Lecture 14: Operational Amplifiers - Part 1

OBJECTIVES:

1. Understand Op Amp Basics
2. Understand the ideal Op Amp model
3. Apply the Ideal Op Amp model to derive inverting and non-inverting op amp circuits

READING

Required :

- Textbook, sections 4.3–4.4, pages 177–190

Optional : None

1 Operational Amplifier (or Op Amp) basics

1.1 The very basics

1. *An Op Amp is an active device*
2. *Because it is an active device it has an external power supply*
3. *Op Amps are 5-terminal devices: 2 inputs, 2 power supplies and 1 output*
4. *OpAmps have very high gain. Theoretically, an open loop gain is ∞ but typical real values are 20k - 200k.*

1.2 Voltage Gain

$$\text{voltage gain} = \frac{V_{out}}{V_{in}} = A \tag{1}$$

$$20\log(A) = 20\log(\frac{V_{out}}{V_{in}}) \tag{2}$$

1.3 Op Amp Schematic & Notation

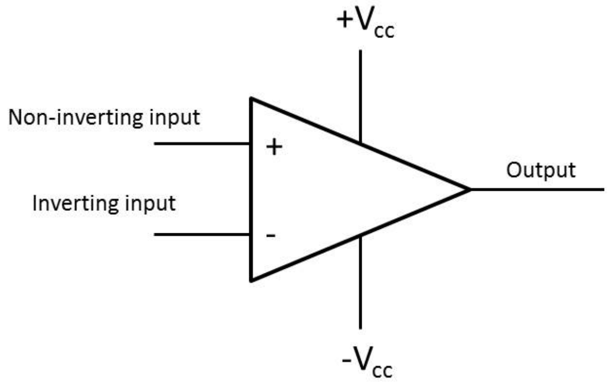

Figure 1: Op Amp showing all five terminals

Figure 1 shows the schematic for an Op Amp with all five terminals labeled. Notice there is both a positive and negative power supply (labeled $+V_{cc}$ and $-V_{cc}$ respectively). Also notice the inputs are labeled as inverting and non-inverting; the reason for this will be obvious soon.

Most of the time in schematics, we suppress the power supplies and illustrate the Op Amp as shown in Figure 2.

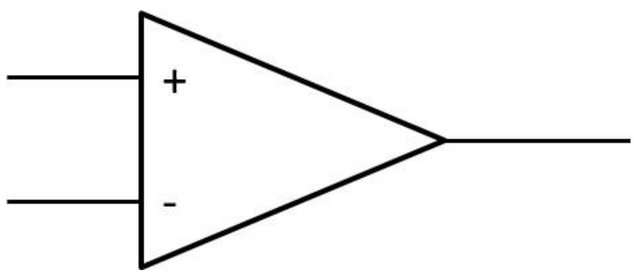

Figure 2: Op Amp showing only input and output terminals

1.4 Op Amp Transfer Characteristics

By transfer charateristics, we are refering to the relationship between the *inverting*, the *non-inverting*, and the *output* terminals. In Figure 3, the voltage (relative to common ground) of each terminal is labeled; the gain A is also shown, typically $A > 10^5$. The relationship between the voltages at the terminals is:

$$v_O = A(v_p - v_n) \tag{3}$$

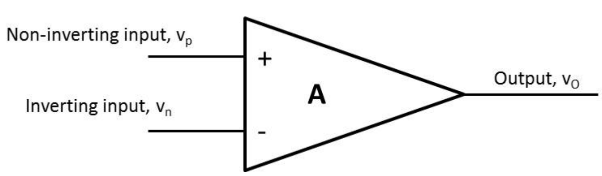

Figure 3: Op Amp with terminal voltages and gain labeled

Op Amps have two operations regions:

Linear mode – The Op Amp is operating lineraly whenever $A|v_p - v_n| < V_{cc}$; the output relationship is $v_O = A(v_p - v_n)$

Saturation mode – The Op Amp is saturated when $A|v_p - v_n| > +V_{cc}$; the output will be $v_O = +V_{cc}$ if $A(v_p - v_n) > +V_{cc}$. If $A(v_p - v_n) < -V_{cc}$ then the output is $v_O = -V_{cc}$.

1.5 The ideal Op Amp

Four our purposes we are going to always assume *Ideal Operational Amplifiers* in our calculation; so we need to give some operating characteristics of an ideal Op Amp.

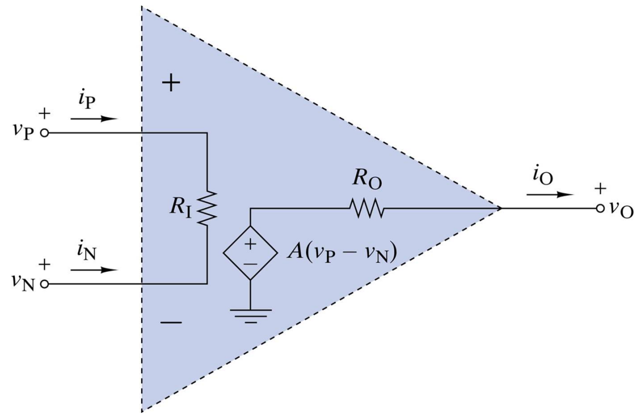

Figure 4: Ideal Op Amp model

Characteristics of ideal Op Amps (reference Figure 4):

1. *Ideal Op Amps have infinite gain*
2. *Zero current flows into the input terminals,* $i_p = i_n = 0$
3. *There is no voltage difference between the input terminals,* $v_p = v_n$
4. *Input resistance is infinite,* $R_I = \infty$
5. *Output resistance is zero,* $R_O = 0$

We will take advantage of these assumptions in the next few sections to develop some common Op Amp circuits:

1. Non-inverting Op Amp (this lesson)
2. Inverting Op Amp (this lesson)
3. Adder (next lesson)
4. Differential Amplifier (next lesson)

2 Non-Inverting Op Amps

Figure 5 shows a schematic for a non-inverting amplifier circuit. In each of the circuits we analyze moving forward it is important to note a couple of things:

1. *The gain,* K*, of the circuit, is not the same as the gain of the amplifier,* A
2. *We will use the term Amplifier or Op Amp to sometimes refer to the device and sometimes to the circuit; pay attention to context*

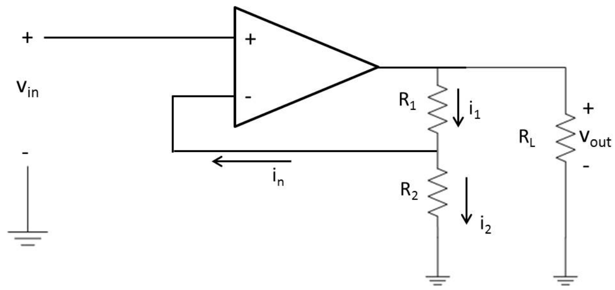

Figure 5: Non-Inverting Op Amp

2.1 Derivation of Non-inverting Op Amp Transfer characteristic

The derviation will continue to refer to Figure 5.

To start the derivation (and almost any Op Amp derivation), we write the KCL equation at the inverting terminal:

$$i_1 = i_2 + i_n \tag{4}$$

but we know, for ideal op amps, $i_n = 0$, so

$$i_1 = i_2 \tag{5}$$

Also take note that current into the non-inverting terminal is also zero; this will become important later when we talk about stage loading.

Next, write an expression for the voltage across R_2

$$V_{R2} = i_2 R_2 \tag{6}$$

but we know that the voltage between the op amp input terminals is zero, so $V_{R2} = V_{in}$ therefore

$$V_{in} = i_2 R_2 \tag{7}$$

We can now solve for the currents, i_1 and i_2

$$i_2 = \frac{V_{in}}{R_2} \tag{8}$$

and since $i_1 = i_2$

$$i_1 = \frac{V_{in}}{R_2} \tag{9}$$

We can know write an expression for V_{out} by recognizing that

$$V_{out} = V_{R1} + V_{R2} \tag{10}$$

which equals (voltage divider rearranged)

$$V_{out} = V_{in} \frac{R1 + R2}{R2} \tag{11}$$

This yields a gain of the non-inverting amplifier of

$$K = \frac{R_1 + R_2}{R_2} = \frac{V_o}{V_i} \tag{12}$$

2.2 Examples

Example 1 - Textbook Exercise 4-15 – If a non-inverting amplifier circuit is operating with $R_1 = 2R_2$ and a $V_{cc} = \pm 12$, what is the gain? For what input voltages is it operating in the linear mode?

