Section 7.1 # 5

Picture: N/A

Goal: The goal is to integrate the function using Integration by Parts.

Set up:

Mathematical Model: $\int u\,dv = uv - \int v\,du$

Work:

$\int te^{-3t}dt \quad u = t \quad du = dt \quad dv = e^{-3t}dt \quad v = -\frac{1}{3}e^{-3t}$

$w = -3t$

$-\frac{1}{3}dw = dt$

$-\frac{1}{3}\int e^{w} = -\frac{1}{3}e^{w} + C$

$\int te^{-3t}dt = -\frac{1}{3}te^{-3t} + \frac{1}{3}\int e^{-3t}dt = \boxed{-\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t} + C}$

Conclusion:

By choosing $t = u$ and $e^{-3t}dt = dv$, integration by parts was able to make a simpler integral because taking the derivative of both sides of $u = t$ takes away the polynomial term so only the exponential term has to be integrated in $\int v\,du$.

Section 7.1, #12

Picture: N/A

Goal: The goal is to integrate the function using integration by parts.

Set up: Mathematical Model: $\int u\,dv = uv - \int v\,du$

Work:

$\int p^5 \ln p\,dp$ $u = \ln p$ $du = \frac{1}{p}dp$ $dv = p^5 dp$ $v = \frac{1}{6}p^6$

$\int p^5 \ln p\,dp = \frac{1}{6}p^6 \ln p - \frac{1}{6}\int p^6 \frac{1}{p}dp = \frac{1}{6}p^6 \ln p - \frac{1}{6}\int p^5 dp = \boxed{\frac{1}{6}p^6 \ln|p| - \frac{1}{36}p^6 + C}$

Conclusion:

By choosing $u = \ln p$ and $dv = p^5 dp$, integration by parts was able to make a simpler integral because the derivative of $\ln p$ is $\frac{1}{p}$, and $\int p^6 \frac{1}{p}dp$ is much easier to integrate than $\int p^5 \ln p\,dp$. If $u = p^5$, the integral from combining v and du would still have $\ln p$ in it, which would not have helped.

Section 7.1, #67

· Picture: N/A

Goal: The goal is to find the distance the particle will travel from time = 0 to time = t by taking the definite integral of the velocity graph.

Set up: Mathematical Model: $\int u\,dv = uv - \int v\,du$

Work:

$v(t) = t^2e^{-t}$ displacement after t seconds $= \int_0^t v(x)\,dx = \int_0^t x^2e^{-x}\,dx$ $x = t$

$u = x^2$ $du = 2x\,dx$ $dv = e^{-x}dx$ $v = -e^{-x}$

$$\int_0^t x^2e^{-x}dx = -x^2e^{-x}\Big|_0^t + 2\int_0^t xe^{-x}dx \qquad w = x \quad dw = dx \quad dz = e^{-x}dx \quad z = -e^{-x}$$

$$= -x^2e^{-x}\Big|_0^t + 2\left(-xe^{-x}\Big|_0^t + \int_0^t e^{-x}dx\right) = -x^2e^{-x}\Big|_0^t + 2\left(-xe^{-x} - e^{-x}\right)\Big|_0^t$$

$$= -t^2e^{-t} - 0 + 2\left(-te^{-t} - e^{-t} + 0 + 1\right) = \boxed{-t^2e^{-t} - 2te^{-t} - 2e^{-t} + 2 \text{ m}}$$

Conclusion:

The question was asking for the specific distance traveled by time t, so a definite integral had to be taken between time = 0 and time = t. Since the polynomial was second degree, integration by parts had to be employed twice. The velocity was in m/s, so the distance is in m.

Section 7.4, #1

Picture: N/A

Goal: The goal is to write out the form of partial fraction decomposition of the given functions.

Set Up:

Mathematical Model: $\frac{R(x)}{Q(x)} = \frac{A}{ax+b} + \frac{B}{cx+d} + \ldots$ where $R(x)$ is less in degree than $Q(x)$ and $Q(x)$ is a product of distinct linear factors; $\frac{R(x)}{Q(x)} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{cx+d} \ldots$ where $R(x)$ is less in degree than $Q(x)$ and $Q(x)$ is a product of some repeated linear factors.

