Lecture 7: Mesh Current Analysis

OBJECTIVES:

1. Demonstrate the ability to write mesh current equations for a given circuit
2. Demonstrate the ability to solve for unknown circuit parameters using mesh current analysis

READING

Required :

- Textbook, sections 3.2, pages 92–100

Optional : None

1 What is Mesh Current Analysis?

Similar to Node Voltage Analysis, it is a more efficient method for solving circuits than being forced to rely on element and connection constraints. What we write instead are equations for *Mesh Currents*. To do this we have to first answer the question, what is a mesh?

A mesh is a loop that does not contain any other loops inside of it

In Figure 1, loops 1 & 2 are meshes, but loop 3 is not.

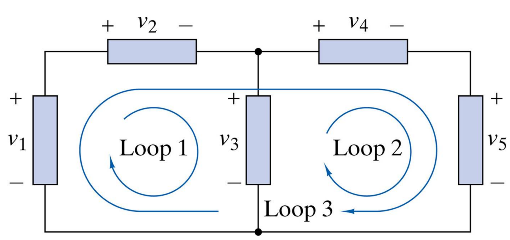


Figure 1: Loops 1 & 2 are meshes; loop 3 is not

2 Circuits with NO CURRENT sources

This section will give steps for wiring Mesh Current equations and then will give examples. The steps will likely not make much sense until we look at a couple of examples.

2.1 Steps for writing Mesh Current Equations

1. *Assign a current to each mesh*
2. *Assign a voltage (magnitude and polarity) to each device in the circuit*
3. *Write Kirchhoff's Voltage Law (KVL) equations for each mesh*
4. *Use device i–v characteristics to rewrite KVL equations from the previous step*
5. *Rewrite equations in standard (matrix) form*

2.2 Examples

Example 1: In the circuit shown in Figure 2, use mesh current analysis to find the voltage drop across the 25 $k\Omega$ resistor and the current out of the (positive terminal) source.

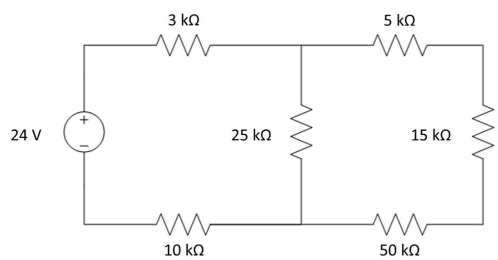
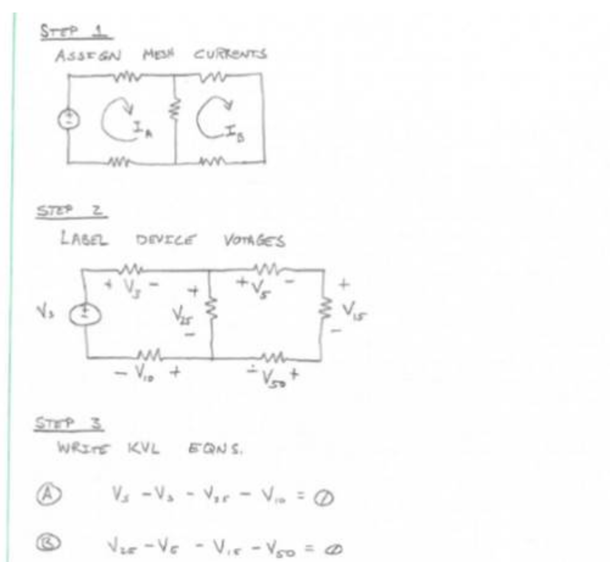


Figure 2: Circuit to accompany example 1



STEP 4
WRITE DEVICE EQNS. (OHM'S LAW)

$V_s = 24$

$V_3 = I_A \cdot 3K\Omega$

$V_{25} = (I_A - I_B) \cdot 25K\Omega$

$V_{10} = I_A \cdot 10K\Omega$

$V_5 = I_B \cdot 5K\Omega$

$V_{15} = I_B \cdot 15K\Omega$

$V_{50} = I_B \cdot 50K\Omega$

STEP 4 (CONT)

Ⓐ $24 - I_A \cdot 3K\Omega - I_A \cdot 25K\Omega + I_B \cdot 25K\Omega - I_A \cdot 10K\Omega = 0$

$24 - I_A(38K\Omega) + I_B(25K\Omega) = 0$

Ⓑ $I_A(25K\Omega) - I_B(25K\Omega) - I_B(5K\Omega) - I_B(15K\Omega) - I_B(50K\Omega) = 0$

$I_A(25K\Omega) - I_B(95K\Omega) = 0$

STEP 5
WRITE IN MATRIX FORM AND SOLVE

$$\begin{bmatrix} 38K\Omega & 25K\Omega \\ 25K\Omega & 95K\Omega \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 24V \\ 0^V \end{bmatrix}$$

$$\begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 0.763 \\ -0.201 \end{bmatrix} mA$$

$$V_{25} = (0.763 - 0.201) \times 10^{-3} \cdot 25 \times 10^3$$

$$\boxed{V_{25} = 14.05\ V}$$

$$\boxed{\text{SOURCE CURRENT} = I_A = 0.763\ mA}$$

Example 2: Let's rework the same example, just add a third mesh. See Figure 3

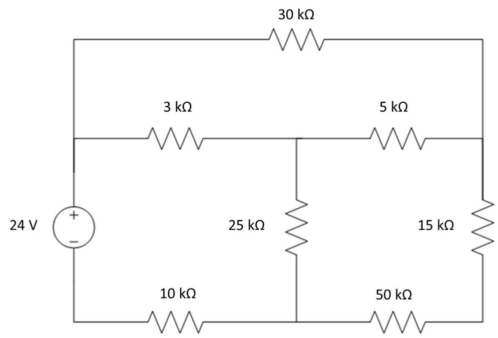
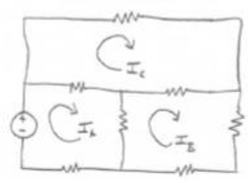
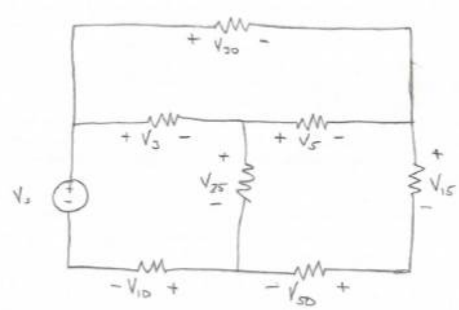


Figure 3: Circuit to accompany example 2

STEP 1
ASSIGN MESH CURRENTS



STEP 2
LABEL DEVICE VOLTAGES



STEP 3
WRITE KVL EQNS.

Ⓐ $V_s - V_3 - V_{25} - V_{10} = 0$

Ⓑ $V_{25} - V_5 - V_{15} - V_{50} = 0$

Ⓒ $V_3 - V_{30} + V_5 = 0$

STEP 4

- WRITE DEVICE EQNS

$V_s = 24V$

$V_3 = (I_A - I_C)\,3K\Omega$

$V_{25} = (I_A - I_B)\,25K\Omega$

$V_{10} = (I_A)\,10K\Omega$

$V_5 = (I_B - I_C)\,5K\Omega$

$V_{15} = (I_B)\,15K\Omega$

$V_{50} = (I_B)\,50K\Omega$

$V_{30} = (I_C)\,50\,K\Omega$

- PLUG INTO EQNS FROM STEP 3

Ⓐ $(I_A - I_C)\,3K\Omega + (I_A - I_B)\,25K\Omega + (I_A)\,10K\Omega = 24V$

$$38K\Omega \cdot I_A - 25K\Omega \cdot I_B - 3K\Omega \cdot I_C = 24V$$

Ⓑ $(I_A - I_B)\,25K\Omega - (I_B - I_C)\,5K\Omega - (I_B)\,15K\Omega - (I_B)\,50K\Omega = \emptyset$

$$25K\Omega \cdot I_A - 95K\Omega \cdot I_B + 5K\Omega\, I_C = \emptyset$$

Ⓒ $(I_A - I_C)\,3K\Omega - (I_C)\,30K\Omega + (I_B - I_C)\,5\,K\Omega = \emptyset$

$$3K\Omega \cdot I_A + 5K\Omega \cdot I_B - 38K\Omega \cdot I_C = \emptyset$$

STEP 5

PUT INTO MATRIX FORM AND SOLVE

$$\begin{bmatrix} 38K\Omega & 25K\Omega & 3K\Omega \\ 25K\Omega & 95K\Omega & 5K\Omega \\ 3K\Omega & 5K\Omega & -38K\Omega \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 24V \\ \emptyset V \\ \emptyset V \end{bmatrix}$$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} \emptyset.761 \\ -\emptyset.203 \\ .0.059 \end{bmatrix} mA$$

WHAT IS THE CURRENT IN THE 5 KΩ RESISTOR?

$$I_B - I_C = -\emptyset.203 - \emptyset.059 = -\emptyset.262\,mA$$

3 Mesh Analysis for circuits with current sources

Recall from last lesson, when we used nodal analysis on circuits with voltage sources, the set up of the problem was a little more challenging, however, the actual solution was simplified. This will same idea applies to using Mesh Analysis on circuits with current sources.

Like last lesson we will introduce 3 techniques for solving these types of circuits.

1. Source transformation
2. Current source in only 1 mesh
3. Supermesh to handle current source in multiple meshes

3.1 Method 1: Source Transformation

If the current source is in parallel with a resistor, we can convert it to an equivalent voltage source. If we eliminate all current sources using this method, we can revert back to the process shown above.

Example 3: Use Mesh Analysis to find the current through the 3 $k\Omega$ resistor in Figure 4

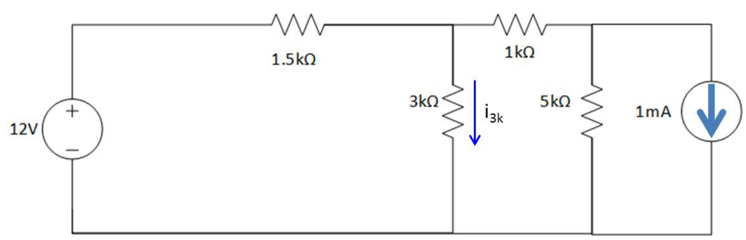
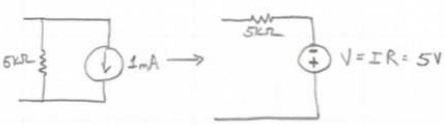
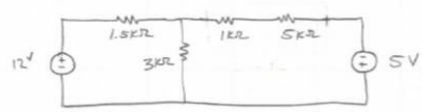


Figure 4: Circuit to accompany example 3

STEP 1: TRANSFORM THE CURRENT SOURCE





STEP 2: WRITE MESH EQUATIONS

(A) $12 - (1.5k\Omega)(I_A) - (3k\Omega)(I_A - I_B) = 0$

(B) $5 - (3k\Omega)(I_B - I_A) - (6k\Omega)I_B = 0$

$$\begin{bmatrix} 4.5k\Omega & -3k\Omega \\ -3k\Omega & 9k\Omega \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 12 \\ 5 \end{bmatrix}$$

$I_A = 3.9\,mA$ $\quad I_B = 1.85\,mA$

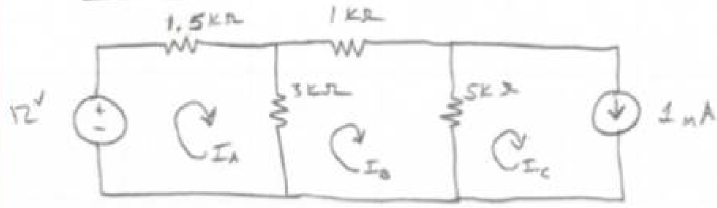
$i_{3K} = I_A - I_B = 2.05\,mA$

3.2 Method 2: Source is part of only 1 mesh

This method is really the simplest of all. If the source is only part of one mesh, we know the mesh current for that mesh; namely the value of the current source.

Example 4: Redo example 3 using method 2.

STEP 1: LABEL MESH CURRENTS



STEP 2: WRITE MESH EQNS.

Ⓐ $12^V - (1.5k\Omega)I_A - 3k\Omega(I_A - I_B) = \emptyset$

Ⓑ $3k\Omega(I_B - I_A) + 1k\Omega\, I_B + 5k\Omega(I_B - I_C) = \emptyset$

Ⓒ $I_C = 1\,mA$ ⟵ (NOT ACTUALLY A MESH EQN)

STEP 3: PLUG Ⓒ INTO Ⓐ & Ⓑ AND WRITE IN MATRIX FORM.

Ⓐ $4.5k\Omega\, I_A - 3k\Omega\, I_B = 12^V$

Ⓑ $-3k\Omega\, I_A + 9k\Omega\, I_B = 5^V$

$$\begin{bmatrix} 4.5k\Omega & -3k\Omega \\ -3k\Omega & 9k\Omega \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 12 \\ 5 \end{bmatrix}$$

$I_A = 3.9\,mA \qquad I_B = 1.85\,mA$

$i_{3k} = 2.05\,mA$

SAME AS BEFORE

3.3 Method 3: Supermeshes

If a current source is shared by 2 meshes, we can combine them into a *Supermesh*. To do this, we identify mesh currents just as we would have before, but we do not write mesh equations for the meshes that share the current source; rather, we just write one supermesh equation. We do not write any mesh equations that include the current source or any other elements that share the branch with the current source. Our last equation simply relates the mesh currents to the current source. Like the other methods, this is best illustrated by an example

