Lesson 16 - Board Work

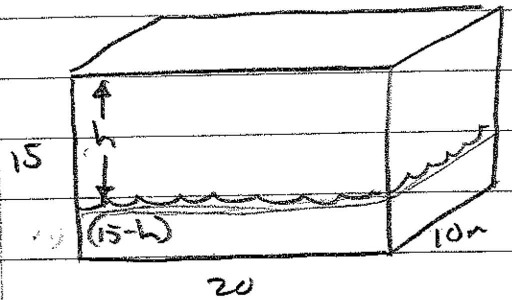
$$S(t) = \int_0^{\infty} e^{-t}\,dt = \lim_{x\to\infty} \int_0^{x} e^{-t}\,dt = \lim_{x\to\infty} \left(-e^{-t}\right)\Big]_0^{x} = \lim_{x\to\infty} \left[\left(-\cancel{\frac{1}{e^{x}}}^{\,0}\right) - \left(-\frac{1}{e^{0}}\right)\right]$$

$$= \boxed{1 \text{ mi}}$$

Note: while $\lim_{t\to\infty} \frac{1}{e^t} = 0$, writing "$\frac{1}{e^{\infty}} = 0$", "$\infty - \infty = 0$", "$\frac{\infty}{\infty} = 1$" is not correct

$\lim_{x\to\infty} \frac{x}{e^x}$ = indeterminate, in the form $\left(\frac{\infty}{\infty}\right)$

apply L-H rule $\Rightarrow \lim_{x\to\infty} \frac{1}{e^x} = 0$



$H^oH = 1000\,kg/m^3$

$\text{mass} = \left(1000\,\frac{kg}{m^3}\right)(20\times10)(dh)$

$\text{force} = 9.8(\text{mass})$

$$W = F\cdot d = 9.8\int_0^{15}((1000)(200)\,dh)\,h$$

$$(2\times10^5)(9.8)\int_0^{15} h\,dh = (2\times10^5)(9.8)\left.\frac{h^2}{2}\right]_0^{15}$$

$$= 1\times10^5(9.8)(15^2-0^2)$$

$$\boxed{W = 220.5\times10^6\,N}$$

starting @ half full

$$W = 1.96\times10^6\int_{7.5}^{15} h\,dh$$

$$= 1.96\times10^6\left(\tfrac{1}{2}\right)h^2\Big]_{7.5}^{15}$$

$$= 9.8\times10^5(15^2-7.5^2) = 9.8\times10^5(168.75)$$

$$\boxed{W = 165.375\times10^6\,N}$$

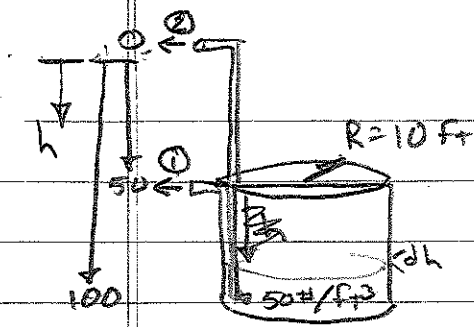
vs, if you just pumped out the first half

$$W = 1.96\times10^6\int_0^{7.5} h\,dh$$

$$= 9.8\times10^5\ h^2\Big|_0^{7.5}$$

$$= 9.8\times10^5(56.25) = \boxed{55.125\times10^6\,N}$$

conveniently: $55.125\times10^6\,N$
$+165.375\times10^6\,N$
$\boxed{= 220.5\times10^6\,N}$



$$\text{Force} = \underbrace{\left(\overset{A}{\pi (10\,ft)^2}\right) \overset{ft}{dh}}_{\text{Volume}} \; \frac{50\,\#}{ft^3}$$

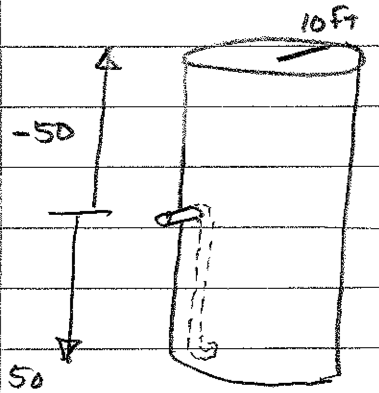
$$W = F \cdot d$$

1) $$W = 5000\pi (\#) \int_0^{50} h\,dh = 2500\pi h^2 \Big|_0^{50}$$

$$= 2500\pi (2500) = \boxed{6.25 \times 10^6 \pi \text{ ft lb}}$$

2) $$W = 5000\pi (\#) \int_{50}^{100} h\,dh = 2500\pi h^2 \Big|_{50}^{100}$$

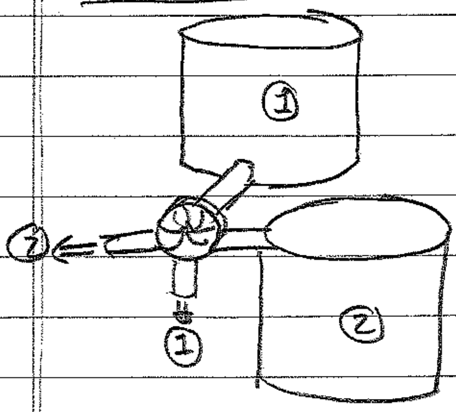
$$2500\pi (10000 - 2500) = \boxed{18.75\pi \times 10^6 \text{ ft lb}}$$



