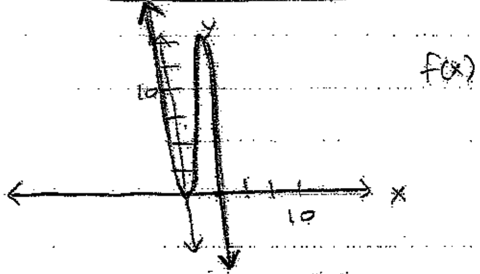
4.9 #23, 25

23. Picture



Goal: To find the antiderivative F of f that satisfies the given condition, $F(0)=4$.

Setup: Given: $F(0)=4$, $f(x)=5x^4-2x^5$

Key Formulas: $F(x)=\int f(x)\,dx$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C$$

Work: $F(x)=\int(5x^4-2x^5)\,dx$

$$\frac{5x^5}{5}-\frac{2x^6}{6}+c$$

$$x^5-\frac{x^6}{3}+C$$

$$F(0)=0^5-\frac{0^6}{3}+C$$

$$F(0)=C$$

$F(0)=4$ so $C=4$

$$\boxed{F(x)=x^5-\tfrac{1}{3}x^6+4}$$

Conclusion: The area under $f(x)=5x^4-2x^5$ is equal to it's antiderivative, which was found to be $x^5-\frac{1}{3}x^6+C$. Then, I solved for C by substituting 0 for x. C was found to be 4 so the antiderivative of f(x) is $x^5-\frac{1}{3}x^6+4$.

25. Picture: N/A

Goal: To find f.

Setup: Given $f''(x)=20x^3-12x^2+6x$

Key Formulas = $f'(x)=\int f''(x)\,dx$ $\quad f(x)=\int f'(x)\,dx$ $\quad \int x^n\,dx=\frac{x^{n+1}}{n+1}+c$

Work: $f''(x)=20x^3-12x^2+6x$

$$f'(x)=\int(20x^3-12x^2+6x)\,dx$$

$$=\frac{20x^4}{4}-\frac{12x^3}{3}+\frac{6x^2}{2}+c$$

$$=5x^4-4x^2+3x^2+c$$

$$f(x)=\int(5x^4-4x^3+3x^2+c)\,dx$$

$$=\frac{5x^5}{5}-\frac{4x^4}{4}+\frac{3x^3}{3}+cx+D$$

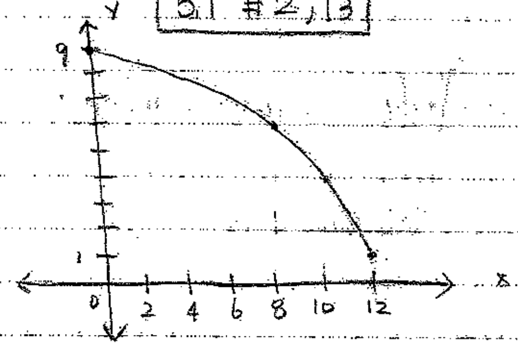
$$=\boxed{x^5-x^4+x^3+cx+D}$$

Conclusion: Since the given equation was $f''(x)$, I integrated the equation two times to obtain f(x). My final answer was $f(x)=x^5-x^4+x^3+cx+D$

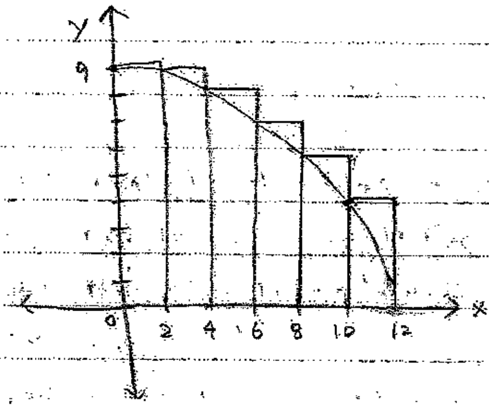
[redacted]

$\boxed{5.1 \ \#2, 13}$

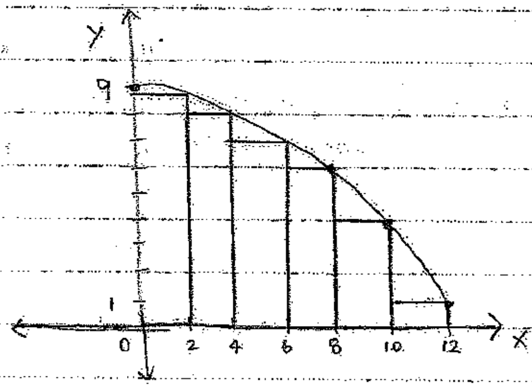
(2.) Picture:



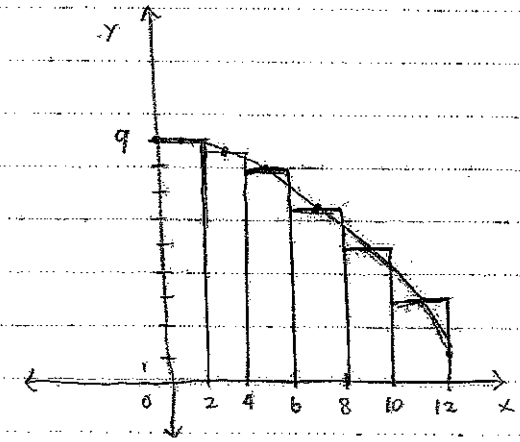
$L_6 \rightarrow$



$R_6 \rightarrow$



$M_6 \rightarrow$



Goal: 1. a) to use 6 rectangles to find estimates of each type for the area under the given graph of f from $x=0$ to $x=12$ using left endpoints, right endpoints, and midpoints.

b) To find if L_6 is an underestimate or overestimate of the true area.

c) To find if R_6 is an underestimate or overestimate of the true area.

d) To find which of the numbers L_6, R_6, M_6 gives the best estimate, and explain.

Setup: Given: $n=6$ $b=12$ $a=0$

Key Formulas: $\Delta x = \frac{b-a}{n} = \frac{12-0}{6} = 2$

$L_6 = \Delta x\,[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)]$

$R_6 = \Delta x\,[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+f(x_6)]$

$M_6 = \Delta x\,[f(\bar{x}_1)+f(\bar{x}_2)+f(\bar{x}_3)+f(\bar{x}_4)+f(\bar{x}_5)+f(\bar{x}_6)]$

Work: ai) $L_6 = \Delta x\,[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)]$

$= 2\,[f(0)+f(2)+f(4)+f(6)+f(8)+f(10)]$

$= 2\,[9+8.8+8.2+7.3+5.9+4.1]$

$= \boxed{86.6}$

aii) $R_6 = \Delta x\,[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+f(x_6)]$

$= 2\,[f(2)+f(4)+f(6)+f(8)+f(10)+f(12)]$

$= 2\,[8.8+8.2+7.3+5.9+4.1+1]$

$= \boxed{70.6}$

aiii) $M_6 = \Delta x\,[f(\bar{x}_1)+f(\bar{x}_2)+f(\bar{x}_3)+f(\bar{x}_4)+f(\bar{x}_5)+f(\bar{x}_6)]$

$= 2\,[f(1)+f(3)+f(5)+f(7)+f(9)+f(11)]$

$= 2\,(8.9+8.5+7.8+6.6+5.0+2.8)$

$= \boxed{79.2}$

b) $\boxed{L_6 \text{ is an overestimate}}$

c) $\boxed{R_6 \text{ is an underestimate}}$

why

on next page

d) $\boxed{M_6}$ explain

Conclusion:

ai) Using 6 subintervals and the left endpoints, the area under the graph from $x=0$ to $x=12$ is 86.6

aii) Using 6 subintervals and the right endpoints, the area under the graph from $x=0$ to $x=12$ is 70.6.

aiii) Using 6 subintervals and the midpoints, the area under the graph from $x=0$ to $x=12$ is 79.2.

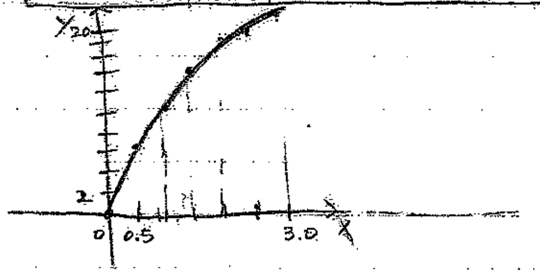
b) Since the function is decreasing, L_6 will be an overestimate. When you draw the bars (6 subintervals) they are above the graph.

c) Since the function is decreasing, R_6 will be an underestimate. When you draw the bars (6 subintervals) they are below the graph.

d) M_6 gives the best estimate because L_6 and R_6 give an over and underestimate. When you look at the graph for M_6, the bars seem to be closer to the graph/area. ✓

13. Picture:

t(s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2



Goal: To find the lower and upper estimates for the distance that she traveled during these 3 seconds.

