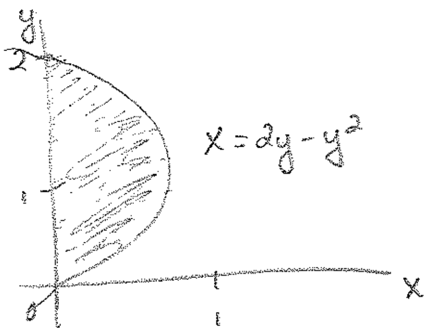
Section 5.4 # 49

Picture:



Goal: The goal is to find the area between the y-axis and the curve from $y = 0$ to $y = 2$.

Set up:

Given: The picture above and $\int_0^2 (2y - y^2)\,dy$.

Mathematical Model: $\int_a^b f(x)\,dx = F(b) - F(a)$

Work:

$$\int_0^2 (2y - y^2)\,dy = \left[y^2 - \tfrac{1}{3}y^3\right]\Big|_0^2 = 4 - \tfrac{8}{3} - 0 + 0 = \boxed{\tfrac{4}{3}}$$

Conclusion:

The area under the curve is $\frac{4}{3}$. This was found by using the Fundamental Theorem of Calculus ($\int_a^b f(x)\,dx = F(b) - F(a)$). To find the antiderivative of the original function, the power rule had to be used.

Section 5.4 # 51

· Picture:

N/A

Goal: The goal is to determine what the definite integral of a rate of change function represents.

Set up:

Given: $W'(t)$ is the rate of growth of a child in pounds per year

Find $\int_5^{10} W'(t)dt$ represents

Mathematical Model: $\int W'(t)dt = W(t) + C$

Work:

N/A

Conclusion:

$\int_5^{10} W'(t)dt$ represents the number of pounds the child gained from year 5 to year 10. Since $W'(t)$ is the rate of change of the child in pounds per year, the antiderivative of the function would just be the pounds gained by the child up to that year. Since the function is being integrated from $t=5$ to $t=10$, it is from year 5 to year 10.

Section 5.5 # 3

Picture:

N/A

Goal: The goal is to integrate the function by using the Substition method.

Set up:

Given: $u = x^3+1$ (inside function)

Mathematical Model: $\int_a^b f(g(x))\, g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$

Work:

$\int \cancel{x^2}(x^3+1)^{1/2}\cancel{dx}$ $\quad u = x^3+1 \quad du = 3x^2 dx \quad \frac{1}{3}du = x^2 dx$

$\downarrow$

$\frac{1}{3}\int u^{1/2}du = \frac{2}{9}u^{3/2}+C = \boxed{\frac{2}{9}(x^3+1)^{3/2} + C}$

Conclusion:

By using x^3+1 as the inside of the composite function, I was able to undo the Chain Rule with the Substitution Rule. The Substitution Rule worked because there was a composite function multiplied by the derivative of the inside function.

Section 5.5 #17

Picture: N/A

Goal: The goal is to evaluate the indefinite integral by using the Substitution Rule.

Set up:

Mathematical Model: $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)\,du$

Work:

$\int \frac{e^u}{(1-e^u)^2} du$ $\quad w = 1-e^u \quad dw = -e^u du \quad -dw = e^u du$

$\downarrow$

$-\int w^{-2} dw = w^{-1} + C = \boxed{(1-e^u)^{-1} + C}$

Conclusion:

By using $1-e^u$ as the inside of the composite function, I was able to undo the Chain Rule with the Substitution Rule. The Substitution Rule worked because there was a composite function multiplied by the derivative of the inside function.

Section 5.5 # 33

Picture:

N/A

Goal: The goal is to evaluate the integral by using the Substitution Rule.

Set up:

Mathematical Model: $\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$

Work:

$\int \frac{\cos x}{\sin^2 x}\,dx$ $\quad u = \sin x \quad du = \cos x\,dx$

$\downarrow$

$\int u^{-2}\,du = -u^{-1} + C = -\frac{1}{\sin(x)} + C = \boxed{-\csc x + C}$

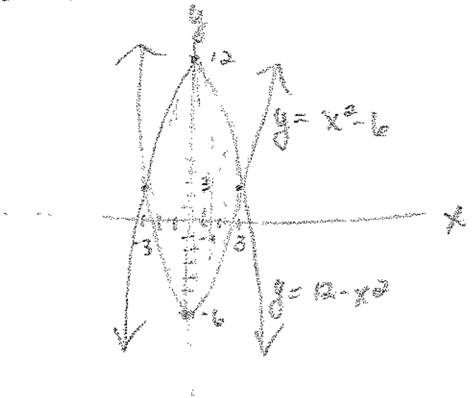
Stop here

Conclusion:

By using sinx as the inside of the composite function, I was able to undo the Chain Rule with the Substitution Rule. I chose sinx instead of cosx because the derivative of cosx is -sinx, and in this instance, sinx is squared and under the fraction bar, so choosing cosx wouldn't have made the integral any simpler.

