Lecture 29: Phasors Circuit Analysis and Design Part 1

OBJECTIVES:

1. Learn to apply circuit analysis techniques we already know to solve circuits in the phasor domain

READING

Required : Textbook, section 8.3, pages 391–402

Optional :

1 Introduction

Over the next couple of lessons we will be applying techniques we have already learned to phasor domain circuits. The only thing that will be different is that we will be working with phasors (complex numbers) instead of real numbers. If you can get over that hump this will feel like a lot of review. We will look at:

1. Parallel and series combinations of **Impedances**
2. Voltage and Current Division
3. Mesh and Node Analyisis
4. Proportionality and Superposition
5. Thevenin and Norton equivalent circuits

All these things still apply to phasor domain circuits. As a reminder, phasor analysis only applies to the steady state sinusoidal responses. Phasor analysis does not tell you anything about the transient response.

2 Resistance and Reactance

In the way of some quick preliminaries we will introduce a new term, **reactance**. We saw in last class that a phasor is just a complex number. Any complex number can be written as $Z = R + jX$ where R is the real part and X is the imaginary part. When we use complex numbers to represent impedances, Z, we call the real part, R, resistance and the imaginary part, X, the reactance. R is always positive, but X can be positive or negative. A positive X is called an *inductive* reactance, while a negative X is a *capacitive* reactance.

3 Series and Parallel Impedances

Good News! We have nothing new to learn here... Remember our derivation about how to add resistances? It all started with KVL, KCL and Ohm's law. Since all those laws apply to impedances, impedances add like resistance.

For series impedances:

$$Z_{eq} = Z_1 + Z_2 + Z_3 + ... + Z_k = \sum_{n=1}^{k} Z_n$$

For parallel impedances:

$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + ... + \frac{1}{Z_k}} = \left[\sum_{n=1}^{k} \frac{1}{Z_n}\right]^{-1}$$

Example 1 – For the circuit shown in Figure 1, find Z_{eq} given $\omega = 2000\ \frac{rad}{s}$

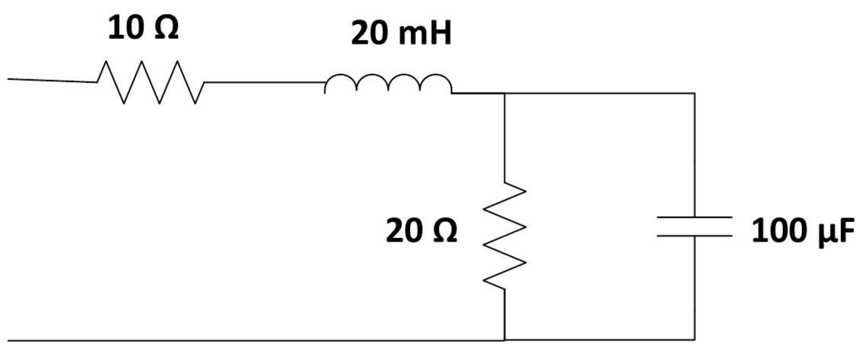


Figure 1: Circuit to accompany Example 1

STEP 1 - CONVERT CAPACITANCE & INDUCTANCE TO IMPEDANCE

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{(2000)(100\times10^{-6})} = -j5 = 5\angle -90^\circ$$

$$Z_L = j\omega L = j(2000)(20\times10^{-3}) = j40 = 40\angle 90^\circ$$

STEP 2 - FIND Z_{eq}

$$20 \| 5\angle -90 = 4.85\angle -76^\circ$$

$$Z_{eq} = 10 + 40\angle 90^\circ + 4.85\angle -76^\circ = 37\angle 72.4^\circ$$

4 Voltage Division

We will use some examples to show that voltage division also works with phasors. Remember, voltage division applies to resistances (now impedances) in series.

Example 2– For the circuits shown in Figures 2 and 3, use voltage division to find the voltages across all resistors and the capacitor.

