Lecture 13: Circuits with Dependent Sources

OBJECTIVES:

1. Understand and Analyze circuits with dependent sources

READING

Required :

- Textbook, sections 4.1–4.2

Optional : None

1 Preliminary Definitions

Active device – Any component that requires an external power supply to operate correctly. *Any device that has a gain greater than 1 is an active device*

Active circuit – Any circuit with at least one active device

Gain – The ratio of our output signal to our input signal (assumes input and output have the same dimensions)

Signal amplification – A gain greater than 1

Dependent source – A voltage or current source whose output is controlled by a voltage or current elsewhere in the circuit

2 Types of dependent sources

Since dependent sources can supply voltage or current and they are controlled by voltage or current elsewhere, we can have 4 types of dependent sources:

1. *Voltage Controlled Voltage Source (VCVS)*
2. *Current Controlled Voltage Source (CCVS)*
3. *Voltage Controlled Current Source (VCCS)*
4. *Current Controlled Current Source (CCCS)*

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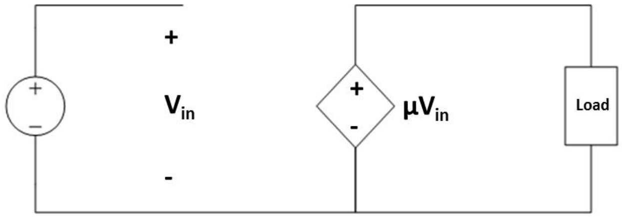


Figure 1: Voltage Controlled Voltage Source, note, the diamond shaped source is the depended one

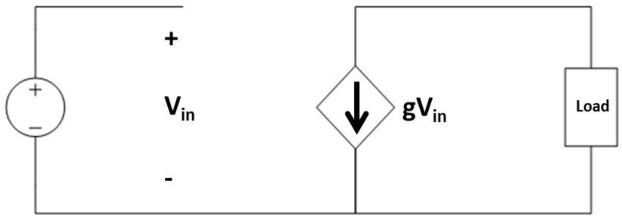


Figure 2: Voltage Controlled Current Source

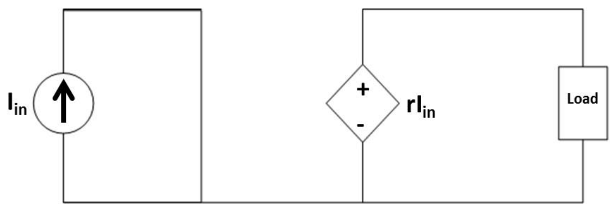


Figure 3: Current Controlled Voltage Source

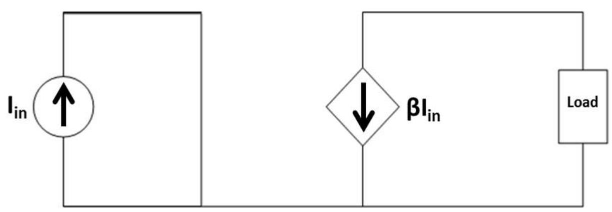


Figure 4: Current Controlled Current Source

Figures 1 – 4 show all the possible dependent source arrangements. It is worth noting that the control signal does not have to be an independent source, but rather can be any current or voltage in the circuit.

Source	In	Out	Gain	Units
VCVS	V	V	μ	unitless
VCCS	V	I	γ	Siemens
CCVS	I	V	r	Ohms
CCCS	I	I	β	unitless

3 Circuit Analysis with Dependent Sources

All circuit analysis techniques we have already learned work with Dependent sources with some caveats....

1. *Dependent sources cannot be turned off independent of the controlling current or voltage; that is to turn off a dependent source, you have to turn off the controlling current or voltage. This is often not practical; therefore, superposition is not normally used for dependent sources*
2. *Circuit reduction and source transformation can be used, provided you do not eliminate the control variable*

Example 1 – Textbook Exercise 4-1 For the circuit shown in Figure 5, find v_o in terms of v_s

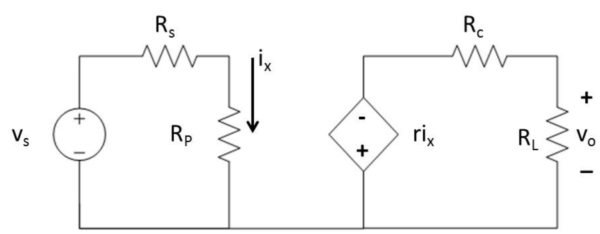


Figure 5: Circuit to accompany Example 1

Note: there is zero current running through the single wire connecting the two loops together.

$$i_x = \frac{V_s}{R_s + R_p} \tag{1}$$

$$ri_x = \frac{rV_s}{R_s + R_p} \tag{2}$$

$$V_o = \frac{R_L}{R_L + R_c}\frac{-rV_s}{R_s + R_p} \quad \text{negative sign due to polarity of source} \tag{3}$$

$$V_o = \frac{-rR_LV_s}{(R_L + R_c)(R_s + R_p)} \tag{4}$$

$$\tag{5}$$

Example 2 For the circuit shown in Figure 6, find v_o in terms of v_s

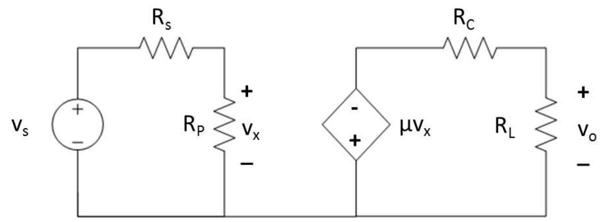


Figure 6: Circuit to accompany Example 2

voltage division: $$V_x = \frac{R_p}{R_s + R_p} V_s \tag{6}$$

voltage of dependent source: $$\mu V_x = \frac{R_p}{R_p + R_s} \mu V_s \tag{7}$$

voltage division to find V_o :

$$V_o = \frac{R_L}{R_L + R_c} \frac{-R_p}{R_s + R_p} \mu V_s \tag{8}$$

$$V_o = \frac{-r V_s R_L R_p}{(R_L + R_c)(R_s + R_p)} \tag{9}$$

$$\tag{10}$$

4 Nodal Analysis

Recall the steps for writing node voltage equations:

1. Identify a **reference node**. You will not write an equation for this node, but the voltages at the other nodes will use this as a reference.
2. Write **KCL equations** at the other $N-1$ nodes
3. Write the currents in the KCL equations in terms of **node voltages** and resistances
4. Rearrange the equations above into **standard form**

Also recall that adding a voltage source to the circuit added additional constraints and we had three methods for dealing with those constraints

1. Source transformation
2. Smart choice of your reference node
3. Define a *Supernode*

All these methods still apply to circuits with dependent sources. We will demonstrate by example:

Example 3 – For the circuit shown in Figure 7,

- Write Node Voltage equations
- Write and expression for v_o as a function of R_F

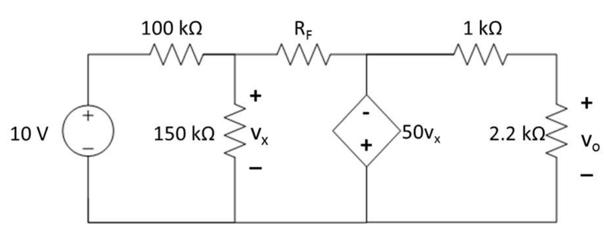


Figure 7: Circuit to accompany Example 3

Step 1. Assign nodes (A-D across top)

Stop 2. Write KCL eqns and constant equations

$$V_A = 10 \tag{11}$$

$$\frac{V_B - V_A}{100k} + \frac{V_B}{150k} + \frac{V_B - (-50)}{R_F} = 0 \qquad \because V_B = V_x \tag{12}$$

$$V_c = -50V_x = -50V_B \tag{13}$$

$$\frac{V_o}{2.2k} + \frac{V_o - V_c}{1k} = 0 \qquad \because V_D = V_o \tag{14}$$

Step 3. Now combine these eqns to get one composed of V_o and $V_x S$

$$\frac{V_x - 10}{100k} + \frac{V_x}{150k} + \frac{V_x - (-50V_x)}{R_F} = 0 \tag{15}$$

$$V_x\left(\frac{1}{100k} + \frac{V_x}{150k} + \frac{51}{R_F}\right) = \frac{10}{100k} \tag{16}$$

$$\frac{V_o}{2.2k} + \frac{V_o - (-50V_x)}{2.2k} = 0 \tag{17}$$

Step 4. Solve for V_x in terms of R_F

$$V_x = \frac{10}{100k}\left(\frac{1}{100k} + \frac{1}{150k} + \frac{1}{R_F}\right) = \frac{10}{100k}\left(\frac{300kR_F}{5R_F + 15.3M}\right) \tag{18}$$

$$V_x = \frac{30R_F}{5R_F + 15.3M} = \frac{6R_F}{R_F + 3.06k} \tag{19}$$

Example 4 – Textbook Exercise 4-2 For the Circuit shown in Figure 8, write an expression for the gain ($K = \frac{v_O}{v_S}$) in terms of R_F then find values for R_F such that $50 < |K| < 10,000$ (given that $\mu = 10^5$).

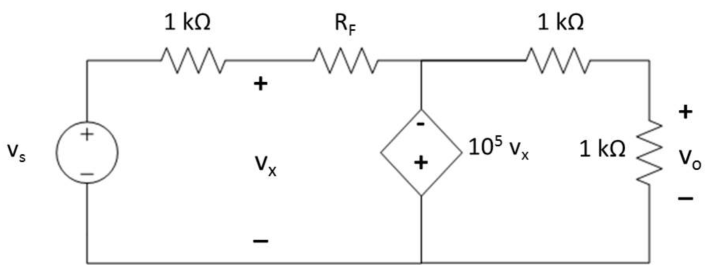


Figure 8: Circuit to accompany Example 4

5 Mesh Analysis

Recall the steps for writing Mesh Current equations:

1. Assign a current to each mesh
2. Assign a voltage (magnitude and polarity) to each device in the circuit
3. Write Kirchhoff's Voltage Law (KVL) equations for each mesh
4. Use device i–v characateristics to rewrite KVL eqations from the previous step
5. Rewrite equations in standard (matrix) form

Futhermore, recall the 3 methods for dealing with current sources

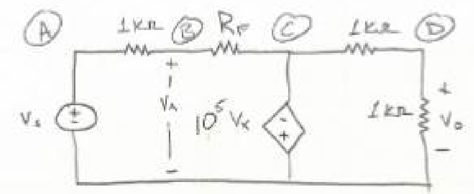
1. Use source transformation to eliminate current sources
2. When possible assign meshes so the source is only part of one mesh
3. Use a *Supermesh*

All these methods can be used when dealing with cicuits with dependent sources. We will demonstrate with examples:

Example 5 – Textbook Exercise 4-8 Use mesh analysis to find the current i_O in Figure 9.

Example 6 – Textbook Exercise 4-9 Use mesh analysis to find the current i_O in Figure 10.

STEP 1 - LABEL NODES



STEP 2 - WRITE NODE EQUATIONS

$$V_A = V_s \qquad V_C = -10^5 V_x$$
$$V_B = V_x \qquad V_D = V_o$$

CONSTRAINTS

B $\quad \dfrac{V_x - V_s}{1k\Omega} + \dfrac{V_x - (-10^5 V_x)}{R_F} = 0$ KCL @ B

D $\quad \dfrac{V_o - (-10^5 V_x)}{1k\Omega} + \dfrac{V_o}{1k\Omega} = 0$ KCL @ D

STEP 3 - SOLVE B FOR V_x

$$V_x \left[\frac{1}{1k} + \frac{1 + 10^5}{R_F} \right] = \frac{V_s}{1k}$$

$\boxed{1 + 10^5 \approx 10^5}$

$$V_x \left[\frac{R_F}{1k\, R_F} + \frac{10^8}{1k \cdot R_F} \right] = \frac{V_s}{1k}$$

$$V_x = \frac{V_s R_F}{R_F + 10^8}$$

STEP 4 - SUBSTITUTE V_x INTO ⑤ & SOLVE FOR $\frac{V_o}{V_s}$

$$\frac{2V_o}{1K} + \frac{10^5 \, V_s R_F}{1K\,(R_F + 10^8)} = 0$$

$$\frac{2V_o}{1K} = \frac{-10^5 \, V_s R_F}{1K\,(R_F + 10^8)}$$

$$\frac{V_o}{V_s} = \frac{-10^8 R_F}{2K\,(R_F + 10^8)}$$

STEP 5 - SET GAIN EQUAL TO "BOUNDARIES & SOLVE

$$50 = \left| \frac{-10^8 R_F}{2K\,(R_F + 10^8)} \right|$$

$$100K = \left| \frac{-10^8 R_F}{R_F + 10^8} \right|$$

$$100K\,(R_F + 10^8) = 10^8 R_F$$

$$10^5 R_F + 10^{13} = 10^8 R_F$$

$$(10^8 - 10^5) R_F = 10^{13}$$

$$\boxed{10^8 - 10^5 \approx 10^8}$$

$$10^8 R_F = 10^{13}$$

$$R_F = 10^5 = 100K\Omega$$

STEP 5 - CONTINUED

$$10{,}000 = \left| \frac{-10^8 R_F}{2k\left(R_F + 10^8\right)} \right|$$

$$2 \times 10^7 = \left| \frac{-10^8 R_F}{R_F + 10^8} \right|$$

$$2 \times 10^7 \left(R_F + 10^8\right) = 10^8 R_F$$

$$2 \times 10^7 R_F + 2 \times 10^{15} = 10^8 R_F$$

$$\left(1 \times 10^8 - 0.2 \times 10^8\right) R_F = 2 \times 10^{15}$$

$$0.8 \times 10^8 R_F = 2 \times 10^{15}$$

$$R_F = 25\,\mathrm{M\Omega}$$

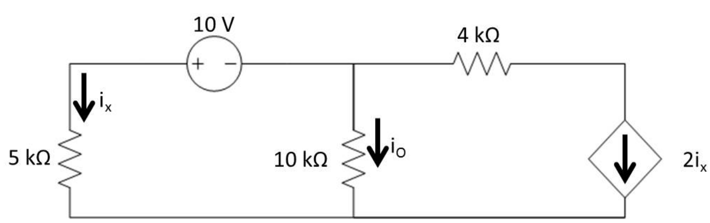


Figure 9: Circuit to accompany Example 5

STEP 1 - ASSIGN MESH CURRENTS

STEP 2 - WRITE EQUATIONS

KVL LOOP A

$$-10V - (i_A - i_B)10K\Omega - (i_A)5K\Omega = 0$$

$$i_B = 2i_x$$

$$i_A = -i_x$$

$$-10V - (-i_x - 2i_x)10K\Omega - (-i_x)5K\Omega = 0$$

STEP 3 - SOLVE FOR i_x

$$3i_x(10K\Omega) + i_x(5K\Omega) = 10V$$

$$30K\, i_x + 5K\, i_x = 10V$$

$$i_x = \frac{10V}{35K} = 286\,\mu A$$

STEP 4 - FIND OTHER CURRENTS

$$i_A = -i_x = -286\,\mu A$$

$$i_B = 2i_x = 572\,\mu A$$

$$i_o = i_A - i_B = -858\,\mu A$$

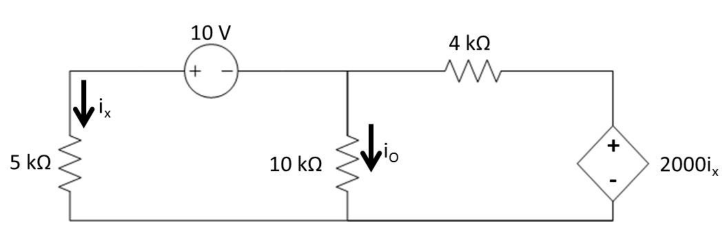
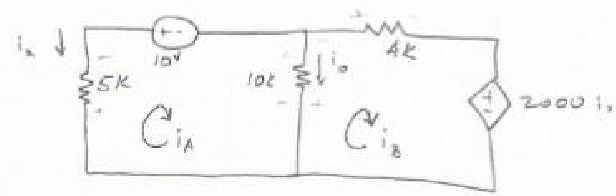


Figure 10: Circuit to accompany Example 6

STEP 1 - ASSIGN MESH CURRENTS



STEP 2 - WRITE EQUATIONS

KVL LOOP A $\quad 5K\, i_A + 10K\, i_A - 10K\, i_B = -10^V$

KVL LOOP B $\quad 10K\, i_B + 4K\, i_B - 10K\, i_A = -2000\, i_x$

$i_A = -i_x$

STEP 3 - WRITE IN MATRIX FORM (SUB $i_A = -i_x$)

$$\begin{bmatrix} 15K & -10K \\ -12K & 14K \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} -10 \\ \emptyset \end{bmatrix}$$

NOTICE A-MATRIX IS NOT SYMMETRIC

STEP 4 - SOLVE FOR CURRENTS

$i_A = -1.56\,mA$

$i_B = -1.33\,mA$

$i_o = i_A - i_B = -230\,\mu A \rightarrow -0.230\,mA$