Lab: Description Logics

Through this lab you will gain a good understanding of the basics of description logics.

Reading Materials

- Lecture slides
- Chapter 1 and 2 in Baader. The Description Logic Handbook: Theory, Implementation, and Applications. Cambridge University Press.
- Table 1: $\mathcal{SROIQ}$ constructors and axioms.

Submission

You should submit a document that contains the answers to the tasks. **Note:** Any form of plagiarism, including using AI to generate answers, will result in failing the lab.

1. Basics

Answer each of the questions below.

1) Why do we say Description Logics (DL) is a concept language? What does "description" mean in DL?

2) In DL, what do the TBox and the ABox stand for?

3) $\mathcal{AL}$ is the base DL in the family of DLs. What are the constructors provided by this language? For each of the constructors, give its semantics using interpretation.

4) We obtain more expressive description languages if we add further constructors to $\mathcal{AL}$. List at least 3 constructors. For each constructor, give its interpretation and the letter that indicates the constructor.

5) Inclusion and equality are the two forms of terminological axioms in DL. What are the differences between these two axioms?

6) In DL, what are the four decidable properties of TBox? What are the reasoning tasks for ABox?

1.

1) Why do we say Description Logics (DL) is a concept language? What does “description” mean in DL?

Description Logic "describes" a set of individuals with the use of concept names (unary predicate FOL), role names(binary predicates in FOL), and Individual names (constants in FOL) from a knowledge base.

2. Terminological box (TBOX)
and
Assertional box (ABOX)

3. AL (Attributive Language)

Constructors	Semantics
Atomic Concept (A)	$A^{\mathcal{I}}$
Universal Concept ($\top$)	$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$
Bottom Concept ($\bot$)	$\bot^{\mathcal{I}} = \emptyset$
Atomic Negation ($\neg A$)	$(\neg A)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus A^{\mathcal{I}}$
Concept Intersection ($C \sqcap D$)	$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \sqcap D^{\mathcal{I}}$
Value Restrictions ($\forall R.C$) (Universal Restrictions)	$(\bullet)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \forall b.(a,b) \in R^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}$
Limited Existential Restrictions ($\exists R.\top$)	$(\exists R.\top)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a,b) \in R^{\mathcal{I}}\}$

4.

Constructors	Semantics
Concept Negation $(\mathcal{C})$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
Union $(\mathcal{U})$	$(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \sqcup D^{\mathcal{I}}$
Full Existential Quantification $(\exists R.C)$	$(\bullet)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\}$
Number Restriction $(\mathcal{N})$	$(\geq n R)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \lvert\{b \mid (a,b) \in R^{\mathcal{I}}\}\rvert \geq n\}$

5. In the most general case, terminological axioms have the form:

Concepts: (C, D)
Roles: (R, S)

$$\underbrace{C \sqsubseteq D \quad (R \sqsubseteq S)}_{\text{Inclusions}} \quad \text{or} \quad \underbrace{C \equiv D \quad (R \equiv S)}_{\text{Equalities}}$$

An interpretation $\mathcal{I}$ satisfies an inclusion: $C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$, it also satisfies an equality $C \equiv D$ if $C^{\mathcal{I}} = D^{\mathcal{I}} \longrightarrow C \sqsubseteq D \Leftrightarrow D \sqsubseteq C$

Two axioms or two sets of axioms are equivalent if they have the same models. An individual belongs to concept C if and only if it belongs to concept D.

Inclusion states necessary conditions for memberships in concept C.

Inclusion: If an individual belongs to concept C, it must also belong to concept D. It's a one-way implication.

Equality: An individual belongs to concept C if and only if it belongs to concept D. It's a two-way implication.

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6. $\mathcal{T}$: (a Tbox) $\mathcal{I}$: an interpretation

Concept: (C, D)

The four decidable properties of Tbox are:

- Satisfiability: A concept C is satisfiable with respect to $\mathcal{T}$ if there exists a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}}$ is nonempty. In this case we say also that $\mathcal{I}$ is a model of C.

- Subsumption: A concept C is subsumed by a concept D with respect to $\mathcal{T}$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model $\mathcal{I}$ of $\mathcal{T}$. In this case we write $C \sqsubseteq_{\mathcal{T}} D$ or $\mathcal{T} \models C \sqsubseteq D$.

- Equivalence: Two concepts C and D are equivalent with respect to $\mathcal{T}$ if $C^{\mathcal{I}} = D^{\mathcal{I}}$ for every model $\mathcal{I}$ of $\mathcal{T}$. In this case we write $C \equiv_{\mathcal{T}} D$ or $\mathcal{T} \models C \equiv D$.

