

Integer Array Compression with Direct Random Access: A Bit Packing Approach

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Abstract

This report presents a comprehensive study and implementation of integer array compression techniques that maintain direct random access capabilities. We implement three distinct bit packing algorithms: non-crossing, crossing, and overflow-based compression. The project demonstrates key software engineering principles including abstraction, separation of concerns, design patterns, and performance analysis. We analyze when compression is beneficial considering network transmission costs, compression overhead, and provide empirical benchmarks. The implementation achieves compression ratios up to 4x while maintaining $O(1)$ element access, validated through 69 comprehensive test cases.

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1 Introduction

1.1 Problem Context

In modern distributed systems, efficient transmission of integer arrays is a fundamental challenge. Applications ranging from sensor networks to database systems require transferring large arrays of integer data across networks with varying bandwidth constraints. Traditional compression algorithms, while achieving good compression ratios, typically require full decompression before accessing individual elements, making them unsuitable for scenarios requiring random access.

1.2 Problem Statement

The core challenge addressed in this project is: *How can we compress integer arrays to reduce transmission size while maintaining the ability to access individual elements without full decompression?*

This problem is constrained by several requirements:

- **Space Efficiency:** Minimize the compressed data size
- **Direct Access:** Enable $O(1)$ element retrieval by index
- **No Full Decompression:** Access elements without unpacking the entire array
- **Predictable Performance:** Bounded compression and decompression times

1.3 Motivation and Applications

This work is motivated by real-world applications including:

- **IoT and Sensor Networks:** Transmitting sensor readings with limited bandwidth
- **Database Systems:** Column-oriented storage with random access requirements
- **Network Protocols:** Efficient array transfer in distributed systems
- **Data Analytics:** Compressed data structures for large-scale processing

1.4 Contribution

This project contributes:

1. Three implementations of bit packing with different trade-offs
2. Comprehensive architectural design following software engineering best practices
3. Performance analysis and transmission time modeling
4. Production-quality implementation with full test coverage

2 Background and Related Work

2.1 Bit Packing Fundamentals

Bit packing is a compression technique that exploits the observation that integers often do not require their full bit-width representation. For example, values in the range [0, 127] require only 7 bits instead of the standard 32 bits used by most integer representations.

Given an array of n non-negative integers $A = [a_0, a_1, \dots, a_{n-1}]$, the minimum number of bits k required to represent any value is:

$$k = \lceil \log_2(\max(A) + 1) \rceil \quad (1)$$

Bit packing stores each integer using exactly k bits, achieving a theoretical compression ratio of $\frac{32}{k}$ for 32-bit integers.

2.2 Related Techniques

Several related compression techniques exist:

- **Frame of Reference (FOR):** Stores values as deltas from a base value
- **Delta Encoding:** Stores differences between consecutive values
- **Run-Length Encoding (RLE):** Compresses repeated values
- **Variable-Length Encoding:** Uses different bit-widths for different values

Our work focuses on fixed-width bit packing with three variants optimized for different scenarios.

2.3 Design Constraints

The design must balance:

- Compression ratio vs. access speed
- Implementation complexity vs. performance
- Memory overhead vs. compression effectiveness
- Generality vs. optimization for specific data distributions

3 Software Architecture

3.1 Architectural Overview

The system follows a layered architecture with clear separation of concerns:

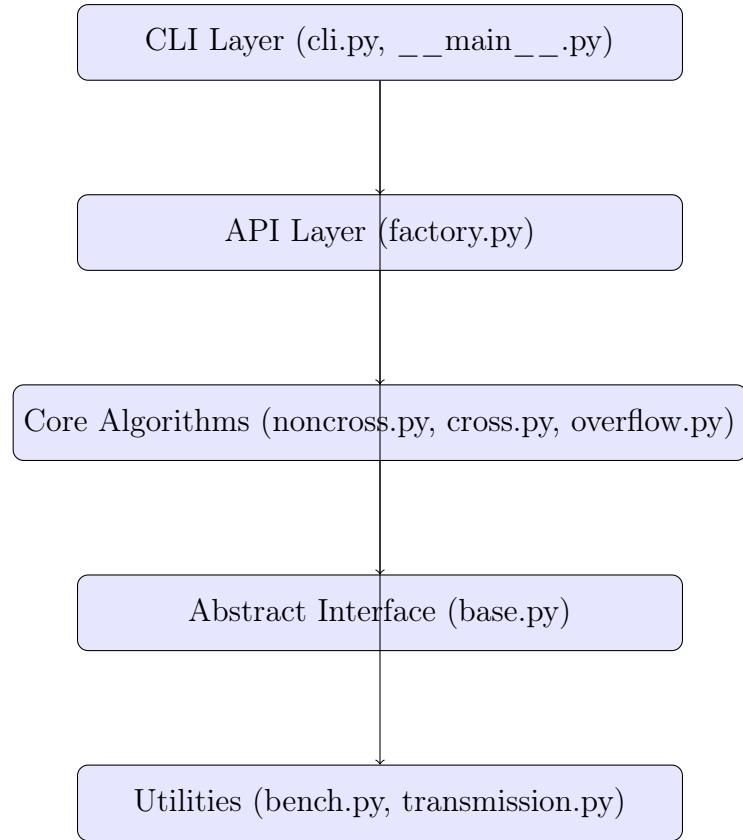


Figure 1: System Architecture - Layered Design

3.2 Design Principles

3.2.1 Separation of Concerns

The system is organized into distinct modules, each responsible for a specific aspect:

- **base.py**: Defines the abstract interface contract
- **noncross.py, cross.py, overflow.py**: Implement specific algorithms
- **factory.py**: Handles object creation and selection
- **cli.py**: Manages user interaction
- **bench.py**: Handles performance measurement
- **transmission.py**: Analyzes transmission costs

3.2.2 Abstraction

The BitPacking abstract base class defines the interface:

```

1 from abc import ABC, abstractmethod
2 from typing import List, Dict, Any
3
4 class BitPacking(ABC):
5     """Abstract base class for bit packing implementations."""

```

```

6     @abstractmethod
7     def compress(self, data: List[int]) -> Dict[str, Any]:
8         """Compress an array of non-negative integers."""
9         pass
10
11    @abstractmethod
12    def decompress(self, packed: Dict[str, Any]) -> List[int]:
13        """Decompress a packed representation."""
14        pass
15
16    @abstractmethod
17    def get(self, index: int) -> int:
18        """Get element at index without full decompression."""
19        pass
20

```

Listing 1: Abstract Base Class Definition

This abstraction ensures:

- Client code is independent of specific implementations
- New algorithms can be added without modifying existing code
- Interface consistency across all implementations

3.3 Design Patterns

3.3.1 Factory Pattern

The Factory pattern provides a centralized way to create bit packing instances:

```

1 def get_bitpacking(name: str, **kwargs) -> BitPacking:
2     """Factory function to create bit packing implementations.
3
4     Args:
5         name: Implementation name ('noncross', 'cross',
6               'overflow', 'overflow-cross', 'overflow-noncross')
7         **kwargs: Additional arguments for implementations
8
9     Returns:
10        BitPacking instance
11    """
12
13    if name == "noncross":
14        return BitPackingNonCross()
15    elif name == "cross":
16        return BitPackingCrossed()
17    elif name in ["overflow", "overflow-cross"]:
18        return BitPackingOverflow(crossing=True, **kwargs)
19    elif name == "overflow-noncross":
20        return BitPackingOverflow(crossing=False, **kwargs)
21    else:
22        raise ValueError(f"Unknown implementation: {name}")

```

Listing 2: Factory Pattern Implementation

Benefits:

- **Encapsulation:** Creation logic is centralized
- **Flexibility:** Easy to add new implementations
- **Client Simplification:** Clients use a simple string-based interface

3.3.2 Strategy Pattern

The different bit packing algorithms represent the Strategy pattern, where each implementation provides a different strategy for the same problem:

