

# COM-711

## SELECTED TOPICS IN COMPUTER VISION

### 2D TRACKING PART 1/2

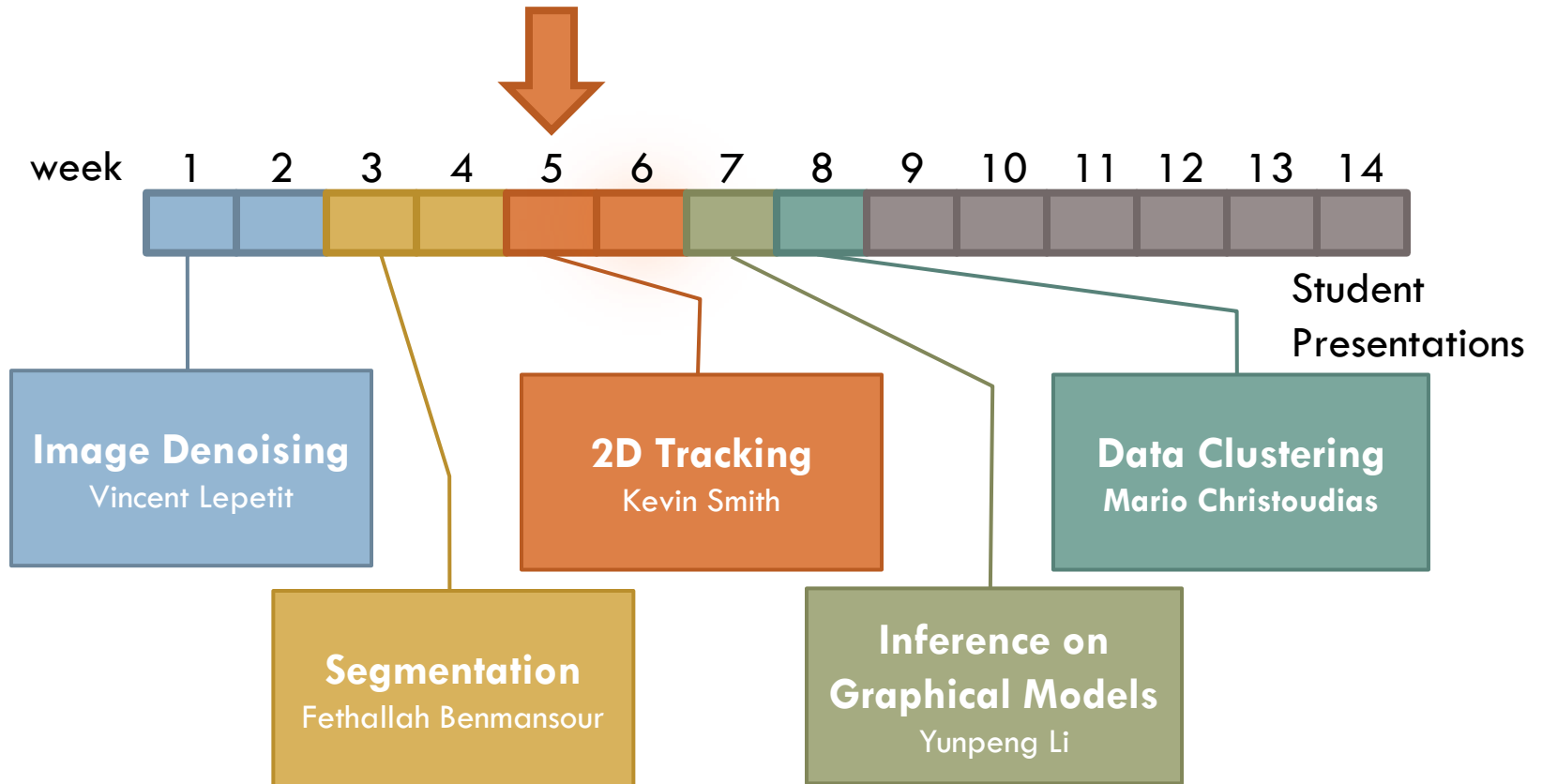
Oct 21, 2011

Kevin Smith

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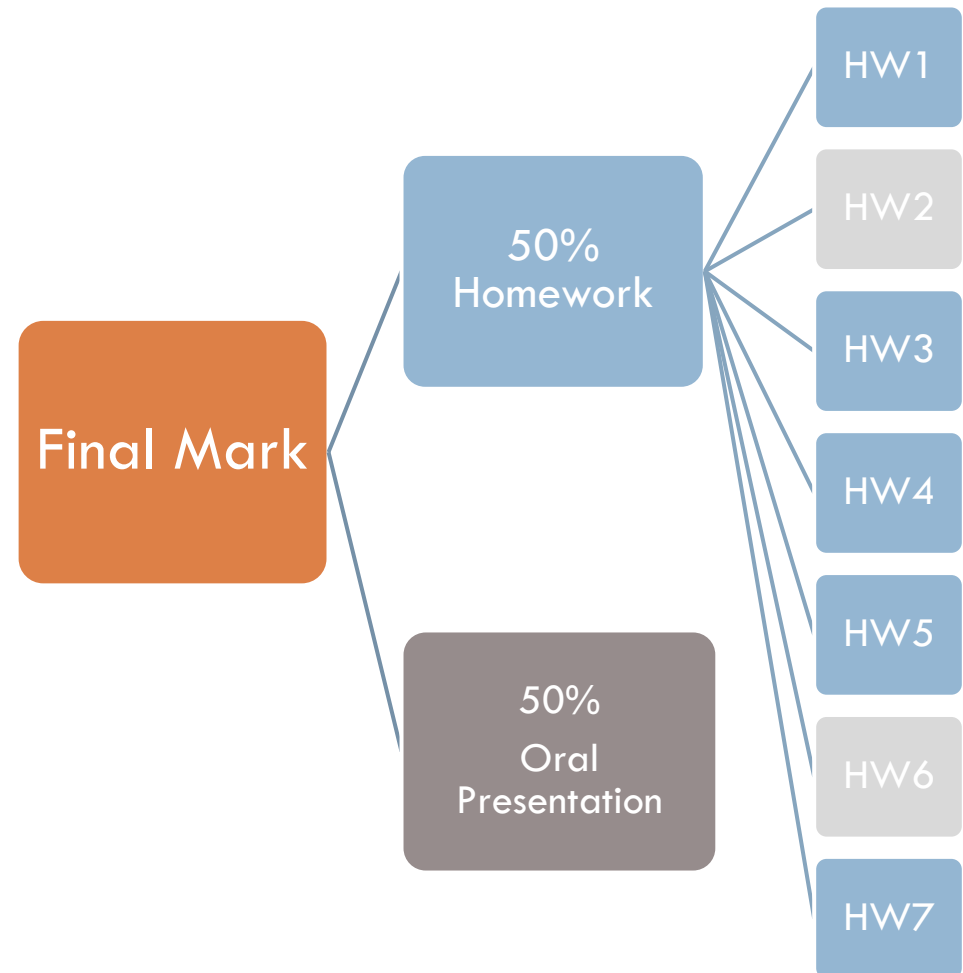
# Course update

## ■ Lectures



# Course update

- Final mark
  - 50% homework
  - 50% presentation
- Homework mark
  - Considers only the best( $N-2$ ) scores from  $N$  total assignments



# Oral presentations

- Each student **will present a published paper** on topics covered in the course to the rest of the class
  - Each student has approx **20 minutes** to speak (including questions)
  - A list of papers you may select from will be posted on the web site <http://cvlab.epfl.ch/teaching/topics/index.php>
  - Alternatively, you may propose a paper to present (subject to approval)
  - Instructors and other students will ask questions about the work
  - Presentations will be held during the last 6 weeks of the course. Time slots will be assigned on a first-come-first serve basis, after the list is posted. A web site will be made available to sign up with your selected paper and time slot

# Oral presentations

- Hints for a good presentation
  - Goal: inform your classmates
  - Start strong
    - Skip the overview and introduction slides
    - Avoid outline slides
    - Start by showing the results/benefits of the work
  - Use formulas only when necessary (fewer is better)
    - Establish a notation the audience understands
    - Describe the intuition behind formulae
  - Concentrate on the novelty of the work
    - Summarize well-known methods to save time
  - Limit the number of slides ( $\sim 1$  slide / minute)
  - Anticipate questions
  - Practice!
    - In front of an audience, if possible

# Course update

## ■ Homework: 7 total assignments

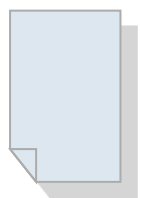
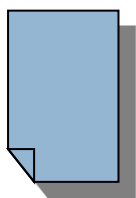
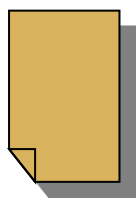


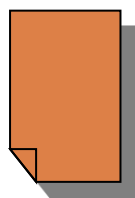
Image denoising



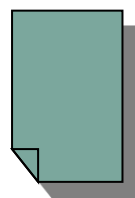
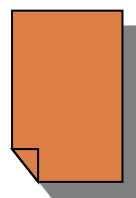
Segmentation



2D Tracking



Inference on  
Graphical  
Models



Data  
Clustering

- Only the best  $N-2$  scores from  $N$  total assignments will be considered (you can “skip” two assignments)
- First 4 assignments available on course web site  
<http://cvlab.epfl.ch/teaching/topics/index.php>

# Homework

- 2D Tracking assignments
  - HW4: Derive the recursive Bayesian filtering equation
  - HW5: Implement a particle filter, use it to track objects in 3 video sequences

# Outline

Introduction to the  
tracking problem

Recursive Bayesian  
filtering

Batch probabilistic  
methods



# Outline

## Introduction to the tracking problem

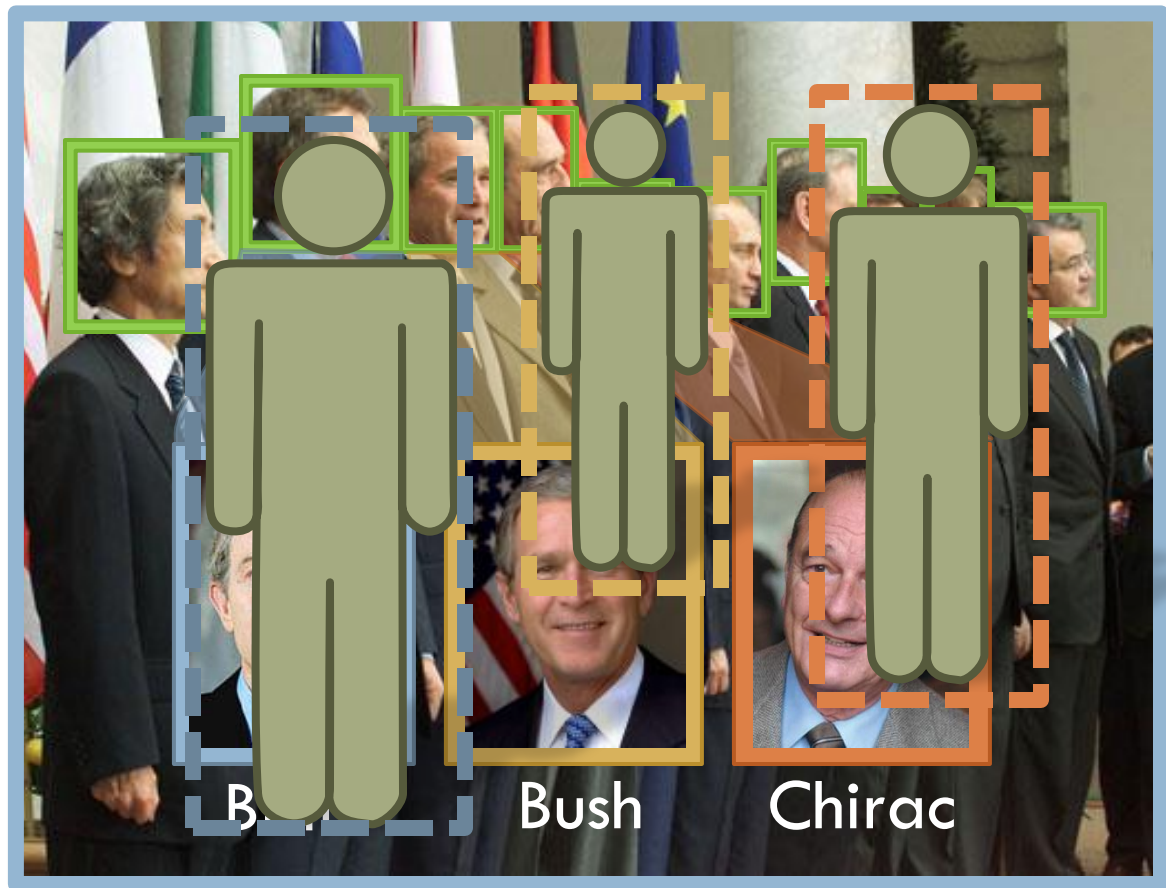
- What is tracking?
- Approaches & assumptions
- Tracking applications
- State of the art & challenges