- Disjointness: Two concepts C and D are disjoint with respect to $\mathcal{T}$ if $C^{\mathcal{I}} \cap D^{\mathcal{I}} = \emptyset$ for every model $\mathcal{I}$ of $\mathcal{T}$.

The reasoning tasks for Abox are:

- Consistency: An ABox $\mathcal{A}$ is consistent with respect to a TBox $\mathcal{T}$, if there is an interpretation that is a model of both $\mathcal{A}$ and $\mathcal{T}$.

$\mathcal{A}$ is therefore consistent if it is consistent with respect to the empty TBox.

- Instance checking: A check to see if assertions are entailed by an Abox.

- Realization: Given an individual a and a set of concepts, find the most specific concepts C from the set such that $\mathcal{A} \models C(a)$.

- Retrieval: Given an ABox $\mathcal{A}$ and a concept C, find all individuals a such that $\mathcal{A} \models C(a)$.

2. Translate the DL statements into Natural Language

Translate the following DL concept descriptions into natural language.

1) $\mathsf{Person} \sqcap \mathsf{Happy}$	Happy person
2) $\mathsf{Person} \sqcap \mathsf{Happy} \sqcap \exists \mathsf{owns.Pet}$	Happy person owns a cat
3) $\mathsf{Person} \sqcap \forall \mathsf{owns.Cat}$	Person owns only cats
4) $\mathsf{Person} \sqcap \neg \mathsf{Happy} \sqcap \exists \mathsf{owns.}(\mathsf{Cat} \sqcap \mathsf{Old})$	Unhappy pet owner who owns an old cat
5) $\mathsf{Person} \sqcap \exists \mathsf{owns.Pet} \sqcap \forall \mathsf{owns.}(\mathsf{Cat} \sqcup \mathsf{Fish})$	Pet owners who only owns cats or fish

3. Modeling in a DL

We obtain more expressive description languages if we add further constructors to $\mathcal{AL}$. $\mathcal{SROIQ}$ is the DL currently underlying the KR language for ontology. The constructors for which each letter stands can be found on the lecture slide. Table 1 give the syntax and semantics of the constructors and axioms in the DL. Now formalize the following statements using the DL. Please state your assumptions on the atomic concepts and roles.

1) Women are persons.	$Women \sqsubseteq Person$
2) Men are persons.	$Man \sqsubseteq Person$
3) Johan is a man.	$Johan \sqsubseteq Man$
4) Bill is a man.	$Bill \sqsubseteq Man$
5) James is a man.	$James \sqsubseteq Man$
6) Johan is not Bill.	$Johan \equiv \neg Bill$
7) Johan is James.	$Johan \equiv James$
8) A parent is a person and has a child.	$Parent \equiv Person \sqcap \exists hasChild$
9) Bill is a child of Johan.	$Bill \sqsubseteq \exists childOf.(Johan)$
10) A sibling is either a brother or a sister.	$Sibling \equiv Brother \sqcup Sister$
11) Alice is her own sibling.	$Alice \sqsubseteq \exists Siblingof.(Alice)$
12) A person is their own relative.	$Person \sqsubseteq \exists hasRelative.Self$

Here I will assume that:
Woman, Man, Person, Parent, Sibling, Brother, Sister, Grandfather, Child, ChildlessPerson, UnmarriedPerson, HappyPerson, Great-Grandfather, GrandParent - Are atomic concepts.

I also assume that:
hasChild, childOf, siblingOf, relativeOf, hasSpouse, hasChildren - Are atomic roles

13) Every grandfather is both a man and has a child who is a parent.
14) A childless person is a person but is not a parent.
15) An unmarried person is a person who does not have a spouse.
16) A happy person is one whose children are all happy persons.
17) A great-grandfather is a father who has more than four children, and all these children are parents.
18) A child is someone who is the inverse of a parent.
19) The relationship of being a father is a sub-role of being a parent.
20) A grandparent is someone's parent's parent.
21) A child cannot be both a son and a daughter at the same time.
22) A parent has a child if and only if they have a son or a daughter.

4. Create a Knowledge Base for Univeristy using $\mathcal{SROIQ}$

Create a DL knowledge base (KB) that describes the education provided by a university in a general way. Try to utilize the constructors and axioms as many as possible from $\mathcal{SROIQ}$. The KB should contain primitive and defined concepts and roles in the TBox, as well as some individuals in the ABox. Please use appropriate symbolic names for concepts, roles, and individuals.

3.

