

---

Lecture 8: Linearity and Superposition**OBJECTIVES:**

1. Understand what is meant by a *Linear* Circuit
2. Understand and be able to apply the principle of superposition

**READING****Required :**

- Textbook, sections 3.3, pages 100–109

**Optional :** None**1 What does it mean for a circuit to be Linear?**

Circuits are linear when they can be modeled using only linear elements and independent sources. Further any linear circuit will obey the following 2 principles:

**Homogeneity** : outputs are proportional to the inputs. This is also referred to as **proportionality**.

**Additivity** : output due to multiple inputs can be found by finding the output due to each individual input and then adding them. This is also referred to as **superposition**; we will devote an entire section to this later.

Mathematically these properties are written as:

$$f(Kx) = Kf(x) \quad \text{proportionality} \quad (1)$$

$$f(x_1 + x_2) = f(x_1) + f(x_2) \quad \text{superposition} \quad (2)$$

## SOLUTION

ECE231: Electrical Circuits and Systems I - Spring 2016

### Lecture 8: Linearity and Superposition

---

**Example 1:** Solve for  $v_{out}$  in the circuit in Figure 1 given  $v_{in} = 1V$ . Then using the proportionality principle, find the output given  $v_{in} = 12V$

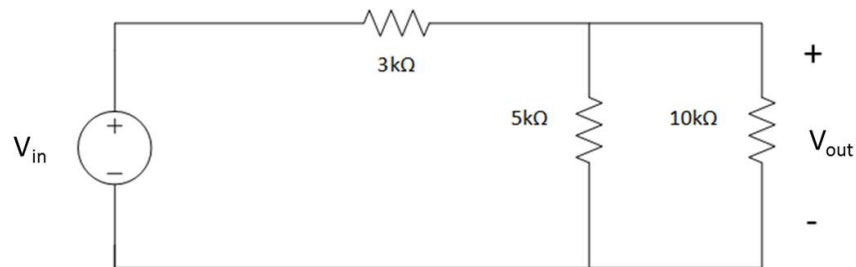


Figure 1: Circuit to accompany example 1

STEP 1: COMBINE THE  $5k\Omega$  &  $10k\Omega$  RESISTORS

$$R_{eq} = \frac{(5 \times 10^3)(10 \times 10^3)}{15 \times 10^3} = 3.33 k\Omega$$

STEP 2: USE VOLTAGE DIVISION TO FIND  $V_{out}$

$$V_{out} = (1V) \left( \frac{3.3k\Omega}{6.3k\Omega} \right) = 0.524 V$$

STEP 3: USE PROPORTIONALITY TO FIND  $V_{out}$  WITH  $V_{in} = 12V$

$$V_{out-12} = \left[ \frac{12V}{1V} \right] (0.524V) = 6.28 V$$

This also provides a useful tool, the voltage divider, which is just a ratio proportion of voltage drops across resistors.

$$\frac{v_{in}}{R_1 + R_2} = \frac{v_{out}}{R_2} \quad (3)$$

$$v_{out} = \frac{R_2}{R_1 + R_2} v_{in} \quad (4)$$

## SOLUTION

### ECE231: Electrical Circuits and Systems I - Spring 2016 Lecture 8: Linearity and Superposition

---

## 2 Superposition (Additivity)

The goal of superposition is to simplify the analysis of circuits with multiple sources by only having to deal with a single source at a time. Notice, I said it is simpler not shorter.

*The steps for circuit analysis using superposition are:*

- 1. Turn off all but one of the independent sources in the circuit*
- 2. Solve for output of the circuit based on the one remaining source*
- 3. Repeat for every source in the circuit*
- 4. Sum the results to get a total output*

**What do we mean by turning off sources?**

**To turn off a voltage source,** we would want the voltage across the two nodes of the source to be zero, so we replace the source with a short circuit

