STAT 461.1002: Homework 1

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1. Three dice are tossed, one red, one blue, and one green. What outcomes make up the event A that the sum of the three faces showing equals 5?

   Answer: $A = \{1r + 1b + 3g,\ 1r + 2b + 2g,\ 1r + 3b + 1g,\ 2r + 1b + 2g,\ 2r + 2b + 1g,\ 3r + 1b + 1g\}$

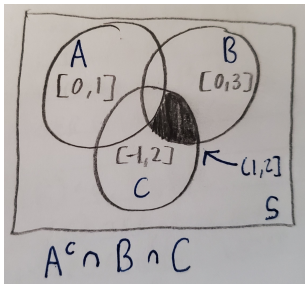
2. Two darts are thrown at the following target:

   (a) Let (u, v) denote the outcome that the first dart lands in region u and the second dart, in region v. List the sample space of (u, v) pairs.
   Answer: $S = \{(1,1),\ (1,2),\ (1,4),\ (2,1),\ (2,2),\ (2,4),\ (4,1),\ (4,2),\ (4,4)\}$

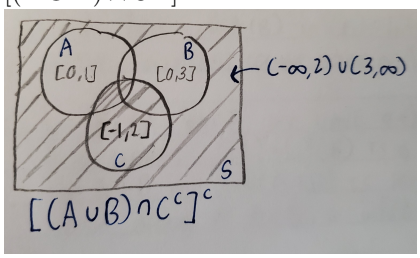
   (b) List the outcomes in the sample space of sums, $u + v$.
   Answer: $S = \{2, 3, 4, 5, 6, 8\}$

3. Define $A = \{x : 0 \leq x \leq 1\}$, $B = \{x : 0 \leq x \leq 3\}$, and $C = \{x : -1 \leq x \leq 2\}$. Draw diagrams showing each of the following sets of points:

   (a) $A^C \cap B \cap C$

   (b) $[(A \cup B) \cap C^C]^C$

4. Suppose that three events A, B, and C are defined on a sample space S. Use the union, intersection, and complement operations to represent each of the following events:

   (a) exactly one event occurs
   Answer: $(A \cup B \cup C) \backslash (A \cap B \cap C)$

   (b) exactly two events occur
   Answer: $[(A \cap B) \backslash C] \cup [(A \cap C) \backslash B] \cup [(B \cap C) \backslash A]$

5. Let A and B be any two events defined on S. Suppose that $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.1$. What is the probability that A or B but not both occur?

   Answer: The probability of A or B but not both is the same as saying $P(A \cup B) - P(A \cap B)) = 0.9 - 0.1 = 0.8$.

6. Suppose that three fair dice are tossed. Let A_i be the event that a 6 shows on the i^{th} die, $i = 1, 2, 3$. Does $P(A_1 \cup A_2 \cup A_3) = \frac{1}{2}$? Explain.

   Answer: Yes, $P(A_1 \cup A_2 \cup A_3) = \frac{1}{2}$. This is because events A_1, A_2, and A_3 are mutually exclusive events, so

$$P(A_1 \cup A_2 \cup A_3) = P(A) + P(B) + P(C) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}.$$

7. An urn contains twenty-four chips, numbered 1 through 24. One is drawn at random. Let A be the event that the number is divisible by 2 and let B be the event that the number is divisible by 3. Find $P(A \cup B)$.

   Answer: $P(A) = \frac{1}{2}$ (obviously) and $P(B) = \frac{1}{3}$ (obviously), so $P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$.

8. Three events A, B, and C are defined on a sample space, S. Given that $P(A) = 0.2$, $P(B) = 0.1$, and $P(C) = 0.3$, what is the smallest possible value for $P[(A \cup B \cup C)^C]$?

   Answer: Smallest possible value of $P[(A \cup B \cup C)^C = 0$.

9. If $P(A) = \frac{1}{2}$ and $P(B^C) = \frac{1}{3}$, can A and B be disjoint? Explain.

   Answer: No, they cannot be disjoint. If we suppose A and B are disjoint, then $P(B)^C$ would also include $P(A)$ in its entirety, so $P(B)^C \geq P(A)$. However, this leads to a contradiction as $P(B)^C = \frac{1}{3}$ and $P(A) = \frac{1}{2}$, so $P(B)^C < P(A)$.

10. **(461 only)** Express the following probability in terms of $P(A)$, $P(B)$, and $P(A \cap B)$: $P(A^C \cap (A \cup B))$.

    Answer: $P(A^C \cap (A \cup B)) = (1 - P(A)) \cap (P(A) + P(B) - P(A \cap B))$.