Lecture 9: Thevenin and Norton Equivalent Circuits

OBJECTIVES:

1. Demonstrate the ability to find Thevenin and Norton equivalent circuits
2. Demonstrate the ability to analyze circuits using Thevenin/Norton equivalent sources

READING

Required :

- Textbook, sections 3.4, pages 109–119

Optional : None

1 Statement of Thevenin's and Norton's Theorems

1.1 Some Preliminary Definitions

Interface – *a connection between two circuits; for our purposes this is normally a two terminal interface*

Source Circuit – *Notionally, the portion of the circuit that provides a signal to the remaining circuit; however, in this context it is really just the portion of the circuit that we will replace with an equivalent circuit. For the theorems to apply, the source circuit must be linear.*

Load Circuit – *Notionally, the portion of the circuit that is consuming power from the source circuit. In this context, it is what ever remails after we have identified our "source" circuit.*

1.2 Statement of Thevenin's Theorem

Thevenin's theorem simply states that the entire source circuit can be replaced with an independent voltage source (v_T) in series with a single resistor (R_T). When this replacement is done, there is no change to the i–v relationship at the source–load *interface*. A Thevenin equivalent circuit is shown in Figure 2(a).

1.3 Statement of Norton's Theorem

Norton's theorem simply states that the entire source circuit can be replaced with an independent current source (i_T) in parallel with a single resistor (R_T). When this replacement is done, there is no change to the i–v relationship at the source–load *interface*. A Norton equivalent circuit is shown in Figure 2(b).

1.4 Another important note

Since both the Thevenin Circuit and the Norton Circuit have identical i–v (at the interface) to the original circuit, they have identical i–v relationships to one another (also at the interface). However, it does not calculate the power dissipated by the source circuit (see p. 109, Figure 3-40 (a) vs (e)).

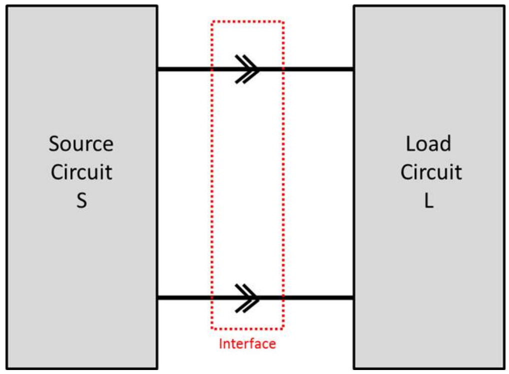


Figure 1: Reference for definitions

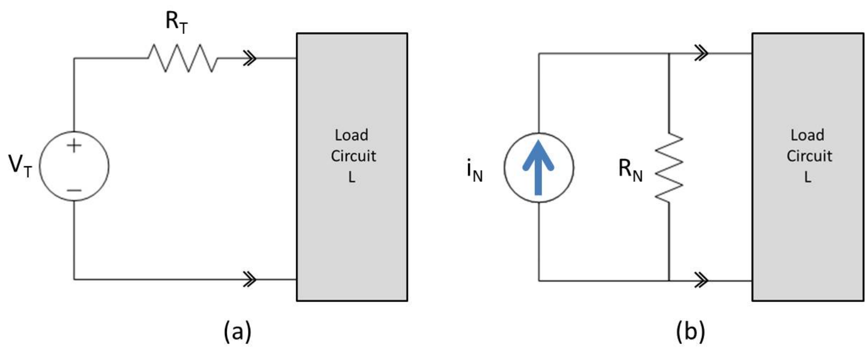


Figure 2: (a) Thevenin and (b) Norton Equivalent Circuits

2 Finding the Thevenin or Norton Equivalent Circuit

How do we go about finding the values of our sources and resistances for the our Thevenin or Norton circuits?

To find a Thevenin voltage, you simply replace the load with an open circuit and find the open circuit voltage, v_{oc}; see Figure 3(a). Then,

$$v_T = v_{oc} \tag{1}$$

To find a Norton current, you simply replace the load with a short circuit and find the short circuit current, i_{sc}; see Figure 3(b). Then,

$$i_N = i_{sc} \tag{2}$$

To find the Thevenin Resistance, there are two methods. The first is to find both v_T and i_N and use

$$R_T = R_N = \frac{v_{oc}}{i_{sc}} = \frac{v_T}{i_N} \tag{3}$$

The other method is called *look-back*. To utilize the look-back method, we short the voltage sources and open the current sources in the **source circuit**. We then calculate the resistance of this new circuit (with the load disconnected). This will give us our R_T; this will be demonstrated in some of our examples below.