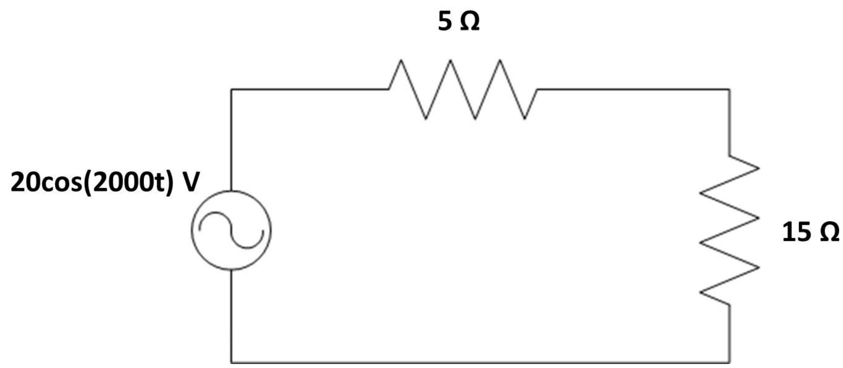


Figure 2: Circuit to accompany example 2

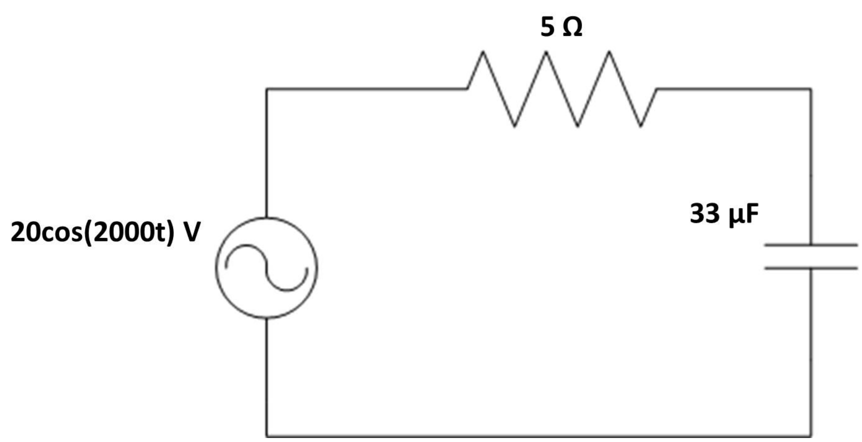
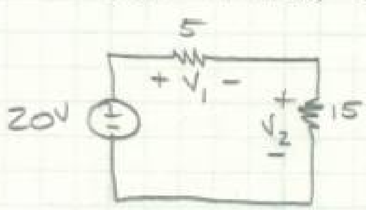


Figure 3: Circuit to accompany example 2

CIRCUIT 1

REDRAW PHASOR CIRCUIT



$V_1 = 20\left[\frac{5}{20}\right] = 5V$

$V_1(t) = 5\cos(2000t)V$

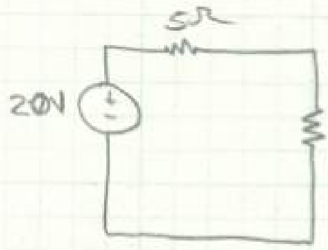
$V_2 = 20\left[\frac{15}{20}\right] = 15V$

$V_2(t) = 15\cos(2000t)$

CIRCUIT 2

DRAW PHASOR CIRCUIT

$\omega = 2000\ \frac{RAD}{s}$



$\frac{-j}{\omega C} = -j15$
$= 15\angle -90°$

$$V_c = 20\left[\frac{15\angle -90°}{5+15\angle -90°}\right] = 18.97\angle -18.3°\ V$$

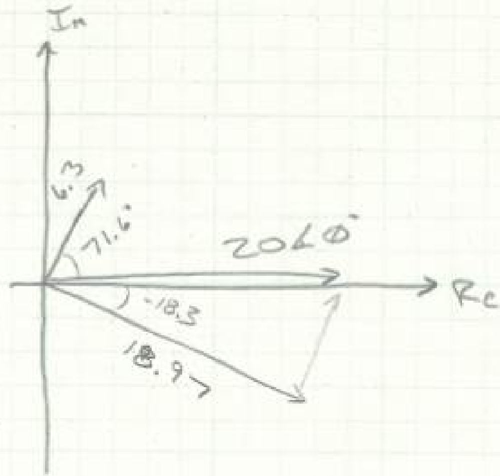
$$V_c(t) = 18.97\cos(2000t - 18.3°)$$

$$V_R = 20\left[\frac{5}{5+15\angle -90°}\right] = 6.3\angle 71.6°$$

$$V_R(t) = 6.3\cos(2000t + 71.6°)\ V$$

CHECK WITH KVL

$$18.97\angle -18.3 + 6.3\angle 71.6° = 20\angle 0°$$



5 Current Division

Now we move onto some examples to show that current division also works with phasors. Remember, current division applies to resistances (now impedances) in parallel.

Example 3– For the circuit in Figure 4, use current division to find the current through the inductor.

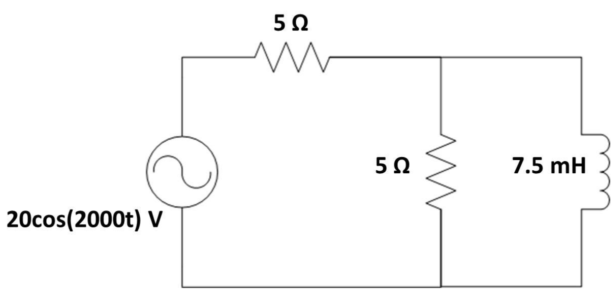
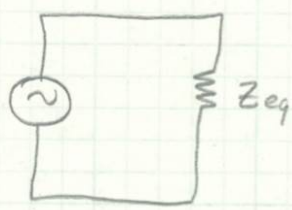


Figure 4: Circuit to accompany example 3

STEP 1 - FIND Z_{eq}



$\omega = 2000$

$$Z_{eq} = 5\Omega + [5\Omega \| j15] \quad j\omega L$$

$$= 5 + \frac{5(j15)}{5+j15}$$

$$= 9.6 \angle 8.9^\circ \Omega$$

STEP 2 - FIND I_{TOTAL}

$$I_{TOTAL} = \frac{V}{Z_{eq}} = \frac{20\angle 0^\circ}{9.6\angle 8.9^\circ} = 2.1\angle -8.9^\circ A$$

