

# Collatz

## Statement

Evirir the dragon has a list of  $n$  integers  $a_1, a_2, \dots, a_n$ . Evirir will repeat the following procedure  $k$  times:

- For each integer  $a_i$  in the list:
  - If  $a_i$  is even, replace  $a_i$  with  $\frac{a_i}{2}$ .
  - Otherwise, replace  $a_i$  with  $3a_i + 1$ .

In the end, what is the sum of all integers in the list?

## Input Format

- The first line consists of two integers  $n$  and  $k$  ( $1 \leq n \leq 1000, 1 \leq k \leq 1000$ ).
- Then  $n$  integers follow:  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq 10^4$ ).

## Output Format

Output a single integer, the sum of the integers in the final list.

## Sample Input 1

```
5 1
1 2 3 4 5
```

## Sample Output 1

```
33
```

## Explanation 1

After performing the procedure once, the list becomes  $\{4, 1, 10, 2, 16\}$ . Summing up, we get  $4 + 1 + 10 + 2 + 16 = 33$ .

## Sample Input 2

```
6 3
3 1 4 1 5 9
```

## Sample Output 2

```
33
```

## Explanation 2

The list is changed like so:  $\{3, 1, 4, 1, 5, 9\} \rightarrow \{10, 4, 2, 4, 16, 28\} \rightarrow \{5, 2, 1, 2, 8, 14\} \rightarrow \{16, 1, 4, 1, 4, 7\}$ . The sum of the integers in the final list is  $16 + 1 + 4 + 1 + 4 + 7 = 33$ .

# Mobile Game

## Statement

Alice is playing a mobile game on a typical Saturday afternoon.

Before the game starts, Alice has  $A$  power level and there are  $N$  enemies with power level  $p_1, p_2, \dots, p_N$ .

Alice can do the following action **zero** or **more** times:

- choose an enemy with strictly less power level than Alice's power level. Kill the enemy and increase Alice's power level by the enemy's power level.

What is the minimum number of enemies Alice need to kill to have at least  $B$  power level? If this is impossible, output  $-1$ .

There are  $T$  test cases to solve. Alice knows you are good in problem solving and requests your help to solve this game.

**Note: You could only kill an enemy once as the enemy does not respawn after death.**

## Input format

The first line contains the number of test cases  $T$  ( $1 \leq T \leq 20$ ). The description of the test cases follows:

- The first line of each test case contains three integers,  $N$  ( $1 \leq N \leq 1000$ ) denoting the number of enemies,  $A$  ( $1 \leq A \leq 10^9$ ) denoting Alice's initial power level and  $B$  ( $1 \leq B \leq 10^9$ ) denoting power level Alice needs to reach.
- The second line of each test case contains  $n$  integers which describe the power levels of the enemies,  $p_i$  ( $1 \leq p_i \leq 10^3$ ) for the  $i^{th}$  enemy.

It is guaranteed that the sum of  $N$  over all test cases does not exceed 1000.

## Output format

For each test case, output a single integer denoting the minimum number of enemies Alice needs to kill to have at least  $B$  power level. If it is impossible, output  $-1$ .

## Sample Input

```
2
5 3 10
4 3 4 1 2
3 20 100
70 86 19
```

## Sample Output

```
3
-1
```

## Explanation

There are two test cases to solve.

For first test case, there are several ways to kill the enemies in which Alice reaches at least 10 power level in three kills. Here is an example, Alice has 3 power level initially and decide to kill the 4<sup>th</sup> enemy to gain 1 power level to 4. Next, Alice kills the 2<sup>nd</sup> enemy and gain 3 power level to 7. Lastly, Alice kills the 1<sup>st</sup> enemy to gain 4 power level to 11, which exceeds the 10 power level.

