$\int u\,dv = uv - \int v\,du$

↳ Log P. ET

Board Work

- Evaluate the integral

1) $\int x^3 \ln(x)\ dx$
2) $\int te^{t^2}\ dt$
3) $\int (x-1)\sin(\pi x)\, dx$
4) $\int ye^{3y}\, dy$
5) $\int z\sin(2z)\ dz$
6) $\int_4^9 \frac{\ln(y)}{\sqrt{y}}\ dy$

- A particle that moves along a straight line has velocity $v(t) = t^2e^{-t}$ meters per second after t seconds. How far will it travel during the first t seconds?

- Find the volume of the solid formed by rotating the region bounded by the curves around the specified axis:
  - $y = x^2 + 1, y = 1, and\ x = 2$: x-axis

Lesson 13 · Boardwork

$$\int u\,dv = uv - \int v\,du$$

LogPoET

1) $\int x^3 \ln(x)\,dx$ let $u = \ln(x)$, $du = \frac{1}{x}dx$; $v = \frac{1}{4}x^4$, $dV = x^3 dx$

$$= (\ln(x))\left(\frac{1}{4}x^4\right) - \frac{1}{4}\int x^4\left(\frac{1}{x}dx\right)$$

$$= \frac{1}{4}x^4\ln(x) - \frac{1}{4}\left(\frac{1}{4}x^4\right) + C$$

$$\boxed{\int x^3 \ln(x)\,dx = \frac{1}{4}x^4\left(\ln x - \frac{1}{4}\right) + C}$$

2) $\int te^{t^2}dt$ let $u = t^2$, $du = 2t\,dt$ $= \int e^u \frac{du}{2} = \frac{1}{2}e^u$

$$\boxed{\int te^{t^2}dt = \frac{1}{2}e^{t^2} + C}$$

3) $\int (x-1)\sin(\pi x)\,dx$ let $u = x-1$, $du = dx$; $v = -\frac{1}{\pi}\cos(\pi x)$, $dv = \sin(\pi x)dx$

$$= (x-1)\left(-\frac{1}{\pi}\cos(\pi x)\right) - -\frac{1}{\pi}\int \cos(\pi x)\,dx$$

$$\boxed{\int (x-1)\sin(\pi x)\,dx = \frac{1}{\pi}\cos(\pi x)(1-x) + \frac{1}{\pi}\left(\frac{1}{\pi}\sin(\pi x)\right) + C}$$

4) $\int ye^{3y}dy$ $u = y$, $du = dy$; $v = \frac{1}{3}e^{3y}$, $dv = e^{3y}dy$

$$= y\left(\frac{1}{3}e^{3y}\right) - \frac{1}{3}\int e^{3y}dy$$

$$= \frac{1}{3}ye^{3y} - \frac{1}{3}\left(\frac{1}{3}e^{3y}\right)$$

$$\boxed{\int ye^{3y}dy = \frac{1}{3}e^{3y}\left(y - \frac{1}{3}\right)}$$

$$\int u\,dv = uv - \int v\,du$$

5) $\int z\sin(2z)\,dz$ $\quad u = z \quad v = -\frac{1}{2}\cos(2z)$

$\quad du = dz \quad dv = \sin(2z)\,dz$

$$= z\left(-\tfrac{1}{2}\cos(2z)\right) - \left(-\tfrac{1}{2}\right)\int \cos(2z)\,dz$$

$$\boxed{\int z\sin(2z)\,dz = -\tfrac{1}{2}z\cos(2z) + \tfrac{1}{2}\sin(2z) + C}$$

6) $\int_4^9 \frac{\ln(y)}{\sqrt{y}}\,dy$ $\quad u = \ln(y) \quad v = 2y^{1/2}$

$\quad du = \frac{1}{y}\,dy \quad dv = y^{-1/2}\,dy$

$$= \ln(y)\left(2y^{1/2}\right)\Big|_4^9 - 2\int_4^9 y^{1/2}\left(\frac{1}{y}\,dy\right)$$