Work:

a) $\frac{1+6x}{(4x-3)(2x+5)} = \boxed{\frac{A}{4x-3} + \frac{B}{2x+5}}$

b) $\frac{10}{5x^2-2x^3} = \frac{10}{x^2(5-2x)} = \boxed{\frac{A}{x} + \frac{B}{x^2} + \frac{C}{5-2x}}$

Conclusion:

Since part a) had no repeating factors, the coefficients were just placed over each individual factor of the denominator. Since part b) had a repeating factor, distinct coefficients had to be placed over each distinct factor and each degree (starting with 1) of the repeated factor.

Section 7.4, # 7

Picture: N/A

Goal: The goal is to evaluate the integral using long division to break it up into more managable parts.

Set Up:

Mathematical Model: $\int f(g(x))g'(x)\,dx = \int f(u)\,du$

Work:

$$\begin{array}{r|l} & x^3+x^2+x+1+\frac{1}{x-1} \\ x-1 & x^4 \\ & -x^4-x^3 \\ \hline & x^3 \\ & -x^3-x^2 \\ \hline & x^2 \\ & -x^2-x \\ \hline & x \\ & x-1 \\ \hline & 1 \end{array}$$

$$\int \frac{x^4}{x-1}\,dx = \int x^3+x^2+x+1+\frac{1}{x-1}\,dx$$

$$= \frac{1}{4}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2+x+C+\int\frac{1}{x-1}\,dx$$

$u = x-1 \quad du = dx$

$$\int \frac{1}{u}\,du = \ln|u|+C = \ln|x-1|+C$$

$$\int \frac{x^4}{x-1}\,dx = \boxed{\frac{1}{4}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2+x+\ln|x-1|+C}$$

Conclusion:

x^4 is a higher degree than $x-1$, so I used long division to break the function into easier parts. Once this was done, a simple substitution could be used to integrate the remaining rational function.

Section 7.4, # 17

Picture: N/A

Goal: The goal is to evaluate the integral by using partial fractions.

Set up:

Mathematical Model: $\frac{R(x)}{Q(x)} = \frac{A}{ax+b} + \frac{B}{cx+d} \cdots$ where $R(x)$ is less in degree than $Q(x)$ and $Q(x)$ is a product of distinct linear factors.

Work:

$$\frac{4y^2-7y-12}{y(y+2)(y-3)} = \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3} = \frac{2}{y} + \frac{9}{5(y+2)} + \frac{15}{y-3}$$

$$4y^2-7y-12 = A(y+2)(y-3) + B(y)(y-3) + C(y)(y+2)$$

let $x=0$, $-12=-6A$ $A=2$ let $x=-2$, $18=10B$ $B=\frac{9}{5}$

let $x=3$, $3=15C$ $C=\frac{1}{5}$

$$\int_1^2 \left(\frac{2}{y} + \frac{9}{5(y+2)} + \frac{1}{5(y-3)}\right)dy$$

$u=y+2$ $w=y-3$

$du=dy$ $dw=dw$

$$\int \frac{9}{5u}\,du + \int \frac{1}{5w}\,dw$$

$$= \frac{9}{5}\ln|u| + \frac{1}{5}\ln|w| + C$$

$$= \frac{9}{5}\ln|y+2| + \frac{1}{5}\ln|y-3| + C$$

$$= \left[2\ln|y| + \frac{9}{5}\ln|y+2| + \frac{1}{5}\ln|y-3|\right]\Big|_1^2$$

$$= 2\ln 2 + \frac{9}{5}\ln 4 + \cancel{\tfrac{1}{5}\ln 1} - \cancel{2\ln 1} - \frac{9}{5}\ln 3 - \frac{1}{5}\ln 2$$

$$= \boxed{\frac{9}{5}\ln\left(\frac{8}{3}\right)}$$

Conclusion:

In order to be integrated, the function had to be broken into smaller fractions. The degree of the denominator was greater than the degree of the numerator, so no long division was required. All of the factors for the numerator were distinct, so the steps for decomposing a Case 1 rational function were followed.

Section 7.8, # 5

Picture: N/A

Goal: The goal is to determine whether the improper integral is convergent or divergent.

