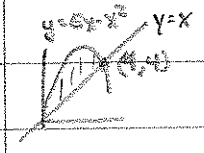
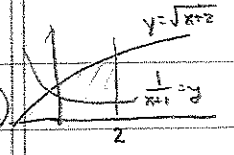
LSN 8
Area Between Curves
Section 6.1

1) 

$$\int_0^4 \left((5x-x^2)-x\right)dx = \int_0^4 (4x-x^2)dx = \left(4\left(\tfrac{1}{2}x^2\right)-\tfrac{1}{3}x^3\right)\Big|_0^4$$

$$= 2(16)-\tfrac{1}{3}(64) = \tfrac{96}{3}-\tfrac{64}{3} = \tfrac{32}{3}$$

$$\boxed{A = \tfrac{32}{3}}$$

2) 

$$A = \int_0^2 \left(\sqrt{x+2} - \frac{1}{x+1}\right)dx = \int_0^2 \left((x+2)^{1/2} - (x+1)^{-1}\right)dx$$

$u = x+2$, $du = dx$ $\qquad$ $u = x+1$, $du = dx$ $\qquad$ $u = x+2$, $du = dx$

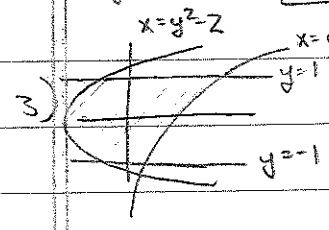
$$= \int u^{1/2}\,du \qquad \int u^{-1}\,du$$

$$= \tfrac{2}{3}u^{3/2} = \tfrac{2}{3}(x+2)^{3/2} \qquad \ln|u| = \ln|x+1|$$

$$A = \left(\tfrac{2}{3}(x+2)^{3/2} - \ln|x+1|\right)\Big|_0^2 = \left[\tfrac{2}{3}4^{3/2} - \ln|3|\right] - \left[\tfrac{2}{3}(2)^{3/2} - \ln\left|\tfrac{1}{1}\right|\right]$$

$$= \tfrac{16}{3} - \ln(3) - \tfrac{4\sqrt{2}}{3} - 0$$

$$\boxed{A = \tfrac{1}{3}(16-4\sqrt{2}) - \ln(3)}$$

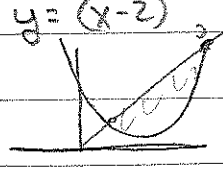
3) 

$$\int_{-1}^{+1} \left[e^y - (y^2-2)\right]dy$$

$$\left(e^y - \tfrac{1}{3}y^3 + 2y\right)\Big|_{-1}^{1} = \left[e^1 - \tfrac{1}{3} + 2\right] - \left[e^{-1} + \tfrac{1}{3} - 2\right]$$

$$= e^1 - e^{-1} - \tfrac{2}{3} + 4$$

$$\boxed{A = e - \frac{1}{e} + \frac{10}{3}}$$

4) 

$$\int_{+1}^{4} \left(x - (x^2-4x+4)\right)dx = \int_1^4 (-x^2+5x-4)dx$$

$$= -\tfrac{1}{3}x^3 + \tfrac{5}{2}x^2 - 4x\Big|_1^4$$

$$\left[-\tfrac{1}{3}(64) + \tfrac{5}{2}(16) - 16\right] - \left[-\tfrac{1}{3} + \tfrac{5}{2}(1) - 4\right]$$

$$\left(\tfrac{1}{3} - \tfrac{64}{3}\right) + \tfrac{5}{2}(16-1) - (16+4)$$

$$-\tfrac{63}{3} + \tfrac{75}{2} - 12 = -\tfrac{126}{6} + \tfrac{225}{6} - \tfrac{72}{6} = \tfrac{27}{6} = \boxed{\tfrac{9}{2}}$$

$$A = \tfrac{9}{2}$$

8) $y = x^2 - 2x$, $y = x + 4$ $\quad A: \int_{-1}^{4} \left((x+4) - (x^2-2x)\right) dx = \int_{-1}^{4} (-x^2 + 3x + 4)\, dx$

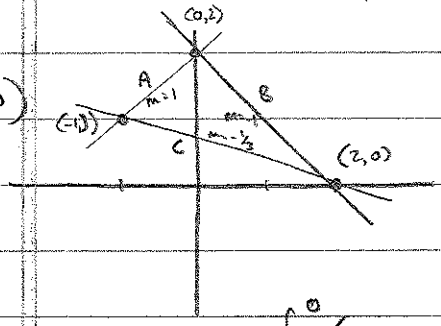
$$A = \left(-\frac{1}{3}x^3 + \frac{3}{2}x^2 + 4x\right)\Big]_{-1}^{4} = \left[-\frac{1}{3}(64) + \frac{3}{2}(16) + 16\right] - \left[-\frac{1}{3}(-1) + \frac{3}{2} - 4\right]$$

$$= -\frac{64}{3} + 24 + 16 - \frac{1}{3} - \frac{3}{2} + 4$$

$$= -\frac{65}{3} - \frac{3}{2} + 44 = -\frac{130}{6} - \frac{9}{6} + \frac{264}{6} = \frac{125}{6}$$

$\therefore$ $\boxed{A = \frac{125}{6}}$

9)



A: $y - 1 = (x - -1)(1)$ $\quad$ B: $y - 2 = (x - 0)(-1)$ $\quad$ C: $y - 0 = (x - 2)(-\frac{1}{3})$

$y = x + 2$ $\quad$ $y = 2 - x$ $\quad$ $y = -\frac{1}{3}x + \frac{2}{3}$

$$A: \int_{-1}^{0} \left((x+2) - \frac{2-x}{3}\right) dx + \int_{0}^{2} \left((2-x) - \frac{2-x}{3}\right) dx$$

$3x + 6 - (2 - x)$ $\qquad$ $6 - 3x$

$$\frac{1}{3}\int_{-1}^{0} (4x + 4)\, dx + \frac{1}{3}\int_{0}^{2} (4 - 2x)\, dx$$

$$A = \frac{1}{3}\left[2x^2 + 4x\right]_{-1}^{0} + \frac{1}{3}\left[4x - x^2\right]_{0}^{2}$$

$$A = \frac{1}{3}\left[0 - (2 - 4)\right] + \frac{1}{3}\left[(8 - 4) - 0\right]$$

$\frac{2}{3}$ $\qquad$ $+$ $\qquad$ $\frac{4}{3} = 2$

$\therefore$ $\boxed{A = 2}$