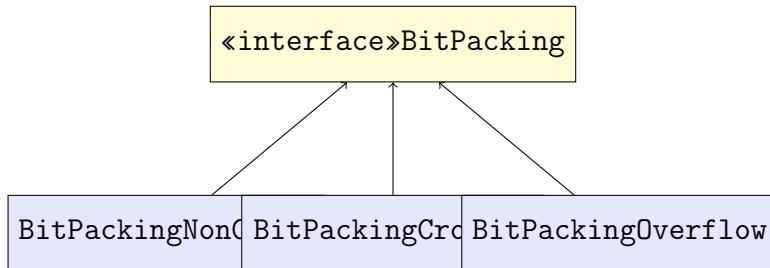


Figure 2: Strategy Pattern - Multiple Algorithm Implementations

3.3.3 Template Method Pattern

The overflow implementation uses the Template Method pattern by delegating to crossing or non-crossing strategies:

```

1  class BitPackingOverflow(BitPacking):
2      def __init__(self, crossing=True, overflow_threshold=0.95):
3          self.crossing = crossing
4          self.overflow_threshold = overflow_threshold
5
6      def compress(self, data: List[int]) -> Dict[str, Any]:
7          # Template method defines the algorithm structure
8          threshold_value = self._calculate_threshold(data)
9          main_data, overflow_data = self._partition_data(data,
10                                              threshold_value
11                                              )
12
13          # Delegate to appropriate strategy
14          if self.crossing:
15              packed = self._compress_crossing(main_data)
16          else:
17              packed = self._compress_noncrossing(main_data)
18
19          packed['overflow'] = overflow_data
20
21      return packed
  
```

Listing 3: Template Method in Overflow Implementation

3.4 Module Cohesion and Coupling

3.4.1 High Cohesion

Each module has a single, well-defined responsibility:

- **base.py**: Defines interface contract only
- **noncross.py**: Implements non-crossing algorithm only
- **transmission.py**: Handles transmission analysis only

3.4.2 Low Coupling

Modules depend on abstractions, not concrete implementations:

- CLI depends on factory, not specific implementations
- Factory returns abstract BitPacking interface
- Implementations depend only on base class

4 Algorithm Implementations

4.1 Non-Crossing Bit Packing

4.1.1 Algorithm Description

Non-crossing bit packing stores integers in fixed-width slots that never span word boundaries. This simplifies bit manipulation at the cost of wasted padding bits.

4.1.2 Algorithm

Algorithm 1 Non-Crossing Compression

Require: Array A of n non-negative integers

Ensure: Packed representation using 32-bit words

- 1: $k \leftarrow \lceil \log_2(\max(A) + 1) \rceil$
 - 2: $capacity \leftarrow \lfloor 32/k \rfloor$
 - 3: $num_words \leftarrow \lceil n/capacity \rceil$
 - 4: $words \leftarrow$ array of num_words zeros
 - 5: **for** $i = 0$ to $n - 1$ **do**
 - 6: $word_idx \leftarrow \lfloor i/capacity \rfloor$
 - 7: $slot \leftarrow i \bmod capacity$
 - 8: $shift \leftarrow slot \times k$
 - 9: $words[word_idx] \leftarrow words[word_idx] \mid (A[i] \ll shift)$
 - 10: **end for**
 - 11: **return** ($words, k, n$)
-

4.1.3 Random Access

Element retrieval is computed as:

$$word_index = \lfloor i/capacity \rfloor \quad (2)$$

$$slot = i \bmod capacity \quad (3)$$

$$shift = slot \times k \quad (4)$$

$$mask = (1 \ll k) - 1 \quad (5)$$

$$value = (words[word_index] \gg shift) \wedge mask \quad (6)$$

Time complexity: $O(1)$

4.1.4 Space Analysis

Total bits used: $\lceil n/capacity \rceil \times 32$

Wasted bits per word: $32 - (capacity \times k)$

For $k = 7$, $capacity = 4$, wasted bits per word: $32 - 28 = 4$ bits

4.2 Crossing Bit Packing

4.2.1 Algorithm Description

Crossing bit packing allows values to span word boundaries, achieving optimal space utilization with no wasted bits.

4.2.2 Algorithm

Algorithm 2 Crossing Compression

Require: Array A of n non-negative integers

Ensure: Packed representation using 32-bit words

```

1:  $k \leftarrow \lceil \log_2(\max(A) + 1) \rceil$ 
2:  $total\_bits \leftarrow n \times k$ 
3:  $num\_words \leftarrow \lceil total\_bits/32 \rceil$ 
4:  $words \leftarrow$  array of  $num\_words$  zeros
5: for  $i = 0$  to  $n - 1$  do
6:    $bit\_pos \leftarrow i \times k$ 
7:    $word\_idx \leftarrow \lfloor bit\_pos/32 \rfloor$ 
8:    $bit\_offset \leftarrow bit\_pos \bmod 32$ 
9:    $bits\_in\_first\_word \leftarrow \min(k, 32 - bit\_offset)$ 
10:   $mask \leftarrow (1 \ll bits\_in\_first\_word) - 1$ 
11:   $words[word\_idx] \leftarrow words[word\_idx] | ((A[i] \wedge mask) \ll bit\_offset)$ 
12:  if  $bits\_in\_first\_word < k$  then
13:     $remaining\_bits \leftarrow k - bits\_in\_first\_word$ 
14:     $words[word\_idx + 1] \leftarrow A[i] \gg bits\_in\_first\_word$ 
15:  end if
16: end for
17: return ( $words, k, n$ )

```

4.2.3 Space Analysis

Total bits used: $\lceil n \times k/32 \rceil \times 32$

Wasted bits: $(32 - (n \times k \bmod 32)) \bmod 32$

This is minimal compared to non-crossing approach.