Set up:

Mathematical Model: $\int_a^\infty f(x)dx = \lim_{t\to\infty}\int_a^t f(x)dx$

Work:

$\int_3^\infty \frac{1}{(x-2)^{3/2}}dx = \lim_{t\to\infty}\int_3^t \frac{1}{(x-2)^{3/2}}dx \qquad u = x-2 \quad du = dx$

$\lim_{t\to\infty}\int_3^t u^{-3/2}du = \lim_{t\to\infty} -2u^{-1/2}\Big|_3^t = \lim_{t\to\infty} -2(x-2)^{-1/2}\Big|_3^t = \lim_{t\to\infty}\left[\cancel{-2(t-2)^{-1/2}} + 2(3-2)^{-1/2}\right] = \boxed{2}$

$\lim_{t\to\infty} -2(t-2)^{-1/2} = 0$ because the denominator gets infinitely large

Conclusion:

The improper integral converges to 2. I know this because the limit of the area under the curve from $x=3$ to $x\to\infty$ is 2. The limit of $-2(t-2)^{-1/2}$ as $t\to\infty$ is 0 because as t gets larger, the value gets smaller until it is almost 0.

Section 7.8, # 13

Picture: N/A

Goal: The goal is to determine whether the integral converges or diverges on the given interval.

Set up:

Mathematical Model: $\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{a} f(x)dx + \int_{a}^{\infty} f(x)dx$

$\int_{-\infty}^{b} f(x)dx = \lim_{t\to-\infty} \int_{t}^{b} f(x)dx \quad \int_{a}^{\infty} f(x)dx = \lim_{t\to\infty} \int_{a}^{t} f(x)dx$

Work:

$\int_{-\infty}^{\infty} xe^{-x^2}dx = \int_{-\infty}^{0} xe^{-x^2}dx + \int_{0}^{\infty} xe^{-x^2}dx = \lim_{t\to-\infty} \int_{t}^{0} xe^{-x^2}dx + \lim_{w\to\infty} \int_{0}^{w} xe^{-x^2}dx$

$u = -x^2$

$-\frac{1}{2}du = xdx \quad -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-x^2} + C$

$\lim_{t\to-\infty} \int_{t}^{0} xe^{-x^2}dx + \lim_{w\to\infty} \int_{0}^{\infty} xe^{-x^2}dx = \lim_{t\to-\infty} -\frac{1}{2}e^{-x^2}\Big|_{-\infty}^{0} + \lim_{w\to\infty} -\frac{1}{2}e^{-x^2}\Big|_{0}^{\infty} = -\frac{1}{2} + \frac{1}{2} = \boxed{0}$

$\lim_{t\to-\infty} -\frac{1}{2}e^{-t^2} = 0$
(as t^2 gets large, the denominator gets large)

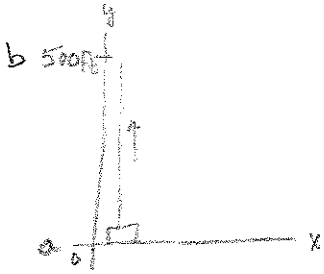
$\lim_{w\to\infty} -\frac{1}{2}e^{-w^2} = 0$
(as w^2 gets large, the denominator gets large)

Conclusion:

The integral converges to 0. This makes sense because the function is odd, and it is being integrated from $x = -a$ to $x = a$. Additionally, the limits in the last part of the problem both go to zero even though $-\infty$ and ∞ have different signs because it is e^{-x^2}, and the term being squared always results in a positive number.

Section 6.4, # 15

Picture:



Goal: The goal is to find the work done to lift the coal out of the shaft.

Set up:

Given: cable weighs 2 lb/ft
coal weighs 800 lbs
shaft is 500 ft deep

Mathematical Model: $W = Fd$, $W = \int_a^b f(x)\,dx$

Work:

$F_{cable} = (500\text{ft} - h\text{ft})\,2\,\text{lbs/ft}$ $F_{coal} = 800\text{ lbs}$ $F_{Total} = (1800\text{lbs} - 2h\text{lbs})$ $d = dh$

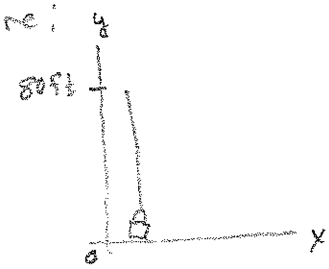
$W = \int_0^{500} (1800 - 2h)\,dh = 1800h - h^2 \Big|_0^{500} = 1800(500) - 500^2 = \boxed{650{,}000 \text{ ft} \cdot \text{lbs}}$

Conclusion:

The force is the weight of what is being pulled up. The weight of the coal is constant, but the weight of the cable varied, so that had to be accounted for. The distance is dh because that is the thickness of the slice of work, and I integrated from 0ft to 500ft because the shaft was 500ft tall.

