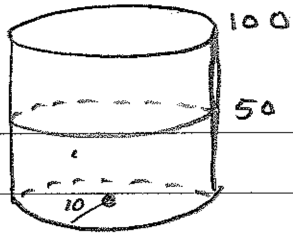
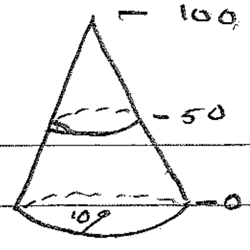
$\rho = 1.55\,slug/ft^3$

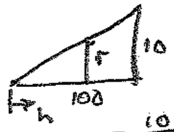
$\rho g = 64.4\,lb/ft^3$



$\downarrow h$

$\rho g = 50\,lb/ft^3$





$r = \frac{10}{100}h$

$F = ma,\ a = g \quad F = \rho g Vol$

$mass = \rho Vol$

$Vol = \pi r^2 (50ft),\ r = 10ft$

$= 5000\pi\,ft^3$

$\boxed{m = \rho Vol = 7750\pi\,(slug)}$

$\boxed{Weight = F = ma = 249550\pi\,lb}$

ground pressure $= F/A$

$A = \pi r^2 = 100\pi$

$\boxed{P = 2500\,lb/ft^2}$

$Vol = \pi \int_{50}^{100} \left(\frac{h}{10}\right)^2 dh = \frac{\pi}{100}\frac{h^3}{3}\Big|_{50}^{100}$

$= \frac{\pi}{300}\left[100^3 - 50^3\right] = 2916.7\pi\,ft^3$

$\boxed{m = \rho Vol = 4520.9\,slug}$

$\boxed{Force = 145\,572.5\pi\,lb}$

$\boxed{P = 1458.3\,lb/ft^2}$

Hydrostatic Pressure $= \rho g h = \left(50\frac{lb}{ft^3}\right)(50ft)$

$\boxed{P_H = 2500\,lb/ft^2}$ depth dependent only

$Force = PA,\ A = \pi r^2 = 100\pi$

$\boxed{F_H = 250\,000\pi\,lb}$ same for cylinder, dif for cone $\Big\}$ WHY?

Pumping: $Work = FD$

$D = h$

$F = ma,\ a = g$

$m = \rho(\pi r^2)dh$

$r = 10$

$W = \pi\rho g r^2 \int_{50}^{100} h\,dh$

$\int_{50}^{100} \frac{h^2}{2} = \frac{1}{2}(100^2 - 50^2) = 3750$

$W = 375\,000\,\pi\rho g\ ft\text{-}lb$

$\boxed{W = 18.75 \times 10^6\pi\ ft\text{-}lb}$

$r = \frac{h}{10}$

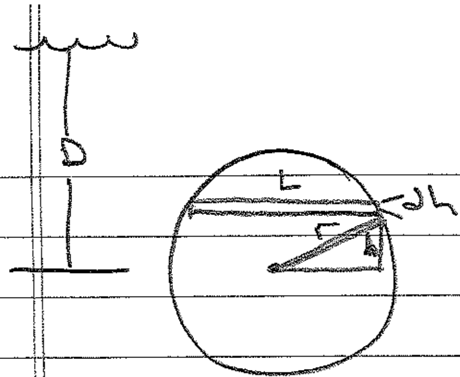
$W = \pi\rho g \int \left(\frac{h}{10}\right)^2 h\,dh$

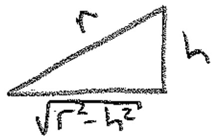
$\frac{\pi\rho g}{100}\frac{h^4}{4}\Big|_{50}^{100}$

$\frac{\pi\rho g}{400}\left[100^4 - 50^4\right] = ?$

$W = 234\,375\,\pi\rho g\ ft\text{-}lb$

$\boxed{W = 11.719 \times 10^6\ ft\text{-}lb}$





$F = PA$

$P = \rho g d$

$d = (D-h)$

$A = 2\sqrt{r^2-h^2}\,dh$

$$F = \int_{-r}^{r} \left[\rho g (D-h)\right]\left[2\sqrt{r^2-h^2}\,dh\right]$$

$$= 2\rho g \left[\int_{-r}^{r} (D-h)(r^2-h^2)^{1/2}\,dh\right]$$

$$= 2\rho g \left[D\int_{-r}^{r} (r^2-h^2)^{1/2}\,dh - \int_{-r}^{r} h(r^2-h^2)^{1/2}\,dh\right]$$

$$\int_{-r}^{r} (r^2-h^2)^{1/2}\,dh = \frac{\pi r^2}{2}$$

$u = r^2 - h^2$

$\frac{du}{-2} = \cancel{(-2)}h\,dh$

$\int u^{1/2}\left(-\frac{du}{2}\right) = -\frac{1}{2}\int u^{1/2}\,du$

$= -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)$

$= -\frac{1}{3}\left(r^2-h^2\right)^{3/2}$

$$= 2\rho g \left[D\frac{\pi r^2}{2} + + \frac{1}{3}(r^2-h^2)^{3/2}\Big|_{-r}^{r}\right]$$

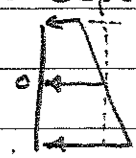
$\frac{1}{3}\left[(r^2-r^2)^{3/2} - (r^2-(-r)^2)^{3/2}\right]$

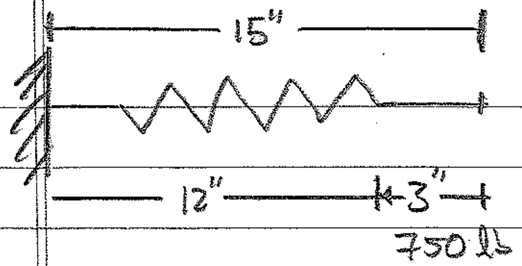
$\frac{1}{3}[\ 0\ ]$

$$\boxed{F = \rho g D \pi r^2}$$

Note: same result as calculating pressure at centroid of the surface, which is at depth D.

$P = \rho g h$ is linear





Hooke's law: p 447

$f(x) = kx \qquad x = \frac{3}{12}\,\text{ft}$

$750\,\text{lb} = \left(\frac{1}{4}\,\text{ft}\right)k$

$3000\,\frac{\text{lb}}{\text{ft}} = k$

$f(x) = 3000x$

$W = FD \;,\; D = dx$

$W = \int_{1/4}^{1/2} \overbrace{3000x}^{\text{lb}}\, \overset{\text{ft}}{dx}$

$= 3000\,\frac{x^2}{2}\Big|_{1/4}^{1/2}$

$= 1500\left(\frac{1}{2}^2 - \frac{1}{4}^2\right)$

$= 1500\left(\frac{1}{4} - \frac{1}{16}\right)$

$= 1500\left(\frac{3}{16}\right)$

$\boxed{W = 843.75\ \text{ft}\cdot\text{lb} \text{ to compress the 15" spring from 12" to 9"}}$