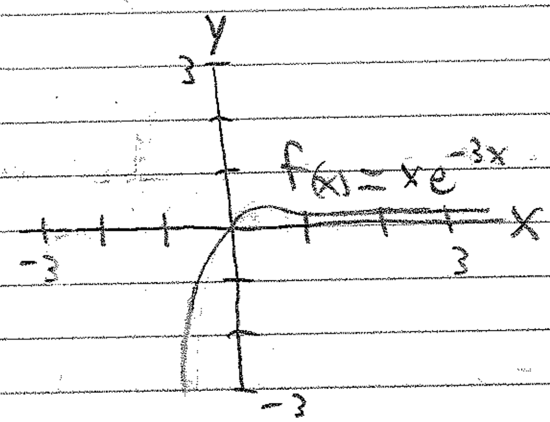
Lesson 12
Section 7.1
Problem #5

Picture:



graphed using calc tool

Goal: Evaluate the integral $\int te^{-3t}\,dt$.

Set Up: given
- the integral, $\int te^{-3t}\,dt$

variables
- t • v • dv
- u • du

equations
- $\int u\,dv = uv - \int v\,du$

Mathematical Model: Formula for integration by parts, $\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx$, and it corresponds to the product rule for differentiation.

Work:

$\int \underbrace{t}_{u}\underbrace{e^{-3t}\,dt}_{v}$, $u = t$, $du = dt$, $dv = e^{-3t}\,dt$, $v = \frac{e^{-3t}}{-3}$, $v = \int dv = \int e^{-3t}\,dt$

$$\int te^{-3t}\,dt = uv - \int v\,du = t\left(\frac{e^{-3t}}{-3}\right) - \int \frac{e^{-3t}}{-3}\,dt = -\frac{1}{3}te^{-3t} + \frac{1}{3}\int e^{-3t}\,dt$$

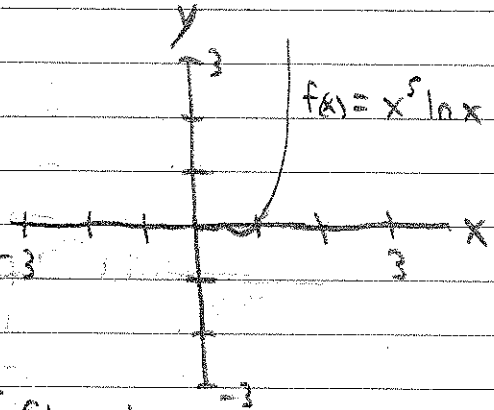
$$\int te^{-3t}\,dt = -\frac{1}{3}te^{-3t} + \frac{1}{3}\left(-\frac{1}{3}\right)\left(e^{-3t}\right) + C = -\frac{1}{3}\left(e^{-3t}\right)\left(t + \frac{1}{3}\right) + C$$

$$\boxed{\int te^{-3t}\,dt = -\frac{1}{3}\left(e^{-3t}\right)\left(t + \frac{1}{3}\right) + C}$$

Conclusion: Using integration by parts, $\int te^{-3t}\,dt$ is equal to $-\frac{1}{3}e^{-3t}\left(t + \frac{1}{3}\right) + C$.

Lesson 12
Section 7.1
Problem #12

Picture:



$f(x) = x^5 \ln x$ graphed using Calc Tool

Goal: Evaluate the integral $\int p^5 \ln(p)\, dp$.

Set up: given | variables | equations

- the integral, $\int p^5 \ln p\, dp$
- p • u • v • du • dv
- $\int u\,dv = uv - \int v\,du$

Mathematical Model: Formula for integration by parts, $\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx$, and it corresponds to the product rule for differentiation.

Work: $\int p^5 \underbrace{\ln(p)}_{u}\, dp$ (dv)

$u = \ln p \quad dv = p^5\,dp \quad v = \int dv = \int p^5\,dp$

$du = \frac{1}{p}\,dp \quad v = \frac{p^6}{6}$

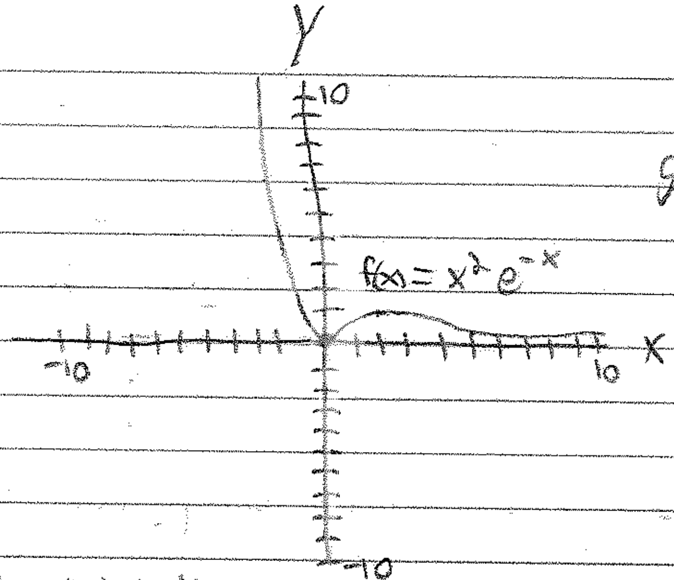
$$\int p^5 \ln p\, dp = uv - \int v\,du = \ln(p)\frac{p^6}{6} - \int \frac{p^6}{6}\left(\frac{1}{p}\,dp\right) = \ln(p)\frac{p^6}{6} - \frac{1}{6}\int p^5\,dp$$

$$\int p^5 \ln p\, dp = \ln(p)\frac{p^6}{6} - \frac{1}{6}\left(\frac{1}{6}\right)p^6 + C = \boxed{\frac{1}{6}p^6\left(\ln(p) - \frac{1}{6}\right) + C}$$

Conclusion: The evaluated integral of $\int p^5 \ln p\, dp$ is $\frac{1}{6}p^6\left(\ln p - \frac{1}{6}\right) + C$. The integral was evaluated by using integration by parts.

Lesson 12
Section 7.1
Problem #67

Picture:



graphed using calc tool

Goal: Determine the total distance a particle travels in t seconds with a velocity of $v(t) = t^2 e^{-t}$ meters per second.

Set up: given
- Velocity, $v(t) = t^2 e^{-t}$
- Units, meters per second
- interval: $[0, t)$ (the limits of integration)

variables
- t • u • v • du • dv
- Velocity • Distance

equations
- $\int_{t_1}^{t_2} |v(t)|\,dt$ = total distance traveled
- $\int u\,dv = uv - \int v\,du$
- $\int_a^b f(x) g'(x)\,dx = f(x)g(x)\Big]_a^b - \int_a^b g(x) f'(x)\,dx$

Mathematical Model: The net change theorem, the integral of a rate of change is the net change: $\int_a^b F'(x)\,dx = F(b) - F(a)$. The formula for integration by parts, $\int f(x) g'(x)\,dx = f(x) g(x) - \int g(x) f'(x)\,dx$, and it corresponds to the product rule for differentiation.

Work:

total distance traveled $= \int_{t_1}^{t_2} |v(t)|\,dt = \int_0^t |t^2 e^{-t}|\,dt$

$u = t^2 \qquad dv = e^{-t}\,dt \qquad v = \int dv = \int e^{-t}\,dt$

$du = 2t\,dt \qquad v = -e^{-t}$

$\int_0^t |t^2 e^{-t}|\,dt = t^2(-e^{-t})\Big]_0^t - \underbrace{\int_0^t -e^{-t}\, 2t\,dt}$ need to do integration by parts AGAIN!

$\int_0^t -e^{-t}\, 2t\,dt$, integration by parts:

$u = 2t \qquad dv = -e^{-t}\,dt \qquad v = \int dv = \int -e^{-t}\,dt$

$du = 2\,dt \qquad v = e^{-t}$

$\int_0^t -e^{-t}\, 2t\,dt = 2t(e^{-t})\Big]_0^t - \int_0^t e^{-t}\, 2\,dt = 2te^{-t} - 2(-e^{-t})\Big|_0^t$

$= 2te^{-t} - \left(2(-e^{-t}) - 2(-e^{-(0)})\right) = 2te^{-t} - (-2e^{-t} + 2) = 2te^{-t} + 2e^{-t} + 2$

