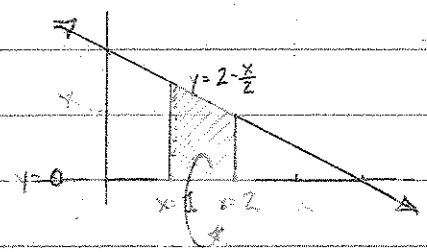
6.2
1, 2, 5

1, 5) Find the volume of the solid bounded by

1) $y = 2 - \frac{x}{2}$, $y = 0$, $x = 1$, $x = 2$ @ x-axis



$r_o = 2 - \frac{x}{2}$ $\quad r_i = 0$ $\quad$ thickness $= dx$

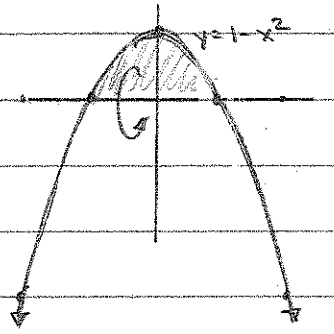
$$\text{Volume} = \pi \int_1^2 \left(r_o^2 - r_i^2\right) dx \qquad r_o^2 = 4 - 2x + \frac{x^2}{4} \qquad r_i^2 = 0$$

$$= \pi \int_1^2 \left(4 - 2x + \frac{x^2}{4}\right) dx = \pi \left(4x - x^2 + \frac{x^3}{12}\right)\Big|_1^2$$

$$= \pi \left[\left(8 - 4 + \frac{8}{12}\right) - \left(4 - 1 + \frac{1}{12}\right)\right] = \pi \left[1 + \frac{7}{12}\right]$$

$$\boxed{V = \frac{19\pi}{12}}$$

2) $y = 1 - x^2$, $y = 0$ @ x-axis



$r_o = 1 - x^2$

$r_i = 0$

$$V = \pi \int_{-1}^{1} \left(1 - x^2\right)^2 dx = 2\pi \int_0^1 \left(1 - x^2\right) dx$$

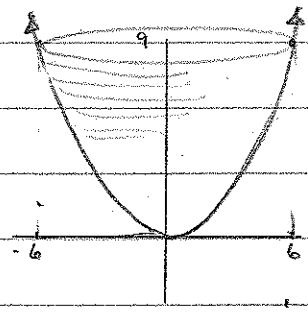
$$= 2\pi \int_0^1 \left(1 - 2x^2 + x^4\right) dx = 2\pi \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right]_0^1 = 2\pi \left[\left(1 - \frac{2}{3} + \frac{1}{5}\right) - 0\right] = 2\pi \left[\frac{15}{15} - \frac{10}{15} + \frac{3}{15}\right]$$

$$= 2\pi \left[\frac{8}{15}\right]$$

$$\boxed{V = \frac{16\pi}{15}}$$

5) $x = 2\sqrt{y}$, $x = 0$, $y = 9$ @ y-axis

$y = \frac{x^2}{4}$



$r_o = x = 2y^{1/2}$

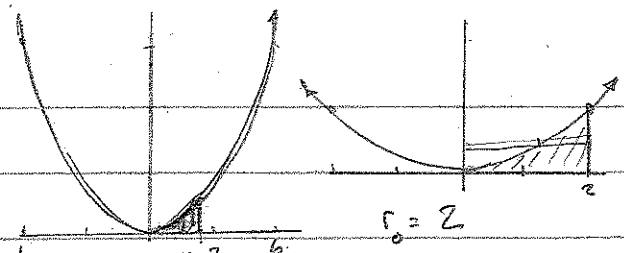
$r_i = 0$

$$V = \pi \int_0^9 \left(2y^{1/2}\right)^2 dy = 4\pi \int_0^9 y\, dy$$

$$= 4\pi \frac{y^2}{2}\Big|_0^9 = 2\pi (81 - 0) = 162\pi$$

$$\boxed{V = 162\pi}$$

10) $y = \frac{1}{4}x^2$, $x = 2$, $y = 0$ @ y-axis



$r_o = 2$

$r_i^2 = x = 2\sqrt{y}$

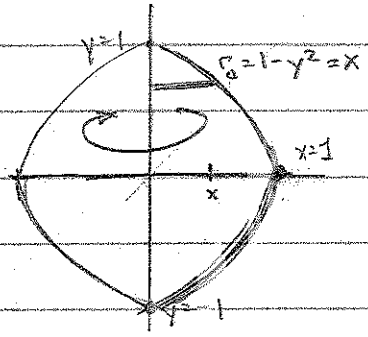
$$V = \pi\int_0^1 \left(2^2 - (2\sqrt{y})^2\right)dy = \pi\int_0^1 (4-4y)\,dy$$

