ECE 231 Quiz 6B (Fall 2016)

NAME: Chess-Key ________/33 = ________%

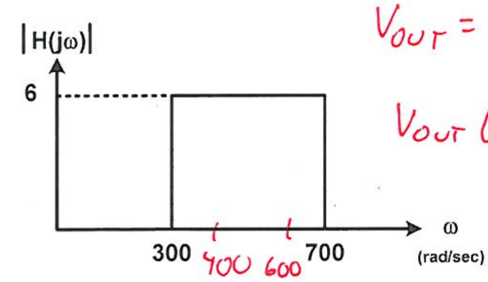
Problem 1: **(1pt each)** Please answer true or false

TRUE FALSE a. The cutoff frequency of the high-pass filter must be ~~above~~ below the cutoff frequency of the low-pass filter to create a band-pass filter by cascading the filters

TRUE FALSE b. Given $|H|_{dB} = 20\log_{10}|H|_{abs}$ and that $|H|_{dB}$ (gain in decibel) is a large negative number, this then means that $|H|_{abs}$ (gain in absolute) is a very small positive number

TRUE FALSE c. The cutoff frequencies for a band-pass filter will only correspond to the cutoff frequencies of cascaded low-pass and high-pass filters if those frequencies are sufficiently far apart

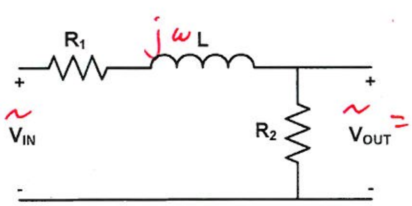
Problem 2: **(4pts)** Given the "ideal" BP filter below and the following input voltage expression, $v_{in}(t) = 2V\cos\left(\cancel{200t + 20^\circ}\right) + 4V\cos\left(400t + 40^\circ\right) + 6V\cos\left(600t\right)$, determine the filter output expression $v_{out}(t)$



$$V_{OUT} = 4V \times 6\cos(400t + 40^\circ) + 6V \times 6\cos(600t)$$

$$V_{OUT}(t) = 24V\cos(400t + 40^\circ) + 36V\cos(600t)$$

Problem 3: **(7pts)** Given the following time-domain circuit, derive the transfer function and then evaluate the peak passband gain (K) and cutoff frequency (ω_c) provided that $R_1 = 400\Omega$, $R_2 = 600\Omega$, $L = 20mH$



$$\tilde{V}_{OUT} = \frac{R_2}{R_1 + R_2 + j\omega L}\tilde{V}_{IN} \rightarrow H(j\omega) = \frac{\hat{V}_{OUT}}{\tilde{V}_{IN}} = \frac{R_2}{R_1 + R_2 + j\omega L} \times \frac{\frac{1}{R_1+R_2}}{\frac{1}{R_1+R_2}}$$

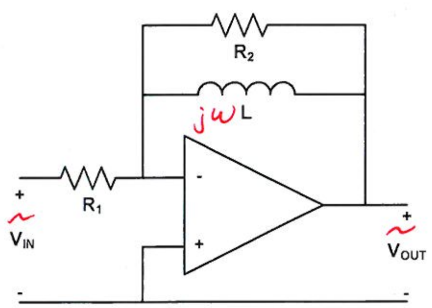
$$H(j\omega) = \frac{\frac{R_2}{R_1+R_2}}{1 + j\omega\frac{L}{R_1+R_2}} \sim \frac{K}{j\frac{\omega}{\omega_c} + 1}$$

$$\rightarrow \quad \omega_c = \frac{R_1 + R_2}{L} = \frac{400 + 600}{.02} = 50 \text{krad/s}$$

$$K = \frac{R_2}{R_1 + R_2} = \frac{600}{400 + 600} = 0.6$$

Problem 4: **(13pts)** Given the following circuit in the time domain

a. **(7pts)** Derive the transfer function and determine whether this is low-pass or high-pass filter



$$H = \frac{\tilde{V}_{OUT}}{\tilde{V}_{IN}} = \frac{-Z_F}{Z_i} = \frac{-(R_2 // j\omega L)}{R_1}$$

$$= \left(\frac{-R_2 \times j\omega L}{R_2 + j\omega L}\right) / R_1 \times \frac{1/R_2}{1/R_2}$$

$$= \frac{-j\omega L/R_1}{1 + j\omega L/R_2} \rightarrow \underline{\text{highpass}}$$

b. **(6pts)** If $R_2 = 80k\Omega$, find L and R_1 so that the peak passband gain ($|K|$) is 10 and the cutoff frequency (ω_c) is 4,000rad/s

$$K j\omega \frac{L}{R_2} = -j\omega \frac{L}{R_1}$$

$$\rightarrow \omega_c = \frac{R_2}{L}$$

$$\rightarrow K = \frac{-R_2}{R_1}$$

$$L = \frac{80K}{4K} = \boxed{20H}$$

$$|K| = \frac{80K}{R_1} = 10 \rightarrow \boxed{R_1 = 8k\Omega}$$

Problem 5: **(6pts)** Given the following transfer function for a real filter: $H(j\omega) = \dfrac{j20\omega}{j\omega + 200}$, if the input to this filter is $v_{in}(t) = 6V\cos(200t + 30^\circ)$, find the magnitude scaling and phase shift that this filter provides and then write the expression for $v_{out}(t)$

$$H(j200) = \frac{j20(200)}{j(200) + 200} = \frac{j4000}{j200 + 200} = \boxed{14.142 \angle 45^\circ}$$

calculator

$$V_{OUT}(t) = 6V \times 14.142 \cos(200t + 30^\circ + 45^\circ)$$

$$\boxed{V_{OUT}(t) = 84.852V \cos(200t + 75^\circ)}$$