

# Generalized Space Shift Keying Modulation for MIMO Channels

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**Abstract**—A fundamental component of spatial modulation (SM), termed generalized space shift keying (GSSK), is presented. GSSK modulation inherently exploits fading in wireless communication to provide better performance over conventional amplitude/phase modulation (APM) techniques. In GSSK, *only* the antenna indices, and not the symbols themselves (as in the case of SM and APM), relay information. We exploit GSSK's degrees of freedom to achieve better performance, which is done by formulating its constellation in an optimal manner. To support our results, we also derive upper bounds on GSSK's bit error probability, where the source of GSSK's strength is made clear. Analytical and simulation results show performance gains (1.5–3 dB) over popular multiple antenna APM systems (including Bell Laboratories layered space time (BLAST) and maximum ratio combining (MRC) schemes), making GSSK an excellent candidate for future wireless applications.

## I. INTRODUCTION

Using multiple antennas in wireless communications allows unprecedented improvements over single antenna systems. One example is the vertical Bell Laboratories layered space-time (V-BLAST) architecture [1], where multiple symbols are multiplexed in space, and transmitted at the same time over all antennas. Due to inter-channel interference (ICI), caused by coupling multiple symbols in time and space, V-BLAST maximum likelihood (ML) detection increases exponentially in complexity with the number of transmit antennas. Hence, practical integration of V-BLAST requires sub-optimal, low complexity receivers [2]. For adequate performance, these receivers require the number of receive antennas to be larger or equal to the number of transmit antennas, which is not practical for downlink transmission to small mobile devices. Consequently, avoiding ICI greatly reduces receiver complexity, and results in performance gains. Also, the V-BLAST algorithms assume that all symbols are transmitted at the same time. Hence, inter-antenna synchronization (IAS) is necessary to avoid performance degradation, which consequently increases transmitter overhead.

*Prior Work:* In [3]–[6], the so-called spatial modulation (SM) seems to be an effective means to remove ICI, and the need for precise time synchronization amongst antennas. SM is a pragmatic approach for transmitting information, where the modulator uses well known APM techniques, such as PSK and quadrature amplitude modulation (QAM), but also employs the antenna index to convey information. Only one antenna

remains active during transmission so that ICI is avoided, and IAS is no longer needed.

Although SM is shown to reduce the receiver's complexity as compared to V-BLAST [3], this is under a sub-optimal SM detection rule that is only valid under some constrained assumptions about the channel. Due to the sub-optimality of the detection, SM does not exhibit the best performance in [3]. As well, the constrained assumptions about the channel questions the validity of the performance comparison with V-BLAST in [6]. The optimal detector for SM is, however, derived in [7] under conventional channel assumptions, and SM is shown to outperform many schemes including V-BLAST (at the expense of increased receiver complexity). Also, the trade-off between the number of transmit antennas versus the APM constellation size is chosen heuristically in [6]. All of this motivates our presentation of a simpler modulation technique, namely GSSK, which can be used to build a stronger SM foundation.

*Contribution:* We analyze GSSK as a fundamental component of SM, in which the spatial domain is exploited to modulate information. The presentation of SM in [6] does not fully explore the idea of using antenna indices as the *only* means to relay information, as is the case for our GSSK scheme. The transmitted *symbols* in GSSK are just a means of identifying the activated antenna. In doing so, we achieve all of the aforementioned advantages comprising SM, while reducing transceiver overhead. GSSK's constellation is thoroughly analyzed, where we present the underlying idea that allows GSSK to outperform APM schemes (such as V-BLAST and MRC). In particular, we show that GSSK takes advantage of the fading process by increasing the constellation's dimension, whose points result to be well spread apart. This analysis opens the door to understanding how SM parameters may be chosen to obtain better performance gains. We also consider the performance of the GSSK scheme, where upper bounds on the bit error rate (BER) are derived. Through our analysis, we design optimal constellations, where it is apparent that tremendous degrees of freedom is available for practical implementation. Simulation results are also presented to support our findings, and illustrate the future research potential of GSSK modulation.

*Organization:* Section II introduces the basic GSSK system model, including a detailed analysis of GSSK's constellation

space. We present analytical results on the bit error probability in Section III, followed by Section IV's development of constellation design rules. Section V then provides simulation results on performance, and we conclude the paper in Section VI.