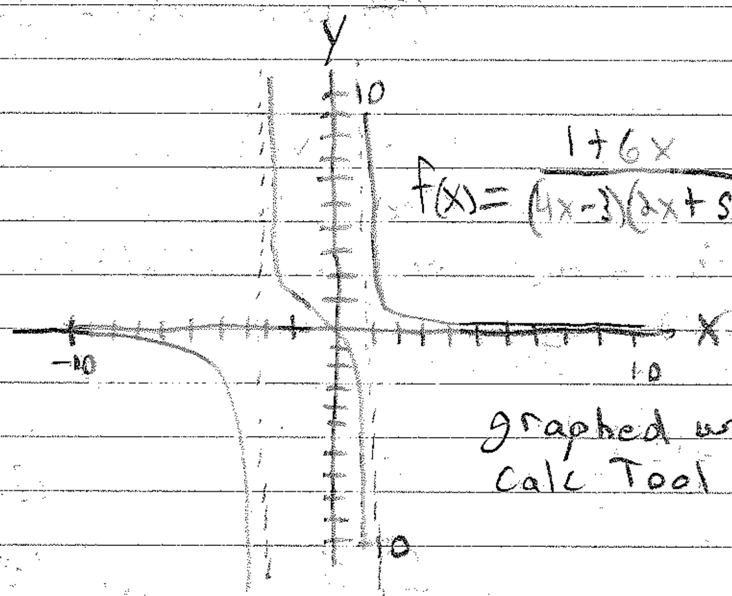
$$\int_0^t |t^2e^{-t}|\,dt = t^2(-e^{-t})\Big]_0^t - \int_0^t -e^{-t}\,2t\,dt$$

$$\int_0^t |t^2e^{-t}|\,dt = -t^2(e^{-t}) - (2te^{-t} + 2e^{-t} - 2)$$
$$= -t^2(e^{-t}) - 2te^{-t} - 2e^{-t} + 2$$

$$\int_0^t |t^2e^{-t}|\,dt = \boxed{-t^2(e^{-t}) - 2te^{-t} - 2e^{-t} + 2} \text{ meters}$$

Conclusion: A particle with a velocity, $v(t) = t^2e^{-t}$ meters per second will travel a total distance of $s(t) = -t^2(e^{-t}) - 2te^{-t} - 2e^{-t} + 2$ meters in t seconds.

Lesson 13
Section 7.4
Problem # 1
a)
Picture:



$$f(x) = \frac{1+6x}{(4x-3)(2x+5)}$$

graphed using Calc Tool

Goal: Write the partial fraction decomposition for the function, $f(x) = \frac{1+6x}{(4x-3)(2x+5)}$.

Set up:

given	variables	equations
$f(x) = \frac{1+6x}{(4x-3)(2x+5)}$	• x • The coefficients of the partial fraction decomposition	• $Q(x) = (a_1x+b_1)(a_2x+b_2)\cdots(a_kx+b_k)$ • $\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \cdots + \frac{A_k}{a_kx+b_k}$ • Degree of $R(x)$ < Degree of $Q(x)$

Mathematical Model: The denominator $Q(x)$ is a product of distinct linear factors, and the partial fraction theorem states

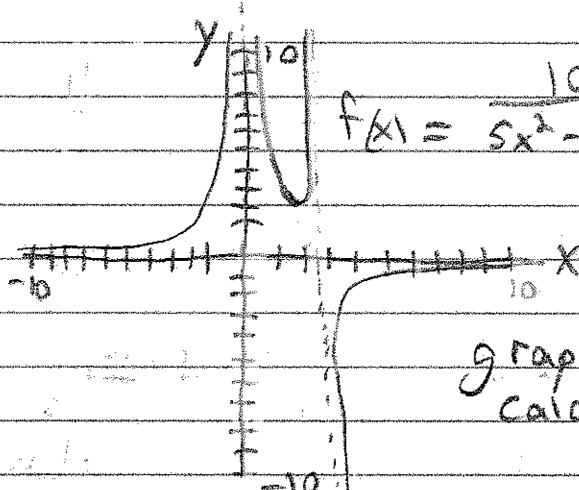
$$\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \cdots + \frac{A_k}{a_kx+b_k}$$

Work: $\dfrac{1+6x}{(4x-3)(2x+5)} = \boxed{\dfrac{A}{4x-3} + \dfrac{B}{2x+5}}$

Conclusion: The partial fraction decomposition of the function, $f(x) = \dfrac{1+6x}{(4x-3)(2x+5)}$ is $\dfrac{A}{4x-3} + \dfrac{B}{2x+5}$.

b)

Picture:



$f(x) = \dfrac{10}{5x^2 - 2x^3}$

graphed Using Calc Tool

Goal: Write the partial fraction decomposition for the function, $f(x) = \dfrac{10}{5x^2 - 2x^3}$.

Set up:

given	Variables	equations
$f(x) = \dfrac{10}{5x^2 - 2x^3}$	• x • The coefficients of the partial fraction decomposition.	• $Q(x) = (a_1x+b_1)(a_2x+b_2)\dots(a_kx+b_k)$ • $\dfrac{R(x)}{Q(x)} = \dfrac{A_1}{a_1x+b_1} + \dfrac{A_2}{a_2x+b_2} + \dots + \dfrac{A_k}{a_kx+b_k}$ • degree of $R(x)$ < degree of $Q(x)$

Mathematical Model: The denominator $Q(x)$ is a product of distinct linear factors, and the partial fraction theorem states,

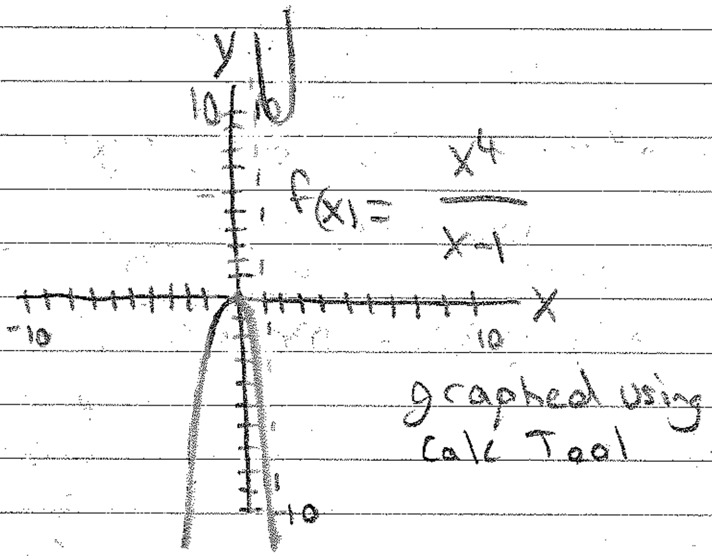
$$\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \dots + \frac{A_k}{a_kx+b_k}.$$

Work: $\dfrac{10}{5x^2-2x^3} = \dfrac{10}{x^2(5-2x)} = \boxed{\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{5-2x}}$

Conclusion: The partial fraction decomposition of the function, $f(x) = \dfrac{10}{5x^2-2x^3}$ is $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{5-2x}$.

Lesson 13
Section 7.4
Problem #7

Picture:



Goal: Evaluate the integral, $\int \frac{x^4}{x-1}\,dx$.

Set Up: given | variables | equations

- $\int \frac{x^4}{x-1}\,dx$
- x
- $Q(x) = (a_1x+b_1)(a_2x+b_2)\cdots(a_kx+b_k)$
- $\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \dots + \frac{A_k}{a_kx+b_k}$
- $R(x)$ degree $<$ $Q(x)$ degree

Mathematical Model: The denominator $Q(x)$ is a product of distinct linear factors, and the partial fraction theorem states:
$\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \dots + \frac{A_k}{a_kx+b_k}$. The degree of $R(x)$ must be less than the degree of $Q(x)$.

Work: $\int \frac{x^4}{x-1}\,dx$ $\frac{R(x)}{Q(x)}$ degree of $R(x) >$ degree of $Q(x)$, $\therefore$ perform long division

$$
\begin{array}{r l}
 & x^3 + x^2 + x + 1 \\
x-1 \,\big)\!\!\! & x^4 + 0 + 0 + 0 + 0 \\
 & \underline{x^4 - x^3} \\
 & x^3 + 0 \\
 & \underline{x^3 - x^2} \\
 & x^2 + 0 \\
 & \underline{x^2 - x} \\
 & x + 0 \\
 & \underline{x - 1} \\
 & 1
\end{array}
$$

remainder $R(x) = \frac{1}{x-1}$

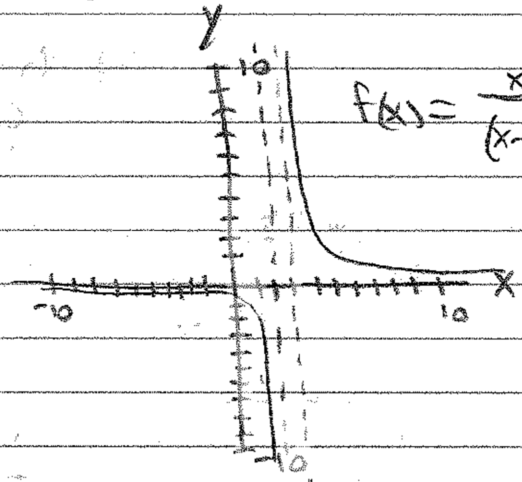
$$\int \frac{x^4}{x-1}\,dx = \int\left(x^3+x^2+x+1+\frac{1}{x-1}\right)dx$$

$$\int \frac{x^4}{x-1}\,dx = \boxed{\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x-1|+C}$$

Conclusion: The evaluation of the integral, $\int \frac{x^4}{x-1}dx$ is:
$\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x-1|+C.$

