Lecture 30: Phasors Circuit Analysis and Design Part 2

OBJECTIVES:

1. Learn to apply circuit analysis techniques we already know to solve circuits in the phasor domain

READING

Required : Textbook, section 8.4–8.5, pages 402–426

Optional :

1 Introduction

Today we continue our analysis of phasor circuits. Last class we did examples of:

1. Parallel and series combinations of **Impedances**
2. Voltage and Current Division
3. Mesh and Node Analyisis

Today we will look at:

1. Proportionality and Superposition
2. Thevenin and Norton equivalent circuits

All these things still apply to phasor domain circuits. As a reminder, phasor analysis only applies to the steady state sinusoidal responses. Phasor analysis does not tell you anything about the transient response.

2 Proportionality

Back in lesson 8 we introduced this introduced this idea of proportionality which states that for a linear circuit, any change to the input has a proportional change to the output. Stated mathematically:

$$y = Kx$$

where x is the input, y is the output, and K is the proportionality factor.

This also applies to phasors and can be written as:

$$\mathbf{Y} = \mathbf{K}\mathbf{X}$$

where Y, X, and K are all in general complex.

To make use of proportionality, the book talks about the *Unit Output Method....* I prefer the *Unit Input Method*. Let's do an example:

Example 1– This example is a modified version of textbook exercise 8-22. For the circuit shown in Figure 1, use the Unit Input Method to find $\mathbf{I_o}$

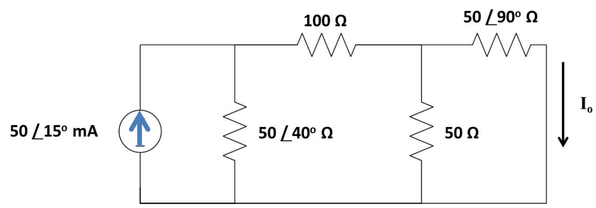
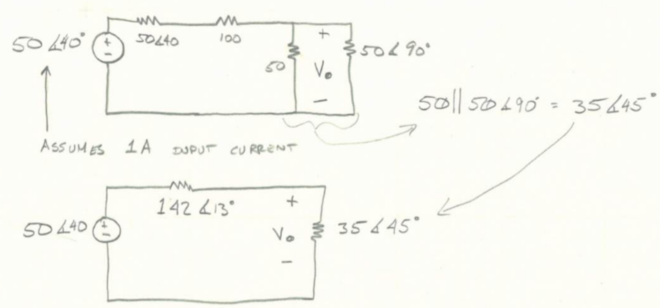


Figure 1: Circuit to accompany example 1

STEP 1 - ASSUME THE INPUT CURRENT IS 1 A & SOLVE FOR I_o

SOURCE TRANSFORM



$$V_o = 50\angle 40^\circ \left[\frac{35\angle 45^\circ}{35\angle 45^\circ + 142\angle 13^\circ}\right] = 10.13\angle 66^\circ$$

$$I_o = \frac{V_o}{50\angle 90^\circ} = 202.7\angle -24^\circ \text{ mA}$$

STEP 2 - MULTIPLY I_o BY ORIGINAL INPUT

$$I_o = (202.7\times 10^{-3}\angle -24^\circ)(50\angle 15^\circ)$$
$$= 10.13\angle -9.1^\circ$$

3 Superposition

Before we jump right into an example, now is a good opportunity to remind you that phasor analysis only works for circuits that are excited by a single sinusoidal frequency. This does not mean we cannot have more than one source, it just means they all have to have the same frequency (ω).

Example 2 – Textbook Exercise 8-23 — For the circuit in Figure 2, use superposition to find $\mathbf{I_x}$.