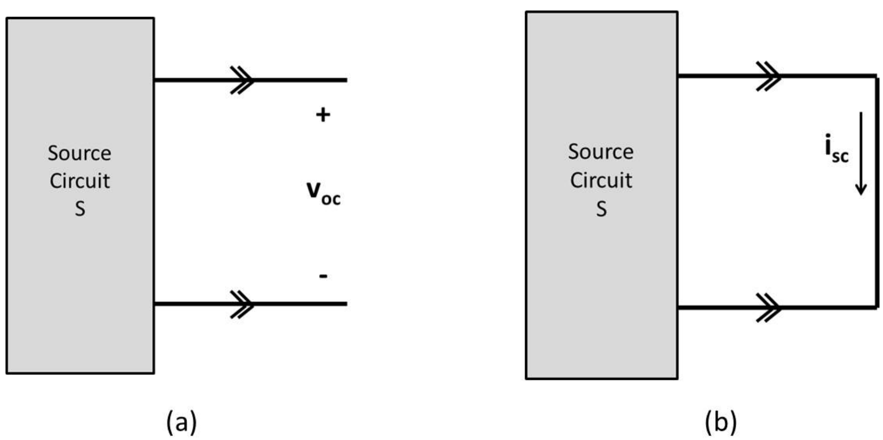


Figure 3: Finding (a) Thevenin Voltages and (b) Norton Currents

3 Examples

3.1 Example 1 - Textbook Exercise 3-28

Find the Thevenin equivalent of the circuit shown in Figure 4. Then find the Norton Equivalent.

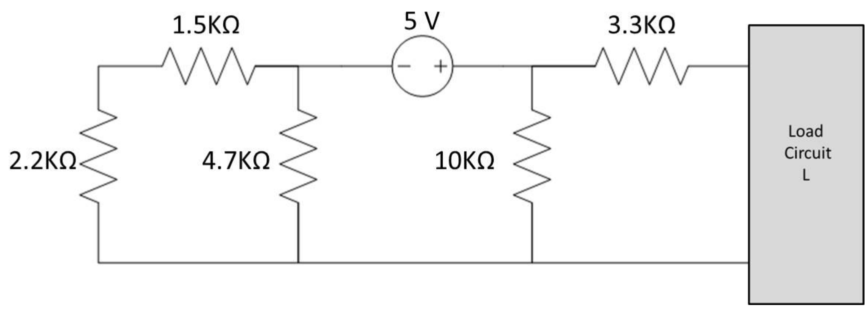
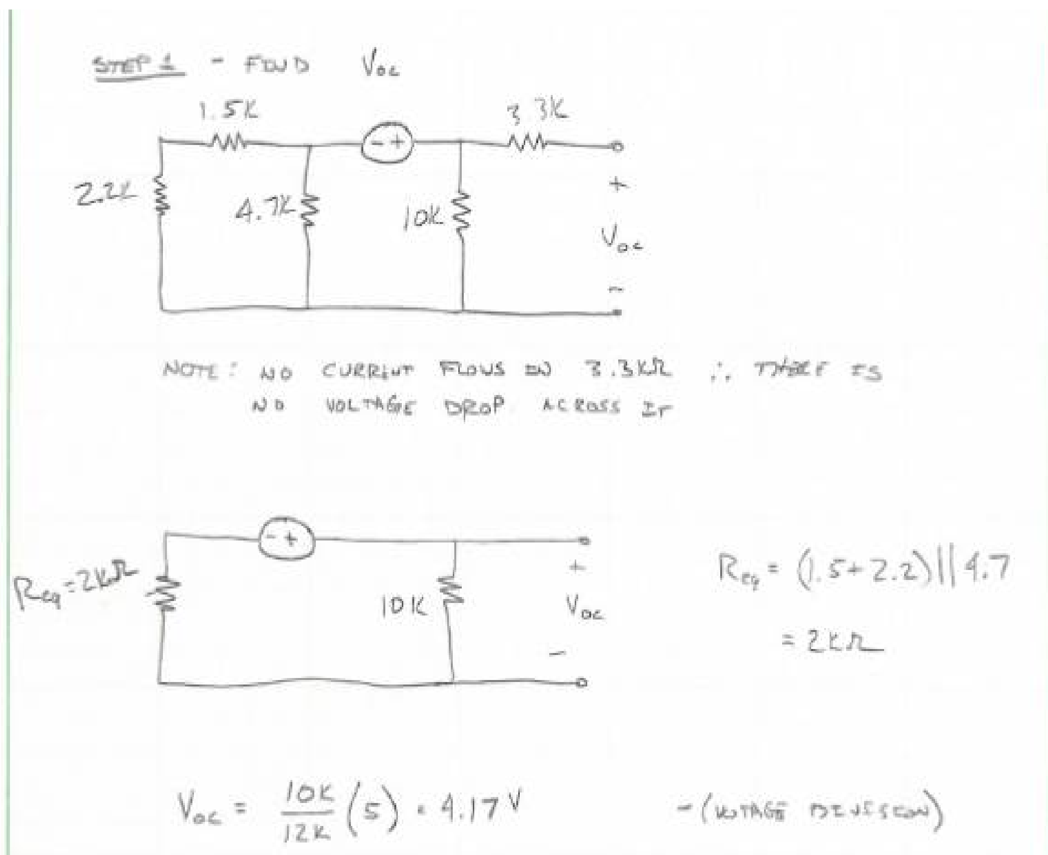