STEP 3 - USE CURRENT DIVISION TO FIND I_L

$$I_L = 2.1\angle -8.9\left[\frac{5}{5+j15}\right] = 664\angle -80.2^\circ \text{ mA}$$

$$i_L(t) = 664 \cos(2000t - 80.2^\circ) \text{ mA}$$

$$I_R = 2.1\angle -8.9\left[\frac{j15}{5+j15}\right] = 2\angle 9.45^\circ A$$

$$i_R(t) = 2 \cos(2000t + 9.45^\circ) A$$

6 Mesh Analysis

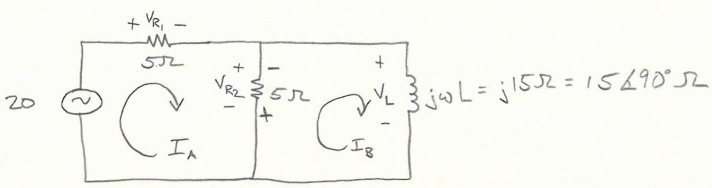
Recall the steps for mesh analysis:

1. Assign a current to each mesh
2. Assign a voltage (magnitude and polarity) to each device in the circuit
3. Write Kirchhoff's Voltage Law (KVL) equations for each mesh
4. Use device i–v characateristics to rewrite KVL eqations from the previous step in terms of mesh currents
5. Rewrite equations in standard (matrix) form & solve

Also recall that the rules above specifically apply to circuits that do not contain any current sources. We offered three techniques for dealing with current sources: source transformation, ensure the current source is only part of one mesh, or use a supermesh.

Example 4– Let's re-solve example 3 using Mesh Analysis.

STEP 1 – DRAW THE 'PHASOR' CIRCUIT AND ASSIGN MESH CURRENTS



STEP 2 – ASSIGN VOLTAGE POLARITIES (SEE ABOVE)

NOTE: V_{R2} HAS A DIFFERENT POLARITY DEPENDING ON WHICH MESH. THIS IS A BY PRODUCT OF THE PASSIVE SIGN CONVENTION

STEP 3 – WRITE KVL EQUATIONS

A $V_{R1} + V_{R2} = 2\emptyset$

B $V_{R2} + V_L = \emptyset$

STEP 4 – USE MESH CURRENTS TO REWRITE EQUATIONS ABOVE

A $5 I_A + 5(I_A - I_B) = 2\emptyset$

B $5(I_B - I_A) + 15\angle 90° \ I_B = \emptyset$

STEP 5 – STANDARD FORM & SOLVE

$$\begin{bmatrix} 10 & -5 \\ -5 & 5+j15 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 2\emptyset \\ \emptyset \end{bmatrix}$$

FROM MATLAB

$I_A = 2.\emptyset 7 \angle -8.9$

$I_B = 658 \angle -80.5° \text{ mA} \leftarrow I_L$

7 Node Voltage Analysis

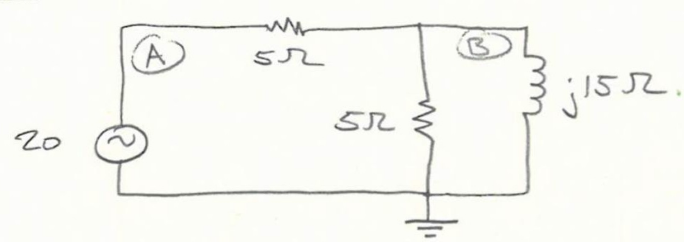
Recall the steps for Node Voltage Analysis:

1. Identify a **reference node**. You will not write an equation for this node, but the voltages at the other nodes will use this as a reference.
2. Write **KCL equations** at the other $N - 1$ nodes
3. Write the currents in the KCL equations in terms of **node voltages** and resistances
4. Rearrange the equations above into **standard form**

Also recall that the steps above apply strictly to circuits with no voltage sources, but we offered three methods for dealing with voltage sources: source transformation, smart choice of a reference node, or super-nodes.

Example 5 – Let's rework example 3 once more, this time with Node Voltage Analysis.

STEP 1 - DRAW THE PHASOR CIRCUIT & IDENTIFY THE REFERENCE NODE



STEP 2/3 - WRITE NODE VOLTAGE EQUATIONS

A $\quad V_A = 20$

B $\quad \dfrac{V_B - 20}{5\Omega} + \dfrac{V_B}{5\Omega} + \dfrac{V_B}{j15} = 0$

STEP 4 - PUT INTO STANDARD FORM & SOLVE

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{2}{5} - j\frac{1}{15} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 20 \\ 4 \end{bmatrix}$$

$V_A = 20 \qquad V_B = 9.8 \angle 9.4^\circ$

$$I_L = \frac{V_B}{j15} = 0.657 \angle -80.5$$