Our gain equation for a non-inverting amplifier is

$$K = \frac{R_1 + R_2}{R_2} \tag{13}$$

If $R_1 = 2R_2$ then the gain becomes

$$K = \frac{2R_2 + R_2}{R_2} = 3 \tag{14}$$

For the amplifier to operate in linear mode.

$$|KV_{in}| < |V_{cc}| \tag{15}$$

therefore

$$|V_{in}| < 4\ V \tag{16}$$

Example 2 – Design an inverting amplifier with a gain of $K = 30\ \ \pm 10\%$ using standard resistance values.

To design the amplifier we really just have to chose values for R_1 and R_2 to satisfy

$$30 \pm 10\% = \frac{R_1 + R_2}{R_2} \tag{17}$$

to that end, let's select $R_2 = 1.5\ k\Omega$ and solve the following exact equation

$$30 = \frac{R_1 + 1.5\ k\Omega}{1.5\ k\Omega} \tag{18}$$

which yields

$$R_1 = 43.5\ k\Omega \tag{19}$$

The closest standard resistance value is $43\ k\Omega$ which gives a gain of

$$K = \frac{43\ k\Omega + 1.5\ k\Omega}{1.5\ k\Omega} = 29.667 \tag{20}$$

This is easily within 10 % of $K = 30$

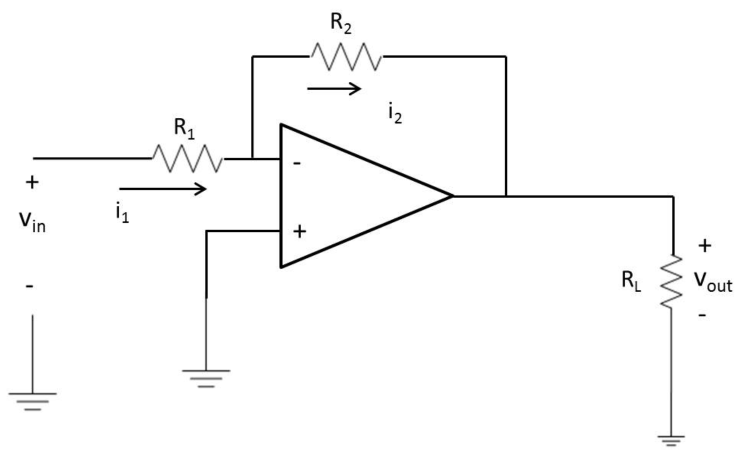

Figure 6: Inverting Op Amp

3 Inverting Op Amps

Figure 6 shows a schematic for a inverting amplifier circuit.

3.1 Derivation of Inverting Op Amp Transfer characteristic

The derviation will continue to refer to Figure 6.

Key parameters not labeled on the diagram (these always apply so I will habitually not label them):

- i_n & i_p are the currents **into** the inverting terminal and non-inverting terminals
- v_n & v_p are the voltages between the inverting and non-inverting terminals and ground

Step 1: Write KCL equation for the inverting terminal

$$i_1 = i_2 + i_n \tag{21}$$

but we know $i_n = 0$ from the ideal Op Amp definition, therefore:

$$i_1 = i_2 \tag{22}$$

Step 2: Next, we can write expressions for i_1 and i_2 in terms of input and output voltages (V_{in} & V_{out})

$$i_1 = \frac{V_{in} - v_n}{R_1} \tag{23}$$

$$i_2 = \frac{v_n - V_{out}}{R_2} \tag{24}$$

but we know $v_n = 0$ from the ideal Op Amp definition, therefore:

$$i_1 = \frac{V_{in}}{R_1} \tag{25}$$

$$i_2 = \frac{-V_{out}}{R_2} \tag{26}$$

Step 3: Since step 1 told us that $i_1 = i_2$, set the expressions from step 2 equal to one another and solve for $K = \frac{V_{out}}{V_{in}}$

$$\frac{V_{in}}{R_1} = \frac{-V_{out}}{R_2} \tag{27}$$

solving for $\frac{V_{out}}{V_{in}}$ *gives*

$$\frac{V_{out}}{V_{in}} = \frac{-R_2}{R_1} = K \tag{28}$$

3.2 Examples

Textbook Design Exercise 4-20 A 2 mV signal needs to be amplified by a gain of $K = -450 \pm 10\%$. Design an appropriate Op Amp circuit using standard resistor values

