Lecture 2: Resistors, Sources, Switches, KCL, KVL

OBJECTIVES:

1. Introduce sources & switches
2. Introduce the concept of *Nodes* & *Loops*
3. Learn and apply Kirchhoff's Voltage & Current Laws (KVL & KCL)
4. Solve circuits using KCL, KVL and Ohm's Law

READING

Required: Textbook, sections 2.1–2.2, pages 15–26

Optional: None

1 A quick review of Resistors

We introduced resistors in last class along with Ohm's Law. Figure 1 shows the resistor current–voltage (i–v) relationship. We will develop i–v relationships for other devices as we go along.

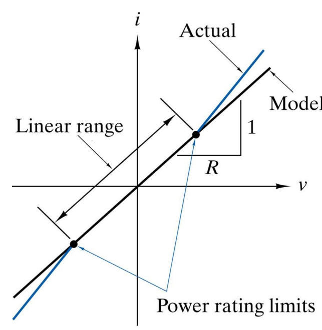

Figure 1: Resistor VI relationsip

2 Open Circuits, Short Circuits and Switches

Open Circuits are a "broken" paths that no current can flow through. Voltage can exist across an open circuit. Open circuits can be modeled as an ∞ resistance. An open circuit is shown in Figure 2(a).

Short Circuits are closed paths that have zero resistance. A short circuit has zero voltage across it. A short circuit is shown in Figure 2(b).

Ideal Switchs are devices that can create an open or short circuit depending on the switch position. See Figure 3 for schematics and i–v characteristics of open and closed *ideal* switches.

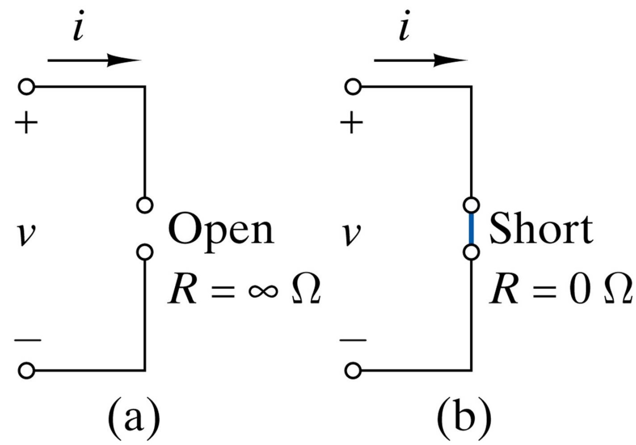

Figure 2: Schematic of (a) an open circuit and (b) a short circuit

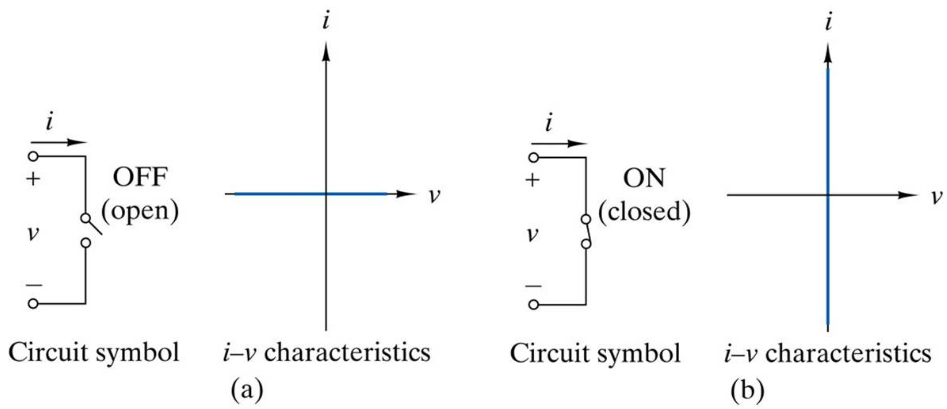

Figure 3: Schematic & i–v relationship of (a) an open switch (open circuit) and (b) a closed switch (short circuit)

3 Ideal Sources

For now we will deal with only ideal voltage sources and ideal current sources. What do we mean by ideal?