*Notation:* Italicized symbols denote scalar values while bold lower/upper case symbols denote vectors/matrices. We use  $(\cdot)^T$  for transpose,  $(\cdot)^H$  for conjugate transpose,  $\binom{\cdot}{\cdot}$  for the binomial coefficient, and  $\|\cdot\|_F$  for the Frobenius norm of a vector/matrix. We use  $\mathcal{CN}(\mathbf{m}, \sigma^2)$  for the complex Gaussian distribution of a random variable, having independent Gaussian distributed real and imaginary parts denoted by  $\mathcal{N}(\mathbf{m}, \frac{\sigma^2}{2})$ , with mean  $\mathbf{m}$  and variance  $\frac{\sigma^2}{2}$ . We use  $P(\cdot)$  for the probability of an event,  $p_Y(\cdot)$  for the probability density function (PDF) of a random variable  $\mathbf{Y}$ , and  $E_{\mathbf{x}}[\cdot]$  for the statistical expectation with respect to  $\mathbf{x}$ . We use  $\text{Re}\{\cdot\}$  for the real part of a complex variable, and  $\mathcal{X}$  to represent a constellation of size  $M$ .

## II. GSSK MODULATION

The general system model consists of a MIMO wireless link with  $N_t$  transmit and  $N_r$  receive antennas, and is shown in Fig. 1. A random sequence of independent bits  $\mathbf{b} = [b_1 \ b_2 \ \dots \ b_k]$  enters a GSSK mapper, where groups of  $m$  bits are mapped to a constellation vector  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_{N_t}]^T$ , with a power constraint of unity (i.e.  $E_{\mathbf{x}}[\mathbf{x}^H \mathbf{x}] = 1$ ). In GSSK, only  $n_t$  antennas remain active during transmission, and hence, only  $n_t$  of the  $x_j$ 's in  $\mathbf{x}$  are nonzero. The signal is transmitted over an  $N_r \times N_t$  wireless channel  $\mathbf{H}$ , and experiences an  $N_r - \dim$  additive white Gaussian (AWGN) noise  $\boldsymbol{\eta} = [\eta_1 \ \eta_2 \ \dots \ \eta_{N_r}]^T$ . The received signal is given by  $\mathbf{y} = \sqrt{\rho} \mathbf{H} \mathbf{x} + \boldsymbol{\eta}$ , where  $\rho$  is the average signal to noise ratio (SNR) at each receive antenna, and  $\mathbf{H}$  and  $\boldsymbol{\eta}$  have independent and identically distributed (iid) entries according to  $\mathcal{CN}(0, 1)$ .

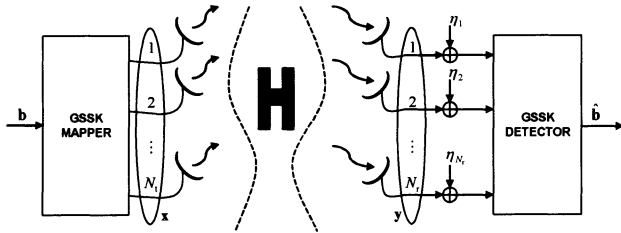


Fig. 1. GSSK system model.

At the receiver side, the GSSK detector estimates the antenna indices that are used during transmission, and demaps the symbol to its component bits  $\hat{\mathbf{b}}$ .

### A. Transmission

The underlying concept in GSSK is using only antenna indices to relay information. In general, combinations of antenna indices can be used. Therefore, for GSSK using  $n_t$  antennas, there are  $M' = \binom{N_t}{n_t}$  possible constellation points.

TABLE I  
EXAMPLE OF THE GSSK MAPPER RULE.