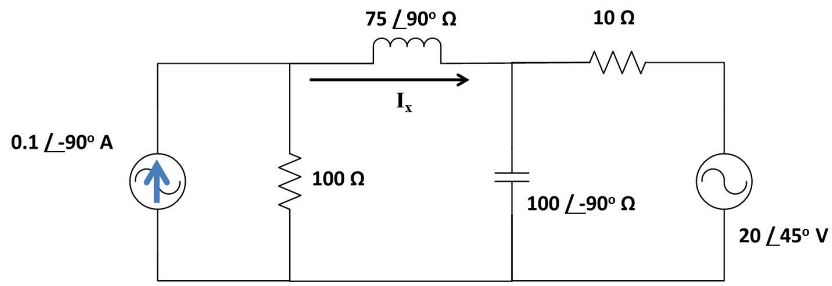
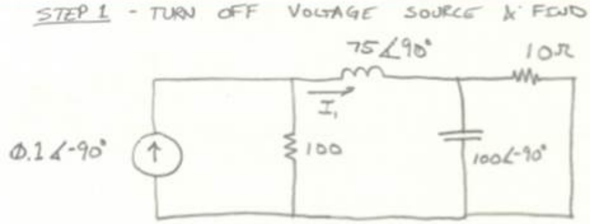
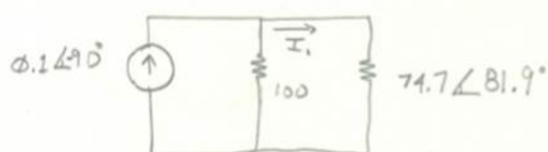


Figure 2: Circuit to accompany example 2

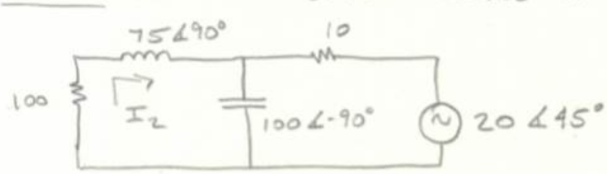
STEP 1 - TURN OFF VOLTAGE SOURCE & FIND I_x

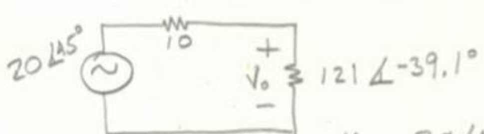




CURRENT DIVISION $I_1 = 0.1\angle -90^\circ \left[\frac{100}{100 + 74.7\angle 81.9^\circ}\right] = 75.45\angle -123.9^\circ \text{ mA}$

STEP 2 - TURN OFF CURRENT SOURCE & FIND I_x





$$V_o = 20\angle 45^\circ \left[\frac{121\angle -39.1^\circ}{10 + 121\angle -39.1^\circ}\right] = 18.8\angle 42^\circ$$

$$I_2 = \frac{-V_o}{100 + j75} = 150.2\angle -174.7 \text{ mA}$$

STEP 3 - SUM CURRENTS

$$I_x = I_1 + I_2 = 206\angle -158^\circ \text{ mA}$$

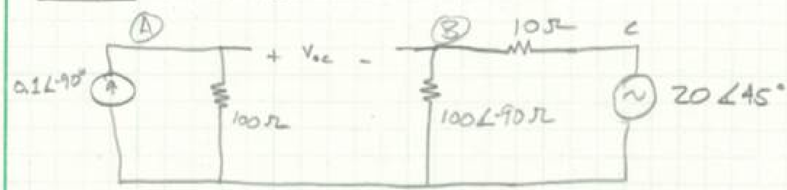
4 Thevenin's and Norton's Theorems

This section will offer a quick review of Thevenin's and Norton's Theorems before we work a couple of examples.

For Thevenins theorem, we will take a *source* circuit that is connected to some *load* circuit and replace the source circuit with a single voltage source in series with a single *impedance*. To do this we must find the value of the source voltage and the *Thevenin impedance*. The source voltage is found by calculating the open circuit voltage if the load is removed. The easiest way to calculate the Thevenin Impedance is by using look-back.

Example 3– Textbook Exercise 8-29 — For the curcuit in example 2 (Figure 2), find the Thevenin circuit seen by the inductor then calculate $\mathbf{I_X}$.

STEP 1 - FIND V_{oc}



USE NODE ANALYSIS $[V_{oc} = V_{AB}]$

A: $V_A = (0.1\angle{-90})(100\Omega) = 10\angle{-90°}$

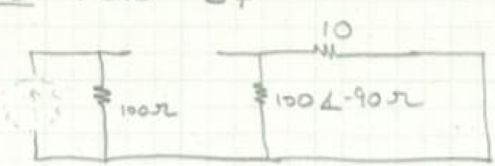
B: $\frac{V_B - 20\angle 45°}{10} + \frac{V_B}{100\angle{-90°}} = 0$