**To turn off a current source,** we want zero current to flow, so we replace it with an open circuit

**Example 2:** Without solving anything else, re-draw the circuit in Figure 2 with each source turned off.

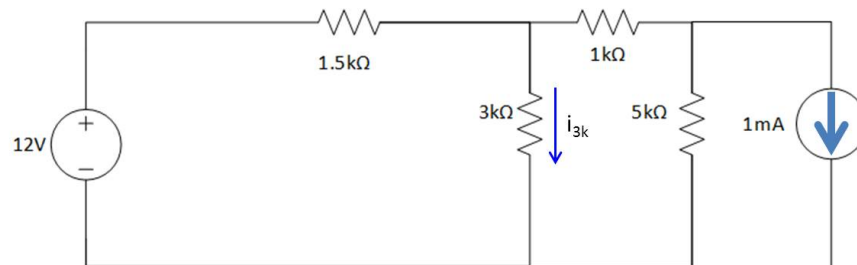
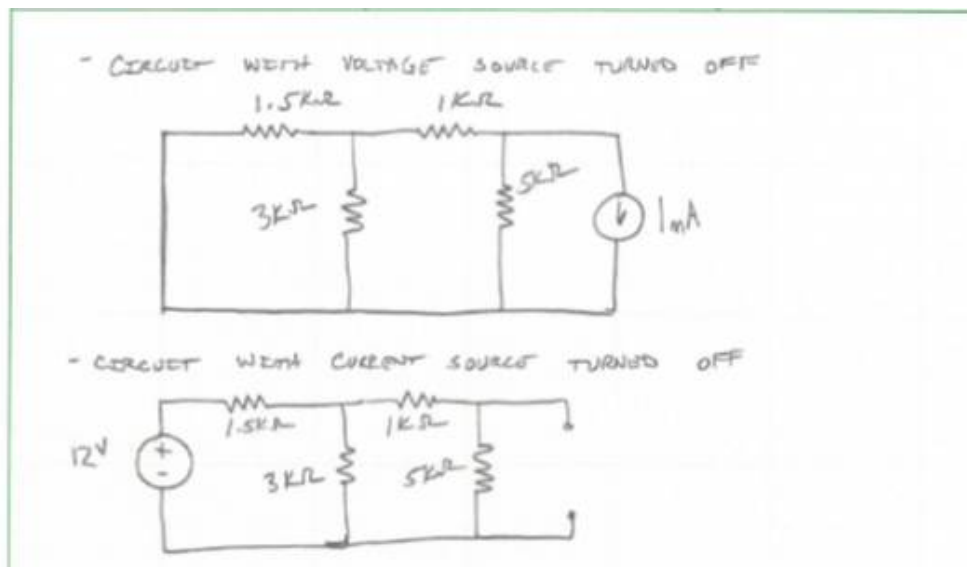


Figure 2: Circuit to accompany example 2



## SOLUTION

### ECE231: Electrical Circuits and Systems I - Spring 2016 Lecture 8: Linearity and Superposition

---

**Example 3:** (Textbook Exercise 3-25) – The circuit in Figure 3 contains two  $R$ - $2R$  modules. Use superposition to find  $v_O$ .

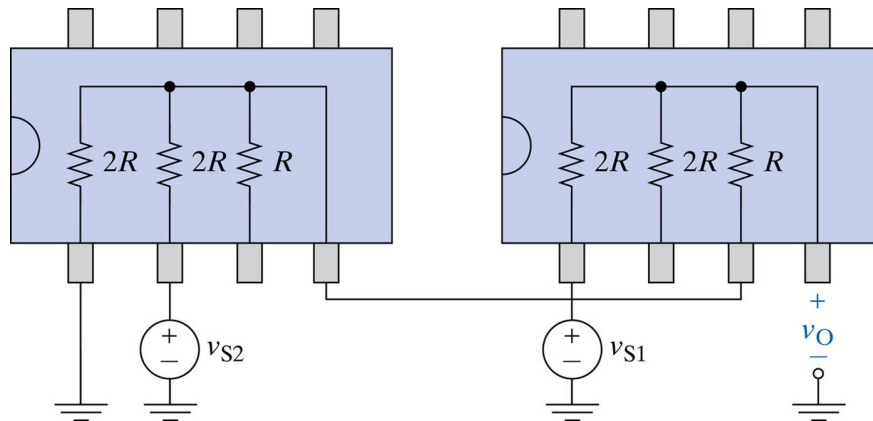
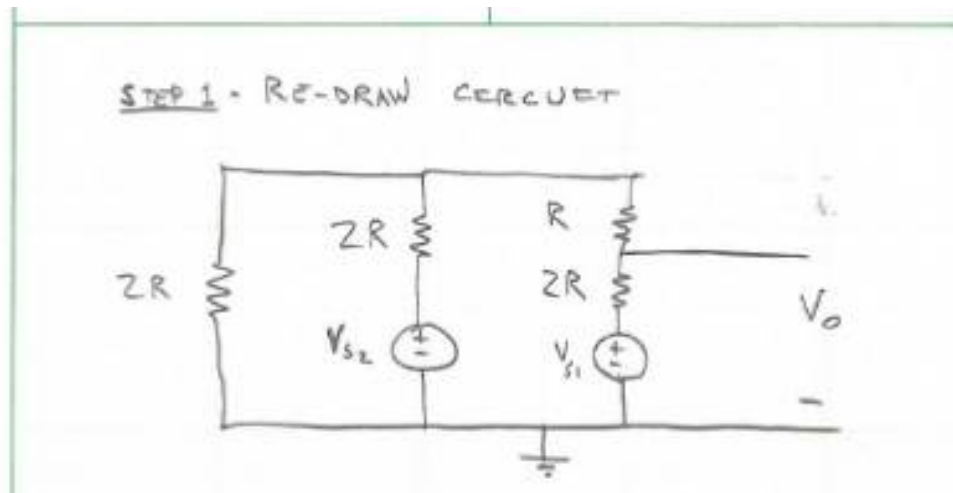