$\mathbf{b} = [b_1 \ b_2 \ b_3]$	$\mathbf{j}$	$\mathbf{x} = [x_1 \ x_2 \ \dots \ x_5]^T$
$[0 \ 0 \ 0]$	(1, 2)	$[\frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}} \ 0 \ 0 \ 0]^T$
$[0 \ 0 \ 1]$	(1, 3)	$[\frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0 \ 0]^T$
$[0 \ 1 \ 0]$	(1, 4)	$[\frac{1}{\sqrt{2}} \ 0 \ 0 \ \frac{1}{\sqrt{2}} \ 0]^T$
$[0 \ 1 \ 1]$	(1, 5)	$[\frac{1}{\sqrt{2}} \ 0 \ 0 \ 0 \ \frac{1}{\sqrt{2}}]^T$
$[1 \ 0 \ 0]$	(2, 3)	$[0 \ \frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}} \ 0 \ 0]^T$
$[1 \ 0 \ 1]$	(2, 4)	$[0 \ \frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0]^T$
$[1 \ 1 \ 0]$	(2, 5)	$[0 \ \frac{1}{\sqrt{2}} \ 0 \ 0 \ \frac{1}{\sqrt{2}}]^T$
$[1 \ 1 \ 1]$	(3, 4)	$[0 \ 0 \ \frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}} \ 0]^T$

For example, with  $n_t = 2$  and  $N_t = 7$ , there are  $M' = 21$  possible combinations. Since we require a constellation size  $M$  in multiples of 2, we only use 16 of the possible 21 combinations. The set of antenna combinations,  $\mathcal{X}$ , may be chosen at random, but we will see in Section IV that more optimal selection rules exist.

Once  $\mathcal{X}$  is formulated, the GSSK's mapper rule is straightforward. Groups of  $m = \log_2(M)$  bits are collected and mapped to a vector  $\mathbf{x}_j$ , where  $\mathbf{j} \in \mathcal{X}$  specifies the antenna combination for the given  $m$  bit pattern. The symbols in  $\mathbf{x}_j$  do not contain information, but can be designed to optimize transmission.<sup>1</sup> The vector  $\mathbf{x}_j$  specifies the activated antennas, during which all other antennas remain idle, and has the following form:

$$\mathbf{x}_j \triangleq \underbrace{\left[ \frac{1}{\sqrt{n_t}} \ 0 \ \dots \ 0 \ \frac{1}{\sqrt{n_t}} \ \dots \ \frac{1}{\sqrt{n_t}} \ 0 \right]^T}_{n_t \text{ of } N_t \text{ non-zero values}}.$$

An example of 8-ary GSSK modulation is given in Table I, where we use  $N_t = 5$ ,  $n_t = 2$ , and  $\mathcal{X}$  is chosen randomly. The output of the channel is therefore given by

$$\mathbf{y} = \sqrt{\rho'} \mathbf{h}_{\mathbf{j}, \text{eff}} + \boldsymbol{\eta}, \quad (1)$$

where  $\rho' = \frac{\rho}{n_t}$ , and  $\mathbf{h}_{\mathbf{j}, \text{eff}} = \mathbf{h}_{\mathbf{j}(1)} + \mathbf{h}_{\mathbf{j}(2)} + \dots + \mathbf{h}_{\mathbf{j}(n_t)}$  ( $\mathbf{j}(\cdot) = j \in \{1, 2, \dots, N_t\}$  specifies the column index of  $\mathbf{H}$ ). We refer to  $\mathbf{h}_{\mathbf{j}, \text{eff}}$  as an effective column, which represents the sum of  $n_t$  distinct columns in  $\mathbf{H}$ .

*Remark 1:* Only  $n_t$  columns of  $\mathbf{H}$  are *activated*, and these columns change depending on the transmitted *information*.

Now that we have seen the mapping rule, let us describe how the receiver estimates the transmit antenna indices  $\mathbf{j}$ .

### B. Detection

The detector's main function is obtaining the antenna indices used at the transmitter. Since the channel inputs are

<sup>1</sup>For our purposes, we consider real values for  $x_j$ , and consider more general (adaptive) symbol design in ongoing work.