Section 6.4, # 16

Picture:



Goal: The goal is to find the work done in pulling the bucket to the top of the well.

Set up:

Given: Bucket weighs 4lbs
Initial weight of water is 40lbs; leaks at .2lbs/s
Bucket pulled up at rate of 2ft/s
Well is 80ft deep

Mathematical Model: $W = Fd$, $W = \int_a^b f(x)\,dx$

Work:

$F_{Bucket} = 4\text{lbs}$

$\frac{.2\text{ bls/s}}{2\text{ ft/s}} = .1\text{ lbs/ft}$ $\quad F_{water} = (40\text{lbs} - (.1\text{ lbs/ft})h\text{ ft})$ $\quad F_{Total} = (44\text{lbs} - .1h\text{ lbs})$ $\quad d = dh$

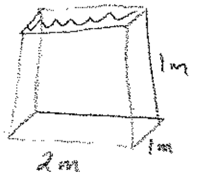
$$W = \int_0^{80} (44 - .1h)\,dh = 44h - .05h^2 \Big|_0^{80} = 44(80) - .05(80)^2 = \boxed{3200\text{ ft}\cdot\text{lbs}}$$

Conclusion:

The force is the weight of what is being pulled up. Since the bucket leaks at .2lbs/s and is being pulled up at 2ft/s, it loses .1lbs/ft when it is being pulled up. When this is added to the weight of the bucket and integrated from 0ft to 80ft, the force over that interval is found. Distance equals dh because that is the thickness of the slice of work.

Section 6.4, # 14

Picture:



Goal: The goal is to determine the amount of work needed to pump half of the water out of the tank.

Set up: Mathematical Model:

$W = \int_a^b f(x)\,dx \quad F = ma \quad m = V\rho \quad V = Ah$ so $F = Ah\rho a$

Given: $\rho_{water} = 1000\ kg/m^3$ and tank is 2m by 1m by 1m, $a = 9.81\ m/s^2$

Work:

$F = Ah\rho a = (2m \cdot 1m)h\,m(1000\ kg/m^3)(9.81\ m/s^2) = 19620h\ N$

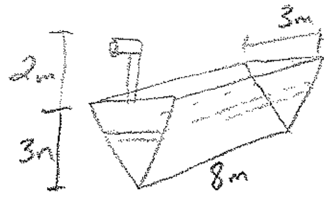
$W = \int_0^{.5} 19620h\,dh = 9810h^2 \Big|_0^{.5} = 9810(.5^2 - 0^2) = \boxed{2452.5\ J}$

Conclusion:

Work is force times distance, and since I took the definite integral of a varying force across a specified distance, in the actual integral distance = dh. I integrated from 0m to .5m because the tank is 1m tall and half the water would be pumped out when the water level is at .5m.

Section 6.4, # 21

Picture:



Goal: The goal is to determine the amount of work needed to pump all of the water out of the tank.

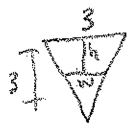
Setup:

Mathematical Model: $W=\int_a^b f(x)\,dx$ $\quad F=ma \quad m=V\rho \quad W=F\cdot d$

Given: Dimensions above, $\rho_{water}=1000\ kg/m^3$, $a=9.81\ m/s^2$

Work:

$V_{slice}=l\cdot w\cdot h = 8m(3m-h)\,dh$ $\qquad F=V\rho a = 8m(3m-h)(1000\,kg/m^3)(9.81\,m/s^2)\,dh$



