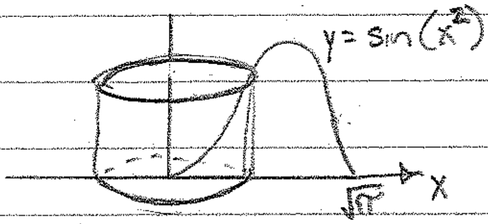
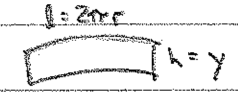
2,3,5) using shells, calculate volume when rotated about y-axis

2)



$$V = \int_0^{\sqrt{\pi}} (2\pi r) h \, dx$$



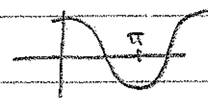
$r = x$

$h = \sin(x^2)$

$$2\pi \int_0^{\sqrt{\pi}} x \sin(x^3) dx \quad \text{let } u = x^2, \quad \frac{du}{2} = \cancel{2}x\,dx$$

$$\frac{2\pi}{2} \int \sin(u) du = \pi(-\cos(u)) + C = \pi(-\cos(x^2)) + C \Big|_0^{\sqrt{\pi}}$$

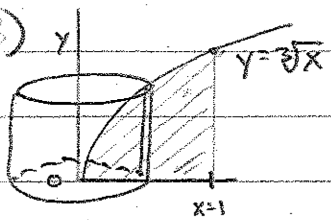
$$V = \pi\left[(-\cos(\pi)) - (-\cos(0))\right]$$



$$V = \pi\left[-(-1) - -1\right] = 2\pi$$

$\boxed{\text{Volume} = 2\pi}$

3)



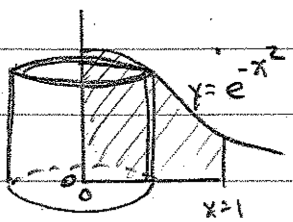
$$\text{Volume} = \int_0^1 2\pi r h \, dx, \text{ where } r = x, \quad h = \sqrt[3]{x}$$

$$V = 2\pi \int_0^1 x \sqrt[3]{x} \, dx = 2\pi \int_0^1 x^{4/3} dx = 2\pi \left(\frac{3}{7} x^{7/3}\right)\Big|_0^1$$

$$V = 2\pi\left(\frac{3}{7}\right)\left[1^{7/3} - 0^{7/3}\right] = \frac{6\pi}{7}$$

$\boxed{\text{Volume} = \frac{6\pi}{7}}$

5)



$$\text{Volume} = \int_0^1 2\pi r h \, dx, \quad r = x \quad h = e^{-x^2}$$

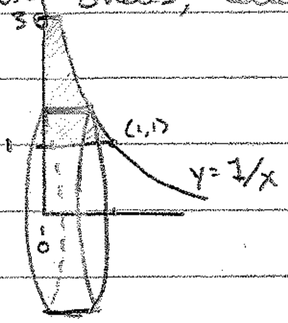
$$V = 2\pi \int_0^1 x e^{-x^2} dx \quad \text{let } u = -x^2, \quad \frac{du}{-2} = -\cancel{2}x\,dx$$

$$V = \frac{2\pi}{-2} \int e^u du = -\pi e^u = -\pi e^{-x^2}$$

$$V = -\pi e^{-x^2}\Big|_0^1 = -\pi(e^{-1} - e^0) = -\pi\left(\frac{1}{e} - 1\right) = \pi\left(1 - \frac{1}{e}\right)$$

$\boxed{\text{Volume} = \pi\left(1 - \frac{1}{e}\right)}$

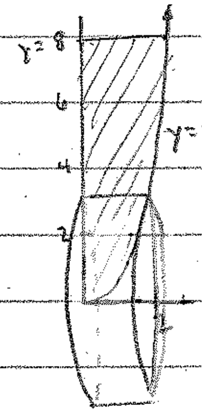
9, 11, 13) using shells, calculate volume of given function rotated about x-axis



9) Volume $= \int (2\pi r) h\,dy$, where $r = y$, $h = x = \frac{1}{y}$

$$V = 2\pi \int_1^3 y \frac{1}{y}\,dy = 2\pi \int_1^3 dy = 2\pi y \Big|_1^3 = 2\pi(3-1) = 4\pi$$