## Recursive Bayesian filtering

## Batch probabilistic methods

# What is tracking?

- detection
- recognition
- tracking



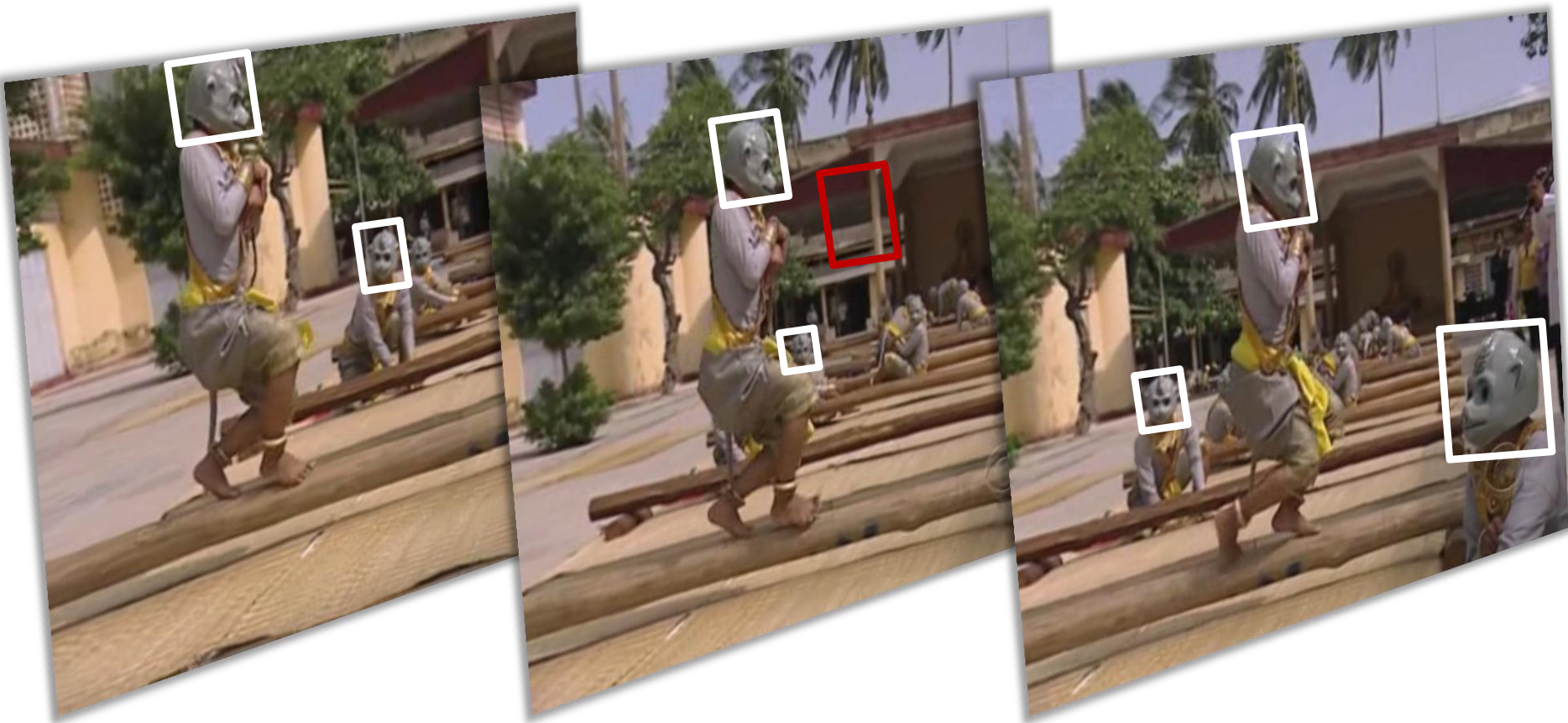
# What is tracking?

- Definition: using **image measurements** and a **predictive dynamic model** to consistently estimate the state(s)  $X_t$  of one or more object(s) over the discrete time steps corresponding to video frames.



# What is tracking?

- Why not just do detection?
  - Estimate the state  $X$  at each time step
  - - inefficient
  - - data association problem



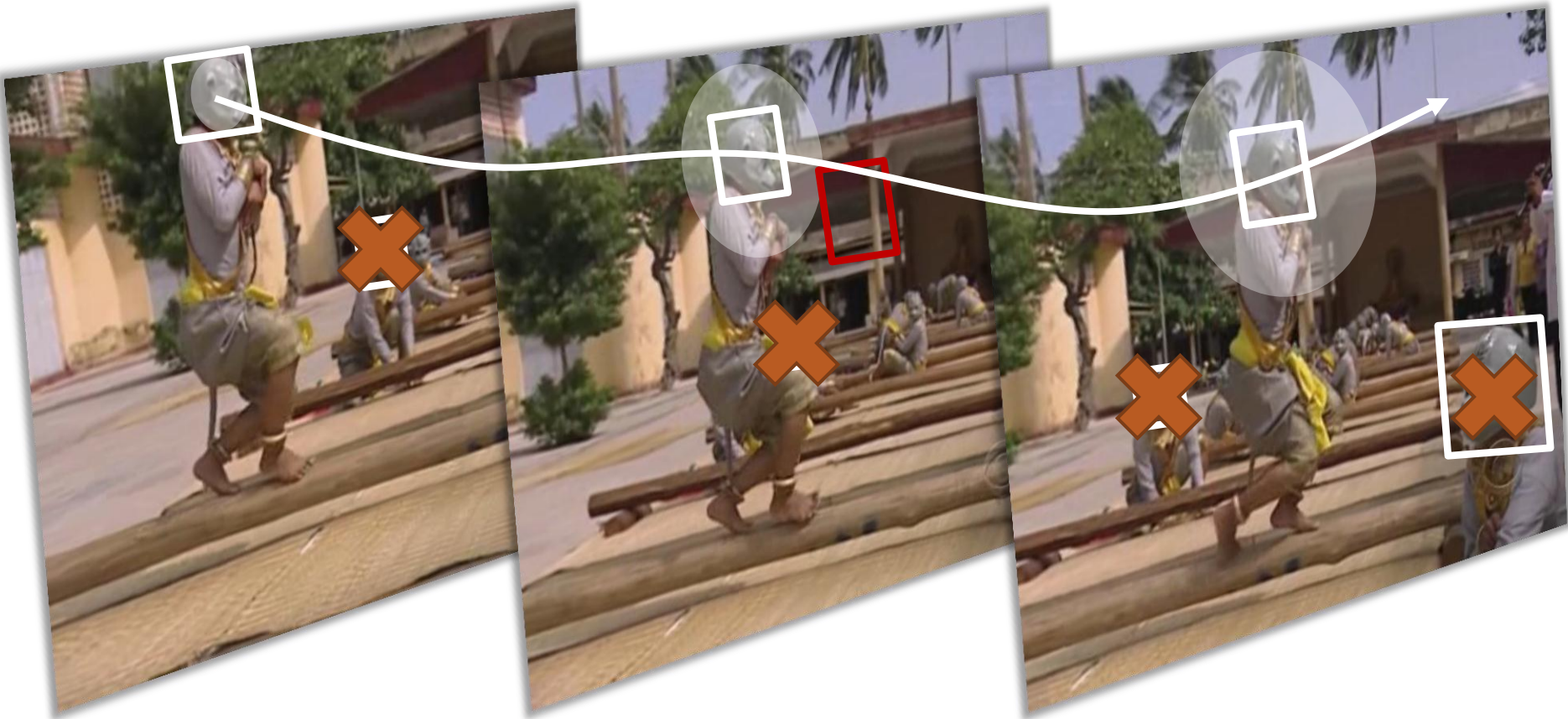


# What is tracking?

## ■ It's better to do tracking

- Maintain an estimate of  $X$  over time, predict the future location

- + efficient, restricts search space
- + smoothes noisy measurements
- - requires knowledge about object behavior



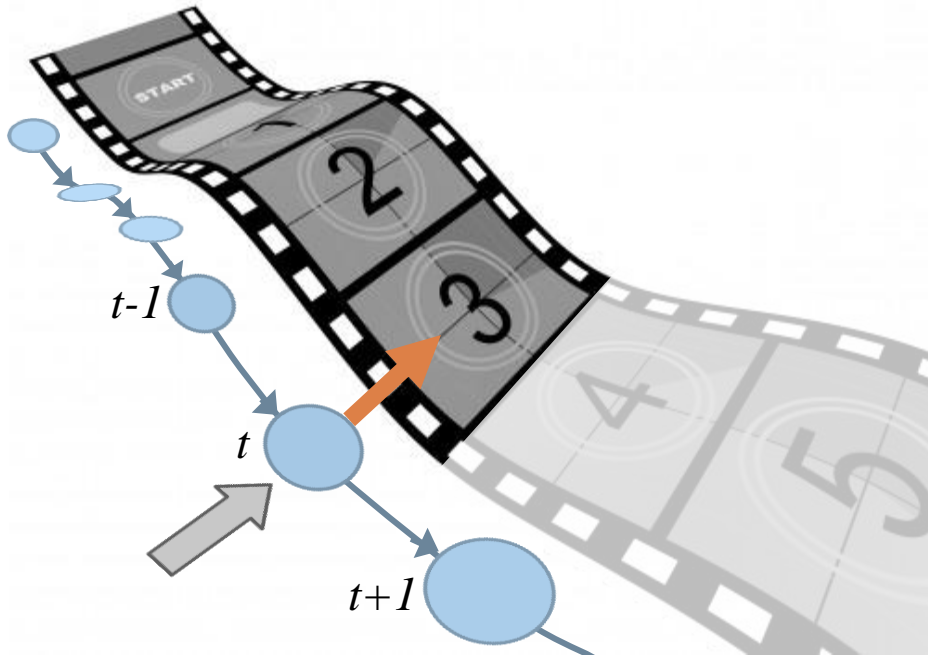
# Tracking assumptions

- Smooth camera
  - No instant transitions between viewpoints
  - Any camera pose/parameter changes are gradual
- Object motion can be modeled
  - Linear models
  - Non-linear mod
- Likelihood of object presence at a location in the image can be modeled
  - Typically uses local image information

# Approaches to tracking


## ■ Sequential

- (recursive, online)
- + Inexpensive  $\rightarrow$  real-time
- - no future information
- - cannot revisit past errors



## ■ Batch Processing

- (offline)
- - Expensive  $\rightarrow$  not real-time\*
- + considers all information
- + can correct past errors

$t=1, \dots, T$  



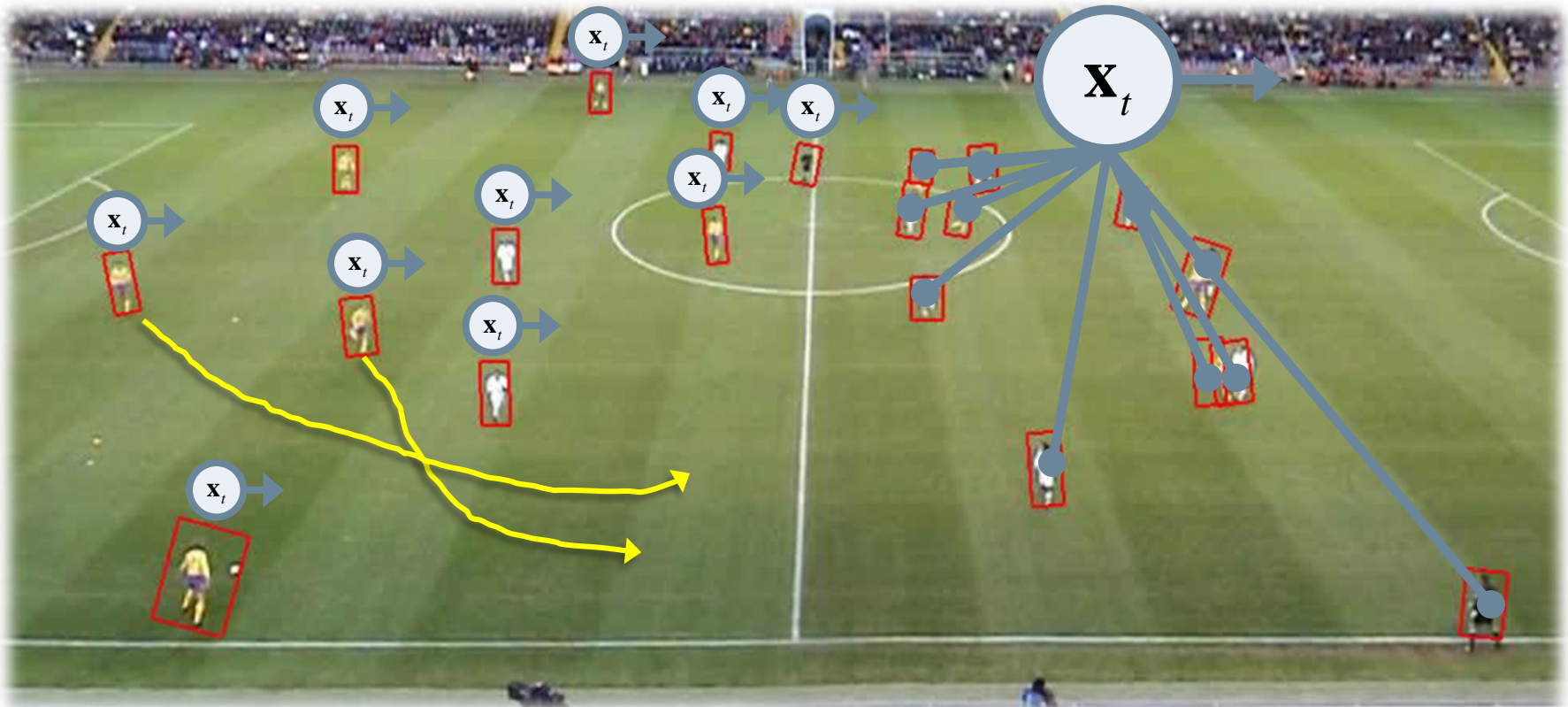
# Approaches to tracking

## ■ Parallel trackers

- several single-object trackers
- computationally less expensive
- how to handle interaction, cross-overs?

