Lecture 15: Operational Amplifiers - Part 2

OBJECTIVES:

1. Starting with the ideal Op Amp model, develop circuits for Summing and Subtracting Amplifiers

READING

Required :

- Textbook, section 4.4, pages 190–201

Optional : None

1 The Summing Amplifier

Often in circuit design, we will need to add to signals together – this CANNOT be down by simply connecting two wires, although many of you in your solutions will try that method. To add two signals we must use a device called an *adder* or *summing amplifier*. In this section we will design a weighted (and inverting) summing amplifier.

1.1 Derivation

Let's start by looking at the inverting Op Amp discussed in last class (see Figure 1)

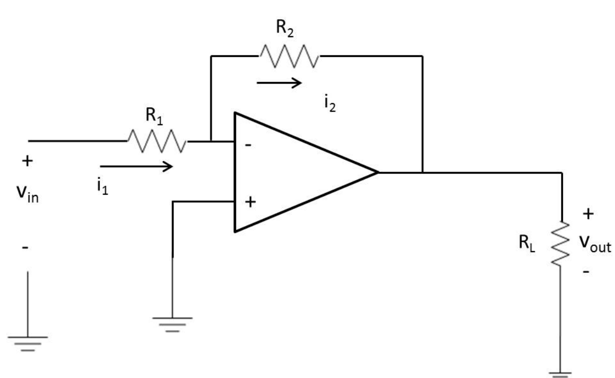


Figure 1: Inverting Op Amp

Recall that the transfer characterisitic for this amplifier is:

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} \tag{1}$$

which can also be written as

$$V_{out} = -\frac{V_{in}}{R_1} R_2 \tag{2}$$

but since $\frac{V_{in}}{R_1} = i_1$ we can say the output voltage is really determined by the input current, i_1.

Are there other ways of increasing the input current without increaseing V_{in}? Consider the circuit shown in Figure 2

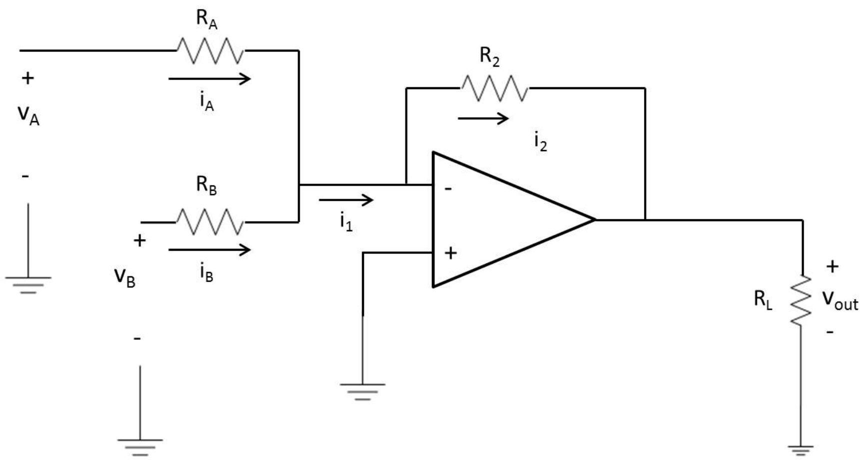


Figure 2: Inverting Summer

The output voltage is still fully determined by the input current, i_1; but now i_1 is the *sum* of two currents, i_A and i_B.

Let's derive a transfer characteristic of this amplifier circuit that relates V_{out} to V_A and V_B.

Start by writing a KCL equation at the inverting terminal

$$i_A + i_B = i_2 + i_n \tag{3}$$

but we know $i_n = 0$ so

$$i_A + i_B = i_2 \tag{4}$$

Since we also know that $v_n = 0$ we can write i_A and i_B interms of V_A and V_B

$$\frac{V_A}{R_A} + \frac{V_B}{R_B} = i_2 \tag{5}$$

We recall from last lesson that

$$V_{out} = -i_2 R_2 \tag{6}$$

whic combining the last 2 equations gives

$$\frac{V_A}{R_A} + \frac{V_B}{R_B} = \frac{-V_{out}}{R_2} \tag{7}$$

which solving for V_{out} gives

$$V_{out} = -\left(\frac{R_2}{R_A} V_A + \frac{R_2}{R_B} V_B\right) \tag{8}$$

Thus the name weighted, inverting summer

This derivation is easily extended to an arbitrary number of inputs.