Lesson 13
Section 7.4
Problem # 19

Picture:



$f(x)=\frac{x^2+1}{(x-3)(x-2)^2}$ graphed using Calc Tool

Goal: Evaluate the integral, $\int \frac{x^2+1}{(x-3)(x-2)^2}dx.$

Set up: given

- $\int \frac{x^2+1}{(x-3)(x-2)^2}$

variables

- x

equations

- $\frac{A_1}{a_1x+b_1}+\frac{A_2}{(a_1x+b_1)^2}+\dots+\frac{A_r}{(a_1x+b_1)^r}$
- $Q(x)=(a_1x+b_1)(a_2x+b_2)\dots(a_kx+b_k)$
- $\frac{R(x)}{Q(x)}=\frac{A_1}{a_1x+b_1}+\frac{A_2}{a_2x+b_2}+\dots+\frac{A_k}{a_kx+b_k}$
- $R(x)$ degree $<$ $Q(x)$ degree

Mathematical Model: The denominator $Q(x)$ is a product of distinct linear factors, and the partial fraction theorem states:
$\frac{R(x)}{Q(x)}=\frac{A_1}{a_1x+b_1}+\frac{A_2}{a_2x+b_2}+\dots+\frac{A_k}{a_kx+b_k}$. The degree of $R(x)$ must be less than the degree of $Q(x)$. Additionally $Q(x)$ is a product of linear factors, some of which are repeated, $\frac{A_1}{a_1x+b_1}+\frac{A_2}{(a_1x+b_1)^2}+\dots+\frac{A_r}{(a_1x+b_1)^r}$.

Work: $\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$$

let $x=2$ $\quad (2)^2+1 = A(2-2)^2 + B(2-3)(2-2) + C(2-3)$

$$4+1 = 0 + 0 + -C$$

$$5 = -C$$

$$\underline{C=-5}$$

let $x=3$ $\quad (3)^2+1 = A(3-2)^2 + B(3-3)(3-2) + C(3-3)$

$$10 = A + 0 + 0$$

$$\underline{A=10}$$

let $x=1$ $\quad (1)^2+1 = 10(1-2)^2 + B(1-3)(1-2) + -5(1-3)$

$$2 = 10 + B(-2)(-1) - 5(-2)$$

$$2 = 10 + 2B + 10$$

$$2 = 20 + 2B$$

$$-20 \qquad -20$$

$$\frac{-18}{2} = \frac{2B}{2}$$

$$\underline{-9 = B}$$

$C=-5 \quad A=10 \quad B=-9$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{10}{x-3} - \frac{9}{x-2} - \frac{5}{(x-2)^2}$$

$$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \left(\frac{10}{x-3} - \frac{9}{x-2} - \frac{5}{(x-2)^2}\right)dx$$

$$\int -\frac{5}{(x-2)^2}\,dx = \int -5(x-2)^{-2}\,dx$$

$$= \frac{-5(x-2)^{-2+1}}{-2+1} = \frac{+5(x-2)^{-1}}{+1}$$

$$= \frac{5}{x-2}$$

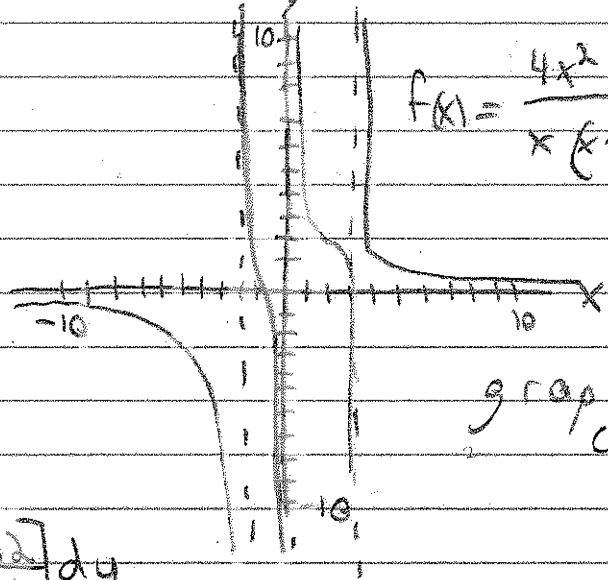
$$= \boxed{10\ln|x-3| - 9\ln|x-2| + \frac{5}{x-2} + C}$$

Conclusion: The evaluated integral, $\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx$, is:

$10\ln|x-3| - 9\ln|x-2| + 5/(x-2) + C$

Lesson 13
Section 7.4
Problem #17

Picture:



$$f(x) = \frac{4x^2 - 7x - 12}{x(x+2)(x-3)}$$

graphed using Calc Tool

Goal: Evaluate the integral, $\int_1^2 \left[\frac{4y^2 - 7y - 12}{y(y+2)(y-3)}\right] dy$

Set up: given | variables | equations

- $\int_1^2 \frac{4y^2 - 7y - 12}{y(y+2)(y-3)} dy$
- y
- $Q(x) = (a_1x + b_1)(a_2x + b_2) \cdots (a_kx + b_k)$
- $\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \cdots + \frac{A_k}{a_kx+b_k}$
- $R(x)$ degree $<$ $Q(x)$ degree

Mathematical Model: The denominator $Q(x)$ is a product of distinct linear factors, and the partial fraction theorem states: $\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \cdots + \frac{A_k}{a_kx+b_k}$. The degree of $R(x)$ must be less than the degree of $Q(x)$.

Work: $\int_1^2 \frac{4y^2 - 7y - 12}{y(y+2)(y-3)} dy \quad \frac{4y^2 - 7y - 12}{y(y+2)(y-3)} = \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3}$

$$4y^2 - 7y - 12 = A(y+2)(y-3) + B(y)(y-3) + C(y)(y+2)$$

let $y = 0$

$$4(0)^2 - 7(0) - 12 = A(0+2)(0-3) + B(0)\underbrace{(0-3)}_{0} + C(0)\underbrace{(0+2)}_{0}$$

$$\frac{-12}{-6} = \frac{-6A}{-6}$$

$$\underline{A = 2}$$

let $y = 3$

$$4(3)^2 - 7(3) - 12 = A(3+2)\cancel{(3-3)} + B(3)\cancel{(3-3)} + C(3)(3+2)$$

$$36 - 21 - 12 = 0 + 0 + 15C$$

$$\frac{3}{15} = \frac{15C}{15}$$

$$\underline{C = \frac{1}{5}}$$

let $y = -2$

$$4(-2)^2 - 7(-2) - 12 = A\cancel{((-2)+2)}((-2)-3) + B(-2)((-2)-3) + C(-2)\cancel{((-2)+2)}$$

$$16 + 14 - 12 = 0 + 10B + 0$$

$$\frac{18}{10} = \frac{10B}{10}$$

$$\underline{B = \frac{9}{5}}$$

$$\frac{4y^2 - 7y - 12}{y(y+2)(y-3)} = \frac{2}{y} + \frac{9/5}{y+2} + \frac{1/5}{y-3}$$

$$\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy = \int_1^2 \left(\frac{2}{y} + \frac{9/5}{y+2} + \frac{1/5}{y-3}\right) dy = \left(2\ln|y| + \frac{9}{5}\ln|y+2| + \frac{1}{5}\ln|y-3|\right)\Big|_1^2$$

$$= \left(2\ln(2) + \frac{9}{5}\ln((2)+2) + \frac{1}{5}\ln|(2)-3|\right) - \left(2\ln(1) + \frac{9}{5}\ln|(1)+2| + \frac{1}{5}\ln|(1)-3|\right)$$

$$= 2\ln(2) + \frac{9}{5}\ln(4) + \frac{1}{5}\underbrace{\ln(1)}_{0} - 2\underbrace{\ln(1)}_{0} - \frac{9}{5}\ln(3) - \frac{1}{5}\ln(2)$$

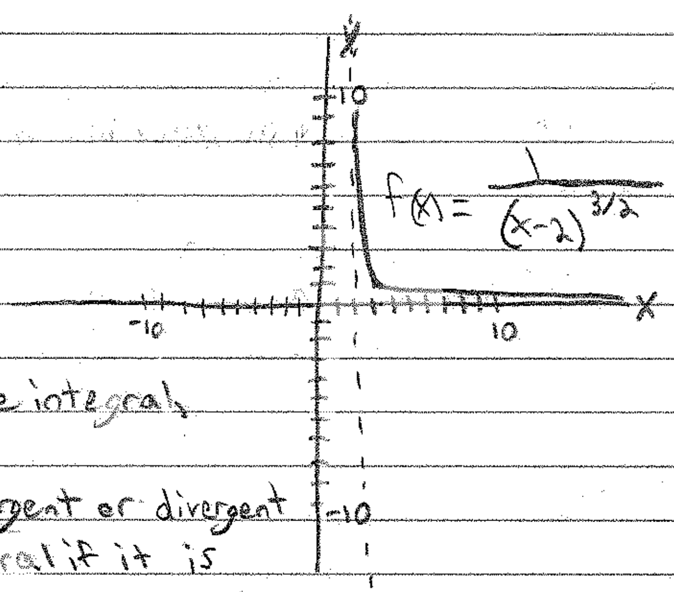
$$= \frac{9}{5}\ln(2) + \frac{9}{5}\ln(4) - \frac{9}{5}\ln(3) = \frac{9}{5}\left(\ln(2) + \ln(4) - \ln(3)\right)$$

$$\boxed{\frac{9}{5}\left(\ln(2) + \ln(4) - \ln(3)\right)}$$

Conclusion: The evaluated integral, $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy$, is $\frac{9}{5}(\ln(2)+\ln(4)-\ln(3))$, which is about 1.77.

