SECTION 4-9: #23, 25.

PROBLEM #23: FIND THE ANTIDERIVATIVE F OF f THAT SATISFIES THE GIVEN CONDITION. CHECK YOUR ANSWER BY COMPARING GRAPHS OF f AND F.

$$f(x) = 5x^4 - 2x^5, \qquad F(0) = 4$$

$$F(x) = 5 \cdot \frac{x^5}{5} - 2 \cdot \frac{x^6}{6} + C$$

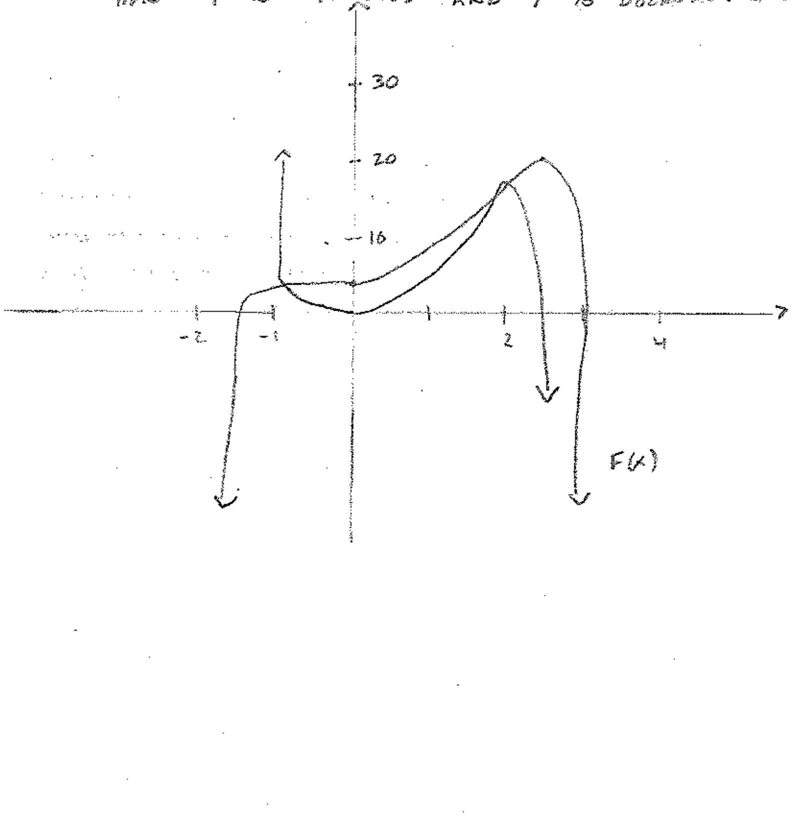
$$F(x) = x^5 - \tfrac{1}{3}x^6 + C$$

$$F(0) = 4 \implies 4 = (0)^5 - \tfrac{1}{3}(0)^6 + C$$

$$4 = C$$

$$\therefore \boxed{F(x) = x^5 - \tfrac{1}{3}x^6 + 4}$$

*THE GRAPH CONFIRMS THE ANSWER BECAUSE WHERE THERE IS A LOCAL MAXIMUM ON F, $f(x) = 0$. F IS ALSO INCREASING WHEN f IS POSITIVE AND F IS DECREASING WHEN f IS NEGATIVE.



PROBLEM #25: FIND f

$$f''(x) = 20x^3 - 12x^2 + 6x$$
$$f = ?$$

$$f'(x) = 20 \cdot \frac{x^4}{4} - 12\frac{x^3}{3} + 6\frac{x^2}{2} + C$$

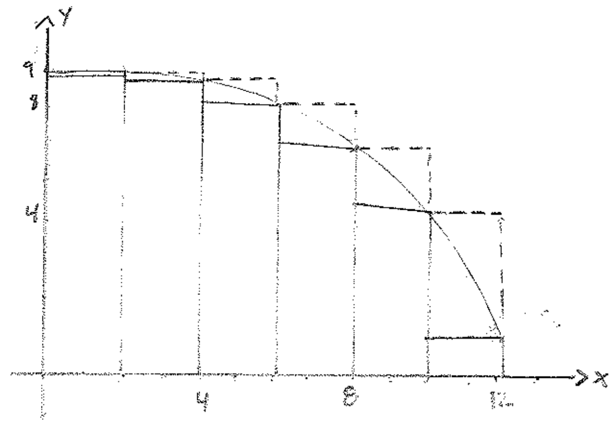
$$f'(x) = 5x^4 - 4x^3 + 3x^2 + C$$

$$f(x) = 5\frac{x^5}{5} - \frac{4x^4}{4} + 3\frac{x^3}{3} + Cx + D$$

$$\boxed{f(x) = x^5 - x^4 + x^3 + Cx + D}$$

SECTION 5-1: # 2, 13

PROBLEM # 2:



A) USE SIX RECTANGLES TO FIND EACH TYPE FOR THE AREA UNDER THE GIVEN GRAPH OF f FROM 0 TO x = 12.

