Board Work

- Write out form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

  1) $\frac{1+6x}{(4x-3)(2x+5)}$

  2) $\frac{10}{5x^2-2x^3}$

  3) $\frac{x}{x^2+x-2}$

  4) $\frac{x^2}{x^2+x-2}$

- Evaluate the integral

  5) $\int \frac{x^4}{x-1}\,dx$

  6) $\int \frac{3t-2}{t+1}\,dt$

  7) $\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$

  8) $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy$

  9) $\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx$

- Find the volume of the resulting solid if the region under the curve $y = \frac{1}{x^2+3x+2}$ from $x = 0$ to $x = 1$ is rotated about the x-axis.

T-14 Boardwork

1) $\frac{1+6x}{(4x-3)(2x+5)} = \frac{A}{4x-3} + \frac{B}{2x+5}$

$1+6x = A(2x+5) + B(4x-3)$

$1+6x = x(2A+4B) + (5A-3B)$

$2(5A-3B = 1)$ $\quad$ $4(5A-3B=1)$

$-5(2A+4B = 6)$ $\quad$ $+3(2A+4B=6)$

$(-6-20)B = 2-30$ $\quad$ $A(20+6) = 4+18 = 22$

$-26B = -28$ $\quad$ $A = \frac{22}{26} = \frac{11}{13}$

$\boxed{B = \frac{14}{13} \quad A = \frac{11}{13}}$

2) $\frac{10}{x^2(5-2x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{5-2x}$

$10 = Ax(5-2x) + B(5-2x) + Cx^2$

$10 = x^2(-2A+C) + x(5A-2B) + (5B)$

$5B = 10$ $\quad$ $\boxed{B = 2}$

$5A-2B = 5A-4 = 0$, $\quad$ $\boxed{A = \frac{4}{5}}$

$-2A+C = -2(\frac{4}{5})+C = 0$, $\quad$ $\boxed{C = -\frac{8}{5}}$

3) $\frac{x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$

$x = A(x-1) + B(x+2) = x(A+B) + (-A+2B)$

$-A+2B = 0$, $\quad A = 2B$

$A+B = 1 = 3B$ $\quad \boxed{B = \frac{1}{3}, \quad A = \frac{2}{3}}$

4) $x^2+x-2 \overline{)\, x^2}$ $\quad$ 1

$-(x^2+x-2)$

$-x+2$

$\frac{-x+2}{x^2+x-2} = 1 + \frac{2-x}{x^2+x-2} = 1 + \frac{2-x}{(x+2)(x-1)}$

$\frac{2-x}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)} = \frac{A(x-1)+B(x+2)}{(x+2)(x-1)}$

$2-x = x(A+B) + (-A+2B)$

$A+B = -1$ $\quad -A = B+1$

$-A+2B = 2$ $\quad (B+1)+2B = 2$

$3B = 1$

$\boxed{B = \frac{1}{3} \quad A = -\frac{4}{3}}$

5) $\int \frac{x^4}{(x-1)}\,dx$

$$(x-1)\overline{)\,x^4}$$

$x^3 + x^2 + x + 1 + \frac{1}{x-1}$

$-(x^4 - x^3)$

x^3

$-(x^3 - x^2)$

$+x^2$

$-(x^2 - x)$

$+x$

$-(x-1)$

$+1$

$$\int \left(x^3 + x^2 + x + 1 + \frac{1}{x-1}\right) dx$$

$u = x-1$, $du = dx$; $\int \frac{1}{u}\,du = \ln|u|$

$$\boxed{\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln(|x-1|)}$$

6) $\int \frac{3t-2}{t+1}\,dt$

$3 - \frac{5}{t+1}$

$t+1\overline{)\,3t-2}$

$-(3t+3)$

-5

$$\int 3\,dt - 5\int \frac{1}{t+1}\,dt$$

$$\boxed{3t - 5\ln|t+1|}$$

7) $\int_0^1 \frac{x-4}{(x-3)(x-2)}\,dx \Rightarrow$ $\frac{x-4}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)} = \frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$

$x-4 = A(x-3) + B(x-2)$

for $x=3$, $-1 = A(0) + B(1)$ $\quad B=-1$

$x=2$, $-2 = A(-1) + B(0)$ $\quad A=2$

$$\int_0^1 \left(\frac{2}{x-2} - \frac{1}{x-3}\right) dx$$

$$-2\int_0^1 \frac{1}{x-2}\,dx + 1\int_0^1 \frac{1}{x-3}\,dx = 2\ln|x-3| - 1\ln|x-2|\Big|_0^1$$

$$\frac{1}{1}\left(\left(2\ln|-2| - 1\ln|-1|\right) - \left(2\ln|-3| - 1\ln|-2|\right)\right)$$

$$\frac{1}{1}\left(2\ln(2) - 1\ln(1) - 2\ln(3) + 1\ln(2)\right)$$

$$= \frac{1}{1}\left(3\ln(2) - 1\ln(1) - 2\ln(3)\right)$$

8) $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy = \int \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3}\,dy$

y^2-y-6

$4y^2-7y-12 = A(y+2)(y-3) + By(y-3) + Cy(y+2)$

$= y^2(A+B+C) + y(-A-3B+2C) + (-6A)$

$3(A+B+C=4)$

$-A-3B+2C=-7$

$-6A=-12$, $(A=2)$

$2A+\emptyset B+5C=5$,

$4+5C=5$, $(C=\frac{1}{5})$

$2+B+\frac{1}{5}=4$

$B=(\frac{9}{5})$

$\int_1^2 \left(2\frac{1}{y} + \frac{9}{5}\frac{1}{y+2} + \frac{1}{5}\frac{1}{y-3}\right)dy$