$$= \pi\left[4y - 2y^2\right]_0^1 = \pi\left[(4-2)-(0-0)\right] = 2\pi$$

$$\boxed{V = 2\pi}$$

21 & 23 in LSN-9 slides

40) $V = \pi\int_{-1}^{1} (1-y^2)^2\,dy$ $\quad r_o = (1-y^2)$ $\quad r_i = 0$



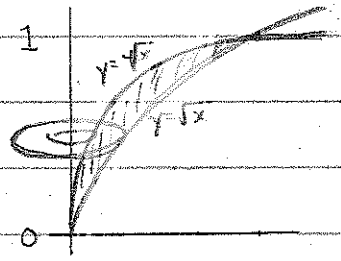
$x = 1 - y^2$

$y^2 = 1 - x$

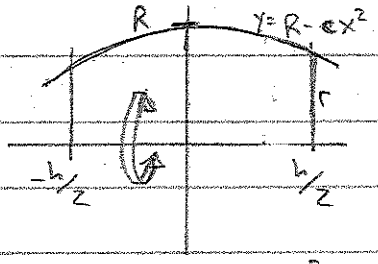
$y = \pm\sqrt{1-x}$

41) $V = \pi\int_0^1 (y^4 - y^8)\,dy$ $\quad r_o^2 = y^4$, $r_o = y^2 = x_o$

$r_i = y^8$, $r_i = y^9 = x_i$



9)



b) $V = 2\pi\int_0^{h/2} (R - cx^2)^2\,dx$

$y(\frac{h}{2}) = r = R - c(\frac{h}{2})^2$

$= R - \frac{ch^2}{4}$

given: $d = \frac{ch^2}{4}$

a) $\boxed{r = R - d}$

$= 2\pi\int_0^{h/2} \left(R^2 - 2Rcx^2 + c^2x^4\right)dx$

$= 2\pi\left[R^2x - \frac{2Rc}{3}x^3 + \frac{c^2}{5}x^5\right]_0^{h/2}$

$\left(\frac{h}{2}\right)^3$ $\quad \left(\frac{h}{2}\right)^5$

$= 2\pi\left[\frac{R^2h}{2} - \left(\frac{ch}{4}\right)\frac{Rh}{3} + \left(\frac{ch^2}{4}\right)^2\frac{h}{10}\right]$

$d = \frac{ch^2}{4}$

$$\boxed{V = \pi h\left[R^2 - \frac{2Rd}{3} + \frac{d^2}{5}\right]}$$

69 contd) $V = \pi h\left[R^2 - \frac{2Rd}{3} + \frac{d^2}{5}\right]$

$r = R - d$, $r^2 = R^2 - 2Rd + d^2$

$= \pi h\left[\begin{array}{l}\left[r^2 - (R^2 - 2Rd + d^2)\right] \\ \quad + \left(R^2 - \frac{2}{3}Rd + \frac{d^2}{5}\right)\end{array}\right\}$ adding "0"

$= \pi h\left[r^2 + (R^2 - R^2) + \left(2Rd - \frac{2}{3}Rd\right) + \left(\frac{d^2}{5} - \frac{d^2}{5}\right)\right]$

$= \pi h\left[r^2 + \frac{4}{3}Rd - \frac{4}{5}d^2\right]$

$= \frac{1}{3}\pi h\left[\begin{array}{l}3r^2 + 4Rd - \frac{12}{5}d^2 \\ \left(-2r^2\right. \\ \left.+2(R^2 - 2Rd + d^2)\right)\end{array}\right\}$ adding "0"

$= \frac{1}{3}\pi h\left[r^2 + 2R^2 + Rd(4-4) + d^2\left(2 - \frac{12}{5}\right)\right]$

$\frac{10}{5}$

$$\boxed{V = \frac{1}{3}\pi h\left[r^2 + 2R^2 - \frac{2}{5}d^2\right]}$$