Setup: Given = table, $a=0$ $b=3.0$, $n=6$

Key Formulas:

$\Delta x = \frac{b-a}{n}$, $\Delta x = \frac{3-0}{6} = 0.5$

$L_6 = \Delta x\,[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)]$

$R_6 = \Delta x\,[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+f(x_6)]$

Work:

$L_6 = \Delta x\,[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)]$

$= 0.5[f(0)+f(0.5)+f(1.0)+f(1.5)+f(2.0)+f(2.5)]$

$= 0.5(0+6.2+10.8+14.9+18.1+19.4)$

$= \boxed{34.7 \text{ ft.}}$

$R_6 = \Delta x\,[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+f(x_6)]$

$= 0.5[f(0.5)+f(1.0)+f(1.5)+f(2.0)+f(2.5)+f(3.0)]$

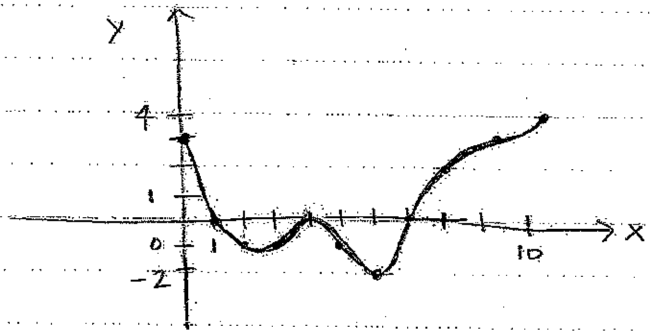
$= 0.5(6.2+10.8+14.9+18.1+19.4+20.2)$

$= \boxed{44.8 \text{ ft.}}$

Conclusion: Using 6 subintervals and left endpoints, the total distance the runner covered was 34.7 feet (during 3 seconds). Using 6 subintervals and right endpoints, the total distance the runner covered in 3 seconds was 44.8 feet. I used left and right endpoints because it would give me the upper and lower estimates. The left endpoints gave me the lower estimate and the right endpoints gave me the upper estimate.

5.2 #5, 8

5. Picture:



Goal: Estimate $\int_0^{10} f(x)dx$ using 5 subintervals with a) right endpoints, b) left endpoints, and c) midpoints

Setup:

Givens: $n=5$, $b=10$, $a=0$

$\Delta x = \frac{b-a}{n}$

$\Delta x = 2$

Key Formulas:

$M_5: \int_0^{10} f(x)dx \approx \Delta x[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)]$

$L_5: \int_0^{10} f(x)dx \approx \Delta x[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)]$

$R_5: \int_0^{10} f(x)dx \approx \Delta x[f(\bar{x}_1)+f(\bar{x}_2)+f(\bar{x}_3)+f(\bar{x}_4)+f(\bar{x}_5)]$

Work:

Right: $\int_0^{10} f(x)dx \approx 2[f(0)+f(2)+f(4)+f(6)+f(8)] \approx 2[(-1)+(0)+(-2)+(2)+(4)]$

$\approx \boxed{6}$

Left: $\int_0^{10} f(x)dx \approx 2[f(2)+f(4)+f(6)+f(8)+f(10)] \approx 2[(3)+(-1)+(0)+(-2)+(2)]$

$\approx 2[2]$

$\approx \boxed{4}$

Midpoint: $\int_0^{10} f(x)dx \approx 2[f(1)+f(3)+f(5)+f(7)+f(9)] \approx 2[(0)+(-1)+(-1)+(0)+(3)]$

$\approx \boxed{2}$

Conclusion:

Using 5 subintervals, the estimated area of f(x) between x=0 and x=10 for $\boxed{\text{right endpoints is 6}}$, $\boxed{\text{left endpoints is 4}}$, and $\boxed{\text{midpoints is 2}}$.

$\boxed{\begin{array}{l} a)\ 6 \\ b)\ 4 \\ c)\ 2 \end{array}}$

8. Picture:

x	3	4	5	6	7	8	9
$f(x)$	-3.4	-2.1	-0.6	0.3	0.9	1.4	1.8

Goal: To estimate $\int_3^9 f(x)\,dx$ using three equal subintervals with a) right endpoints b) left endpoints, and c) midpoints. Also, figure out whether your estimates are less than or greater than the exact value of the integer, knowing that it is an increasing function.