assumed equally likely, the optimal detector is ML, which is given by

$$\begin{aligned} \mathbf{k} &= \arg \max_{\mathbf{j}} p_{\mathbf{Y}}(\mathbf{y} | \mathbf{x}_{\mathbf{j}}, \mathbf{H}) \\ &= \arg \min_{\mathbf{j}} \left\| \mathbf{y} - \sqrt{\rho'} \mathbf{h}_{\mathbf{j}, \text{eff}} \right\|_{\text{F}}^2 \\ &= \arg \max_{\mathbf{j}} \text{Re} \left\{ \left( \mathbf{y} - \frac{\sqrt{\rho'}}{2} \mathbf{h}_{\mathbf{j}, \text{eff}} \right)^H \mathbf{h}_{\mathbf{j}, \text{eff}} \right\}, \quad (2) \end{aligned}$$

where  $\mathbf{k} \in \mathcal{K}$  represents the estimated antenna indices, and  $p_{\mathbf{Y}}(\mathbf{y} | \mathbf{x}_{\mathbf{j}}, \mathbf{H})$  is given by

$$p_{\mathbf{Y}}(\mathbf{y} | \mathbf{x}_{\mathbf{j}}, \mathbf{H}) = \frac{1}{\pi^{N_r}} \exp \left( - \left\| \mathbf{y} - \sqrt{\rho'} \mathbf{H} \mathbf{x}_{\mathbf{j}} \right\|_{\text{F}}^2 \right). \quad (3)$$

*Remark 2:* The detection rule is a maximization problem over *all* effective columns of  $\mathbf{H}$  (there are  $M$  of them). Essentially, these effective columns act as *random* constellation points for GSSK modulation.

### C. Constellation

We now look into GSSK's constellation in more detail and highlight some of its strength. Consider a fixed channel realization  $\mathbf{H}$ , and the effective  $N_r$ -dim constellation symbol  $\mathbf{x}_{\mathbf{j}, \text{eff}} = \mathbf{H} \mathbf{x}_{\mathbf{j}}$ , shown in Fig. 2 for APM and GSSK. In APM, the effective constellation  $\mathcal{X}^{\text{eff}}$  is composed of scaled versions of the vector  $\mathbf{h} \mathbf{x}_{\mathbf{j}}$ . But in GSSK,  $\mathcal{X}^{\text{eff}}$  is made up of scaled versions of *all* effective columns of  $\mathbf{H}$ . Decisions for APM are performed in the 1-dim complex space, independent of which antenna is used (since after matched filtering, the sufficient statistics are scalar). On the other hand, GSSK decisions are made in the  $N_r$ -dim space. We therefore expect GSSK to outperform APM schemes for increasing  $M$  and  $N_r$ . Also, better performance is achieved in GSSK for channel realizations having effective columns that are widely spread apart in the  $N_r$ -dim space, which depend on the stochastic properties of the channel and can be capitalized upon for adaptive forms of GSSK. As well,  $\mathcal{X}$  can be formulated to optimize the distance spectrum of GSSK's  $\mathcal{X}^{\text{eff}}$ , by exploiting GSSK's degrees of freedom, namely  $n_t$  and  $N_r$ .

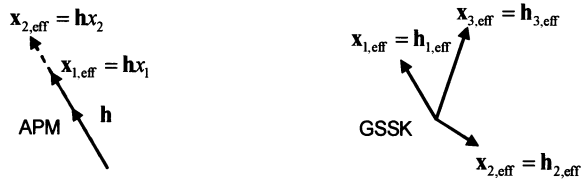


Fig. 2. Illustration of the effective constellation space  $\mathcal{X}^{\text{eff}}$ .

*Remark 3:* It is using the column indices of  $\mathbf{H}$  as the source of information that results in the improved constellation space, and *not* the fact that different columns are being used for transmission. For example, if APM with transmission on alternating antenna indices is considered,  $\mathcal{X}^{\text{eff}}$  would not change since at any given time, the receiver explores only all possible transmit *symbols*. The actual *antenna indices* are

assumed to be known at the receiver, and remains the same regardless of the transmitted information. If, on the other hand, both the antenna indices and the symbols conveyed information, the modulation scheme would not be APM, but rather a form of SM. In this case,  $\mathcal{X}^{\text{eff}}$  is similar to GSSK, but with the possibility of having more than one scaled version of  $\mathbf{x}_{\mathbf{j}, \text{eff}}$  along the same direction.