Figure 4: Example 1 Circuit

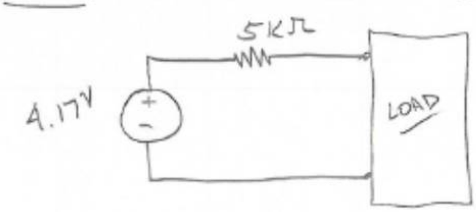


STEP 2 - USE LOOK-BACK TO FIND R_T

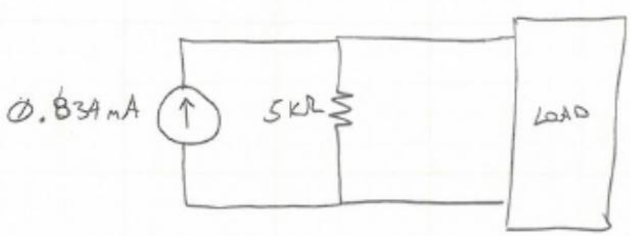
$$R_T = (2 \| 10) + 3.3$$

$$R_T = 5\,k\Omega$$

STEP 3 - DRAW THEVENIN CIRCUIT



STEP 4 - USE SOURCE TRANSFORMATION TO DRAW NORTON CIRCUIT



$$i = \frac{V_T}{R_T} = \frac{4.17}{5K} = 0.834\,mA$$

3.2 Example 2 - Textbook Exercise 3-30

Find the Thevenin and Norton equivalents of the circuit shown in Figure 5. Then find the voltage, current and power delivered to a 50Ω load.

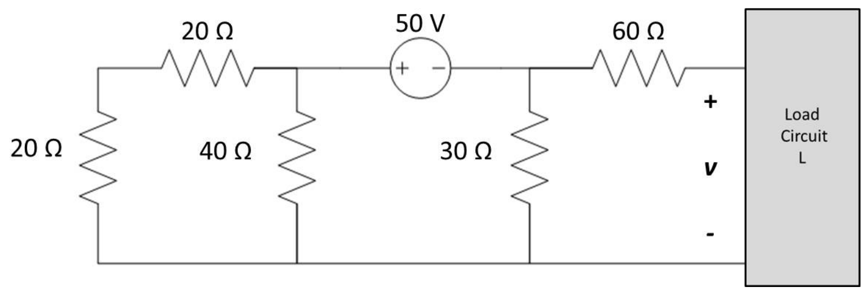
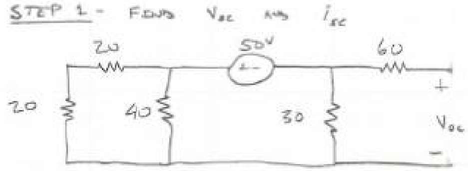
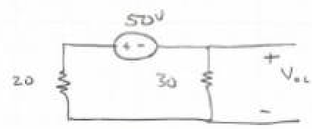