Lesson 14
Section 7.8
Problem #5

Picture:



graphed using Calc Tool

Goal: Determine if the integral $\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx$, is convergent or divergent and evaluate the integral if it is convergent.

Set up: given | Variables | equations

- $\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx$
- x
- $\int_a^{\infty} f(x)\,dx = \lim_{t \to \infty} \int_a^t f(x)\,dx$

Mathematical Model: If $\int_a^t f(x)\,dx$ exists for every number $t \geq a$, then $\int_a^{\infty} f(x)\,dx = \lim_{t \to \infty} \int_a^t f(x)\,dx$ provided this limit exists (as a finite number). The improper integrals $\int_a^{\infty} f(x)\,dx$ and $\int_{-\infty}^{b} f(x)\,dx$ are called convergent if the corresponding limit exists and divergent if the limit does not exist.

Work: $$\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx = \lim_{b \to \infty} \int_3^b \frac{1}{(x-2)^{3/2}}\,dx$$

$$\lim_{b \to \infty} \int_3^b (x-2)^{-\frac{3}{2}}\,dx = \lim_{b \to \infty} \left(\frac{(x-2)^{-\frac{3}{2}+\frac{2}{2}}}{-\frac{3}{2}+\frac{2}{2}} \right) \Bigg|_3^b = \lim_{b \to \infty} \left(\frac{(x-2)^{-1/2}}{-1/2} \right) \Bigg|_3^b$$

$$= \lim_{b \to \infty} \left(\frac{-2}{\sqrt{x-2}} \right) \Bigg|_3^b = \lim_{b \to \infty} \left(\frac{-2}{\sqrt{(b)-2}} - \frac{-2}{\sqrt{(3)-2}} \right)$$

$$= \lim_{b \to \infty} \left(\frac{-2}{\sqrt{b-2}} + 2 \right) = \lim_{b \to \infty} \left(\frac{-2}{\sqrt{b-2}} \right) + 2$$ As b approaches ∞, $\frac{-2}{\sqrt{b-2}}$ goes to zero.

$= 0 + 2 = 2$ $\therefore$ the integral is convergent because the limit exists.

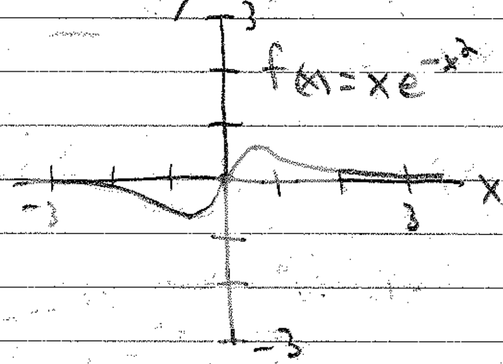
$\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx = \boxed{2}$ $\boxed{\text{The integral is convergent.}}$ because the limit exists and a value/number is retrieved from evaluating the integral.

Conclusion: The integral, $\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx$ is convergent because the limit exists and the evaluated integral, $\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx$ is 2.

Lesson 14
Section 7.8
Problem #13

Picture:

graphed using Calc. Tool.



Goal: Determine if the integral, $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$ is convergent or divergent and evaluate the integral if it is convergent.

Set up:

given	variables	equations
$\int_{-\infty}^{\infty} xe^{-x^2}\,dx$	x	$\int u\,dv = uv - \int v\,du$
		$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{\infty} f(x)\,dx$
		$\int_a^{\infty} f(x)\,dx = \lim_{t \to a} \int_a^t f(x)\,dx$
		$\int_{-\infty}^{b} f(x)\,dx = \lim_{t \to -\infty} \int_t^b f(x)\,dx$

Mathematical Model: The improper integrals $\int_a^{\infty} f(x)\,dx$ and $\int_{-\infty}^{b} f(x)\,dx$ are called convergent if the corresponding limit exists and divergent if the limit does not exist. If both $\int_a^{\infty} f(x)\,dx$ and $\int_{-\infty}^{a} f(x)\,dx$ are convergent, then we define $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{\infty} f(x)\,dx$.

<u>Work:</u> $\int_{-\infty}^{\infty} xe^{-x^2}\,dx = \int_{-\infty}^{0} xe^{-x^2}\,dx + \int_{0}^{\infty} xe^{-x^2}\,dx$

$\int_{-\infty}^{0} xe^{-x^2}\,dx = \lim_{a\to-\infty} \int_{a}^{0} xe^{-x^2}\,dx$ let $u = x^2$

$du = 2x\,dx$

$\lim_{a\to-\infty} \int_{a}^{0} xe^{-x^2}\,dx = \lim_{a\to-\infty} \int_{a}^{0} xe^{-u}\frac{du}{2x}$ $dx = \frac{du}{2x}$

$= \lim_{a\to-\infty} \int_{a}^{0} \frac{1}{2}e^{-u}\,du = \lim_{a\to-\infty} \left(-\frac{1}{2}e^{-u}\right)\Big|_{a}^{0} = \lim_{a\to-\infty} \left(-\frac{1}{2}e^{-x^2}\right)\Big|_{a}^{0}$

$= \lim_{a\to-\infty} \left(\frac{-1}{2e^{x^2}}\right)\Big|_{a}^{0} = \lim_{a\to-\infty} \left[\left(\frac{-1}{2e^{(0)^2}}\right) - \left(\frac{-1}{2e^{(a)^2}}\right)\right] = -\frac{1}{2} + \lim_{a\to-\infty} \left(\frac{1}{2e^{a^2}}\right)$

as a approaches $-\infty$, $\frac{1}{2e^{a^2}}$ goes to zero. $-\frac{1}{2} + \lim_{a\to-\infty} \underbrace{\left(\frac{1}{2e^{a^2}}\right)}_{0} = -\frac{1}{2}$

$\int_{0}^{\infty} xe^{-x^2}\,dx = \lim_{b\to\infty} \int_{0}^{b} xe^{-x^2}\,dx$ let $u = x^2$

$du = 2x\,dx$

$= \lim_{b\to\infty} \int_{0}^{b} xe^{-u}\frac{du}{2x} = \lim_{b\to\infty} \int_{0}^{b} \frac{1}{2}e^{-u}\,du$ $dx = \frac{du}{2x}$

$= \lim_{b\to\infty} -\frac{1}{2}e^{-u}\Big|_{0}^{b} = \lim_{b\to\infty} -\frac{1}{2}e^{-x^2}\Big|_{0}^{b} = \lim_{b\to\infty} \left(\frac{-1}{2e^{x^2}}\right)\Big|_{0}^{b}$

$= \lim_{b\to\infty} \left(\frac{-1}{2e^{(b)^2}} - \frac{-1}{2e^{(0)^2}}\right) = \lim_{b\to\infty} \frac{-1}{2e^{b^2}} + \frac{1}{2}$ As b approaches ∞,

$\frac{-1}{2e^{b^2}}$ goes to zero. $\lim_{b\to\infty} \underbrace{\cancel{\frac{-1}{2e^{b^2}}}}_{0} + \frac{1}{2} = \frac{1}{2}$

$\int_{-\infty}^{0} xe^{-x^2}\,dx = -\frac{1}{2}$ and $\int_{0}^{\infty} xe^{-x^2}\,dx = \frac{1}{2}$

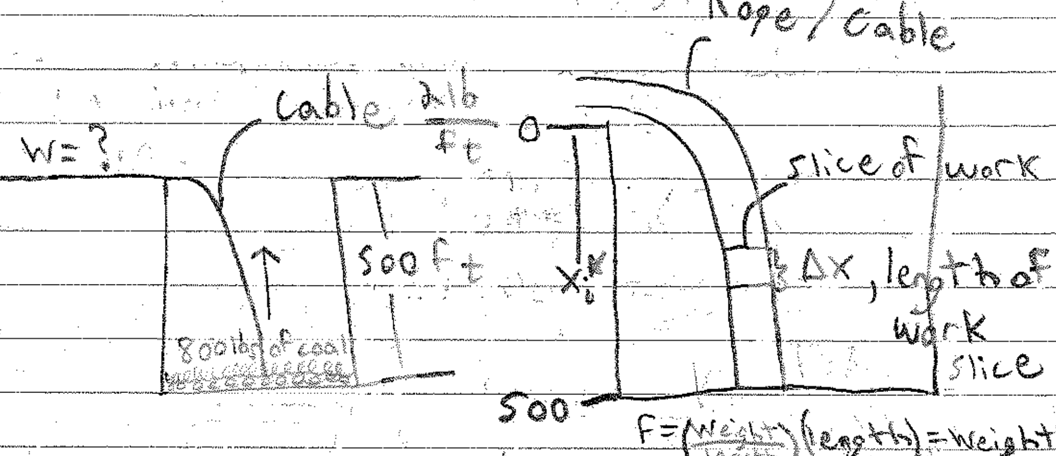
$\int_{-\infty}^{\infty} xe^{-x^2}\,dx = \int_{-\infty}^{0} xe^{-x^2}\,dx + \int_{0}^{\infty} xe^{-x^2}\,dx = -\frac{1}{2} + \frac{1}{2} = 0$

$\boxed{\int_{-\infty}^{\infty} xe^{-x^2}\,dx = 0 \;\therefore\; \text{the integral is convergent because the limit exists.}}$

<u>Conclusion:</u> The integral $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$ is convergent because the limit exists and the evaluated integral, $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$ is zero.