Setup:

Givens: $n=3$ $b=9$ $a=3$

$\Delta x = \frac{b-a}{n} = 2$

Key Formulas:

R_3: $\int_3^9 f(x)\,dx \approx \Delta x\,[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)]$

L_3: $\int_3^9 f(x)\,dx \approx \Delta x\,[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+f(x_6)]$

M_3: $\int_3^9 f(x)\,dx \approx \Delta x\,[f(\bar{x}_1)+f(\bar{x}_2)+f(\bar{x}_3)]$

Work:

Right: $\int_3^9 f(x)\,dx \approx 2[f(3)+f(5)+f(7)]$

$\approx 2[(-3.4)+(-0.6)+(0.9)] \approx \boxed{-6.2}$

Left: $\int_3^9 f(x)\,dx \approx 2[f(5)+f(7)+f(9)]$

$\approx 2[(-0.6)+(0.9)+(1.8)] \approx \boxed{4.2}$

Midpoint: $\int_3^9 f(x)\,dx \approx 2[f(4)+f(6)+f(8)]$

$\approx 2[(-2.1)+(0.3)+(1.4)] \approx \boxed{-0.8}$

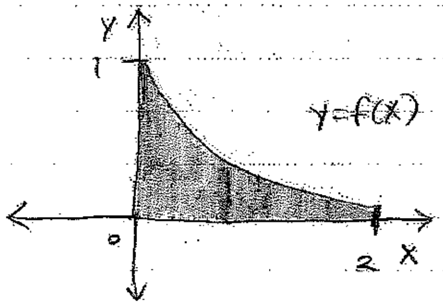
Conclusion:

Using 3 equal subintervals, the estimated area under $f(x)$ between $x=3$ and $x=9$ for right endpoints is -6.2, left endpoints is 4.2, and midpoints is -0.4. a) -6.2 b) 4.2 c) -0.8

For right endpoints, the estimate is greater than the exact value. For left endpoints, the estimate is less than the exact value. For midpoint, the estimate is probably very close to the exact value because the left endpoint is an underestimate and the right endpoint is an overestimate. The midpoint is very close to the graph because it is between the over and under estimates.

7.7 #2, 11

2. Picture



Goal: To find which rule produced each product
(left, right, Trapezoidal, Midpoint Rule → 0.7811, 0.8675, 0.8632, 0.9540).
and to find between which 2 approximations

Setup: Givens: 0.7811, 0.8675, 0.8632, 0.9540
- same number of subintervals were used
- f is decreasing and concave up

Work/Conclusion: a) Since f is decreasing, and concave up the left endpoint rule would give an overestimate. Therefore, it would be the largest estimate of 0.9540.

Since f is decreasing and concave up, the right endpoint rule would give an underestimate. Therefore, it would be the smallest estimate of 0.7811.

Since f is concave up, when you connect the top sides of the trapezoid when dividing the area into subintervals, there will be some space of the trapezoid outside the graph. Therefore, it would be a slight overestimate. The value that is less than 0.9540 is 0.8675, so the Trapezoidal Rule produced an area of 0.8675.

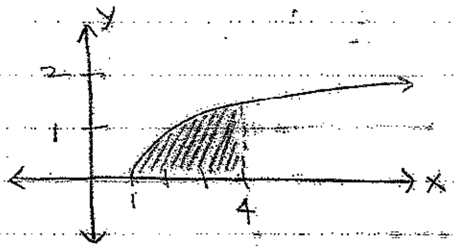
Since f is concave up, the Midpoint rule will have some space outside and inside the graph. Also, since f is decreasing, it is an underestimate because the tangent line is under the curve. The Midpoint Rule has a product that is slightly an underestimate, so it is 0.8632. (0.8632 is greater than 0.7811 but less than 0.8675).

$\boxed{L = 0.9540 \quad R = 0.7811 \quad T = 0.8675 \quad M = 0.8632}$

b) The true value of $\int_0^2 f(x)dx$ lies between the Midpoint Rule and Trapezoidal Rule approximations. Since the Left and Midpoint Rules gave underestimates, and the Trapezoidal and Right Rules gave overestimates, the actual value will be between the slight underestimate and slight overestimate approximations, which is between the Trapezoidal and Midpoint Rules.

11.

Picture:



Goal: Approximate given integral (with specified value of n) using a) The Trapezoidal Rule b) the Midpoint Rule c) Simpson's Rule.

