Lecture 15: Operational Amplifiers - Part 2

OBJECTIVES:

1. Starting with the ideal Op Amp model, develop circuits for Summing and Subtracting Amplifiers

READING

Required :

- Textbook, section 4.4, pages 190–201

Optional : None

1 The Summing Amplifier

Often in circuit design, we will need to add to signals together – this CANNOT be done by simply connecting two wires, although many of you in your solutions will try that method. To add two signals we must use a device called an *adder* or *summing amplifier*. In this section we will design a weighted (and inverting) summing amplifier.

1.1 Derivation

Let's start by looking at the inverting Op Amp discussed in last class (see Figure **??**)

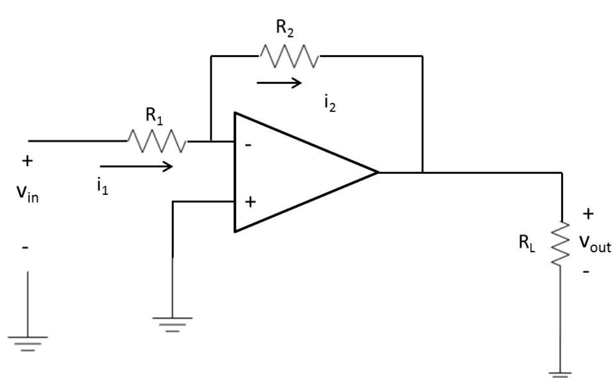


Figure 1: Inverting Op Amp

Recall that the transfer characterisitic for this amplifier is:

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} \tag{1}$$

which can also be written as

$$V_{out} = -\frac{V_{in}}{R_1} R_2 \tag{2}$$

but since $\frac{V_{in}}{R_1} = i_1$ we can say the output voltage is really determined by the input current, i_1.

Are there other ways of increasing the input current without increaseing V_{in}? Consider the circuit shown in Figure **??**

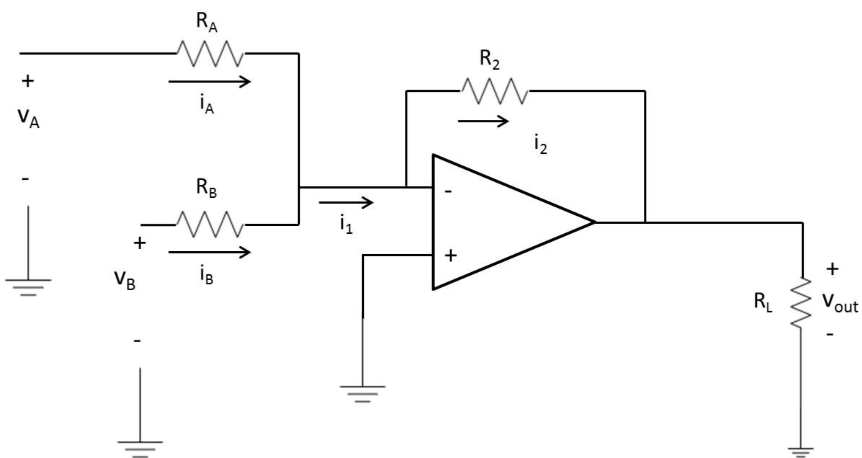


Figure 2: Inverting Summer

The output voltage is still fully determined by the input current, i_1; but now i_1 is the *sum* of two currents, i_A and i_B.

Let's derive a transfer characteristic of this amplifier circuit that relates V_{out} to V_A and V_B.

Start by writing a KCL equation at the inverting terminal

$$i_A + i_B = i_2 + i_n \tag{3}$$

but we know $i_n = 0$ so

$$i_A + i_B = i_2 \tag{4}$$

Since we also know that $v_n = 0$ we can write i_A and i_B interms of V_A and V_B and $V_B = 0 = V_n$

$$\frac{V_A - V_n}{R_A} + \frac{V_B - V_n}{R_B} = \frac{V_A}{R_A} + \frac{V_B}{R_B} = i_2 \tag{5}$$

simplifying

$$\frac{V_A}{R_A} + \frac{V_B}{R_B} = \frac{-V_{out}}{R_2} \tag{6}$$

which solving for V_{out} gives

$$V_{out} = -\left(\frac{R_2}{R_A} V_A + \frac{R_2}{R_B} V_B\right) \tag{7}$$

Thus the name weighted, inverting summer

This derivation is easily extended to an arbitrary number of inputs.