A block diagram of a summer is shown in Figure 3. It should be noted that the gains (K_1 and K_2) are typically negative in our implementation

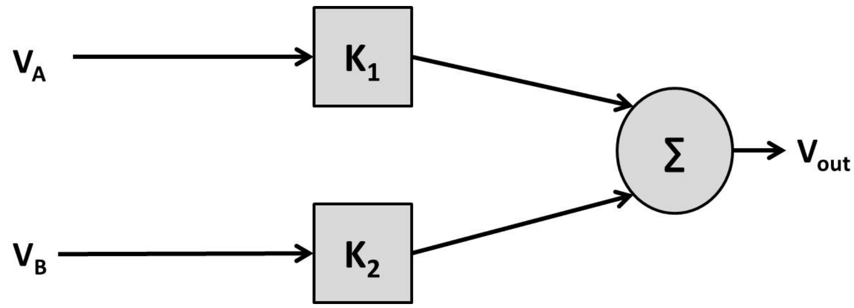


Figure 3: Summer Block Diagram

1.2 Examples

Textbook Design Example 4-16 Design an inverting summer (refer to Figure 2) that implements the following transfer characteristic:

$$V_{out} = -(5V_A + 13V_B) \tag{9}$$

Looking back at the transfer characteristic for an inverting summer we know we need the resistances to satisfy the following relationships

$$\frac{R_2}{R_A} = 5 \tag{10}$$

$$\frac{R_2}{R_B} = 13 \tag{11}$$

If we limit ourself to standard resistance values, we can start by selecting a value for R_2. Lets let $R_2 = 91\ k\Omega$. We can then solve for R_A and R_B and select the closest standard value. It is always good to verify that you are within about 5% of your desired gain. We select $R_A = 18\ k\Omega$ and $R_B = 6.8\ k\Omega$. These values yield gains of

$$K_1 = \frac{91\ k\Omega}{18\ k\Omega} = 5.0556 \tag{12}$$

$$K_2 = \frac{91\ k\Omega}{6.8\ k\Omega} = 13.3824 \tag{13}$$

Textbook Exercise 4-23

(a) For the design above find V_{out} if $V_A = 2\ V$ and $V_B = -0.5\ V$

(b) If $V_A = 500\ mV$ and $V_{CC} = 15\ V$, what is the maximum value of V_B for linear operation?

part (a) is just plug and chug

$$V_{out} = -\left(K_1V_A + K_2V_B\right) = -\left(5.05 \times 2\ V + 13.38 \times (-0.5)\right) = -3.41\ V \tag{14}$$

for part (b) we know $|V_{out}| \leq 15\ V$. If we set $V_{out} = 15\ V$ in the transfer characteristic equation and solve for V_B we get:

$$V_B = -\frac{V_{out} + K_1V_A}{K_2} = -\frac{15\ V + 5.05 \times 500\ mV}{13.38} = -1.31\ V \tag{15}$$

If we set $V_{out} = -15\ V$ in the transfer characteristic equation and solve for V_B we get:

$$V_B = -\frac{V_{out} + K_1V_A}{K_2} = -\frac{-15\ V + 5.05 \times 500\ mV}{13.38} = 932\ mV \tag{16}$$

After checking both edges, pick the largest. Intuition may have allowed you to select the largest without doing both calculations.

2 The Differential Amplifier or Subtractor

To understand the differential amplifier, let's start with the block diagram shown in Figure 4. Notice this block diagram is very similar to the summer block diagram, except that one of the inputs in into an *inverting terminal*.

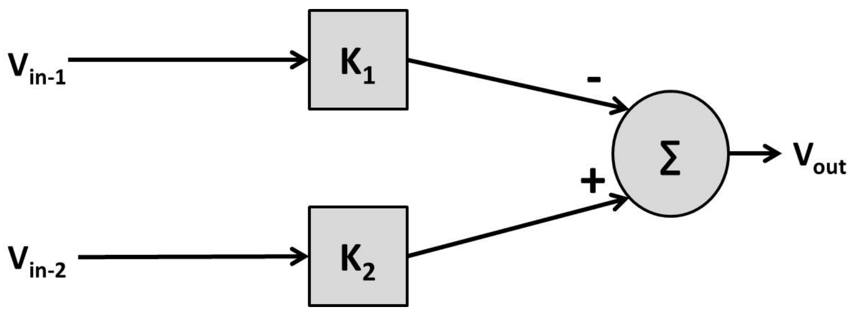


Figure 4: Differential Amplifier Block Diagram

2.1 Derivation

Rather than try to design a differential amplifier, I will give you the circuit (see Figure 5) and then we can derive the transfer characteristic.