Lesson 15
Section 6.4
Problem #15

Picture:



Goal: Find the work done when a 2lb/ft cable lifts 800 lbs of coal out of a 500 ft mine shaft.

Setup: given	Variables	equations
• Mine shaft 500 ft deep	• Work (W)	• W = Fd (work = force × distance)
• 800 lbs of coal, what needs to be lifted	• height/distance (h)	• F = mg = weight
• Cable weighs 2lb/ft	• Force (F)	• $W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx$
• Coal needs to be lifted 500 ft.		

Mathematical Model: The work done in moving the object from a to b as the limit of $W \approx \sum_{i=1}^{n} f(x_i^*)\Delta x$ as $n \to \infty$, and the limit of the Riemann sum is a definite integral and so: $W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx$.

Work: $W = Fd$

$$F = \underbrace{(2\,\Delta x)}_{\text{force/weight of cable}} + \underbrace{800}_{\text{Force/weight of coal}} \quad d = \underbrace{x_i^*}_{\text{distance the slice of work has to move}}$$

$$\text{Force due to Cable} = \left(\frac{\text{weight}}{\cancel{\text{length}}}\right)(\cancel{\text{length}}) = \text{weight}$$

$$= \frac{2\text{lb}}{\cancel{\text{ft}}}(\Delta x \cancel{\text{ft}}) = 2\,\Delta x \text{ lbs}$$

$$\boxed{W = \lim_{n\to\infty} \sum_{i=1}^{n} (2(x_i^*) + 800)\,\Delta x = \int_0^{500} (2x + 800)\,dx}$$

$$W = \int_0^{500} (2x+800)\,dx = x^2 + 800x \Big|_0^{500}$$

$$W = (500)^2 + 800(500) - ((0)^2 + 800(0))$$

$$W = 250{,}000 + 400{,}000$$

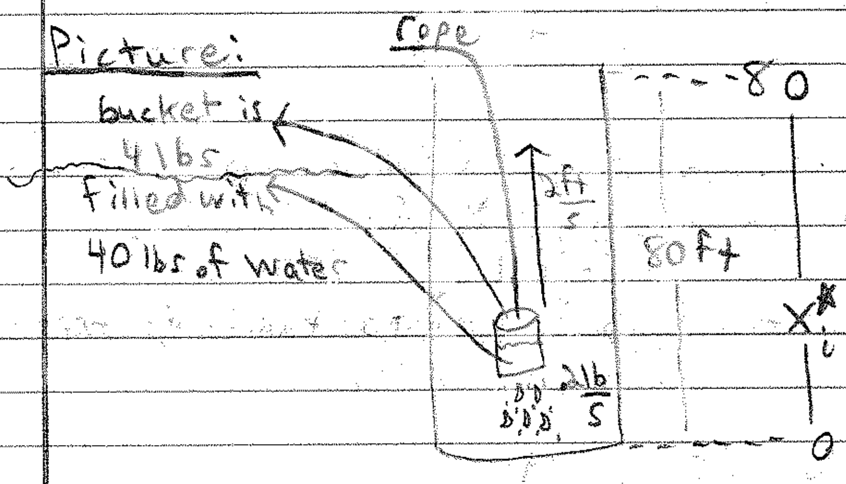
$$W = 650{,}000 \text{ ft}\cdot\text{lbs}$$

$$\boxed{W = 650{,}000 \text{ ft}\cdot\text{lbs}}$$

Conclusion: The work done when a 2 lb/ft cable lifts 800 lbs of coal out of a 500ft mine shaft is 650,000 ft·lbs.

Lesson 15
Section 6.4
Problem #16

Picture:



Goal: Find the work done in pulling a leaking bucket to the top of an 80ft well by setting up a Riemann Sum and then expressing work as a definite integral, and evaluating the work integral.

Set Up: given variables equations

- Bucket weighs 4 lbs
- Well of 80 ft (height)
- Bucket initially has 40 lbs of water
- Bucket is pulled up at a rate of $2\frac{ft}{s}$
- Water leaks out of the bucket at a rate of $0.2\frac{lb}{s}$

- X (height/position/ distance travelled)

- $W = Fd$ work = force (distance)
- $F = mg$ = weight
- $W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*) \Delta x = \int_a^b f(x)\,dx$
- $\left(\frac{\text{weight}}{\text{length}}\right)(\text{length}) = \text{Weight}$

Mathematical Model: The work done in moving the object from a to b as the limit of $W = \sum_{i=1}^{n} f(x_i^*) \Delta x$ as $n\to\infty$, and the limit of the Riemann sum is a definite integral such that,

$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*) \Delta x = \int_a^b f(x)\,dx.$

Work:

$$W = Fd$$

Force = (Weight of bucket) + (Weight of water)

Distance = x_i^* (height/position (X)) Total distance = 80 ft

Force = 4 lbs + (Initial weight of H_2O − Weight of H_2O that leaks out)

$$= 4\text{lbs} + \left(40\text{lbs} - 0.2\frac{lb}{s}\left(\frac{1s}{2ft}\right) x_i^* ft\right)$$

Force = $4\text{lbs} + \left(40\text{lbs} - \frac{.2\,lb}{2} x_i^*\right)$ x_i^* = distance travelled in feet

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*) \Delta x = \int_a^b f(x)\,dx$$

$$\boxed{W = \lim_{n\to\infty} \sum_{i=1}^{n} 4 + \left(40 - \frac{.2}{2} x_i^*\right) \Delta x = \int_0^{80} 4 + \left(40 - \frac{.2}{2} x\right) dx}$$

$$W = \int_0^{80} 4 + \left(40 - \frac{.2}{2} x\right) dx = \int_0^{80} (44 - .1x)\,dx = 44x - \frac{.1}{2}x^2 \Big|_0^{80}$$

$$= 44(80) - \frac{.1(80)^2}{2} - \left(\underbrace{44(0)}_{0} - \underbrace{\frac{.1(0)^2}{2}}_{0}\right) = 44(80) - \frac{.1(80)^2}{2} = 3520 - 320$$

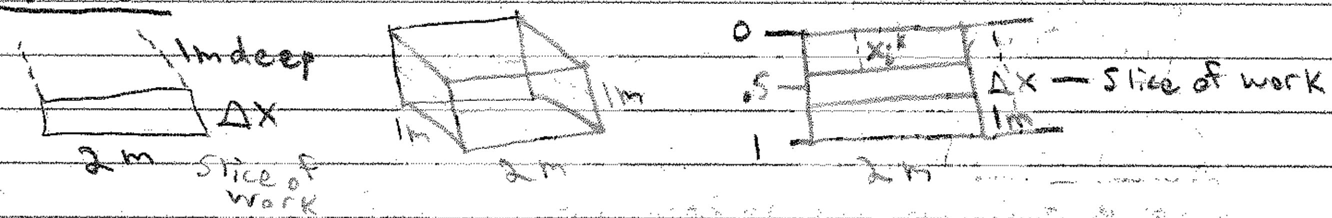
$W = 3520 - 320 = 3200$ ft·lbs

$\boxed{W = 3200 \text{ ft·lbs}}$

Conclusion: The work done in pulling a leaking bucket to the top of an 80 ft well is 3200 ft·lbs.

Lesson 15
Section 6.4
Problem #19

Picture:



Goal: Calculate the work needed to pump half of the water out of 2m long, 1m wide, and 1m deep aquarium.