Setup: Given: $\int_1^4 \sqrt{\ln x}\, dx$, $n=6$

$b=4 \quad a=1$

Key Formulas: $\Delta x = \frac{b-a}{n} \qquad \Delta x = \frac{4-1}{6} = 0.5$

$T_6 = \int_1^4 \sqrt{\ln x}\, dx \approx \frac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+2f(x_4)+2f(x_5)+f(x_6)]$

$M_6 = \int_1^4 \sqrt{\ln x}\, dx \approx \Delta x[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)]$

$S_6 = \int_1^4 \sqrt{\ln x}\, dx \approx \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+4f(x_5)+f(x_6)]$

Work:

a) For trapezoidal rule:

Trapezoidal rule: $n+1 = 7$ terms

x	f(x)
1.0	0
1.5	0.63676
2.0	0.83255
2.5	0.95723
3.0	1.04815
3.5	1.11927
4.0	1.17741

$\int_1^4 \sqrt{\ln x}\, dx \approx T_6 = \frac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+2f(x_4)+2f(x_5)+f(x_6)]$

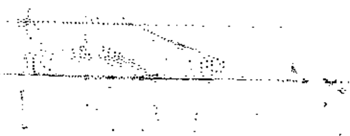
$= \frac{0.5}{2}[(0)+2(0.63676)+2(0.83255)+2(0.95723)+2(1.04815)+2(1.11927)+(1.17741)]$

$= \boxed{2.59133}$

b) For Midpoint rule:

x	f(x)
1.25	0.47238
1.75	0.74807
2.25	0.90052
2.75	1.00578
3.25	1.08566
[illegible]	[illegible]

[illegible]



[illegible] $\rightarrow$ C.78 [illegible]

[illegible]

$$\int_1^4 \sqrt{\ln x}\,dx \approx M_6 = \Delta x\left[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)\right]$$

$$= 0.5\left[0.47238+0.74807+0.90052+1.00578+1.08566+1.14968\right]$$

$$= \boxed{2.68105}$$

c) For Simpson's Rule:
7 terms (n+1)

X	f(x)
1.0	0
1.5	0.63676
2.0	0.83255
2.5	0.95723
3.0	1.04815
3.5	1.11927
4.0	1.17741

$$\int_1^4 \sqrt{\ln x}\,dx \approx S_6 = \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+4f(x_5)+f(x_6)\right]$$

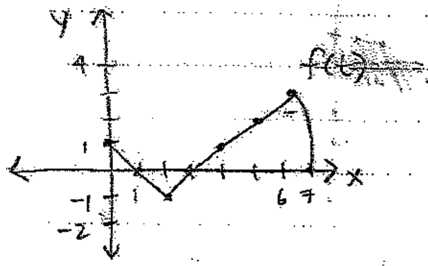
$$= \frac{0.5}{3}\left[(0)+4(0.63676)+2(0.83255)+4(0.96723)+2(1.04815)+4(1.11927)+1.17741\right]$$

$$= \boxed{2.63198}$$

Conclusion: The area under $f(x)=\sqrt{\ln x}$ between $x=1$ and $x=4$ using the Trapezoidal Rule is approximately 2.59133. Using the Midpoint Rule, the approximate area is 2.68105 and using Simpson's Rule, the approximate area is 2.63198. Therefore, the actual area is between 2.59133 and 2.68105.

5.3 #2, 21, 37

(2.) Picture:



Goal: a) Evaluate $g(x)$ for $x=0,1,2,3,4,5$, and 6
b) Estimate $g(7)$
c) Find where g has a max value and min value
d) Graph g

Setup:

Given: $g(x) = \int_0^x f(t)\,dt$

Key Information: $g(x) = \int_0^x f(t)\,dt =$ area under graph between $x=0$ and $x=x$.

Area of Δ': $\frac{bh}{2}$ $\qquad g'(x) = f(x)$

Work/Conclusion:

a) $g(x) = \int_0^x f(t)\,dt$

The area of $f(x)$ which is equal to $g(x)$ from $x=0$ to $x=a$, is as listed on the left. I used the area formula for a triangle to get the areas underneath the graph.