$= \left(2\ln|y| + \frac{9}{5}\ln|y+2| + \frac{1}{5}\ln|y-3|\right)\Big|_1^2$

$= \left(2\ln(2) + \frac{9}{5}\ln(4) + \frac{1}{5}\ln(1)\right) - \left(2\ln(1) + \frac{9}{5}\ln(3) + \frac{1}{5}\ln(2)\right)$

$= \frac{1}{5}\left(9\ln(2) + 9\ln(4) - 9\ln(1) - 9\ln(3)\right)$

$\int_1^2 (\quad)\,dy = \frac{9}{5}\left(\ln(4) - \ln(3) + \ln(2) - \ln(1)\right)$

9) $\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$= \frac{A(x-2)^2 + B(x-2)(x-3) + C(x-3)}{(x-3)(x-2)^2}$

$\therefore\ x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$

$x^2+1 = x^2(A+B) + x(-4A-5B+C) + (4A+6B-3C)$

4(① $A+B=1$, $A=1-B$

② − $4A-5B+C=0$

+ ③ $4A+6B-3C=1$)

② + ③ $\Rightarrow$ $B-2C=1$, $B=1+2C$

4① + ② $0A-B+C=4$, $B=C-4$

$1+2C=C-4$

$C=-5$

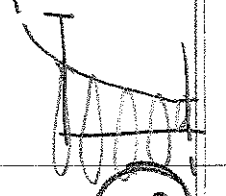
$B=-9$

$A=10$

$10\int \frac{1}{x-3}\,dx - 9\int \frac{1}{x-2}\,dx - 5\int \frac{1}{(x-2)^2}\,dx$

$u=x-2$, $du=dx$, $\int u^{-2}\,du = -u^{-1}$

$10\ln|x-3| - 9\ln|x-2| + 5(x-2)^{-1}$



(10) $\pi\int_0^1 \frac{1}{(x^2+3x+2)^2}\,dx = \pi\int \frac{1}{(x+1)^2(x+2)^2}\,dx$

$(x+1)(x+2)$

$\pi\int \frac{1}{(x+1)^2(x+2)^2}\,dx = \pi\int\left(\frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+2)} + \frac{D}{(x+2)^2}\right)dx$

$= \pi\int \frac{A(x+1)(x+2)^2 + B(x+2)^2 + C(x+2)(x+1)^2 + D(x+1)^2}{(x+1)^2(x+2)^2}\,dx$

$= \pi\int \frac{A(x+1)(x^2+4x+4) + B(x^2+4x+4) + C(x+2)(x^2+2x+1) + D(x^2+2x+1)}{(x+1)^2(x+2)^2}\,dx$

$= \pi\int \frac{1}{(x+1)^2(x+2)^2}\left[A(x^3+4x^2+4x + x^2+4x+4) + B(x^2+4x+4) + C(x^3+2x^2+x + 2x^2+4x+2) + D(x^2+2x+1)\right]dx$

$1 = x^3(A+C) + x^2(5A+B+4C+D) + x(8A+4B+5C+2D) + (4A+4B+2C+1D)$

① $1 = 4A+4B+2C+1D$

② $0 = A+C$ $\quad A=-C$

③ $0 = 5A+B+4C+0$

④ $0 = 8A+4B+5C+2D$

②·① $2-0 = (8-8)A + (8-4)B + (4-5)C + (2-2)D$

$2 = 4B - C = 4B + A$

$B = \frac{2+C}{4}$

①-③ $1 = A(4-5) + B(4-1) + C(2-4) + (1-1)D$

$1 = -A + 3B - 2C + 0D$

$1 = -(-C) + 3\left(\frac{2+C}{4}\right) - 2C$

$4 = 4C + 6 + 3C - 8C$

$-2 = -C$ $\quad C = 2$

$A = -2$

$B = 1$

$0 = 5(-2) + 1 + 4(2) + D$

$0 = -10 + 1 + 8 + D$

$D = 1$

$V = \pi\int_0^1 \left(\frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{(x+2)} + \frac{1}{(x+2)^2}\right)dx$

$\int u^{-2}\,du = -u^{-1}$

$= \pi\left(-2\ln|x+1| - \frac{1}{x+1} + 2\ln|x+2| - \frac{1}{x+2}\right)\Big|_0^1$

$\pi\left[\left(-2\ln(2) - \frac{1}{2} + 2\ln(3) - \frac{1}{3}\right) - \left(-2\ln(1) - 1 + 2\ln(2) - \frac{1}{2}\right)\right]$

$\pi\left[-4\ln(2) - 2\ln(1) + 2\ln(3) + \frac{2}{3}\right]$

$V = \pi(.0913)$

$V = .2868$