## III. PERFORMANCE ANALYSIS

### A. Error Probability

GSSK's performance is derived using the well known union bounding technique [8, p. 261-262]. The average BER for GSSK is union bounded as

$$\begin{aligned} P_{\text{e,bit}} &\leq E_{\mathbf{x}_{\mathbf{j}}} \left[ \sum_{\mathbf{k}} N(\mathbf{j}, \mathbf{k}) P(\mathbf{x}_{\mathbf{j}} \rightarrow \mathbf{x}_{\mathbf{k}}) \right] \\ &= \sum_{\mathbf{j}} \sum_{\mathbf{k}} \frac{N(\mathbf{j}, \mathbf{k})}{M} P(\mathbf{x}_{\mathbf{j}} \rightarrow \mathbf{x}_{\mathbf{k}}), \quad (4) \end{aligned}$$

where  $N(\mathbf{j}, \mathbf{k})$  is the number of bits in error between the constellation vector  $\mathbf{x}_{\mathbf{j}}$  and  $\mathbf{x}_{\mathbf{k}}$ , and  $P(\mathbf{x}_{\mathbf{j}} \rightarrow \mathbf{x}_{\mathbf{k}})$  denotes the pairwise error probability (PEP) of deciding on  $\mathbf{x}_{\mathbf{k}}$  given that  $\mathbf{x}_{\mathbf{j}}$  is transmitted. By using (2), the PEP conditioned on  $\mathbf{H}$  is given by

$$P(\mathbf{x}_{\mathbf{j}} \rightarrow \mathbf{x}_{\mathbf{k}} | \mathbf{H}) = P(d_{\mathbf{k}} > d_{\mathbf{j}} | \mathbf{H}) = Q(\sqrt{\kappa}),$$

where  $d_{\mathbf{j}} = \text{Re} \left\{ \left( \mathbf{y} - \frac{\sqrt{\rho'}}{2} \mathbf{h}_{\mathbf{j}, \text{eff}} \right)^H \mathbf{h}_{\mathbf{j}, \text{eff}} \right\}$  and  $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$ . We define  $\kappa$  as

$$\kappa \triangleq \frac{\rho'}{2} \left\| \mathbf{h}_{\mathbf{j}, \text{eff}} - \mathbf{h}_{\mathbf{k}, \text{eff}} \right\|_{\text{F}}^2 = \sum_{n=1}^{2N_r} \alpha_n^2, \quad (5)$$

where  $\alpha_n \sim \mathcal{N}(0, \sigma_\alpha^2)$ ,  $\sigma_\alpha^2 = \frac{\rho d(\mathbf{j}, \mathbf{k})}{4n_t}$ , and  $d(\mathbf{j}, \mathbf{k})$  is the number of distinct columns of  $\mathbf{H}$  between  $\mathbf{h}_{\mathbf{j}, \text{eff}}$  and  $\mathbf{h}_{\mathbf{k}, \text{eff}}$ .

*Remark 4:* The metric affecting the system performance is the distance between the effective columns of  $\mathbf{H}$ . We hinted at this observation earlier in Section II-C, when analyzing GSSK's constellation. Also, we can choose  $\mathbf{h}_{\mathbf{j}, \text{eff}}$ 's (i.e.  $\mathcal{X}$ ) such that large  $d(\mathbf{j}, \mathbf{k})$ 's are obtained, which translates into achieving better performance.

The random variable  $\kappa$  in (5) is chi-squared distributed with  $s = 2N_r$  degrees of freedom, and PDF  $p_\kappa(v)$  given by [8, p. 41]. The PEP can then be formulated as

$$\begin{aligned} P(\mathbf{x}_{\mathbf{j}} \rightarrow \mathbf{x}_{\mathbf{k}}) &= E_\kappa [P(\mathbf{x}_{\mathbf{j}} \rightarrow \mathbf{x}_{\mathbf{k}} | \mathbf{H})] \\ &= \int_{v=0}^{\infty} Q(\sqrt{v}) p_\kappa(v) dv, \quad (6) \end{aligned}$$

which has a closed form expression given in [9, Eq. (64)]. Thus,

$$P(\mathbf{x}_{\mathbf{j}} \rightarrow \mathbf{x}_{\mathbf{k}}) = \gamma_{\alpha}^{N_r} \sum_{k=0}^{N_r-1} \binom{N_r-1+k}{k} [1-\gamma_{\alpha}]^k, \quad (7)$$

