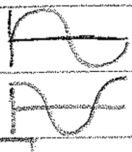
1) $\int_0^{\pi/4} \tan(x)\,dx$ $\tan(x)=\frac{\sin(x)}{\cos(x)}$ $\sin(0)=0$

$\cos(0)=0$

$\boxed{\text{No discontinuity on } [0, \tfrac{\pi}{4}] \;\therefore\; \text{proper}}$ $\sin(\frac{\pi}{4})=\cos(\frac{\pi}{4})$

2) $\int_0^{\pi} \tan(x)\,dx$



$\sin(0)=0$ $\sin(\pi)=0$

$\cos(0)=0$ $\cos(\pi)=-1$

$\boxed{\text{discontinuity @ } x=\tfrac{\pi}{2} \;\therefore\; \text{improper}}$ $\cos(\frac{\pi}{2})=0$

3) $\int_{-1}^{1} \frac{1}{(x-2)(x+1)}\,dx$ $\boxed{\text{IMPROPER: discontinuity @ } x=2,-1 \quad -1 \in [-1,1]}$

4) $\int_0^{\infty} e^{-x^3}\,dx = \int_0^{\infty} \frac{1}{e^{x^3}}\,dx$ $\boxed{\text{IMPROPER INFINITE LIMIT OF INTEGRATION}}$

5) $\int_{-\infty}^{0} \frac{1}{3-4x}\,dx \Rightarrow$ let $u = 3-4x$ $\Rightarrow \lim_{t\to-\infty} \left(-\frac{1}{4}\right)\int_t^0 \frac{1}{u}\,du$

$du = -4\,dx$, $dx = \frac{du}{-4}$ $\quad \int \frac{1}{u}\,du = \ln|u|$

$= \lim_{t\to-\infty} \left(-\frac{1}{4}\right) \ln|3-4x| \Big|_t^0$

$= \lim_{t\to-\infty} \left(-\frac{1}{4}\right)\left[\ln|3| - \ln|3-4t|\right]$ $\quad \infty$

$\boxed{\text{DNE} \therefore \text{Divergent}}$

6) $\lim_{t\to\infty}\int_2^t \frac{1}{(v+3)(v-1)}\,dv = \lim_{t\to\infty}\int_2^t \left(\frac{A}{v+3} + \frac{B}{v-1}\right) dv$ $\quad 1 = A(v-1) + B(v+3)$

@$v=1$, $1 = 0A + 4B$ $\quad \boxed{B = \frac{1}{4}}$

$\lim_{t\to\infty}\int_2^t \left(\frac{-1/4}{v+3} + \frac{1/4}{v-1}\right) dv$ $\quad$ @$v=-3$, $1 = -4A$ $\quad \boxed{A = -\frac{1}{4}}$

$\lim_{t\to\infty}\left[\frac{1}{4}\ln|v-1| - \frac{1}{4}\ln|v+3|\right]_2^t = \lim_{t\to\infty}\frac{1}{4}\left[\ln\left|\frac{v-1}{v+3}\right|\right]_2^t$

Note: divergent @ $-3, +1$ but are not included on $[2, \infty)$

$= \lim_{t\to\infty}\frac{1}{4}\left[\ln\left|\frac{t-1}{t+3}\right| - \ln\left|\frac{1}{5}\right|\right]$

$= \lim_{t\to\infty}\frac{1}{4}\left[\ln\left|\frac{\frac{t}{t} - \frac{1}{t}}{\frac{t}{t} + \frac{3}{t}}\right| - \left(\ln|1| - \ln|5|\right)\right]$

$= \frac{1}{4}\left[\ln(1) - \ln(1) + \ln(5)\right] = \boxed{\frac{\ln(5)}{4}}$

7) $\int_2^3 (3-x)^{-1/2}\,dx$, $u = 3-x$, $du = -dx$ $\quad \lim_{t\to 3}\int_2^t u^{-1/2}(-du) = \lim_{t\to 3}\left(-2u^{1/2}\right)\Big|_2^t$

$= -2\lim_{t\to 3}(3-x)^{1/2}\Big]_2^t = -2\lim_{t\to 3}\left[(3-t)^{1/2} - (1)^{1/2}\right]$

$= -2[0-1] = \boxed{2}$

LSN 14
BW

$$\lim_{x\to\infty}\int_0^x \frac{10t}{e^{t^2}}\,dt \Rightarrow \begin{matrix} u=-t^2 \\ du=-2t\,dt \\ -\frac{du}{2}=t\,dt \end{matrix} \Rightarrow \lim_{x\to\infty}(10)\int e^u\left(-\frac{du}{2}\right)$$

$$= \lim_{x\to\infty}(-5)e^u = \lim_{x\to\infty}(-5)e^{-t^2}\Big|_0^x$$

$$= \lim_{x\to\infty}(-5)\left(\cancel{\frac{1}{e^{x^2}}}^{\,0} - 1\right) = \boxed{5}$$

$$\int_0^\infty 3te^{-t}\,dt \left\{\begin{matrix} u=t & dv=e^{-t}dt \\ du=dt & v=-e^{-t}\end{matrix}\right\} \Rightarrow 3\left(\underbrace{-te^{-t} - -\int e^{-t}dt}_{-te^{-t}+-e^{-t}}\right)$$

$$\lim_{x\to\infty}(-3)(e^{-t})(t+1) = \lim_{x\to\infty}(-3)\left[\underbrace{e^{-x}(x+1)}_{\lim_{x\to\infty}\frac{x+1}{e^x}\Rightarrow \text{L'Hopital's Rule} = \lim_{x\to\infty}\frac{1}{e^x}=0} - e^0(0+1)\right] = \boxed{3}$$

$$\lim_{t\to-\infty}\int_t^0 \frac{e^x}{1+e^x}\,dx \Rightarrow \left\{\begin{matrix} u=1+e^x \\ du=e^x dx\end{matrix}\right\} \Rightarrow \lim_{t\to-\infty}\int_t^0 \frac{1}{u}\,du = \lim_{t\to-\infty}\ln|u|$$

$$= \lim_{t\to-\infty}\ln|1+e^x|\Big|_t^0 = \lim_{t\to-\infty}\left[\ln(1+e^0) - \ln(1+e^t)\right]$$

$$= \ln(2) - \ln(1+\cancel{0}^{\,0})$$

$$= \boxed{\ln(2)}$$