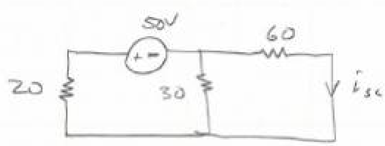
Figure 5: Example 2 Circuit

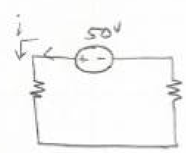
STEP 1 - FIND V_{oc} AND i_{sc}





$$\boxed{V_{oc} = -50\left(\frac{30}{50}\right) = -30V}$$





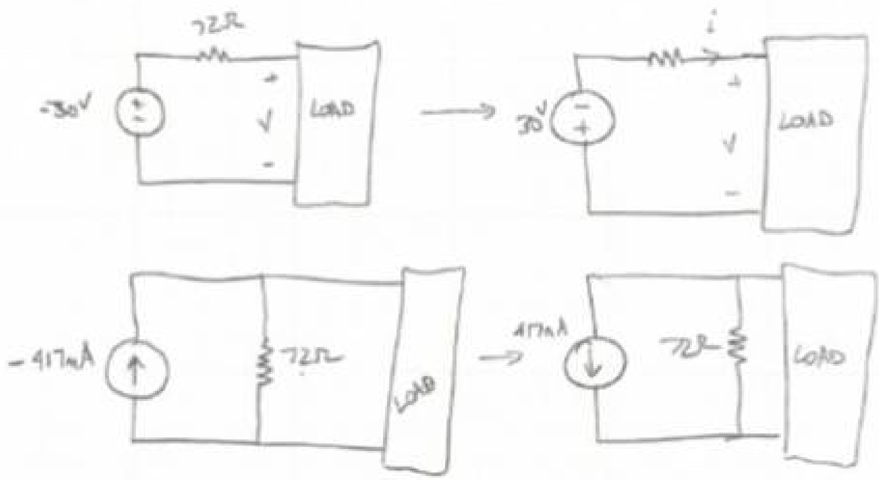
$30 \| 60 = 20$ $\quad$ $i = \frac{50}{40} = 1.25A$

$$i_{sc} = -1.25\left(\frac{30}{90}\right) = -417 \text{ mA}$$ - (CURRENT DIVISION)

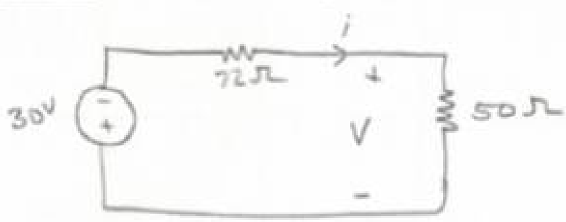
STEP 2 - FIND R_T

$$R_T = \frac{V_{oc}}{i_{sc}} = \frac{-30V}{-417mA} = 72\Omega$$

STEP 3 - DRAW BOTH CIRCUITS



STEP 4 - EVALUATE CIRCUIT FOR 50Ω LOAD



$$V = -30V\left(\frac{50}{122}\right) = -12.3V$$

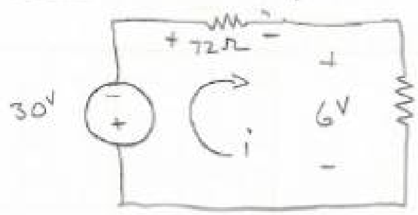
$$i = \frac{-12.3V}{50\Omega} = -245\,mA$$

$$P = 3W$$

3.3 Example 3 - Textbook Exercise 3-31

Find the current and power delivered to an unknown load in Figure 5 if $v = +6\ V$.

STEP 1 - FIND LOOP CURRENT



KVL

$$-30V - i\,72 - 6V = 0$$

$$-i\,72 = 36V$$

$$i = -0.5A$$

$$\therefore P = Vi = (6)(-0.5) = -3W$$

THIS MEANS OUR "LOAD" IS SUPPLYING POWER.... SO THE "LOAD" CONTAINS A SOURCE....

