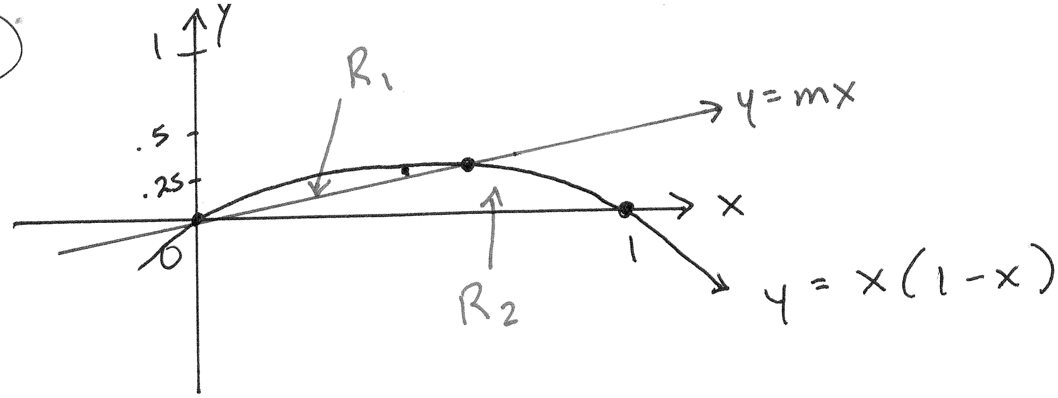
#1




- Solve for $m$
- Find Intersection between $y=mx$ and $y=x(1-x)$

$$\frac{m\cancel{x}}{\cancel{x}} = \frac{\cancel{x}(1-x)}{\cancel{x}} \qquad m = 1-x$$
$$x = 1-m$$

- Find Area between curves

$$R_1 = \int_0^{1-m} (x-x^2) - mx\,dx = \frac{x^2}{2} - \frac{x^3}{3} - \frac{mx^2}{2}\Big|_0^{1-m}$$

$$= \frac{(1-m)^2}{2} - \frac{m(1-m)^2}{2} - \frac{(1-m)^3}{3}$$

$$R_2 = \int_0^{1-m} mx\,dx + \int_{1-m}^{1} x - x^2\,dx = \frac{mx^2}{2}\Big|_0^{1-m} + \frac{x^2}{2} - \frac{x^3}{3}\Big|_{1-m}^{1}$$

$$= \frac{m(1-m)^2}{2} + \left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{(1-m)^2}{2} - \frac{(1-m)^3}{3}\right)$$

$$= \frac{m(1-m)^2}{2} - \frac{(1-m)^2}{2} + \frac{(1-m)^3}{3} + \frac{1}{6}$$

Set $R_1$ and $R_2$ equal to one another and solve for $m$

$$R_1 = R_2$$

$$\frac{(1-m)^2}{2} - \frac{m(1-m)^2}{2} - \frac{(1-m)^3}{3} = \frac{m(1-m)^2}{2} - \frac{(1-m)^2}{2} + \frac{(1-m)^3}{3} + \frac{1}{6}$$

$$\frac{\cancel{2}}{\cancel{2}}(1-m)^2 - \frac{\cancel{2}m}{\cancel{2}}(1-m)^2 - \frac{2(1-m)^3}{3} = \frac{1}{6}$$

