Max Flow - Fulkerson Alg.: Longest Path Problem. Let E_i = earliest time to get to place (use D.P.), E_{ij} = earliest time to complete ij. Find $E_i = \max[E_i + e_{ij}, E_i + e_{ik}, E_i + e_{il}]$, stop after all nodes have been included, then choose longest path. Can also find L_i = lowest time to complete = $\min L_i <$ or $> t_{ij}$.
Make a chart: Activity $\quad ij \quad t_{ij} \quad E_i \quad E_{ij} \quad L_i \quad TF_{ij}$, where $TF_{ij} = L_i - E_{ij}$ = Flexibility. A **critical path** is a path with 0 flexibility.
Ex: Form LP to find the max # of barrels/hr sent from so (source) to si (sink). Note that flow must pass thru some/all of stations 1, 2, and 3. Let x_{ij} = barrels/hr thru arc (i,j), x_0 = barrels entering si.

$$\begin{array}{llll} \text{LP: } \max z = x_0 & & & \\ \text{s.t. } x_{so,1} \le 2 & x_{so,2} \le 3 & x_{1,2} \le 3 & x_{2,si} \le 2 \\ x_{1,3} \le 4 & x_{3,si} \le 1 & x_0 = x_{so,1} + x_{so,2} & x_{so,1} = x_{1,2} + x_{1,3} \\ x_{so,2} + x_{12} = x_{2,si} & x_{13} = x_{3,si} & x_{3,si} + x_{2,si} = x_0 & x_{ij} \ge 0 \end{array}$$

Inventory - EOQ: Purchase Cost = $C(z) = K + cz$, Holding Cost = $H(z) = hz$, h = hold cost, $Q^* = \sqrt{\frac{2ak}{h}}$ = how much, $\frac{Q^*}{a}$ = time between, k = fixed cost, c = var cost, a = rate needed.
Ex: Let Demand = 4000gallons/month, Charge = \$50 + \$0.7/gallon refill, Annual Hold Fee = \$0.3.
Order Size = $Q^* = \sqrt{\frac{2(4000)(50)}{0.025}}$ = 4000 gallons, # of orders/year= $\frac{12(4000)}{4000} = 12$, Time b/w Orders= $\frac{4000}{12(4000)} = \frac{1}{12} \rightarrow$ once a month, Lead Time of 2 weeks: $\frac{12a \cdot LT}{52} = 1846.15 \rightarrow$ reorder at 1846.15 gallons.
EOQ Delayed Case: $Q^* = \sqrt{\frac{2ak}{h} \cdot \frac{p+h}{p}} \rightarrow$ order, $S^* = \sqrt{\frac{2ak}{h} \cdot \frac{p}{p+h}} \rightarrow$ max inventory, $Q^* - S^*$ = max shortage, Penalty $= pz$.
Dynamic Program: Start at the end. State: stages, states, recursion. $V_i(x) = \max/\min x$ s.t. $x_i \le x(r_i(x_i) + v_i + (x - x_i))$.
1) Start at the end w/ all possibilities, 2) Move to previous step and test all possibilities which include step before vals, in the 1st step do once, w/ all utility. *Can use DP for shortest/longest path also (easier).
NLPs: 1) $\overline{\nabla}(f(x) + \sum_{i=1} \lambda_i \overline{\nabla} g_i(x)) = 0$, 2) Feasible $g_i(x) \le 0$, $i = 1, \ldots, m$, 3) $\lambda \ge 0$, 4) $\lambda_i \cdot g_i(x) = 0 = i, \ldots, m$.
Ex: