Lecture 10: Maximum Power Transfer

OBJECTIVES:

1. Derive and understand the conditions for Maximum Power Transfer

READING

Required :

- Textbook, sections 3.5, pages 122–125

Optional : None

1 Maximum Power Transfer

As discussed in last lesson, an interface is a connection between two circuits where a signal level can be observed, measured or specified. But what is the maximum signal level that can be transfered across an interface? That is what we will explore this lesson. For the purposes of this discussion, we will only consider examples where both the source circuit and the load circuit are linear resistive circuits; see Figure 1.

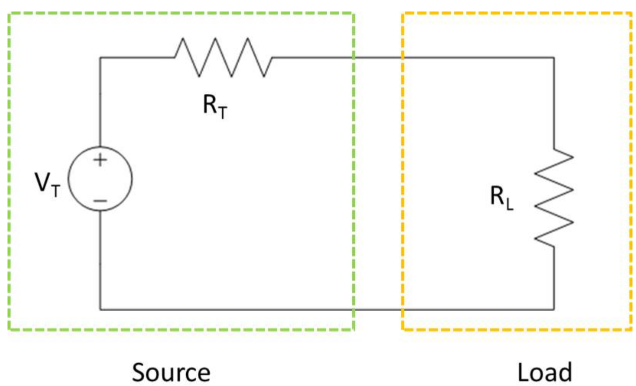


Figure 1: Circuit showing linear source and load

What do we mean by maximum power transfer?

This is the condition when the power consumed by the load is at its maximum.

What are the conditions for maximum power transfer?

Assume in this discussion that we cannot control R_T. This entire discussion will reference Figure 1

Start by writing an equation for the voltage across the load resistor:

$$v_{Load} = \frac{R_L}{R_L + R_T} V_T \tag{1}$$

You should be able to easily recognize that v_{Load} gets larger as R_L gets larger; if $R_L = \infty$ then $v_{Load} = V_T$ which is the max value it can take on.

Next, let's find the current in the loop:

$$i_{Load} = \frac{V_T}{R_L + R_T} \tag{2}$$

It should be easy to see that i_{Load} grows as R_L get smaller. If $R_L = 0$ this $i_{Load} = i_{sc} = \frac{V_T}{R_T}$

But

$$P_{Load} = v_{Load} i_{Load} \tag{3}$$

which, using previous equations, is:

$$P_{Load} = \frac{R_L}{R_L + R_T} V_T \frac{V_T}{R_L + R_T} \tag{4}$$

$$P_{Load} = \frac{R_L V_T^2}{(R_L + R_T)^2} \tag{5}$$

How can we maximize this equation?

Take the derivative (with respect to R_L) and set it equal to zero!

To take this derivative we need to use the quotient rule:

If

$$f(x) = \frac{g(x)}{h(x)} \tag{6}$$

then

$$\frac{\partial f(x)}{\partial x} = \frac{h(x)\partial g(x) - g(x)\partial h(x)}{h^2(x)} \tag{7}$$

So,

$$\frac{\partial P}{\partial R_L} = \frac{(R_L + R_T)^2 V_T^2 - 2R_L V_T^2 (R_L + R_T)}{(R_L + R_T)^4} \tag{8}$$

which will simplify to

$$\frac{\partial P}{\partial R_L} = V_T^2 \frac{R_T - R_L}{(R_L + R_T)^3} \tag{9}$$

If we set that equal to zero

$$V_T^2 \frac{R_T - R_L}{(R_L + R_T)^3} = 0 \tag{10}$$

Then solve for R_L

$$R_L = R_T \tag{11}$$

Now found the value for R_T that give max power, what is the maximum power delivered?
Plug $R_L = R_T$ into the power equation and solve to get

$$P_{Load} = \frac{R_T V_T^2}{(2R_L)^2} = \frac{R_T V_T^2}{4R_T^2} = \frac{V_T^2}{4R_T} = P_{MAX} \tag{12}$$

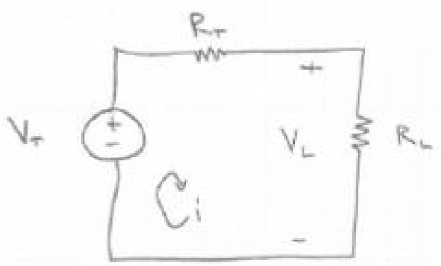
$$P_{MAX} = \frac{V_T^2}{4R_T} = \frac{V_T i_N}{4} = \frac{i_N^2 R_T}{4} \tag{13}$$

Note, P_{MAX} does not imply maximum efficiency too (see book p. 117 for more details).

2 Examples

2.1 Example 1 - Textbook Exercise 3-33

A source circuit delivers 4 V to a 50Ω load and 5 V to a 75Ω load. Find the maximum voltage, current, and power available from the source.



