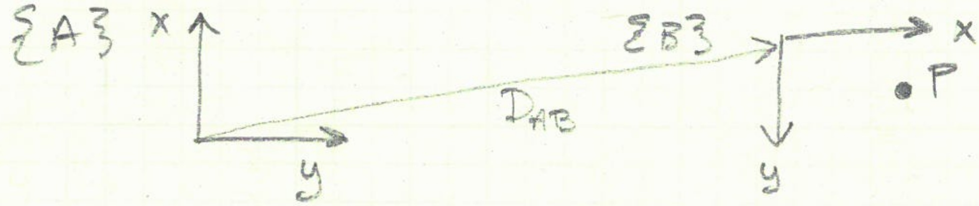
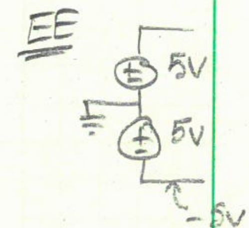
- Talk about robotics and why serial manipulators are important
- <u>Serial</u> - Kinematic chain or a series (one after the other) of links.
- <u>parallel</u> - usually used in manufacturing or amusement park rides.

Coordinate Frames

- we are going to talk about coordinate frames.
  - let's start simple and do 2D frames.



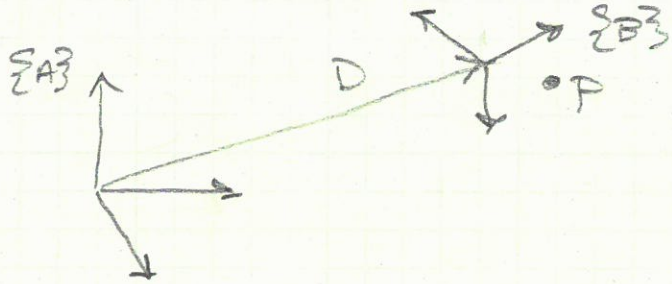


- we generally want to know a couple things
  - where are we located relative to a frame
    - point $P^B = [x, y]$
  - the orientation between our current frame and another frame
    - use a rotation matrix R^A_B to convert between frames

      $\leftarrow 2\times2 \quad \begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix} = R$
    - Questions:
      - is there a difference between x-axis^A and x-axis^B? if so, what?
      - Converting between frames is like converting Euro's to US dollars
  - where are we located relative to another frame
    - $P^A = D_{AB} + R^A_B P^A$

      D_{AB} ← distance between frames

- Now this idea of position and orientation with respect to a frame is called a <u>pose</u>
- Of course we can also expand this to 3D



$$P^A = D + R^A_B P^B$$

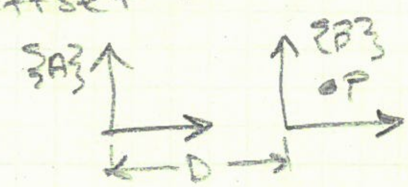
P^A is $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

R^A_B is 3x3, more complex. We will talk about Euler angles latter

- Properties of R
  - composed only of sin and cos terms
  - magnitude of rows and colums is 1
  - the inverse of R is equal R^T
    - nice, no math!

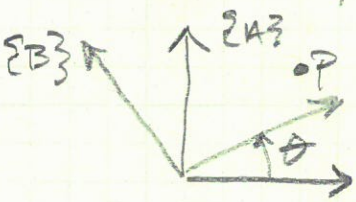
Conseptual Review:

- {A} and {B}, no rotation between them but offset:



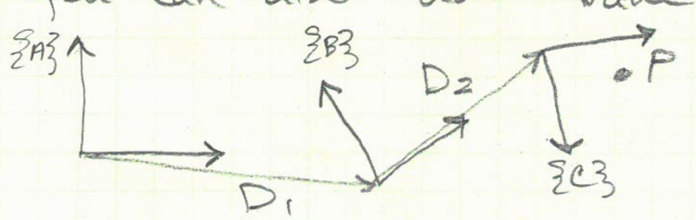
$$P^A = D + \cancel{R}^{\,1} P^B$$

- No offset, but a rotation



$$P^A = R P^B$$

- You can also add frames:



← this could be a serial robot with 3 link

$$P^A = D_1 + R^A_B P^B$$
$$= D_1 + R^A_B \left[D_2 + R^B_C P^C \right]$$

- <u>Serial manipulator</u> - this is what our robot arm is. Think serial like serial data: one wire

- Need to understand (remember) simple vector/matrix ops
  - add
  - multiply
  - divide or inverse
  - transpose