The source provides the same current or voltage output independent of the load it is connected to. Another way of saying this is that the i–v characteristic is always a constant. For a current source, $i(v) = constant$. for a voltage source, $v(i) = constant$.

Figure 4 shows schematics of ideal voltage sources and the i–v characteristic.

Figure 5 shows the schematics of an ideal current sources and the i–v characteristic.

Thought Question: Can a car battery be modeled as an ideal voltage source? Why or why not?

No. When you start your car, starter draws very high current from the battery (around 100-250A). When the current draw is this high, the voltage from the battery will decrease to 8-10V. Since the voltage is not constant, but rather is dependent on current, it is not an ideal source

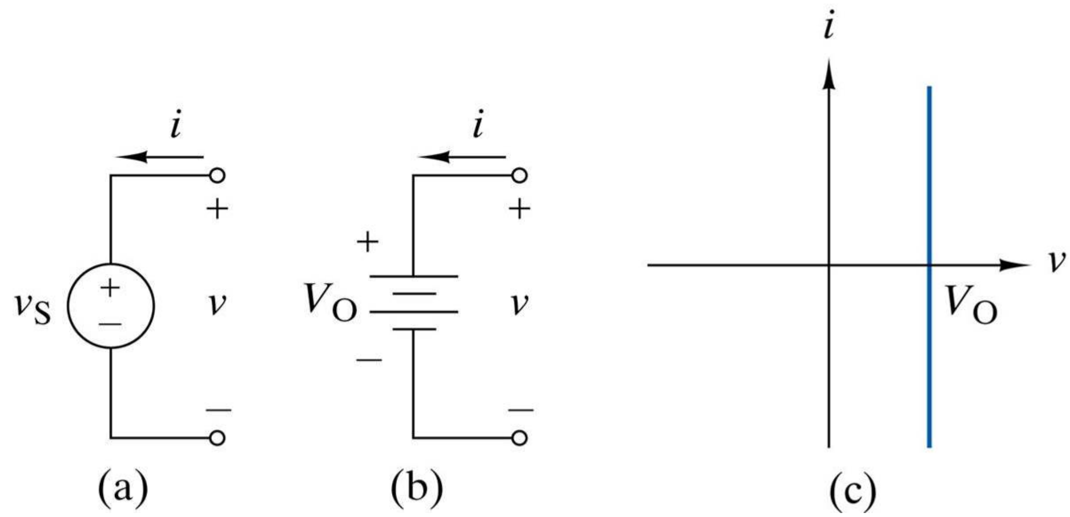

Figure 4: (a) An ideal voltage source, (b) an ideal battery, and (c) their i–v characteristic

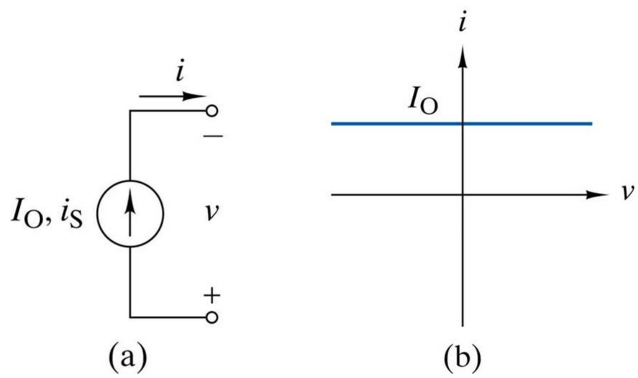

Figure 5: (a) An ideal current source, and (b) its i–v characteristic

4 Nodes

What is a **node**?

1. *A node is the junture of* two or more *devices*
2. *The node includes all the wire running between devices*

You need to be able to count nodes in a circuit. The easiet way to do that is to "erase" all the devices, but leave the wire. The "clumps" of wire left behind are your nodes. Figure 6 demonstrates this procedure.