3.4 Example 4

Use Thevenin's theorem to find v_o in Figure 6

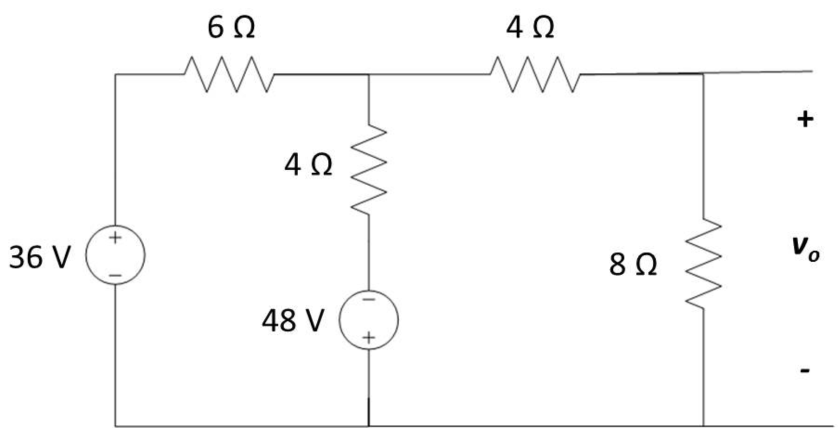
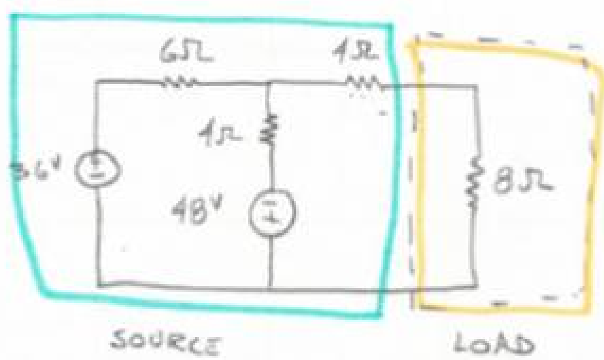


Figure 6: Example 4 Circuit

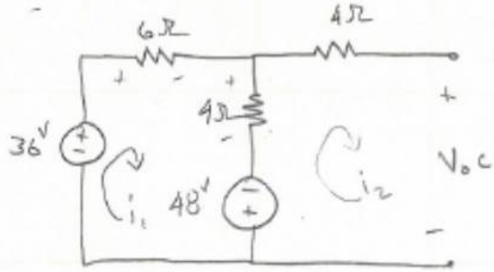
- NOTICE THE OUPUT VOLTAGE IS ACROSS THE 8Ω RESISTOR

  SO $R_L = 8\Omega$

- "BREAK" THE CIRCUIT AS FOLLOWS



- FIND V_{oc}



$i_2 = 0$

KVL (MESH ANALYSIS)