13. $\text{Grandfather} \equiv \text{Man} \sqcap \exists \text{hasChild}.\text{Parent}$

14. $\text{ChildlessPerson} \equiv \text{Person} \sqcap \neg\exists \text{hasChild}.\top$

15. $\text{UnmarriedPerson} \equiv \text{Person} \sqcap \neg\exists \text{hasSpouse}.\top$

16. $\text{HappyPerson} \equiv \text{Person} \sqcap \forall \text{hasChildren}.(\text{Person} \sqcap \text{Happy})$

17. $\text{GreatGrandFather} \equiv \text{Father} \sqcap > 4.\text{hasChild} \sqcap \forall \text{hasChild}.\text{Parent}$

18. $\text{hasChild} \equiv \text{hasParent}^{-}$

19. $\text{Father} \sqsubseteq \text{Parent}$

20. $\text{hasGrandparent} \equiv \text{hasParent} \circ \text{hasParent}$

21. Disjoint(:hasSon, :hasDaughter)

22. $\text{Parent} \equiv \exists \text{hasChild}.\text{Child} \leftrightarrow \exists \text{hasChild}.(\text{Man} \sqcup \text{Woman})$

4.

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$$\text{University} \xrightarrow{1..\infty} \text{Program} \xrightarrow{1..\infty} \text{courses} \xrightarrow{1..\infty} \text{Examinator}$$

courses → Teacher

courses → Assistants

courses → Students

courses → Learning outcomes

TBOX

$$\text{JönköpingUniversity} \sqsubseteq \text{University}$$

$$\text{CivilengineeringInCS} \sqsubseteq \text{Program}$$

$$\text{KnowledgeRepresentationAndSW} \sqsubseteq \text{Course}$$

$$\text{hasCourse} \sqsubseteq (\text{Program} \times \text{Course})$$

$$\text{Program} \equiv \geq 1.\exists.\text{hasCourse}$$

$$\text{Course} \equiv \geq 1.\exists.\text{hasTeachers} \sqcap \geq 1.\exists.\text{hasStudents}$$

$$\text{Teacher} \sqsubseteq \exists.\text{teaches}.\text{Course} \sqcap \exists.\text{worksAt}.\text{University}$$

$$\text{teaches} \sqsubseteq (\text{Teacher} \times \text{Course})$$

$$\text{enrolledIn} \sqsubseteq (\text{Student} \times (\text{Course} \sqcup \text{Program}))$$

ABOX

Student(Martin)

Teacher(He Tan)

enrolledIn (Martin, Knowledge Repr.)

teaches (He Tan, Knowledge Repr.)

attends (Martin, Jönköping University)

enrolledIn (Martin, CivilengineeringInCS)

hasCourse (CivilengineeringInCS, KnowledgeRepr.)

	Syntex	**Semantics**
$\mathcal{SROIQ}$ **constructors**		
universal concept	$\top$	Δ^I
bottom concept	$\bot$	$\bot$
atomic concept	A	A^I
intersection	$C \sqcap D$	$C^I \cap D^I$
union	$C \sqcup D$	$C^I \cup D^I$
complement	$\neg A$	$\Delta^I \backslash A^I$
universal restriction	$\forall R.C$	$\{a \in \Delta^I \| \forall b.(a,b) \in R^I \rightarrow b \in C^I\}$
existential restriction	$\exists R.C$	$\{a \in \Delta^I \| \exists b.(a,b) \in R^I \rightarrow b \in C^I\}$
at-least restriction	$\geq n\ \exists R.C$	$\{a \in \Delta^I \| \#\{b \| (a,b) \in R^I \rightarrow b \in C^I\} \geq n\}$
at-most restriction	$\leq n\ \exists R.C$	$\{a \in \Delta^I \| \#\{b \| (a,b) \in R^I \rightarrow b \in C^I\} \leq n\}$
local reflexivity	$\exists R.Self$	$\{a \| (a,a) \in R^I\}$
nominal	$\{a\}$	$\{a^I\}$
atomic role	R	R^I
inverse role	R	R^-
universal role	U	$\Delta^I \times \Delta^I$
individual name	a	a^I
$\mathcal{SROIQ}$ **axioms**		
concept assertion	$C(a)$	$a^I \in C^I$
role assertion	$R(a,b)$	$(a^I, b^I) \in R^I$
individual equality	$a \approx b$	$a^I = b^I$
individual inequality	$a \neq b$	$a^I \neq b^I$
concept inclusion	$C \sqsubseteq D$	$C^I \subseteq D^I$
concept eqivalence	$C \equiv D$	$C^I = D^I$
role inclusion	$R \sqsubseteq S$	$R^I \subseteq S^I$
role equivalence	$R \equiv S$	$R^I = S^I$
complex role inclusion	$R_1 \circ R_2 \sqsubseteq S$	$R_1 \circ R_2 \subseteq S$
role disjointness	$Disjoint(R,S)$	$R_1 \cap R_2 = \emptyset$

Table 1: $\mathcal{SROIQ}$ constructors and axioms