$$(1-m)^2 - m(1-m)^2 - \frac{2(1-m)^3}{3} = \frac{1}{6}$$

$$(1-m)^2(1-m) - \frac{2(1-m)^3}{3} = \frac{1}{6}$$

$$\frac{3(1-m)^3}{3} - \frac{2(1-m)^3}{3} = \frac{1}{6}$$

$$\frac{(1-m)^3}{3} = \frac{1}{6}$$

$$(1-m)^3 = 1/2$$

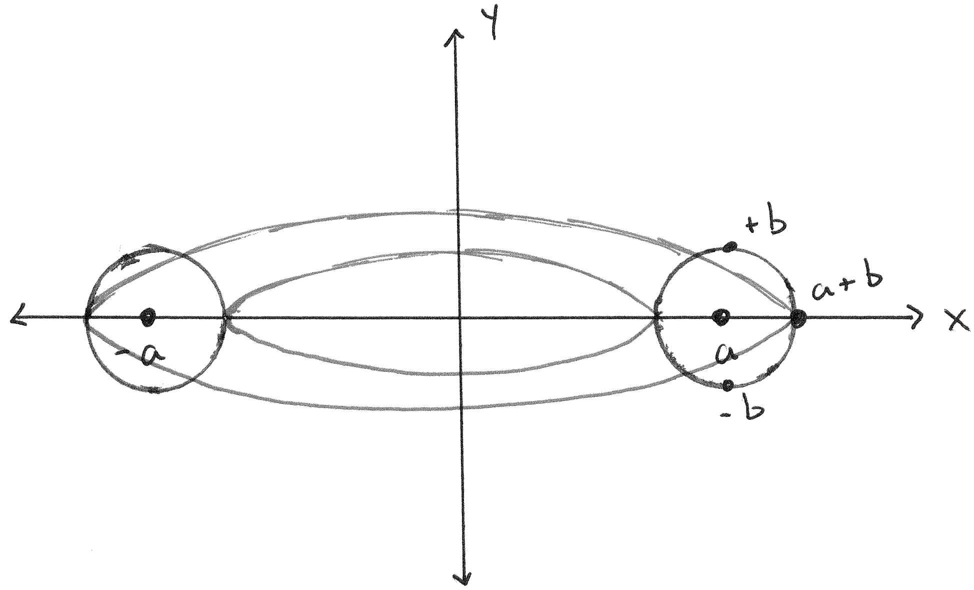
$$1 - m = \sqrt[3]{1/2}$$

$$\boxed{m = 1 - \sqrt[3]{1/2}}$$

$$\boxed{y = \left(1 - \sqrt[3]{\tfrac{1}{2}}\right)x}$$

\# 2

a.)




Using Washer Method, rotating around y-axis,
integrating with respect to height (y)

$(x-a)^2 + y^2 = b^2$ solve for $x$

$(x-a)^2 = b^2 - y^2$

$x - a = \pm\sqrt{b^2 - y^2}$

$x = a + \sqrt{b^2 - y^2}$ Outer Radius

and

$x = a - \sqrt{b^2 - y^2}$ Inner Radius

$$\int_a^b \text{Area}_{outer}\, dy - \int_a^b \text{Area}_{Inner}\, dy = \int_a^b \pi\left[r_{outer}^2 - r_{Inner}^2\right] dy$$

#2

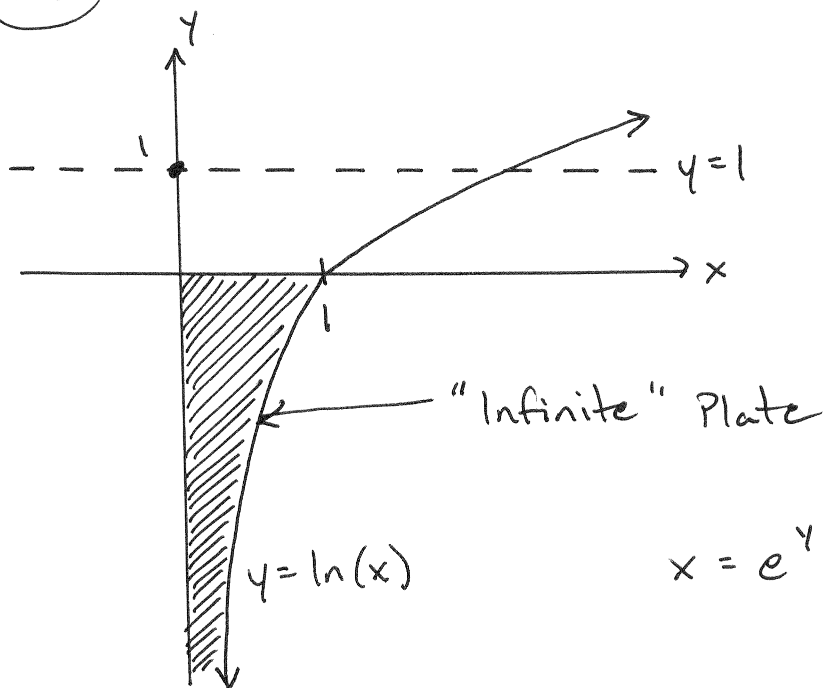
b.) $$\int_{-b}^{b} \pi\left[\left(a+\sqrt{b^2-y^2}\right)^2-\left(a-\sqrt{b^2-y^2}\right)^2\right]dy$$

$$= \pi \int_{-b}^{b} \left(a^2+2a\sqrt{b^2-y^2}+(b^2-y^2)\right)-\left(a^2-2a\sqrt{b^2-y^2}+(b^2-y^2)\right)dy$$

$$\boxed{= \pi \int_{-b}^{b} 4a\pi\sqrt{b^2-y^2}\,dy}$$

c.) $$2a\pi \underbrace{\int_{-b}^{b} 2\sqrt{b^2-y^2}\,dy}_{\text{this integral represents the area of a circle}} = 2a\pi\left(\pi(b)^2\right) = \boxed{2ab^2\pi^2}\ \text{units}^3$$

#3




$x = e^y$ is length of Horizontal slice

$dy$ is width of slice

$$\text{Force} = \text{Pressure} \times \text{Area}$$

$$= \rho g(\text{depth}) \times \text{Area}$$

$$= \int_{-\infty}^{0} (1000 \, {}^{kg}/_{m^3})(9.8 \, {}^{m}/_{s^2}) \underbrace{(1-y)}_{\text{depth}} \underbrace{(e^y)}_{\text{length}} \underbrace{dy}_{\text{width}}$$