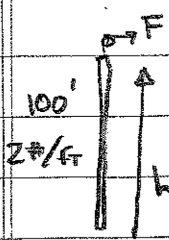
Net work if full tank drained/pumped from midpoint

$$W = 2500\pi h^2 \Big|_{-50}^{50}$$

$$= 2500\pi \left((50^2) - (-50)^2 \right) = 0$$



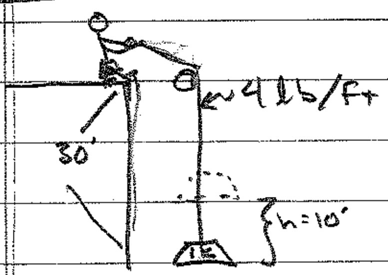
so, could draining tank 1 drive a pump to completely empty tank 2 ?



$W = F \cdot D \qquad F = (200\# - 2\# h) \text{ or } (100-h)2\#$

$$W = \int_0^{100} 2(100-h)\,dh = 2\left(100h - \frac{h^2}{2}\right)\Big|_0^{100}$$

$$= 2\left[\left(100 \cdot 100 - \frac{100^2}{2}\right) - (0)\right] = \boxed{10^{000} \text{ ft} \cdot \text{lb}}$$

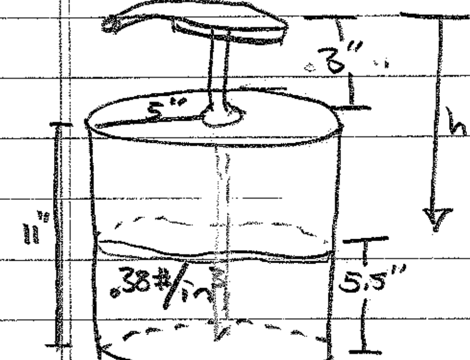


$F = 1000\# + (30-h)4\,\#/\text{ft}$

$W = FD$

$$= \int_0^{10} (1120 - 4h)\,dh$$

$$= 1120h - \frac{h^2}{2}\Big|_0^{10} = \left((11^{200} - 50) - 0\right) = \boxed{11^{150} \text{ ft} \cdot \text{lb}}$$



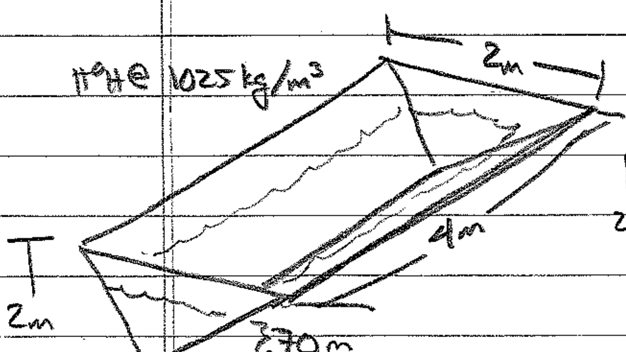
$F = \pi(5")^2\left(.38 \frac{\text{lb}}{\text{in}^3}\right) dh$

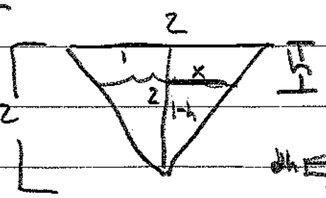
$D = 8.5 \to 14"$

$$W = FD = (.38\pi)(25)\int_{8.5}^{14} h\,dh$$

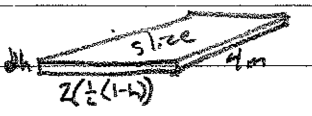
$$= \frac{1}{2}(.38\pi)(25)\; h^2 \Big|_{8.5}^{14}$$

$$= 4.75\pi(14^2 - 8.5^2) = \boxed{587.8\pi \text{ (in} \cdot \text{lb)}}$$





$\frac{1}{2} = \frac{x}{1-h}, \quad x = \frac{1}{2}(1-h)$



$$F = \underbrace{\underbrace{(1-h)(4)(dh)}_{\text{Vol}}\left(1025 \frac{\text{kg}}{\text{m}^3}\right)}_{\text{mass}}\underbrace{\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}_{\text{accel}}$$

$$W = F \cdot D = 4(1025)(9.81)\int_{.3\text{m}}^{1\text{m}} (1-h)h\,dh$$

$$W = 40221\left[\frac{h^2}{2} - \frac{h^3}{3}\right]_{.3}^{1} = 40221\left[\underbrace{\left(\frac{1}{2} - \frac{1}{3}\right)}_{1/6} - \underbrace{\left(\frac{.09}{2} - \frac{.027}{3}\right)}_{.036}\right] = \boxed{5255.5 \text{ N} \cdot \text{m}}$$