(i) L_6 (SAMPLE POINTS ARE LEFT ENDPOINTS

(ii) R_6 (SAMPLE POINTS ARE RIGHT ENDPOINTS

(iii) SAMPLE PNTS ARE MIDPOINTS

$L_6 = 2(9 + 8.8 + 8.2 + 7.2 + 6 + 4)$

$L_6 \approx 86.4 \text{ UNITS}^2$

$\boxed{L_6 \approx 86.4 \text{ UNITS}^2}$

$R_6 = 2(8.8 + 8.2 + 7.2 + 6 + 4 + 1)$

$R_6 \approx 70.4 \text{ UNITS}^2$

$\boxed{R_6 \approx 70.4 \text{ UNITS}^2}$

$M_6 = 2(8.9 + 8.5 + 7.8 + 6.8 + 5 + 2.9)$

$M_6 \approx 79.8 \text{ UNITS}^2$

$\boxed{M_6 \approx 79.8 \text{ UNITS}^2}$

B) IS L_6 AN UNDERESTIMATE OR OVERESTIMATE OF THE TRUE AREA?

> L_6 IS AN OVERESTIMATE OF THE TRUE AREA

→ THERE IS AREA BEING CALCULATED ABOVE THE CURVE

C) IS R_6 AN UNDERESTIMATE OR OVERESTIMATE OF THE TRUE AREA?

> R_6 IS AN UNDERESTIMATE OF THE TRUE AREA.

→ AREA BELOW THE CURVE NOT CALCULATED.

D) WHICH OF THE NUMBERS L_6, R_6, OR M_6 GIVES US THE BEST ESTIMATE? EXPLAIN.

> M_6 GIVES THE BEST ESTIMATE. THIS IS BECAUSE AT SOME POINTS IT IS AN OVERESTIMATE AND OTHERS IT IS AN UNDERESTIMATE. IT IS ALSO ABOUT HALFWAY BETWEEN L_6 AND R_6.

PROBLEM #13:

THE SPEED OF A RUNNER INCREASED STEADILY DURING THE FIRST THREE SECONDS OF A RACE. HER SPEED AT HALF-SECOND INTERVALS IS GIVEN THE TABLE. FIND LOWER AND UPPER ESTIMATES FOR THE DISTANCE THAT SHE TRAVELED DURING THESE THREE SECONDS.

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
V(ft/s)	0.	6.2	10.8	14.9	18.1	19.4	20.2

$L_6 = .5(0 + 6.2 + 10.8 + 14.9 + 18.1 + 19.4)$

$L_6 = (69.4)(.5)$

$L_6 = 34.7$

$\boxed{L_6 = 34.7 FT}$

$R_6 = .5(6.2 + 10.8 + 14.9 + 18.1 + 19.4 + 20.2)$

$R_6 = .5(89.6)$

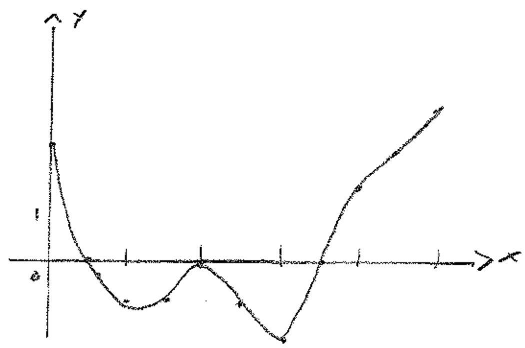
$R_6 = 44.8$

$\boxed{R_6 = 44.8 FT}$

SECTION 5.2: # 5, 8:

PROBLEM #5:

THE GRAPH OF A FUNCTION f IS GIVEN. ESTIMATE $\int_0^{10} f(x)dx$ USING FIVE SUBINTERVALS WITH A) RIGHT ENDPOINTS, B) LEFT ENDPOINTS AND C) MIDPOINTS.



