Lesson 30; Section 11.8, Question 15

Goal: Find the radius of convergence and Interval of Convergence

Setup: $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$

Work: $\lim_{n\to\infty} \left| \frac{(x-2)^n(x-2)}{(n+1)^2+1} \cdot \frac{n^2+1}{(x-2)^n} \right| < 1$

$= \lim_{n\to\infty} \left| \frac{(x-2)\left(\frac{n^2}{n^2}+\frac{1}{n^2}\right)}{\frac{(n+1)^2}{n^2}+\frac{1}{n^2}} \right| < 1$

$= \lim_{n\to\infty} \left| \frac{(x-2)\cdot 1+\frac{1}{n^2}}{\left(1+\frac{1}{n}\right)^2+\frac{1}{n^2}} \right| < 1 = |x-2|<1$

~~∴ radius is 1 & center is 2~~

$-1 < x-2 < 1$
$+2 \quad +2 \quad +2$

$1 < x < 3$

$x=1$

$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$

Alternating Series ✓

① $\frac{1}{(n+1)^2+1} \overset{?}{\leq} \frac{1}{n^2+1}$

$n^2+1 \overset{?}{\leq} (n+1)^2+1$

$n^2 \overset{?}{\leq} n^2+2n+1$

$-1 \leq 2n$

$-\frac{1}{2} \leq n$ true for all $n \geq -\frac{1}{2}$ ✓

② $\lim_{n\to\infty} \frac{1}{n^2+2n+2} = 0$ ✓

∴ converges @ $x=1$ ✓

$x=3$

$\sum_{n=0}^{\infty} \frac{(1)^n}{n^2+1}$ comparison test: $b_n = \sum \frac{1}{n^2}$

$\frac{1}{n^2} \overset{?}{>} \frac{1}{n^2+1}$

$n^2+1 \overset{?}{\geq} n^2$

$1 \overset{?}{\geq} 0$ true ✓

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ P-series $P=2$ ∴ converge

∴ $\sum_{n=0}^{\infty} \frac{1^n}{n^2+1}$ converges ✓

Conclusion:
Radius of convergence: 1
Interval of convergence: [1, 3]

Very Nice