$-\infty$ ← Improper Integral

$$\lim_{t\to\infty} \int_{-t}^{0} 9800(1-y)(e^y)\,dy = \lim_{t\to\infty} 9800 \int_{-t}^{0} e^y - ye^y \, dy$$

Use Integration by Parts

$$= \lim_{t\to\infty} 9800\left[ e^y \Big|_{-t}^{0} - (ye^y - e^y)\Big|_{-t}^{0} \right] = 9800 \lim_{t\to\infty} 2e^y - ye^y \Big|_{-t}^{0}$$

$$= 9800 \lim_{t\to\infty} (2e^0 - 0e^0) - (2e^{-t} - (-t)e^{-t}) = 9800 \lim_{t\to\infty} 2 - \frac{2}{e^t} - \underbrace{\frac{t}{e^t}}_{\text{Use L' Hospital's Rule}}$$

$$= 9800(2 - 0 - 0) \boxed{= 19600 \text{ Joules}}$$

#4

a.) $A(y) = \frac{\pi}{9} y^2 = \pi r^2 = \pi \left( \frac{1}{3} y \right)^2$

$\frac{\Delta \text{ radius}}{\Delta \text{ height}} = \frac{4-0}{12-0} = \frac{1}{3}$ $\qquad r = \frac{1}{3} y$

b.) $y(t)$ is height of water at time $t$

$V(t)$ is the volume of water at time $t$

Water drains through a hole w/ area $a$

Torricelli's Law (pg. 603, Stewart)

$\frac{dV}{dt} = -a\sqrt{2gh}$ $\qquad h = y$

$g$ = gravity

$a$ = area of hole

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt} = A(h) \frac{dh}{dt}$ (pg. 604, Stewart)

$$\frac{dy}{dt} = \frac{\frac{dV}{dt}}{A(y)} = \frac{-a\sqrt{2gh}}{\frac{\pi}{9} y^2} = \frac{-\frac{1}{144}\sqrt{2(32.2)y}}{\frac{\pi}{9} y^2}$$

$a = 2 \text{ in}^2 = \frac{2}{144} \text{ ft}^2$ $\qquad \frac{dy}{dt} = -\frac{9}{72\pi} \cdot \sqrt{2(32.2)} \cdot y^{-3/2}$

Using Separation of Variables

$\int y^{3/2} \, dy = \int \frac{-\sqrt{64.4}}{8\pi} \, dt$ $\qquad y^{5/2} = \frac{-5\sqrt{64.4}}{16 \pi} t + C_2$

$\frac{2}{5} y^{5/2} = \frac{-\sqrt{64.4}}{8\pi} t + C_1$

Using $y(0) = 12$

$(12)^{5/2} = -\frac{5\sqrt{64.4}}{16\pi}(0) + C_2$ $\qquad C_2 = 498.83$ or $12^{5/2}$

b.) Continued . . .

$$y(t) = \frac{-5\sqrt{64.4}}{16\pi} \times t + 12^{5/2}$$

c.) The tank is empty when $y = 0$

$$y(t) = 0 = \frac{-5\sqrt{64.4}}{16 \cdot \pi} \times t + 12^{5/2}$$

$$t = \frac{(12^{5/2})(16\pi)}{5\sqrt{64.4}} = 624.8993 \text{ seconds}$$

#5 Surface Area Eq. found on pg. 547, Stewart

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = x^{1/2} - \frac{1}{3} x^{3/2}$$

$$\frac{dy}{dx} = \frac{1}{2} x^{-1/2} - \frac{1}{2} x^{1/2}$$

$$S = \int_{x=1}^{x=3} \underbrace{2\pi \left(x^{1/2} - \frac{1}{3} x^{3/2}\right) \sqrt{1 + \left(\frac{1}{2} x^{-1/2} - \frac{1}{2} x^{1/2}\right)^2}}_{f(x)}\, dx$$

Simpson's Rule

$$\Delta x = \frac{3-1}{10} = \frac{1}{5}$$

$$\approx \frac{1/5}{3} \Big[ f(1) + 4f(1.2) + 2f(1.4) + 4f(1.6) + 2f(1.8) + 4f(2) + \ldots$$

$$2f(2.2) + 4f(2.4) + 2f(2.6) + 4f(2.8) + f(3) \Big]$$

$$\approx \frac{1}{15} \Big[ 4.189 + 4(4.147) + 2(4.021) + 4(3.812) + 2(3.519) + 4(3.142)$$

$$+ 2(2.681) + 4(2.136) + 2(1.508) + 4(0.796) + 0 \Big]$$

$$\approx 5.58505$$