Section 6.1 # 13

Picture:



Goal: The goal is to sketch the region enclosed by the curves and integrate in order to find the area between them.

Set up:

Mathematical Model: $A = \int_a^b [f(x) - g(x)]\,dx$ when $f(x) \geq$ for all x on $[a,b]$ and $f(x)$ and $g(x)$ are continuous.

Work:

$12 - x^2 = x^2 - 6 \quad 2x^2 = 18 \quad x^2 = 9 \quad x = \pm 3$

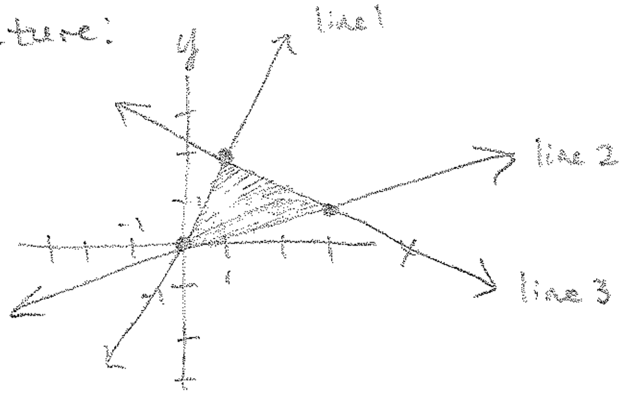
$A = \int_{-3}^{3} (12 - x^2 - x^2 + 6)\,dx = \int_{-3}^{3} (18 - 2x^2)\,dx = \left[18x - \frac{2}{3}x^3\right]\Big|_{-3}^{3} = 54 - 18 + 54 - 18 = \boxed{72}$

Conclusion:

The two functions intersected at $x = 3$ and $x = -3$, so these became my bounds of integration. $y = 12 - x^2$ is greater than $x^2 - 6 = y$ on the entire interval, so I set up the integral as $A = \int_{-3}^{3} (12 - x^2 - x^2 + 6)\,dx$, where $y = x^2 - 6$ is subtracted from $y = 12 - x^2$.

Section 6.1 # 29

Picture:



Goal: The goal is to find the area of the triangle made by the intersecting lines above.

Set up:

Mathematical Model: $A = \int_a^b [f(x) - g(x)]\,dx$ when $f(x) \geq g(x)$ for all x on $[a, b]$ and $f(x)$ and $g(x)$ are continuous.

Work:

$m_1 = \frac{0-2}{0-1} = 2 \quad y - 0 = 2(x-0) \quad y = 2x$

$m_2 = \frac{0-1}{0-3} = \frac{1}{3} \quad y - 0 = \frac{1}{3}(x-0) \quad y = \frac{1}{3}x$

$m_3 = \frac{1-2}{3-1} = -\frac{1}{2} \quad y - 2 = -\frac{1}{2}(x-1) \quad y = -\frac{1}{2}x + \frac{5}{2}$

$$A = \int_0^1 \left(2x - \frac{1}{3}x\right)dx + \int_1^3 \left(-\frac{1}{2}x + \frac{5}{2} - \frac{1}{3}x\right)dx = \int_0^1 \frac{5}{3}x\,dx + \int_1^3 \left(-\frac{5}{6}x + \frac{5}{2}\right)dx$$

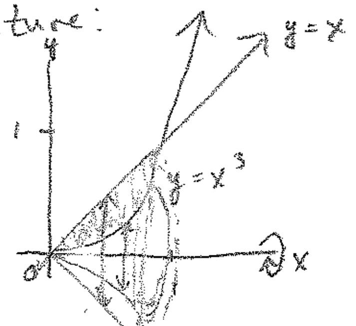
$$= \frac{5}{6}x^2\Big|_0^1 + \left[-\frac{5}{12}x^2 + \frac{5}{2}x\right]\Big|_1^3 = \frac{5}{6} - 0 - \frac{15}{4} + \frac{15}{2} + \frac{5}{12} - \frac{5}{2} = \boxed{\frac{5}{2}}$$

Conclusion:

The area of the triangle is $\frac{5}{2}$. First the equations of the lines had to be found. Then, the integral had to be broken into two parts with the break at the "center" vertex of the triangle. Once both integrals were added together, the area of the whole triangle was determined.

Section 6.2 # 7

Picture:



Goal: The goal is to find the volume of the solid made by rotating a given area around the x-axis.