A block diagram of a summer is shown in Figure **??**. It should be noted that the gains (K_1 and K_2) are typically negative in our implementation

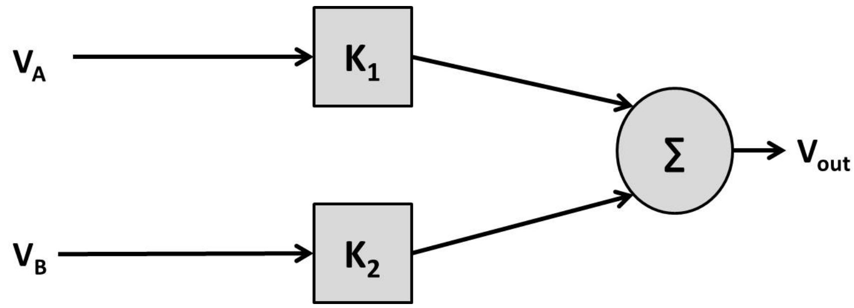


Figure 3: Summer Block Diagram

1.2 Examples

Textbook Design Example 4-16 Design an inverting summer (refer to Figure ??) that implements the following transfer characteristic:

$$V_{out} = -(5V_A + 13V_B) \tag{8}$$

Looking back at the transfer characteristic for an inverting summer we know we need the resistances to satisfy the following relationships

$$\frac{R_2}{R_A} = 5 \tag{9}$$

$$\frac{R_2}{R_B} = 13 \tag{10}$$

If we limit ourself to standard resistance values, we can start by selecting a value for R_2. Lets let $R_2 = 91\ k\Omega$. We can then solve for R_A and R_B and select the closest standard value. It is always good to verify that you are within about 5% of your desired gain. We select $R_A = 18\ k\Omega$ and $R_B = 6.8\ k\Omega$. These values yield gains of

$$K_1 = \frac{91\ k\Omega}{18\ k\Omega} = 5.0556 \tag{11}$$

$$K_2 = \frac{91\ k\Omega}{6.8\ k\Omega} = 13.3824 \tag{12}$$

Textbook Exercise 4-23

(a) For the design above find V_{out} if $V_A = 2\ V$ and $V_B = -0.5\ V$

(b) If $V_A = 500\ mV$ and $V_{CC} = 15\ V$, what is the maximum value of V_B for linear operation?

part (a) is just plug and chug

$$V_{out} = -\left(K_1V_A + K_2V_B\right) = -\left(5.05 \times 2\ V + 13.38 \times (-0.5)\right) = -3.41\ V \tag{13}$$

for part (b) we know $|V_{out}| \leq 15\ V$. If we set $V_{out} = 15\ V$ in the transfer characteristic equation and solve for V_B we get:

$$V_B = -\frac{V_{out} + K_1V_A}{K_2} = -\frac{15\ V + 5.05 \times 500\ mV}{13.38} = -1.31\ V \tag{14}$$

If we set $V_{out} = -15\ V$ in the transfer characteristic equation and solve for V_B we get:

$$V_B = -\frac{V_{out} + K_1V_A}{K_2} = -\frac{-15\ V + 5.05 \times 500\ mV}{13.38} = 932\ mV \tag{15}$$

After checking both edges, pick the largest. Intuition may have allowed you to select the largest without doing both calculations.

2 The Differential Amplifier or Subtractor

To understand the differential amplifier, let's start with the block diagram shown in Figure **??**. Notice this block diagram is very similar to the summer block diagram, except that one of the inputs in into an *inverting terminal*.

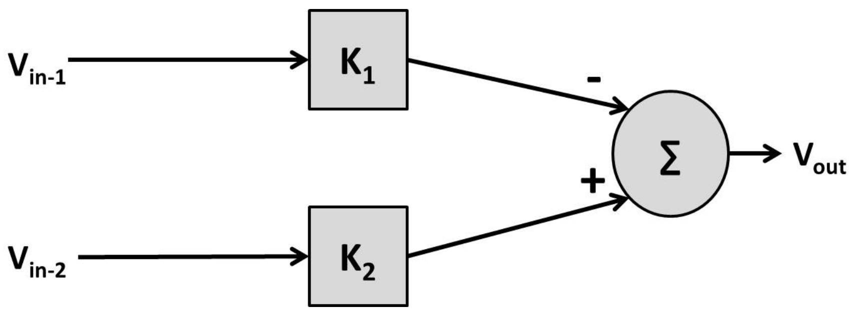


Figure 4: Differential Amplifier Block Diagram

2.1 Derivation

Rather than try to design a differential amplifier, I will give you the circuit (see Figure **??**) and then we can derive the transfer characteristic.

