Branch and Bound – Number of selected children

Heuristics – Case: Fruit fly
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Goal of experiment:
In this experiment the number of selected children are changed in the branch and bound (BnB) algorithm: depth-first search which prunes using breakpoints. It will show what the influence of this constraint is on the run time and the number of inversions made to find the solution.

Methods:
The BnB depth-first search with breakpoints searches for a path from a given genome to a solution genome. The path consists of reversions of gene blocks in a genome. From the given root genome, every possible reversion is generated: the children. Depending on the number of selected children, one, two, or three children with the least breakpoints are added to the stack. The upper bound is updated every time a solution is found.

Results:

Table1. Branch and bound: Influence of the number of selected children on the runtime and number of inversions from Drosophila Melanogaster to the solution.

Number of selected children (fruit flies)	Number of inversions to solution	Runtime
1	17	< 1 sec
2	13	1 – 3 min
3	13	> 4 hrs

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Branch and Bound: depth-first search using breakpoints.

Genome fruitfly: [23, 1, 2, 11, 24, 22, 19, 6, 10, 7, 25, 20, 5, 8, 18, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Solution found in generation: 17, but still searching for a better solution...

Best solution found in generation:  17
Path to solution:
Inversion:  0 [23, 1, 2, 11, 24, 22, 19, 6, 10, 7, 25, 20, 5, 8, 18, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Inversion:  1 [23, 1, 2, 11, 24, 22, 19, 20, 25, 7, 10, 6, 5, 8, 18, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Inversion:  2 [23, 1, 2, 11, 24, 22, 19, 20, 25, 7, 10, 6, 5, 4, 3, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  3 [11, 2, 1, 23, 24, 22, 19, 20, 25, 7, 10, 6, 5, 4, 3, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  4 [7, 25, 20, 19, 22, 24, 23, 1, 2, 11, 10, 6, 5, 4, 3, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  5 [10, 11, 2, 1, 23, 24, 22, 19, 20, 25, 7, 6, 5, 4, 3, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  6 [10, 11, 12, 13, 14, 15, 16, 17, 21, 3, 4, 5, 6, 7, 25, 20, 19, 22, 24, 23, 1, 2, 18, 8, 9]
Inversion:  7 [10, 11, 12, 13, 14, 15, 16, 17, 25, 7, 6, 5, 4, 3, 21, 20, 19, 22, 24, 23, 1, 2, 18, 8, 9]
Inversion:  8 [10, 11, 12, 13, 14, 15, 16, 17, 22, 19, 20, 21, 3, 4, 5, 6, 7, 25, 24, 23, 1, 2, 18, 8, 9]
Inversion:  9 [10, 11, 12, 13, 14, 15, 16, 17, 18, 2, 1, 23, 24, 25, 7, 6, 5, 4, 3, 21, 20, 19, 22, 8, 9]
Inversion: 10 [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 3, 4, 5, 6, 7, 25, 24, 23, 1, 2, 22, 8, 9]
Inversion: 11 [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 2, 1, 23, 24, 25, 7, 6, 5, 4, 3, 8, 9]
Inversion: 12 [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 2, 1, 23, 24, 25, 3, 4, 5, 6, 7, 8, 9]
Inversion: 13 [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 24, 23, 1, 2, 3, 4, 5, 6, 7, 8, 9]
Inversion: 14 [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 1, 2, 3, 4, 5, 6, 7, 8, 9]
Inversion: 15 [25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 1, 2, 3, 4, 5, 6, 7, 8, 9]
Inversion: 16 [25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
Inversion: 17 [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]

Runtime:  0.09604063099959603
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Figure 1. Result of BnB depth-first search when selecting one child.

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Branch and Bound: depth-first search using breakpoints.

Genome fruitfly: [23, 1, 2, 11, 24, 22, 19, 6, 10, 7, 25, 20, 5, 8, 18, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Solution found in generation: 18, but still searching for a better solution...
Solution found in generation: 17, but still searching for a better solution...
Solution found in generation: 16, but still searching for a better solution...
Solution found in generation: 15, but still searching for a better solution...
Solution found in generation: 14, but still searching for a better solution...
Solution found in generation: 13, but still searching for a better solution...