Since the required gain is negative is should be clear we need an inverting *amplifier. Recall, the gain for an inverting amplifier is*

$$K = \frac{-R_2}{R_1} \tag{29}$$

We can play around with different values of R_1 *and* R_2 *to find a value of* K *that is close to* -450*. The closest I was able to come up with was* $R_1 = 1.8\ k\Omega$ *and* $R_2 = 820\ k\Omega$ *which gives a gain of*

$$K = \frac{-820\ k\Omega}{1.8\ k\Omega} = -455.556 \tag{30}$$

easily inside the 10% *tolerance.*

Notice the chosen resistors are in the range of 1 $k\Omega$ *to* 1 $M\Omega$*. Bigger or smaller than that can have some adverse effects that we will discuss later.*

Example 3 - Textbook Exercise 4-19–Find V_{out} of an inverting amplifier with $R_1 = 10\ k\Omega$ and $R_2 = 33\ k\Omega$ when $V_{in} = 2\ V$. What about when $V_{in} = 4\ V$ and $V_{in} = 6\ V$. $V_{cc} = \pm 15\ V$

First calculate the gain

$$K = \frac{-33\ k\Omega}{10\ k\Omega} = -3.3 \tag{31}$$

For $V_{in} = 2\ V$*,*

$$V_{out} = -3.3 \times 2\ V = -6.6\ V \tag{32}$$

For $V_{in} = 4\ V$*,*

$$V_{out} = -3.3 \times 4\ V = -13.2\ V \tag{33}$$

For $V_{in} = 6\ V$*,*

$$V_{out} = -3.3 \times 6\ V = -19.8\ V \tag{34}$$

but this saturates the amplifier and we only get $-15\ V$ *output.*

3.3 Input loading of in Inverting Amplifier

Since an inverting amplifier has an external resistor (with a current flowing) on the input, we will have to look at the effects of *loading*. Any resistance attached to the input will "add" to the input resistance (R_1)and change the gain of the amplifier circuit. Before we move to an example, notice that this is not a problem with non-inverting amplifiers since there is not an external resistor attached to the input.

Example 4 A non-ideal 12 V source that has an internal resistance of 50Ω is attached to a Inverting Op Amp with $R_1 = 1\ k\Omega$ and $R_2 = 10\ k\Omega$. What is the ideal gain, K, of the amplifier? What are the actual values of K and V_{out}? If $R_1 = 100\ \Omega$ and $R_2 = 1\ k\Omega$ What is the ideal gain? What are the actual values of K and V_{out}?

The ideal gain of the amplifier is for the $R_1 = 1\ k\Omega$ and $R_2 = 10\ k\Omega$ is

$$K = \frac{-10\ k\Omega}{1\ k\Omega} = -10 \tag{35}$$

To calculate the actual value of the gain, you must recognize that the internal resistance of the source adds in series to the input resistor giving

$$K = \frac{-10\ k\Omega}{1\ k\Omega + 50\Omega} = -9.524 \tag{36}$$

which yields a $V_{out} = 114.4\ V$

The ideal gain of the amplifier is for the $R_1 = 100\ \Omega$ and $R_2 = 1\ k\Omega$ is

$$K = \frac{-1\ k\Omega}{100\ \Omega} = -10 \tag{37}$$

To calculate the actual value of the gain, you must recognize that the internal resistance of the source adds in series to the input resistor giving

$$K = \frac{-1\ k\Omega}{100\ \Omega + 50\Omega} = -6.667 \tag{38}$$

which yields a $V_{out} = 80\ V$

Notice the effects of loading *on an inverting amplifier.*

3.4 Summary

Type	K	Account for input loading
Non-inverting	$\frac{R_1+R_2}{R_2}$	No
Inverting	$\frac{-R_2}{R_1}$	Yes

Table 1: Op Amp Characteristics