$\frac{3}{3}=\frac{w}{3-h}$

$w=3-h$

$d=(2m+h)$

$W=\int_0^3 8m(3m-h)(1000\,kg/m^3)(9.81\,m/s^2)(2m+h)\,dh$

$=\int_0^3(-78480h^2+78480h+470880)\,dh$

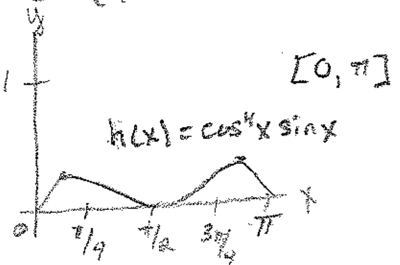
$=\left[-26160h^3+39240h^2+470880h\right]\Big|_0^3 = \boxed{1{,}059{,}480\ J}$

Conclusion:

For this tank, the height and width changed as you go up (compared to a rectangular tank), so the width had to be in terms of height. This was found using similar triangles. The height of the slice was dh, so the displacement needed a value. All of the water had to go at least 2m, so the displacement was 2m+h, h being the level of the water from the top of the tank. Since the extra 2m was accounted for in the displacement, I integrated from 0m to 3m, or the height of the tank.

Section 6.5, # 7

Picture:



Goal: The goal is to find the average value of the function on the given interval.

Set up:

Mathematical Model: $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$

Work:

$h(x) = \cos^4 x \sin x \quad [0, \pi]$

$u = \cos x$ $\quad \int u^4 du = \frac{1}{5}u^5 + C$

$-du = \sin x$

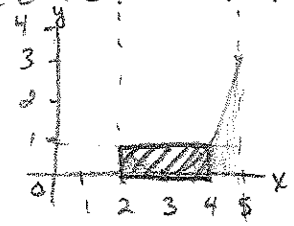
$h_{ave} = \frac{1}{\pi - 0}\int_0^{\pi} \cos^4 x \sin x \, dx = \frac{1}{\pi}\left[-\frac{1}{5}\cos^5 x\right]\Big|_0^{\pi} = -\frac{1}{5\pi}\left(\cos^5(\pi) - \cos^5(0)\right) = \boxed{\frac{2}{5\pi}}$

Conclusion:

The average value on this interval is $\frac{2}{5\pi}$. That means that a rectangle of height $\frac{2}{5\pi}$ and width $(\pi - 0)$ would have the same area as the area under $h(x)$ from $[0, \pi]$.

Section 6.5, #9

Picture: $f(x) = (x-3)^2$ $[2,5]$



part (c)

Goal: The goal is to find the average value of f on the interval, find c such that $f(c) = f_{ave}$, and sketch the graph of f and a rectangle with equal area.

Setup:

Mathematical Model: $f(c) = f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$

Work:

a) $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{5-2}\int_2^5 (x-3)^2\,dx$ $\quad u = x-3$, $du = dx$ $\quad \int u^2 = \frac{1}{3}u^3 + C$

$= \frac{1}{3}\left[\frac{1}{3}(x-3)^3\right]\Big|_2^5 = \frac{1}{9}(2^3 - (-1)^3) = \boxed{1}$

b) $(x-3)^2 = 1$

$x-3 = 1$ and $x-3 = -1$

$\boxed{x = 4 \text{ and } x = 2}$ So $c = 2, 4$

Conclusion:

The average value for the function is 1. This means that there is a rectangle of height 1 and width (5-2) that can be drawn that has the same area as the area under the curve from [2,5]. x=4 and x=2 are the x-values at which f(x) equals the average value (also where the rectangle crosses the function).

Section 8.3, # 1

Picture:

Goal: The goal is to determine the hydrostatic pressure on the bottom of the aquarium, the hydrostatic force on the bottom of the aquarium. and the hydrostatic force on one of the ends of the aquarium.

