3 October 2014 Section 7.1 Homework

5) Evaluate the integral

$\int te^{-3t}\,dt$

$f(x)g(x) = \int f(x)g'(x) + f'(x)g(x)\,dx$

$f(x)g(x) = \int f(x)g'(x)dx + \int f'(x)g(x)\,dx$

$f(x)g(x) - \int f'(x)g(x)\,dx = \int f(x)g'(x)\,dx$

$\int u\,dv = uv - \int v\,du$

$\int te^{-3t}\,dt$

$u = t \qquad v = -\frac{1}{3}e^{-3t}$

$du = dt \qquad dv = e^{-3t}$

$t\left(-\frac{1}{3}e^{-3t}\right) - \int -\frac{1}{3}e^{-3t}\,dt$

$t\left(-\frac{1}{3}e^{-3t}\right) + \frac{1}{3}\int e^{-3t}\,dt$

$t\left(-\frac{1}{3}e^{-3t}\right) + \frac{1}{3}\left(-\frac{1}{3}\right)e^{-3t} + C$

$t\left(-\frac{1}{3}e^{-3t}\right) - \frac{1}{9}e^{-3t} + C$

$\boxed{\left(-\frac{1}{3}\right)te^{-3t} - \left(\frac{1}{9}\right)e^{-3t} + C}$

67) A particle that moves along a straight line has velocity $v(t) = t^2 e^{-t}$ meters per second after t seconds. How far will it travel during the first t seconds?

$f(x)g(x) = \int f(x)g'(x) + f'(x)g(x)\,dx$

$f(x)g(x) = \int f(x)g'(x)\,dx + \int f'(x)g(x)\,dx$

$f(x)g(x) - \int f'(x)g(x)\,dx = \int f(x)g'(x)\,dx$

$\int u\,dv = uv - \int v\,du$

$v(t) = x'(t)$

$v(t) = t^2 e^{-t}$

$x(t) = \int_0^t v(z)\,dz$

$x(t) = \int_0^t z^2 e^{-z}\,dz$

$u = z^2 \qquad v = -e^{-z}$

$du = 2z\,dz \qquad dv = e^{-z}$

$\left[-z^2 e^{-z}\right]_0^t - \int -e^{-z}\,2z\,dz$

$\left[-z^2 e^{-z}\right]_0^t - 2\int -e^{-z}\,z\,dz$

$\left\{-t^2 e^{-t} - \left((0)^2 e^{-0}\right) = -t^2 e^{-t}\right\}$

$-t^2 e^{-t} + 2\int_0^t e^{-z}\,z\,dz$

$u = z \qquad v = -e^{-z}$

$du = dz \qquad dv = e^{-z}$

$-t^2 e^{-t} + 2\left(\left[-z e^{-z}\right]_0^t + \int_0^t e^{-z}\,dz\right)$

$-t^2 e^{-t} + 2\left(-te^{-t} - (0) + \left(-e^{-t} + 1\right)\right)$

$-t^2 e^{-t} - 2te^{-t} - 2e^{-t} + 2 = \boxed{2 - e^{-t}\left(t^2 + 2t + 2\right)\,m}$

3 October 2014 Section 7.4 Homework

1) Write out the form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

(a) $\dfrac{1+6x}{(4x-3)(2x+5)}$ (b) $\dfrac{10}{5x^2-2x^3}$

$$F(x)=\frac{P(x)}{Q(x)}=S(x)+\frac{R(x)}{Q(x)} \qquad \frac{A}{(ax+b)^i} \qquad \frac{Ax+B}{(ax^2+bx+c)}$$

(a) $\dfrac{1+6x}{(4x-3)(2x+5)}=\dfrac{A}{(4x-3)}+\dfrac{B}{(2x+5)}$

$$\boxed{1+6x=\frac{A}{(4x-3)}+\frac{B}{(2x+5)}}$$

(b) $\dfrac{10}{5x^2-2x^3}=\dfrac{10}{x^2(5-2x)}$

$$=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(5-2x)}$$

$$\boxed{10=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(5-2x)}}$$

(7) Evaluate the integral.