where  $\gamma_\alpha = \frac{1}{2} \left(1 - \sqrt{\frac{\sigma_\alpha^2}{1+\sigma_\alpha^2}}\right)$ , and is a function of  $\mathbf{j}$  and  $\mathbf{k}$ . Plugging (7) into (4), we obtain

$$P_{e,\text{bit}} \leq \frac{1}{M} \sum_{\mathbf{j}} \sum_{\mathbf{k}} N(\mathbf{j}, \mathbf{k}) \gamma_\alpha^{N_r} \sum_{k=0}^{N_r-1} \binom{N_r-1+k}{k} [1-\gamma_\alpha]^k. \quad (8)$$

### B. Diversity

In order to clearly show the system diversity, we re-derive the error probability with a loose upper bound. Specifically, we use  $Q(x) \leq \frac{1}{2} \exp\left(-\frac{x^2}{2}\right)$  [8, p. 54], and upper bound (6) by

$$\begin{aligned} P(\mathbf{x}_j \rightarrow \mathbf{x}_j) &\leq \int_0^\infty \frac{\exp\left(-\frac{v}{2}\right) v^{N_r-1} \exp\left(-\frac{v}{2\sigma_\alpha^2}\right)}{2^{N_r+1} \sigma_\alpha^{N_r} \Gamma(N_r)} dv \\ &= \frac{1}{2} (\sigma_\alpha^2 + 1)^{-N_r} \\ &\leq 2^{2N_r+1} \left(\frac{d(\mathbf{j}, \mathbf{k})}{n_t}\right)^{-N_r} \rho^{-N_r}. \end{aligned}$$

Therefore, the bit error probability is given by

$$P_{e,\text{bit}} \leq C \rho^{-N_r}, \quad (9)$$

where  $C = \sum_{\mathbf{j}} \sum_{\mathbf{k}} \frac{N(\mathbf{j}, \mathbf{k})}{M} 2^{2N_r+1} \left(\frac{d(\mathbf{j}, \mathbf{k})}{n_t}\right)^{-N_r}$ . We clearly see from (9) that a diversity order of  $N_r$  is achieved, which is the same as that of an MRC-APM system using  $N_r$  receive antennas.

## IV. OPTIMAL CONSTELLATION DESIGN

We consider the optimal formulation of GSSK's constellation  $\mathcal{X}$ , in terms of minimizing the bit error rate. The probability of bit errors given by (8) is minimized by the following joint optimization problem:

$$\hat{\theta}_{\text{opt}} = \arg \min_{\theta_{\text{opt}}} P_{e,\text{bit}}, \quad (10)$$

where  $\hat{\theta}_{\text{opt}} = (\hat{N}_t, \hat{n}_t, \hat{\mathcal{X}}, \hat{\mu})$ ,  $\theta_{\text{opt}} = (N_t, n_t, \mathcal{X}, \mu)$  with constraint  $n_t < N_t$  and  $M' \geq M$ ,  $\mu$  is the labeling rule for the constellation  $\mathcal{X}$  (i.e. the rule for labeling  $m$  bits to a symbol vector  $\mathbf{x}_j$ ), and  $P_{e,\text{bit}}$  is given by (8). We note that in (8),  $N(\mathbf{j}, \mathbf{k})$  is affected by  $\mu$ , whereas  $\gamma_\alpha$  is affected by  $n_t$ ,  $N_t$ , and  $\mathcal{X}$ . To simplify the design process, we present two sub-optimal constellation design rules.

In the first algorithm, we simplify the optimization problem by considering relatively high SNRs. Hence, (10) reduces to

$$\hat{\theta}_1 = \arg \min_{\theta_1} \sum_{\mathbf{j}} \sum_{\mathbf{k}} N(\mathbf{j}, \mathbf{k}) \gamma_\alpha, \quad (11)$$

where we only keep the terms contributing most to  $P_{e,\text{bit}}$ , and  $\hat{\theta}_1$  and  $\theta_1$  have the same parameters as  $\hat{\theta}_{\text{opt}}$  and  $\theta_{\text{opt}}$ , respectively. Still, the optimization is fairly complex (remember, for each  $N_t$  chosen, there are  $N_t - 1$  possibilities for  $n_t$ , from which there are  $\binom{N_t}{n_t}$  available antenna combinations to formulate  $\mathcal{X}$ ). Therefore, to simplify the optimization further,