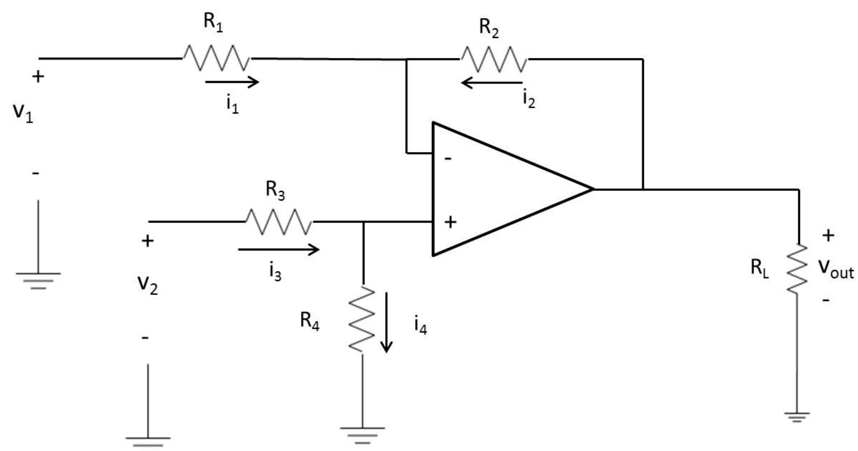
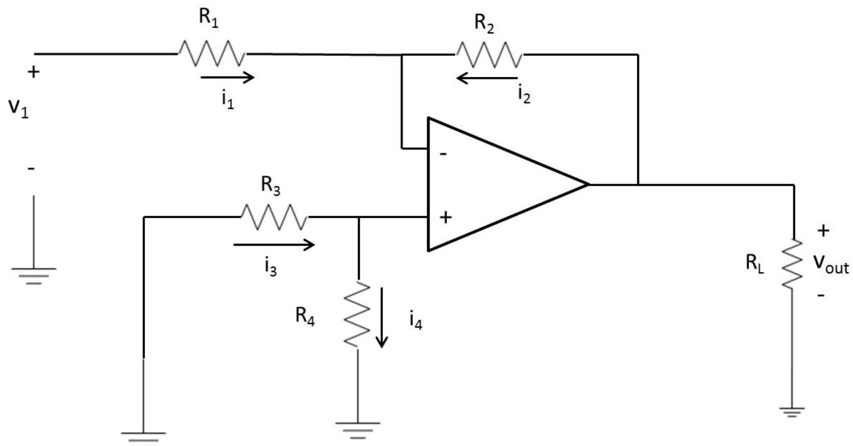


Figure 5: Differential Amplifier Circuit Diagram

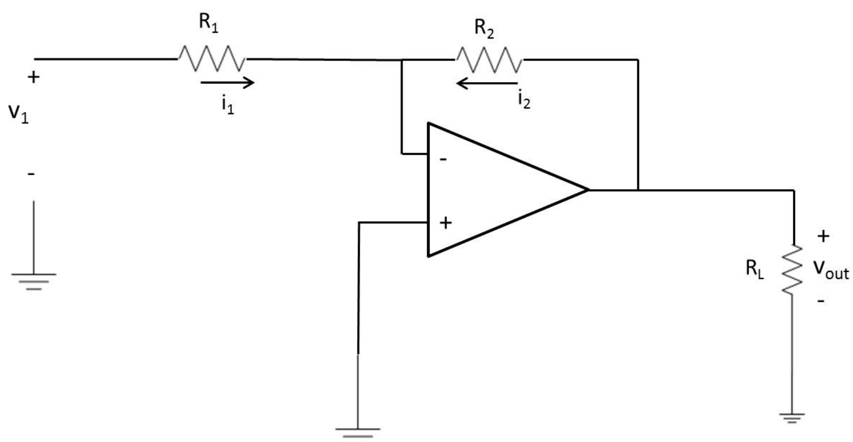
The derivation below will take advantage of the *Superposition* principle.

We will start by *turning off* V_2 (remember to turn off a voltage source you short it) and finding the output resulting from V_1.