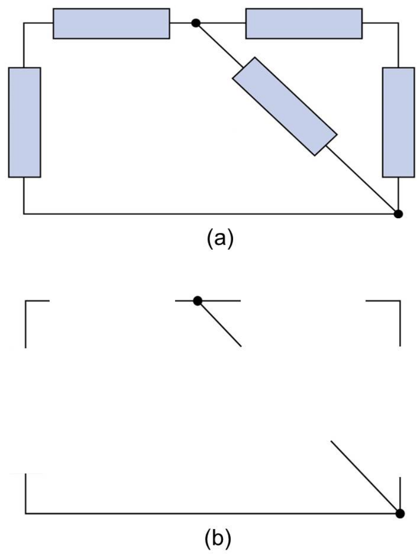

Figure 6: (a) A simple circuit, (b) the same circuit with devices removed leaving only the 4 nodes

5 Loops

What is a **loop**?

1. *A loop is a closed path through a circuit*
2. *A loop cannot pass through any node twice; other than starting and finishing at the same node (which is the definition of a closed path)*

You also need to be able to identify and count loops in a circuit. See Figure 7 for a simple example and Figure 8 for a more complicated example.

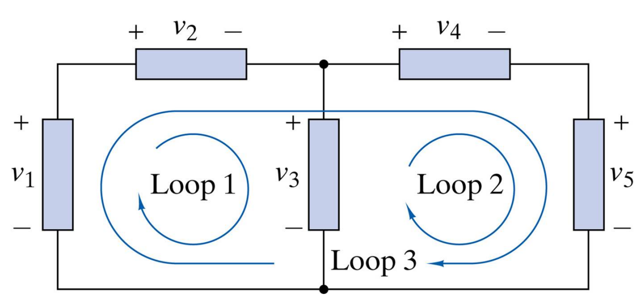

Figure 7: Example circuit with 3 loops

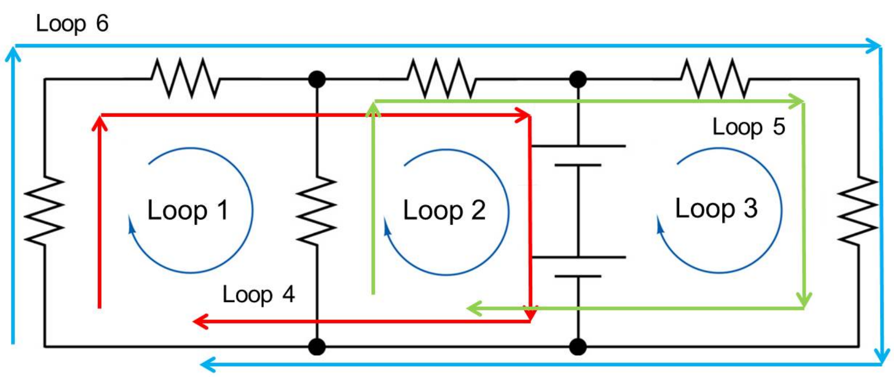

Figure 8: Example circuit with 6 loops

6 Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law in words

1. *Algebraic sum of all currents at a node equals zero*
2. *Sum of currents entering a node equals the sum of currents leaving the node*
3. *Goes-intas = Goes-outas*

KCL in mathematics

$$\sum_{Node} i = 0 \tag{1}$$

$$\sum i_{in} = \sum i_{out} \tag{2}$$

$$\sum goesintas = \sum goesoutas \tag{3}$$

Write a KCL equation for the center node in Figure 9

$$\begin{aligned} i_1 - i_2 - i_3 &= 0 \\ i_1 &= i_2 + i_3 \end{aligned} \tag{4}$$