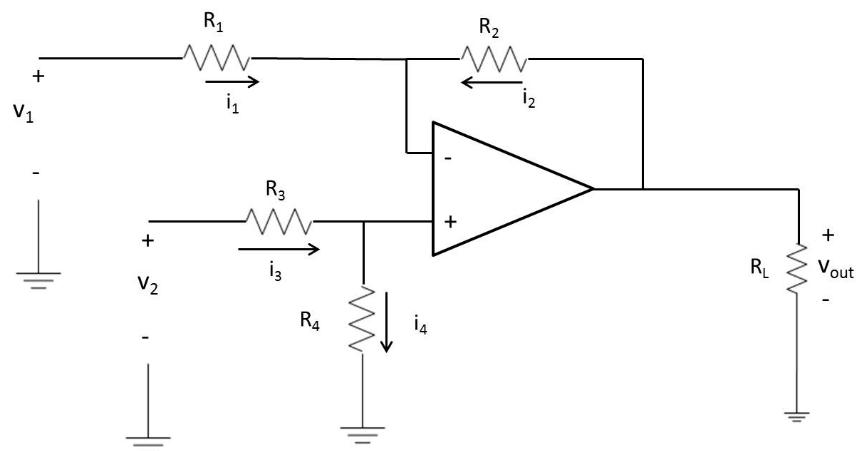
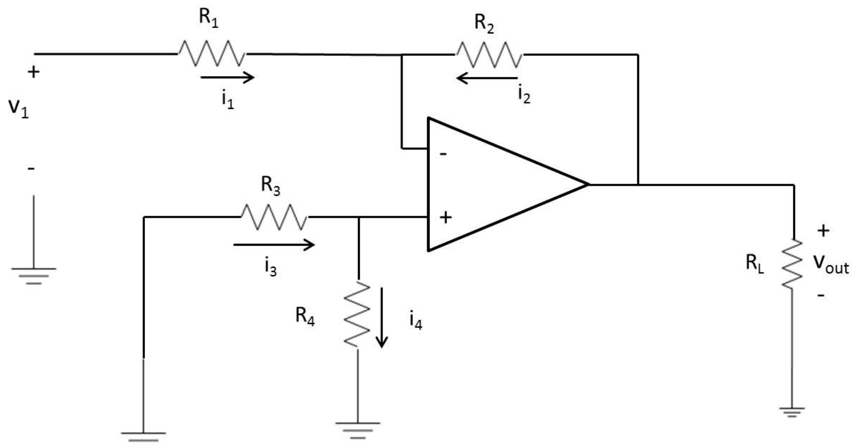


Figure 5: Differential Amplifier Circuit Diagram

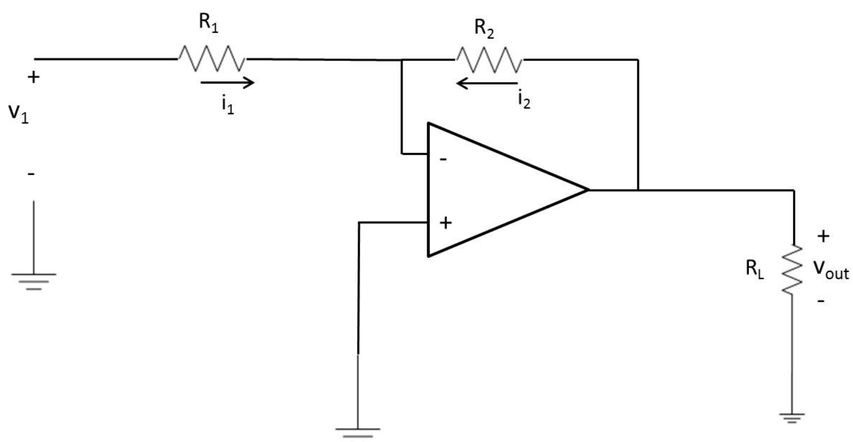
The derivation below will take advantage of the *Superposition* principle.

We will start by *turning off* V_2 (remember to turn off a voltage source you short it) and finding the output resulting from V_1.