Set up:

Mathematical Model: $P_{Hydro} = \text{depth} \cdot \text{weight density}$ $F_{Hydro} = P_{Hydro} \cdot A$ $A = l \cdot w$

Given: $\delta_{water} = 62.5 \text{ lbs/ft}^3$ and dimensions above $F_{Hydro} = \int_a^b f(x)\,dx$

Work:

a) $P = \text{depth} \cdot \text{weight density} = (3\text{ft})(62.5 \text{ lbs/ft}^3) = \boxed{187.5 \text{ lbs/ft}^2}$

b) $F = PA = Plw = (187.5 \text{ lbs/ft}^2)(5\text{ft})(2\text{ft}) = \boxed{1875 \text{ lbs}}$

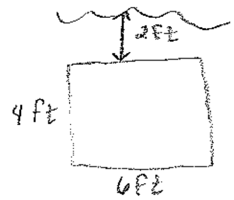
c) $F = \int_a^b h \cdot \text{density} \cdot w \cdot \Delta h = \int_0^3 h(62.5)(2)\,dh = 62.5h^2\Big|_0^3 = \boxed{562.5 \text{ lbs}}$

Conclusion:

The hydrostatic force on the end of the tank differs from that of the bottom because pressure and area change with depth. This is why the integral with respect to depth was taken and the height for volume was dh.

Section 8.3, # 3

Picture:



Goal: The goal is to set up a Riemann sum that approximates the hydrostatic force on the plate and then express the force as a definite integral and evaluate that integral.

Setup:

Mathematical Model: $F = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx$ $\quad \Delta x = \frac{b-a}{n}$

$F = \rho g A d \quad A = l \cdot w$

Given: $\delta_{water} = 62.5\ \text{lbs/ft}^3$ and dimensions above

Work:

$F = \rho g A d \quad \rho g = 62.5\ \text{lbs/ft}^3 \quad d = h \quad A = l \cdot w \quad w = \Delta h \quad l = 6ft$

Riemann Sum: $\lim_{n\to\infty} \sum_{i=1}^{n} (62.5\ \text{lbs/ft}^3)(6ft)h\,\Delta h$

Integral: $\int_2^6 (62.5\ \text{lbs/ft}^3)(6ft)h\,dh = 187.5h^2\Big|_2^6 = 187.5(36-4) = \boxed{6000\text{lbs}}$

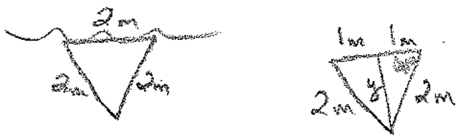
What's "h" here?

Conclusion:

The force can be approximated with a Riemann sum by choosing a value for n, the number of slices. As n gets larger, the approximation gets more accurate until $n\to\infty$ and the approximation approaches the definite answer. For the Riemann sum, Δh, the width would be $\frac{b-a}{n}$; however, for the integral this value becomes infinitely small and turns into dh.

Section 8.3, # 7

Picture:



Goal: The goal is to set up a Riemann Sum that approximates the hydrostatic force on the plate and then express the force as a definite integral and evaluate that integral.

Setup:

Mathematical Model: $F = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx \quad \Delta x = \frac{b-a}{n}$

$F = \rho g A d \quad A = l \cdot w$

Given: $\rho = 1000\,\text{kg/m}^3$ $g = 9.8\,\text{m/s}^2$ and dimensions above

Work:

$F = \rho g A d \quad \rho = 1000\,\text{kg/m}^3 \quad g = 9.8\,\text{m/s}^2 \quad \text{depth} = h \quad A = l \cdot w \quad w = \Delta h$

$\sin 60 = \frac{y}{2m} \quad y = \sqrt{3}\,m \qquad \frac{\Delta l}{\Delta h} = \frac{2m - 0m}{\sqrt{3}m - 0m} = \frac{2\sqrt{3}}{3} \quad \Delta l = \frac{2\sqrt{3}}{3}h \quad l = \left(2m - \frac{2\sqrt{3}}{3}h\right)$

Riemann Sum: $\lim_{n \to \infty} \sum_{i=1}^{n} (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)\left(2m - \frac{2\sqrt{3}}{3}h\right)h\,\Delta h$

Integral: $\int_0^{\sqrt{3}} (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)\left(2m - \frac{2\sqrt{3}}{3}h\right)h\,dh = \int_0^{\sqrt{3}} (19600h - 11316h^2)\,dh$

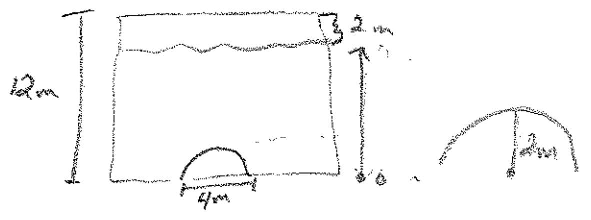
$= 9800h^2 - 3772h^3 \Big|_0^{\sqrt{3}} = \boxed{9800\text{ N}}$

Conclusion:

The force can be approximated with a Riemann sum by choosing a value for, n, the number of slices. As n gets larger, the approximation gets more accurate until $n \to \infty$ and the approximation approaches the definite answer. The altitude of the equilateral triangle had to be found so that l could be put in terms of h and so that I could get my upper bound of integration.