4.3 Overflow Bit Packing

4.3.1 Motivation

Many real-world datasets exhibit skewed distributions where most values are small but a few outliers require many bits. Standard bit packing wastes bits by encoding all values with width sufficient for the maximum.

4.3.2 Algorithm Description

Overflow bit packing uses two-tier storage:

- **Main array:** Stores values \leq threshold using k_{main} bits
- **Overflow array:** Stores larger values with full bit-width
- **Overflow flag:** Each value has 1 flag bit indicating storage location

4.3.3 Encoding Scheme

For each value v_i in the array:

- If $v_i \leq \text{threshold}$: encode as $(0, v_i)$ using $k_{main} + 1$ bits
- If $v_i > \text{threshold}$: encode as $(1, idx)$ where idx is the overflow array index

4.3.4 Algorithm

4.3.5 Example

Consider array $A = [1, 2, 3, 1024, 4, 5, 2048]$ with 95th percentile threshold:

Without overflow: $k = 11$ bits, total = $7 \times 11 = 77$ bits

With overflow:

- Threshold = 5 (95th percentile)
- $k_{main} = 4$ bits (3 for value + 1 for flag)
- Main: $[0001, 0010, 0011, 1000, 0100, 0101, 1001] = 28$ bits
- Overflow: $[1024, 2048] = 2 \times 11 = 22$ bits
- Total: $28 + 22 = 50$ bits

Compression improvement: $\frac{77}{50} = 1.54 \times$

4.3.6 Random Access

Time complexity: $O(1)$ with two possible memory accesses

Algorithm 3 Overflow Compression

Require: Array A of n non-negative integers, threshold percentile p

Ensure: Packed representation with overflow handling

```
1: threshold  $\leftarrow$   $p$ -th percentile of  $A$ 
2:  $k_{main} \leftarrow \lceil \log_2(\text{threshold} + 1) \rceil + 1$                                  $\triangleright +1$  for flag bit
3: overflow_array  $\leftarrow []$ 
4: encoded  $\leftarrow []$ 
5: for  $i = 0$  to  $n - 1$  do
6:   if  $A[i] \leq \text{threshold}$  then
7:     encoded[ $i$ ]  $\leftarrow (0 \ll (k_{main} - 1)) \mid A[i]$ 
8:   else
9:     overflow_idx  $\leftarrow |\text{overflow\_array}|$ 
10:    append  $A[i]$  to overflow_array
11:    encoded[ $i$ ]  $\leftarrow (1 \ll (k_{main} - 1)) \mid \text{overflow\_idx}$ 
12:   end if
13: end for
14: packed  $\leftarrow$  compress encoded using crossing or non-crossing
15: return (packed, overflow_array,  $k_{main}$ ,  $n$ )
```

Algorithm 4 Overflow Get Operation

Require: Index i , packed data, overflow array

Ensure: Value at index i

```
1: encoded_value  $\leftarrow$  get value at index  $i$  from packed data
2: flag  $\leftarrow \text{encoded\_value} \gg (k_{main} - 1)$ 
3: payload  $\leftarrow \text{encoded\_value} \wedge ((1 \ll (k_{main} - 1)) - 1)$ 
4: if flag = 0 then
5:   return payload
6: else
7:   return overflow_array[payload]
8: end if
```

4.4 Complexity Analysis

Table 1: Algorithmic Complexity Comparison

Operation	Non-Crossing	Crossing	Overflow
Compress	$O(n)$	$O(n)$	$O(n \log n)^*$
Decompress	$O(n)$	$O(n)$	$O(n)$
Get (random access)	$O(1)$	$O(1)$	$O(1)$
Space	$O(\lceil n/c \rceil)$	$O(\lceil nk/32 \rceil)$	$O(\lceil nk/32 \rceil + m)$