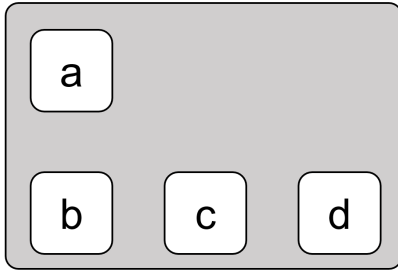
For second test case, the only enemy Alice could kill is the last enemy in which Alice gains 19 power level from 20 to 39. However, after that Alice could not kill any enemies, hence, it is impossible to reach 100 power level.

# Innovation

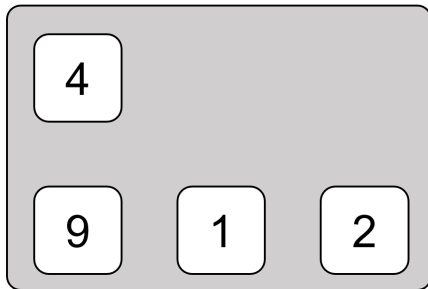
## Statement

Note: You do not need to know the rules of the original Innovation to solve this problem.

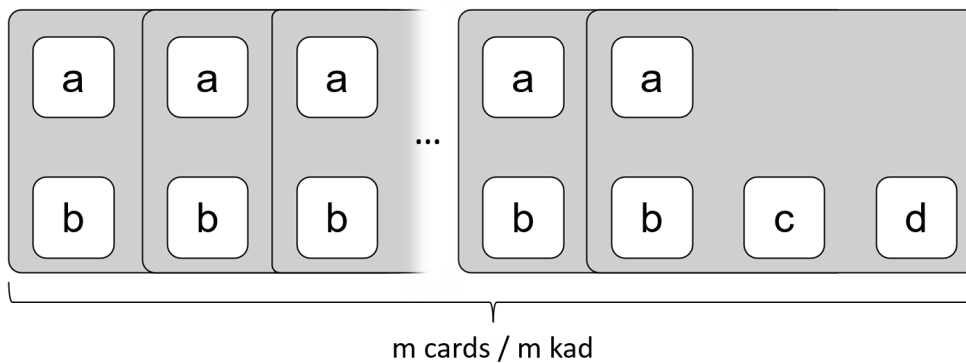
Evirir the dragon is playing a variant of the board game Innovation! Currently, Evirir has  $n$  cards, and on top of each card are 4 (non-negative) integers:  $a$ ,  $b$ ,  $c$ , and  $d$ , laid out like this:



Here is an example card (2nd card in sample input 1):



Each card has its own values of  $a$ ,  $b$ ,  $c$  and  $d$ . Now, Evirir will choose  $m$  cards, and put them down like this in any order they want:



After this, Evirir will gain points equal to the sum of all **visible** numbers. What is the maximum amount of points Evirir can gain if he chooses the  $m$  cards and arrange them optimally?

## Input Format

Let  $a_i$ ,  $b_i$ ,  $c_i$ , and  $d_i$  denote the values of  $a$ ,  $b$ ,  $c$ , and  $d$  on the  $i$ -th card. The input will be formatted as follows:

```
n m
a1 b1 c1 d1
```

```
a2 b2 c2 d2
...
an bn cn dn
```

Constraints:

- $1 \leq m \leq n \leq 20000$ .
- $0 \leq a_i, b_i, c_i, d_i \leq 10^9$ .

## Output Format

Output a single integer, the maximum amount of points Evirir can gain.

## Sample Input 1

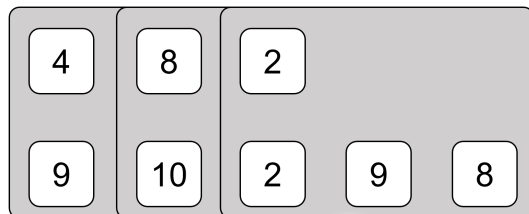
```
5 3
3 5 6 6
4 9 1 2
1 2 3 4
2 2 9 8
8 10 2 3
```

## Sample Output 1

52

## Sample 1 Explanation

Evirir can put the 2nd, 5th, and 4th card from bottom to top as follows:



The sum of visible numbers is 52. It can be proven that this is the maximum amount of points Evirir can get.