Section 8.3, #14

Picture:



Goal: The goal is to determine the hydrostatic force on the gate.

Set up:

Mathematical Model: $F_{Hydro} = \rho g A d = \int_a^b f(x)\,dx$ $\quad r^2 = x^2+y^2$ $\quad \cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$

Given: $\rho = 1000\ Kg/m^3$ $\quad g = 9.8\ m/s^2$ $\quad A = l\cdot w$ $\quad \sin 2\theta = 2\sin\theta\cos\theta$

Work:

$A = l\cdot w \quad w = dh \quad r^2 = x^2+y^2 \quad x = \sqrt{r^2-y^2},\ l = 2\sqrt{4-h^2} \quad d = 10-h$

$2\int_0^2 (1000\ Kg/m^3)(9.8\ m/s^2)(\sqrt{4-h^2})(10-h)\,dh = \int_0^2 19600(10-h)\sqrt{4-h^2}\,dh$

$= 19600\left(\int_0^2 10\sqrt{4-h^2}\,dh - \int_0^2 h\sqrt{4-h^2}\,dh\right)$ $\quad h = 2\sin\theta \quad dh = 2\cos\theta\,d\theta \quad \frac{h}{2} = \sin\theta$

$2,\ h,\ \theta,\ \sqrt{4-h^2}$

$u = 4-h^2 \quad -\frac{1}{2}du = h\,dh$ $\qquad 10\int_0^2 \sqrt{4-4\sin^2\theta}\,(2\cos\theta\,d\theta)$ $\qquad \theta = \sin^{-1}\left(\frac{h}{2}\right)$

$-\frac{1}{2}\int_0^2 u^{1/2}\,du = -\frac{1}{2}\left(\frac{2}{3}\right)u^{3/2}\Big|_0^2$ $\qquad \cos\theta = \frac{\sqrt{4-h^2}}{2}$

$= -\frac{1}{3}(4-h^2)^{3/2}\Big|_0^2$ $\qquad = 40\int_0^2 \cos^2\theta\,d\theta = 20\int_0^2 (1+\cos(2\theta))\,d\theta$

$= 20\left[\theta + \frac{1}{2}\sin(2\theta)\right]\Big|_0^2 = 20\left[\theta + \cos\theta\sin\theta\right]\Big|_0^2$

$19600\left[20\left(\sin^{-1}\left(\frac{h}{2}\right) + \frac{h\sqrt{4-h^2}}{4}\right)\Big|_0^2 + \frac{1}{3}(4-h^2)^{3/2}\Big|_0^2\right]$ $\qquad = 20\left[\sin^{-1}\left(\frac{h}{2}\right) + \left(\frac{h}{2}\right)\left(\frac{\sqrt{4-h^2}}{2}\right)\right]\Big|_0^2$

$= 19600\left(20\sin^{-1}(1) + 0 - 0 - 0 + 0 - \frac{1}{3}(4)^{3/2}\right) = \boxed{563485\ N}$

Conclusion:

The hydrostatic force on the gate is pressure multiplied by area. Pressure varies with depth, so an integral with respect to height was ~~taken~~. I chose the ground as ~~the~~ origin and integrated from 0 to 2 because the gate is 2m tall. Since it is a semi-circle, I used the equation of a circle, solved for x, and multiplied by 2 to get both sides of the axis for ~~the~~ whole length. Depth is 10-h because at the origin, d=10 and as you go up from the origin, depth decreases by h. The integral had to be broken into two to be solved, one by trig substitution, and the other by u-substitution. For the trig substitution, it was of the form $\sqrt{a^2-x^2}$, so I set $h = a\sin\theta = 2\sin\theta$. The reference right triangle and a few double angle formulas helped me to get from θ back to h.