$\boxed{\text{Volume} = 4\pi}$

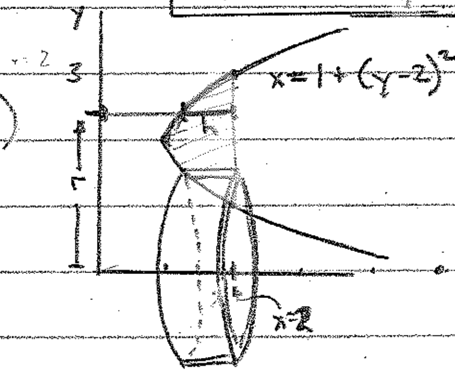


11) Volume $= \int_0^8 2\pi r h\,dy$, where $r = y$, $h = x = y^{1/3}$

$$V = 2\pi \int_0^1 y\, y^{1/3}\,dy = 2\pi \int_0^1 y^{4/3}\,dy = 2\pi\left(\frac{3}{7} y^{7/3}\right)\Big|_0^8$$

$$V = \frac{6\pi}{7}\left((2^3)^{7/3} - 0\right) = \frac{6\pi}{7}(2^7) = \frac{6\pi}{7}(128) = \frac{768\pi}{7}$$

$\boxed{\text{Volume} = \frac{768\pi}{7}}$



13) Volume $= \int (2\pi r) h\,dy$ where $r = y$, $h = [2 - (1 + (y-2)^2)]$

$(y^2 - 4y + 4)$

$(y^2 - 4y + 5)$

$h = [-y^2 + 4y - 3]$

$$V = 2\pi \int_1^3 y\left(2 - (1 + (y-2)^2)\right) dy$$

$$V = 2\pi \int_1^3 (-y^3 + 4y^2 - 3y)\,dy$$

$$V = 2\pi\left(-\frac{1}{4}y^4 + \frac{4}{3}y^3 - \frac{3}{2}y^2\right)\Big|_1^3$$

$$V = 2\pi\left[\left(-\frac{1}{4}(81) + \frac{4}{3}(27) - \frac{3}{2}(9)\right) - \left(-\frac{1}{4}1 + \frac{4}{3}1 - \frac{3}{2}1\right)\right]$$

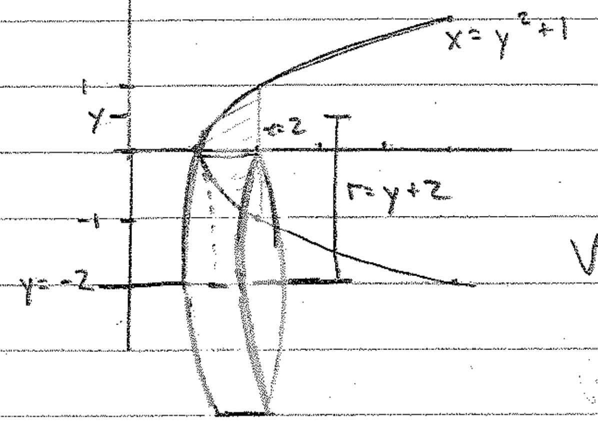
$$2\pi\left[-\frac{81}{4} + 36 - \frac{27}{2} + \frac{1}{4} - \frac{4}{3} + \frac{3}{2}\right]$$

$$2\pi\left[\frac{1}{4}(-81 + 144 - 54 + 1 + 6) - \frac{4}{3}\right] = 2\pi\left[\frac{16}{4} - \frac{4}{3}\right] =$$

$$= 2\pi\left[\frac{12}{3} - \frac{4}{3}\right] = 2\pi\left(\frac{8}{3}\right)$$

$\boxed{\text{Volume} = \frac{16\pi}{3}}$

20)



$V = \int 2\pi r h \, dy$ where $r = y+2$

$h = 2 - x = 2 - (y^2+1)$

$h = 2 - y^2 - 1 = 1 - y^2$

$$V = 2\pi \int_{-1}^{1} (y+2)(1-y^2)\, dy$$

$$2\pi \int_{-1}^{1} (y - y^3 + 2 - 2y^2)\, dy$$

$$V = 2\pi \left(\frac{1}{2}y^2 - \frac{1}{4}y^4 + 2y - \frac{2}{3}y^3 \right) \Big|_{-1}^{1}$$

$$V = 2\pi \left[\left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{4}} + 2 - \frac{2}{3} \right) - \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{4}} - 2 + \frac{2}{3} \right) \right]$$

$$= 2\pi \left[2 - \frac{2}{3} - -2 - \frac{2}{3} \right] = 2\pi \left[4 - \frac{4}{3} \right] = 2\pi \left(\frac{8}{3} \right)$$

$$\boxed{\text{Volume} = \frac{16\pi}{3}}$$