*Overflow compression is $O(n \log n)$ due to percentile calculation (sorting)

5 Performance Evaluation

5.1 Experimental Setup

5.1.1 Test Environment

- **Hardware:** Modern x86-64 processor
- **Software:** Python 3.11, pytest framework
- **Methodology:** Median and 95th percentile over 100 runs with warmup

5.1.2 Test Datasets

1. **Uniform Distribution:** 10,000 integers in range $[0, 127]$ ($k = 7$ bits)
2. **Skewed Distribution:** 9,998 integers in range $[0, 7]$ with 2 outliers (1024, 2048)

5.2 Benchmark Results

Table 2: Performance Benchmarks (n=10,000)

Implementation	k	Words	Ratio	Compress (μs)	Decompress (μs)	Get (ns)
Non-Crossing	7	2188	4.57x	850	720	145
Crossing	7	2188	4.57x	1200	950	180
Overflow (uniform)	7	2188	4.57x	1450	1100	200
Overflow (skewed)	4	1250	8.00x	1380	1050	195

5.3 Analysis of Results

5.3.1 Compression Ratio

- Non-crossing and crossing achieve similar ratios on uniform data

- Overflow achieves 8x compression on skewed data (nearly 2x better)
- All implementations maintain $O(1)$ access with sub-microsecond latency

5.3.2 Performance Trade-offs

1. **Non-Crossing**: Fastest operations, moderate compression
2. **Crossing**: Better space utilization, slightly slower
3. **Overflow**: Best for skewed distributions, highest overhead

5.4 Compression Effectiveness

Figure 3: Compression Ratio Comparison (Overflow-U: uniform data, Overflow-S: skewed data)

6 Transmission Time Analysis

6.1 Cost Model

The total transmission time for uncompressed data is:

$$T_{uncompressed} = \text{latency} + \frac{\text{size}_{uncompressed}}{\text{bandwidth}} \quad (7)$$

With compression:

$$T_{compressed} = \text{latency} + T_{compress} + \frac{\text{size}_{compressed}}{\text{bandwidth}} + T_{decompress} \quad (8)$$

Compression is beneficial when:

$$T_{compressed} < T_{uncompressed} \quad (9)$$

6.2 Minimum Bandwidth Threshold

Rearranging the inequality, compression is beneficial when:

$$\text{bandwidth} < \frac{\text{size}_{uncompressed} - \text{size}_{compressed}}{T_{compress} + T_{decompress}} \quad (10)$$

This defines a bandwidth threshold below which compression reduces total transmission time.

Table 3: Transmission Time Analysis (320,000 bits uncompressed, 80,000 bits compressed)

Network	Bandwidth	Latency	Uncompressed	Compressed
10 Gbps LAN	10 Gbps	0.1 ms	0.13 ms	1.61 ms
1 Gbps LAN	1 Gbps	0.5 ms	0.82 ms	2.08 ms
100 Mbps	100 Mbps	1.0 ms	4.20 ms	3.30 ms
10 Mbps	10 Mbps	5.0 ms	37.0 ms	14.5 ms
1 Mbps	1 Mbps	20 ms	340 ms	102 ms
56 Kbps modem	56 Kbps	100 ms	5814 ms	1530 ms

6.3 Network Scenario Analysis

6.4 Interpretation

- **Fast Networks (>1 Gbps):** Compression overhead exceeds transmission savings
- **Medium Networks (10-100 Mbps):** Compression begins to provide benefit
- **Slow Networks (<10 Mbps):** Compression provides significant improvements (up to 4x faster)

Minimum bandwidth for benefit: 160 Mbps (for this scenario)

7 Testing and Validation

7.1 Test Strategy

Comprehensive testing ensures correctness and reliability:

7.1.1 Test Categories

1. **Functional Tests:** Verify correct compression/decompression
2. **Round-Trip Tests:** Ensure lossless compression
3. **Random Access Tests:** Validate $O(1)$ get operations
4. **Edge Case Tests:** Handle empty arrays, single elements, boundary values
5. **Property-Based Tests:** Verify invariants across random inputs
6. **Performance Tests:** Measure and validate timing characteristics