LOOP 1

$$36^V - 6i_1 - 4i_1 + 48V = 0$$

$$10i_1 = 84V$$

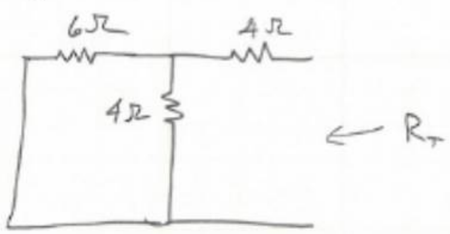
$$i_1 = 8.4\,A$$

LOOP 2

$$-48 + (8.4A)(4\Omega) - V_{oc} = 0$$

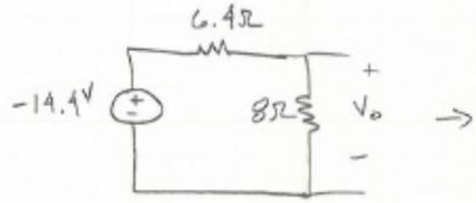
$$V_{oc} = -14.4\,V$$

- FIND R_T USING LOOK BACK



$$R_T = (6 \| 4) + 4 = 6.4\Omega$$

- FIND V_o USING VOLTAGE DIVISION



$$\rightarrow \quad V_o = -14.4^V \left(\frac{8}{14.4}\right) = -8V$$

3.5 Example 5

Use Norton's theorem to find v_o in Figure 7

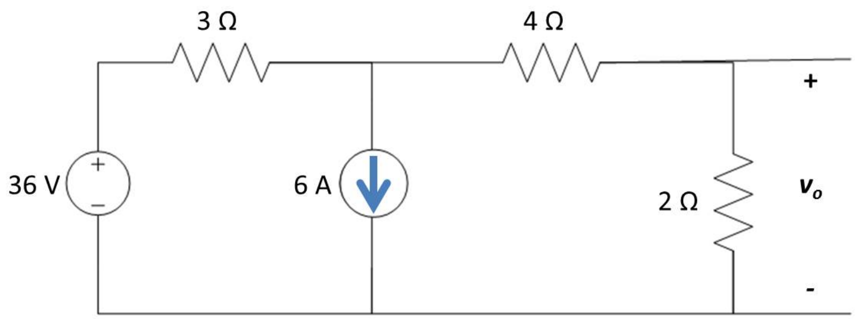
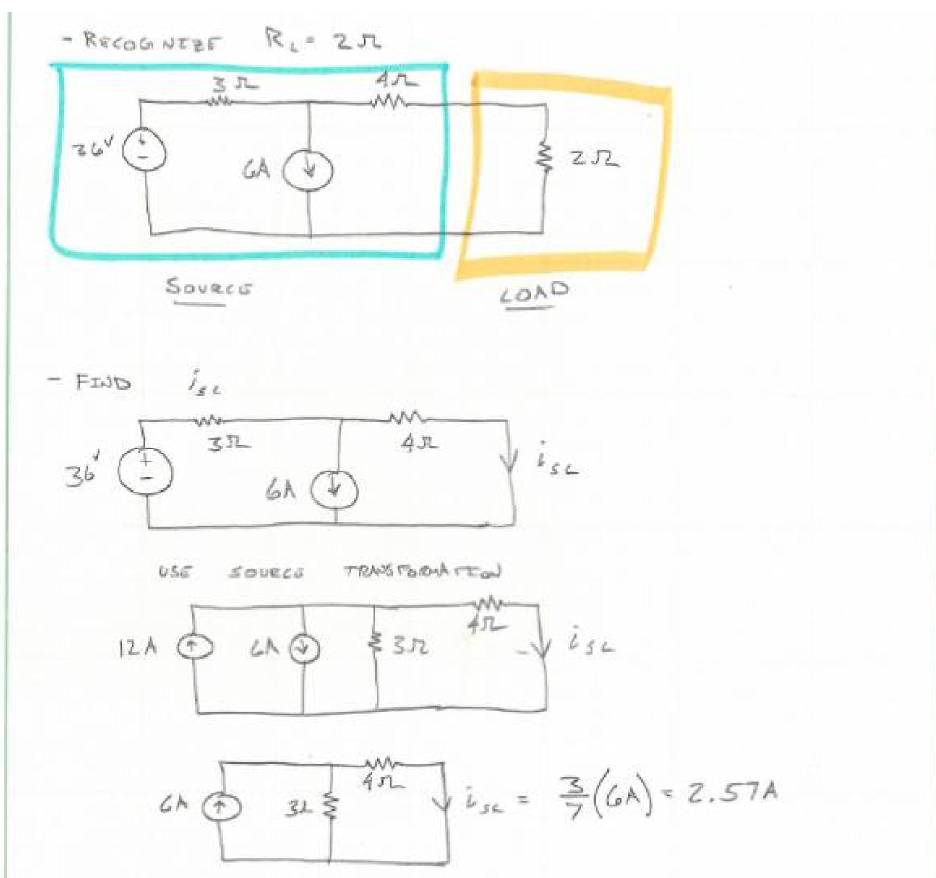
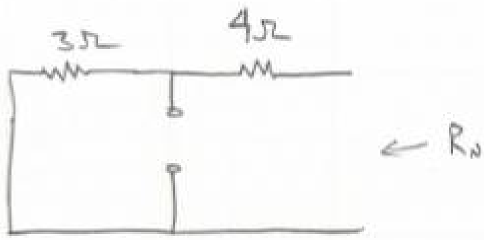


Figure 7: Example 5 Circuit

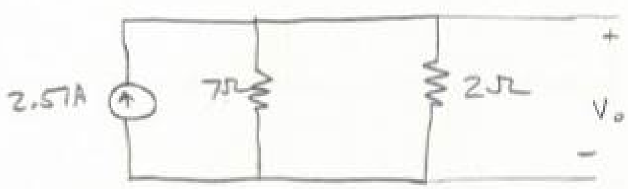


- USE LOOK BACK TO FIND R_N

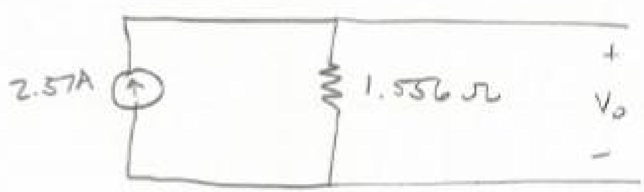


$R_N = 7\Omega$

- DRAW NORTON CIRCUIT



$7\Omega \| 2\Omega = 1.556\Omega$



$V_o = (1.556)(2.57) = 4V$