$g(0) = \int_0^0 f(t)\,dt = 0$

$g(1) = \int_0^1 f(t)\,dt = \frac{bh}{2} = \frac{1}{2}$

$g(2) = \int_0^2 f(t)\,dt = \frac{bh}{2} - \frac{bh}{2} = \frac{1}{2} - \frac{1}{2} = 0$

$g(3) = \int_0^3 f(t)\,dt = \frac{bh}{2} - \frac{bh}{2} - \frac{bh}{2} = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = -\frac{1}{2}$

$g(4) = \int_0^4 f(t)\,dt = \frac{bh}{2} - \frac{bh}{2} - \frac{bh}{2} - \frac{bh}{2} = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = 0$

$g(5) = \int_0^5 f(t)\,dt = \frac{bh}{2} - \frac{bh}{2} - \frac{bh}{2} + \frac{bh}{2} = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \frac{2(2)}{2} = \frac{3}{2}$

$g(6) = \int_0^6 f(t)\,dt = \frac{bh}{2} - \frac{bh}{2} - \frac{bh}{2} + \frac{bh}{2} = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \frac{3(3)}{2} = 4$

b) $g(7) = \int_0^7 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt + \int_2^3 f(t)\,dt + \int_3^6 f(t)\,dt + \int_6^7 f(t)\,dt$

$\approx \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \frac{9}{2} + 2.2 \approx 6.2$ (area)

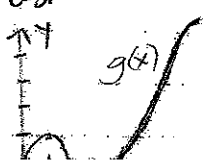
The area of $f(x)$ from $x=0$ to $x=7$ is about 6.2.

2.2 because there are about 2.2 squares under $\int_6^7 f(t)\,dt$.

c) g has a max value at $t=7$, because at $t=7$, there is a maximum area. Therefore, the max is when $t=7$, which is 6.2 ($g(7)$). g has a minimum value at $t=3$, which is -0.5 because the minimum area occurs at $t=3$.

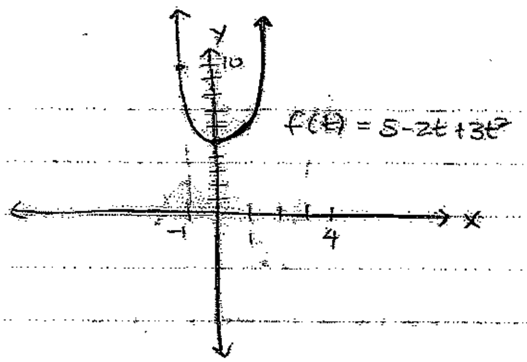
Since $g'(x) = f(x)$

d)



✓

21. Picture:



Goal: Evaluate $\int_1^4 (5-2t+3t^2)\,dt$

Setup: Given = $\int_1^4 (5-2t+3t^2)\,dt$

Key Formulas = $\int_a^b f(x)\,dx = F(b) - F(a)$

Work: $\int_1^4 (5-2t+3t^2)\,dt$

$5t - \frac{2t^2}{2} + \frac{3t^3}{3} + C$

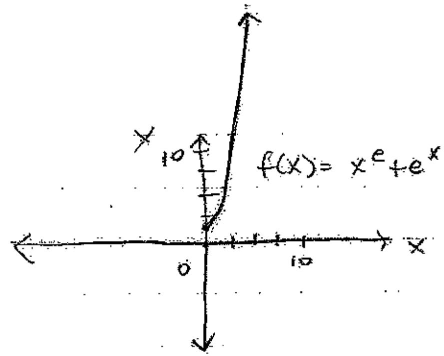
$5t - t^2 + t^3 \Big|_{t=1}^{t=4}$

$(20-16+64)-(5-1+1)$

$\boxed{63}$

Conclusion: The area under $f(t) = 5-2t+3t^2$ between $x=1$ and $x=4$ is $\boxed{63.}$

37. Picture:



Goal: Evaluate $\int_0^1 (x^e + e^x)\,dx$

Setup: Given = $\int_0^1 (x^e + e^x)\,dx$

Key Formulas = $\int_a^b f(x)\,dx = F(b) - F(a)$

$\int e^x dx = e^x + c$

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Work: $\int_0^1 (x^e + e^x)\,dx$

$$\frac{x^{e+1}}{e+1} + e^x \Big|_{x=0}^{x=1}$$

$$\left(\frac{(1)^{e+1}}{e+1} + e\right) - (1)$$

$$\boxed{\frac{1}{e+1} + e - 1}$$

Conclusion: The area under $f(x) = x^e + e^x$ between $x=0$ and $x=1$ is $\frac{1}{e+1} + e - 1$.