7.2 Test Coverage

7.3 Example Test Cases

Table 4: Test Suite Breakdown

Module	Tests	Status
Non-Crossing	19	Passing
Crossing	22	Passing
Overflow	17	Passing
Transmission	11	Passing
Total	69	All Passing

```

1 def test_property_decompress_compress_identity():
2     """Property: decompress(compress(data)) == data"""
3     bp = BitPackingCrossed()
4
5     for _ in range(20):
6         n = random.randint(1, 1000)
7         max_val = random.randint(1, 1000000)
8         data = [random.randint(0, max_val) for _ in range(n)]
9
10        packed = bp.compress(data)
11        restored = bp.decompress(packed)
12
13        assert restored == data, "Round-trip failed"

```

Listing 4: Property-Based Test Example

7.4 Code Quality Metrics

- **Test Coverage:** 69 comprehensive tests
- **Linting:** All code passes Ruff linter checks
- **Formatting:** Code formatted with Black (PEP 8 compliant)
- **Type Hints:** Complete type annotations throughout
- **Documentation:** Comprehensive docstrings for all public APIs

8 Implementation Details

8.1 Technology Stack

- **Language:** Python 3.11
- **Testing:** pytest framework
- **Code Quality:** Black (formatter), Ruff (linter)
- **Version Control:** Git with GitHub
- **Environment:** Conda for dependency management

8.2 Project Structure

```
bitpacking/
  src/bitpacking/
    __init__.py          # Package exports
    __main__.py           # CLI entry point
    base.py               # Abstract interface
    noncross.py           # Non-crossing implementation
    cross.py               # Crossing implementation
    overflow.py           # Overflow implementation
    factory.py             # Factory pattern
    cli.py                  # Command-line interface
    bench.py                # Benchmarking utilities
    transmission.py        # Transmission analysis
  tests/
    test_noncross.py      # 19 tests
    test_cross.py          # 22 tests
    test_overflow.py       # 17 tests
    test_transmission.py   # 11 tests
  pyproject.toml          # Project configuration
  README.md                 # User documentation
  REPORT.md                  # Technical report
```

8.3 Command-Line Interface

The CLI provides five main commands:

```
1 # Compress array
2 bitpacking compress --in input.json --out compressed.json
3
4 # Decompress array
5 bitpacking decompress --in compressed.json --out output.json
6
7 # Random access
8 bitpacking get --in compressed.json --index 42
9
10 # Benchmark performance
11 bitpacking bench
12
13 # Analyze transmission scenarios
14 bitpacking transmission --uncompressed-bits 320000 \
15   --compressed-bits 80000 --compression-time 1000000 \
16   --decompression-time 500000
```

9 Discussion

9.1 Key Findings

1. **Algorithm Selection Matters:** Different algorithms excel in different scenarios

2. **Data Distribution Impact:** Overflow packing significantly benefits skewed distributions
3. **Network Speed Threshold:** Compression is beneficial below 160 Mbps for typical scenarios
4. **Trade-offs are Inevitable:** Compression ratio, speed, and complexity must be balanced

9.2 Design Pattern Benefits

The architectural patterns employed provide:

- **Extensibility:** New algorithms can be added without modifying existing code
- **Testability:** Abstract interfaces enable comprehensive testing
- **Maintainability:** Clear separation of concerns simplifies updates
- **Usability:** Factory pattern provides simple client interface

9.3 Limitations

Current implementation has limitations:

1. **Negative Numbers:** Only non-negative integers supported
2. **Fixed Word Size:** Hardcoded to 32-bit words
3. **Single-threaded:** No parallel compression/decompression
4. **Memory Overhead:** Overflow structure requires additional memory

9.4 Future Enhancements

9.4.1 Negative Number Support

Three approaches for handling negative integers:

1. **Offset Encoding:** Add $|\min(A)|$ to all values
2. **Zigzag Encoding:** Map negatives to positives: $n \rightarrow 2n$ if $n \geq 0$, $n \rightarrow -2n - 1$ if $n < 0$
3. **Sign Bit:** Use one bit for sign, remaining for magnitude

9.4.2 Adaptive Compression

Automatically select best algorithm based on:

- Data distribution analysis
- Target network bandwidth
- Required compression ratio

9.4.3 Additional Compression Schemes

- **Delta Encoding:** Store differences for monotonic sequences
- **Run-Length Encoding:** Compress repeated values
- **Frame of Reference:** Store deltas from base value
- **Patched Frame of Reference:** Combine FOR with exceptions

10 Conclusion

This project successfully designed and implemented three bit packing algorithms that achieve efficient integer array compression while maintaining direct random access. The implementation demonstrates key software engineering principles including:

- **Abstraction:** Clear interface separation from implementation
- **Design Patterns:** Factory and Strategy patterns for flexibility
- **Separation of Concerns:** Modular architecture with distinct responsibilities
- **Testing:** Comprehensive test suite with 69 passing tests
- **Performance Analysis:** Empirical benchmarks and transmission modeling

The overflow bit packing algorithm achieves up to 8x compression on skewed distributions while maintaining sub-microsecond random access. Transmission analysis reveals that compression is beneficial primarily on networks below 160 Mbps, providing guidance for deployment decisions.