Figure 3: Circuit to accompany example 3. NOTE: In images taken from the textbook, crossed wires are only connected if they are *dotted*. For circuits that I draw on the board or ones I generate for notes, crossed wires are connections. You will need to learn to infer from context..... If there are some *dotted* nodes, crossed wires without dots are not connected; if there are no dots, crossed wires are connected.

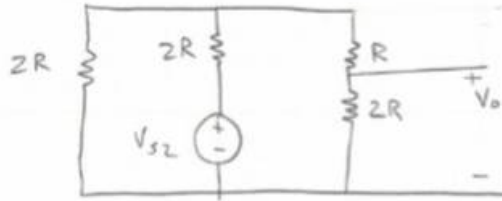


## SOLUTION

### ECE231: Electrical Circuits and Systems I - Spring 2016 Lecture 8: Linearity and Superposition

---

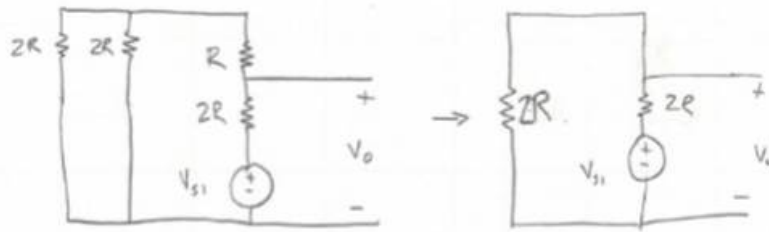
STEP 2 - "TURN OFF"  $V_{s1}$  & SOLVE FOR  $V_o$



2R || 3R =  $\frac{6}{5}R$

$$V' = \left( \frac{\frac{6}{5}R}{\frac{6}{5}R + \frac{16}{5}R} \right) V_{s2} = \frac{3}{8} V_{s2}$$
$$V_o = \frac{2}{3} V' = \frac{2}{3} \left( \frac{3}{8} V_{s2} \right) = \frac{1}{4} V_{s2}$$

STEP 3 - "TURN OFF"  $V_{s2}$  & SOLVE FOR  $V_o$



- USE VOLTAGE DIVISION TO FIND  $V_{2R}$

$$V_{2R} = \frac{2R}{4R} V_{s1} = \frac{V_{s1}}{2}$$

$$V_o = V_{2R}$$

STEP 4 - ADD OUTPUTS FROM STEPS 2 & 3

$$V_o = \frac{1}{2} V_{s1} + \frac{1}{4} V_{s2}$$

## SOLUTION

ECE231: Electrical Circuits and Systems I - Spring 2016

### Lecture 8: Linearity and Superposition

**Example 4:** (Textbook Exercise 3-26) – Repeat the above example, but replace  $v_{s2}$  in Figure 3 with a current source,  $i_{s2}$  with the reference arrow pointed toward ground

