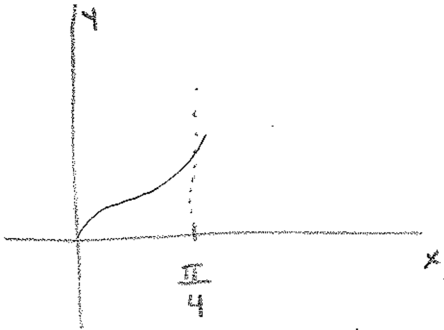
Boardwork
Lesson 1.4

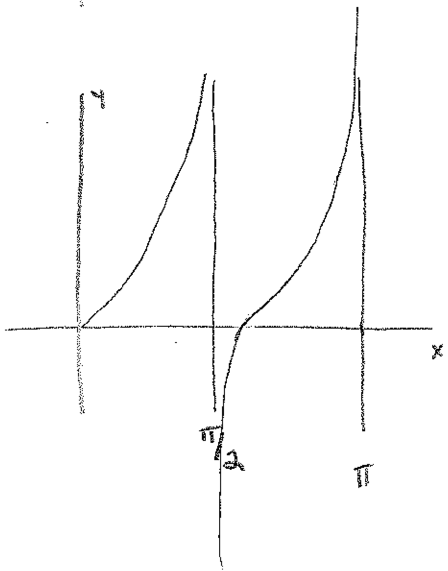
1) $\int_0^{\pi/4} \tan(x)\,dx$

Proper



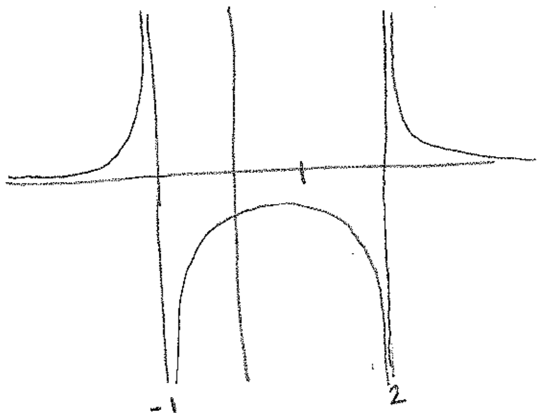
2) $\int_0^{\pi} \tan(x)\,dx$

Improper : Type 2
Discontinuity at $\pi/2$



3) $\int_{-1}^{1} \frac{1}{x^2 - x - 2}\,dx$

Improper : Type 2
Discontinuity at -1



4) $\int_0^{\infty} e^{-x^3}\,dx$

Improper : Type 1
Infinite Limit

5) $\displaystyle\int_{-\infty}^{0} \frac{1}{3-4x}\,dx = \lim_{a\to\infty}\int_{a}^{0} \frac{1}{3-4x}\,dx$

Type 1: Infinite Limit

$$= \lim_{a\to\infty} \left[\frac{-1}{4}\ln|3-4x|\right]_a^0$$

$$= \lim_{a\to\infty} \left[\frac{-1}{4}\ln|3| + \frac{1}{4}\ln|3-4a|\right]$$

$$= \frac{-\ln|3|}{4} + \infty = \boxed{\infty} \text{ Divergent}$$

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6) $\displaystyle\int_{2}^{\infty} \frac{dv}{v^2+2v-3} = \lim_{b\to\infty}\int_{2}^{b} \frac{1}{v^2+2v-3}\,dv$