Set up:

Given: Region enclosed by $y = x$ and $y = x^3$ when $x \geq 0$, about x-axis

Mathematical Model: $V = \pi \int_a^b [(f(x))^2 - (g(x))^2]\,dx$ where $f(x)$ is the outer radius and $g(x)$ is the inner radius

Work:

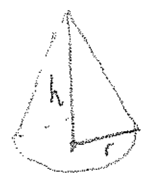
$x = x^3 \quad x^3 - x = 0 \quad x(x^2 - 1) = 0 \quad x = 0, 1, \cancel{-1}$

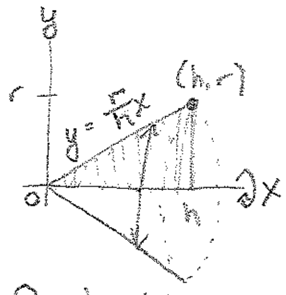
$V = \pi \int_0^1 (x^2 - x^6)\,dx = \pi \left[\frac{1}{3}x^3 - \frac{1}{7}x^7\right]\Big|_0^1 = \pi\left(\frac{1}{3} - \frac{1}{7}\right) = \boxed{\frac{4\pi}{21}}$

Conclusion: Because there is an outer and inner radius, washers are made when the area is rotated about the x-axis, making a solid with a hole in the center. $(x^3)^2$ was subtracted from $(x)^2$ because $y = x$ was the outer and $y = x^3$ was the inner radius from 0 to 1. The bounds 0 to 1 were found by setting the two functions equal to each other to see where they intersect, and $x = -1$ was thrown out because the question wanted only the positive interval.

Section 6.2 # 47

Picture:





Goal: The goal is to find the volume of the cone in terms of r and h.

Set up: $V = \int_a^b A(x)\,dx \quad A = \pi r^2$ - Mathematical Model

Work:

$m = \frac{0-r}{0-h} = \frac{r}{h} \qquad y = \frac{r}{h}x$

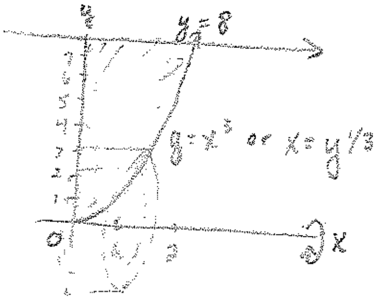
$V = \pi \int_0^h \left(\frac{r}{h}x\right)^2 dx = \frac{\pi r^2}{h^2}\int_0^h x^2\,dx = \frac{\pi r^2}{h^2}\left[\frac{1}{3}x^3\right]\Big|_0^h = \boxed{\frac{\pi}{3} r^2 h}$

Conclusion:

Whenever a right triangle with a leg on an axis is revolved around that axis, a right circular cone is formed. Thus, to find the volume of the cone, the disc method of integration can be used.

Section 6.3 # 11

Picture;



Goal:

The goal is to find the volume of the given area when it is rotated around the x-axis by using the cylindrical shells method.

Set up:

Mathematical Model: $V = 2\pi\int_a^b rh\,dr$

Work:

$y = x^3 \quad x = y^{1/3}$

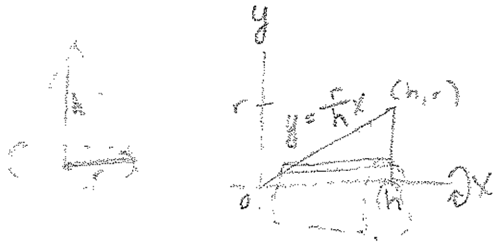
$V = 2\pi\int_0^8 y(y^{1/3})dy = 2\pi\int_0^8 y^{4/3}dy = 2\pi\left[\frac{3}{7}y^{7/3}\right]\Big|_0^8 = 2\pi\left(\frac{3}{7}(8)^{7/3} - 0\right) = \boxed{\frac{768\pi}{7}}$

Conclusion:

Since the cylindrical shells method was used, rotation around around the x-axis requires integrating with respect to y. So, $y = x^3$ was changed to $x = y^{1/3}$, and the volume was solved for by using $2\pi y$ as the circumference of a shell, dy as the thickness, and $y^{1/3}$ as the radius.

Section 6.3 # 47

Picture:



Goal:

The goal is to find the volume of a right circular cone in terms of r and h using the shells method of integration.

Set up:

Mathematical Model: $V = 2\pi \int_a^b rh\,dr$

Work:

$y = \frac{r}{h}x \quad x = \frac{h}{r}y$

$V = 2\pi \int_0^r y\left(\frac{h}{r}y\right)dy = \frac{2\pi h}{r}\int_0^r y^2\,dy = \frac{2\pi h}{r}\left[\frac{1}{3}y^3\right]\Big|_0^r = \frac{2\pi h}{r}\left(\frac{1}{3}r^3 - 0\right) = \boxed{\frac{\pi}{3}r^2h}$

Conclusion:

Whenever a right triangle with a leg on an axis is revolved around that axis, a right circular cone is formed with base radius r (height of other leg) and height h (length of leg along axis. This triangle can then be put into a volume formula to determine the volume of the cone formed by revolving the triangle around an axis.