- Process:
  - Start with a simplified version

* art skills are not important

- Euler angles not really used here
  - Example of what is called a rotation matrix
- Rotation Matrix

$$R^{TO}_{From} = [3 \times 3] \qquad R^B_A = (R^A_B)^{-1} = (R^A_B)^T$$

look Ma' no inversion

- Position + Orientation * properties of <u>orthonormal</u>

<u>pose</u>: common robotics term for the combined position/orientation of something (robot) relative to a reference frame

$\{X\} \equiv$ reference frame name, here it is X

$$P^A = R^A_B \, P^B \leftarrow \text{in ref } \{B\}$$

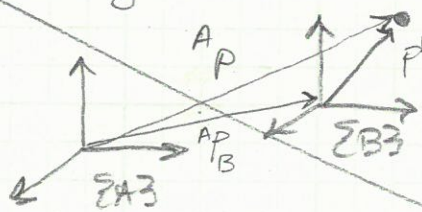
R^A_B: transition from {B} to {A}

P^A: values in {A}

$P^A \equiv$ vector measured in ref{A}

$P^W \equiv$ vector measured in ref {W}

~~- Adding 2 Frames together~~



~~$^{A}P_{B} \equiv$ {B} location in {A}, this is called a translation~~

~~Now what we want is P^{A}, where is the dot in {A}.~~

~~$P^{A} = {}^{A}P_{B} + R^{A}_{B} P^{B}$~~

~~translation~~

~~rotation, to align {B} with {A}~~

~~* you can not add $^{A}P_{B}$ and P^{B} together, they are defined in different frames~~

- To be more compact, we will use a homogerious matrix

$$T^{A}_{B} = \begin{bmatrix} R^{A}_{B} & P^{A}_{B} \\ 0\ 0\ 0 & 1 \end{bmatrix} \Leftarrow 4\times4$$

$R^{A}_{B} \equiv 3\times3$ matrix

$P^{A}_{B} \equiv 3\times1$ vector

bottom row is always [0 0 0 1]
This will mean something when we get to vision

$$P^{A} = T^{A}_{B} P^{B}$$

- You can also these together:

$$\underbrace{T^{A}_{C}}_{} = \underbrace{T^{A}_{B}}_{\{B\}\to\{A\}} * \underbrace{T^{B}_{C}}_{\{C\}\to\{B\}} \xRightarrow{\text{think}} T^{A}_{\cancel{B}} * T^{\cancel{B}}_{C} = T^{A}_{C}$$

they cancle

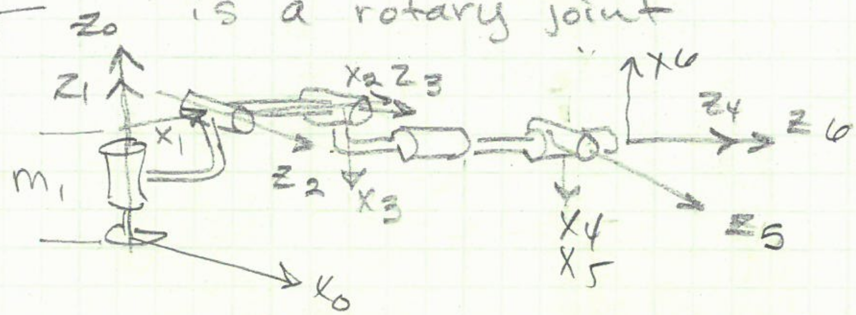
- DH
  - I suggest you read and follow the examples

- DH Process (my summary)

    ① put Z-axis through axis of rotation

    ② put X-axis along the common perpendicular between the z-axes

    ③ Starting @ 0, assign each frame a number
    - {0} is the base reference frame which generally doesn't move
    - {1} is often co-aligned w/ {0} if it is a rotary joint



$X_4 \equiv Z_4 \perp Z_5$, X_4 @ intersection

$X_5 \equiv Z_5 \perp Z_6$, X_5 @ intersection

$X_6 \equiv$ anywhere $\perp Z_6$, put @ end effector location

$X_6 \equiv Z_6$ is the last coordinate frame, pick a $\perp$ to Z_6 that is useful

Move to end

	a_{i-1}	twist α_{i-1}	d_i	θ_i
1	0	0°	m_1	θ_1
2	m_2	90°	0	θ_2
3	m_3	0°	0	θ_3
4	m_5	-90	m_4	θ_4
5	0	90	0	θ_5
6	0	-90	m_6	θ_6

* I replaced d's w/ m's because it is confusing otherwise