## ■ Joint state

- single multi-object representation
- computationally expensive
- principled interaction models

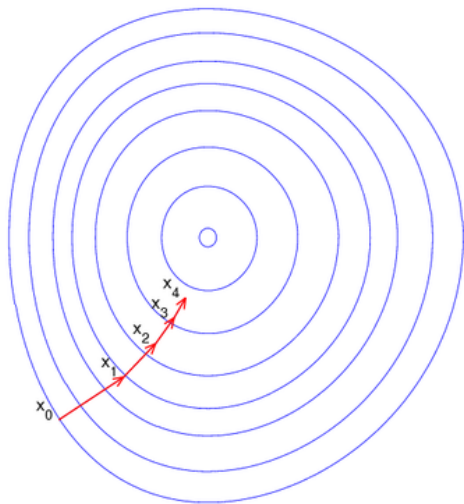




# Approaches to tracking

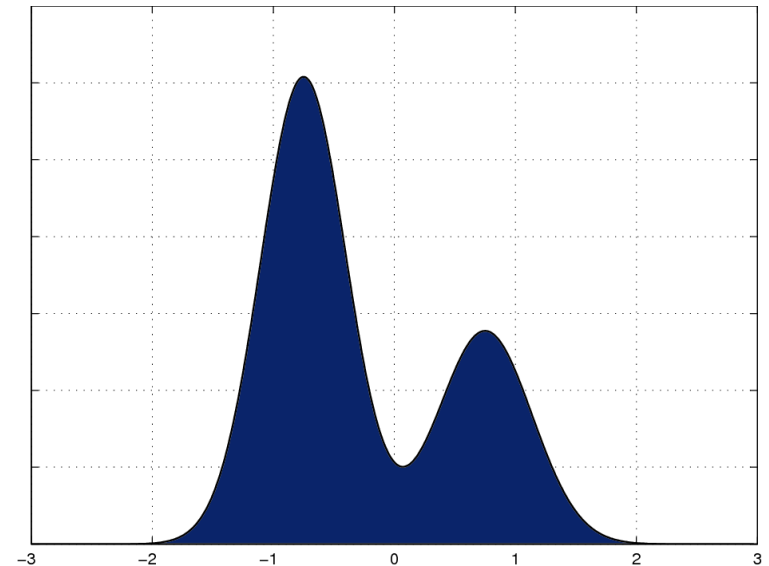
## ■ Non-probabilistic

- + quick convergence\*
- + efficient
- - stuck in local minima
- - does not model multiple objects

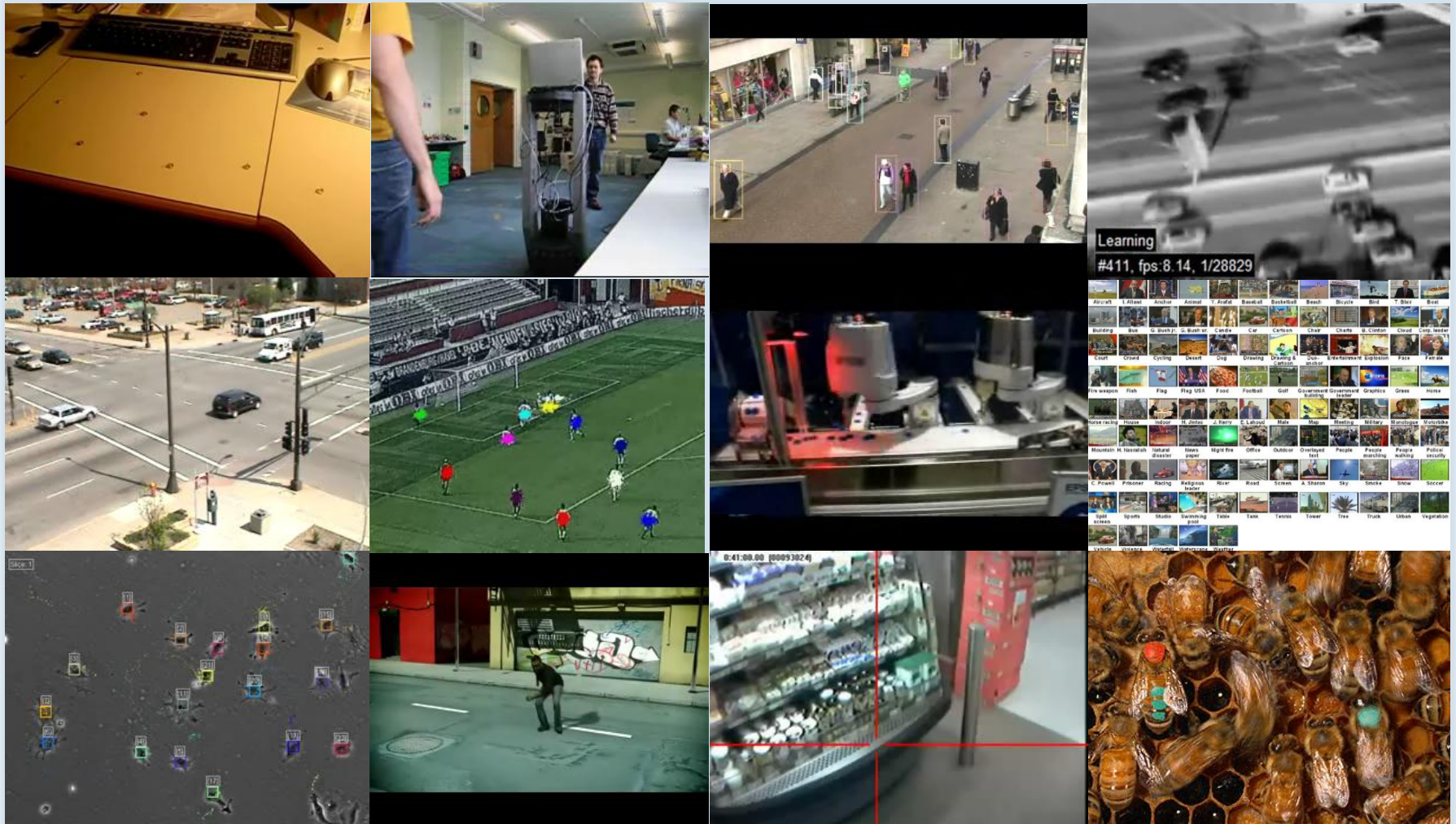


## ■ Probabilistic

- + flexible, principled
- + multi-modal
- - slower
- - interpretation

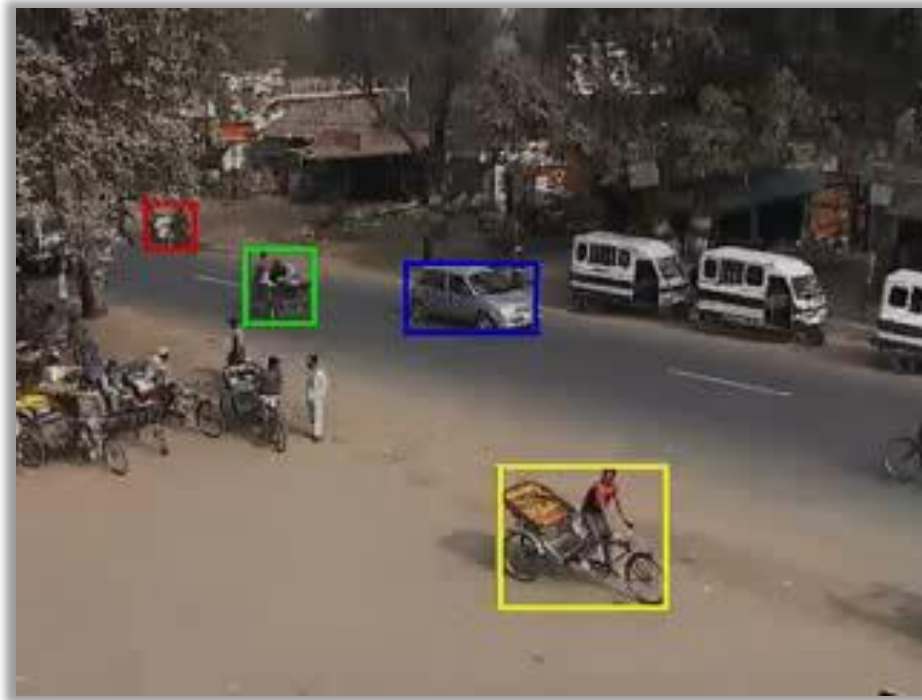
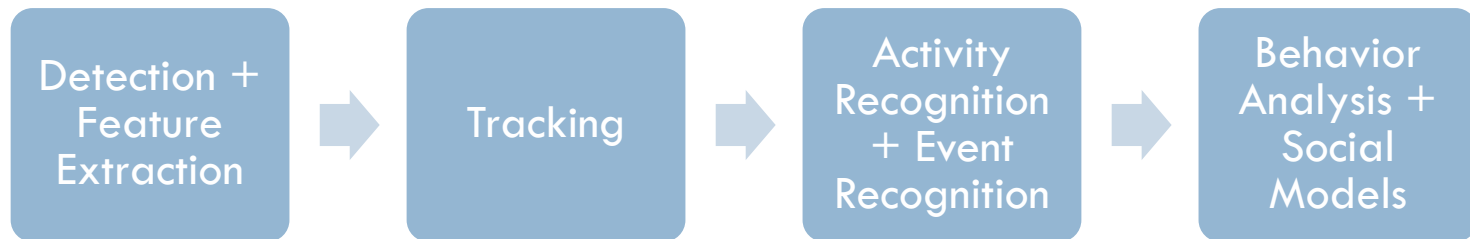


# Tracking applications



# Tracking applications

- Tracking is an essential step in many computer vision based applications



# Tracking applications

## ■ Sports



P. Nillius, J. Sullivan, S. Carlsson, [Multi-Target Tracking - Linking Identities using Bayesian Network Inference](#)  
Computer Vision and Pattern Recognition (CVPR), 2006



# Tracking applications

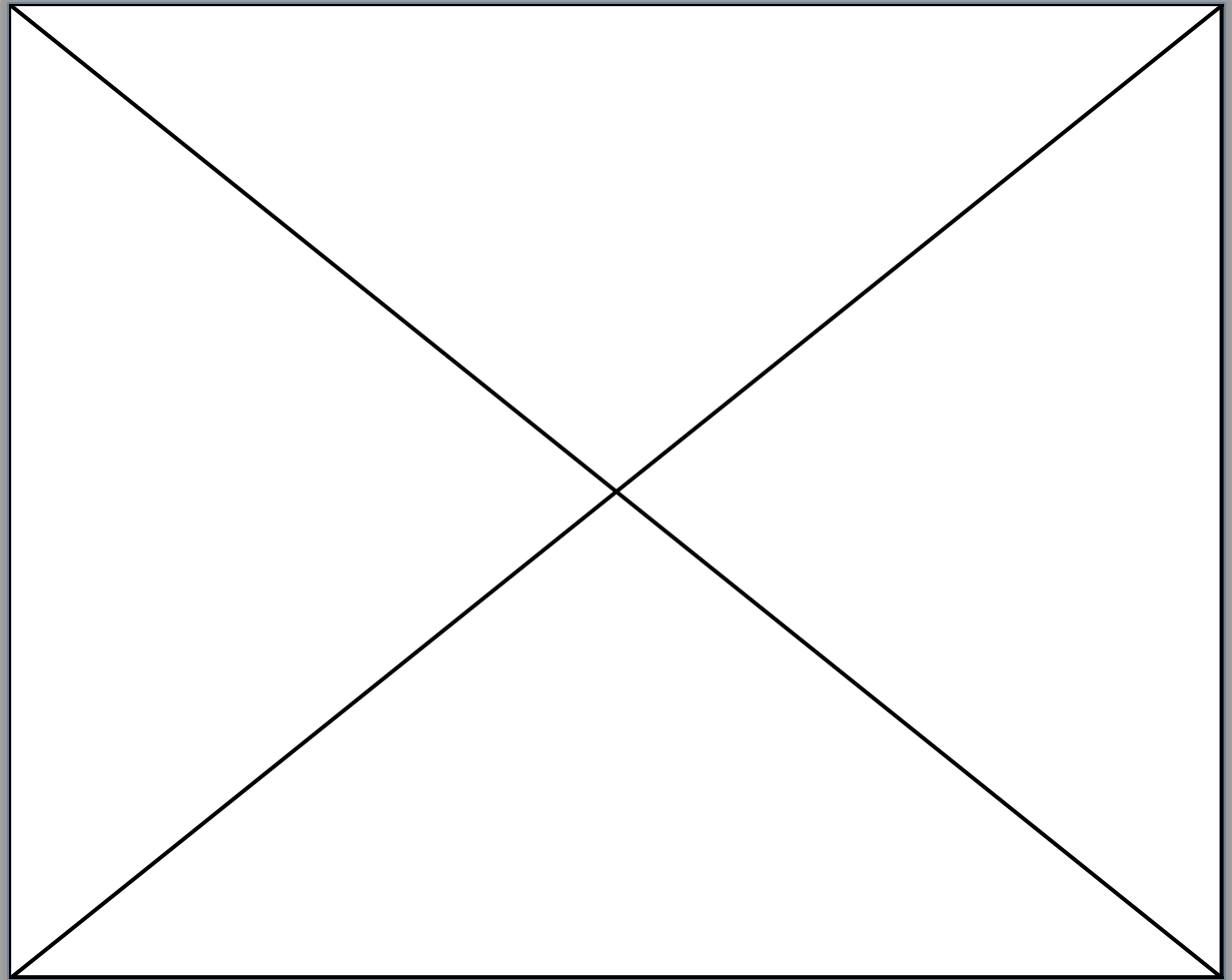
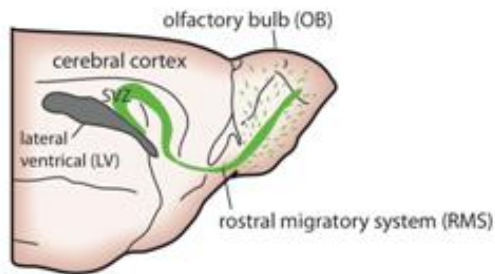
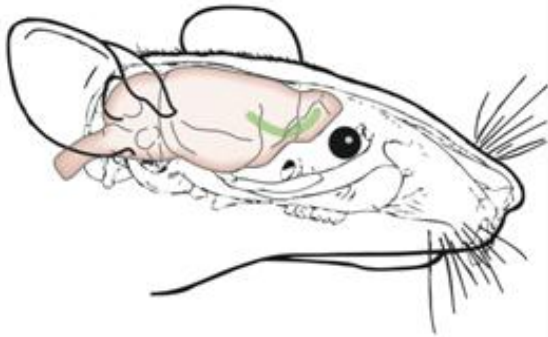
## ■ Surveillance



K. Smith, P. Quelhas, and D. Gatica-Perez, [Detecting Abandoned Luggage Items in a Public Space](#), Performance Evaluation of Tracking and Surveillance (PETS) Workshop at CVPR, New York, NY, June 18 2006