Best solution found in generation:  13
Path to solution:
Inversion:  0 [23, 1, 2, 11, 24, 22, 19, 6, 10, 7, 25, 20, 5, 8, 18, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Inversion:  1 [23, 1, 2, 11, 24, 22, 19, 6, 10, 7, 25, 20, 5, 4, 3, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  2 [24, 11, 2, 1, 23, 22, 19, 6, 10, 7, 25, 20, 5, 4, 3, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  3 [24, 11, 2, 1, 23, 22, 19, 20, 25, 7, 10, 6, 5, 4, 3, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  4 [1, 2, 11, 24, 23, 22, 19, 20, 25, 7, 10, 6, 5, 4, 3, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  5 [1, 2, 3, 4, 5, 6, 10, 7, 25, 20, 19, 22, 23, 24, 11, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  6 [1, 2, 3, 4, 5, 6, 7, 10, 25, 20, 19, 22, 23, 24, 11, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  7 [1, 2, 3, 4, 5, 6, 7, 24, 23, 22, 19, 20, 25, 10, 11, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  8 [1, 2, 3, 4, 5, 6, 7, 20, 19, 22, 23, 24, 25, 10, 11, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion:  9 [1, 2, 3, 4, 5, 6, 7, 20, 19, 11, 10, 25, 24, 23, 22, 21, 17, 16, 15, 14, 13, 12, 18, 8, 9]
Inversion: 10 [1, 2, 3, 4, 5, 6, 7, 20, 19, 11, 10, 9, 8, 18, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25]
Inversion: 11 [1, 2, 3, 4, 5, 6, 7, 17, 16, 15, 14, 13, 12, 18, 8, 9, 10, 11, 19, 20, 21, 22, 23, 24, 25]
Inversion: 12 [1, 2, 3, 4, 5, 6, 7, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 18, 19, 20, 21, 22, 23, 24, 25]
Inversion: 13 [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]

Runtime:  67.84605488299985
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Figure 2. Result of BnB depth-first search when selecting two children.

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Branch and Bound: depth-first search using breakpoints.

Genome fruitfly: [23, 1, 2, 11, 24, 22, 19, 6, 10, 7, 25, 20, 5, 8, 18, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Solution found in generation: 22, but still searching for a better solution...
Solution found in generation: 21, but still searching for a better solution...
Solution found in generation: 19, but still searching for a better solution...
Solution found in generation: 18, but still searching for a better solution...
Solution found in generation: 17, but still searching for a better solution...
Solution found in generation: 16, but still searching for a better solution...
Solution found in generation: 15, but still searching for a better solution...
Solution found in generation: 14, but still searching for a better solution...
Solution found in generation: 13, but still searching for a better solution...

Best solution found in generation:  13
Path to solution:
Inversion:  0 [23, 1, 2, 11, 24, 22, 19, 6, 10, 7, 25, 20, 5, 8, 18, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Inversion:  1 [24, 11, 2, 1, 23, 22, 19, 6, 10, 7, 25, 20, 5, 8, 18, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Inversion:  2 [24, 11, 2, 1, 23, 22, 19, 20, 25, 7, 10, 6, 5, 8, 18, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Inversion:  3 [24, 25, 20, 19, 22, 23, 1, 2, 11, 7, 10, 6, 5, 8, 18, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Inversion:  4 [24, 25, 20, 19, 18, 8, 5, 6, 10, 7, 11, 2, 1, 23, 22, 12, 13, 14, 15, 16, 17, 21, 3, 4, 9]
Inversion:  5 [24, 25, 17, 16, 15, 14, 13, 12, 22, 23, 1, 2, 11, 7, 10, 6, 5, 8, 18, 19, 20, 21, 3, 4, 9]
Inversion:  6 [24, 25, 17, 16, 15, 14, 13, 12, 22, 23, 1, 2, 11, 10, 7, 6, 5, 8, 18, 19, 20, 21, 3, 4, 9]
Inversion:  7 [24, 25, 17, 16, 15, 14, 13, 12, 22, 23, 1, 2, 11, 10, 5, 6, 7, 8, 18, 19, 20, 21, 3, 4, 9]
Inversion:  8 [24, 25, 17, 16, 15, 14, 13, 12, 22, 23, 1, 2, 11, 10, 5, 6, 7, 8, 9, 4, 3, 21, 20, 19, 18]
Inversion:  9 [24, 25, 17, 16, 15, 14, 13, 12, 22, 23, 1, 2, 3, 4, 9, 8, 7, 6, 5, 10, 11, 21, 20, 19, 18]
Inversion: 10 [24, 25, 17, 16, 15, 14, 13, 12, 11, 10, 5, 6, 7, 8, 9, 4, 3, 2, 1, 23, 22, 21, 20, 19, 18]
Inversion: 11 [1, 2, 3, 4, 9, 8, 7, 6, 5, 10, 11, 12, 13, 14, 15, 16, 17, 25, 24, 23, 22, 21, 20, 19, 18]
Inversion: 12 [1, 2, 3, 4, 9, 8, 7, 6, 5, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]
Inversion: 13 [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]

Runtime:  13837.757065755
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Figure 3. Result of BnB depth-first search when selecting three children.

Discussion:

Selecting one child has a fast runtime of less than one second and finds a solution in 17 inversions (Figure 1). While selecting two children has a slightly longer runtime of 2.6 minutes, but finds a solution in just 13 inversions (Figure 2). Selecting three children, increased the runtime up to 4 hours, and returned a solution of 13 inversions (Figure 3). The runtime increases exponentially by selecting more children.

Conclusion:

Selecting two children is more optimal than selecting one or three, when taking in consideration the number of inversions and the runtime.