

$$1) \quad x^2 + 4x + 3 \sqrt{\frac{x-4}{x^3}} + \frac{13x+12}{x^2+4x+3}$$

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$$x-4 + \frac{13x+12}{(x+3)(x+1)} = \boxed{x-4 + \frac{A}{x+3} + \frac{B}{x+1}}$$

$$2) \quad -6 \int_2^3 \frac{1}{x^2-1} dx = \int_2^3 \frac{-6}{(x+1)(x-1)} dx = \int \left(\frac{A}{x+1} + \frac{B}{x-1} \right) dx$$

$$-6 = A(x-1) + B(x+1)$$

$$x(A+B) = 0x \quad A = -B$$

$$(B-A) = -6$$

$$B - (-B) = -6 = -2B \quad B = 3 \quad A = -3$$

$$\int_2^3 \left(\frac{3}{x+1} - \frac{3}{x-1} \right) dx = \left[3 \ln|x+1| - 3 \ln|x-1| \right]_2^3$$

$$= 3 \ln \left| \frac{x+1}{x-1} \right|_2^3 = 3 \left[\ln \left| \frac{4}{2} \right| - \ln \left| \frac{3}{1} \right| \right] = \boxed{3 \ln \left(\frac{2}{3} \right)}$$

$$3) \quad \int_1^\infty 2x e^{-x^2} dx = \lim_{t \rightarrow \infty} \int_1^t 2x e^{-x^2} dx$$

$$u = -x^2 \quad du = -2x dx$$

$$\int -e^u du = -e^u + C$$

$$\lim_{t \rightarrow \infty} \left(-e^{-x^2} \right) \Big|_1^t = \lim_{t \rightarrow \infty} \left[-e^{-t^2} + e^{-1} \right] = \lim_{t \rightarrow \infty} \left[\frac{1}{e} - \frac{1}{e^{t^2}} \right] = \boxed{\frac{1}{e}}$$

$$4) \quad \int_0^8 \frac{7}{\sqrt{x}} dx = \lim_{t \rightarrow 0^+} \left(7 \int_t^8 x^{-1/2} dx \right) = \lim_{t \rightarrow 0^+} \left(7 \left(2x^{1/2} \right) \right) \Big|_t^8$$

$$= \lim_{t \rightarrow 0^+} \left[14 \left(8^{1/2} - t^{1/2} \right) \right] = \boxed{28\sqrt{2}}$$

$$5) \quad r(t) = 1000 t e^{-t/2} \text{ (people/day)}$$

$$S(t) = 1000 \int_0^\infty t e^{-t/2} dt = \lim_{x \rightarrow \infty} \int_0^x t e^{-t/2} dt$$

$$\Rightarrow u = t \quad du = dt \quad dv = e^{-t/2} dt \quad v = -2e^{-t/2}$$

$$\lim_{x \rightarrow \infty} \left(1000 \left[-2te^{-t/2} + 2 \int e^{-t/2} dt \right] \right) = \lim_{x \rightarrow \infty} \left(1000 \left[-2te^{-t/2} + 2(-2e^{-t/2}) \right] \right)$$

$$= \lim_{x \rightarrow \infty} \left(-2000 \left[e^{-t/2} (t+2) \right] \right) = \lim_{x \rightarrow \infty} \left(-2000 \left[e^{-x/2} (x+2) - e^{-0/2} (0+2) \right] \right) = (-2000)(-2)$$

$$\boxed{S(t) = +4000}$$