$$\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy$$

$$F(x) = \frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)} \qquad \frac{A}{(ax+b)^2} \qquad \frac{Ax+B}{(ax^2+bx+c)}$$

$$\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy = \frac{A}{y} + \frac{B}{(y+2)} + \frac{C}{(y-3)}$$

$$4y^2-7y-12 = A(y+2)(y-3) + B(y)(y-3) + C(y)(y+2)$$

$$= A(y^2-y-6) + B(y^2-3y) + C(y^2+2y)$$

$$y^2(A+B+C) \quad y(-A-3B+2C) \quad y^0(-6A)$$

$4 = A+B+C$

$-7 = -A-3B+2C$

$-12 = -6A \rightarrow A=2$

$4 = 2+B+C$ $\qquad B+\left(\frac{1}{5}\right) = 2$

$B+C = 2 \rightarrow B = 2-C$ $\qquad B = 2-\frac{1}{5} \rightarrow B = \frac{10}{5}-\frac{1}{5}$

$-7 = -(2)-3(2-C)+2C$ $\qquad B = \frac{9}{5}$

$-5 = -6+3C+2C \rightarrow 1 = 5C$

$C = \frac{1}{5}$

$$\int_1^2 \frac{2}{y} + \frac{9}{5(y+2)} + \frac{1}{5(y-3)}\,dy$$

$$= \left[2\ln|y| + \left(\tfrac{9}{5}\right)\ln|y+2| + \left(\tfrac{1}{5}\right)\ln|y-3|\right]_1^2$$

$$= \left[2\ln|2| + \left(\tfrac{9}{5}\right)\ln|4| + \left(\tfrac{1}{5}\right) + \ln|-1| - \left(2\ln|1| + \left(\tfrac{9}{5}\right)\ln|3| + \left(\tfrac{1}{5}\right)\ln|-2|\right)\right]$$

$$= \left[2\ln(2) + \left(\tfrac{9}{5}\right)\ln(4) + \left(\tfrac{1}{5}\right)(0) - 2(0) - \left(\tfrac{9}{5}\right)\ln(3) - \left(\tfrac{1}{5}\right)\ln(2)\right]$$

$$= \left[2\ln(2) + \left(\tfrac{9}{5}\right)\ln(4) - \left(\tfrac{9}{5}\right)\ln(3) - \left(\tfrac{1}{5}\right)\ln(2)\right]$$

$$= 2\ln(2) + \frac{9}{5}\left(\ln(4) - \ln(3)\right) - \left(\frac{1}{5}\right)\ln(2)$$

$$= \ln(2)\left(2-\frac{1}{5}\right) + \frac{9}{5}\left(\ln(4)-\ln(3)\right)$$

$$= \frac{9}{5}\left(\ln(2)+\ln(4)-\ln(3)\right) = \frac{9}{5}\ln\frac{(2)\times(4)}{(3)} = \boxed{\frac{9}{5}\ln\left(\frac{8}{3}\right)}$$

19) Evaluate the integral

$$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx$$

$$f(x)=\frac{P(x)}{Q(x)}=S(x)+\frac{R(x)}{Q(x)} \qquad \frac{A}{(ax+b)^i} \qquad \frac{Ax+B}{(ax^2+bx+c)}$$

$$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \frac{A}{(x-3)}+\frac{B}{(x-2)}+\frac{C}{(x-2)^2}\left[(x-3)(x-2)^2\right]$$

$$x^2+1 = A(x-2)^2+B(x-3)(x-2)+C(x-3)$$

$$= A(x-2)(x-2)+B(x-3)(x-2)+C(x-3)$$

$$= A(x^2-4x+4)+B(x^2-5x+6)+C(x-3)$$

$$= Ax^2-4Ax+4A+Bx^2-5Bx+6B+Cx-3C$$

$$= x^2(A+B)+x(-4A-5B+C)+(4A+6B-3C)$$

$$1=A+B$$

$$0=-4A-5B+C$$

$$1=4A+6B-3C$$

$1=A+B \rightarrow A=1-B$ $\qquad$ $1=4(1-B)+6B-3C$

$0=-4(1-B)-5B+C$ $\qquad$ $1=4-4B+6B-3C$

$0=-4+4B-5B+C$ $\qquad$ $1=4+2B-3C$

$4-C=4B-5B$ $\qquad$ $1=4+2(C-4)-3C$

$4-C=-B \rightarrow B=-4+C$ $\qquad$ $1=4+2C-8-3C \rightarrow 1=-4-C$

$B=C-4$ $\qquad$ $-C=5 \rightarrow C=-5$

$B=(-5)-4 \rightarrow B=-9$

$A=1+9 \rightarrow A=10$

$$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx=\int \frac{10}{(x-3)}-\frac{9}{(x-2)}-\frac{5}{(x-2)^2}\,dx$$

