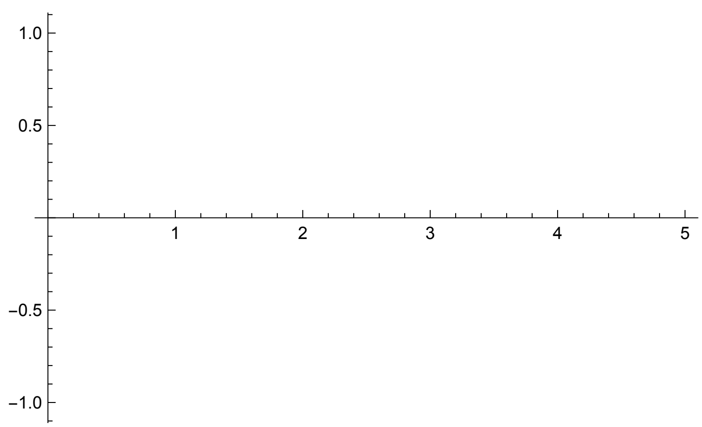
Out[22]=


In[28]:=
```
Clear[func, Y, x];
```

$$\texttt{func[Y\_] = Integrate}\left[\texttt{Exp}\left[-\frac{y}{2}\right]\texttt{BesselJ[0, x y] Exp[2}\,\pi\,\texttt{I y Y], \{y, 0, }\infty\texttt{\},}\right.$$
$$\left.\texttt{Assumptions} \to \texttt{\{Element[Y, Reals], Element[x, Reals], x > 0\}}\right]\texttt{ // Simplify}$$

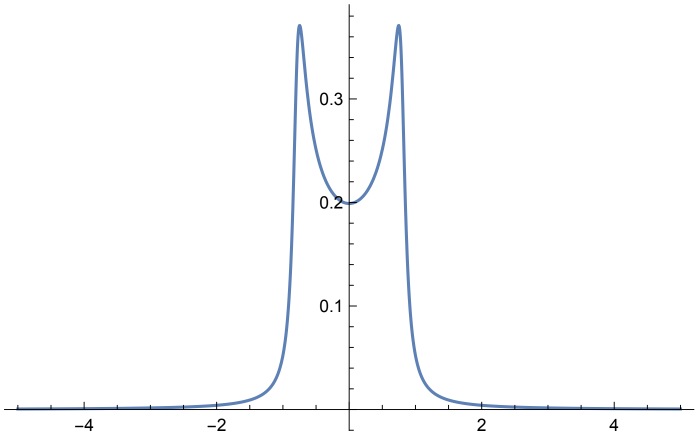
Out[29]=
$$\frac{2\,i}{(i+4\,\pi\,Y)\,\sqrt{1-\frac{4\,x^2}{(i+4\,\pi\,Y)^2}}}$$

In[63]:=
$$\texttt{Plot}\left[\texttt{Re}\left[\texttt{func[Y]} - \frac{2}{\sqrt{(1-4\,\pi\,i\,Y)^2+4\,x^2}}\right]\texttt{ /. x} \to \texttt{1, \{Y, 0, 5\}}\right]$$

Out[63]=


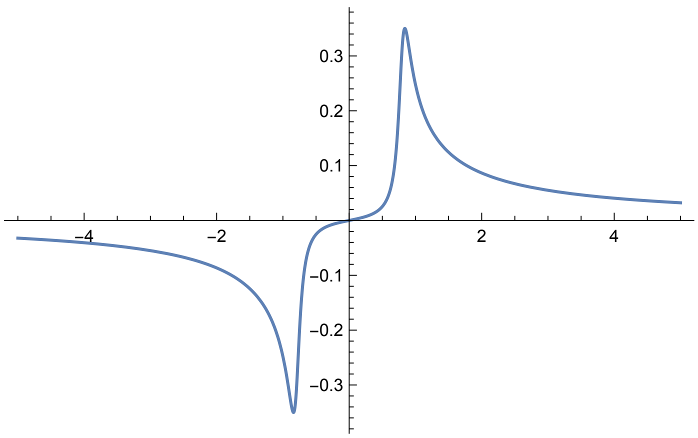
In[42]:= `Plot[Re[N[func[Y]]] /. x → 5, {Y, -5, 5}, PlotRange → All]`

Out[42]=



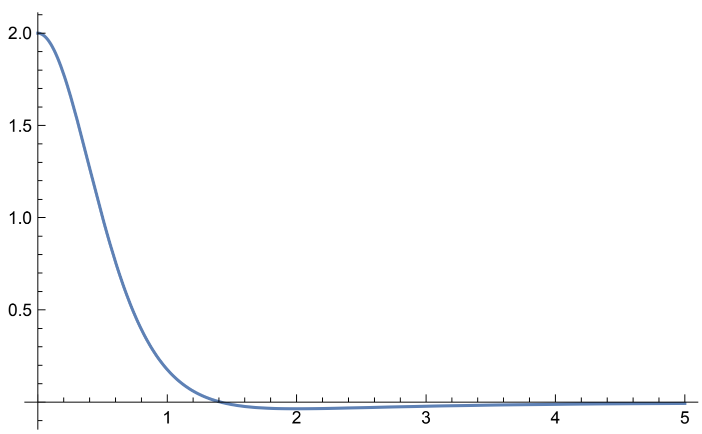
In[43]:= `Plot[Im[N[func[Y]]] /. x → 5, {Y, -5, 5}, PlotRange → All]`

Out[43]=



In[45]:= $\mathtt{Plot}\left[\frac{2 - x^2}{\left(1 + x^2\right)^{5/2}}, \{x, 0, 5\}, \mathtt{PlotRange} \to \mathtt{All}\right]$

Out[45]=



In[64]:= `f0 = Integrate[Exp[-y / 2] Exp[2 π I y Y], {y, 0, ∞}, Assumptions → Element[Y, Reals]]`

Out[64]= $\frac{2\,i}{i + 4\,\pi\,Y}$

In[65]:=
```
f1 = Integrate[ y Exp[-y / 2] Exp[2 π I y Y], {y, 0, ∞}, Assumptions → Element[Y, Reals]]
```

Out[65]= $-\frac{4}{(\mathbb{i} + 4\,\pi\,Y)^2}$

In[66]:=
```
f2 = Integrate[ y^2 Exp[-y / 2] Exp[2 π I y Y],
    {y, 0, ∞}, Assumptions → Element[Y, Reals]] // Simplify
```

Out[66]= $\frac{2}{\left(\frac{1}{2} - 2\,\mathbb{i}\,\pi\,Y\right)^3}$

Rewrite by hand and so check my results

In[68]:= $f0 - \frac{2}{1 - 4\,\pi\,I\,Y}$ // Simplify

Out[68]= 0

In[69]:= $f1 - \frac{4}{(1 - 4\,\pi\,I\,Y)^2}$ // Simplify

Out[69]= 0

In[71]:= $f2 - \frac{16}{(1 - 4\,\pi\,I\,Y)^3}$ // Simplify

Out[71]= 0