A) $\int_0^{10} f(x)dx \approx R_5 = \Delta x\left[f(2)+f(4)+f(6)+f(8)+f(10)\right]$

$\Delta x = \frac{10-0}{5} = 2$

$R_5 = 2(-1+0-2+2+4) = 2(3)$

$\boxed{R_5 = 6}$

B) $\int_0^{10} f(x)dx \approx L_5 = \Delta x\left[f(0)+f(2)+f(4)+f(6)+f(8)\right]$

$L_5 = 2(3-1+0-2+2)$

$\boxed{L_5 = 4}$

C) $\int_0^{10} f(x)dx \approx M_5 = \Delta x\left[f(1)+f(3)+f(5)+f(7)+f(9)\right]$

$M_5 = 2(0-1-1+0+3)$

$= 2(1)$

$\boxed{M_5 = 2}$

PROBLEM #8:

THE TABLE GIVES THE VALUES OF A FUNCTION OBTAINED FROM AN EXPIRIMENT. USE THEM TO ESTIMATE $\int_3^9 f(x)\,dx$ USING 3 EQUAL SUBINTERVALS WITH A) RIGHT ENDPOINTS, B) LEFT ENDPOINTS, AND C) MIDPOINTS. IF THE FUNCTION IS KNOWN TO BE AN INCREASING FUNCTION, CAN YOU SAY WHETHER YOUR ESTIMATES ARE LESS THAN OR GREATER THAN THE EXACT VALUE OF THE INTEGRAL?

x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-.6	.3	.9	1.4	1.8

A) $\int_3^9 f(x)\,dx \approx R_3 = \Delta x\,[f(5) + f(7) + f(9)]$

$R_3 = 2[-.6 + .9 + 1.8] = 4.2$

8 THIS AGAIN

$\boxed{R_3 = 4.2}$

B) $\int_3^9 f(x)\,dx \approx L_3 = \Delta x\,[f(-3.4) + f(5) + f(7)]$

$L_3 = 2[-3.4 + -.6 + .9]$

$L_3 = (-3.1)2$

$\boxed{L_3 = -6.2}$

C) $\int_3^9 f(x)\,dx \approx M_3 = \Delta x\,[f(4) + f(6) + f(8)]$

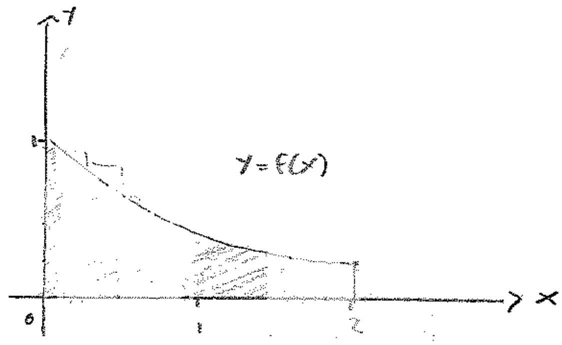
$M_3 = 2[-2.1 + .3 + 1.4]$

$\boxed{M_3 = -.8}$

* NO, YOU CAN'T TELL BECAUSE THE FUNCTION GOES UNDER THE X-AXIS, WHICH MEANS AT SOME POINTS THE LEFT ENDPOINT IS LESS THAN THE EXACT INTERVAL AND SOME POINTS IT IS ABOVE THE EXACT INTERVAL.

SECTION 7-7: 2, 11

PROBLEM #2: THE LEFT, RIGHT, TRAPEZOIDAL, AND MIDPOINT RULE APPROXIMATIONS WERE USED TO ESTIMATE $\int_0^2 f(x)dx$, WHERE f IS THE FUNCTION WHOSE GRAPH IS SHOWN. THE ESTIMATES WERE .7811, .8675, .8632, AND .9540, AND THE SAME NUMBER OF SUBINTERVALS WERE USED IN EACH CASE.



A) WHICH RULE PRODUCED WHICH ESTIMATE?

$\boxed{\text{LEFT ENDPOINT RULE: .9540}}$

↳ IT'S AN OVERESTIMATE

$\boxed{\text{RIGHT ENDPOINT RULE: .7811}}$

↳ IT'S AN UNDERESTIMATE

Trap ~~MIDPOINT~~ RULE: $\frac{.7811 + .9540}{2} = .8675$

Trap $\boxed{\text{~~MIDPOINT~~ RULE: .8675}}$

$\boxed{\text{~~TRAPEZOIDAL~~ RULE: .8632}}$
Midpoint

B) BETWEEN WHICH TWO APPROXIMATIONS DOES THE TRUE VALUE OF $\int_0^2 f(x)dx$ LIE?

THE TRUE VALUE OF $\int_0^2 f(x)dx$ LIES BETWEEN THE APPROXIMATIONS OF .8675 AND .8632.