Set up: given
- Dimensions of tank (1m deep, 1m wide, 2m long)
- Full of water
- Pump only half of the water out
- Density of water is $1000 \frac{kg}{m^3}$

Variables
- x (height)/(distance)

equations
- $W = Fd$ work = force × distance
- $F = ma$
- $m = dV$
- $W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)dx$
- a = min distance b = max distance
- $V = Ah$

Mathematical model: The work done in pumping water out of a tank from a to b is the limit of $W = \sum_{i=1}^{n} f(x_i^*)\Delta x$ as $n\to\infty$, and the limit of the Riemann sum is a definite integral such that,

$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)dx.$

Work: $W = Fd$

$F = ma$

$F = d(V)(a)$

$= d\,A(h)(g)$

$= 1000\frac{kg}{m^3}(l)(w)(h)(g) = 1000\frac{kg}{m^3}(2m)(\Delta x)(1m)\left(9.8\frac{m}{s^2}\right)$

$F = 19{,}600\,\Delta x\,N \qquad d = x_i^*$

$$\boxed{W = \lim_{n\to\infty}\sum_{i=1}^{n} 19{,}600(x_i^*)\,\Delta x = \int_0^{1/2} 19{,}600\,x\,dx}$$

$$W = \frac{19{,}600x^2}{2}\Big|_0^{1/2} = \frac{19{,}600}{2}\left(\tfrac{1}{2}\right)^2 - \cancel{\frac{19{,}600}{2}(0)^2}\ 0$$

$$\boxed{W = 2450\text{ J}}$$

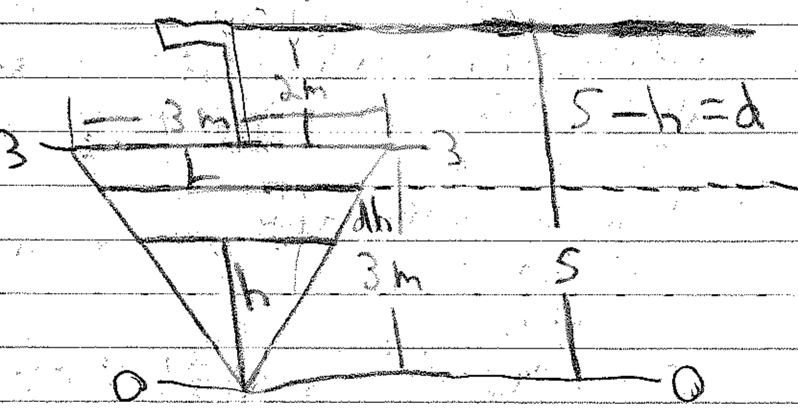
Conclusion: The work needed to pump half of the water out of the 1m deep, 1m long, 2m wide aquarium is 2,450 J.

Lesson 15

Section 6.4

Problem #21

Picture:



Goal: Find the work required to pump the water out of the spout.

Setup: given

- Tank is full of water
- Pump the water out of the spout
- Density of water is $1000\,kg/m^3$
- Shape of the tank with its dimensions
- 2m tall spout, 3m high tank, 8m deep tank, and 3m wide tank (at the maximum width)
- Triangular Prism Tank

Variables

- h (height) / (distance)

equations

- $W = Fd$ work = Force × distance
- $F = ma$ · $m = dV$
- $W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)dx$
- a = min distance b = max distance
- $V = Ah$ $A = l(w)$
- $\frac{\Delta L}{\Delta h}$ (similar triangles)

Mathematical Model: The work done in pumping water out of a tank from a to b is the limit of $W = \sum_{i=1}^{n} f(x_i^*)\Delta x$ as $n\to\infty$, and the limit of the Riemann sum is a definite integral such that,

$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)dx$.

Work: $W = Fd$

$F = ma$

$F = dVa = dAha = d(l)(w)(h)(g)$

$F = \left(1000\frac{kg}{m^3}\right)(l)\,dh\,(8m)\left(9.8\frac{m}{s^2}\right)$

By similar triangles:

$\frac{\Delta l}{\Delta h} = \frac{3-0}{3-0} = \frac{3}{3} = 1$

$\Delta l = \Delta h$

$l = h$

$F = \left(1000\frac{kg}{m^3}\right)(h)\,dh\,(8m)\left(9.8\frac{m}{s^2}\right)$

$F = 78{,}400(h)\,dh$ N

$d = 5-h$

$$W = \int_0^3 78{,}400\,h(5-h)\,dh = 78{,}400\int_0^3 (5h-h^2)\,dh$$

$$W = 78{,}400\left(\frac{5}{2}h^2 - \frac{h^3}{3}\right)\Big|_0^3 = 78{,}400\left[\left(\frac{5}{2}(3)^2 - \frac{(3)^3}{3}\right) - \left(\frac{5}{2}(0)^2 - \frac{(0)^3}{3}\right)\right]$$

$$W = 78{,}400(22.5-9) = 1058400\text{ J}$$

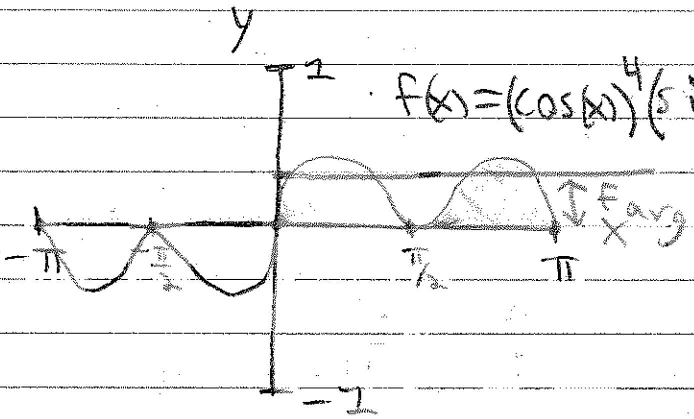
$$\boxed{W = 1{,}058{,}400\text{ J}}$$

Conclusion: The work required to pump the water out of the spout is, 1,058,400 J.

Lesson 16
Section 6.5
Problem #7

Picture:

graphed using Calc tool



Goal: Find the average value of the function $h(x) = \cos^4 x \sin x$ on the interval $[0, \pi]$.

Set up: given
- $h(x) = \cos^4 x \sin x$
- Interval $[0, \pi]$

variables
- x

equations
- $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$

Mathematical Model: The average value of f on the interval $[a,b]$ is, $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$.

Work: $h(x) = \cos^4 x \sin x \quad [0, \pi]$

$$h_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{\pi - 0}\int_0^{\pi} \cos^4 x \sin x \, dx$$

let $u = \cos x$

$du = -\sin(x)\,dx$

$dx = \frac{du}{-\sin(x)}$

$u(a) = \cos(0) = 1$

$u(b) = \cos(\pi) = -1$

$$h_{ave} = \frac{1}{\pi - 0}\int_0^{\pi} u^4 \cancel{\sin x}\,\frac{du}{-\cancel{\sin(x)}} = \frac{-1}{\pi}\int_1^{-1} u^4\,du$$

$$h_{ave} = \frac{-1}{\pi}\left(\frac{u^5}{5}\right)\Big|_1^{-1} = \frac{-1}{\pi}\left(\frac{(-1)^5}{5} - \frac{(1)^5}{5}\right)$$

$$h_{ave} = \frac{-1}{\pi}\left(\frac{-1}{5} - \frac{1}{5}\right) = \frac{-1}{\pi}\left(\frac{-2}{5}\right) = \frac{2}{5\pi}$$

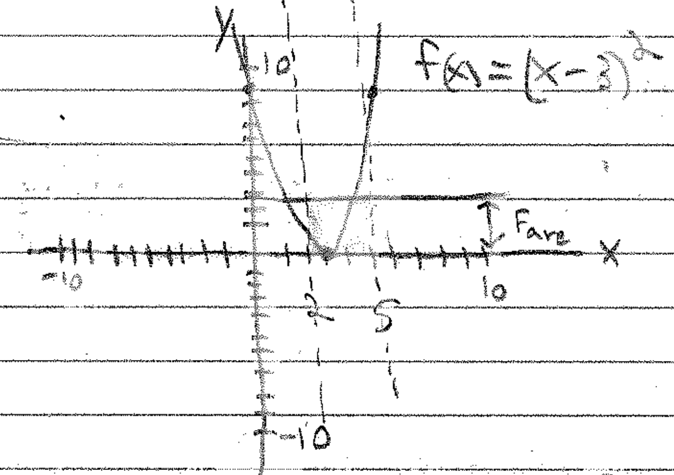
$$\boxed{h_{ave} = \frac{2}{5\pi}}$$

Conclusion: The average value of $h(x) = \cos^4 x \sin x$ on the interval $[0, \pi]$ is $\frac{2}{5\pi}$.

Lesson 16
Section 6.5
Problem #9

Picture:

graphed using Calc Tool



Goal: Find the average value of $f(x) = (x-3)^2$ on the interval $[2, 5]$. Calculate the value of c such that, $f(c) = f_{ave}$. Finally, sketch a graph of $f(x) = (x-3)^2$ and a rectangle whose area is the same as the area under the graph of $f(x) = (x-3)^2$.

Set up: given
- $f(x) = (x-3)^2$
- Interval $[2, 5]$
- $f_{ave} = f(c)$

variables
- x
- c

equations
- $f_{ave} = \frac{1}{b-a} \int_a^b f(x)\,dx$
- $f(c) = f_{ave} = \frac{1}{b-a} \int_a^b f(x)\,dx$
- $\int_a^b f(x)\,dx = f(c)(b-a)$

Mathematical Model: The mean value theorem for integrals if f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that $f(c) = f_{ave} = \frac{1}{b-a} \int_a^b f(x)\,dx$ that is,

$\int_a^b f(x)\,dx = f(c)(b-a)$.