# Tracking applications

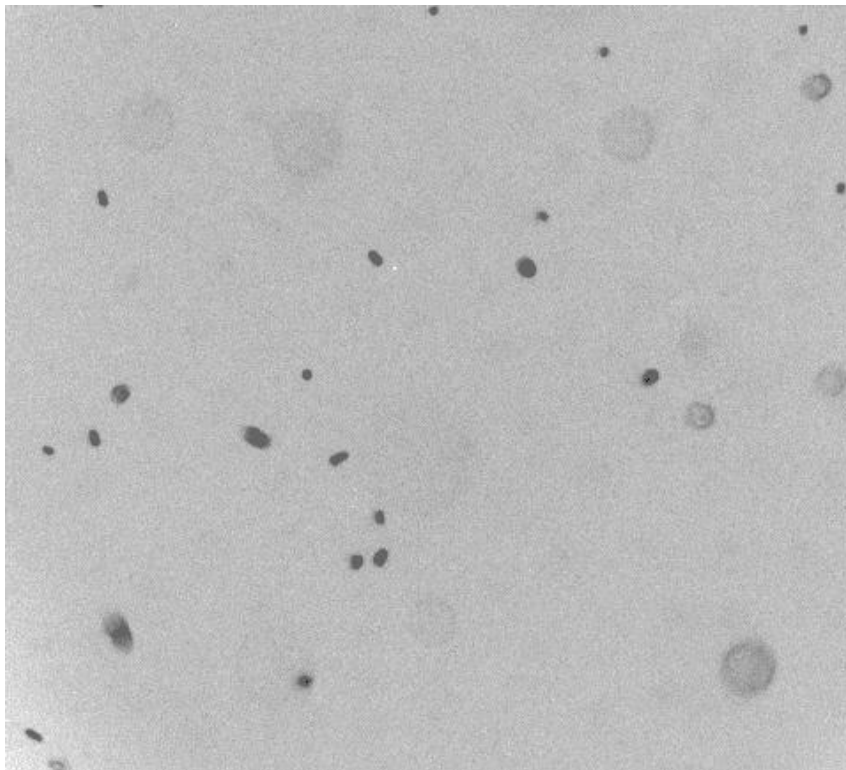
## ■ Biological research



# Tracking applications

## ■ Biological Research

- Goal: develop a method to “trap” Salmonella bacteria



# What is the state of the art?

- Despite being classic computer vision problem, tracking is **largely unsolved**
  - Some limited successes
  - No general-purpose tracker
  - No standard data corpus for comparison
  - No standard evaluation methodology
  - Challenging problems remain

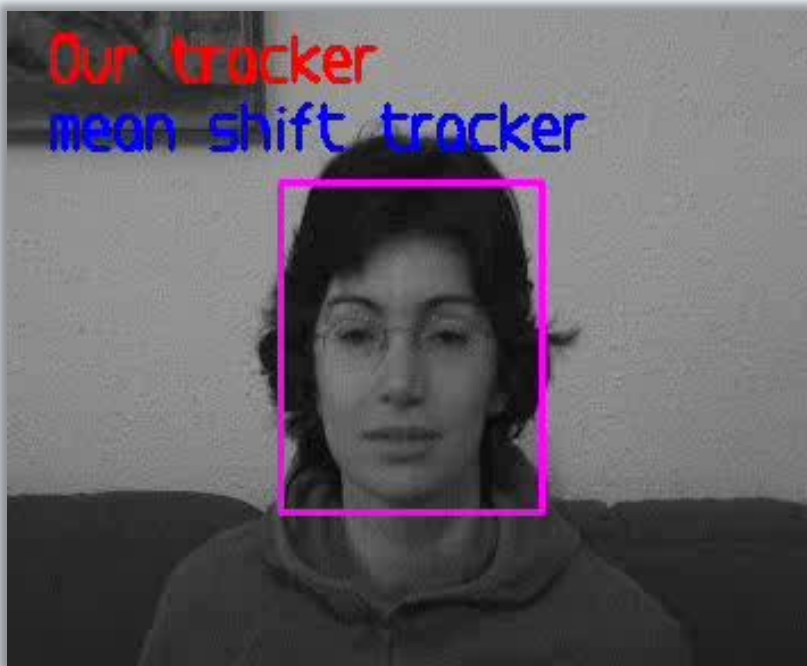


# Obstacles to tracking



- appearance change
- occlusion
- distraction
- illumination change
- difficult motion
- multiple objects
- scale change
- efficient solution

# Obstacles to tracking



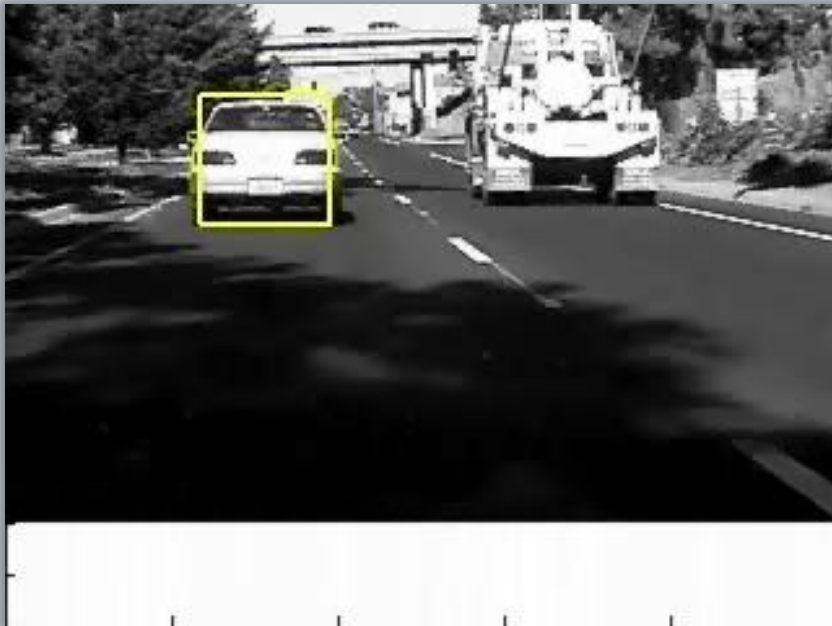
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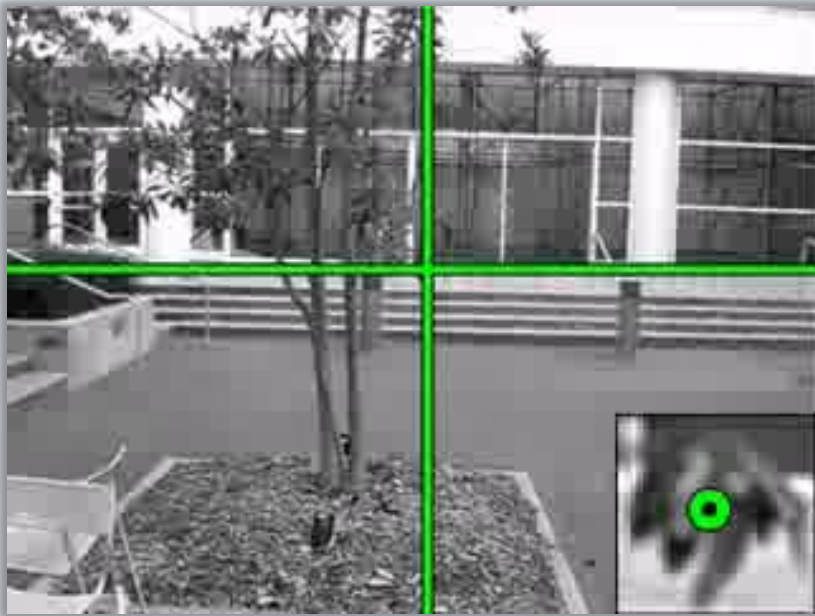
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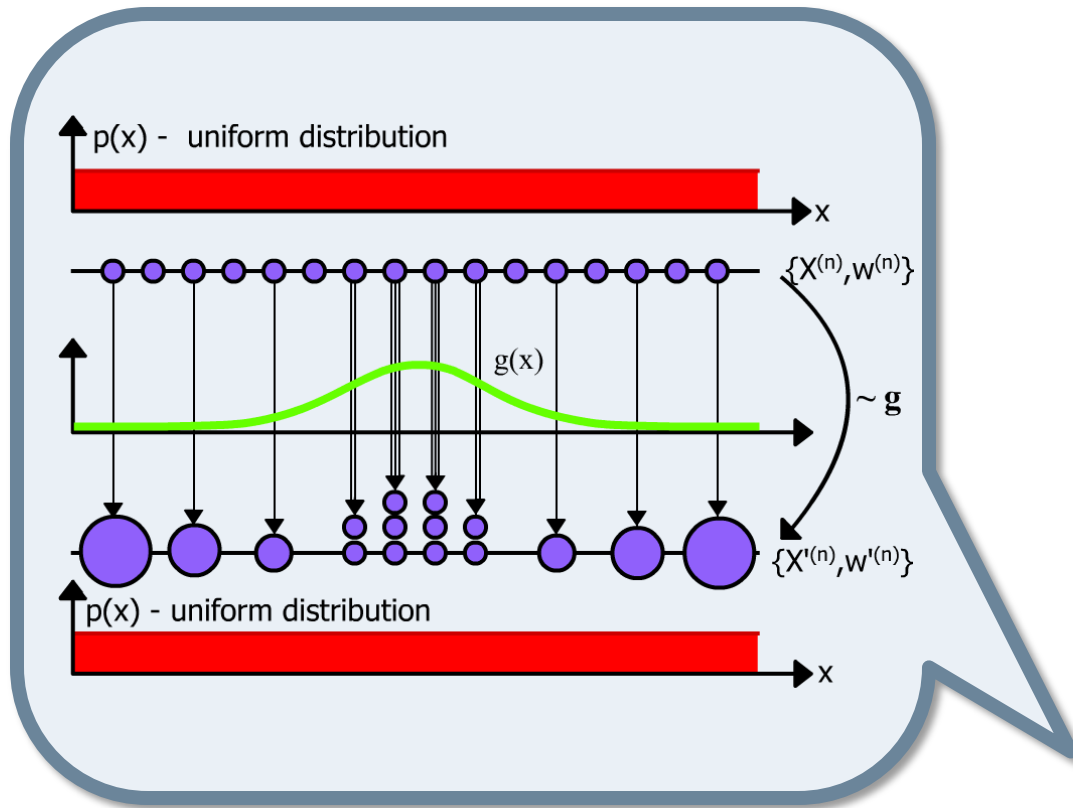
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- efficient solution



# Outline

Introduction to the tracking problem

Recursive Bayesian filtering

- Background & formulation
- Kalman filter
- Particle filter

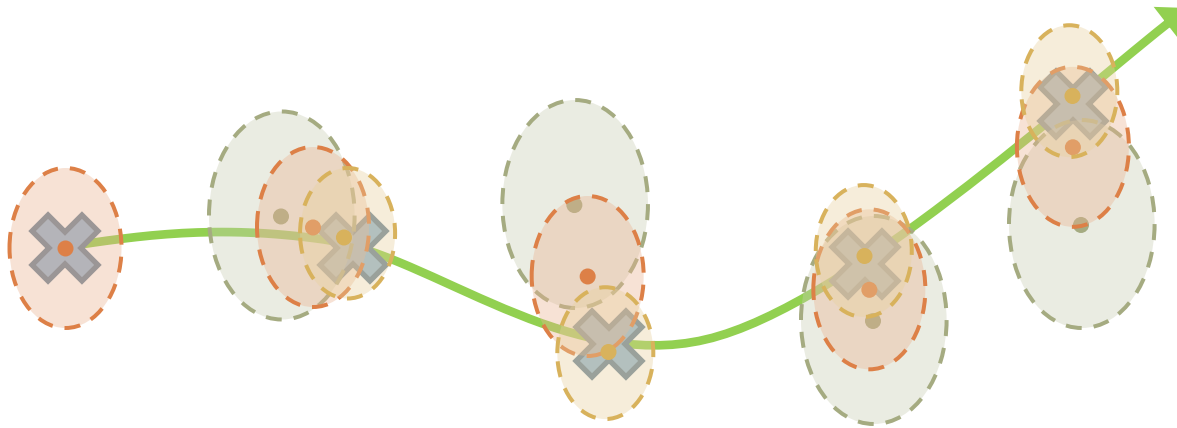
Batch probabilistic methods

# Recursive Bayesian filtering

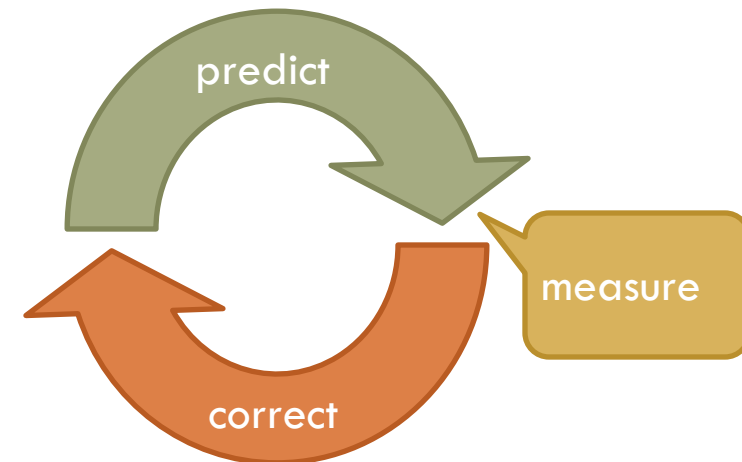
- How is it characterized?
  - Sequential
  - Parallel trackers OR joint modeling of multiple objects
  - Probabilistic
- Popular examples
  - Kalman filter
  - Particle filter