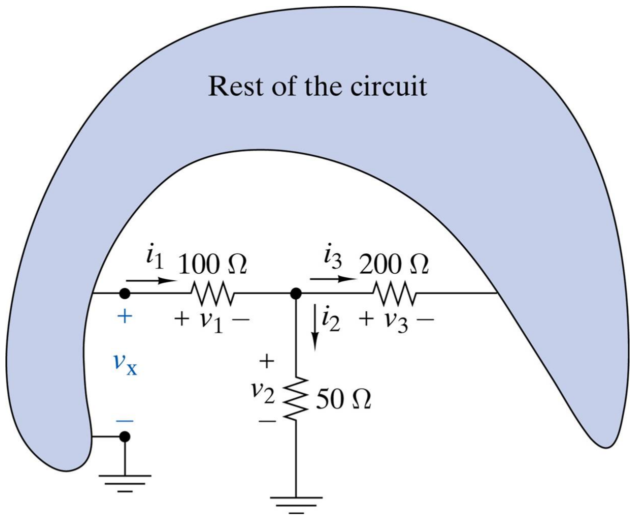

Figure 9: KCL Example

KCL Example—This is problem 2-20 from the textbook.

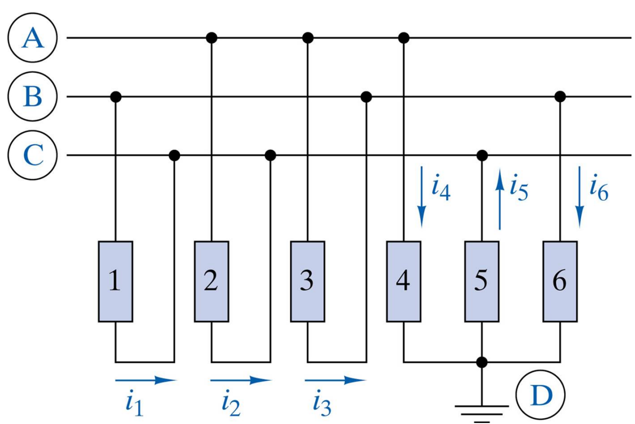

Figure 10: A second KCL Example

The circuit in Figure 10 is organized around 3 signal lines A, B & C (assume lines are open circuits off both edges).

- How many nodes are in this circuit?
- Write KCL equations for the circuit
- If $i_1 = -20\ mA$, $i_2 = -12\ mA$, and $i_3 = 50\ mA$; find i_4, i_5, & i_6

There are 4 nodes in this circuit (each signal line and the ground)

KCL equations are:

$$\begin{aligned} i_2 + i_3 + i_4 &= 0 \\ i_3 &= i_1 + i_6 \\ i_1 + i_2 + i_5 &= 0 \\ i_4 + i_6 &= i_5 \end{aligned} \tag{5}$$

We can use i_1 and i_2 to find i_5

$$\begin{aligned} i_5 &= -(i_1 + i_2) \\ i_5 &= -(-20\ mA + (-12\ mA)) \\ i_5 &= 32\ mA \end{aligned}$$

We can use i_2 and i_3 to find i_4

$$\begin{aligned} i_4 &= -(i_2 + i_3) \\ i_4 &= -(-12\ mA + 50\ mA) \\ i_4 &= -38\ mA \end{aligned}$$

We can use i_4 and i_5 to find i_6

$$\begin{aligned} i_6 &= i_5 - i_4 \\ i_6 &= 32\ mA - (-38\ mA) \\ i_6 &= 70\ mA \end{aligned}$$

What you may have noticed in our solution above was that we were able to write 4 KCL equations; one for each node. What may not be obvious is that only 3 of the equations were linearly independent. The proof of this is below:

We can rewrite our 4 equations in matrix form

$$\begin{bmatrix} 0 & -1 & -1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

If we focus on just the coeffcient matrix, we can use row reduction techniques to rewrite the matrix as

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The fact that the matrix can be row-reduced to give a zero row tells you that one of the equations is not linearly independent.