$i = \frac{V_L}{R_L}$

KVL

$$V_T - iR_T - V_L = 0$$

① $V_T - \frac{4}{50} R_T - 4 = 0$

② $V_T - \frac{5}{75} R_T - 5 = 0$

$$\begin{bmatrix} 1 & -0.08 \\ 1 & -0.067 \end{bmatrix} \begin{bmatrix} V_T \\ R_T \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$V_T = 10.15^V$$

$$R_T = 77\Omega$$

MAX VOLTAGE $= V_{oc} = 10.15^V$

MAX CURRENT $= i_{sc} = \frac{V_{oc}}{R_T} = 0.131 = 131\,mA$

MAX POWER $= \frac{(10.15)^2}{4 \cdot 77} = 0.334$ W

2.2 Example 2

For the circuit shown in Figure 2, find the value of R_L that gives maximum power transfer and find the maximum amount of power absorbed by R_L.

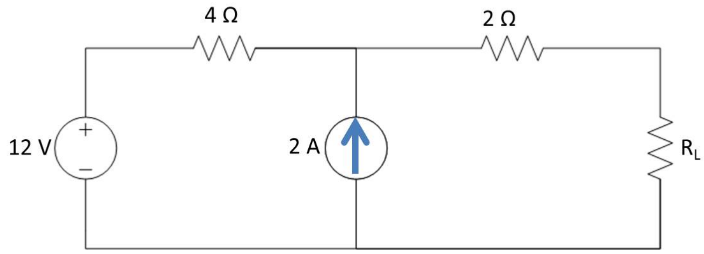
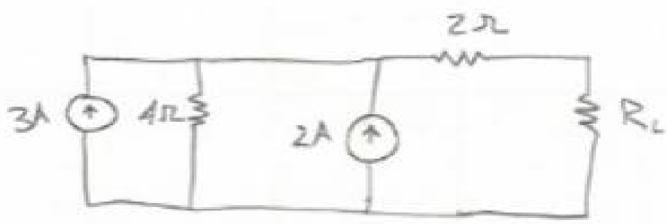
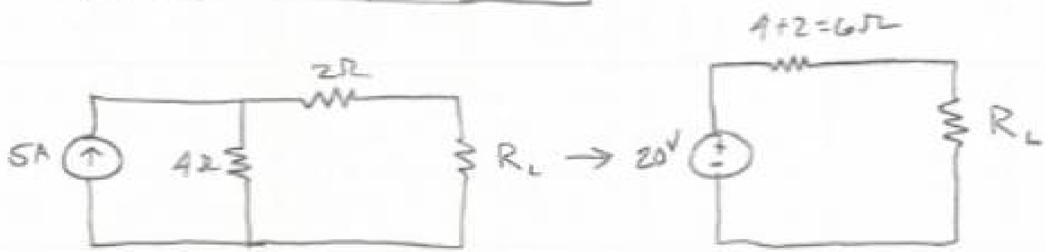


Figure 2: Example 2 Circuit

- USE SOURCE TRANSFORMATION TO SIMPLIFY THE CIRCUIT





R_L FOR MAXIMUM POWER TRANSFER $= R_T$

$$R_L = 6\Omega$$

$$P_{MAX} = \frac{(20V)^2}{4R_T} = 16.7\ W$$

2.3 Example 3

For the circuit shown in Figure 3, find the value of R_L that gives maximum power transfer and find the maximum amount of power absorbed by R_L.

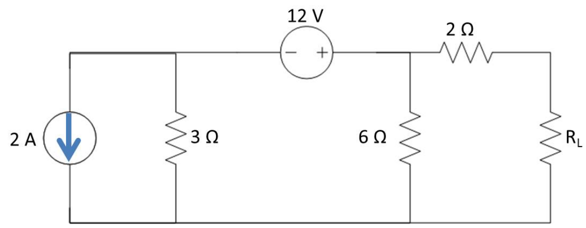
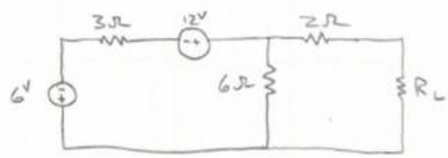
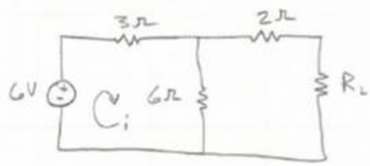


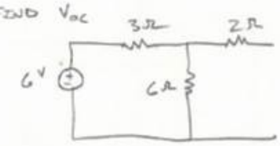
Figure 3: Example 3 Circuit

- USE SOURCE TRANSFORM TO SIMPLIFY THE CIRCUIT



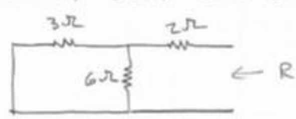


- FIND V_{oc}



$$V_{oc} = \frac{6}{9}(6) = 4^V$$

- FIND R_T USING LOOK BACK



$$R_T = (3 \parallel 6) + 2 = 4\,\Omega$$

$$\boxed{R_L = 4\,\Omega}$$

$$P_{MAX} = \frac{(4^V)^2}{4 \cdot 4} = 1W$$