# Recursive Bayesian filtering

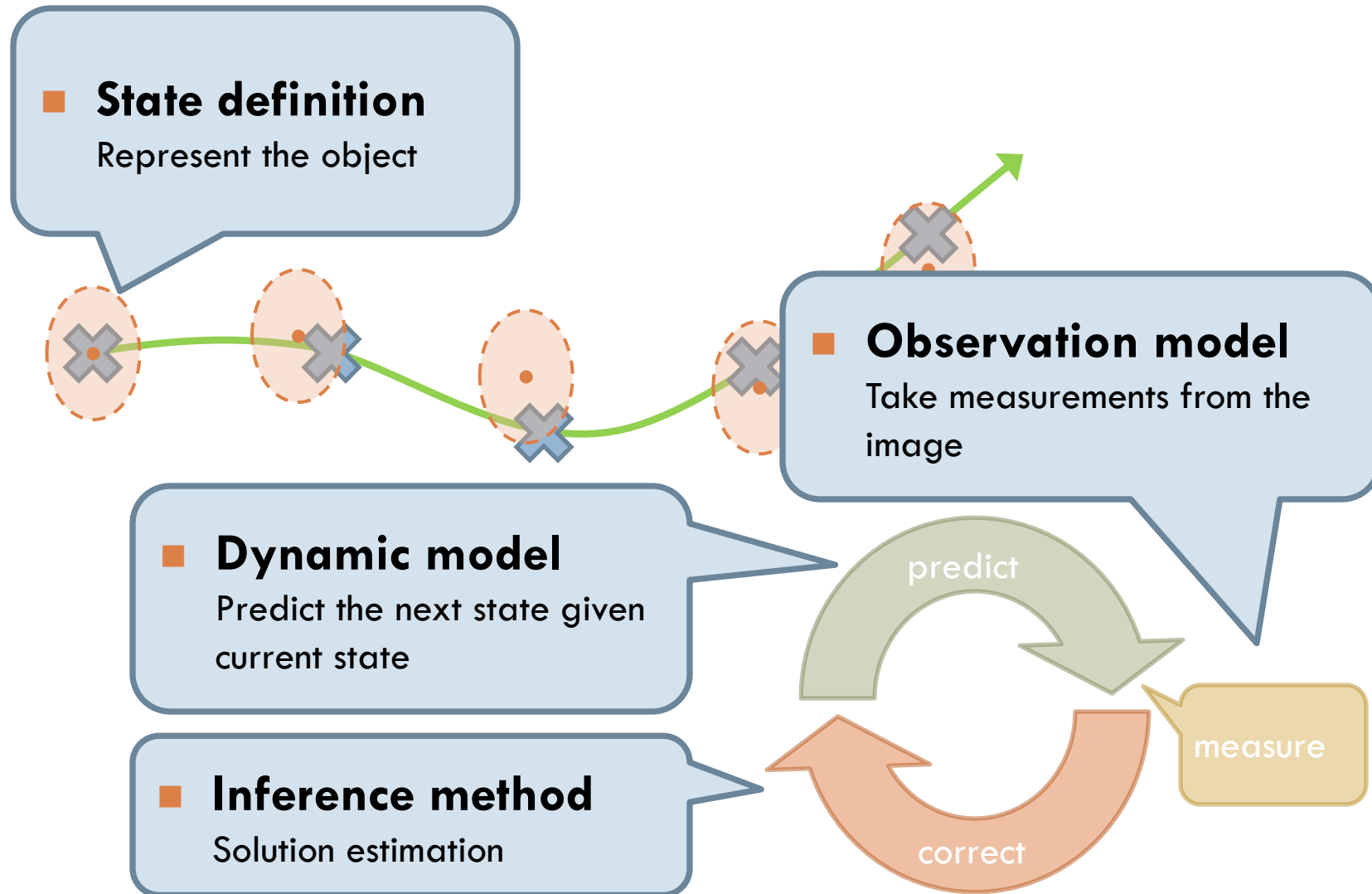
- **Key idea 1:** Probability distributions represent our belief as to the state of the object



- **Key idea 2:** Recursive cycle
  1. **Predict** from motion model
  2. **Measurement** from image
  3. **Correct** the prediction...repeat



# Recursive Bayesian filtering



# Tracking ingredients

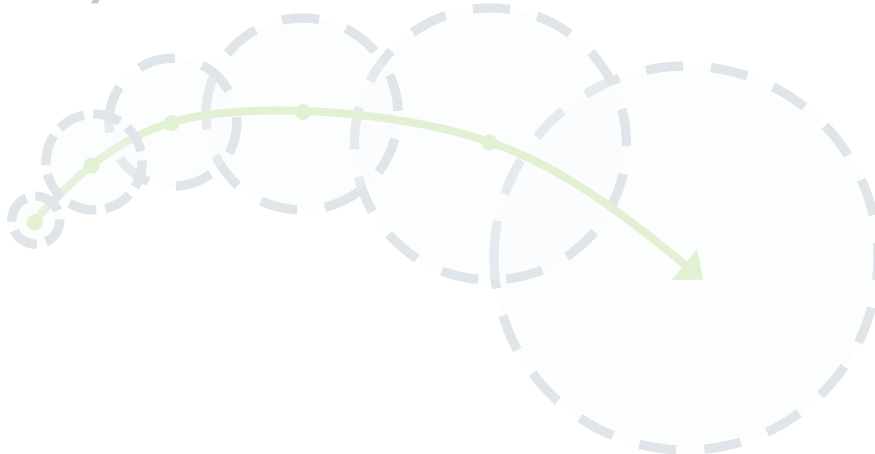
## ■ State Definition



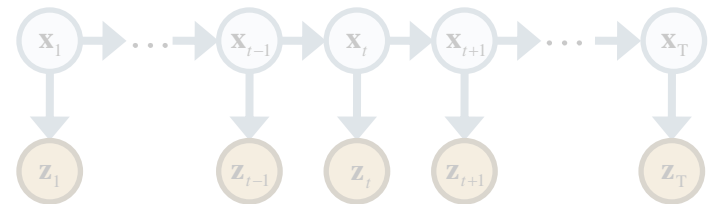
## ■ Observation Model



## ■ Dynamic Model



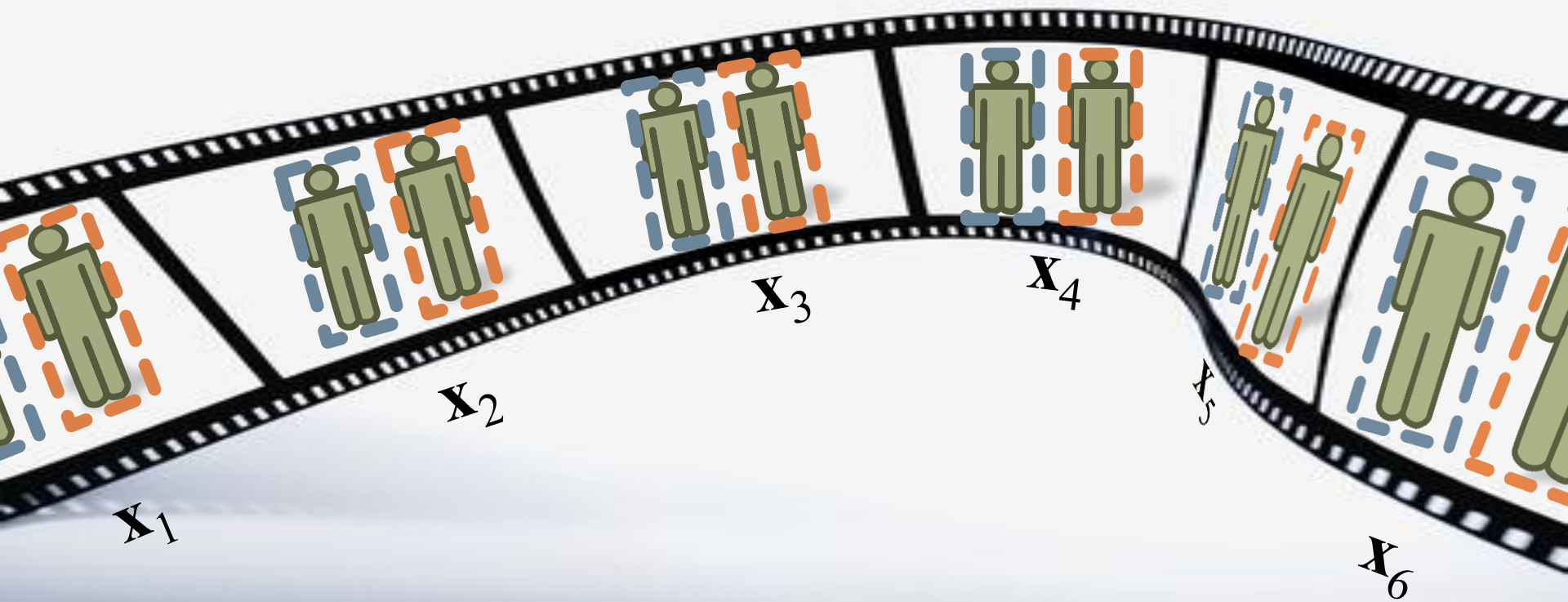
## ■ Inference



# State definition

- Describes properties of the tracked object(s) at an instant in time
- Defines solution space

$$X_t = \{\mathbf{x}_1, \dots, \mathbf{x}_{t-1}, \mathbf{x}_t\}$$



# State definition

- Decomposed for time step  $t$ ,  $\mathbf{X}_t$  can parameterize the object in many ways, often via:

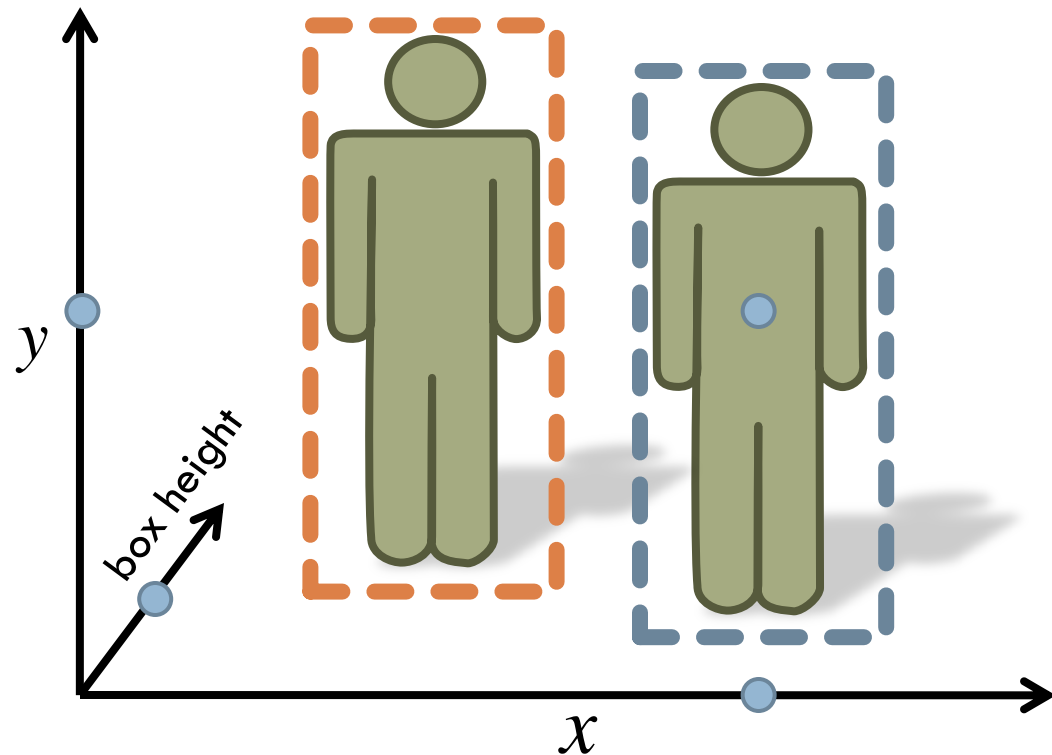
- location
- velocity
- size
- shape
- identity
- switching model