STEP 1 - DRAW THE CIRCUIT

STEP 2 - "TURN OFF"  $V_{s1}$

USE CURRENT DIVIDER TO FIND CURRENT IN OUTPUT RESISTOR

$$i_2 = -i_{s2} \left[ \frac{2R}{5R} \right] = -\frac{2i_{s2}}{5}$$

$$V_o = -2R \cdot \frac{2}{5} i_{s2} = -\frac{4}{5} i_{s2} R$$

STEP 3 "TURN OFF"  $i_{s2}$

$$V_o = \frac{3R}{5R} V_{s1} = \frac{3}{5} V_{s1}$$

STEP 4 - ADD OUTPUTS FROM PREVIOUS STEPS

$$V_o = \frac{3}{5} V_{s1} - \frac{4}{5} i_{s2} R$$

## SOLUTION

### ECE231: Electrical Circuits and Systems I - Spring 2016 Lecture 8: Linearity and Superposition

**Example 5:** Use Superposition to find  $v_O$  in Figure 4

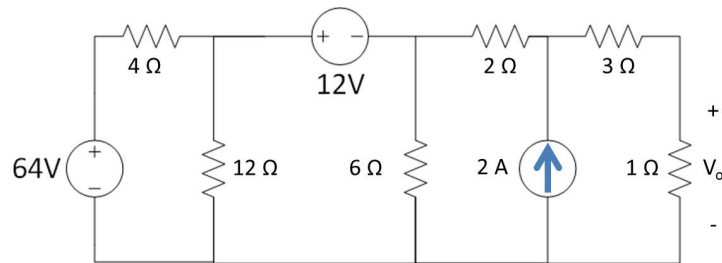


Figure 4: Circuit to accompany example 5

STEP 1 - TURN OFF BOTH VOLTAGE SOURCES

$R_g = 4\Omega$

$R_g = (4 \parallel 12 \parallel 6) + 2 = 4\Omega$

USE CURRENT DIVISION TO FIND RIGHT SIDE CURRENT

$$i_r = 2A \left( \frac{1}{8} \right) = 1A$$

USE OHM'S LAW TO FIND  $V_o$

$$V_o = (1A)(1\Omega) = 1V$$

STEP 2 - TURN OFF CURRENT SOURCE & 12V SOURCE

$V' = (2.4/6.4) 64V = 24V$

USE VOLTAGE DIVISION TO FIND  $V'$

APPLY VOLTAGE DIVISION AGAIN TO FIND  $V_o$

$$V_o = \frac{1}{6} (24V) = 4V$$

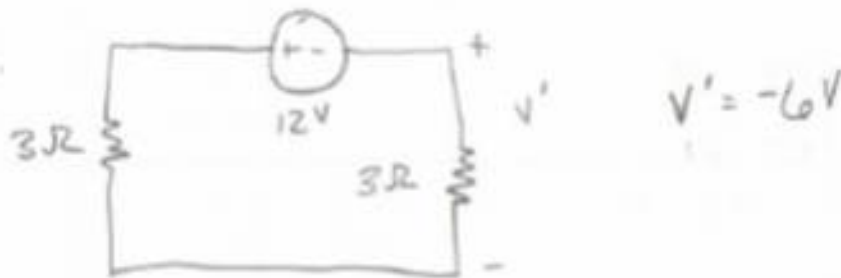
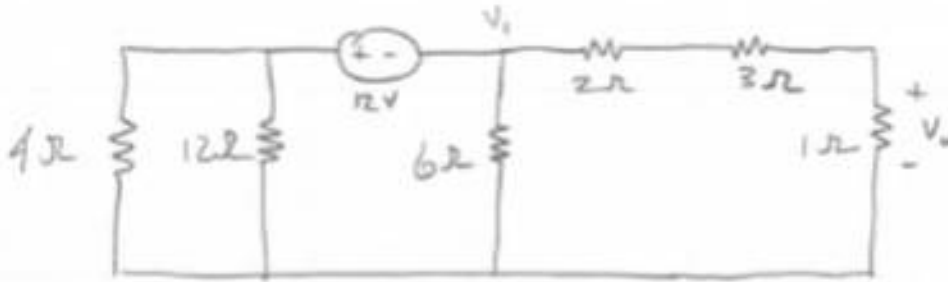
## SOLUTION

ECE231: Electrical Circuits and Systems I - Spring 2016

### Lecture 8: Linearity and Superposition

---

STEP 3 - TURN OFF CURRENT SOURCE &  $6A$  SOURCE



USE VOLTAGE DIVISION TO FIND  $V_o$

$$V_o = \frac{1}{6}(-6V) = -1V$$

STEP 4 - SUM PREVIOUS STEPS TO GET  $V_o$

$$V_o = 1V + 4V - 1V = 4V$$