Work:

$f(x) = (x-3)^2 \quad , [2,5]$

a) $f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{5-2}\int_2^5 (x-3)^2 dx$

$f_{ave} = \frac{1}{3}\left(\frac{(x-3)^3}{3}\right)\Big|_2^5 = \frac{1}{3}\left(\frac{((5)-3)^3}{3} - \frac{((2)-3)^3}{3}\right) = \frac{1}{3}\left(\frac{(2)^3}{3} - \frac{(-1)^3}{3}\right)$

$f_{ave} = \frac{1}{3}\left(\frac{8}{3} - \frac{-1}{3}\right) = \frac{1}{3}\left(\frac{8}{3}+\frac{1}{3}\right) = \frac{1}{3}\left(\frac{9}{3}\right) = \frac{1}{3}(3) = 1$

$\boxed{f_{ave} = 1}$

b) $f_{ave} = f(c) \quad 1 = (c-3)^2 \quad \sqrt{1} = \sqrt{(c-3)^2}$

$\pm 1 = c-3$

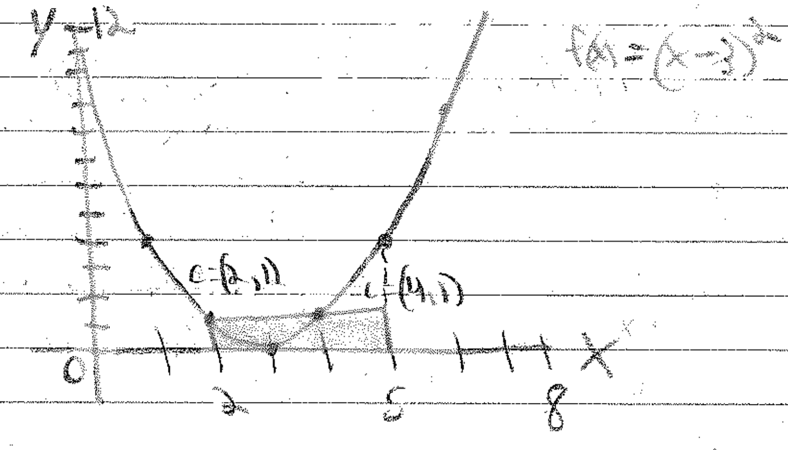
$1 = c-3 \qquad -1 = c-3$

$+3 \quad +3 \qquad +3 \quad +3$

$c = 4 \qquad c = 2$

$\boxed{c = 4 \text{ or } c = 2}$

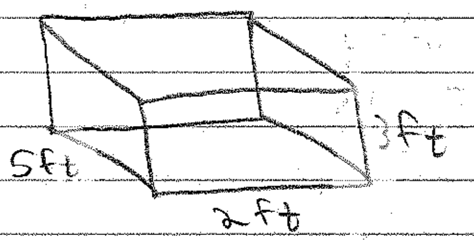
c)



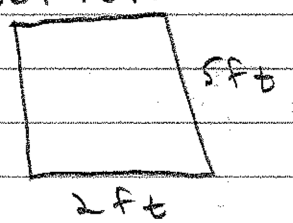
Conclusion: The average value of $f(x) = (x-3)^2$ on the interval $[2,5]$ is 1. The value of c, such that $f_{ave} = f(c)$ is $c = 4$ or $c = 2$. The area under the graph $f(x) = (x-3)^2$ on the interval $[2,5]$ has the same area of a rectangle whose area is 3 square units.

Lesson 17 and 18
Section 8.3
Problem # 1

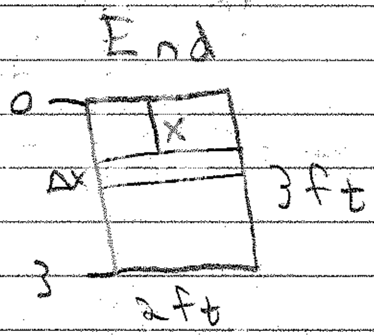
Picture:



Bottom



End



Goal: Find the hydrostatic pressure on the bottom of the aquarium, and the hydrostatic force on the bottom. Additionally, find the hydrostatic force on one end of the aquarium.

Set up: given
- Dimensions of aquarium (5ft long, 2ft wide, 3ft deep)
- Aquarium is full of water

Variables
- x (depth/distance)

equations.
- $F = mg = \rho g A d$
- $V = Ad$
- $m = \rho V = \rho A d$
- $P = \frac{F}{A} = \rho g d$
- $F = PA$
- $P = \rho g d = \delta d$

Mathematical Model: The force exerted by the fluid on the plate is ~~therefore~~, $F = mg = \rho g A d$, where g is the acceleration due to gravity. The pressure P on the plate is defined to be the force per unit area: $P = \frac{F}{A} = \rho g d$. Adding the forces and taking the limit as $n \to \infty$, the total hydrostatic force is obtained.

Work:

a) $P = \frac{F}{A} = \rho g d = \delta d \quad P = \delta d = \left(62.5 \frac{lb}{ft^{3}}\right)(3\,ft) = 187.5 \frac{lb}{ft^2}$

$$\boxed{P = 187.5 \frac{lb}{ft^2}}$$

b) $F = mg = \rho V g = \rho A d g = \delta A d = \delta (l)(w)(d)$

$F = \left(62.5 \frac{lb}{ft^3}\right)(5\ ft)(2\ ft)(3\ ft) = 1875\ lbs$

$\boxed{F = 1{,}875\ lbs}$

c) $F = PA = \delta d (l)(w) = \left(62.5 \frac{lb}{ft^3}\right)(x)(2)\Delta x$

$F = \int_0^3 (62.5)(2)\, x\, dx = \int_0^3 125x\, dx = 125\int_0^3 x\, dx$

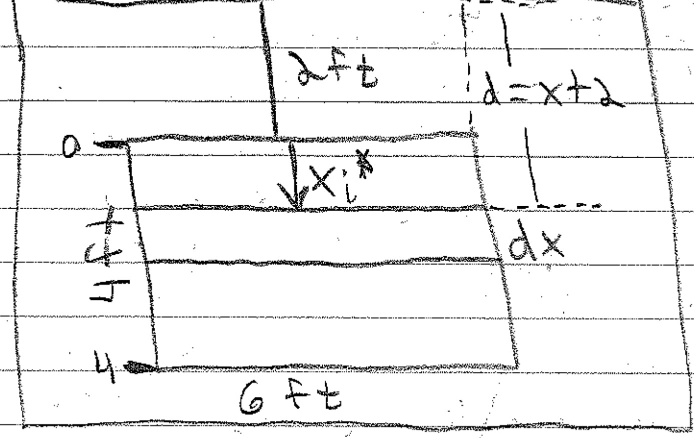
$F = 125\left(\frac{x^2}{2}\right)\Big|_0^3 = 125\left(\frac{(3)^2}{2} - \cancel{\frac{(0)^2}{2}}_{0}\right) = 125\left(\frac{9}{2}\right) = 562.5$

$\boxed{F = 562.5\ lbs}$

Conclusion: The hydrostatic pressure on the bottom of the aquarium is $187.5 \frac{lbs}{ft^2}$. The hydrostatic force on the bottom of the aquarium is 1,875 lbs. The hydrostatic force on one end of the aquarium is 562.5 lbs.

Lesson 17 and 18
Section 8.3
Problem # 3

Picture:



Goal: Determine how to calculate the hydrostatic force against one side of a vertical plate submerged in water by a Riemann sum. Then express the force as an integral and evaluate it.

Set up: given	Variables	equations
• Vertical plate	• x (depth)	• $F = mg = \rho g A d$ • $V = Ad$
• Shape of plate and dimensions (rectangle, 4 ft by 6 ft)		• $m = \rho V = \rho A d$ • $P = \frac{F}{A} = \rho g d = \delta d$
• Plate is submerged 2 ft underwater		• $F = PA$ • $F = \delta A d$
		• $F = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x = \int_a^b f(x) dx$

Mathematical Model: The force exerted by the fluid on the plate is: $F = mg = \rho g A d = \delta A d$, where g is the acceleration due to gravity. The pressure P on the plate is defined to be the force per unit area: $P = F/A = \rho g d$. Adding the forces and taking the limit as $n \to \infty$, the total hydrostatic force is obtained.