## Sample Input 2

```
4 2
2 0 0 100
0 3 0 0
4 0 0 0
0 5 0 0
```

## Sample Output 2

107

# Tichu

## Statement

You have  $N$  cards in your hand.  $N - K$  cards have integers on them. The remaining  $K$  cards are wild cards that can each represent a card of any integer you choose.

A **run** of length  $m$  is a sequence of  $m$  cards with the integers  $(a, a + 1, a + 2, \dots, a + m - 1)$  for some integer  $a$ . For example, the sequences  $(7, 8, 9)$ ,  $(1, 2, 3, 4, 5)$ , and  $(6)$  are runs, but  $(2, 3, 5, 6)$ ,  $(1, 6)$ , and  $(4, 3, 2, 1)$  are not runs.

What is the longest **run** you can make with these cards? You may rearrange the cards in your hand however you like to form this run.

## Input Format

- The first line consists of an integer  $N$  ( $1 \leq N \leq 10^5$ ), followed by another integer  $K$  ( $0 \leq K \leq N$ ).
- The second line contains  $N - K$  integers, representing the cards in your hand that aren't wild cards:  $C_1, C_2, \dots, C_{N-K}$  ( $1 \leq C_i \leq 10^9$ ).

## Output Format

Output a single integer, the length of the longest run you can form from the cards you have.

## Sample Input

```
11 2
2 5 4 3 3 8 7 11 15
```

## Sample Output

```
8
```

## Explanation

If you make the wildcards a 6 and a 9, you can make the run  $(2, 3, 4, 5, [6], 7, 8, [9])$ , which has length 8. You cannot make a longer run.

# Rectangles

## Statement

You have  $N$  **red** rectangles. Rectangle  $i$  has height  $h_i$  and width  $w_i$ . The  $N$  rectangles have their bottom side on the x-axis and are placed side by side (red rectangle  $i$  and  $i + 1$  share a side).

You are required to draw at most  $K$  blue rectangles such that every **red** rectangle is inside of **exactly one blue** rectangle. You are able to determine the positions and the dimensions of the **blue** rectangles freely. Hence, you want to find the minimum total area of blue rectangles to cover all  $N$  **red** rectangles.

## Input format

The first line consists two integers  $N$  and  $K$  ( $2 \leq N \leq 200$ ) ( $1 \leq K \leq 10^9$ ).

$N$  lines follow, the  $i$ -th line contains two integers  $h_i$  and  $w_i$  ( $1 \leq h_i, w_i \leq 1000$ ).

The bounds and points of each task are given below:

Task 1 (15 points):  $N \leq 7, K \leq 7$

Task 2 (20 points):  $N \leq 200, K = 1$

Task 3 (20 points):  $N \leq 200, K = 2$

Task 4 (15 points):  $N \leq 200, K = 998244353$

Task 5 (30 points):  $N \leq 200, K \leq N - 1$

## Output format

One integer denoting the minimum area of blue rectangles to cover all  $N$  **red** rectangles using at most  $K$  **blue** rectangles as described in the statement.

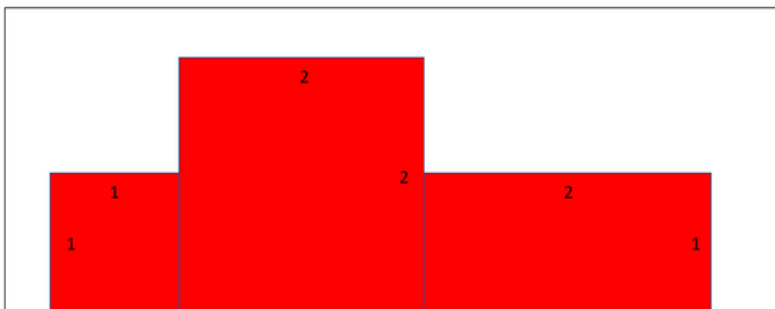
## Sample Input

```
3 2
1 1
2 2
1 2
```

