

1. Prove using the definition that $\lim_{n \rightarrow \infty} \frac{n-3}{5n-1} = \frac{1}{5}$. Write your work clearly, scan it in ONE pdf file and submit it here.

Proof. Let $\epsilon > 0$. Then we must show that there exists a δ such that $0 < \delta < n$ implies $\left| \frac{n-3}{5n-1} - \frac{1}{5} \right| < \epsilon$. Rewriting the expression in absolute values gives us the following:

$$\left| \frac{n-3}{5n-1} - \frac{1}{5} \right| = \left| \frac{5n-15-5n+1}{25n-5} \right| = \left| \frac{-14}{5(5n-1)} \right| = \frac{14}{5|5n-1|} < \frac{14}{5n-1} < \epsilon.$$

If this is the case, then $\left| \frac{n-3}{5n-1} - \frac{1}{5} \right| < \epsilon$ when $\frac{14}{5n-1} < \epsilon$, or whenever $n > \frac{14}{5\epsilon} + 1$. Thus, we may choose $\delta = \frac{14}{5\epsilon} + 1$.

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