Example 5: Find the voltage drop across the 1 $k\Omega$ resistor in Figure 5. Use Mesh Analysis method 3

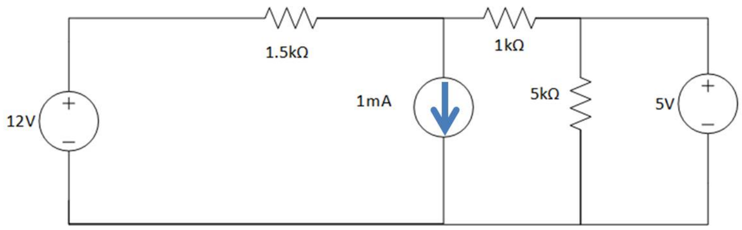
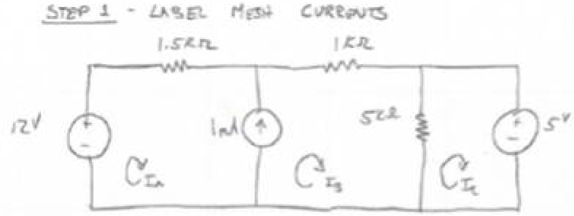


Figure 5: Circuit to accompany example 5

STEP 1 - LABEL MESH CURRENTS



STEP 2 - WRITE MESH EQNS

SUPERMESH A/B $12 - (1.5k\Omega)(I_A) - 1k\Omega\, I_B - 5k\Omega\,(I_B - I_C) = 0$

(C) $-5k\Omega\,(I_C - I_B) - 5V = 0$

$I_B - I_A = 1mA \longrightarrow I_A = I_B - 1mA$

$$\begin{bmatrix} 7.5k\Omega & -5k\Omega \\ -5k\Omega & 5k\Omega \end{bmatrix} \begin{bmatrix} I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 13.5 \\ -5V \end{bmatrix}$$

$I_B = 3.4\,mA$

$I_C = 2.4\,mA$

$\therefore I_A = 2.4\,mA$

STEP 3: FIND VOLTAGE ACROSS $1k\Omega$

$V_{1k} = (I_B)(1k\Omega) = 3.4V$

Example 6: Use mesh analysis to find the voltage across the $1A$ source in the circuit shown in Figure 6

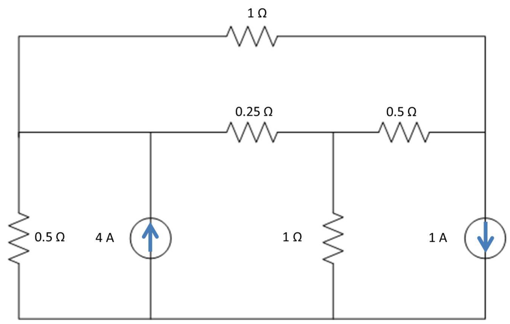
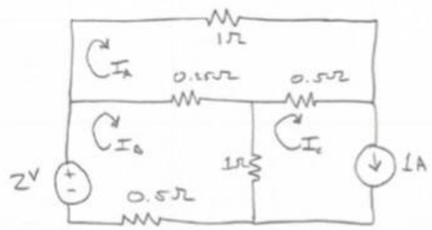


Figure 6: Circuit to accompany example 6

STEP 1: CONVERT 4A CURRENT SOURCE INTO A VOLTAGE SOURCE.



STEP 2- WRITE MESH EQUATIONS

Ⓐ $-(1\Omega)(I_A) - (0.5\Omega)(I_A - I_C) - (0.25\Omega)(I_A - I_B) = 0$

Ⓑ $2^V - (0.25\Omega)(I_B - I_A) - (1\Omega)(I_B - I_C) - (0.5\Omega)\, I_B = 0$

Ⓒ $I_C = 1A$

STEP 3- WRITE IN MATRIX FORM

$$\begin{bmatrix} -1.75 & 0.25 \\ 0.25 & -1.75 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} -0.5 \\ -3 \end{bmatrix} \qquad \begin{matrix} I_A = 542mA \\ I_B = 1.79A \end{matrix}$$

STEP 4- FIND VOLTAGE ACROSS 1A SOURCE

$$\begin{aligned} V_{1A} &= (I_A - I_C)(0.5\Omega) + (I_B - I_C)(1\Omega) \\ &= (.542 - 1)(0.5) + (1.79 - 1)(1\Omega) \\ &= 0.561\ V \end{aligned}$$

Notice that this matches our results we got using nodal analysis in lesson 6