With V_2 turned off the circuit becomes:



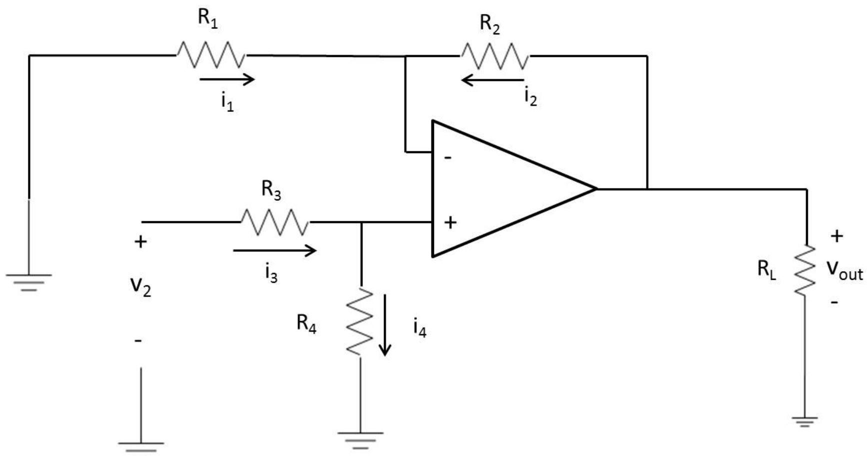
With V_2 off i_3 and i_4 are zero and the circuit becomes which is easily recognized as an inverting amplifier.



The transfer characteristic is

$$V_{out-1} = -\frac{R_2}{R_1}V_1 \tag{16}$$

Continuing the derivation we now *turn off* V_1. The cicuit would now look like



We now can find the voltage at the non-inverting terminal, v_p using voltage division

$$v_p = \frac{R_4}{R_3 + R_4} V_2 \tag{17}$$

and we know that $v_p = v_n$ therefore

$$v_n = \frac{R_4}{R_3 + R_4} V_2 \tag{18}$$

Now do a voltage divider

$$V_{out-2} = \frac{R_1 + R_2}{R_1} V_n \tag{19}$$

$$V_{out-2} = \frac{R_1 + R_2}{R_1} \frac{R_4}{R_3 + R_4} V_2 \tag{20}$$

$$\tag{21}$$

Now that we have the output from V_1 and V_2 we can add them

$$V_{out} = V_{out-1} + V_{out-2} = K_1 V_{out-1} + K_2 V_{out-2} \tag{22}$$

$$V_{out} = -\frac{R_2}{R_1} V_1 + \left[\frac{R_4}{R_3 + R_4}\right] \left[\frac{R_1 + R_2}{R_1}\right] V_2 \tag{23}$$

so referring back to the block diagram

$$K_1 = \frac{R_2}{R_1} \tag{24}$$

$$K_2 = \left[\frac{R_4}{R_3 + R_4}\right] \left[\frac{R_1 + R_2}{R_1}\right] \tag{25}$$

Note: We dropped the negatives sign because it is accounted for in the block diagram at the summing junction

2.2 Examples

Textbook Exercise 4-25

(a) Find the transfer characteristic of a differential amplifier if: $R_1 = 10\ k\Omega$, $R_2 = 40\ k\Omega$, $R_3 = 10\ k\Omega$, and $R_4 = 15\ k\Omega$.

(b) If $V_{CC} = \pm 15\ V$ and $V_1 = 3\ V$ what is the allowable range of V_2 for linear operation?

part (a) is plug and chug.... The transfer characteristic written in terms of gains, K_1 and K_2 is :

$$V_{out} = -K_1 V_1 + K_2 V_2 \tag{26}$$

solve for gains, K_1 and K_2

$$K_1 = \frac{40\ k\Omega}{10\ k\Omega} = 4 \tag{27}$$

$$K_2 = \left[\frac{15\ k\Omega}{10\ k\Omega + 15\ k\Omega}\right]\left[\frac{10\ k\Omega + 40\ k\Omega}{10\ k\Omega}\right] = 3 \tag{28}$$

Plugging these gains into the transfer characteristic gives

$$V_{out} = -4V_1 + 3V_2 \tag{29}$$

For part (b) we start by solving the transder characteristic for V_2

$$V_2 = \frac{V_{out} + 4V_1}{3} \tag{30}$$

we now solve for the two cases: (1) $V_{out} = 15\ V$, (2) $V_{out} = -15\ V$

$$V_2 = \frac{-15 + (4 \times 3\ V)}{3} = -1\ V \tag{31}$$

$$V_2 = \frac{15 + (4 \times 3\ V)}{3} = 9\ V \tag{32}$$

therefore

$$-1\ V \leq V_2 \leq 9\ V \tag{33}$$