$$\mathbf{X}_t = x$$

$$\mathbf{X}_t = (x, y)$$

$$\mathbf{X}_t = (x, y, h)$$

$$\mathbf{X}_t = \{\mathbf{X}_t^1, \mathbf{X}_t^2\}$$



# State definition

## ■ Object defined by a point

- position
- velocity
- acceleration

$$\mathbf{x}_t = (x, y)$$

$$\mathbf{x}_t = (x, y, \dot{x}, \dot{y})$$

$$\mathbf{x}_t = (x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y})$$





# State definition

## ■ Bounding box

- position
- height
- aspect
- velocity

$$\mathbf{x}_t = (x, y)$$

$$\mathbf{x}_t = (x, y, h, a)$$

$$\mathbf{x}_t = (x, y, \dot{x}, \dot{y}, h, a)$$



# State definition

## ■ Ellipse

- location
- eccentricity
- major axis

$$\mathbf{x}_t = (x, y, m, e)$$

$$\mathbf{x}_t = (x, y, a, b)$$



# State definition

## ■ Active contour

- b-splines
- control points
- spline length
- $H$  basis functions

$$\mathbf{x}_t = (X(s), Y(s))$$

$$X(s) = H(s)X, 0 \leq s \leq N$$

$$Y(s) = H(s)Y$$

$$X = \{x^1, x^s, \dots, x^N\}$$

$$Y = \{y^1, y^s, \dots, y^N\}$$



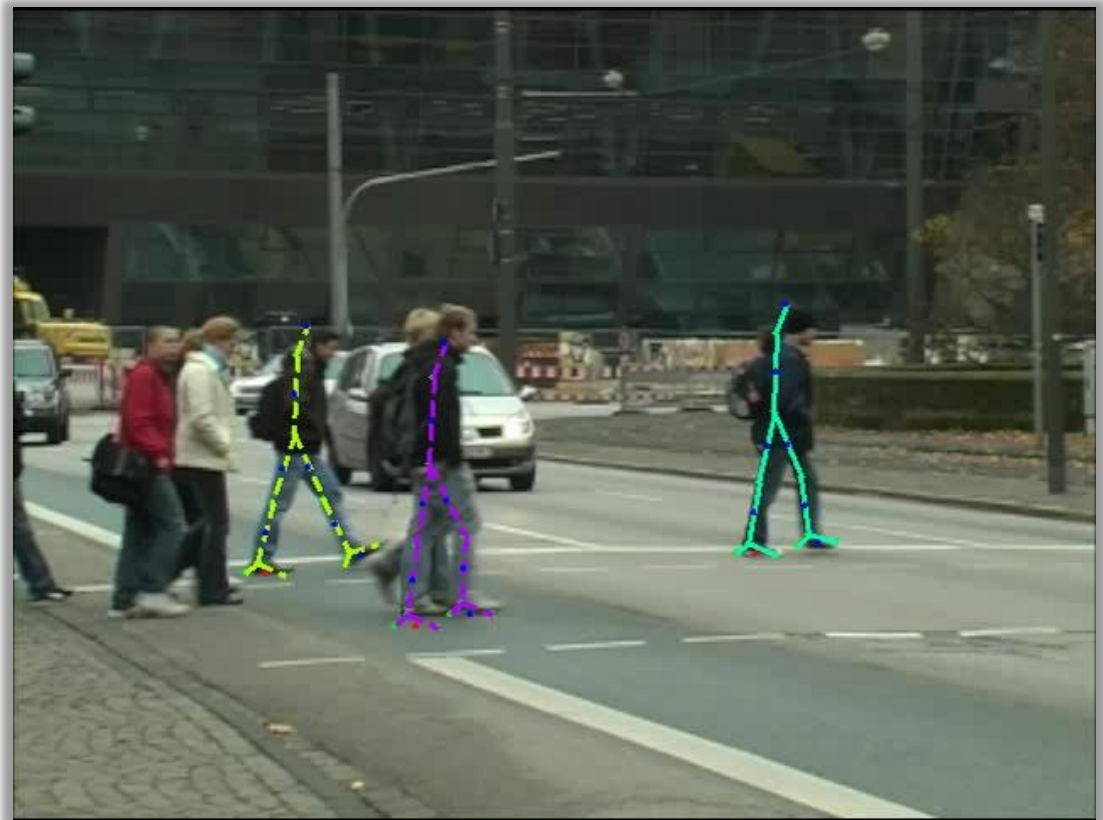
# State definition

## ■ Articulated & Part-based Models

- set of vertices
- locations
- scales
- constraints

$$\mathbf{X}_t = \{v^1, v^i, \dots, v^N\}$$

$$v^i = (x^i, y^i, s^i)$$

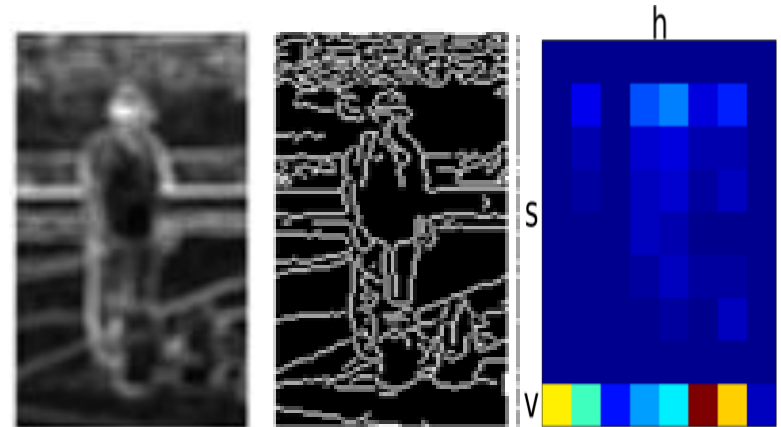


# Tracking ingredients

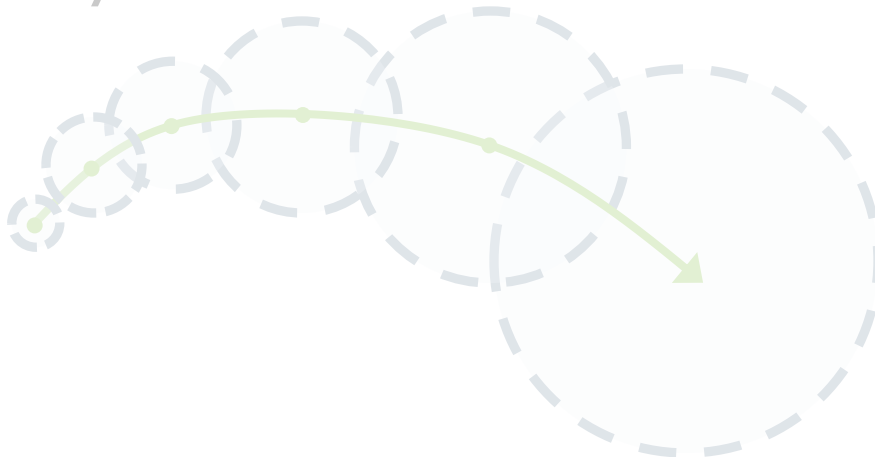
## State Definition



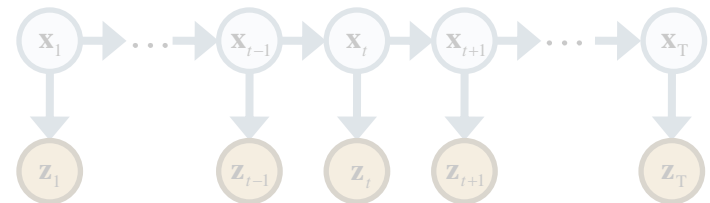
## Observation Model



## Dynamic Model



## Inference



# Observation model

- Notation - observations

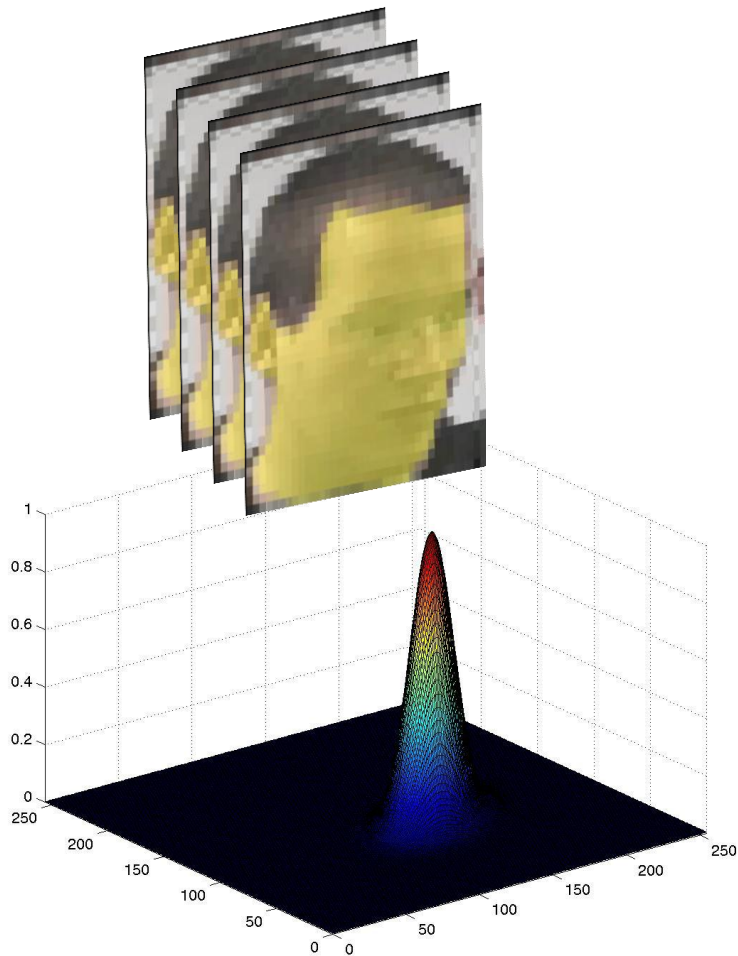
$$\mathbf{Z}_t = \{\mathbf{z}_1, \dots, \mathbf{z}_{t-1}, \mathbf{z}_t\}$$

- Returns the likelihood that a state hypothesis gave rise to the observed image data

$$p(\mathbf{z}_t \mid \mathbf{x}_t)$$

# Observation model

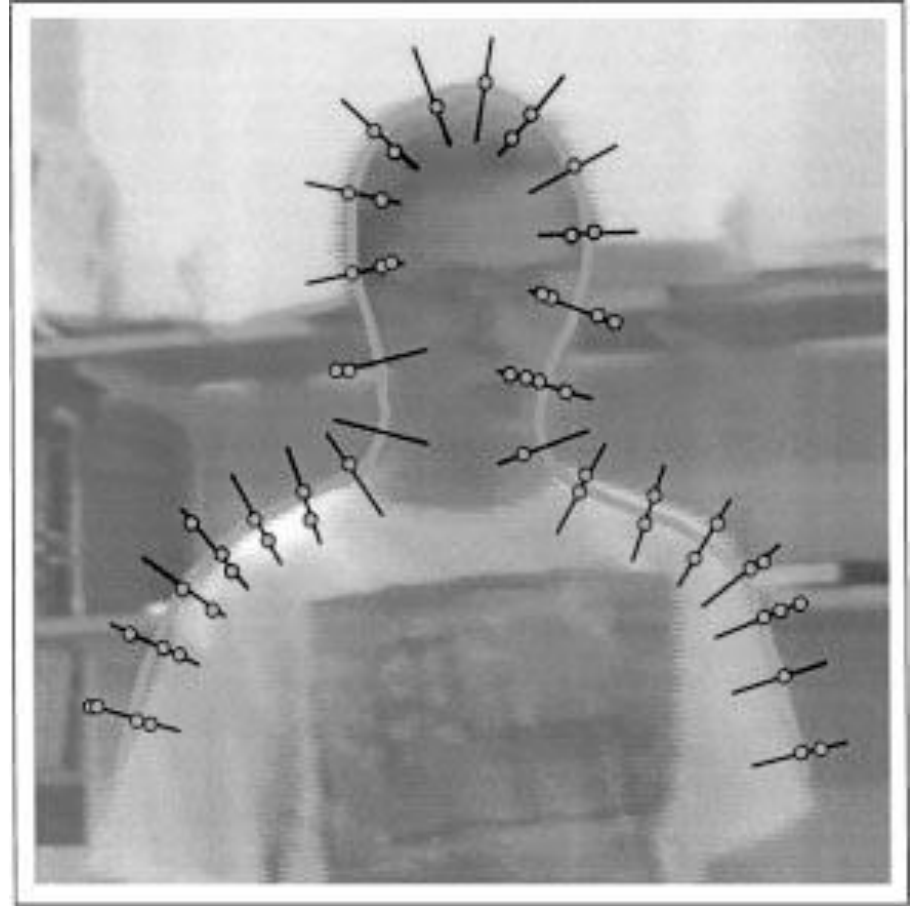
## ■ Modeling skin color





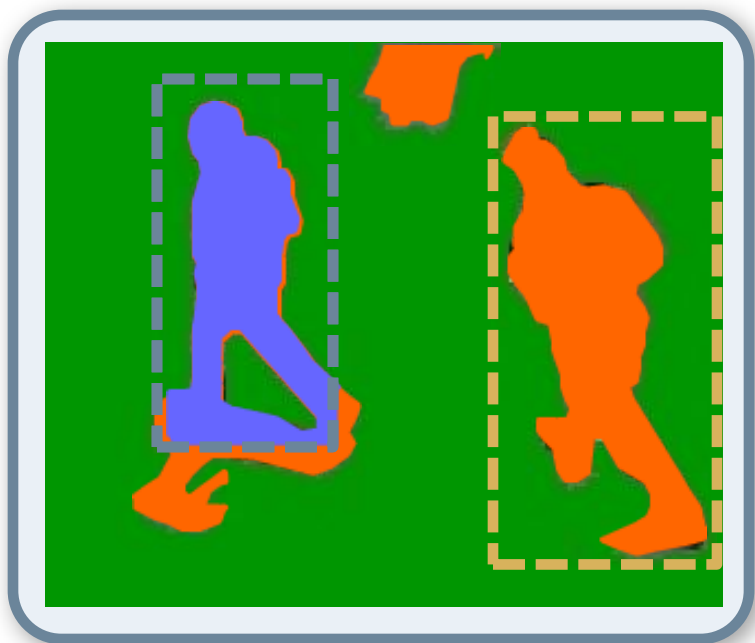
# Observation model

- Sum of measurements taken from lines perpendicular to a contour



# Observation model

- Background/foreground silhouette modeling



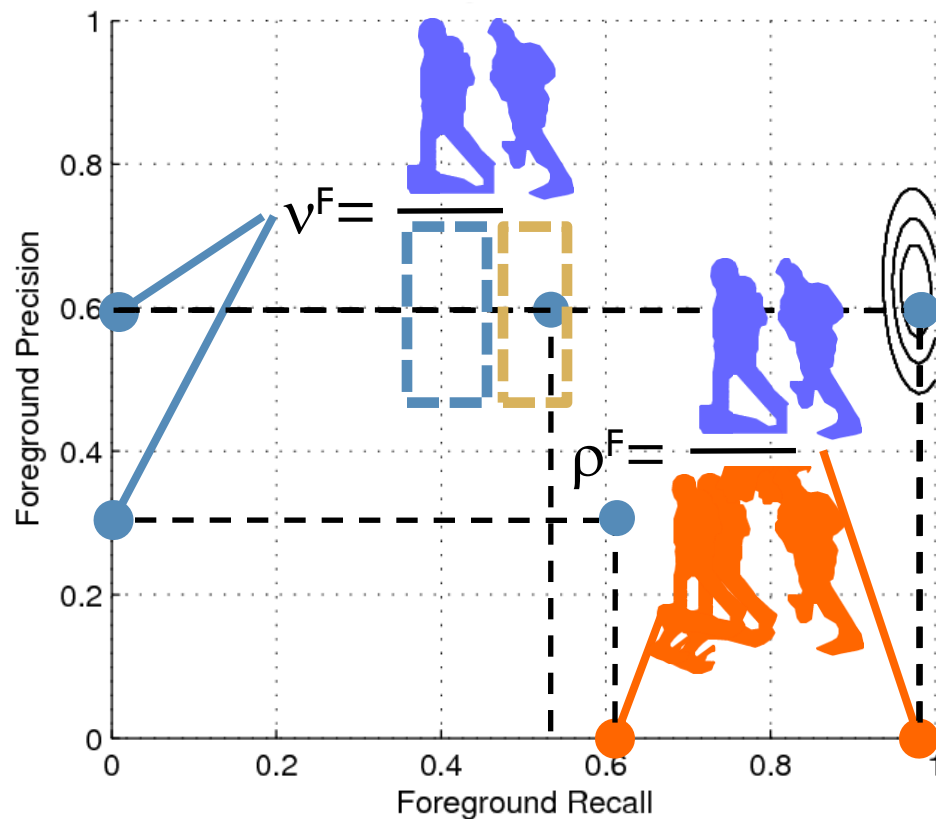
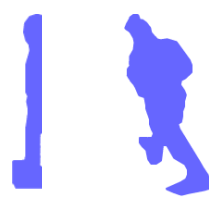
foreground