$$=10\ln|(x-3)|-9\ln|(x-2)|-5(-1)(x-2)^{-1}+C$$

$$=\boxed{10\ln|(x-3)|-9\ln|(x-2)|+\frac{5}{(x-2)}+C}$$

3 October 2014 Section 7.8 Homework

5) Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx$$

$$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx \qquad \int_{-\infty}^{b} f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx$$

$$\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx = \lim_{t\to\infty}\int_3^t \frac{1}{(x-2)^{3/2}}\,dx$$

$$\lim_{t\to\infty}\int_3^t (x-2)^{-3/2}\,dx$$

$$\lim_{t\to\infty}\left[-2(x-2)^{-1/2}\right]_3^t$$

$$\lim_{t\to\infty}\left[-2(t-2)^{-1/2} - (-2(3-2)^{-1/2})\right]$$

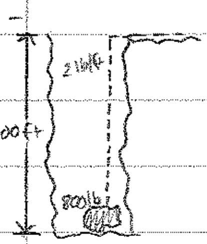
$$\lim_{t\to\infty}\left[\frac{-2}{\sqrt{t-2}} + \frac{2}{\sqrt{1}}\right]$$

$$\boxed{\lim_{t\to\infty}\left[\frac{-2}{\sqrt{t-2}} + 2\right] = 0+2 \rightarrow \text{Convergent}}$$

3 October 2014 Section 6.4 Homework

15) A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mine shaft 500 ft deep. Find the work done.

Work = Force × Distance   Force = Mass × Acceleration



$W = FD$

$D = dh$

$F = ma$

$a = N/A$

$m = (2\,lb/ft)(500 - h) + 800 = 1000 - 2h + 800$

$= 1{,}800 - 2h$

$$W = \int_0^{500} 1{,}800 - 2h\, dh$$

$$= \left[1{,}800h - \frac{2h^2}{2}\right]_0^{500}$$

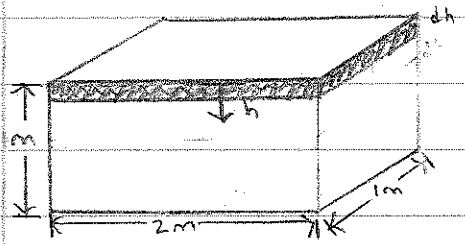
$$= \left[1{,}800h - h^2\right]_0^{500}$$

$$= 900{,}000 - 250{,}000 = \boxed{650{,}000 \text{ ft-lb}}$$

The work required to lift 800 lb of coal up a 500 ft deep mine shaft using a 2 lb/ft is 650,000 ft-lb. This value is represented by the integral $W = \int_0^{500} 1{,}800 - 2h\, dh$

19) An aquarium 2m long, 1m wide, and 1m deep is full of water. Find the work needed to pump half of the water out of the aquarium. (use the fact that the density of water is 1000 kg/m^3)

Work = Force x Distance   Force = mass x Acceleration



$w = FD$

$D = h$

$F = ma$

$a = 9.81 m/s^2 = g$

$m = \rho V = \rho(1)(2)(dh)$

$$w = \int_0^{1/2} \rho(1)(2)\, gh\, dh$$

$$w = 2\rho g \int_0^{1/2} h\, dh$$

$$= 2\rho g \left[\frac{h^2}{2}\right]_0^{1/2}$$

$$= 2\rho g \left[\frac{1}{8} - 0\right]$$

$$= 2m(1{,}000\ kg/m^3)(9.81 m/s^2)\left[\frac{1}{8}\right]$$

$$= \boxed{2450\ J}$$

The work required to pump half of the water out of a full 1m x 2m x 1m aquarium is equal to 2,450 J and is represented by integral expression: $w = 2\rho g \int_0^{1/2} h\, dh$.

3 October 2014 Section 6.5 Homework

7) Find the average value of the function on the given integral.