we consider only minimizing the parameter  $\gamma_\alpha$  instead. Hence, the second constellation design rule is given by

$$\begin{aligned} \hat{\theta}_2 &= \arg \max_{\theta_2} \sum_{\mathbf{j}} \sum_{\mathbf{k}} \sigma_\alpha^2 \\ &= \arg \max_{\theta_2} \frac{1}{n_t} \sum_{\mathbf{j}} \sum_{\mathbf{k}} d(\mathbf{j}, \mathbf{k}) \end{aligned} \quad (12)$$

where  $\hat{\theta}_2 = (\hat{N}_t, \hat{n}_t, \hat{\mathcal{X}})$ ,  $\theta_2 = (N_t, n_t, \mathcal{X})$ . Therefore, for a given  $N_t$  and  $n_t$  value, we maximize the values of  $d(\mathbf{j}, \mathbf{k})$ . This maximization can be interpreted as having antenna combinations that are as different from one another as possible. Widely varying antenna combinations imply having a large  $M'$ , which can be increased by choosing a larger  $N_t$ , and thus increasing the transmitter's overhead. Note that a large  $n_t$  will also help, but will have adverse effects as well due to the  $\frac{1}{n_t}$  factor in (12). Depending on system requirements (i.e. low hardware overhead, or high performance), the range of requirements for  $N_t$  and  $n_t$  can be specified, from which the optimal combination of  $N_t$  and  $n_t$  can be obtained by (12) through computer search.

*Remark 5:* The optimal set of parameters may not be unique, since several sets may result in identical performance. Also, this trade-off between transmitter complexity and performance provides design flexibility, which can be exploited in adaptive type systems.

Figure 3 illustrates GSSK's performance bounds given by (8), with  $M = 8$  and  $M = 32$ , and for varying  $N_t$ 's. For each plot,  $n_t$  and  $\mathcal{X}$  are obtained by computer simulations using (12).

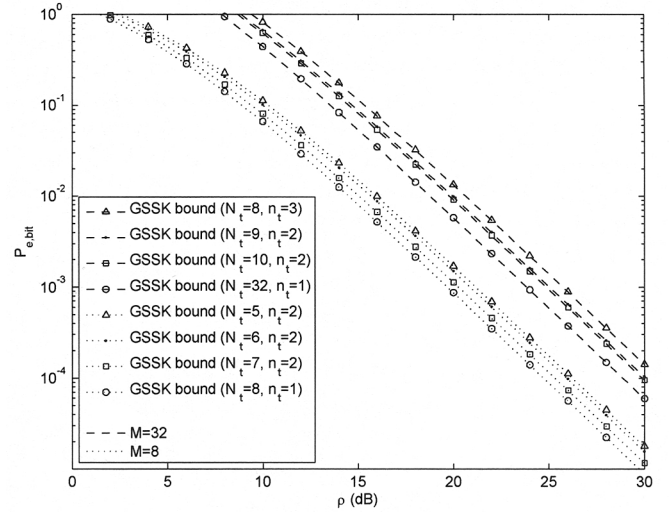


Fig. 3. GSSK bounds for varying  $M$ ,  $N_t$ , and  $n_t$  ( $N_r = 2$ ).

As expected, the performance degrades as  $N_t$  decreases, and as  $n_t$  increases. We also note that certain antenna transitions do not gain much in performance. For example, only a few tenths of a dB is gained from the transition of  $N_t = 9$  to  $N_t = 10$  ( $M = 32$ ).

## V. SIMULATION RESULTS

In this section, we compare GSSK's performance for varying  $N_t$  and  $n_t$ . Monte Carlo simulations are run for at least  $10^5$  channel realizations. We use Gray (or quasi-Gray) mapping when appropriate (i.e. for PSK and QAM modulation).