Type 1: Infinite Limit

$$= \lim_{b\to\infty}\int_{2}^{b} \frac{1}{(v-1)(v+3)}\,dv$$

Side work:
$A(v+3) + B(v-1) = 1$
$v=-3,\ B = -\frac{1}{4}$
$v=1,\ A = \frac{1}{4}$

$$= \lim_{b\to\infty}\int_{2}^{b} \frac{\frac{1}{4}}{v-1} + \frac{-\frac{1}{4}}{v+3}\,dv$$

$$= \lim_{b\to\infty}\left[\frac{1}{4}\ln|v-1| - \frac{1}{4}\ln|v+3|\right]_2^b$$

$$= \lim_{b\to\infty}\left[\left(\frac{1}{4}\ln|b-1| - \frac{1}{4}\ln|b+3|\right) - \left(\frac{1}{4}\ln|1| - \frac{1}{4}\ln|5|\right)\right]$$

$$= \lim_{b\to\infty}\left[\frac{1}{4}\ln|b-1| - \frac{1}{4}\ln|b+3| + \frac{1}{4}\ln|5|\right]$$

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$$= \lim_{b\to\infty}\left[\frac{1}{4}\ln\left|\frac{b-1}{b+3}\right| + \frac{1}{4}\ln|5|\right]$$

$$= \lim_{b\to\infty}\left[\frac{1}{4}\ln\left|\frac{\frac{b}{b}-\frac{1}{b}}{\frac{b}{b}+\frac{3}{b}}\right| + \frac{1}{4}\ln|5|\right] = \boxed{\frac{1}{4}\ln|5|}$$

Converges

7) $\int_2^3 \frac{1}{\sqrt{3-x}}\,dx = \lim_{b\to 3^-} \int_2^b \frac{1}{\sqrt{3-x}}\,dx$

Type 2:
Discontinuous

$= \lim_{b\to 3^-} \left[-2\sqrt{3-x}\right]_2^b$

Side work:

$u = 3-x$

$du = -dx$

$-du = dx$

$\int \frac{1}{\sqrt{u}}(-1)\,du$

$= -2\sqrt{u}$

$= -2\sqrt{3-x}$

$= \lim_{b\to 3^-} \left[(-2\sqrt{3-b}) - (-2\sqrt{1})\right]$

$= \lim_{b\to 3^-} \left[-2\sqrt{3-b} + 2\right] = 0 + 2 = \boxed{2}$

converges

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8) $\int_0^\infty \frac{10t}{e^{t^2}}\,dt = \lim_{b\to\infty} \int_0^b \frac{10t}{e^{t^2}}\,dt$

Side work:

$u = t^2 \quad du = 2t\,dt$

$5du = 10t\,dt$

$\int e^{-u}\,5\,du$

$= -5e^{-t^2} = -\frac{5}{e^{t^2}}$

$= \lim_{b\to\infty} \left[-\frac{5}{e^{t^2}}\right]_0^b$

$= \lim_{b\to\infty} \left[\frac{-5}{e^{b^2}} + \frac{5}{e^0}\right]$

$= 0 + 5 = \boxed{5 \text{ miles}}$

9) $\int_0^\infty \frac{3t}{e^t}\,dt = \lim_{b\to\infty} \int_0^b \frac{3t}{e^t}\,dt$

Side work:

$U = 3t \quad dv = e^{-t}dt$

$du = 3dt \quad v = -e^{-t}$

$3te^{-t} - \int -3e^{-t}dt$

$3te^{-t} - 3e^{-t}$

$= \lim_{b\to\infty} \left[-3te^{-t} - 3e^{-t}\right]_0^b$

$= \lim_{b\to\infty} \left[(-3be^{-b} - 3e^{-b}) - (0 - 3e^0)\right]$

$= \lim_{b\to\infty} \left[-3be^{-b} - 3e^{-b} + 3\right]$

$= \lim_{b\to\infty} \left[-3be^{-b} + 3\right]$

L'Hospital's Rule

$\infty(0)$

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convert to $\frac{\infty}{\infty}$

$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

$= \lim_{b\to\infty} \left[\frac{-3b}{\frac{1}{1/e^b}} + 3\right] = \lim_{b\to\infty} \left[\frac{-3b}{e^b} + 3\right]$

$= \lim_{b\to\infty} \left[\frac{-3}{e^b} + 3\right] = \boxed{3 \text{ liters}}$