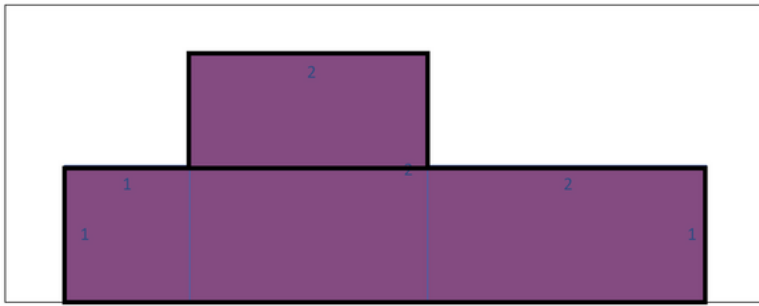
## Sample Output

8

Explanation: The diagram looks like this:

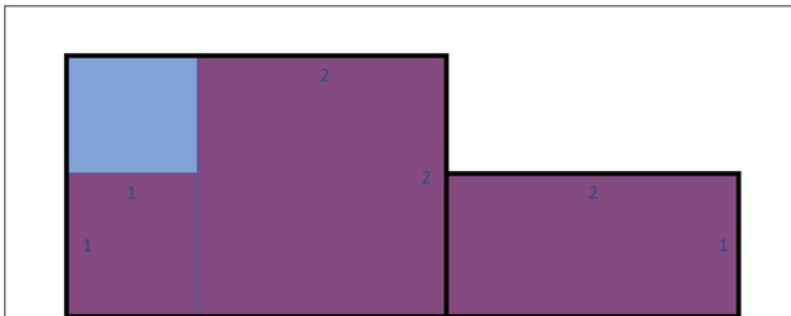


Note that you cannot make the following arrangement of blue rectangles:



This is an invalid arrangement, as **red** rectangle 2 is covered by both **blue** rectangle 1 and **blue** rectangle 2. A **red** rectangle can only be inside one **blue** rectangle only.

The answer can be obtained by the following arrangement:





## Sum<sup>k</sup>

### Statement

Given an array  $A$  of  $N$  integers  $a_1, a_2, \dots, a_n$ . Define the score of a subset  $S$  of  $A$  as the sum of the elements in  $S$  raised to the power of  $K$ . That is, for a subset  $S = \{s_1, s_2, \dots, s_m\}$ , the score of  $S$  is  $(s_1 + s_2 + \dots + s_m)^K$ . Find the sum of scores over all possible non-empty subsets of  $A$  modulo 998244353.

$X$  modulo  $M$  is the remainder when  $X$  is divided by  $M$ .

### Input Format

- The first line consists of two integers  $N$  and  $K$ .
- Then  $N$  integers follow:  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq MAXA$ ).

The bounds and points of each test case are given in the following table.

Task	N	K	MAXA	Points
1	10	1	100	5
2	10	2	100	5
3	18	2	$10^9$	10
4	1000	2	$10^9$	10
5	100000	2	$10^9$	10
6	100000	3	$10^9$	10
7	200	200	$10^9$	25
8	777	150	$10^9$	25

### Output Format

Output a single integer, the sum of scores over all subsets of  $A$  modulo 998244353.

### Sample Input 1

```
3 2
1 2 3
```

### Sample Output 1

```
100
```

### Explanation

There are 7 possible non empty subsets:  $\{1\}$ ,  $1^2 = 1$   $\{2\}$ ,  $2^2 = 4$   $\{3\}$ ,  $3^2 = 9$   $\{1, 2\}$ ,  $3^2 = 9$   $\{1, 3\}$ ,  $4^2 = 16$   $\{2, 3\}$ ,  $5^2 = 25$   $\{1, 2, 3\}$ ,  $6^2 = 36$  Totaling up,  $1 + 4 + 9 + 9 + 16 + 25 + 36 = 100$ .

### Sample Input 2

```
2 1
3 3
```

### Sample Output 2

```
12
```