PROBLEM #11:

USE A) THE TRAPEZOIDAL RULE, B) THE MIDPOINT RULE, AND C) SIMPSON'S RULE TO APPROXIMATE THE GIVEN INTEGRAL WITH THE SPECIFIED VALUE OF n.

$$\int_1^4 \sqrt{\ln x}\, dx, \quad n = 6$$

$$\Delta x = \frac{1}{2}$$

A) $T_6 = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + f(x_6)\right]$

$T_6 = \frac{1/2}{2}\left[f(1) + 2f(1.5) + 2f(2) + 2f(2.5) + 2f(3) + 2f(3.5) + f(4)\right]$

$T_6 = \frac{1}{4}(10.36533565)$

$T_6 = 2.591333912$

$\boxed{T_6 \approx 2.59}$

B) $M_6 = \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4) + f(\bar{x}_5) + f(\bar{x}_6)\right]$

$M_6 = \frac{1}{2}\left[f(1.25) + f(1.75) + f(2.25) + f(2.75) + f(3.25) + f(3.75)\right]$

$M_6 = \frac{1}{2}(5.362091008)$

$M_6 = 2.681045504$

$\boxed{M_6 \approx 2.68}$

C) $S_6 = \frac{1}{6}\left[f(1) + 4f(1.5) + 2f(2) + 4f(2.5) + 2f(3) + 4f(3.5) + f(4)\right]$

$S_6 = \frac{1}{6}\left[15.7918579\right]$

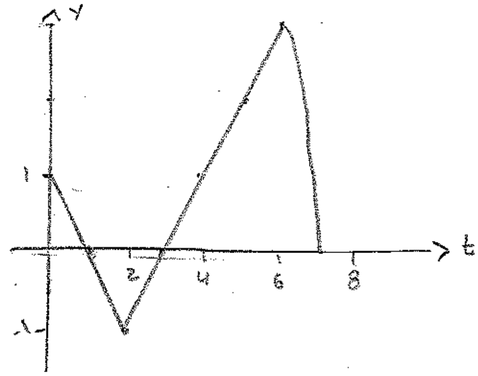
$S_6 = 2.631976317$

$\boxed{S_6 \approx 2.63}$

SECTION 5-3: # 7, 21, 37:

PROBLEM #7:

LET $g(x) = \int_0^x f(t)\,dt$, WHERE f IS THE FUNCTION WHOSE GRAPH IS SHOWN.



A) EVALUATE $g(x)$ FOR $x = 0, 1, 2, 3, 4, 5$ AND 6

$g(0) = \int_0^0 f(t)\,dt = 0$ $\boxed{g(0) = 0}$

$g(1) = \int_0^1 f(t)\,dt = 1/2$ $\boxed{g(1) = 1/2}$

$g(2) = \int_0^2 f(t)\,dt = 0$ $\boxed{g(2) = 0}$

$\boxed{g(3) = -1/2}$

$\boxed{g(4) = 0}$

$g(5) = 1.5$ $\boxed{g(5) = 1.5}$

$\boxed{g(6) = 4}$

B) ESTIMATE $g(7)$

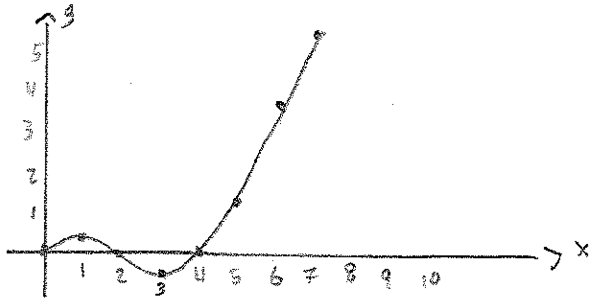
$\boxed{g(7) \approx 5.9}$

C) WHERE DOES g HAVE A MAXIMUM VALUE? WHERE DOES IT HAVE A MINIMUM VALUE?

MAXIMUM VALUE = 5.9 @ $x = 7$

MINIMUM VALUE = $-1/2$ @ $x = 3$.

D) SKETCH A ROUGH GRAPH OF g.



PROBLEM #21:

EVALUATE THE INTEGRAL.

$\int_1^4 (5 - 2t + 3t^2)\,dt$

$[5t - t^2 + t^3]_1^4 = F'(a) - F'(b)$

$= [5(4) - (4)^2 + (4)^3] - [5(1) - (1)^2 + (1)^3]$

$= 68 - 5 = 63$

$\boxed{\int_1^4 (5 - 2t + 3t^2)\,dt = 63}$

PROBLEM#37:

EVALUATE THE INTEGRAL

$$\int_0^1 (x^e + e^x)\,dx$$

$$\Downarrow$$

$$\frac{x^{e+1}}{e+1} + e^x$$

$$\left[\frac{x^{e+1}}{e+1} + e^x\right]_0^1 = F'(a) - F'(b)$$

$$\left(\frac{1}{e+1} + e\right) - (0+1) = \frac{1}{e+1} + e - 1$$

$$\boxed{\int_0^1 (x^e + e^x)\,dx = \frac{1}{e+1} + e - 1 \quad \text{or} \quad = 1.987}$$