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1.

(a) The number of spatial increments by which a receptive field is moved.

(b) kernel size, termination of neighborhood to perform convolution

(c) The same weights and a single bias are used to generate the convolution values corresponding to all locations in the input image.

(d) First layer feature maps : 60×60

First layer pooled feature maps : 30×30

Second layer feature maps : 24×24

Second layer pooled feature maps : 12×12

(e) First layer : $(7 \times 7) \times 6 + 6 = 300$

Second layer : $6 \times (7 \times 7) \times 12 + 12 = 3540$

(f)
$$\delta_{x,y}(\ell) = \frac{\partial E}{\partial z_{x,y}(\ell)} = \sum_u \sum_v \frac{\partial E}{\partial z_{u,v}(\ell+1)} \cdot \frac{\partial z_{u,v}(\ell+1)}{\partial z_{x,y}(\ell)}$$

$$= \sum_u \sum_v \delta_{u,v}(\ell+1) \cdot \frac{\partial z_{u,v}(\ell+1)}{\partial z_{x,y}(\ell)}$$

$$= \sum_u \sum_v \delta_{u,v}(\ell+1) \cdot \frac{\partial}{\partial z_{x,y}(\ell)} \left[\sum_l \sum_k w_{l,k}(\ell+1) \cdot h(z_{u-l,v-k}(\ell)) + b(\ell+1) \right]$$

$$= \sum_u \sum_v \delta_{u,v}(\ell+1) \left[\sum_{u-x} \sum_{v-y} w_{u-x,v-y}(\ell+1) h'(z_{x,y}(\ell)) \right]$$

$$= \sum_u \sum_v \delta_{u,v}(\ell+1) w_{u-x,v-y}(\ell+1) h'(z_{x,y}(\ell))$$

$$= h'(z_{x,y}(\ell)) \sum_u \sum_v \delta_{u,v}(\ell+1) w_{u-x,v-y}(\ell+1)$$

$$= h'(z_{x,y}(\ell)) \left[\delta_{x,y}(\ell+1) \star w_{-x,-y}(\ell+1) \right]$$

2.

(a) $F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx = \int_{-W/2}^{W/2} A e^{-j2\pi ut} dt$

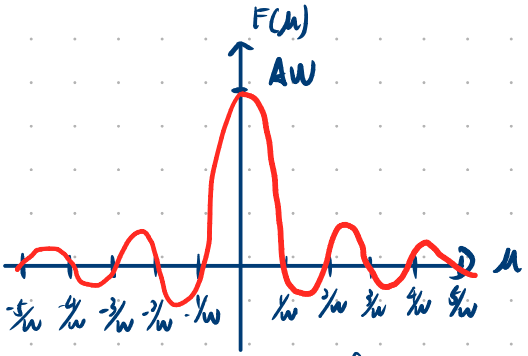
$$= \frac{A}{-j2\pi u} \left[e^{-j2\pi ut} \right]_{-W/2}^{W/2} = \frac{-A}{j2\pi u} \left[e^{-jW\pi u} - e^{jW\pi u} \right]$$

$$= \frac{A}{j2\pi u} \left[e^{jW\pi u} - e^{-jW\pi u} \right]$$

$$= \frac{A}{j2\pi u} \cdot 2j \cdot \sin(\pi u W) = A \cdot \frac{\sin(\pi u W)}{\pi u}$$

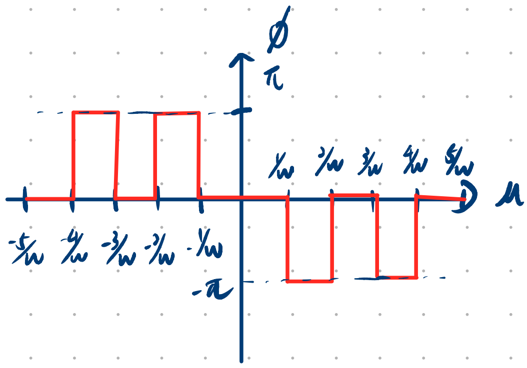
$$= AW \cdot \frac{\sin(\pi u W)}{\pi u W} = AW \cdot \operatorname{sinc}(uW)$$

(b)



Fourier Transform

$\Rightarrow$



Phase

The phase of a positive real number is zero.

The phase of a negative real number is π

The phase of zero is defined to be zero

3.

(a) $F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$

$\Rightarrow F(0,0) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{0} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$ ——— ①

$\because$ The averge of a function can be define as $\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$

$\Rightarrow \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) = \frac{1}{MN} F(0,0)$

$\therefore$ The average of a function is $\frac{1}{MN} F(0,0)$ where

F is the DFT of the function.