7 Kirchhoff's Voltage Law (KVL)

Kirchhoffs Voltage Law in words:

1. *The algebraic sum of voltages around any closed path equals zero*
2. *The sum of voltage rises around a loop equals the sum of voltage drops around the same loop*
3. *If it goes up, it must come down*

KVL in mathmatics:

$$\sum_{Loop} v = 0 \tag{6}$$

$$\sum v_{rises} = \sum v_{losses} \tag{7}$$

KVL Example 1: Write KVL equations (for the two labeled loops) and find unknown voltages for the circuit shown in Figure 11. [Exercise 2-5 from text]

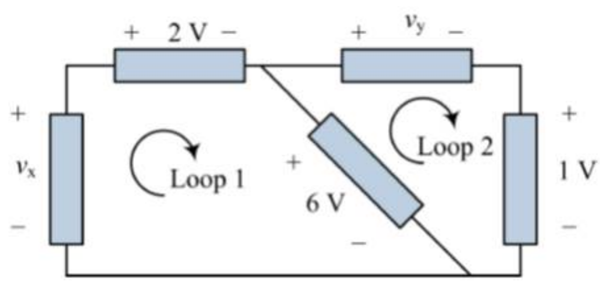

Figure 11: A KVL Example

We start by writing the KVL equations.

Loop 1:

$$v_x - 2\ V - 6\ V = 0 \tag{8}$$

Loop 2

$$6\ V - v_y - 1\ V = 0 \tag{9}$$

We can now solve each equation to see that $v_x = 8\ V$ and $v_y = 5\ V$.

KVL Example 2: Find v_x, v_y, and v_z in the circuit shown in Figure 12. [Exercise 2-6 from text]. This example is a little unique in that we are going to define a loop where one does not physically exist.

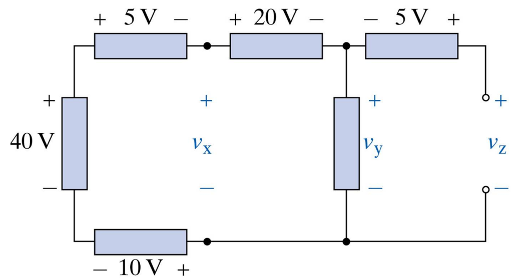

Figure 12: A second KVL Example

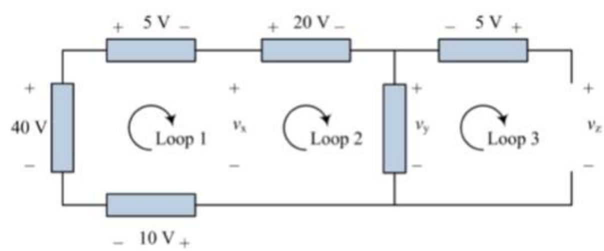

Figure 13: Circuit above with loops labeled

To start the problem lets label some loops... This is the tricky part. Notice we defined 2 loops inside a "single loop". Because we defined v_x the way we did, we can account for this rise/loss in each "loop".

Next we write the KVL equations.

Loop 1:

$$40\ V - 5\ V - v_x - 10\ V = 0 \tag{10}$$

Loop 2

$$v_x - 20\ V - v_y = 0 \tag{11}$$

Loop 3

$$v_y + 5\ V - v_z = 0 \tag{12}$$

We can now solve each equation to see that $v_x = 25\ V$, $v_y = 5\ V$, and $v_z = 10\ V$.

8 Review Questions

1. What does it really mean is we calculate a negative voltage or current?

   When we get a negative number for voltage or current, it just means our initial "assigned" voltage polarity or current direction was wrong. You can flip the polarity/direction and drop the negative.

2. What is the current through an open circuit?

   Zero

3. What is the voltage across a short circuit?

   Zero, there is not voltage across a short circuit. Since voltage is measure between two nodes and a short circuit does not seperate any nodes, you cannot measure a voltage across it.