tracker

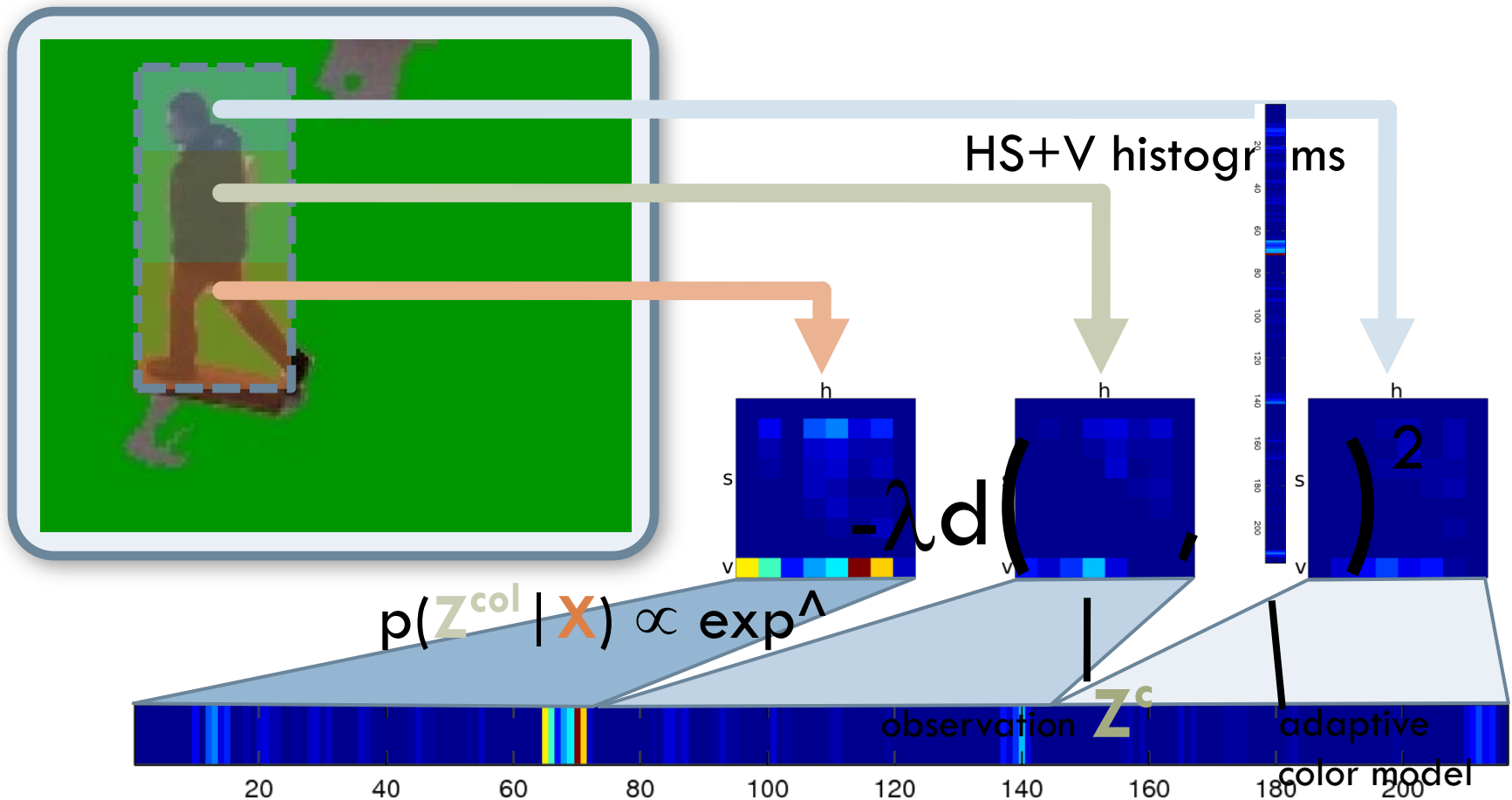


intersection



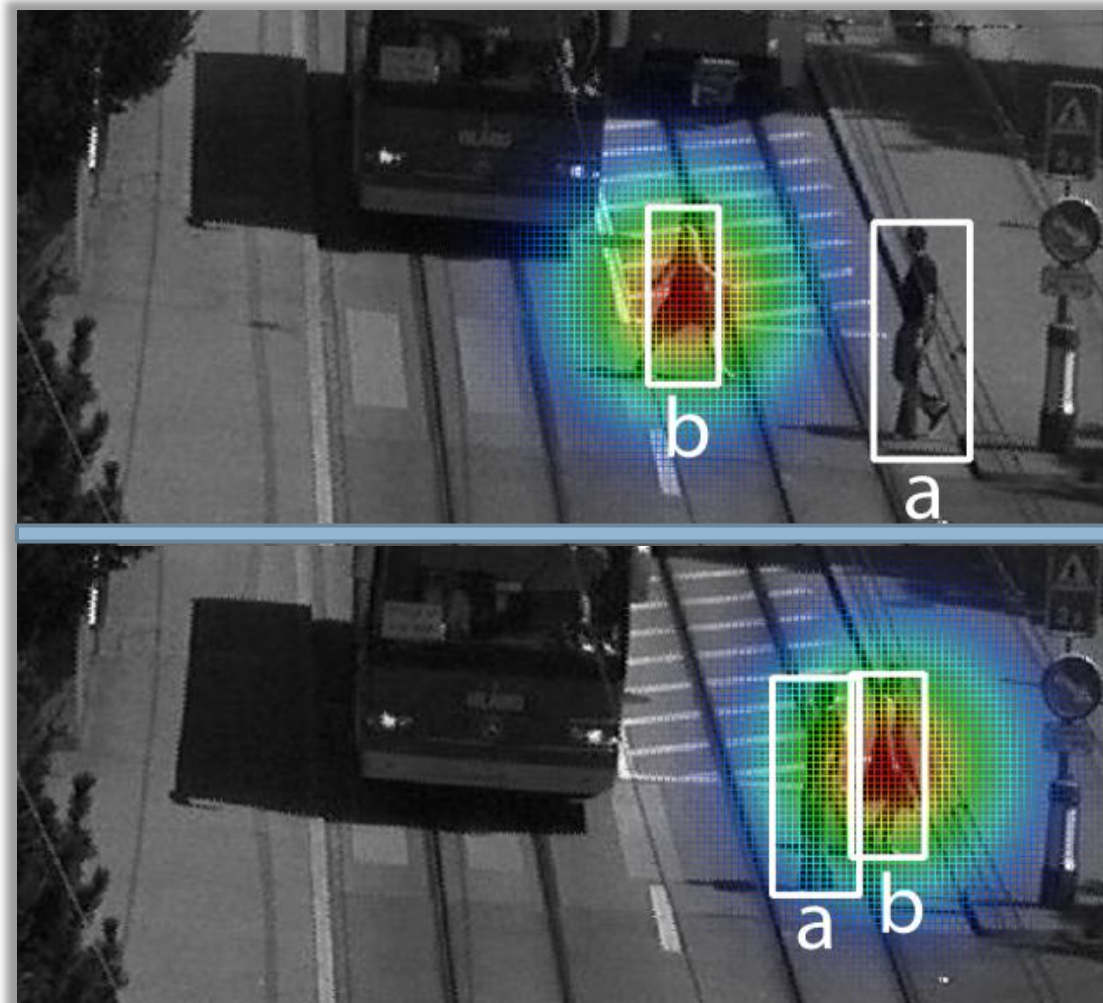
# Observation model

## Parts-based color model



# Observation model

- Detector confidence
- HOG based sliding window detector

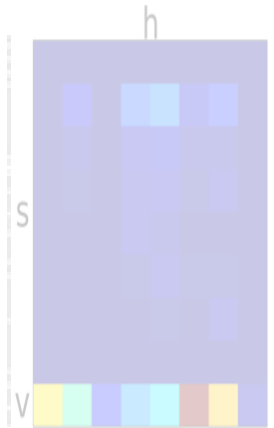


# Tracking ingredients

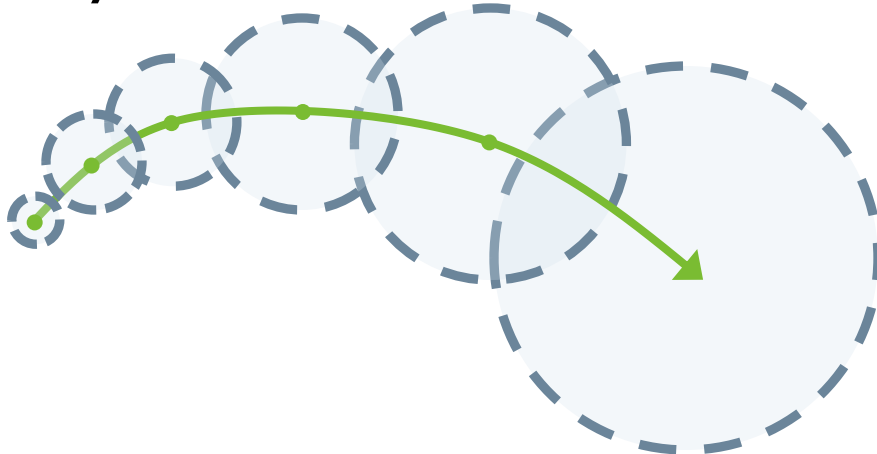
## State Definition



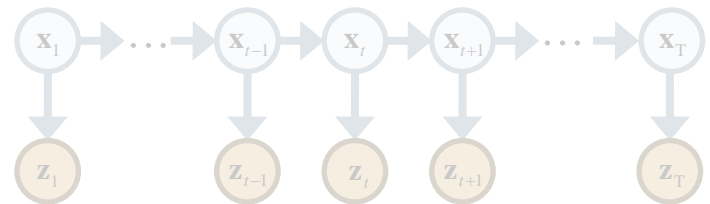
## Observation Model



## Dynamic Model



## Inference



# Dynamic model

- Current state is predicted from previous state

$$p(\mathbf{x}_t \mid \mathbf{x}_{t-1}) = N(\mathbf{F}_t \mathbf{x}_{t-1}, \Sigma_{F_t})$$

$$\mathbf{x}_t \sim N(\mathbf{F}_t \mathbf{x}_{t-1}, \Sigma_{F_t}) \quad \text{to obtain samples}$$

- Autoregressive linear dynamic model

$$\mathbf{x}_t = \mathbf{F}_t \mathbf{x}_{t-1} + \mathbf{w}_t$$

Predicted state  
time  $t$

State transition  
model

Previous state  
time  $t-1$

Noise term  
 $\mathbf{w}_t \sim N(0, \mathbf{Q}_t)$

# Dynamic model

- 1<sup>st</sup> order autoregressive
  - models position & velocity

$$\mathbf{x}_t = \mathbf{F}_t \mathbf{x}_{t-1} + \mathbf{w}_t$$

State  
vector

$$\mathbf{x}_t = \begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix}$$

$$\begin{pmatrix} x_t \\ y_t \\ \dot{x}_t \\ \dot{y}_t \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & & 1 & \\ & 1 & & 1 \\ & & 1 & \\ & & & 1 \end{pmatrix}}_{\mathbf{F}_t} \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \dot{x}_{t-1} \\ \dot{y}_{t-1} \end{pmatrix} + \mathbf{w}_t$$

State transition  
model

Previous  
state



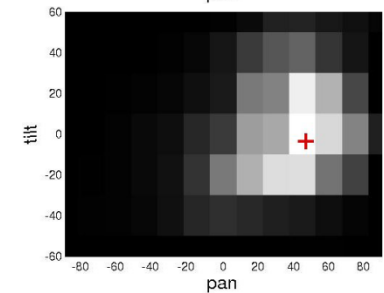
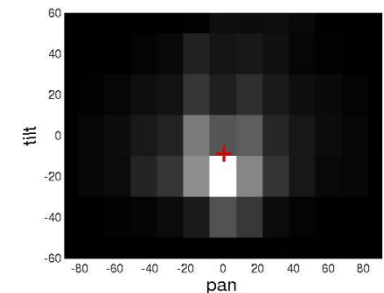
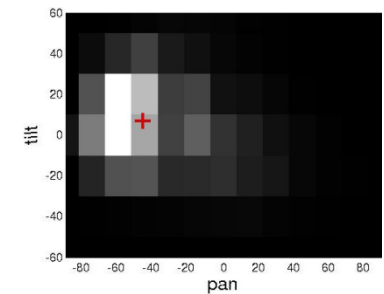
# Dynamic model

- Nonlinear dynamic models
  - Discrete state transitions



Discrete pose states

Transition probabilities

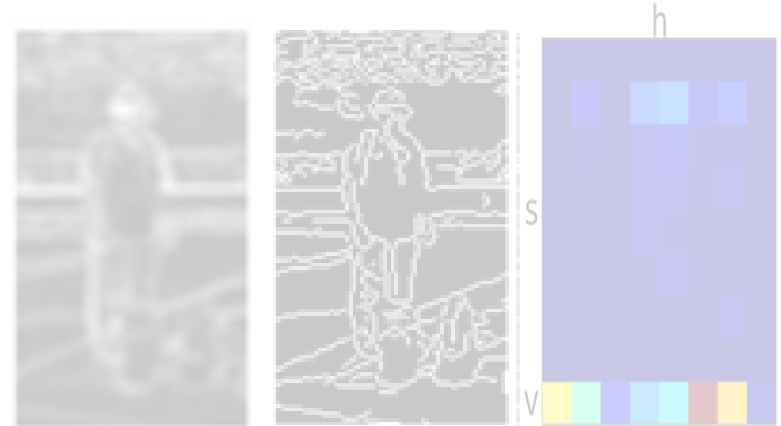


# Tracking ingredients

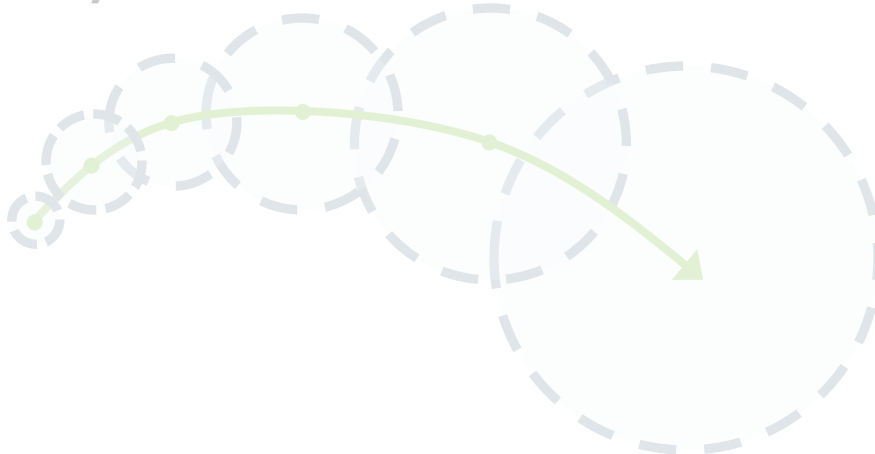
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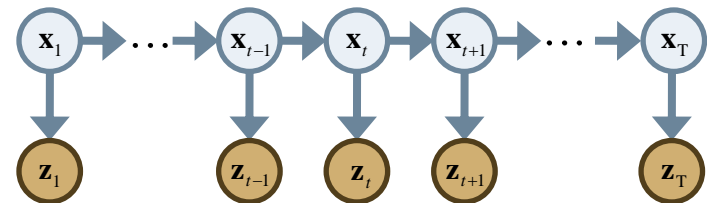
## ■ Observation Model