With V_2 turned off the circuit becomes:



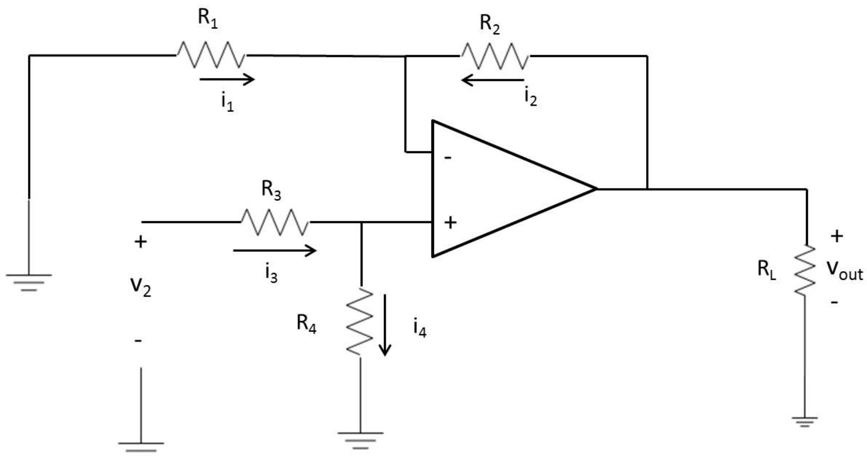
With V_2 off i_3 and i_4 are zero and the circuit becomes which is easily recognized as an inverting amplifier.



The transfer characteristic is

$$V_{out-1} = -\frac{R_2}{R_1}V_1 \tag{17}$$

Continuing the derivation we now *turn off* V_1. The cicuit would now look like



We now can find the voltage at the non-inverting terminal, v_p using voltage division

$$v_p = \frac{R_4}{R_3 + R_4} V_2 \tag{18}$$

and we know that $v_p = v_n$ therefore

$$v_n = \frac{R_4}{R_3 + R_4} V_2 \tag{19}$$

From here we can solve for i_1

$$i_1 = -V_2 \frac{R_4}{(R_3 + R_4) R_1} \tag{20}$$

and since $i_1 = -i_2$

$$i_2 = V_2 \frac{R_4}{(R_3 + R_4) R_1} \tag{21}$$

which means

$$V_{out-2} = V_2 \left\{ \frac{R_4}{R_3 + R_4} V_2 + \frac{R_4 R_2}{(R_3 + R_4) R_1} \right\} = V_2 \left[\frac{R_4}{R_3 + R_4} \right] \left[\frac{R_1 + R_2}{R_1} \right] \tag{22}$$

Now that we have the output from V_1 and V_2 we can add them

$$V_{out} = V_{out-1} + V_{out-2} \tag{23}$$

$$V_{out} = -\frac{R_2}{R_1} V_1 + \left[\frac{R_4}{R_3 + R_4} \right] \left[\frac{R_1 + R_2}{R_1} \right] V_2 \tag{24}$$

so referring back to the block diagram,

$$K_1 = \frac{R_2}{R_1} \tag{25}$$

Note: We dropped the negatives sign because it is accounted for in the block diagram at the summing junction

$$K_2 = \left[\frac{R_4}{R_3 + R_4} \right] \left[\frac{R_1 + R_2}{R_1} \right] \tag{26}$$

2.2 Examples

Textbook Exercise 4-25

(a) Find the transfer characteristic of a differential amplifier if: $R_1 = 10\ k\Omega$, $R_2 = 40\ k\Omega$, $R_3 = 10\ k\Omega$, and $R_4 = 15\ k\Omega$.

(b) If $V_{CC} = \pm 15\ V$ and $V_1 = 3\ V$ what is the allowable range of V_2 for linear operation?

part (a) is plug and chug.... The transfer characteristic written in terms of gains, K_1 and K_2 is :

$$V_{out} = -K_1 V_1 + K_2 V_2 \tag{27}$$

solve for gains, K_1 and K_2

$$K_1 = \frac{40\ k\Omega}{10\ k\Omega} = 4 \tag{28}$$

$$K_2 = \left[\frac{15\ k\Omega}{10\ k\Omega + 15\ k\Omega}\right]\left[\frac{10\ k\Omega + 40\ k\Omega}{10\ k\Omega}\right] = 3 \tag{29}$$

Plugging these gains into the transfer characteristic gives

$$V_{out} = -4V_1 + 3V_2 \tag{30}$$

For part (b) we start by solving the transder characteristic for V_2

$$V_2 = \frac{V_{out} + 4V_1}{3} \tag{31}$$

we now solve for the two cases: (1) $V_{out} = 15\ V$, (2) $V_{out} = -15\ V$

$$V_2 = \frac{-15 + (4 \times 3\ V)}{3} = -1\ V \tag{32}$$

$$V_2 = \frac{15 + (4 \times 3\ V)}{3} = 9\ V \tag{33}$$

therefore

$$-1\ V \leq V_2 \leq 9\ V \tag{34}$$