$h(x) = \cos^4 x \sin x \quad [0, \pi]$

Average value $= \frac{1}{b-a} \int_a^b f(x)\,dx$

$= \frac{1}{\pi} \int_0^{\pi} \cos^4(x) \sin(x)\,dx$ $\quad u = \cos(x)$

$du = -\sin(x)\,dx$

$dx = \frac{du}{-\sin(x)}$

$\frac{1}{\pi} \int_0^{\pi} (u)^4 \cancel{\sin(x)} \frac{du}{-\cancel{\sin(x)}}$

$-\frac{1}{\pi} \int_1^{-1} (u)^4\,du$

$-\frac{1}{\pi} (-) \left[\frac{u^5}{5} \right]_{-1}^{1}$

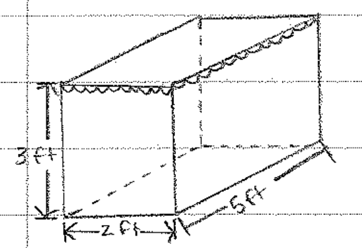
$\frac{1}{\pi} (-) \left[\frac{1}{5} - \frac{(-1)}{5} \right]$

$\frac{1}{\pi} \left[\frac{1}{5} + \frac{1}{5} \right]$

$\frac{1}{\pi} \left[\frac{2}{5} \right] = \boxed{\frac{2}{5\pi}}$

7 October 2014 Section 8.3 Homework

1) An aquarium 5ft long, 2ft wide, and 3ft deep is full of water. Find (a) the hydrostatic pressure on the bottom of the aquarium, (b) the hydrostatic force on the bottom, and (c) the hydrostatic force on one end of the aquarium.



$P = \frac{F}{A} = \rho g h$

$F = mg = \rho g A d$

$P = \rho g h$

$\rho g = 62.5\,lb/ft^3$

$h = 3$

(a) $P = 62.5\,lb/ft^3\,(3\,ft)$

$= \boxed{187.5\,lb/ft^2}$

(b) $F = PA$

$P = 187.5\,lb/ft^2$

$A = 2ft(5ft)$

$F = (187.5\,lb/ft^2)(2ft)(5ft) = \boxed{1{,}875\,lb}$

(c) $F = PA$

$P = 62.5(3-y)$

$A = 2dy$

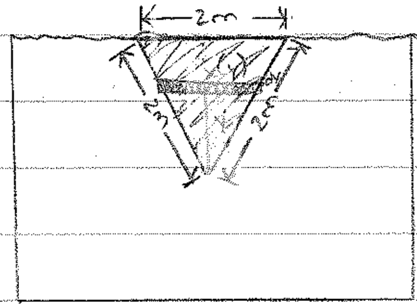
$\int_0^3 62.5(3-y)(2)\,dy = 2(62.5)\int_0^3 (3-y)\,dy$

$= 125\left[3y - \frac{y^2}{2}\right]_0^3$

$= 125\left[9 - \frac{9}{2}\right]$

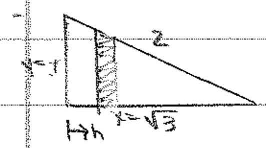
$= 125\left[\frac{18}{2} - \frac{9}{2}\right] = 125\left[\frac{9}{2}\right] = \boxed{562.5\,lb}$

7) A vertical plate is submerged (or partially) in water and has the indicated shape. Explain how to approximate the hydrostatic force against one side of the plate by a Riemann sum. Then express the force as an integral and evaluate it.



$F = PA$ $\rho g = 62.5\ lb/ft^3$

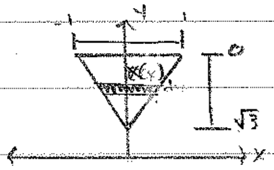
$P = \frac{F}{A} = \rho g h$ $g = 32.2\ ft/s^2$ or $9.8\ m/s^2$



$1^2 + b^2 = 4$

$b^2 = 4 - 1 \Rightarrow b^2 = 3$

$b = \sqrt{3}$



$y(x) = \frac{-1}{\sqrt{3}}x + 2$

$y(2) = \frac{-1}{\sqrt{3}}(2) + 2$

$= \frac{-2}{\sqrt{3}} + 2 = 2 - \frac{2}{\sqrt{3}}$

$A = y(x)\,dx$

$= 2 - \frac{2}{\sqrt{3}}x\,dx$

$F = PA$

$P = \rho g h = 9{,}800(x)$

$A = 2 - \frac{2}{\sqrt{3}}\,dx$

$F = \int_0^{\sqrt{3}} 9{,}800(x)\left(2 - \frac{2}{\sqrt{3}}\right)dx$

$F = 9{,}800\int_0^{\sqrt{3}} \left(2x - \frac{2}{\sqrt{3}}x\right)dx$

$= 9{,}800\left[\frac{2x^2}{2} - \frac{2x^3}{3\sqrt{3}}\right]_0^{\sqrt{3}} = 9{,}800\left[(3) - \frac{2(\sqrt{3})^3}{3\sqrt{3}} - 0\right]$

$= 9{,}800[3 - 2] = \boxed{9{,}800\ N}$