## ■ Dynamic Model



## ■ Inference



# Recursive Bayesian filtering

## ■ Filtering equation:

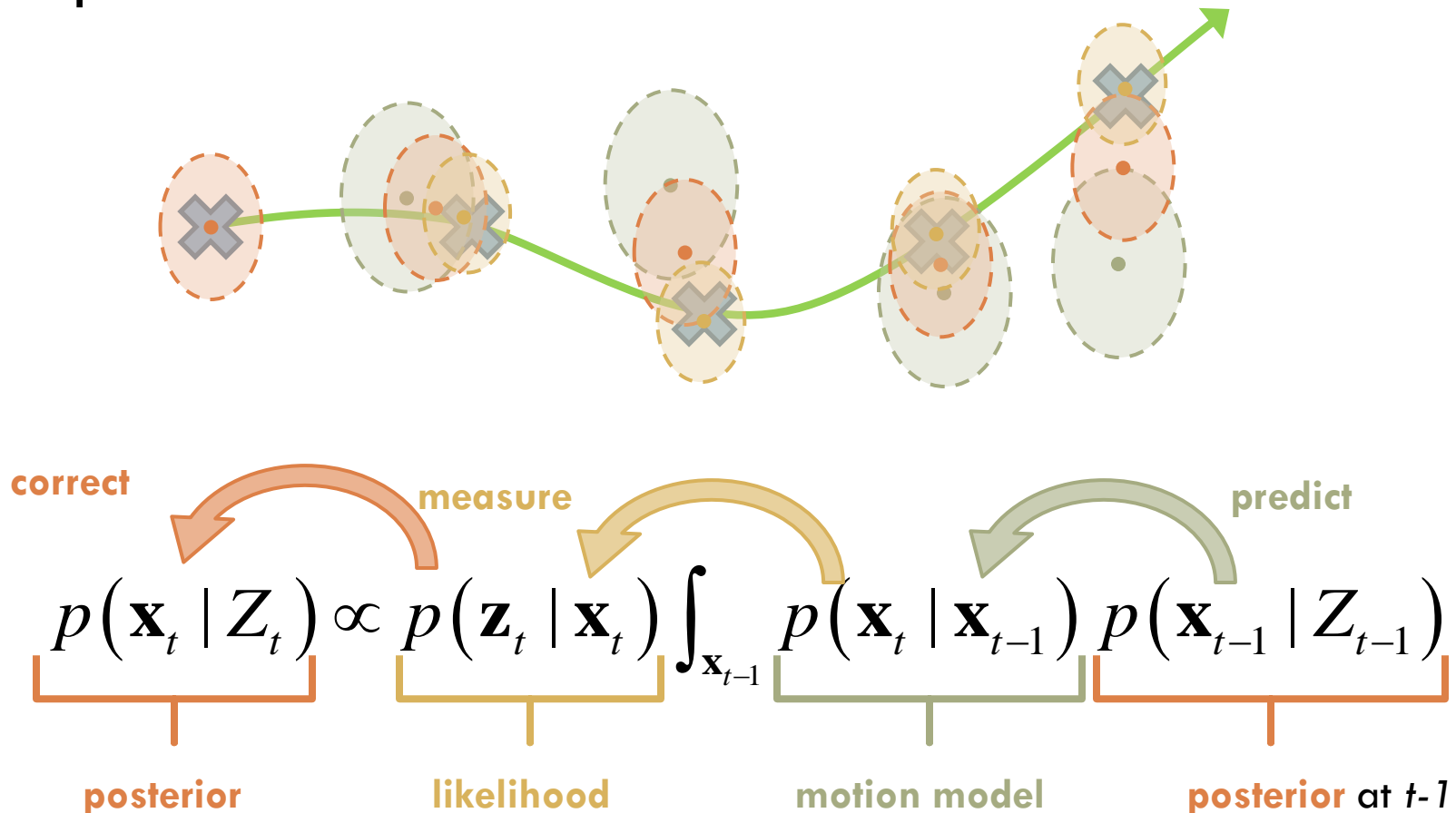
$$\underbrace{p(\mathbf{x}_t | Z_t)}_{\text{posterior estimate}} \propto \underbrace{p(\mathbf{z}_t | \mathbf{x}_t)}_{\text{likelihood or observation model}} \int_{\mathbf{x}_{t-1}} \underbrace{p(\mathbf{x}_t | \mathbf{x}_{t-1})}_{\text{motion model or dynamic model}} \underbrace{p(\mathbf{x}_{t-1} | Z_{t-1})}_{\text{posterior estimated at } t-1}$$

## ■ Definitions

- State from 1 to time  $t$ :  $X_t = \{\mathbf{x}_1, \dots, \mathbf{x}_{t-1}, \mathbf{x}_t\}$
- Observations from 1 to time  $t$ :  $Z_t = \{\mathbf{z}_1, \dots, \mathbf{z}_{t-1}, \mathbf{z}_t\}$


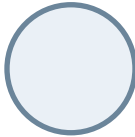
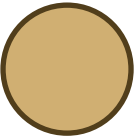
# Recursive Bayesian filtering

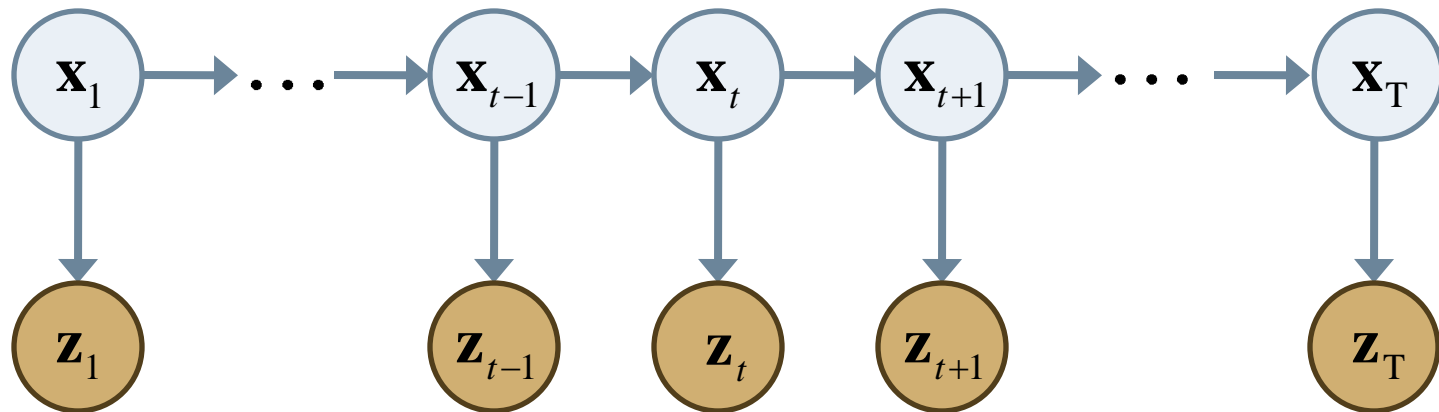
- Use probability distributions to model the tracking problem



# Modeling the tracking problem

## ■ Model the problem as a Hidden Markov Model (HMM)

- Dependency 
- Variables: hidden , or observed 



## ■ Assumptions

- Dynamics form a Markov chain  $p(\mathbf{x}_t | X_{t-1}) = p(\mathbf{x}_t | \mathbf{x}_{t-1})$
- Independent observations  $p(\mathbf{z}_t | \mathbf{x}_t, X_{t-1}, Z_{t-1}) = p(\mathbf{z}_t | \mathbf{x}_t)$

# Recursive Bayesian filtering

## Derivation setup

### ■ Notation

■ State from 0 to time  $t$ :  $X_t = \{\mathbf{x}_1, \dots, \mathbf{x}_{t-1}, \mathbf{x}_t\}$

■ Observations from 0 to time  $t$ :  $Z_t = \{\mathbf{z}_1, \dots, \mathbf{z}_{t-1}, \mathbf{z}_t\}$

### ■ Assumptions

■ Dynamics form a Markov chain  $p(\mathbf{x}_t | X_{t-1}) = p(\mathbf{x}_t | \mathbf{x}_{t-1})$

■ Independent observations  $p(\mathbf{z}_t | \mathbf{x}_t, X_{t-1}, Z_{t-1}) = p(\mathbf{z}_t | \mathbf{x}_t)$

# Useful probability relations

- Conditional probability

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B | A) = \frac{P(A \cap B)}{P(A)}$$

- Bayes theorem

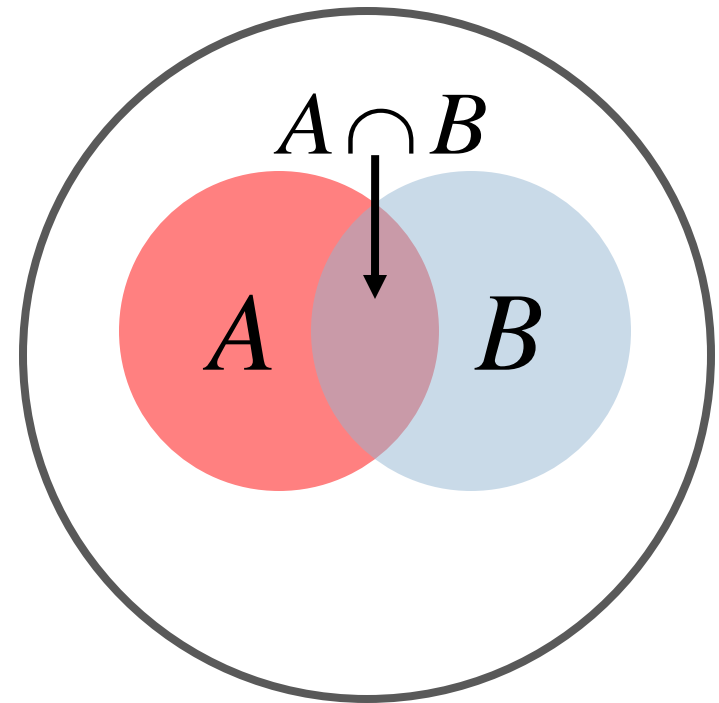
$$P(A | B) = \frac{P(B | A)P(A)}{P(B)}$$

- Marginal probability (discrete)

$$P(A) = \sum_B P(A, B) = \sum_B P(A | B)P(B)$$

$$P(A \cap B) = P(A, B) = P(A \text{ and } B)$$

$$P(A \cup B) = P(A \text{ or } B)$$





# Recursive Bayesian filtering

## Derivation setup

starting from this relation

$$p(X_t, Z_t) = p(\mathbf{x}_t, \mathbf{z}_t, X_{t-1}, Z_{t-1})$$

derive the recursive Bayesian filtering equation

$$p(\mathbf{x}_t | Z_t) \propto p(\mathbf{z}_t | \mathbf{x}_t) \int_{\mathbf{x}_{t-1}} p(\mathbf{x}_t | \mathbf{x}_{t-1}) p(\mathbf{x}_{t-1} | Z_{t-1})$$

# Recursive Bayesian filtering

- Derivation removed (homework assignment)

# Recursive Bayesian filtering

- Derivation removed (homework assignment)

# Homework

## ■ HW4: Derive the recursive Bayesian filtering equation

### COM-711 Homework 4

Kevin Smith

October 21, 2011

This assignment is due **December 2, 2011**. Turn in your work electronically to Kevin Smith at `kevin.smith@lmc.biol.ethz.ch`. You may submit your work as a PDF file or as a scanned handwritten document.

### 1 Derive the recursive Bayesian filtering distribution

In lecture 5, we discussed how to arrive at the formula underlying probabilistic tracking algorithms including the particle filter and the Kalman filter. Your task is to start from the joint relation

$$p(X_t, Z_t) = p(\mathbf{x}_t, \mathbf{z}_t, X_{t-1}, Z_{t-1}), \quad (1)$$

and derive the recursive Bayesian filtering distribution, which is given by

$$p(\mathbf{x}_t | Z_t) \propto p(\mathbf{z}_t | \mathbf{x}_t) \int_{\mathbf{x}_{t-1}} p(\mathbf{x}_t | \mathbf{x}_{t-1}) p(\mathbf{x}_{t-1} | Z_{t-1}), \quad (2)$$

where  $\mathbf{x}_t$  is the state of the object(s) at time step  $t$ ,  $\mathbf{z}_t$  is the observation at time  $t$ ,  $X_t = \{\mathbf{x}_1, \dots, \mathbf{x}_{t-1}, \mathbf{x}_t\}$  is the set of all object(s) states, and  $Z_t = \{\mathbf{z}_1, \dots, \mathbf{z}_{t-1}, \mathbf{z}_t\}$  is the set of all current and previous observations. Keep in mind our assumptions: that the dynamic process is Markovian (the current state depends only on the previous state)

$$p(\mathbf{x}_t | X_{t-1}) = p(\mathbf{x}_t | \mathbf{x}_{t-1}), \quad (3)$$

and that the current observation is independent from the other observations.