$\int y^{-1/2}\,dy$

$$= \left[\ln(9)(6) - \ln(4)(4)\right] - 2\left[2y^{1/2}\right]_4^9 + C$$

$\ln 3^2 \quad \ln 2^2$

$2\ln 3 \quad 2\ln 2$

$$\left[12\ln(3) - 8\ln(2)\right] - 4\left[3-2\right] + C$$

$$\boxed{\int_4^9 \frac{\ln(y)}{\sqrt{y}}\,dy = 12\ln(3) - 8\ln(2) - 4 + C = 3.64 + C}$$

7) $s(t) = \int_0^t t^2 e^{-t}\,dt$ $\quad u = t^2 \quad v = -e^{-t}$

$\quad du = 2t\,dt \quad dv = e^{-t}\,dt$

$$= t^2\left(-e^{-t}\right)\Big|_0^t - \int_0^t \left(-e^{-t}\right)(2t\,dt)$$

$(t^2(-e^{-t}) - 0) \quad + 2\int t e^{-t}\,dt$ $\quad u = t \quad v = -e^{-t}$

$\quad du = dt \quad dv = e^{-t}\,dt$

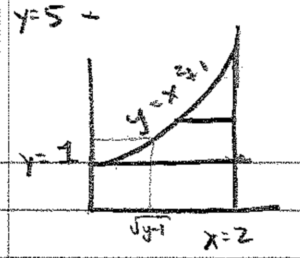
$$= -t^2e^{-t} + 2\left[-te^{-t}\Big|_0^t - -\int_0^t e^{-t}\,dt\right]$$

$$= -t^2e^{-t} - 2te^{-t} + 2\left(-e^{-t}\right)\Big|_0^t$$

$$= -t^2e^{-t} - 2te^{-t} - 2\left(e^{-t} - e^0\right)$$

$$-2e^{-t} + 2$$

$$\boxed{s(t) = \int_0^t t^2e^{-t}\,dt = 2 - e^{-t}\left(t^2 + 2t + 2\right)}$$



$y = x^2 + 1$

$x^2 = y - 1$

$x = \sqrt{y-1}$

$x \in (0, 2)$

SHELLS

$(y^3 - y^2)^{\frac{1}{2}}$

$$V = \int_1^5 (2\pi y)(2 - \sqrt{y-1})\,dy$$

$$= 2\pi \int_1^5 2y\,dy - 2\pi \int_1^5 y\sqrt{y-1}\,dy$$

$$= 2\pi y^2 \Big|_1^5 - 2\pi\Big(\text{let } u = y \quad du = dy \quad v = \tfrac{2}{3}(y-1)^{3/2} \quad dv = (y-1)^{1/2}dy\Big)$$

$$= 2\pi(25-1) - 2\pi\left[y\left(\tfrac{2}{3}(y-1)^{3/2}\right)\Big|_1^5 - \tfrac{2}{3}\int_1^5 (y-1)^{3/2}\,dy\right]$$

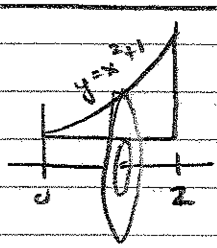
$$= 2\pi(24) - 2\pi\left[\tfrac{2}{3}\left(5(4^{3/2}) - 1(0^{3/2})\right) - \tfrac{2}{3}\cdot\tfrac{2}{5}(y-1)^{5/2}\right]_1^5$$

$$= 48\pi - 2\pi\left[\tfrac{2}{3}(40 - 0) - \tfrac{4}{15}\left(\underset{2^5}{4^{5/2}} - 0^{5/2}\right)\right]$$

$$= 48\pi - \frac{160\pi}{3} + \frac{8\pi}{15}(32)$$

$$= \frac{\pi}{15}(720 - 800 + 256) = \boxed{V = \frac{176\pi}{15}}$$

DISCS



$$V = \pi\int (r_o^2 - r_i^2)\,dx$$

$r_o = x^2 + 1$

$r_i = 1$

$$V = \pi\int_0^2 \left((x^2+1)^2 - 1^2\right)dx$$

$$= \pi\int_0^2 (x^4 + 2x^2 + \cancel{1 - 1})\,dx$$

$$= \pi\left(\tfrac{1}{5}x^5 + \tfrac{2}{3}x^3\right)\Big|_0^2$$

$$= \pi\left(\frac{32}{5} + \frac{8}{3}\right)$$

$$= \frac{\pi}{15}(96 + 80) = \boxed{\frac{176\pi}{15} = V}$$