(b) ① The sample sum of a odd function is 0 $\Rightarrow \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\text{odd}]=0$

② The product of an odd function and an even function is an odd function

$$\Rightarrow \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\text{even}\cdot\text{odd}] = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\text{odd}] = 0$$

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(1) $f(x,y)$ is real and even $\Rightarrow F(u,v)$ is also real and even

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi(ux/M + vy/N)}$$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi ux/M}\; e^{-j2\pi vy/N}$$

$e^{-j\theta} = \cos\theta - j\sin\theta$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\,[\cos(2\pi ux/M) - j\sin(2\pi ux/M)]\,[\cos(2\pi vy/N) - j\sin(2\pi vy/N)]$$

$f(x,y)$ is real and even
$\cos\theta$ is even ; $\sin$ is odd

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} [\text{even}]\,[\text{even} - j\,\text{odd}]\,[\text{even} - j\,\text{odd}]$$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} [\text{even}]\,[\text{even}\cdot\text{even} - 2j\,\text{even}\cdot\text{odd} - \text{odd}\cdot\text{odd}]$$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\text{even}] - 2j\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\text{even}\cdot\text{odd}] - \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\text{even}]$$

from ②

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\text{even}] - \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\text{even}] \Rightarrow F(u,v) \text{ is real and even } \#$$

(2) $F(u,v)$ is real and even $\Rightarrow f(x,y)$ is also real and even

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi(ux/M + vy/N)}$$

$$= \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi ux/M}\; e^{j2\pi vy/N}$$

$$= \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} [\text{even}]\,[\text{even} + j\,\text{odd}]\,[\text{even} + j\,\text{odd}]$$

$$= \frac{1}{MN}\left\{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}[\text{even}] + 2j\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}[\text{even}\cdot\text{odd}] - \sum_{u=0}^{M-1}\sum_{v=0}^{N-1}[\text{even}]\right\}$$

from ②

$$= \frac{1}{MN}\left\{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}[\text{even}] - \sum_{u=0}^{M-1}\sum_{v=0}^{N-1}[\text{even}]\right\} \Rightarrow f(x,y) \text{ is real and even } \#$$

(c) $\because F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M + vy/N)}$

$\therefore F(u-u_0, v-v_0) \Leftrightarrow f(x,y)e^{j2\pi(u_0x/M + v_0y/N)}$

Let $u_0 = M/2$, $v_0 = N/2$

$\therefore F(u-\frac{M}{2}, v-\frac{N}{2}) \Leftrightarrow \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{j\pi(x+y)} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)(-1)^{(x+y)}$

$F(u-\frac{M}{2}, v-\frac{N}{2}) = \mathfrak{F}[f(x,y)(-1)^{(x+y)}]$ #

(d) 讓相鄰週期在算 convolution 的時候不要互相影響
problem ⇒ 會有 frequency leakage 的現象 (出現假的高頻訊號)

4.
(a) 2M, 2N

(e) real, symmetric

(f) $H(u,v)F(u,v)$

(g) real

(h) top, left quadrant

5.

(a)

i	n_i
0	1
1	2
2	4
3	5
4	6
5	5
6	1
7	1

$S_0 = 7 \times \frac{1}{25} = \frac{7}{25} \sim 0$

$S_1 = 7 \times \frac{3}{25} = \frac{21}{25} \approx 1$

$S_2 = 7 \times \frac{7}{25} = \frac{49}{25} \approx 2$

$S_3 = 7 \times \frac{12}{25} = \frac{84}{25} \approx 3$

$S_4 = 7 \times \frac{18}{25} = \frac{126}{25} \approx 5$

$S_5 = 7 \times \frac{23}{25} = \frac{161}{25} \approx 6$

$S_6 = 7 \times \frac{24}{25} = \frac{168}{25} \approx 7$

$S_7 = 7 \times \frac{25}{25} = \frac{175}{25} \approx 7$

3	6	5	7	6
6	5	3	5	6
5	5	2	3	7
3	2	2	3	6
0	1	1	2	5

(b) $G(z) = (L-1)\sum_{i=0}^{L-1} P_i(i) = S$

$\Rightarrow z = G^{-1}(S)$

Image B

i	n_i
0	2
1	2
2	4
3	2
4	5
5	5
6	3
7	2

$S_0 = 7\times\frac{2}{25} = \frac{14}{25} \approx 1$

$S_1 = 7\times\frac{4}{25} = \frac{28}{25} \approx 1$

$S_2 = 7\times\frac{8}{25} = \frac{56}{25} \approx 2$

$S_3 = 7\times\frac{10}{25} = \frac{70}{25} \approx 3$

$S_4 = 7\times\frac{15}{25} = \frac{105}{25} \approx 4$

$S_5 = 7\times\frac{20}{25} = \frac{140}{25} \approx 6$

$S_6 = 7\times\frac{23}{25} = \frac{161}{25} \approx 6$

$S_7 = 7\times\frac{25}{25} = \frac{175}{25} \approx 7$

S

3	6	5	7	6
6	5	3	5	6
5	5	2	3	7
3	2	2	3	6
0	1	1	2	5

$\Downarrow\ z = G^{-1}(S)$

3	5	4	7	5
5	4	3	4	5
4	4	2	3	7
3	2	2	3	5
0	0	0	2	4