$V_B\left[\frac{1}{10} + \frac{1}{100\angle{-90}}\right] = 2\angle 45°$

$V_B = 19\angle 39.25$

$\therefore V_{oc} = 27.34\angle{-124.3°} = -15.4 - j22.6\ V.$

STEP 2 - FIND Z_T

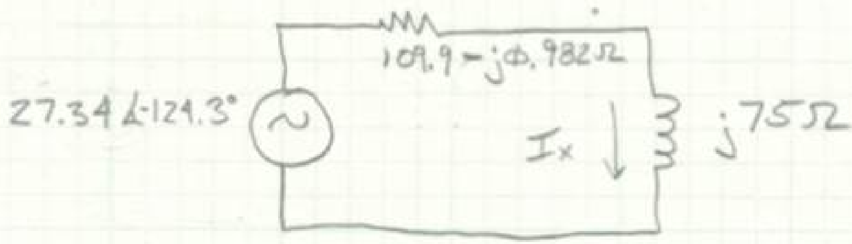


$Z_T = 100 + [10 \,||\, 100\angle{-90}]$

$= 100 + \left[\frac{1000\angle{-90}}{10 - j100}\right] = 109.9 - j0.982\Omega$

$= 109.9\angle{\sim 0°}$

STEP 3 - DRAW THEVENIN CIRCUIT



STEP 4 - SOLVE FOR I_x

$$Z_{eq} = 109.9 - j0.982 + j75$$
$$= 109.9 + j74.018 = 132.5\angle 34^\circ$$

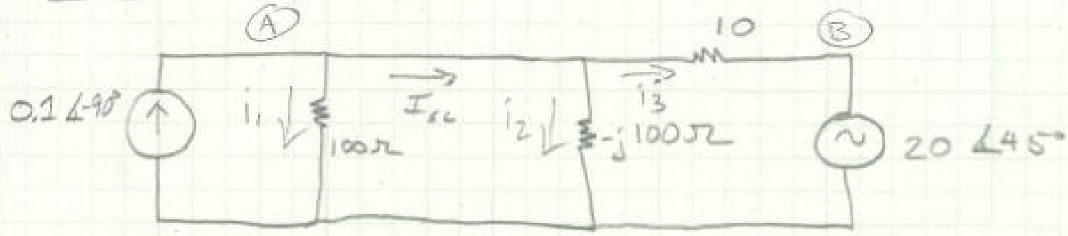
$$I_x = \frac{V_{oc}}{Z_{eq}} = \frac{27\angle -124.3^\circ}{132.5\angle 34^\circ} = 0.203\angle -158^\circ$$

$\boxed{\text{MATCHES SUPER-POSITION SOLUTION}}$

Norton's Theorem is like Thevenin's except we find a current source in parallel with an impedance.

Example 4 – Rework Example 3 with Norton's Theorem.

STEP 1 - FIND I_{sc}



NODE ANALYSIS

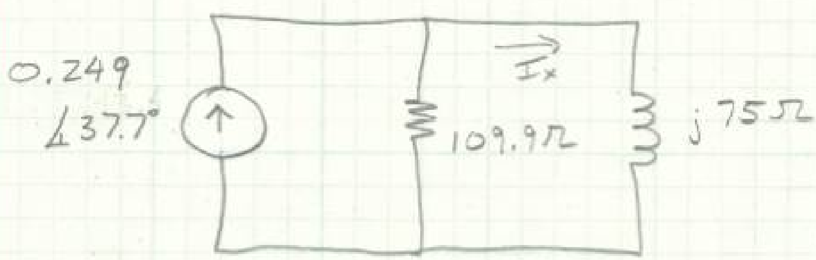
$$\text{(A)}\quad -0.1\angle 90^\circ + \frac{V_A}{100} + \frac{V_A}{-j100} + \frac{V_A - 20\angle 45^\circ}{10} = 0$$

$$V_A\left[\frac{1}{100} + \frac{1}{-j100} + \frac{1}{10}\right] = 2\angle 45^\circ + 0.1\angle -90^\circ$$

$$V_A = 17.5\angle 37.7^\circ$$

$$I_{sc} = \frac{V_A}{-j100} + \frac{V_A - 20\angle 45^\circ}{10} = 0.2488\angle -37.7^\circ$$

STEP 2 - USING Z_T FROM EXAMPLE 3, DRAW NORTON CIRCUIT



STEP 3 - USE CURRENT DIVIDER TO FIND I_x

$$I_x = 0.249\angle 37.7^\circ \left[\frac{109.9}{109.9 + j75\,\Omega}\right]$$

$$I_x = 206 \angle -157.6 \text{ mA}$$