Work: $F = mg = \rho V g = \rho A d g = \rho g A d = \delta A d$

$$F = \delta A d = \left(62.5 \frac{lb}{ft^3}\right)(l)(w)(d) = 62.5 \frac{lb}{ft^3}(6ft)\Delta x (x+2)$$

$$F = 62.5(6)(x+2)\Delta x$$

$$\boxed{F = \lim_{n \to \infty} \sum_{i=1}^{n} (62.5)(6)(x_i^* + 2)\Delta x = \int_0^4 (62.5)(6)(x+2)dx}$$

$$F = \int_0^4 375(x+2)dx = \int_0^4 375x + 750\, dx = \left(\frac{375x^2}{2} + 750x\right)\Big|_0^4$$

$$F = \frac{375(4)^2}{2} + 750(4) - \left(\underbrace{\frac{375(0)^2}{2}}_{0} + \underbrace{750(0)}_{0}\right) = 3000 + 3000 = 6000$$

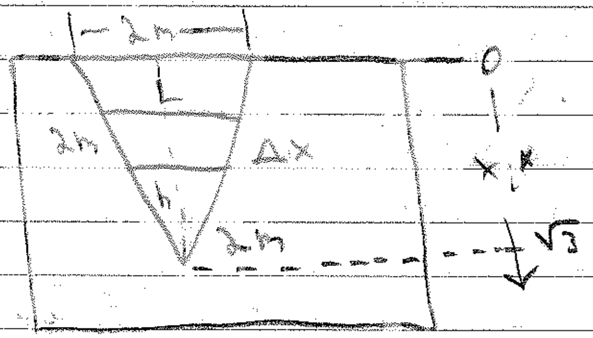
$$\boxed{F = 6{,}000 \text{ lbs}}$$

Conclusion: The hydrostatic force can be approximated by a Riemann sum by setting up a vertical axis with $x = 0$ at the

start of the shape's surface and x is the depth as it increases in the downward direction. The area of the slice of pressure is l(w), which is $6\Delta x$. The pressure on the slice of pressure is $\delta(d)$, which is $62.5\ \frac{lb}{ft^3}(x_i^*+2)$. The hydrostatic force on the strip is
$P(A) = 62.5(x_i^*+2)(6\Delta x)$, and the total hydrostatic force is the Riemann sum. The total hydrostatic force is 6,000 lbs.

Lesson 17 and 18
Section 8.3
Problem #7

Picture:



Goal: Determine how to calculate the hydrostatic force against one side of a vertical plate submerged in water by a Riemann sum. Then express the force as an integral and evaluate it.

Set up: given | Variables | equations

- vertical plate
- Shape of plate and dimensions (triangle, base of 2ft, side lengths of 2ft.)
- Plate is submerged at water level.

Variables:
- x (depth)

equations:
- $F = mg = \rho g A d$ • $V = Ad$ • $m = \rho V = \rho A d$
- $P = \frac{F}{A} = \rho g d = \delta d$ • $F = PA = \delta A d$
- $F = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx$

Mathematical Model: The force exerted by the fluid on the plate is: $F = mg = \rho g A d = \delta A d$, where g is the acceleration due to gravity. The pressure P on the plate is defined to be the force per unit area: $P = F/A = \rho g d$. Adding the forces and taking the limit as $n\to\infty$, the total hydrostatic force is obtained.

Work: $F = mg = \rho V g = \rho A d g$ $\quad F = \rho g A d = \rho g\, l(w)\, d$

$F = (1000\, kg/m^3)(9.8\, \frac{m}{s^2})(l)\, \Delta x\, (x_i^*)$ $\qquad A = l(w)$

By similar Triangles:

$F = (1000)(9.8)\left(\frac{2\sqrt{3} - 2x_i^*}{\sqrt{3}}\right)\Delta x\, (x_i^*)$ $\qquad \frac{2}{L} = \frac{\sqrt{3}}{\sqrt{3} - x_i^*}$ $\qquad 2 = \left(\frac{\sqrt{3}}{\sqrt{3} - x_i^*}\right)L$

$F = 9{,}800\left(\frac{2\sqrt{3}}{\sqrt{3}} - \frac{2x_i^*}{\sqrt{3}}\right)\Delta x\, (x_i^*)$

$F = 9{,}800\left(2 - \frac{2x_i^*}{\sqrt{3}}\right)\Delta x\, (x_i^*)$ $\qquad L = 2\left(\frac{\sqrt{3} - x_i^*}{\sqrt{3}}\right) = \frac{2\sqrt{3} - 2x_i^*}{\sqrt{3}}$

$$\boxed{F = \lim_{n \to \infty} \sum_{i=1}^{n} (9{,}800)\left(2 - \frac{2x_i^*}{\sqrt{3}}\right)(x_i^*)\,\Delta x = \int_0^{\sqrt{3}} (9{,}800)(x)\left(2 - \frac{2x}{\sqrt{3}}\right)dx}$$

$$F = 9{,}800 \int_0^{\sqrt{3}} \left(2x - \frac{2}{\sqrt{3}}x^2\right)dx = 9{,}800\left(x^2 - \frac{2}{3\sqrt{3}}x^3\right)\Big|_0^{\sqrt{3}}$$

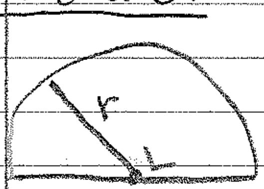
$$F = 9{,}800\left((\sqrt{3})^2 - \frac{2}{3\sqrt{3}}(\sqrt{3})^3\right) = 9{,}800\left(3 - \frac{2}{3}(3)\right) = 9{,}800(3-2) = 9{,}800(1)$$

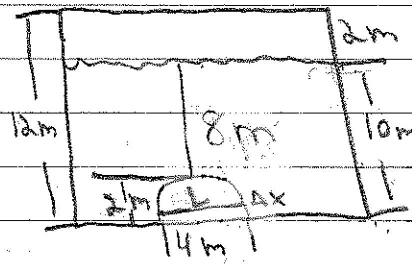
$\boxed{F = 9{,}800\, N}$

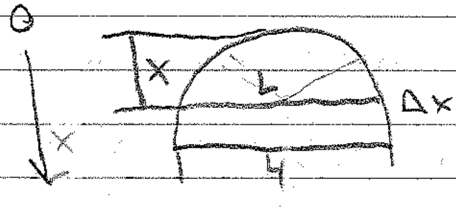
Conclusion: The hydrostatic force can be approximated by a Riemann sum by setting up a vertical axis with $x=0$ at the start of the shape's surface and x_i^* is the depth as it increases downward. The area of the slice of pressure is $\left(2 - \frac{2}{\sqrt{3}}x_i^*\right)\Delta x$. The pressure is $\rho g d$, which is $9{,}800\, x_i^*$. The hydrostatic force on the slice is $9{,}800\left(2 - \frac{2}{\sqrt{3}}x_i^*\right)(x_i^*)\Delta x$. The total hydrostatic force is the Riemann sum. The total hydrostatic force is 9,800 N.

Lesson 17 and 18
Section 8.3
Problem #14

Picture:







Goal: Find the hydrostatic force against the semicicrcular gate.

Set up: given | Variables | equations

• shape of gate (semicircular)
• Diameter of circle (4 m)
• The depth the gate is submerged (10 m) under water

• x (depth)

• $F = mg = \rho g A d$ • $V = Ad$ • $m = \rho V = \rho A d$
• $P = F/A = \rho g d = \delta d$ • $F = PA = \delta A d$

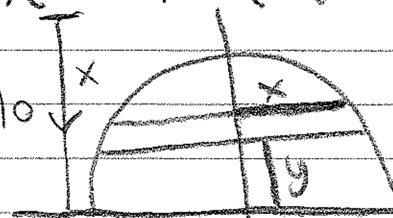
Mathematical Model: The force exerted by the fluid on the plate is: $F = mg = \rho g A d = \delta A d$, where g is the acceleration due to gravity. The pressure P on the plate is defined to be the force per unit area: $P = F/A = \rho g d$. Adding the forces and taking the limit as $n \to \infty$, the total hydrostatic force is obtained.

Work: $F = mg = \rho V g = \rho g A d = \rho g (l)(w) d$

$F = \left(1000 \frac{kg}{m^3}\right)\left(9.8 \frac{m}{s^2}\right)(l)(\Delta x)(x) = 9{,}800(x)(l)\,dx$

$r^2 = x^2 + y^2$

$x = \sqrt{r^2 - y^2}$



$l = 2x$

$y = 10 - x$

$l = 2x = 2\sqrt{r^2 - y^2}$

$l = 2\sqrt{(2)^2 - (10 - x)^2}$

$$l = 2\sqrt{(2)^2 - (10-x)^2} = 2\sqrt{4-(10-x)^2}$$

$$F = 9{,}800\, x\,(l)\, dx = 9{,}800(x)\left(2\sqrt{4-(10-x)^2}\right) dx$$

$$F = \int_8^{10} 9{,}800(x)\left(2\sqrt{4-(10-x)^2}\right) dx \qquad (10-x)(10-x)$$

$$x^2 - 20x + 100$$

$$F = \int_8^{10} 9{,}800(x)\left(2\sqrt{4-x^2+20x-100}\right) dx$$

$$\boxed{F = \int_8^{10} 9{,}800(x)\left(2\sqrt{-x^2+20x-96}\right) dx}$$

Conclusion: The hydrostatic force is

$$\int_8^{10} 9{,}800(x)\left(2\sqrt{-x^2+20x-96}\right) dx.$$