In Fig. 4, we demonstrate GSSK's performance versus V-BLAST, MRC, and SM. We target  $m = 3$  bits/s/Hz transmission, and consider  $N_r = 4$ . For reference, we use three different transmission setups. The first one is APM, 8-QAM transmission with  $N_t = 1$  (single antenna transmission), and  $M = 8$ . The second is V-BLAST with BPSK modulation,  $N_t = 3$ , and ordered successive interference cancellation (OSIC) with the minimum mean squared error (MMSE) receiver [2]. Third, SM [3] using optimal detection [7] with BPSK modulation, and  $N_t = 4$  antennas is shown.

We plot GSSK for two different constraints. The first is with  $N_t = M = 8$ , and hence,  $n_t = 1$ . The second is with  $N_t = 5$ ,  $n_t = 2$ , and  $\mathcal{X}$  obtained from (12). The bounds of (8) are also plotted for comparison. The simulation and analytical results are a close match, especially for high SNRs. The bounds are tighter for  $n_t = 2$  than  $n_t = 1$ , and this is due to the fact that summing over all possible constellation points (for the union bound) is more justified in the case of  $n_t = 2$ . When  $n_t = 1$ , the constellation points are all unique (i.e. distinct columns of  $\mathbf{H}$ ), resulting in a union bound that is more loose (in this case, the nearest neighbor approximation is better suited). On the other hand, with  $n_t = 2$ , the constellation points share common columns of  $\mathbf{H}$ , and a larger number of constellation points have an effect on performance. Therefore, the union bound for these higher  $n_t$  values result in tighter bounds.

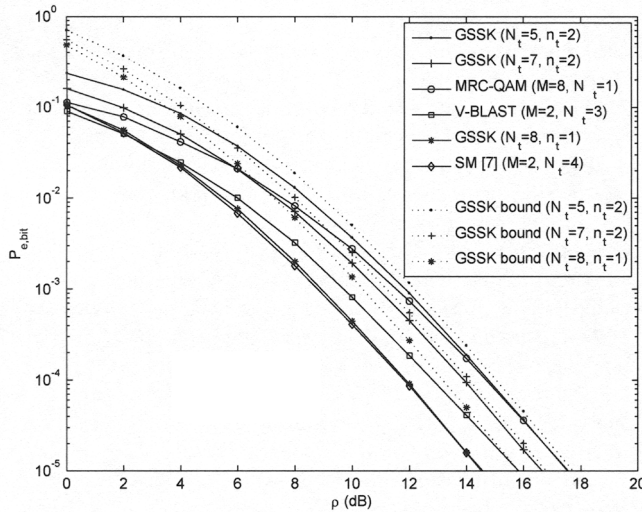


Fig. 4. BER performance of GSSK versus MRC-QAM, V-BLAST, and SM, for  $m = 3$  bits/s/Hz transmission ( $N_r = 4$ ).

GSSK's performance improvements is clearly shown in the figure, where we observe gains of 3 dB over APM, and 1 dB over V-BLAST (for  $P_{e,bit} = 10^{-5}$ ). GSSK has almost identical performance to that of SM, but with lower complexity. This

complexity reduction is attributed to the fact that symbols do not carry information (as in SM, MRC, and V-BLAST), therefore reducing the optimization problem's overhead when detecting the message. We also note that, as expected, GSSK's performance degrades as  $N_t$  is decreased, but still outperforms APM in most cases.

## VI. CONCLUSION

In this paper, we introduced a new modulation method (referred to as GSSK) for MIMO wireless links by exploiting the inherent fading process. Rather than transmitting information through symbols, the transmitter antenna indices were used as the *sole* information conveying mechanism. Throughout the paper, we laid out GSSK fundamentals as the building ground for hybrid modulation schemes (which combine GSSK and APM) such as in [6]. We presented GSSK improvements over APM (up to 3 dB), and derived closed form upper bounds on the bit error probability. All of SM's merits mentioned in [6] are *also* inherent in GSSK (at similar performance), but with lower computational overhead, and greater design flexibility. These advantages make GSSK a promising candidate for low complexity transceivers in next generation communication systems. Future research directions will involve the adaptive case, where GSSK's constellation can take advantage of channel conditions. We also intend to investigate GSSK's robustness to non-ideal channel conditions, and practical GSSK implementation issues in current MIMO communication standards.

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