

- Serial manipulator - this is what our robot arm is. Think Serial like Serial data: one wire
- Need to understand (remember) simple vector/matrix ops
  - add
  - multiply
  - divide or inverse
  - transpose
- Process :
  - Start with a simplified version



\* art skills are not important

- Euler angles, not really used here
- Example of what is called a rotation matrix
- Rotation Matrix

$$R_{\text{From}}^{\text{To}} = [3 \times 3]$$

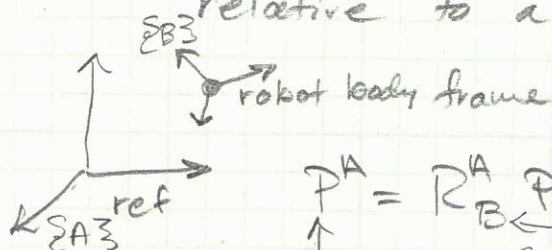
$$R_A^B = (R_B^A)^{-1} = (R_B^A)^T$$

look Ma! no inversion

- Position + Orientation

\* properties of orthonormal

pose : common robotics term for the combined position/orientation of something (robot) relative to a reference frame.



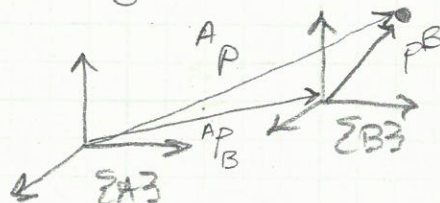
$\{X\} \equiv$  reference frame name, here it is X

$$P^A = R_B^A P^B \leftarrow \begin{array}{l} \text{in ref } \{B\} \\ \text{transition from } \{B\} \text{ to } \{A\} \\ \text{values in } \{A\} \end{array}$$

$P^A \equiv$  vector measured in ref  $\{A\}$

$P^W \equiv$  vector measured in ref  $\{W\}$

# - Adding 2 Frames together



${}^A P_B \equiv \{B\}$  location in  $\{A\}$ , this is called a translation

Now what we want is  $P^A$ , where is the dot in  $\{A\}$ .

$$P^A = \underset{\substack{\uparrow \\ \text{translation}}}{A P_B} + \underset{\substack{\uparrow \\ \text{rotation, to align } \{B\} \\ \text{with } \{A\}}}{R_B^A} P^B$$

\* you can not add  ${}^A P_B$  and  $P^B$  together, they are defined in different frames

rotation, to align  $\{B\}$  with  $\{A\}$

- To be more compact, we will use a homogeneous matrix

$$T_B^A = \begin{bmatrix} R_B^A & P_B^A \\ 0 & 0 & 0 & 1 \end{bmatrix} \leftarrow 4 \times 4$$

$R_B^A \equiv 3 \times 3$  matrix

$P_B^A \equiv 3 \times 1$  vector

$\uparrow$  bottom row is always  $[0 \ 0 \ 0 \ 1]$

This will mean something when we get to vision

$$P^A = T_B^A P^B$$

- You can also these together:

$$T_C^A = T_B^A * T_C^B$$

think  $\Rightarrow$

$$T_B^A * T_C^B = T_C^A \quad \leftarrow \text{they cancel}$$

$\{B\} \rightarrow \{A\}$

$\{C\} \rightarrow \{B\}$

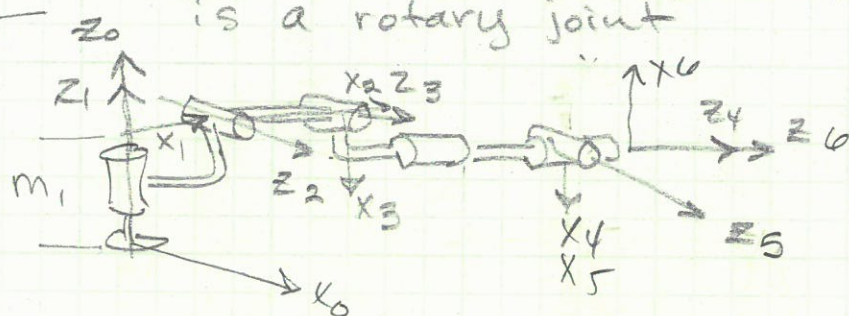
- DH

- I suggest you read and follow the examples



# - DH Process (my summary)

- ① put Z-axis through axis of rotation
- ② put X-axis along the common perpendicular between the Z-axes
- ③ starting @ 0, assign each frame a number
  - $\{0\}$  is the base reference frame which generally doesn't move
  - $\{1\}$  is often co-aligned w/  $\{0\}$  if it is a rotary joint



$X_4 \equiv Z_4 \perp Z_5$ ,  $X_4$  @ intersection

$X_5 \equiv Z_5 \perp Z_6$ ,  $X_5$  @ intersection

$X_6 \equiv$  anywhere  $\perp Z_6$ , put @ end effector location

$X_6 \equiv Z_6$  is the last coordinate frame,  
pick a  $\perp$  to  $Z_6$  that is useful

	$a_{i-1}$	twist $\alpha_{i-1}$	$d_i$	$\theta_i$
1	0	0	$m_1$	$\theta_1$
2	$m_2$	$90^\circ$	0	$\theta_2$
3	$m_3$	$0^\circ$	0	$\theta_3$
4	$m_4$	$-90^\circ$	0	$\theta_4$
5	0	$90^\circ$	0	$\theta_5$
6	0	$-90^\circ$	$m_6$	$\theta_6$

\* I replaced d's w/ m's because it is confusing otherwise

move to end