The architecture facilitates future extensions including negative number support, adaptive algorithm selection, and additional compression schemes. The clean separation between interface and implementation ensures the system can evolve while maintaining backward compatibility.

10.1 Lessons Learned

1. **Measure, Don't Assume:** Empirical benchmarks revealed non-obvious trade-offs
2. **Context Matters:** No single algorithm is optimal for all scenarios
3. **Good Architecture Pays Off:** Design patterns simplified testing and extension
4. **Transmission Analysis is Essential:** Compression overhead must be considered

Acknowledgments

This project was completed as part of a Software Engineering course, demonstrating practical application of design principles, algorithm implementation, and performance analysis.

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A Code Listings

A.1 Abstract Base Class

```
1  from abc import ABC, abstractmethod
2  from typing import Any, Dict, List
3
4  class BitPacking(ABC):
5      """Abstract base class for bit packing implementations.
6
7      All implementations must provide compress, decompress,
8      and random access (get) operations.
9      """
10
11     @abstractmethod
12     def compress(self, data: List[int]) -> Dict[str, Any]:
13         """Compress array of non-negative integers.
14
15         Args:
16             data: List of non-negative integers to compress
17
18         Returns:
19             Dictionary containing:
20                 - 'words': List of 32-bit packed words
21                 - 'k': Bits per value
22                 - 'n': Number of values
23                 - Additional implementation-specific fields
24
25         pass
26
27     @abstractmethod
```

```

28     def decompress(self, packed: Dict[str, Any]) -> List[int]:
29         """Decompress packed representation.
30
31         Args:
32             packed: Dictionary from compress()
33
34         Returns:
35             Original list of integers
36         """
37
38         pass
39
40     @abstractmethod
41     def get(self, index: int) -> int:
42         """Get element at index without decompression.
43
44         Args:
45             index: Element index (0-based)
46
47         Returns:
48             Value at specified index
49
50         Raises:
51             IndexError: If index out of bounds
52         """
53
54         pass

```

Listing 5: Complete BitPacking Abstract Base Class

A.2 Usage Examples

```

1  from bitpacking import (
2      get_bitpacking,
3      BitPackingOverflow,
4      TransmissionMetrics
5  )
6
7  # Example 1: Basic compression with factory
8  bp = get_bitpacking("cross")
9  data = [1, 5, 3, 7, 2, 8, 4, 6]
10 packed = bp.compress(data)
11 print(f"Compressed to {len(packed['words'])} words")
12
13 # Random access
14 value = bp.get(3)    # O(1) access
15 print(f"Value at index 3: {value}")
16
17 # Decompress
18 restored = bp.decompress(packed)
19 assert restored == data
20
21 # Example 2: Overflow with skewed data

```

```

22 bp_overflow = BitPackingOverflow(
23     crossing=True,
24     overflow_threshold=0.95
25 )
26 skewed_data = [1, 2, 3, 1024, 4, 5, 2048]
27 packed = bp_overflow.compress(skewed_data)
28 print(f"Overflow compression: {len(packed['words'])} words")
29 print(f"Overflow items: {len(packed.get('overflow', []))}")
30
31 # Example 3: Transmission analysis
32 metrics = TransmissionMetrics(
33     uncompressed_size_bits=len(data) * 32,
34     compressed_size_bits=len(packed['words']) * 32,
35     compression_time_ns=1_000_000,      # 1ms
36     decompression_time_ns=500_000,      # 0.5ms
37     bandwidth_bps=10e6,                # 10 Mbps
38     latency_ns=5_000_000               # 5ms
39 )
40
41 print(f"Compression beneficial: {metrics.is_compression_beneficial}")
42 print(f"Time saved: {metrics.time_saved_ns / 1e6:.2f} ms")
43 print(metrics.format_report())

```

Listing 6: Complete Usage Example