

$$16) \quad \frac{dy}{dt} = 100 - y \Rightarrow \frac{1}{100-y} dy = dt$$

$$u = 100 - y \\ du = -dy \\ - \int \frac{1}{u} du$$

$$- \ln |u|$$

$$- \ln |100 - y| = t + C_1$$

$$\ln |100 - y| = C_2 - t$$

$$|100 - y| = e^{C_2} e^{-t} = C_3 e^{-t}$$

$$100 - y = C_4 e^{-t}$$

$$y = 100 - C_4 e^{-t}$$

$$\text{if } y(0) = 10 = 100 - C_4(1), \quad C_4 = 90, \quad y = 100 - 90 e^{-t}$$

$$\text{if } y(0) = 135 \quad C_4 = -35 \quad y = 100 + 35 e^{-t}$$

17)

67°F



Let  $H$  = heat/energy/temp

Newton's law of cooling/heating: the rate of change ( $\frac{dH}{dt}$ ) of a body is proportional to the difference between the temperature of the body & its surroundings

$$A) \quad \frac{dH}{dt} = k(H - 67)$$

$$\frac{1}{H-67} dH = k dt$$

$$\ln |H - 67| = kt + C_1$$

$$|H - 67| = e^{kt} e^{C_1} = C_2 e^{kt}$$

$$H - 67 = C_3 e^{kt}$$

$$B) \quad H = C_3 e^{kt} + 67$$

$$H(0) = 36 = C_3 e^0 + 67$$

$$C_3 = -31$$

$$H(t) = 67 - 31 e^{kt}$$

$$\frac{dH}{dt} = k(67 - H)$$

$$\frac{1}{67-H} dH = k dt$$

$$+ \ln |67 - H| = -(kt + C_1) = C_2 - kt$$

$$|67 - H| = e^{C_2} e^{-kt} = C_3 e^{-kt}$$

$$67 - H = C_4 e^{-kt}$$

$$H = 67 - C_4 e^{-kt}$$

$$H(0) = 36 = 67 - C_4 e^0$$

$$C_4 = 31$$

$$H(t) = 67 - 31 e^{-kt}$$

at  $t=4$

$$H(4) = 43 = 67 - 31 e^{4k}$$

$$31 e^{4k} = 24$$

$$e^{4k} = 24/31$$

$$k = \ln\left(\frac{24}{31}\right) \Rightarrow$$

$$H(4) = 67 - 31 e^{-4k} = 43$$

$$-31 e^{-4k} = -24$$

$$e^{-4k} = \frac{24}{31}$$

$$H(t) = 67 - 31 e^{\ln\left(\frac{24}{31}\right)t} \leftarrow -k = \ln\left(\frac{24}{31}\right) \quad k = -\ln\left(\frac{24}{31}\right)$$

$$H(4) = 67 - 31 e^{\ln\left(\frac{24}{31}\right)(4)}$$